Abstract

This paper studies deviations from Ricardian equivalence for deficit-financed tax cuts of various durations in an incomplete-markets overlapping-generations (OLG) economy with imperfect altruism. The novel feature is that generations’ degrees of altruism are calibrated to aggregate data on inter-vivos transfers. The results of this economy are contrasted with two workhorse models, a dynastic economy and a standard OLG economy, both of which make strong implicit assumptions on altruism. Deviations from Ricardian equivalence in the imperfect-altruism economy are even larger than those in the standard OLG economy, welfare implications, however, tend to be closer to the dynastic economy. JEL Codes: D64, H31, H62.
1 Introduction

Government budget deficits are not only a contentious issue in policy debates – economic theory also provides two starkly different views. The Samuelson-Diamond OLG model (see, for example, Diamond (1965)) predicts that budget deficits enrich current generations at the expense of future generations. On the other hand, Barro (1974) argues that Ricardian equivalence (RE) holds in an OLG economy in which there are altruistically-motivated transfers such as gifts and bequests. Barro (1974)’s result requires the assumption that transfers occur due to altruism, but does not require a specific degree of altruism. Two workhorse models in macroeconomics, the infinite horizon and the OLG models, make strong implicit assumptions on the magnitude of altruism. The former implicitly assumes that households within the same family line (e.g. young and old households) have perfect altruism and the latter that altruism is entirely absent. In reality, we would suspect that altruism is neither perfect nor entirely absent but somewhere in between. For policy analysis it is important to know what the predictions of a model with more plausible magnitudes of altruism are. Furthermore, comparing the predictions of such a model with those from the two workhorse models may then provide a guide as to which of the two workhorse models is a more suitable approximation to reality.

In order to quantify deviations from RE, this paper maintains Barro (1974)’s key assumption of altruistically-motivated transfers. Crucially, however, in order to obtain realistic intensities of altruism for the young and the old generations their altruism parameters are calibrated to data on aggregate inter-vivos transfers. As in Nishiyama (2002), the data reported by Gale & Scholz (1994), which is based on the Survey of Consumer Finances (SCF), is used for the calibration of the altruism

1In terms of terminology, gifts and inter-vivos transfers are used interchangeably. The Latin phrase inter vivos means between the living, and so such a transfer refers to one made during one’s lifetime.

2Equilibrium transfers instead must be an interior solution for all current and future generations – Barro (1974) refers to this as universally operative transfer motives. A simple example may highlight this point. Suppose that in a two-period OLG economy generation A gets a tax-cut in the first period, which has to be paid back in the second period by generation B. If generation A had planned to give a transfer motivated by altruism to generation B prior to the tax cut then generation A would increase the transfer to generation B by the amount of the tax-cut. Consequently, government-induced transfers are privately undone and there is no response in aggregate consumption. Note that there is no assumption made on the size of altruism. Of course, if altruism is entirely absent, as in the Samuelson-Diamond OLG model, there is no chance for this type of neutralization to occur. The tax cut acts like an increase in generation A’s income and current aggregate consumption increases.
parameters. The calibrated magnitudes of altruism are imperfect (that is, an individual cares more about her own well-being than about someone else’s well-being) and asymmetric, since the old generation has a larger degree of altruism than does the young generation. This asymmetry is driven by the fact that the transfers observed in the data from the old generation to the young generation are larger than those from the young generation to the old generation. Altruism is far from perfect since observed aggregate transfers are just not large enough to justify large magnitudes of altruism. The model’s predictions are also in line with empirical evidence on inter-vivos transfers based on micro-level data. This data suggests that inter-vivos transfers are especially likely to occur when the recipient is liquidity-constrained. It also suggests that transfers are increasing in both the donor’s wealth and labor income and decreasing in the recipient’s labor income (see, for example, Cox & Jappelli (1990), McGarry & Schoeni (1995) and Berry (2008)).

Deficit-financed tax cuts experiments are computed over four different durations, ranging from a two-year tax-cut to a 25-year tax-cut. Each tax-cut regime is followed by a tax-hike regime of equal duration. As in Heathcote (2005), deviations from RE are quantified by measuring how aggregate consumption changes when the tax rate changes. This paper puts a strong emphasis on the study of the transition path of aggregate consumption for the period during which additional deficits finance part of government expenditures. Furthermore, the deviations from RE are gauged in comparison to two standard workhorse models, both of which arise as specific restrictions on the altruism parameters in the model. In what will be referred to as the no-altruism (NA) economy, altruism is absent for both the young and the old generations, whereas, in the perfect-altruism (PA) economy, altruism is perfect for both the young and the old generations. All three economies are calibrated to match the same wealth-to-GNP ratio and fraction of wealth-poor households.

A priori, one would suspect that deviations from RE are largest in the NA economy, smallest in the PA economy, with those from the imperfect-altruism (IA) economy lying somewhere between the two workhorse models, depending on the extent of altruism. The main finding of the paper, however, is that deviations from RE during a deficit-financed tax cut in the IA economy are often even larger than those in the NA economy. This result is driven by the following two phenomena. First, the fraction of borrowing-constrained households is largest in the IA economy since, in the
equilibrium of this economy, there is an incentive to become constrained as transfers only flow when the recipient is constrained. This incentive is absent from the other two economies. The second reason stems from imperfect altruism and lack of commitment, both of which imply strategic considerations in the consumption-savings decision. Both households that are current recipients of gifts and households that expect to receive transfers in the future, if they remain poor enough, face a disincentive to save out of the higher disposable income during the tax-cut regime. These households anticipate that they will be “bailed out” with higher private transfers when taxes increase in the future and, as a result, they increase consumption even more upon the tax cut. This type of moral hazard is absent in the PA and NA economies, in which savings increase at an earlier point in time.

This same relationship does not hold, however, for ex-ante welfare implications. That is, the ex-ante welfare gains from deficit-financed tax cuts are not largest in the IA economy but rather lie in-between those of the NA and the PA economies. In addition, they tend to lie closer to those from the PA economy. Since deficits crowd out gifts, and gifts are received by households whose consumption policies differ substantially from those in the NA economy, the welfare effects do not lie above those from the NA economy. Gifts, which are free, are replaced with deficits, which must be repaid, and so recipients of private transfers lose most. This effect dampens the ex-ante welfare implications and is a repercussion that is entirely absent from the NA economy.

The various experiments are conducted in a small open endowment economy populated by a large number of young households and a large number of old households. One young household and one old household make up a family. An old household faces a mortality hazard given by a Poisson rate. When the old household dies, the young household becomes an old household, and a new young household enters the economy; in this way, a new family is formed. Households within the same family are imperfectly altruistic toward each other. Households face an idiosyncratic labor income process but do not face aggregate uncertainty. Markets to insure against idiosyncratic risks are absent. Households decide about consumption, about savings in a riskless asset (subject to a no-borrowing constraint), and about a non-negative transfer to the other agent. Commitment is absent. The government finances a constant stream of expenditure using lump-sum taxes and government debt.
Computing an imperfect-altruism model, in which generations overlap for multiple time periods and there is a lack of commitment, is not a trivial task. In this regard, this paper builds on Barczyk & Kredler (2011a) and Barczyk & Kredler (2011b). These authors study a dynamic Markovian game of two infinitely-lived altruistic agents without commitment. In the former paper, the authors provide a theoretical characterization of possible equilibria, which points to one stable equilibrium in which transfers only flow when the recipient is borrowing-constrained. Their latter paper is more computational in nature; it studies the transfers-when-constrained equilibrium and provides a numerical algorithm. As such, it is the basic building block for the IA economy studied here. In terms of studying deviations from RE with incomplete markets this paper is principally related to Heathcote (2005) and Laitner (1992). Both of these papers, however, employ a dynastic economy (i.e. they implicitly assume perfect altruism)\(^3\). Altig & Davis (1988) study the consequences of interactions between borrowing constraints and intergenerational altruism on an array of inter- and intra-generational redistributive policies. Households overlap for three periods, commitment is assumed, and the analysis is of theoretical nature. In this paper households overlap for multiple time periods, commitment is absent, and the analysis is quantitative. Laitner (1988) studies an OLG economy in which altruism is imperfect, commitment is absent, and there is heterogeneity in lifetime earnings ability. Laitner (1988) shows that transfer motives fail to be universally operative in this environment so that RE does not hold but does not analyze its quantitative importance. Furthermore, and importantly, this paper allows generations to overlap for multiple time periods, while Laitner (1988) only allows generations to overlap for one period. This opens up the study of a transition period, which is the main focus of the paper, along with the study of long-run steady-states\(^4\).

\(^3\)This modelling approach has the advantage that the individual household problems can be pooled and solved as a joint-maximization problem. This is a significant simplification of perfect altruism. Another strand in the literature employs other forms of altruism such as the joy-of-giving type (also referred to as impure altruism, since the donor derives pleasure directly from the act of giving and does not depend on the well-being of the recipient); see, for example, Abel & Bernheim (1991) and Andreoni (1989). The problem with this approach is that it is inconsistent with empirical evidence on inter-vivos transfers.

\(^4\)Abel (1987), who theoretically studies the operativeness of the gift and bequest motives, calls on future research to study the transition path in heterogeneous economies in his concluding remarks. In his conclusion, Laitner (1988) suggests an expansion of the model to many-period lives as being a key issue.
The remainder of the paper proceeds as follows. Section 2 provides the physical environment, the equilibrium definition, and a description of the incentives households face. The calibration of the economy is discussed in section 3. Section 4 presents the main results. Section 5 concludes.

2 Model Framework

2.1 The Environment

The economy is a small open endowment economy. Time $t$ is continuous. A household in the economy faces two life-cycle stages, $s \in \{1, 2\}$. When $s = 1$ the household is in the young life-cycle stage and when $s = 2$ it is in the old life-cycle stage. At each point in time there is a large (measure one) number of young households and a large (measure one) number of old households. One young household and one old household make up a family. An old household faces a mortality hazard given by a Poisson rate, $\delta$. When the old household dies, the young household becomes an old household, and a new young household enters the economy; in this way, a new family is formed.

Households face an idiosyncratic labor income process but do not face aggregate uncertainty. At each point in time, a household obtains an exogenous endowment, $y^s$, according to a Poisson rate, $\xi$, and transitions, $y^s \in \{y_1, y_2, y_3\}$, where $y_1 < y_2 < y_3$. The initial income realization of a new young household follows a three-state Markov chain. I use $\pi_{ij}$ to denote the probability that he obtains the income realization $j$, given that the old household in the family has the income realization $i$. Furthermore, the new young household obtains an initial wealth endowment that is proportional to his income realization, $m \cdot y^1_j$, where $m$ is the factor of proportionality. This endowment acts as a modelling short-cut to account for initial wealth observed in the data while not modelling the origination of this wealth (more on this in section 3). Finally, when an old household dies, any wealth it has left is automatically bequeathed to the young household.

The market arrangement is as in standard incomplete-markets models. There is a single asset that pays a time-invariant rate of interest, $r$. A household can hold a non-negative amount, $w^s \geq 0$, of the asset. Markets to insure against idiosyncratic
risks are absent.

Government consumption is constant, \(\{G\}\). It is financed through a proportional lump-sum tax, \(\phi_t\), and through deficits, \(dD/dt = \dot{D}_t\), where \(D\) stands for government debt. The flow version of the government budget constraint is given by
\[
\dot{D}_t = rD_t + G - \phi_tY, \text{ given } D_0 = D, \tag{1}
\]
where \(Y\) is the (time-invariant) aggregate gross endowment.

At each point in time, households choose a consumption rate, \(c^s \geq 0\), and a non-negative transfer rate, \(g^s \geq 0\) (\(g\) stands for “gift”). These choices then imply the following savings rate
\[
\dot{w}^s = rw^s + y^s + g^{s'} - c^s - g^s, \quad s \in \{1, 2\}, \tag{2}
\]
where \(s'\) denotes the life-cycle stage of the family counterpart.

There are two important qualifications to equation (2). First, it describes the law of motion of wealth while households remain in their current life-cycle stage. That is, it does not describe a discrete change in the wealth level, which occurs if a household receives a bequest. Second, when \(w^s = 0\) feasibility has to be enforced in the following way
\[
c^s + g^s \leq y^s + g^{s'}, \quad s \in \{1, 2\}.
\]
That is, a household cannot spend more than it receives.

A household’s flow utility is given by
\[
U^s(c^s, c^{s'}) = u(c^s) + \alpha^s u(c^{s'}), \quad \alpha^s \in [0, 1], \quad s \in \{1, 2\},
\]
where \(\alpha^s\) is the degree of altruism for a household in life-cycle stage \(s\). The preferences of a household are represented by
\[
E_0 \int_0^T e^{-\rho t} U^s(c^s_t, c^{s'}_t) dt + [1 - I_{s=2}(1 - \alpha^2)] e^{-\rho \tau} V^e(Z_\tau, y_1), \quad s \in \{1, 2\},
\]
where \(I_{s=2}\) is an indicator variable. It is equal to zero for a young household, \(s = 1\), and is equal to one for an old household, \(s = 2\). Life-time utility is decomposed
into what accrues over the current life-cycle stage, $s$, and what accrues afterwards (continuation value). Note that the continuation value of an old household is the same as one for a young household, except that $V^e$ is multiplied by $\alpha^2$.\(^5\)

The integration starts with the current date (here, time 0) and ends with the random time of death of the old household, $\tau$. The latter follows an exponential distribution.\(^6\) The discount rate is denoted by $\rho$. The variable $Z_\tau$ denotes the level of wealth the currently-young household would own given the death of the old household. That is, it is the sum of his own life-cycle savings and the (non-negative) wealth left behind by the old household.

In order to define the function $V^e$, let $V^2(w^1, w^2, y^1, y^2)$ be the value of an old household with wealth $w^2$ and income $y^2$, along with his young household’s wealth $w^1$ and income $y^1$. Then,

$$V^e(Z, y^1_i) = \sum_j \pi_{ij} V^2 (m \cdot y^1_j, Z, y^1_j, y^2_i).$$

$V^e$ is therefore the expected value of a currently-young household, whose family has wealth $Z$ and current income realization $y^1_i$, becoming an old household.

The PA model is obtained when setting $\alpha^1 = 1 = \alpha^2$ and the NA model when $\alpha^1 = 0 = \alpha^2$. In addition to serving as benchmarks-of-comparison, the PA and the NA economies will be useful in disentangling the effects from imperfect altruism and incomplete markets when performing the computational experiments.

### 2.2 Equilibrium Definition

When altruism is imperfect and commitment is absent, strategic considerations in the consumption-savings decisions arise. In order to deal with this implication, the concept of a Markov-perfect equilibrium is used.

A Markov strategy is a pair of non-negative functions $\{c^1(t, x), g^1(t, x)\}$ for a

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\(^5\)It is important to emphasize that while this function can be interpreted as a “bequest motive,” it should not be confused with an ad hoc bequest function as, for example, in the case of warm-glow altruism. For example, when $\alpha^2 = 1$, then the preferences are simply the one of an infinitely-lived household.

\(^6\)That is $F(t) = Pr(\tau < t) = 1 - e^{-\delta t}$, where $\delta$ is the mortality hazard alluded to above. Recall that the death of an old household implies that the young household in the family becomes an old household and that a new young household enters the economy, and so a new family is formed.
young household and a pair of non-negative functions \( \{c^2(t, x), g^2(t, x)\} \) for an old household. The payoff-relevant state includes time, \( t \), and \( x = (w^1, w^2, y^1, y^2) \), where \( w^1 \) is the young’s household wealth, \( w^2 \) is the old’s household wealth, \( y^1 \) is the young’s household income, and \( y^2 \) is the old’s household income. Strategies are unrestricted in that there is no upper bound on consumption and transfer functions at any point in the state space. When one of the households has zero assets (e.g. the young household), feasibility is enforced by setting realized consumption equal to

\[
c^{1*}(t, 0, w^2, y^1, y^2) = \min \left\{ c^1(t, 0, w^2, y^1, y^2), y^1 + g^2(t, 0, w^2, y^1, y^2) \right\}.
\]

The above equality states that the young household cannot eat more than what resources are available, but can announce plans to do so. Realized consumption for the old household when it has zero assets is defined in an analogous manner.

When the other player’s strategy is Markov, the best-response problem of each player is a dynamic-programming problem and best responses will be Markov as well. Let \( V^s(t, x) \) be the value for a household in life-cycle stage \( s \) and state \( (t, x) \). Denote by \( s' \) the life-cycle stage in which is the other household of the family. Given the strategy \( \{c^{s'}(t, x), g^{s'}(t, x)\} \) of the other household in the family, \( V^s(t, x) \) and its partial derivatives satisfy the following partial differential equation

\[
\rho V^s = \max_{c^s \geq 0, g^s \geq 0} \left\{ U^s(c^s, c^{s'}) + \dot{w}^{s'} V^s_{w^{s'}} + \dot{w}^s V^s_{w^s} + \xi (V^s(\hat{y}^s, \cdot) - V^s) + \xi (V^s(\hat{y}^{s'}, \cdot) - V^s) + \delta ([1 - I_{s=2}(1 - \alpha^2)]|V^e - V^s|) \right\},
\]

where \( V^e = \sum_j \pi_{ij} V^2 \).

This equation is known as the Hamilton-Jacobi-Bellman equation (HJB). Subscripts denote partial derivatives (e.g. \( V^s_{w^s} = \partial V^s/\partial w^s \)), and for better readability, the dependence of the value function and the policies on \( (t, x) \) is suppressed. The interpretation of equation (3) is deferred to subsection 2.3.

A Markov-perfect equilibrium is given by a set of functions for young households, \( \{c^1(\cdot), g^1(\cdot), V^1(\cdot)\} \), a set of functions for old households, \( \{c^2(\cdot), g^2(\cdot), V^2(\cdot)\} \), and distributions of households over the state-space, \( \{\lambda_i\} \), such that, given the world
interest rate, \( r \), the government policy rules, \( \{\phi_t, D_t, G\} \), and an initial distribution, \( \lambda_0 \), the following restrictions hold:

1. Given \( \{c^2(\cdot), g^2(\cdot)\} \), \( \{c^1(\cdot), g^1(\cdot), V^1(\cdot)\} \) solves (3) for \( s = 1 \), subject to feasibility.

2. Given \( \{c^1(\cdot), g^1(\cdot)\} \), \( \{c^2(\cdot), g^2(\cdot), V^2(\cdot)\} \) solves (3) for \( s = 2 \), subject to feasibility.

3. The government budget constraint – equation (1) – holds.

4. The probability measure, \( \lambda \), follows the law of motion induced by \( \{c^1(\cdot), g^1(\cdot), c^2(\cdot), g^2(\cdot)\} \).

As is well-known, a Markov-perfect equilibrium is subgame perfect.

### 2.3 Best Responses: Hamilton-Jacobi-Bellman Equations

We now turn our attention to the interpretation of equation (3). To this end, we write out the savings rates, given by equation (2), for \( s = 1 \) and \( s = 2 \), and group terms according to whether or not they directly factor into the household’s current choices:

\[
pV^s = \alpha^s u(c^s) + \left( rw^s + g^s - c^s - g^s \right) V^s_{w^s} + (rw^s + y^s + g^s) V^s_{w^s} + \xi \left( V^s\left( \hat{y}^s, \cdot \right) - V^s \right) + \xi \left( V^s\left( \hat{y}^s', \cdot \right) - V^s \right) + \delta \left( [1 - I_{s=2}(1 - \alpha^2)] V^e - V^s \right) + \max_{g^s \geq 0} \{g^s \left( V^s_{w^s} - V^s_{w^s'} \right) \} + \max_{c^s \geq 0} \{u(c^s) - c^s V^s_{w^s} \}; \quad s \in \{1, 2\}. \tag{4}
\]

The left-hand side of equation (4) is the flow value of the households’ optimal program. In order for the allocation to be optimal, this value must equal the value of the right-hand side. The last line on the right-hand side collects the terms which are relevant for the household’s current choices. All the other terms are predetermined given the current state.
A household’s value is affected by its own income uncertainty as well as by the income uncertainty faced by its related household. In the HJB, this manifests itself through two jump-terms; the former through the term “jump in $y^s$” and the latter through the term “jump in $y^s'$”. These jump-terms measure the differences from current values for the household if either its own income jumps or its counterpart’s income jumps. The current values are denoted $y^s$ and $y^s'$, respectively, and the jump terms are denoted $\hat{y}^s$ and $\hat{y}^s'$. A household also faces the uncertainty of a change in the life-cycle stage; the term “jump in $s$” accounts for the effect this uncertainty has on the household’s value. For a household in $s = 1$ this term becomes $\delta (V^e - V^1)$. A young household becomes an old household at the Poisson rate $\delta$. Conditional on this event, he obtains the value $V^e$ and loses the value $V^1$. For a household in $s = 2$, the term becomes $\delta (\alpha^2 V^e - V^2)$. An old household faces the mortality hazard $\delta$. Conditional on his death, the old household loses $V^2$, but its current well-being is still affected by how well-off the young household will be, $\alpha^2 V^e$.

The last line collects the household’s current choices. The first-order condition for consumption is given by

$$u_c(c^s) = V^s_w,$$

which states that the marginal utility of current consumption is equal to the marginal value of saving. An important feature is that current actions of the other player do not have to be explicitly contemplated by the decision maker. Thus, over a short amount of time best-response functions are constants and optimal consumption can be obtained, as in a standard consumption-savings problem, without having to compute best responses for each action of the other player. The term $\mu^s \equiv V^s_{w^s'} - V^s_{w^s}$ is referred to as the transfer motive since it measures the marginal benefit to a donor of transferring an additional unit of resources to the recipient. Transfers will be set to zero whenever the transfer motive, $\mu^s$, is negative. If $\mu^s = 0$, then the household is (locally) indifferent with regard to the intra-family wealth distribution so that any transfer flow is consistent with optimality. When $\mu^s > 0$, a situation that will not take place in the equilibrium, the transfer $g^s$ will be large enough to instantaneously

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7This feature of the decision problem is a crucial simplification of continuous time with respect to discrete time. The technical reasons are that second-order effects vanish as time becomes continuous and that flow utility $u(c^s) + \alpha^s u(c^s')$ is separable in $c^s$ and $c^s'$. For a more extensive discussion refer to section 2 in Barczyk & Kredler (2011a).
achieve $\mu^s = 0$. Finally, note that in the PA economy ($\alpha^1 = 1 = \alpha^2$) the transfer motive holds with equality – that is, $\mu^1 = \mu^2$. Households of the same family are indifferent (locally and globally) with regards to the intra-family wealth distribution.

In the equilibrium it will turn out that inter-vivos transfers only flow when the recipient is borrowing-constrained. In that region of the state space transfer motives take on a different form – they resemble those in the static transfer model. Suppose, for example, that the young household is constrained, the old household is unconstrained, and transfers are zero:

$$u_c(y^1) > V_{w1}^1, \quad V_{w2}^2 = u_c(c^2), \quad \text{and} \quad g^2 = 0.$$ 

Since transfers are zero, it must be the case that

$$u_c(c^2) > \alpha^2 u_c(y^1).$$

Otherwise, the old household would choose a positive amount of transfers in order to equalize her $c^2$- and $g^2$-margins

$$V_{w2}^2 = u_c(c^2) = \alpha^2 u_c(y^1 + g^2).$$

This shows that the old household can “dictate” the young household’s consumption of the young household, such that $c^1 = y^1 + g^2$. Although the constrained case is actually more intricate than presented here, this simple example conveys the intuition well.\(^8\)

### 2.4 Savings Incentives: Euler Equations

The Euler equations reveal the various savings incentives that households face and how policies of the other player influence decision making. In addition, contrasting the Euler equations for the IA economy with those from the PA and NA economies highlights key differences in the savings incentives among these economies.

In order to obtain the Euler equation of a household in life-cycle stage $s$, take the

\(^8\)For a complete analysis, refer to sections 3.2 (one household is broke) and A.3 (both households are broke) in Barczyk & Kredler (2011b). These sections also form the basis for the computation of inter-vivos transfers in the numerical algorithm.
derivative of HJB (4) with respect to $w^s$

$$
\mathcal{A}u_c(c^s) = (\rho - r)u_c(c^s) - \delta \{ [1 - I_{s=2}(1 - \alpha^2)]V^s_w - u_c(c^s) \} + 
\begin{align*}
\text{representative household} & \quad [V^s_w - \alpha^s u_c(c^s)]c_w^s + \mu^s g_w^s, \\
\text{jump in } s & \quad \alpha^s u_c(c^s) \end{align*}
$$

The operator $\mathcal{A}$ is defined as the “expected time derivative”. The terms $c_w^s$ and $g_w^s$ denote the partial derivatives of the consumption policy and of the transfer policy for the counterpart household in life-cycle stage $s'$, respectively, with respect to the wealth of the household in life-cycle stage $s$. Economically speaking, this measures how the other player’s consumption and transfer react to an increase in the household’s wealth. Note that the term transfer-induced incentive also includes $-\mu^s g_w^s$; if $g_w^s > 0$, then $\mu^s = 0$ and the term vanishes. Thus, in order to highlight this term, it is assumed that $\mu^s < 0$.

When dividing both sides of the Euler equation by $u_c(c^s)$, the terms on the right-hand side determine the (expected) growth rate of marginal utility. If a household marginally increases wealth, then there is the standard trade-off between lower current consumption and higher future consumption, as shown by the term “representative household”. But, there are additional effects, the most important ones contained in the term referred to in the equation as “altruistic-strategic distortion”. When the household has a higher level of wealth, the consumption of the other player reacts by $c_w^s$. Suppose that $c_w^s > 0$. As a consequence, there is a marginal benefit of $\alpha^s u_c(c^s)$ which stems from the consumption of the other player. The additional consumption of the other player provides an incentive to save, and so $\alpha^s u_c(c^s)$ enters the Euler equation with the same sign as does the interest rate. An increase in consump-

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9 The operator $\mathcal{A}$ (the infinitesimal generator) is defined for a differentiable function $f(x)$ as

$$
\mathcal{A}f(x_t) = \lim_{\Delta t \to 0} \frac{1}{\Delta t} E_t[f(x_{t+\Delta t}) - f(x_t)]
= f_w^s \dot{w}_t^s + f_w^{s'} \dot{w}_t^{s'} + \xi [f(\dot{y}^s, \cdot) - f(y^s, \cdot)] + \xi [f(\dot{y}^{s'}, \cdot) - f(y^{s'}, \cdot)].
$$

10 This is reasonable since a higher level of wealth for one household implies that transfers are more likely and larger and the likelihood that the other household has to provide transfers in the future is reduced.
tion of the other player, however, means that he will have fewer resources. Thus, the increase in consumption by the other player comes at a cost of $V_{w}^{s}$. The term values what would happen if the equilibrium path was left and another equilibrium path, in which the other player has $c_{w}^{s'}$ less wealth, was entered into. A disincentive to save results, and so the term enters the Euler equation with the same sign as does the discount rate. The term referred to as “transfer-induced incentive” in the equation involves the transfer motive, $\mu^{s}$, and the reaction in the transfer function of the other player to an increase in the level of wealth, $g_{w}^{s'}$. The transfer motive is negative since the household currently does not provide transfers (otherwise this term would be simply zero). As long as the other household does not provide any transfers, $g_{w}^{s'} = 0$, and this term vanishes. If it is assumed that $g_{w}^{s'} > 0$ – that is, the other player rewards thrift by providing an increasing schedule in savings of the other – then this term provides an incentive to save, just as the donor intended to, and it enters the equation with the same sign as does the interest rate.

The Euler equation for the NA economy is obtained by substituting $\alpha^{1} = 0 = \alpha^{2}$ in the Euler equation (5). The payoff relevant state for an old household does not include resources from the young household in the family, and so the last line vanishes. For the old household, the term “jump in $s$” simply becomes $\delta u_{c}(c^{s})$, and it can be seen that the Euler equation is just as in Yaari (1965) – that is, uncertain lifetimes increase the discount rate $\rho$ by the mortality hazard $\delta$:

$$\mathcal{A}u_{c}(c^{2}) = (\rho + \delta - r)u_{c}(c^{2}).$$

Since, in the event of death, the young household receives an accidental bequest, the old household’s wealth continues to be a payoff relevant state for the young household. The young household’s Euler equation becomes:

$$\mathcal{A}u_{c}(c^{1}) = (\rho - r)u_{c}(c^{1}) - \delta[V_{Z}^{e} - u_{c}(c^{1})].$$

For the PA economy, $\alpha^{1} = 1 = \alpha^{2}$. The two individual household problems can then be pooled into a single joint-maximization problem. The dynasty’s payoff relevant state variables are given by their joint level of wealth $W_{t} = w_{t}^{1} + w_{t}^{2}$ and the joint level of the income realizations $Y_{t} = y^{1} + y^{2}$. Thus, consumption and saving decisions depend only on total family resources, but not on how they are distributed.
Furthermore, $V_{w_1}^2 = V_{w_2}^2 = V_{w_1}^1 = u'(c^1)$, and so there are neither altruistic-strategic distortions nor transfer-induced incentives. The Euler equation thus becomes

$$A u_c(c) = (\rho - r) u_c(c) - \delta [V_Z^e - u_c(c)].$$

A final remark concerns the issue of commitment. With commitment it would be the case that $c_{w,s}^* = 0 = g_{w,s}^*$ and the behavior of the economy would lie “in-between” the PA and the NA economies. Without commitment, however, the nature of the model fundamentally alters. For the PA and the NA economies, and in the case of commitment, the Euler equations are ordinary-differential equations, that do not take into account what would happen off the equilibrium path. In the IA economy, the Euler equations are partial-differential equations, signifying that off-equilibrium information enters.

### 3 Calibration

The key parameters for the IA economy are the discount rate, $\rho$, and the young household’s and old household’s degrees of altruism, $\alpha^1$ and $\alpha^2$, respectively. For the NA and PA economies, the key parameter is the discount rate, $\rho$. The discount rate is jointly identified by the wealth-to-GNP ratio and the fraction of wealth-poor households. The degrees of altruism are primarily pinned down by aggregate transfer-to-wealth ratios of the young generation and the old generation. Before a discussion of the calibration of these parameters in more detail occurs, the parameters common to the three economies will be discussed.

The young life-cycle stage approximately corresponds to the 25 to 50 age range, and the old life-cycle stage corresponds to the 50 to 75 age range. The expected duration of a life-cycle stage is therefore 25 years, implying a mortality hazard, $\delta$, of 4%. It follows that two generations overlap, on average, for 25 years.

The labor income process follows a three-state Markov chain, calibrated to the U.S. income distribution for households of ages 25 to 65. An income realization can be low, medium, or high. When a household is in a low or in a high income state, it can only switch to the medium level. Furthermore, the household’s income can jump up or down with equal likelihood. From the middle income state, income can switch
to the high or the low levels with equal probability. There is therefore only one rate parameter to be calibrated. It is chosen in order to match a persistence parameter for the labor income process of 0.9. The income for an old household consists of a weighted average of the realization of the labor income process and its corresponding social security income. The latter is computed as the replacement rate multiplied by income, where the replacement rates are taken from Mitchell & Phillips (2006). The weights capture the average duration an old household spends earning an income from work and from social security.

The probability that a young household enters the economy with income realization $j$, given that the old household has income realization $i$, is denoted by $\pi_{ij}$. Analogous to the labor income process, if the parent household is in a low or in a high income state, a young household is equally likely to enter the economy with a medium level of income. If the parent household is in the medium income state, the new young household will enter the economy with a low or a high level of earnings with equal probability. Again, only one parameter needs to be calibrated. This parameter is chosen in order to match an inter-generational correlation of labor income of 0.4. Finally, the initial wealth endowment of a new young household is proportional to his income realization (i.e. $m \cdot y^1_1$), with the factor of proportionality denoted by $m$. This short-cut allows the model to account for the initial wealth observed in the data for households aged 25 to 30 (see, for example, Budria-Rodriguez et al. (2002)), without speculating on the origination of this initial wealth.

The felicity function is given by

$$u(c) = \frac{c^{1-\gamma}}{1-\gamma},$$

where $\gamma$ is the coefficient of relative risk aversion. The felicity function is identical for both old and young households and the coefficient of relative risk aversion is assumed to be 2. The interest rate is assumed to be $r = 4\%$. Table 1 summarizes the parameters the economies have in common.

The values for the calibration targets are shown in table 2. For the IA economy, the parameters $\{\rho, \alpha^1, \alpha^2\}$ are chosen in order to match four calibration targets (i.e. the calibration is over-identified): a wealth-to-GNP ratio of 3.4, a fraction of 20% of households without wealth (Wealth-poor-%), a transfer-to-wealth ratio for the young
Table 1: Parameters common across economies.

<table>
<thead>
<tr>
<th>Parameter</th>
<th>γ</th>
<th>r</th>
<th>δ</th>
<th>ξ</th>
<th>π</th>
<th>m</th>
</tr>
</thead>
<tbody>
<tr>
<td>Value</td>
<td>2</td>
<td>4%</td>
<td>4%</td>
<td>10%</td>
<td>0.36</td>
<td>1</td>
</tr>
</tbody>
</table>

Parameters and their values that the three economies have in common. \( \gamma \) is the coefficient of relative risk aversion, \( r \) is the interest rate, \( \delta \) is the mortality hazard, \( \xi \) is the earnings hazard, \( \pi \) is the earnings-heritability probability, and \( m \) is the factor of proportionality, which, together with the income realization, determine the initial wealth endowment of a new young household.

generation of 0.03%, and a transfer-to-wealth ratio for the old generation of 0.32%.

For both the NA and PA economies, the discount rate \( \rho \) is calibrated by matching a wealth-to-GNP ratio of 3.4 and a fraction of 20% of households without wealth.

Table 2: Calibration targets.

<table>
<thead>
<tr>
<th>Target</th>
<th>NA</th>
<th>IA</th>
<th>PA</th>
</tr>
</thead>
<tbody>
<tr>
<td>Wealth/GNP</td>
<td>3.4</td>
<td>3.4</td>
<td>3.4</td>
</tr>
<tr>
<td>Wealth-poor-%</td>
<td>20%</td>
<td>20%</td>
<td>20%</td>
</tr>
<tr>
<td>Young: Transfer/Wealth</td>
<td>0.03%</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Old: Transfer/Wealth</td>
<td>0.32%</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

Calibration targets and their values for the three economies. Wealth/GNP is the wealth-to-GNP ratio, Wealth-poor-% is the fraction of households without wealth, and Transfer/Wealth is the transfer-to-wealth ratio.

The value of 20% of households without wealth is based on Jappelli (1990), who report that in the 1983 Survey of Consumers Finances (SCF), 12.5% of households have a request for credit rejected and a further 6.5% does not apply for it, expecting to be rejected. Gale & Scholz (1994) document data on intended intergenerational transfers using the 1983-86 SCF. Intended transfers are defined to include financial support given to other households, trust accumulations, and life insurance payments to children. Bequests are excluded since they are not necessarily intentional. The annual flow of intended transfers as a percentage of aggregate net worth is 0.53% \(^{11}\) and is made up of 0.35% of support given to adult family members, 0.12% of trusts, \(^{11}\)This number appears to be very small, since it is an annual flow. Converting this flow into a stock, Gale & Scholz (1994) argue that intended transfers are the source of at least 20% of aggregate net worth.
and 0.05% of life insurance. The 0.35% of support given to adult family members as a percentage of aggregate net worth is the most appropriate counterpart in the data to the flow of annual gifts as a percentage of aggregate net worth generated by the IA economy.\textsuperscript{12} Furthermore, the authors report that this ratio consists of 0.32% of support given to young households and of 0.03% of support given to old households.\textsuperscript{13} The young and the old generations’ transfer-to-wealth ratios are primarily responsible for the identification of their respective degrees of altruism.

The wealth-to-GNP ratio would clearly be enough to identify the discount rate, and as such, the calibration of the parameters in the three economies is over-identified. Wealth-poor households are, however, an important driver of deviations from RE (e.g. borrowing-constrained households). Since this paper is concerned with quantifying deviations from RE, and comparing these deviations among the three economies, it proves useful to include the fraction of wealth-poor households as an additional calibration target. This inclusion helps to ensure a realistic fraction of borrowing-constrained households and a more meaningful comparison across the three economies. The parameters are chosen to minimize the equally weighted sum of squared differences between the moments generated by the model and the data.

Table 3 shows the resulting values of the parameters. The discount rate, $\rho$,

<table>
<thead>
<tr>
<th>Parameter</th>
<th>NA</th>
<th>IA</th>
<th>PA</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\rho$</td>
<td>3.15%</td>
<td>3%</td>
<td>5.14%</td>
</tr>
<tr>
<td>$\alpha^1$</td>
<td>0</td>
<td>0.12</td>
<td>1</td>
</tr>
<tr>
<td>$\alpha^2$</td>
<td>0</td>
<td>0.28</td>
<td>1</td>
</tr>
</tbody>
</table>

is largest in the PA economy, since the effective discount rate in the IA and NA economies also take into account the mortality hazard, $\delta$. For the IA and NA economies, the discount rates are relatively similar, with values of 3% and 3.15%, respectively. The value for the young household’s altruism parameter is $\alpha^1 = 0.12$ and the one

\textsuperscript{12}In section 4.4, I will change these values from the stated values here to account for likely under-reporting in the SCF data. The calibration will be redone in order to check the robustness of the results (the SCF only reports support given if its value is at least $3000). Qualitatively the results do not change after redoing the calibration.)

\textsuperscript{13}Nishiyama (2002) also uses the data reported by Gale & Scholz (1994) to calibrate the degree of altruism, but restricts the altruism parameter of the young household to zero.
for the old household is $\alpha^2 = 0.28$. The interpretation for the value of the degree of altruism depends on the value of the coefficient of relative risk aversion (here $\gamma = 2$). This can be easily seen from a static setting, in which transfers flow as long as consumption inequality exceeds $A = \alpha^{-1/\gamma}$. Taking $A$ as a more meaningful measure for the economic interpretation of the $\alpha$’s (namely, as a measure of how much consumption inequality would be tolerated by an altruist in a static setting), the current calibration yields $A^1 = 2.9$ and $A^2 = 1.9$ (see section A.2 in the appendix for more details).

4 Results

The novel feature of this paper is that deviations from RE are quantified in a setting where imperfectly-altruistic generations overlap for multiple periods. An important strength of the model is that it predicts inter-vivos transfers to occur in identifiable circumstances. In contrast, in the NA economy, gifts are a priori excluded for all families, while in the PA economy, transfers are such that family households always have equal consumption. The differences in consumption and transfer behavior will play a central role for deviations from RE in the deficit-financed tax cuts that will be studied.

The following time line illustrates the timing of taxes for a generic deficit-financed tax cut:

<table>
<thead>
<tr>
<th>Announcement</th>
<th>0</th>
<th>$S_1$</th>
<th>$S_1 + S_2$</th>
<th>Time</th>
</tr>
</thead>
<tbody>
<tr>
<td>Balanced budget</td>
<td>Tax cut</td>
<td>Tax hike</td>
<td>Balanced budget</td>
<td></td>
</tr>
</tbody>
</table>

Initially, the economy is assumed to be in the stationary equilibrium. In this equilibrium, lump-sum taxes finance the entire expenditure stream, \( \{G\} \). The deficit-financed tax cut is announced prior to its implementation at time 0. The time of the announcement is indicated by the two dots. The tax cut lasts for $S_1$ years, during which time deficits pay for the shortfall in government revenues. The tax hike

\[14\] The degree of relative risk aversion is, in this particular respect, irrelevant for a perfect altruist, since in this case $A = 1$. 

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implemented afterwards is such that the debt accumulated over-and-above its steady-state value is paid off over the following $S_2$ years. Finally, the tax rate returns to its steady-state value indefinitely (see section A.1 of the appendix for the computational strategy).

4.1 Stationary Equilibrium

The key features of the stationary equilibrium are as follows. First, donors delay inter-vivos transfers until recipients are borrowing constrained. Second, the consumption paths of gift recipients jump downward when entering a transfer region. Finally, household policies become increasingly similar to those in the NA economy in regions where the intra-family wealth distribution is more balanced. This equilibrium is qualitatively as in Barczyk & Kredler (2011b). They study this type of equilibrium in an economy with two infinitely-lived altruistic agents in detail and provide a numerical algorithm. In order to understand the upcoming main results of the paper, a brief discussion behind the intuition of the key features is warranted.

The donor is faced with incentives to delay transfers until the recipient is borrowing-constrained. Conversely, as long as the wealth of both households in a family are positive, $w^1 > 0$ and $w^2 > 0$, each household’s transfer motive in the HJB (4) is negative, $\mu^1 < 0$ and $\mu^2 < 0$. Each household has a larger marginal value with respect to its own savings than with the other household’s savings. This is obviously also the case for the NA economy; it is not, however, the case for the PA economy, in which each household’s transfer motive is zero, $\mu^1 = 0 = \mu^2$, throughout the state space. PA economy households are indifferent with respect to the intra-family wealth distribution, valuing only the size of family resources, and not how these resources are distributed across the two households.\textsuperscript{15} The economic intuition for

\textsuperscript{15}An interesting implication of this result is that it suggests a failure of neutrality for certain types of unexpected redistributive policies in the IA economy, but not in the PA economy, despite incomplete markets. Suppose there is a one-time unexpected lump-sum transfer of wealth, say $T r$, from the young generation to the old generation. Since, in the IA economy, the value function of the old is increasing in the direction $(w_1 - T r, w_2 + T r)$, the old household would have no reason to privately undo the government’s induced transfer. In contrast, in the PA economy, this redistribution would be neutral since only total dynasty resources matter, and not how they are distributed. Furthermore, we would expect the welfare consequences of this redistribution to differ vis-à-vis the NA model. While the value functions in the IA economy do change, this change is attenuated, since households care about both capital stocks, i.e. $V^2_{w_2} > 0$ and $V^2_{w_1} > 0$, whereas households in the NA economy do not attribute any value to someone else’s resources. Neutrality of this redistribution, in terms of
the delay in transfers in the IA economy is as follows. Due to imperfect altruism, a recipient household does not fully internalize the effects its consumption behavior has on family resources. The household would consume an “early” transfer at a faster rate than is in the donor’s interest. After consuming the transfer, the recipient household would come back and ask for more; after all, the recipient knows that the donor cannot credibly commit to not providing a transfer yet again. When a household is borrowing-constrained, however, the donor has control over the recipient’s consumption behavior – in fact, the donor household uses transfers in order to temporarily implement his preferred allocation.

The second feature present in the stationary equilibrium is that a consumption path displays a discontinuity in the form of a downward jump when a household enters a transfer region. The intuition behind this feature is closely related to the *Samaritan’s dilemma*, known from two-period models (for example, see Lindbeck & Weibull (1988)). A household that obtains transfers in the second period anticipates that an additional unit saved will be “taxed” by the donor through a reduction in transfers. Consequently, the future transfer recipient over-consumes in the initial period. In contrast, when the household does not expect to receive transfers, the marginal value of saving equals the marginal utility of consumption, and so consumption in the first period is as in the standard selfish case.

These two features are illustrated in figure (1), which portrays a history of consumption, wealth, and transfers for two different families. The left-hand side top and bottom graphs belong to one family and the right-hand side ones to the other family. The top panel displays the histories for consumption and gifts, while the bottom panel displays the histories for wealth and bequests. The dashed lines refer to the young households and the solid lines refer to the old households. In both families, the old household dies in the year 2026 (demarcated by vertical solid lines).

The purpose of the figure is to demonstrate the various kinds of transfer behavior that are feasible, as well as the discontinuity of the consumption path that occurs when a household enters a transfer region. To that end, the families are endowed with different initial wealth endowments and earning streams. As can be seen for the family shown on the left-hand side, the old household’s wealth is depleted by 2012. The young household, on the other hand, accumulates wealth. Between 2000 and allocations and welfare, only holds in the PA economy.
The figure portrays a history of consumption, wealth, and transfers for two different families. The left-hand side top and bottom graphs belong to one family and the right-hand side ones to the other family. The top panel displays the histories for consumption and gifts, while the bottom panel displays the histories for wealth and bequests. The dashed lines refer to the young households and the solid lines refer to the old households. In both families, the old household dies in the year 2026, demarcated by vertical solid lines.

In 2011, gifts from the young household to the old household do not occur. Following 2012, the old household obtains inter-vivos transfers from the young household (the red dashed line, which is enlarged 5x for expositional purposes). These transfers continue and increase over time as the young household accumulates more wealth. In 2026, flows of gifts stop due to the death of the old household; there is no bequest. Finally, the drop of the old household’s consumption path (the analogue to the Samaritan’s dilemma) is clearly visible in 2012.

For the family displayed on the right-hand side, the young household’s wealth decreases quickly and is exhausted by 2002. The old household’s wealth decreases gradually until 2026, at which point the remaining wealth is bequeathed to the young household, and the young household’s wealth jumps up. The young household obtains gifts from the old household (the solid red line, which is enlarged 20x for expositional purposes) beginning in 2002. These transfers are, however, temporary.
They eventually cease due to a decrease in the old household’s wealth level. Though gifts are temporary, the young household obtains a bequest upon the old household’s death. In addition, upon the receipt of inter-vivos transfers, the young household’s consumption jumps down.

Figure (2) shows that household policies become increasingly similar to those in the NA economy in regions where the intra-family wealth distribution is more balanced. The figure compares the savings behavior of the NA economy (the black arrows) and the IA economy (the red arrows) for various combinations of families’ earnings profiles. The horizontal axis represents old households’ wealth and the vertical axis represents young households’ wealth. In the top panel, the first graph shows a family where both the young and the old households have low earnings, while the second graph shows a family where the young household has high earnings and the old household has low earnings. In the bottom panel, the first graph shows a family where the young household has low earnings and the old household has high earnings, while the second graph shows a family where both households have high earnings. A point in the figure corresponds to state $x = (y^1, y^2, w^1, w^2)$. The arrows emanating from $x$ represent the young and old households’ change in wealth (the continuous state), $\dot{w}^1$ and $\dot{w}^2$, when in that state, for both the IA economy and the NA economy. Along the horizontal axis only the old household owns wealth and along the vertical axes only the young household owns wealth. If a ray that emanates from the origin rotates from the horizontal axis towards the vertical axis is considered, then as the ray rotates up to the 45 degree line, the intra-family wealth distribution becomes increasingly balanced. The figure shows that, along this rotation, the savings behavior in both the IA economy and the NA economy become increasingly similar.

The intuition behind this resemblance is that inter-vivos transfers become increasingly unlikely as both households’ shares of wealth become progressively similar. This resemblance to the NA economy intensifies since households in the IA economy come to rely only on themselves. Conversely, for a relatively unequal intra-family wealth distribution, gifts in the near future are more likely, and IA economy households’ current consumption-savings decisions are more strongly influenced by this possibility. This consideration does not enter NA economy households’s decision-making.
Wealth evolution in the NA economy (black) and the IA economy (red).

### 4.2 Deficit-Financed Tax Cuts

As in Heathcote (2005), deviations from RE are quantified by measuring how aggregate consumption changes when the tax rate changes. In particular, I divide the difference in aggregate consumption between the tax-cut (tax-hike) regime and aggregate consumption in the stationary equilibrium by the change in aggregate disposable income (the change in the respective tax revenue). The resulting measure is what Heathcote (2005) calls the *propensity to consume of income tax* (PCT):

$$\text{PCT}_t \equiv \frac{C_t - \bar{C}}{\Delta Y^d} = \frac{\int_X c(t, x)\lambda(t, x)dx - \int_X \bar{c}(x)\bar{\lambda}(x)dx}{\Delta Y^d},$$

where \(\int_X dx\) stands for integrating over the state-space \(X\), \(\Delta Y^d\) denotes the change in aggregate disposable income, \(C_t = \int_X c(t, x)\lambda(t, x)dx\) is time-\(t\) aggregate consumption, and \(\bar{C} = \int_X \bar{c}(x)\bar{\lambda}(x)dx\) is steady-state aggregate consumption. If RE holds, then aggregate consumption remains unchanged, \(C_t = \bar{C}\), and the PCT equals zero. If all households are hand-to-mouth consumers, aggregate consumption changes one-for-one with the change in the tax revenue, \(C_t - \bar{C} = \Delta Y^d\), and the PCT equals
Recalling the time line illustrating the timing of taxes at the beginning of section 4, a deficit-financed tax cut will have the following structure. First, the tax-cut regime will last as long as the tax-hike regime. Second, each deficit-financed tax cut will be announced one year prior to its implementation; this attempts to capture the fact that there is a lag between the announcement of a policy and its implementation (the results do not significantly change when altering the length of this lag). Third, deficits will finance 3% of stationary government consumption; this ensures realistic debt-to-GNP ratios in all the experiments. Finally, the debt, which has been accumulated over-and-above the steady-state level, will be repaid in equal payments over the duration of the tax-hike regime.

I compute experiments over four different durations. In what I refer to as the short-term experiment, there is a two year tax-cut followed by a two year tax-hike. The next is a medium-term one, corresponding to a four year tax-cut and a four year tax-hike. One interpretation of the short- and medium-term experiment durations is to think of them as corresponding to the U.S. presidential election cycle. The third is a long-term one, which corresponds to a 25 year tax-cut and 25 year tax-hike; this experiment is intended to mimic the expected length of time young and old generations overlap at the time of implementation of the deficit-financed tax cut. In the fourth and final experiment, there is an eight year tax-cut followed by an eight year tax-hike. I refer to this experiment as a medium/long-term one and its inclusion offers a case that lies in between the medium- and the long-term experiments.

Figure 3 illustrates the various economies’ PCTs over time for the four deficit-financed tax cuts. Each change in government-financing policy is announced in 2000. The tax-cut regime is implemented in 2001. This implementation is indicated by the first vertical dashed line. The second vertical dashed line marks the onset of the tax-hike regime. The horizontal lines at zero are the PCTs over time when RE holds, the red lines are the PCTs over time for the PA economy, the black lines are the PCTs over time for the NA economy, and the blue dashed lines are the PCTs over time for the IA economy.

In 2000 the PCTs for all three economies jump up and the ensuing trajectories of the PCTs between 2000 and 2001 are positive.\footnote{In order to compute the PCTs for the period between the announcement and the implementation,
Figure 3: PCTs over time.

PCTs over time for the four durations of the deficit-financed tax cut. The dashed vertical lines mark the onset of the tax-cut regime and the tax-hike regime, respectively. The horizontal lines at zero are the PCTs over time when RE holds, the red lines are the PCTs over time for the PA economy, the black lines are the PCTs over time for the NA economy, and the blue dashed lines are the PCTs over time for the IA economy.
The initial jumps of the PCTs across the four experiments, and the ensuing trajectories of the PCTs between 2000 and 2001, indicate that they increase as the duration of the deficit-financed tax cut lengthens. This is because the farther into the future is the tax-hike, the larger is the extent to which the tax-cut is internalized as a permanent increase in income. Mortality risk makes it less likely that a current household is responsible for the entire tax burden. In addition, income uncertainty makes it more likely that a household’s borrowing constraint will be binding at some point during the deficit-financed tax cut. Both of these truncate the effective planning horizon of households. In the NA economy, for example, the tax-cut would be completely internalized as an increase in permanent income if the tax-cut was certain to last beyond the current households’ lifetimes. Due to the fact that households’ planning horizons are confined to their own lifetime, households in such an economy would not take into account any of the effects stemming from the tax-hike. A binding borrowing constraint similarly truncates the effective planning horizon of a household in any of the three economies. Specifically, the truncation is such that economic consequences that lie beyond the time of the binding borrowing constraint are not internalized. As the duration of the experiment lengthens, unconstrained households increasingly expect to be constrained at some point in the future during the experiment. From figures 4a and 4b, it can be seen that, for the short- and the medium-term experiments, the PCTs’ trajectories for the IA economy roughly coincide with those from the NA economy. This resemblance disappears as the experiment lengthens, as can be seen in figures 4c and 4d. For the medium/long-term experiment, the IA economy’s PCT lies in-between the NA economy’s and the PA

I use the change in aggregate disposable income of the tax-cut regime.
economy’s PCTs. For the long-term experiment, the IA economy’s PCT practically coincides with the PA economy’s PCT.

The tax-cut is implemented in 2001. Once again, the PCTs in all three economies jump up. This time, however, the jumps are due to constrained households. Households that remain constrained after receiving the tax-cut act like hand-to-mouth consumers and consume the entire tax-cut. The differing magnitudes in the jump of the PCTs can be accounted for by considering the fraction of borrowing-constrained households in the stationary equilibrium of the respective economy. In the IA economy this fraction is 22.07% (of these, 15.73% are young households and 6.34% are old households). In the NA economy, this fraction stands at 14.56% (of these, 6.82% are young households and 7.74% are old households). Finally, in the PA economy, there are 13.79% of constrained households (here, there is an equal 6.89% fraction of young and old constrained households). This fraction is largest for the IA economy because, in the equilibrium of this economy, transfers only flow when the recipient is constrained, which provides an incentive to become constrained. This incentive is absent from the other two economies. There is, however, an important caveat: not all constrained households in the IA economy increase their consumption one-for-one upon receiving the tax-cut. Since the tax-cut also crowds out private transfers, households which are recipients of private transfers prior to the tax-cut (8.41% of young households and 1.43% of old households) receive less transfers when the tax-cut takes place. Transfer recipients’ consumption may therefore be unchanged or may only change by a small fraction of the tax-cut. In the PA and the NA economies, the fraction of constrained households reveals how many households act as hand-to-mouth consumers. Conversely, in the IA economy, relative resources of the households in the family play an important role.

As we have discussed so far, the differences in the PCTs’ levels are largely explained by the fractions of constrained households, which themselves depend on the extent of altruism, and on the length of the experiment. The level of the PCT increases in the fraction of constrained households since these households act like hand-to-mouth consumers. The level also increases in the duration of the deficit-financed tax cut because of the increased likelihood for an unconstrained household to be constrained at some point in the future. For the duration of the tax-cut regime, summing up the effects from the unconstrained and the constrained households, the
trajectories of the PCTs in the IA economy are strictly above those from the NA economy and from the PA economy for the short, medium, and medium/long term tax-cuts. A striking feature is the following: the PCTs in the IA economy decrease at a much slower rate than those in the other two economies. This feature is particularly evident in the long-term experiment, shown in figure 4d, where the trajectory of the PCT in the IA economy is initially below the one from the NA economy but eventually surpasses it. The economic intuition behind the differences in the shapes of the PCTs’ trajectories has to do with imperfect altruism and lack of commitment. The possibility of future transfers provides a disincentive to save out of the higher disposable income for current recipients of gifts or for households which expect to receive transfers in the future if they remain poor enough throughout the tax-cut regime. With time, the aggregate consumption policies in all economies start to prescribe consumption rates which are below those from the stationary economy in anticipation of the tax hike (we will return to this point shortly). This process occurs at a slower rate in the IA economy, however, than it does in the other two economies. As the economy approaches the date of the tax increase, it becomes increasingly likely that current households will carry the burden of paying back the accumulated debt through higher taxes. Only in the IA economy are there households which can count on being “bailed out” with private transfers when taxes increase. Households without altruism, as well as households with perfect altruism, do not have this type of moral hazard, and they therefore increase their savings at an earlier point in time.

Table 4 presents the average PCTs over the tax-cut regime in order to offer a more succinct view of the differences of the PCTs’ trajectories. Unsurprisingly, this average increases in the duration of the experiment for all three economies. What is more significant is that the average PCT in the IA economy is larger than the average PCTs for the other two economies in all of the experiments. This difference is particularly stark for the short-, medium-, and the medium/long-term changes in the government-financing policy. For these durations, the PCT in the IA economy is 29%, 30%, and 24% larger than the PCT in the NA economy and 52%, 58%, and 61% larger than the PCT in the PA economy. For the long-term deficit-financed tax cut, the average PCT of the IA economy becomes very similar to that of the NA economy; the average is merely 7% higher. In addition, the difference to the PA
economy becomes less pronounced; the average is 44% higher.\footnote{\textbf{17}I computed a variety of comparative statics exercises to study the sensitivity of this result. Holding the other parameters constant I computed the same exercise as shown here for various values for the altruism parameters in an empirically plausible range. Qualitatively the results are unchanged and quantitatively there are only some minor changes.}

Table 4: Average PCTs (in terms of $’s) for the tax-cut regime.

<table>
<thead>
<tr>
<th>Experiment</th>
<th>RE</th>
<th>NA</th>
<th>IA</th>
<th>PA</th>
</tr>
</thead>
<tbody>
<tr>
<td>Short-run</td>
<td>0</td>
<td>0.091</td>
<td>0.117</td>
<td>0.077</td>
</tr>
<tr>
<td>Medium-run</td>
<td>0</td>
<td>0.102</td>
<td>0.133</td>
<td>0.084</td>
</tr>
<tr>
<td>Medium/Long-run</td>
<td>0</td>
<td>0.140</td>
<td>0.174</td>
<td>0.108</td>
</tr>
<tr>
<td>Long-run</td>
<td>0</td>
<td>0.323</td>
<td>0.347</td>
<td>0.241</td>
</tr>
</tbody>
</table>

In order to further our understanding behind the driving forces of the aggregate consumption dynamics during the experiment, it is helpful to decompose the PCTs for a given point in time. Furthermore, this will aid us in understanding the jumps of the PCTs and their ensuing trajectories, starting with the tax-hike regime.

There are two main reasons why aggregate consumption during the experiment differs from its stationary value. First, consumption policies may change over time. For a given state, optimal consumption rates differing from the stationary aggregate consumption policy may be prescribed. Second, the distribution of households over the state-space may change. Due to changes in consumption-savings policies, the distribution of households across the state-space during the experiment may differ from the stationary density. Formally, the PCT can be decomposed into the following three components

\[
PCT_t \equiv \frac{C_t - \bar{C}}{\Delta Y^d} = \frac{\int_X \tilde{c}_t(x)\tilde{\lambda}(x)dx}{\Delta Y^d} + \frac{\int_X \bar{c}(x)\tilde{\lambda}_t(x)dx}{\Delta Y^d} + \frac{\int_X \tilde{c}_t(x)\bar{\lambda}(x)dx}{\Delta Y^d},
\]

where \(\int_X dx\) stands for integrating over the state-space \(X\), \(C_t = \int_X c(t, x)\lambda(t, x)dx\) is time-\(t\) aggregate consumption, \(\bar{C} = \int_X \bar{c}(x)\bar{\lambda}(x)dx\) is steady-state aggregate consumption, \(\Delta Y^d\) denotes the change in aggregate disposable income, \(\tilde{c}_t(x) = c(t, x) - \bar{c}(x)\) is the difference between the time-\(t\) aggregate consumption policy and the stationary consumption policy evaluated at state \(x\), and \(\tilde{\lambda}_t(x) = \lambda(t, x) - \bar{\lambda}(x)\) is the difference between the time-\(t\) density and the stationary density evaluated at state \(x\).
A positive value of $\tilde{c}_t(x)$ means that the time-$t$ consumption policy stipulates a higher consumption rate than does the steady-state consumption policy for a family with state $x$. The aggregation of $\tilde{c}_t(x)$ over the state space, $X$, using the stationary density, $\int_X \tilde{c}_t(x) \tilde{\lambda}(x) dx$, tells us how much changes in policies contribute to the difference between time-$t$ aggregate consumption and stationary aggregate consumption. I will refer to $\int_X \tilde{c}_t(x) \tilde{\lambda}(x) dx$ as the policy-change component. In order to obtain the contribution of the policy-change component to the PCT it is divided by the change in aggregate disposable income, $\Delta Y^d$. I will refer to the resulting number as the PCT-adjusted policy-change component.

A positive value of $\tilde{\lambda}_t(x)$ tells us that the time-$t$ mass of families with state $x$ is larger than it is under the stationary density. Using $\tilde{\lambda}_t(x)$ to sum up the steady-state consumption policy, $\tilde{c}(x)$, across the state space, $\int_X \tilde{c}(x) \tilde{\lambda}(t, x) dx$, provides the contribution to the difference between time-$t$ aggregate consumption and stationary aggregate consumption due to a change in the density. For example, a positive value of $\int_X \tilde{c}(x) \tilde{\lambda}(t, x) dx$ means that a larger mass of households must be in regions of the state space where the steady-state consumption policy prescribes larger consumption rates. That is, time-$t$ aggregate wealth is larger than steady-state aggregate wealth due to changes in consumption-savings policies. I will call $\int_X \tilde{c}(x) \tilde{\lambda}(t, x) dx$ the density-change component. Again, in order to obtain the contribution of this term to the PCT it is divided by the change in aggregate disposable income, $\Delta Y^d$. I will call the resulting number the PCT-adjusted density-change component.

The third term of equation (6) is of an order lower than the other two terms and is verified to be negligible in the computations. Thus, the PCT at a given point in time is approximately equal to the sum of the PCT-adjusted policy-change component and the PCT-adjusted density-change component:

$$\text{PCT}_t = \frac{C_t - \bar{C}}{\Delta Y^d} \approx \frac{\text{Policy Change}}{\Delta Y^d} + \frac{\text{Density Change}}{\Delta Y^d},$$

where

$$\text{Policy Change} \equiv \int_X \tilde{c}(x) \tilde{\lambda}_t(x) dx \quad \text{and} \quad \text{Density Change} \equiv \int_X \tilde{c}_t(x) \tilde{\lambda}(x) dx.$$
term experiment. The discussion would not substantially differ for the other experiments. Figure 4 shows this decomposition for the three economies for the short-term experiment. The dashed vertical lines mark the onset of the tax-cut regime and of the tax-hike regime, respectively. The trajectories of the PCTs are shown by the thick solid lines. The dashed graphs are the PCT-adjusted density-change components and the narrow solid lines are the PCT-adjusted policy-change components.

The trajectories of the PCTs between 2000 and 2001 are entirely due to changes in the optimal policies. This can be seen from the fact that the thin and the thick solid lines coincide over this period. The policy-change component is a forward looking component and accounts for changes in household’s consumption-savings choices. Since the consumption rate is a flow variable, the policy-change component can vary rapidly. The density-change component is a backward looking one and accounts for past decisions on the current level of wealth. It responds slowly since wealth is a stock variable.

In 2001, when the tax-cut is implemented, the policy-change components of the PCTs jump up. These jumps are due to changes in the constrained households’ policies. For constrained households, consumption policies change one-for-one with the tax-cut. Throughout the tax-cut regime, the policy-change component for constrained households remains constant (i.e. the term policy change for these households is simply a horizontal line). As can be seen from the figure, however, the policy-change component decreases as the tax-hike regime approaches. Evidently, this must be due to changes in consumption policies of the unconstrained households.

Figure 5 shows the policy-change component for unconstrained households with positive wealth throughout the short-term experiment. The policy-change component for unconstrained households decreases throughout the tax-cut regime and rapidly turns negative (for longer-term experiments the point at which the policy-change component turns negative happens later). Thus, unconstrained households time-\(t\) policies start to prescribe lower consumption rates than do the stationary policies. Households increase their savings in order to buffer for the possibility of having to pay back the accumulated debt. Consequently, more households are in states with higher wealth than when in the stationary equilibrium. As these households save more, aggregate wealth in the economy increases. This is reflected by the fact that
Figure 4: Decomposition of the PCTs.

Decomposition of the PCTs for the short-term experiment, see equation (6). The dashed vertical lines mark the onset of the tax-cut regime and the tax-hike regime, respectively. The dashed graphs trace out the PCT-adjusted density-change component, the narrow solid lines represent the PCT-adjusted policy-change component, and the thick solid lines are the trajectories of the PCTs, i.e. the sum of the dashed lines and the narrow solid lines.

The density-change component throughout the tax-cut regime is positive and increasing. Over time an increasing part of the aggregate consumption is explained by households having higher wealth.

Furthermore, as can be seen in figure 5, the policy-change component for households with positive wealth at a given point in time during the tax-cut regime is smallest in the PA economy. In anticipation of the tax-hike, PA economy households increase their savings relatively more than do households in the other two economies. The policy-change component for households in the IA economy more closely resembles the one from the NA economy, but is even larger (this observation does change qualitatively in the long-term experiment: in figure 4d, it can be seen that between 2000 and 2001, the PA economy’s policy-change component roughly coincides with the one from the IA economy; after some time, the PA economy’s policy-change component will, however, be lower again). Households in the IA economy
Policy-change component for (unconstrained) households with positive wealth in the short-term experiment, see equation (6). The dashed vertical lines mark the onset of the tax-cut regime and the tax-hike regime, respectively.

save the least in preparation for the tax-hike regime.

In 2003, the tax-hike regime is implemented. The lump-sum tax increases above its stationary equilibrium value. The additional tax revenue is used to pay off the debt, which has been accumulated over-and-above the steady-state level of debt and its associated interest payments. The additional debt is repaid in equal payments and the duration of the tax-hike regime equals the length of the tax-cut regime. The change in aggregate disposable income, $\Delta Y^d$, becomes negative and is in absolute terms larger than under the tax-cut regime, since both the debt and the accumulated interest must be repaid. Obviously, a larger tax-hike is required for longer durations of the tax-cut. This induces a purely mechanical change in the computation of the PCT (see equation (6)).

When the tax-hike takes place, as can be seen from figure 4, the PCT-adjusted policy-change components jump up. This is due to both unconstrained and constrained households. The policy-change component for unconstrained households is negative, and since households have expected the change in the financing policy,
consumption policies for unconstrained households change continuously (see figure 5). There is now, however, a negative change in aggregate disposable income, $\Delta Y^d < 0$, so that the PCT-adjusted policy-change component for unconstrained households switches signs. For constrained households, the policy-change component is positive before reaching the onset of the tax-hike regime. Once the tax-hike takes place, constrained households’ consumption policies change discontinuously. In particular, their consumption policies prescribe a lower consumption rate than is the case in the stationary equilibrium. It follows that the policy-change component for constrained households is negative for the duration of the tax-hike regime and the PCT-adjusted policy-change component is positive.

The density-change components are positive before the onset of the tax-hike regime. Since the density changes continuously and since there is a negative change in aggregate disposable income, the PCT-adjusted density-change components switch from positive values to negative values, as can be seen in figure 4. In sum, the PCT-adjusted policy-change components outweigh the PCT-adjusted density-change components, and so the PCTs continue to be positive throughout the tax-hike regime.

Constrained households’ consumption policies change one-for-one with the tax-cut. Since disposable income during the tax-hike regime is below steady-state disposable income, constrained households’ consumption policies prescribe a consumption rate below the steady-state value. The PCT-adjusted policy-change component for these households is constant and positive throughout the tax-hike regime (i.e. a positive horizontal line). As can be seen in figure 4, however, the PCT-adjusted policy-change component decreases throughout the tax-hike regime. This must be due to the unconstrained households. Consumption policies for unconstrained households prescribe increasingly higher consumption rates. These time-$t$ consumption rates are still below the steady-state consumption rates, but gradually approach them (see figure 5). This happens since, once the tax-hike regime ends, the tax rate permanently decreases to its stationary value, and consumption policies revert to the stationary policies. As these households save less, aggregate wealth in the economy decreases and the mass of households retreats to lower wealth states. This is reflected by the term density change.

A noticeable feature of figure 4 is that the PCTs’ trajectories during the tax-hike regime are more similar across the three economies than they were during the
tax-cut regime (see also figure 4a). What is especially striking is the close resemblance between the PA economy’s PCT-adjusted policy-change component and the IA economy’s one. Figure 5 reveals that at the beginning of the tax-hike regime, the policy-change component for unconstrained households in the PA economy is smallest. This, however, translates into the largest PCT-adjusted policy-change component for unconstrained households. Adding to this the constrained households’ PCT-adjusted policy-change component yields the one observed in figure 4. Thus, despite the fact that there is a smaller fraction of constrained households in the PA economy than there is in the IA economy, the PCT-adjusted policy-change components are roughly of the same size for both economies. The unconstrained households’ PCT-adjusted policy-change components make up for this difference. For the NA economy, the fraction of constrained households is close to the one in the PA economy and the unconstrained households’ PCT-adjusted policy-change component is larger than the one in the PA economy. This explains why the PCT in the NA economy is relatively smaller than in the other two economies.

Table 5: Average PCTs (in terms of $’s) for the tax-hike regime.

<table>
<thead>
<tr>
<th>Experiment</th>
<th>RE</th>
<th>NA</th>
<th>IA</th>
<th>PA</th>
</tr>
</thead>
<tbody>
<tr>
<td>Short</td>
<td>0</td>
<td>0.076</td>
<td>0.093</td>
<td>0.089</td>
</tr>
<tr>
<td>Medium</td>
<td>0</td>
<td>0.074</td>
<td>0.091</td>
<td>0.086</td>
</tr>
<tr>
<td>Medium/Long</td>
<td>0</td>
<td>0.081</td>
<td>0.099</td>
<td>0.089</td>
</tr>
<tr>
<td>Long</td>
<td>0</td>
<td>0.275</td>
<td>0.277</td>
<td>0.233</td>
</tr>
</tbody>
</table>

Referring back to figure 3, it can be seen that the similarity of the PCTs’ trajectories across the three economies is also present for the experiments of longer durations. For convenience, the average PCTs over the duration of the tax-hike regime are summarized in table 5. The average PCTs reflect the similarity.

4.3 Welfare Implications

A complementary way of studying deviations from RE is through the lens of a welfare analysis. I perform welfare calculations by computing the consumption equivalent variation (CEV) of a deficit-financed tax cut for a “newborn” household under the veil of ignorance.
Suppose a household can either be born into the stationary economy or into the same economy, but subject to the deficit-financed tax cut. In either case, the household does not know what its type will be – that is, the household does not know at which density point it will be. The CEV is the percentage of annual consumption a household in the stationary economy would have to be compensated with in order for that household to be as well off as in the deficit-financed tax cut economy’s equilibrium. Table 6 provides the CEVs for the three economies for the various durations of the deficit-financed tax cut.

Table 6: Consumption equivalent variations under the veil of ignorance (%).

<table>
<thead>
<tr>
<th>Experiment</th>
<th>RE</th>
<th>NA</th>
<th>IA</th>
<th>PA</th>
</tr>
</thead>
<tbody>
<tr>
<td>Short</td>
<td>0</td>
<td>0.0049</td>
<td>0.0035</td>
<td>0.0027</td>
</tr>
<tr>
<td>Medium</td>
<td>0</td>
<td>0.0152</td>
<td>0.0085</td>
<td>0.0066</td>
</tr>
<tr>
<td>Medium/Long</td>
<td>0</td>
<td>0.0498</td>
<td>0.0263</td>
<td>0.0211</td>
</tr>
<tr>
<td>Long</td>
<td>0</td>
<td>0.2384</td>
<td>0.1473</td>
<td>0.1249</td>
</tr>
</tbody>
</table>

The CEVs are zero in an economy with RE. Due to market incompleteness, the deficit-financed tax cut enlarges the set of feasible consumption paths a household can choose in all three economies. Households can use the tax-cut as an additional channel through which they may smooth their consumption path, while still being free to save some or all of it. As a result, the CEVs are positive and increase in the duration of the experiment in all three economies.

Contrasting the CEVs across the economies, it can be seen that in all the computational experiments, the CEVs are largest in the NA economy, smallest in the PA economy, and the CEVs from the IA economy lie in between. Though the PCTs of the IA economy are larger than those belonging to the other two economies, it is striking to note that this same relationship does not hold for welfare implications. In particular, as judged from the aggregate consumption dynamics discussed previously, one would have expected a closer resemblance between the IA economy’s welfare implications and those of the NA economy’s. This, however, is not the case.

In the short-term experiment, the IA economy’s CEV lies half-way between the NA economy’s CEV (it is 28.6% below) and the PA economy’s CEV (it is 29.6% above). In the medium-term experiment the IA economy’s CEV is 44.1% below the NA economy’s CEV and 28.8% above the PA economy’s CEV. Only in the
medium/long-term experiment does the IA economy’s similarity to the NA economy increase; the IA economy’s CEV is 17.9% below the NA’s CEV and 24.6% above the PA economy’s CEV. Finally, in the long-term experiment, the IA economy’s CEV more closely resembles the PA economy’s CEV; it is 17.9% larger and 38.2% smaller than the NA economy’s CEV.

In the IA economy, there are households (especially young households) that are recipients of private transfers. One effect of the deficit-financed tax cut on recipients of inter-vivos transfers is the exchange of gifts, which are free of any future financial obligations, for a tax-cut. This tax-cut, however, comes with a future obligation to repay the accumulated debt with higher taxes. While households which receive private transfers throughout the entire experiment are largely unaffected, this tax-cut-for-private-transfers swap matters for households for which transfers stop flowing either prior to or during the tax-hike regime. These households will have had their inter-vivos transfers crowded out by the tax-cut. While this would leave their consumption profile unaltered, it would present a financial obligation in the form of a higher tax burden. For some transfer recipients, this mechanism implies a welfare loss. This loss depresses the CEV in the IA economy, and since it is absent from the NA economy, the resemblance between the two economies dwindles.

### 4.4 Robustness: An Alternative Calibration

Are the results with respect to the IA economy robust to changes in the key parameters? Would the results change dramatically if the calibration of the economy were to rely on different values of the calibration targets? In this section, I attempt to address these questions.

I compute the PCTs and the CEVs with a re-calibrated version of the IA economy. The new values of the key parameters are chosen such that the model matches larger values of the transfer-to-wealth ratios for both the young and the old. The rationale behind the larger values of these calibration targets is as follows. The SCF only reports private transfer amounts of $3000 and above. By comparing the transfer data from the SCF with other data sources, Gale & Scholz (1994) conclude that inter-vivos transfers are likely to be one-third larger than the amount of inter-vivos transfers reported in the SCF. Therefore, the transfer-to-wealth ratios of both the
young and the old are increased accordingly. The values of the two other calibration targets (the wealth-to-GNP ratio and the wealth-poor-%) remain unchanged. The newly calibrated values of the key parameters are summarized by table 7.

Table 7: Alternative Values of Key Parameters

<table>
<thead>
<tr>
<th>Parameter</th>
<th>NA</th>
<th>IA</th>
<th>PA</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\rho$</td>
<td>3.15%</td>
<td>3%</td>
<td>5.14%</td>
</tr>
<tr>
<td>$\alpha^1$</td>
<td>0</td>
<td>0.15</td>
<td>1</td>
</tr>
<tr>
<td>$\alpha^2$</td>
<td>0</td>
<td>0.31</td>
<td>1</td>
</tr>
</tbody>
</table>

The discount rate $\rho = 3\%$, and so is unchanged from the previous calibration. This is driven by the unchanged values of the wealth-to-GNP ratio and the wealth-poor-%, both of which are primarily responsible for its identification. As expected, the replication of larger values for the transfer-to-wealth ratios leads to an increase in the degrees of altruism for both young and old households. The young household’s degree of altruism increases from $\alpha^1 = 0.12$ to $\alpha^1 = 0.15$ and the old household’s degree of altruism increases from $\alpha^2 = 0.28$ to $\alpha^2 = 0.31$.

The features of the new stationary equilibrium are qualitatively the same as those in the original stationary equilibrium (see section 4.1). In particular, transfers continue to only flow to constrained households. There are, however, more states in which transfers flow in the new calibration. This creates an additional incentive to become constrained. Consequently, the fraction of borrowing-constrained households increases slightly from 22.07% to 23.58%.

Table 8: Average PCTs (in terms of $'s) for the tax-cut regime.

<table>
<thead>
<tr>
<th>Experiment</th>
<th>RE</th>
<th>NA</th>
<th>IA</th>
<th>PA</th>
</tr>
</thead>
<tbody>
<tr>
<td>Short</td>
<td>0</td>
<td>0.091</td>
<td>0.118</td>
<td>0.077</td>
</tr>
<tr>
<td>Medium</td>
<td>0</td>
<td>0.102</td>
<td>0.134</td>
<td>0.084</td>
</tr>
<tr>
<td>Medium/Long</td>
<td>0</td>
<td>0.140</td>
<td>0.174</td>
<td>0.108</td>
</tr>
<tr>
<td>Long</td>
<td>0</td>
<td>0.323</td>
<td>0.344</td>
<td>0.241</td>
</tr>
</tbody>
</table>

A convenient way to gauge the robustness of the computational experiments is to compare the average PCTs resulting from the alternative calibration with those from the previous calibration (for the counterpart to figure 3, see figure 6). The average PCTs for the duration of the tax-cut regime are presented in table 8 (compare
Table 9: Average PCTs (in terms of $’s) for the tax-hike regime.

<table>
<thead>
<tr>
<th>Experiment</th>
<th>RE</th>
<th>NA</th>
<th>IA</th>
<th>PA</th>
</tr>
</thead>
<tbody>
<tr>
<td>Short</td>
<td>0</td>
<td>0.076</td>
<td>0.096</td>
<td>0.089</td>
</tr>
<tr>
<td>Medium</td>
<td>0</td>
<td>0.074</td>
<td>0.094</td>
<td>0.086</td>
</tr>
<tr>
<td>Medium/Long</td>
<td>0</td>
<td>0.081</td>
<td>0.102</td>
<td>0.089</td>
</tr>
<tr>
<td>Long</td>
<td>0</td>
<td>0.275</td>
<td>0.274</td>
<td>0.233</td>
</tr>
</tbody>
</table>

Table 10: Consumption equivalent variations under the veil of ignorance (%).

<table>
<thead>
<tr>
<th>Experiment</th>
<th>RE</th>
<th>NA</th>
<th>IA</th>
<th>PA</th>
</tr>
</thead>
<tbody>
<tr>
<td>Short</td>
<td>0</td>
<td>0.0049</td>
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<td>0.0027</td>
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<td>Long</td>
<td>0</td>
<td>0.2384</td>
<td>0.1362</td>
<td>0.1249</td>
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</tbody>
</table>

to table 4). The results are practically unchanged. In the short- and medium-term experiments, the average PCTs are marginally larger than before. This is a consequence of the larger fraction of borrowing-constrained households now present. The average PCT for the long-term deficit-financed tax cut is somewhat below the one from the benchmark calibration. A higher degree of altruism, holding the discount rate constant, renders households relatively more patient. This effect dampens the PCT and plays an increasingly important role as the duration of the deficit-financed tax cut lengthens. This can be seen in figure 7d, which shows that the initial jump of the PCT in 2000 is below its corresponding value from the previous calibration (compare to figure 4d). The average PCTs over the duration of the tax-hike regime are presented in table 9 (compare to table 5). Again, quantitatively the results only differ minutely. The short-, medium-, and medium/long-term experiments’ average PCTs are slightly larger. In addition, the long-term experiment’s average PCT is slightly below its corresponding value in the previous calibration.

Finally, the welfare implications, as measured by the CEVs, are shown in table 10 (compare to table 6). As before, the CEVs from the IA economy lie in between those from the PA and NA economies. The results only differ slightly. All the CEVs from the IA economy, however, are less similar than those from the NA economy and approach the PA economy CEVs.

In addition to the alternative calibration presented here, a variety of comparative
Statics exercises have been conducted, in which a symmetric altruism restriction was made, such that $\alpha^1 = \alpha^2$. Specifically, the average PCTs in the IA economy have been computed, while allowing for the degree of (symmetric) altruism to vary in an empirically plausible range, holding all other parameters unchanged. The result that the average PCT over the duration of the tax-cut regime in the IA economy is larger than the one in the NA economy holds for $\alpha$’s in a fairly wide range. For example, in the long-term experiment, the result holds as long as $\alpha < 0.5$.

5 Conclusions

Standard workhorse models in macroeconomics implicitly assume that altruism is either entirely absent or perfect, leaving the implications of intermediate degrees of altruism on deviations from RE largely unexplored. Such intermediate altruism levels are in line with the empirical evidence on inter-vivos transfers. In the data, transfers do occur, contradicting the implication of the standard OLG economy, but seem to take place only in special circumstances, suggesting that family members not always act as to equalize consumption as predicted by the dynastic model. In particular, their occurrence is especially likely when the recipient is borrowing-constrained, as predicted by the intermediate altruism model.

A priori, one would suspect that the response of aggregate consumption to a deficit-financed tax cut in the intermediate altruism model is in between that of the standard OLG and that of the dynastic economy. Surprisingly, this paper finds that in a plausibly calibrated imperfect altruism economy deviations from RE, in terms of changes in aggregate quantities, are often even stronger than in the standard OLG economy. The reasons have primarily to do with the larger number of constrained households and the strategic considerations present in this economy. However, the deviations from RE in terms of the PCT are misleading insofar as they do not translate to welfare implications. Indeed, in terms of welfare the imperfect altruism economy tends to be closer to the dynastic economy.
References


### Appendix

#### A.1 Computational Strategy

In the following the numerical strategy to compute deficit-financed tax cut equilibria is briefly outlined. The first step is to obtain the stationary equilibrium – the definition
is as in section 2.2 except that time is not a state variable. This equilibrium is computed by adapting the numerical algorithm provided by Barczyk & Kredler (2011b)’s computational online appendix. This numerical algorithm is closely related to value function iteration. A key ingredient is the use of a form of the Markov-chain approximation method for continuous-time control problems (for a more technical treatment on this see for example Dupuis & Kushner (2001)).

One specific problem that arises in the current setting, which is absent in Barczyk & Kredler (2011b), is to obtain the continuation value for very large bequests. When such bequests are made, I often have to obtain \( v^e \) for levels of wealth that lie far outside the grid. An excellent method to extrapolate the function is by exploiting homogeneity. For families with large levels of wealth we can safely neglect the income dimension and thus assume that value functions and policies are homogenous. Consumption and transfer policies are linear in wealth, which translates into value functions being of form \( W^{1-\gamma} \) in total family wealth (for details, see the homogeneous altruism setting in Barczyk & Kredler (2011a)). The old household’s value function is then given by

\[
v(w, w', y, y') = \tilde{v}(P) W^{1-\gamma}, \quad \text{where} \quad P = \frac{w}{W}, \quad \text{and} \quad W = w + w'.
\]

The function \( \tilde{v} \) can then be calculated from the outermost grid points\(^{18} \) from

\[
\tilde{v}(P) = v(w, w', y, y') W^{\gamma-1}.
\]

This gives us \( \tilde{v}(P) \) on a finite grid; intermediate values can be approximated by linear interpolation. The \( P \) which realizes upon death of the parent household is given by

\[
P = \frac{w + w'}{w'(y_j) + w + w'}.
\]

The methods which are used to solve for the stationary equilibrium can also be used to compute policy functions throughout the transition. Due to the absence of aggregate risk, the distribution of households over the state space is not a payoff-relevant state for the household. It follows that the policies as of time \( S_1 + S_2 \) are

\(^{18}\)i.e. the grid points where either the parent or the child household hold (or both) hold the maximal wealth \( W \) on the grid.
given by the ones from the stationary equilibrium. The policy functions throughout 
the deficit-financed tax cut are obtained by backward iteration on the HJBs, as given 
by equation (4). The backward iteration is initialized with the stationary value func-
tions. Using the techniques from Barczyk & Kredler (2011b) the policy functions at 
time $S_1 + S_2 - \Delta t$ are computed, where $\Delta t$ is a small increment of time. Iterating 
backward in this way until the announcement date provides the policies and value 
functions for the young and the old households throughout the transition. This is 
the same procedure as computing the stationary equilibrium except that time is an 
additional state variable and the iteration ends at the time of the announcement.

What remains are the densities throughout the transition, $\{\lambda_t\}$. These are com-
puted as follows. Prior to the announcement date, the economy is assumed to be in 
the stationary equilibrium. The aggregate state of the economy is therefore given by 
the stationary density. The stationary density is mapped forward using the transition 
probabilities implied by the policies obtained throughout the deficit-financed tax cut.

### A.2 Altruism Parameter

The purpose of this section is to provide an interpretation of the altruism parameter 
and it’s relationship with the coefficient of relative risk aversion.

Consider a typical per-period utility function of, say, old household $i$. It is as-
sumed to be additively separable in its own consumption and consumption of the 
young household $j$, i.e.

$$U^i(c_i, c_j) = u(c_i) + \alpha u(c_j), \quad \alpha \in [0, 1]$$

where $u(\cdot)$ is a CRRA utility. If $\alpha = 1$ we speak of perfect altruism and when $\alpha = 0$ 
we speak of no altruism. But what are reasonable values for this parameter? We 
can get a sense of what “reasonable” may mean by considering the following FOC, 
which in equilibrium, has to hold in the static model

$$u_c(c_i) \geq \alpha u_c(c_j) \quad \Rightarrow \quad \alpha \leq \left(\frac{c_i}{c_j}\right)^{-\gamma}$$

If the inequality points the other way, the old household would provide the young 
household with a transfer since the additional utility she obtains if the young house-
hold consumes is larger than from her own consumption. On the other hand if the old household does not have enough resources to equalize this margin her marginal utility is strictly larger than the marginal utility she would obtain from consumption by the young household, in which case there are no transfers.

When the margin is equalized we can think about parameter values for $\alpha$ as the answer to the following thought experiment: What degree of consumption inequality does an (imperfect) altruist tolerate before she decides to transfer resources, which can only be consumed, from her own consumption? If, for example, a reasonable answer appears to be 2 and $\gamma = 2$ then we can infer $\alpha = (2)^{-2} = 0.25$. With logarithmic utility the interpretation is particularly simple since $\alpha_i = c_j/c_i = 0.5$: Agents who have a degree of altruism of degree 0.5 provide voluntary transfers when consumption inequality exceeds 2.

Another interesting observation which becomes evident is that it is not the absolute value of $\alpha$ which in itself is meaningful, but its size in conjunction with the coefficient of relative risk-aversion. From the example we see that an agent with $\gamma = 2$ and $\alpha = 0.25$ tolerates the same degree of consumption inequality as an agent with $\gamma = 1$ and $\alpha = 0.5$. Thus, in order to speak about degrees of altruism we have to keep in mind that this is only meaningful in the context of specifying a degree of risk-aversion.
Figure 6: Alternative calibration: PCTs over time.

PCTs over time for the four durations of the deficit-financed tax cut. The dashed vertical lines mark the onset of the tax-cut regime and the tax-hike regime, respectively. The horizontal lines at zero are the PCTs over time when Ricardian equivalence holds, the red lines are the PCTs over time for the PA economy, the black lines are the PCTs over time for the NA economy, and the blue dashed lines are the PCTs over time for the IA economy.