Fiscal Externalities, Liquidity Constraints and Grants to Post-Secondary Students

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May 22, 2013

Abstract

Two of the most frequently discussed reasons for government financial aid to students enrolled in post-secondary education (PSE) are liquidity constraints and fiscal externalities. The purpose of this paper is to explore these motivations, and to analyze their consequences for government policy. I present a simple model of PSE in which both are allowed to be present, and apply the “sufficient statistics” method of Chetty (2009) to solve for an equation for the welfare gain from increasing aid, allowing me to determine whether government financial aid should be more or less generous. I then use statistical extrapolations and a calibration and simulation of my model to estimate the optimal level of student grants. I find that financial aid should be more generous, and that the optimal amount of aid roughly corresponds to eliminating tuition at public universities. I also find that these results are largely unchanged if students are assumed to face no liquidity constraints, suggesting that the latter issue is of second-order importance, whereas general equilibrium effects of tuition subsidies on relative or absolute wages can significantly affect the results. Further analysis of these issues using structural estimation would also be useful.

*I am very grateful to David Lee and Raj Chetty for helpful comments and suggestions, as well as to the participants of the Industrial Relations Sections Graduate Lunch and Public Finance Working Group seminars at Princeton University. Any errors or omissions are the responsibility of the author.
1 Introduction

Post-secondary education (PSE) has been the focus of a considerable amount of research by economists, especially in recent decades in North America as tuition and fee increases have motivated increased public discussion and concern about the affordability of post-secondary study and appropriate government policy in this area. Persistent public concern about the ability of students and their families to finance the cost of PSE has been mirrored by an economic literature studying the existence and magnitude of student borrowing constraints. At the same time, a growing literature seeks to provide insights into how education subsidy policy can be used to offset tax distortions affecting individuals’ education choices, a phenomenon which may be called a fiscal externality.

Although the literature on PSE is far more voluminous than this limited overview suggests, these two strands of the literature are of particular relevance, as liquidity constraints and fiscal externalities are among the most significant potential reasons for government financial aid to post-secondary students. Bovenberg and Jacobs (2005), for instance, cite fiscal externalities and capital market imperfections (along with spillovers and redistributive motives) as two of the most prominent justifications for subsidies to education. The purpose of my paper is to explore these two motivations, and to analyze their consequences for welfare analysis of government financial aid policy.

I will present a simple model of PSE in which both liquidity constraints and fiscal externalities are allowed to be present, and I apply the “sufficient statistics” method recommended by Chetty (2009) to solve the model for an equation for the marginal welfare gain from increasing grants to PSE students. I can then evaluate this welfare derivative and determine whether government grants should be more or less generous; I also use two methods, namely statistical extrapolations of my welfare equation and a calibration and simulation of my model, to estimate the optimal level of student grants. My conclusions are that financial aid should be more generous, and that my preferred estimate is that the optimal policy corresponds to eliminating tuition at the median public university. This result is similar to the finding in Saez (2002) that an Earned Income Tax Credit is optimal when low-income behavioural responses to taxation are concentrated on the extensive margin; here, the decision to attend PSE is an important extensive margin, and thus a large transfer to individuals who undertake that action may be efficient. My results are robust to a variety of alterations
and extensions, and they are largely unchanged if I assume that students face no liquidity constraints; however, the results are sensitive to the nature of general equilibrium effects of PSE graduate supply on relative and absolute wages.

The main contribution of my paper is to provide credible numerical estimates that can inform policy discussion, as practical policy questions have been neglected in the existing literature. Analysis of this sort helps us to understand what we do and don’t know, and in which areas we need to know more; specifically, my analysis indicates a baseline result of abolishing public tuition, and then provides indications of whether or not these results are sensitive to particular factors. I show that varying the level of risk-aversion or liquidity constraints do not greatly affect my conclusions; the latter point suggests that the sizable empirical literature seeking evidence for or against such constraints may be of limited policy relevance. On the other hand, I demonstrate that alternative estimates of the size of general equilibrium effects can make a big difference to the results. I also find that further structural work that takes into account multiple dimensions of heterogeneity would be valuable, as it would provide answers to additional policy questions and allow us to consider tuition subsidy programs targetted more effectively at marginal students, with reduced distributional costs and greater potential of Pareto gains. I intend to study both of these issues more closely in future work.

An additional contribution is to show how the sufficient statistics method can be adapted to the context of PSE, and to use this method to provide results which are robust to much of the variety of assumptions and modelling strategies found in the literature; my paper represents the first attempt that I am aware of to perform sufficient statistics analysis of PSE.

Finally, a secondary contribution comes from the analysis of my simple model, where I show that an absence of causal effects of family income on post-secondary enrollment is not consistent with an absence of liquidity constraints. Previous empirical work has often made the assumption that income effects on enrollment are necessary and sufficient evidence of borrowing constraints, but I demonstrate that in an unconstrained population, income may be expected to have a negative effect on enrollment.

The rest of the paper is organized as follows. Section 2 provides a brief analysis of how each of the liquidity constraint and fiscal externality motivations may justify government funding of students, and discusses the relevant literatures. Section 3 lays out my simple
model, and solves it for a sufficient statistics condition for the optimum. Section 4 provides the main numerical results. Section 5 then performs the experiment of shutting down the liquidity constraint and fiscal externality motivations for financial aid one by one, while section 6 studies heterogeneity and section 7 considers general equilibrium effects. Section 8 provides a conclusion.

2 The Role of Liquidity Constraints and Fiscal Externalities in Post-Secondary Education

Significant literatures study the existence and magnitude of fiscal externalities and liquidity constraints in post-secondary education. I therefore begin by discussing these literatures and their main findings, to provide motivation for the analysis that follows. Subsequently, I will explain how I combine the insights from these literatures into a novel framework for the analysis of tuition subsidy policy.

2.1 Fiscal Externalities

The term “fiscal externality” has been used to describe a variety of concepts in different literatures, so I begin by defining the term as used in this paper: I refer to effects of government programs upon labour market outcomes, which in turn affect government tax revenues and expenditures, influencing the government’s ability to fund its entire range of programs. In this way, one program has an “external” effect on the funding for other programs: if a program affects individuals’ decisions in such a way as to increase their total income, for example, this increases income tax revenues and allows for more spending in other areas, or lower tax rates, with added benefits to society.\(^1\)

Alternatively, I can describe the phenomenon at the individual level: when choices of an individual affect their own taxable income, and thereby the tax revenues collected from that worker, those choices have external effects on other people through the tax system. In the context of PSE, obtaining education is believed to augment workers’ skills and increase their

\(^1\)To the extent that the revenue increase is experienced by the same level of government that implements the program in question, the effect is not external in the usual sense. However, if different levels of government are involved, this phenomenon becomes very similar to that described in the tax competition literature, as summarized by Wilson (1999). Lawson (2012) provides a more detailed history of the concept of fiscal externalities.
future earnings, so the decision to attend university or college provides external benefits to all workers through an increase in future tax revenues. Programs that influence such individual choices, therefore, interact with this already existing externality or distortion from taxes. This is the context in which fiscal externalities are usually described when it comes to education policy: a program that offsets a pre-existing tax distortion, for example a tuition subsidy to encourage students to attend higher education and increase their skills and productivity, can increase efficiency.

The equality of these two concepts is pointed out in de Bartolome (1999): if an individual’s action increases tax revenues, this benefits other agents, while at the same time this implies that the individual wasn’t receiving the full returns to that action, indicating a distortion to incentives and an insufficient amount of that action being performed. Thus, if a government needs to raise revenue for necessary public expenditures or redistribution, and is restricted to using a distortionary proportional tax, this will generate a distortion on the education margin which may be mitigated through the use of a subsidy to education. A fiscal externality, therefore, is also an application of the Theory of the Second Best: a pre-existing inefficiency in the form of some exogenously required government spending alters the efficient policy in another area.

The idea of fiscal benefits from education is not a new one, and economists have been explicitly stating it as a motivation for subsidizing education at least as far as Singer (1972), who discusses how education can generate external benefits through the taxes paid on increased income and a reduction in expenditures on the social safety net. After Guesnerie and Roberts (1984) showed that quantity controls can be welfare-enhancing in a Second Best world, Del Rey (2001) and Greco (2011) apply this insight to education, showing that public provision can reduce tax-induced inefficiencies. Meanwhile, the use of publicly-provided private goods such as education to weaken self-selection constraints in redistribution is studied

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2 My analysis focusses primarily on 4-year university attendance, but I will use “PSE” throughout this paper to refer to post-secondary education, rather than “university” or “college,” as the latter term in particular has different meanings in different countries.

3 The distortion arises because a tax reduces the returns and indirect costs of education, but the direct cost of any goods invested in education is not reduced, as pointed out by Trostel (1993).

4 It is necessary to assume the necessity both of this government expenditure and of a distortionary tax system; in other words, I require an “irreducible distortion,” or a distortion that is not itself generated by policy. Otherwise, the advice of Browning (1999) to remove the initial policy generating the distortion and thereby to obtain the first-best would be valid. The generality of Browning (1999)’s analysis is limited by the fact that it is based on a special case in which the policy generating the distortion and the policy instrument which could be used to offset it are identical in practical terms.
by Boadway and Marchand (1995), among others.

A large literature, meanwhile, studies policies of the type I will be focussing on: financial incentives for post-secondary education. Simulations in Trostel (1993) show that proportional income taxation could have a significant negative effect on investment in human capital. As a result, Trostel (1996) proposes subsidizing education to offset this distortion, calculating an optimal percentage subsidy on financial investments in education approximately equal to the marginal tax rate. Subsequently, Trostel (2010) quantifies the fiscal benefits of PSE to government, estimating that net government spending per university degree is negative in the United States, as direct expenditures of about $71000 (in present-value 2005 dollars) are more than offset by expenditure savings of $56000 and increased tax revenues amounting to $197000.

In the more recent theoretical literature, Bovenberg and Jacobs (2005) present a model in which it is optimal to use a subsidy to education to perfectly offset income taxes, returning human capital investment to the first-best amount. They describe this result as an extension of the production efficiency theorem of Diamond and Mirrlees. Numerous papers then follow which qualify this finding by seeking conditions under which education should be effectively subsidized or taxed, i.e. under which the quantity of education should be induced to move above or below the first-best amount. Richter (2009), Richter and Braun (2010), and Braun (2010) all find that education should be subsidized beyond the first-best if the human capital accumulation function has an increasing elasticity with respect to education, which Braun (2010) argues is likely. Finally, the way in which the returns to education interact with other characteristics and decisions is analyzed by Maldonado (2008) and Jacobs and Bovenberg...

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5 Similar findings for high school graduation are discussed in a New York Times editorial, Levin and Rouse (January 25, 2012), which points out that reducing undereducation in that context would pay for itself. Damon and Glewwe (2011) also undertake a related examination of the costs and benefits of state financing of public universities in Minnesota, finding that the fiscal benefits outweigh the costs; however, they neglect federal taxes and conclude that a large portion of the gains come from wage spillovers from education workers, which I will consider in section 7.2.

6 Therefore, from the government’s perspective, a university degree is a money-generating machine, as each dollar spent at present is more than recovered later. However, this does not mean that a tuition subsidy is necessarily a money-generating machine in the sense that an increase in subsidies would not require a tax increase and would therefore be “free” in fiscal terms. Increasing student grants does encourage more students to attend university, but it also requires increased payments to inframarginal students, so what matters is how sensitive enrollment is to tuition. In my baseline numerical case, I do find that a marginal increase in tuition subsidies from the current level is “free,” but as grants increase, this quickly ceases to be true. This means that taxes eventually have to increase, and since I assume a simple proportional tax, this will lead to negative redistributional consequences, making it even more striking that a utilitarian social planner would find significant welfare gains from raising grants.
(2011). Both find that education should be effectively taxed if it is complementary with ability, while the latter paper also shows that education should be subsidized relative to first-best if it is complementary with labour effort, leaving as an empirical matter which of these effects dominates.

These theoretical papers, however, do not attempt any numerical evaluation. Thus, Trostel (1996) remains one of the few attempts to derive numerical results for optimal subsidies to post-secondary education, along with Bohacek and Kapicka (2008), who find relatively small optimal subsidies of less than 20% for nearly all individuals.\(^7\)

Given the sparse nature of the literature in this area, there is a need for a renewed effort at calculating the practical policy implications of fiscal externalities in PSE. Additionally, the description above highlights that the literature is characterized by considerable variety in assumptions, modelling strategies, and findings, and so the sufficient statistics approach holds promise in providing a framework for analysis and numerical results which are robust to much of this variety.

### 2.2 Liquidity Constraints

The possibility that borrowing constraints may prevent some young people from making efficient choices about their education is one of the most frequently mentioned justifications for government financial aid to students, especially students from low-income families,\(^8\) and there is a considerable literature studying the presence and magnitude of these constraints. Often, this analysis takes the form of looking for causal effects of family income on PSE enrollment, on the grounds that students from higher-income families will be less likely to face constraints. Some studies have also looked for effects of income on the frequency of delayed or part-time enrollment, as well as the relative sensitivity of enrollment to tuition versus expected returns to education.\(^9\)

\(^7\)The interpretation of this result is complicated by the fact that there are no direct costs of schooling in their model, only the indirect cost of foregone earnings. Bohacek and Kapicka (2008) find that education subsidies have small welfare effects if taxes are set efficiently, but large effects with an exogenous inefficient tax. Given that my model assumes a potentially inefficient proportional tax, the same could be true here, although a progressive income tax would be likely to strengthen my results. I do not analyze the optimal tax system along with education subsidies, as that would be complicated within the sufficient statistics approach.

\(^8\)See, for example, Kane (1999).

\(^9\)Delayed and part-time enrollment are inefficient in simple models of education choice, so although imperfect information about earning capabilities could be responsible, they are sometimes taken as evidence of constraints on education choices; see, for example, Ellwood and Kane (2000) and Kane (1996). Greater sensitivity of enrollment to tuition than to measures of future (or expected) returns to education is also
However, the existence of liquidity constraints among students remains the subject of a persistent empirical controversy, as a number of papers have argued that constraints among PSE students are small or negligible. Cameron and Taber (2004) and Shea (2000), for example, find little or no effect of income on enrollment, and a series of papers by James Heckman and various co-authors, as summarized in Cunha, Heckman, Lochner, and Masterov (2006), find that the income-enrollment relationship in the 1979 NLSY is close to zero after controlling for various measures of skill and family background. They argue that this means that the income gap in enrollment is primarily caused by long-run family factors and not short-run liquidity constraints. Kane (2006), however, points out that this evidence cannot rule out liquidity constraints, because current income is not a perfect proxy for the availability of financial resources for education, and measures of family background may also serve as (imperfect) proxies for family resources. Therefore, a distinction between “long-run” and “short-run” factors in the data may not be particularly meaningful.

Other papers do claim to find positive effects of income on enrollment, including Acemoglu and Pischke (2001) and Coelli (2011). Belley and Lochner (2007) repeat the analysis of Heckman and co-authors on data from the 1997 NLSY, and show that income has become a much more important determinant of enrollment since NLSY79.

Furthermore, any empirical analysis is complicated by the fact that an absence of income effects on enrollment is not a sufficient condition for a lack of liquidity constraints, as it is possible for students’ consumption or other choices (such as working while in school) to be constrained even if the enrollment decision is not significantly affected, as pointed out by Belley and Lochner (2007) and Lochner and Monge-Naranjo (2008). Additionally, later in the paper I will provide further evidence against the idea that liquidity constraints imply positive effects of income on enrollment, by demonstrating that, in a simple model, the causal effect of income on enrollment should be negative in the absence of constraints.

My interpretation of the literature is that, although there is suggestive evidence on both

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10 Shea (2000) does find that income has a significant effect for children from the poverty subsample of the PSID and for children whose fathers have less than 12 years of schooling.
11 In fact, Coelli (2011) finds that parental job loss leads to a significant reduction in enrollment, and cannot conclusively disentangle income loss from other consequences of job loss.
12 Policy simulations in Keane and Wolpin (2001) predict exactly this type of response.
13 Also, Brown, Scholz, and Seshadri (2009) argue that by modelling the financial relationship between child and parents, it can be shown that constrained students need not necessarily be poor, and they provide suggestive evidence along these lines.
sides, it is difficult to find a definitive test for liquidity constraints among PSE students, as argued by Kane (2006). Therefore, the empirical controversy is likely to persist. I will therefore consider a range of possibilities in my numerical analysis, and my results may shed some light on the relevance of this issue for optimal financial aid policy.

I should note that, for simplicity, my analysis only considers optimizing a program of grants to students; I abstract away from guaranteed loans and changes in loan policy. This is not as restrictive as it may seem, given that in my analysis I will allow borrowing limits to be determined by data on the magnitude of liquidity constraints. Therefore, if guaranteed loans are effective in reducing constraints, then the borrowing limits I calculate should be quite loose, and I do consider a case in which there are no liquidity constraints, in which case further loans will have no effect. Even in the cases in which I assume significant borrowing constraints, one justification for holding loan policy fixed is that several papers, notably Keane and Wolpin (2001) and Johnson (2012), argue that raising borrowing limits in programs like GSL will have very little effect on enrollment or graduation, because of a precautionary savings motive, though debt-aversion could have a similar effect. These papers find that students do not want to build up excessive amounts of debt early in life for fear of a negative income shock shortly after entering the labour market. Therefore, if liquidity constraints do exist for such a large and lumpy investment even in the presence of guaranteed loans, I assume that this market failure can only be addressed through grants to students.

2.3 My Analysis of Grants to Students

My discussion of the literatures on fiscal externalities and liquidity constraints in PSE has shown that, although a great deal of effort has been put into studying the existence and theoretical properties of these phenomena, very little work has been done on their practical policy implications. Therefore, the main contribution of this paper will be to demonstrate the effects of fiscal externalities and liquidity constraints on optimal tuition subsidy policy.

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14 The baseline version of my model does not include any uncertainty and therefore cannot be used to model such a precautionary savings motive, though an extension in appendix C.3 does consider uncertainty about post-first-period earnings.

15 Upon entering the job market, the individual would no longer qualify for new low-interest loans, making a negative shock especially costly. Rothstein and Rouse (2011) show that student loans can affect early career choices in a way that is consistent with credit constraints.

16 Johnson (2012) specifically argues that tuition subsidies will be much more effective than loans in raising university completion.
To do so, I will use a simple and intuitive model of PSE that allows me to highlight the essential tradeoffs of financial aid policy. My analysis represents the first application of the “sufficient statistics” method proposed by Chetty (2009) to the context of post-secondary education; I use this method to derive an expression for the derivative of social welfare with respect to student grants. The benefits of this approach are described in Chetty (2009): I obtain simple and credible identification, with the ability to make welfare predictions. However, there are also costs: the welfare derivative, evaluated using current estimates of a set of sufficient statistics, is only valid locally. In order to make out-of-sample predictions and solve for the optimal policy, some variety of extrapolation is required, and Chetty (2009) proposes two options: statistical extrapolation of the sufficient statistics, and using the statistics to calibrate and simulate a structural model. I will implement both approaches, allowing me to demonstrate the robustness of my results to alternative assumptions.

The model and analysis is deliberately kept simple; rather than calibrating or estimating a completely realistic structural model with multiple dimensions of heterogeneity, I focus on a basic model as a first-order approximation to reality, emphasizing a few key features. However, I do not regard my analysis as a substitute to structural estimation, but rather as a complement, and I intend to undertake future research using discrete-choice estimation of a model with observed and unobserved heterogeneity using data from the NLSY, allowing me to answer additional policy questions.17 In the current paper, my goal is to act upon the request in Lipsey (2007) for Second-Best policy analysis that aims for “piecemeal improvements in context-specific situations,” by providing clear numerical results from an intuitive model of PSE.

In discussing fiscal externalities and liquidity constraints, I abstract away from non-monetary motivations for government support of students. My intention is to be both simple and conservative, and so I ignore other potential positive externalities from PSE, such as social benefits from better-educated citizens, as mentioned by Kane (1999) and McPherson and Schapiro (2006), and discussed in detail by Lochner (2011).

17As noted in the introduction, one important question is whether tuition subsidies can be targetted specifically at marginal individuals, who are highly responsive to student grants. A structural approach that allows for multiple dimensions of heterogeneity would allow me to consider this and perhaps seek Pareto-improving reforms to financial aid policy. In the current paper, I limit myself to considering universal grants paid for by a simple proportional tax, which generally imposes negative redistributinal effects, as everyone pays more in taxes, including high-school graduates, while the higher-income university graduates are more than compensated with increased grants, making it even more remarkable that large welfare gains can be generated by PSE grants.
3 A Simple Model of University Education

In this section, I will present my model of post-secondary education, followed by the calculations leading to an expression for the derivative of social welfare with respect to student grants.

3.1 Model Setup

Time is finite, and consists of twelve periods, each corresponding to 4 years, representing a normal working life of 48 years (say, from age 18 to 65 inclusive).\(^{18}\) In the first period, each individual has a choice of attending university or working at wage \(Y_{01}\), and this choice is represented by \(s = \{0, 1\}\), where 1 indicates attendance. In periods \(t = 2, \ldots, 12\), the individual works at a wage \(Y_{st}\) that depends on the education choice in the first period, where \(Y_{1t} > Y_{0t}\).\(^{19}\) I therefore model only the attainment of a 4-year university degree, and I assume a single jurisdiction, with a single government planner, abstracting from issues of coordination of policy in a federation and from mobility of students across countries.\(^{20}\)

I assume that the real interest and discount rates are both equal to 3% per year, and since a period is equal to 4 years, I will use \(r = 0.12\) for the interest rate and \(\beta = \frac{1}{1.12} \approx 0.893\) for the discount rate. I also allow for real wage growth, calculated from the average net compensation series used by the Social Security Administration for the computation of the national average wage index, deflated using the CPI; the average real growth rate over 1991-2008 is almost exactly 1%, so I allow wages to grow at \(g = 0.04\) per period.

The individual’s utility from consumption \(c\) while in university is \(u(c)\), whereas it is \(v(c)\) while employed, allowing for direct utility or disutility from university attendance as well as different utility from consumption in the two states. Both utility functions obey the usual properties of \(u', v' > 0\) and \(u'', v'' < 0\), and I denote individuals by \(i\). If an individual chooses not to attend university, then since the interest and discount rates are equal, they will simply set consumption to a constant value \(c_{0i}^{0}\) in each period, and receive lifetime utility of \(U_{0i} = \sum_{t=1}^{12} \beta^{t-1} v(c_{0i}^{0})\). If they do attend university, they will set per-period post-schooling

\(^{18}\)In principle, this could be simplified to a 2-period model, but the periods would be of drastically different lengths, making discounting and the comparison of incomes and utility in the two periods more complicated; the analytical simplicity of a 2-period approach is offset by greater ease of interpretation with 12 periods.

\(^{19}\)These wages are assumed to be exogenous, and there is no uncertainty. I allow for uncertainty in future incomes in an extension in appendix C.3.

\(^{20}\)These issues are studied by Del Rey (2001) and Poutvaara (2008), among others.
consumption \( c^1_{vi} \) to some constant value, and choose some value \( c_{ui} \) of consumption while in school, receiving lifetime utility of \( U_{1i} + \epsilon_i \), where \( U_{1i} = u(c_{ui}) + \sum_{t=2}^{12} \beta^{t-1}v(c^1_{vt}) \) and where \( \epsilon_i \) captures any idiosyncratic portion of the utility or disutility from schooling.

The individual’s budget constraints (one for each value of \( s \)) can be written in the following way:

\[
\sum_{t=1}^{12} \left( \frac{1}{1+r} \right)^{t-1} c^0_{vi} = (1-\tau) \sum_{t=1}^{12} \left( \frac{1}{1+r} \right)^{t-1} Y_{0t} \\
c_{ui} + \sum_{t=2}^{12} \left( \frac{1}{1+r} \right)^{t-1} c^1_{vi} = (b-e) + (1-\tau) \sum_{t=2}^{12} \left( \frac{1}{1+r} \right)^{t-1} Y_{1t}
\]

where \( e \) is the direct cost of university to the individual,\(^{21} \tau \) is the marginal tax rate,\(^{22} \) and \( b \) is the government grant given to students. For simplicity, I restrict attention to a lump-sum grant for now, though I will consider a 2-tier grant scheme in section 6.1.\(^{23} \) To simplify the notation, let me define \( R_x = \sum_{t=x}^{12} \left( \frac{1}{1+r} \right)^{t-1} \) and \( \gamma_x = \sum_{t=x}^{12} \left( \frac{1}{1+r} \right)^{t-1} \); then the budget constraints can be written as:

\[
R_1 c^0_{vi} = (1-\tau)\gamma_1 Y_{01} \\
c_{ui} + R_2 c^1_{vi} = (b-e) + (1-\tau)\gamma_2 Y_{11}.
\]

I also allow students to face a liquidity constraint, which will take the form of a limit \( A_i \) to the debt that a student may accumulate:\(^{24} \)

\[
c_{ui} - (b-e) \leq A_i.
\]

Therefore, the individual’s maximization problem is to choose \( \{s_i, c^0_{vi}, c^1_{vi}, c_{ui}\} \) to maximize \( V_i = s_i (U_{1i} + \epsilon_i) + (1-s_i)U_{0i} \):

\[
V_i = s_i [u(c_{ui}) + R_2 v(c^1_{vi}) + \epsilon_i - \lambda_{1i}(c_{ui} + R_2 c^1_{vi} - (b-e) - (1-\tau)\gamma_2 Y_{11}) - \mu_i(c_{ui} - (b-e) - A_i)]
\]

\(^{21}\)This cost is assumed to be exogenous and constant, which implies that there is no institutional response to \( b \). Long (2004) provides some evidence that private universities do respond at least partially to increased government aid, including reducing their own institutional aid. The analysis here more closely corresponds to a mandated reduction of tuition at public colleges and universities, offset by increased appropriations.

\(^{22}\)In this simplified example, with income dependent only on education, a non-linear income tax could perfectly replicate my combination of a linear tax and education subsidy. However, with a more complex income process, this would no longer hold. I keep the model simple in assuming linearity both of the income tax and the education subsidy, though similar general results would follow with a non-linear tax.

\(^{23}\)Partly due to the rising importance of merit aid and tax credits, government financial aid is not universally directed at low-income families; McPherson and Schapiro (2006) state that governments provide “rather little” in the form of grants to low-income students. Work such as Courant, McPherson, and Resch (2006) emphasize that students seem to be much more responsive to simple, broad-based financial aid policies than to more complicated aid programs requiring action such as special applications.

\(^{24}\)Given the lack of uncertainty when employed, this constraint will never bind on an individual after completing university, nor on an individual who chooses not to attend university.
\[+(1 - s_i)[R_1 v(c^0_{vi}) - \lambda_0 i(R_1 c^0_{vi} - (1 - \tau)\gamma_1 Y_01)].\]

The government chooses \(b\) and \(\tau\) subject to a budget constraint:

\[Sb + G = \tau[S\gamma_2 Y_{11} + (1 - S)\gamma_1 Y_{01}] = \tau\bar{Y}\]

where \(S = E(s_i)\) is the expectation of \(s_i\) across the population, or the fraction of the population attending university, \(G\) is the discounted total of other (exogenous) government spending over the 12 periods,\(^{25}\) and \(\bar{Y}\) is mean total discounted lifetime income. If \(V_i\) is total lifetime utility of individual \(i\), and social welfare \(V\) is utilitarian with equal weights, then \(V = E(V_i)\) and the social welfare gain from increasing \(b\) is:

\[dV/db = E\left(\frac{\partial V_i}{\partial b}\right) + E\left(\frac{\partial V_i}{\partial \tau}\right) d\tau/db.\] (1)

### 3.2 Welfare Calculations

I will now solve the model for an empirically-implementable version of (1). First, I evaluate the terms in (1), making use of the (unwritten) first-order conditions of the individual’s maximization problem:

\[
\frac{\partial V_i}{\partial b} = s_i(\lambda_{1i} + \mu_i) = s_i u'(c_{ui})
\]

\[
\frac{\partial V_i}{\partial \tau} = -s_i\gamma_1 Y_{01} + (1 - s_i)\lambda_0 i\gamma_1 Y_{01} = -s_i\gamma_2 Y_{11} v'(c^1_{vi}) - (1 - s_i)\gamma_1 Y_{01} v'(c^0_{vi})
\]

\[d\tau/db = S \bar{Y} \left[1 + \varepsilon_{Sb} - \left(1 + \frac{G}{Sb}\right) \varepsilon_{Yb}\right].\] (2)

where \(\varepsilon_{ab}\) represents the (total derivative) elasticity of \(a\) with respect to \(b\). Defining \(E_0[\cdot]\) and \(E_1[\cdot]\) as expectations over individuals for which \(s_i = 0\) and \(s_i = 1\) respectively, the welfare derivative is:

\[dV/db = SE_1[u'(c_{ui})] - [S\gamma_2 Y_{11} E_1[v'(c^1_{vi})] + (1 - S)\gamma_1 Y_{01} E_0[v'(c^0_{vi})]] S \bar{Y} \left[1 + \varepsilon_{Sb} - \left(1 + \frac{G}{Sb}\right) \varepsilon_{Yb}\right].\]

Notice that \(\varepsilon_i\) only affects the choice of \(s_i\), and that the debt limit has no effect on the consumption of those who do not attend university; therefore, \(c^0_{vi} = c^0_v\) is constant across individuals and \(E_0[v'(c^0_{vi})] = v'(c^0_v)\). Next, notice that for some quantity \(c^*\), I can write

\(^{25}\)Because \(G\) is exogenous, I do not need to account for it in individual welfare.
\[ S\gamma_2 Y_{11} E_1 [v'(c_1^{v_i})] + (1 - S)\gamma_1 Y_{01} v'(c_0^v) = \bar{Y} v'(c^*), \]
where I expect \( c_1^{v_i} > c^* > c_0^v \) and therefore \( v'(c^*) < v'(c_0^v) \). The expression for \( \frac{dV}{db} \) thus becomes:

\[
\frac{dV}{db} = SE_1 \left[ \frac{u'(c_{ui})}{v'(c_0^v)} - S v'(c^*) \left[ 1 + \varepsilon_{Sb} - \left( 1 + \frac{G}{Sb} \right) \varepsilon_{Yb} \right] \right].
\]

Finally, to normalize the welfare gain into a dollar amount, define \( \frac{dW}{db} \equiv \frac{dV}{v'(c_0^v)} \); this expresses the welfare gain in terms of an equivalent amount of additional consumption among non-graduates.\(^{26}\) Therefore:

\[
\frac{dW}{db} = S E_1 \left[ \frac{u'(c_{ui})}{v'(c_0^v)} - S v'(c^*) \left[ 1 + \varepsilon_{Sb} - \left( 1 + \frac{G}{Sb} \right) \varepsilon_{Yb} \right] \right] 
\simeq S \left[ \frac{E_1 [u'(c_{ui})] - v'(c_0^v)}{v'(c_0^v)} \left[ 1 + \varepsilon_{Sb} - \left( 1 + \frac{G}{Sb} \right) \varepsilon_{Yb} \right] \right].
\]

By making the assumption that \( v'(c^*) \simeq v'(c_0^v) \) above, I am overstating the relative importance of the fiscal effect; given that the optimum will be occur where \( \frac{dr}{db} \) is positive, this will tend to lead to an underestimate of the optimal \( b \).

\(^{(3)}\) provides a simple and intuitive illustration of the welfare consequences of tuition subsidies, or indeed of any government income transfer program: the ratio of marginal utilities measures the welfare gain or loss from taking a dollar from one group and giving it to another, while the subsequent terms express the additional revenue cost or saving to the government from behavioural responses, i.e. it shows how leaky the bucket is. A formula of this sort can generally be adapted to consider any transfer program, where the welfare gain from redistribution is measured against the resource cost of distortions. In my case, there may be a welfare gain from redistributing towards students if they are borrowing constrained, or there may well be a cost, as I am considering a redistribution away from lower-income uneducated individuals towards higher-income graduates; however, the revenue effect of tuition subsidies is likely to be positive, at least at baseline, so that there is a fiscal benefit of grants to students. Therefore, even if tuition subsidies redistribute in the “wrong” direction, they may increase total welfare because the gains to graduates can be considerably larger than the losses to the uneducated.

By design, this model has been conspicuous in its simplicity, but the result is very general; a far more general analysis in Lawson (2012) produces an expression in exactly this

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\(^{26}\)Since presumably \( v'(c_0^v) > v'(c_{vi}^v) \) for all \( i \), this is less than the dollar amount I would get if I normalized by the mean marginal utility while employed.
form. What the simplicity buys me is, on the one hand, ease of interpretation, and on the other, a starting point for the next step in my analysis: the replacement of the ratio of marginal utilities with some empirically observable quantity. For sufficient statistics analysis of unemployment insurance, Chetty (2008) suggests a decomposition of the marginal utility term into “liquidity” and “moral hazard” effects, and I will follow the same approach here.\(^\text{27}\)

For simplicity, let me first assume that debt limits are the same for all individuals; the result is robust to a distribution of debt limits under certain assumptions, as I show in appendix A, but the intuition is clearer in the simplest case. Thus, since the only heterogeneity enters in the form of \(\epsilon_i\), consumption choices if schooling is undertaken are identical for all individuals, i.e. \(c_{ui} = c_u, c_{vi} = c_v^1\) for all \(i\). An individual chooses to attend university if \(U_1 + \epsilon_i \geq U_0\), or:

\[
\epsilon_i \geq R_1 v(c_v^0) - u(c_u) - R_2 v(c_v^1).
\]

I assume that \(\epsilon_i\) follows some continuously differentiable distribution \(F(\epsilon)\), with a density given by \(f(\epsilon)\). It follows that \(S = 1 - F[R_1 v(c_v^0) - u(c_u) - R_2 v(c_v^1)]\), and therefore:

\[
\frac{\partial S}{\partial b} = f(\epsilon^*) u'(c_u)
\]

\[
\frac{\partial S}{\partial a_1} = f(\epsilon^*) [u'(c_u) - v'(c_v^0)]
\]

\[
\frac{\partial S}{\partial w_1} = -f(\epsilon^*) v'(c_v^0)
\]

where \(\epsilon^*\) is the critical value, and where \(a_1\) and \(w_1\) are artificial concepts representing dollars of additional income in the first period, with \(a_1\) being an unconditional lump-sum of cash and \(w_1\) an amount of additional after-tax employment income. It follows that I can rewrite (3) in the following way:

\[
\frac{dW}{db} \approx S \left[ L - \epsilon_s Sb + \left( 1 + \frac{G}{Sb} \right) \epsilon \bar{y}_b \right]
\]

\[(4)\]

where \(L = \frac{\partial S}{\partial a_1} - \frac{\partial S}{\partial b} - \frac{\partial S}{\partial w_1} \).

The \(\frac{\partial S}{\partial a_1}\) in the numerator of \(L\) corresponds to the “liquidity” effect in Chetty (2008), whereas the \(\frac{\partial S}{\partial b} - \frac{\partial S}{\partial a_1} = -\frac{\partial S}{\partial w_1}\) in the denominator is the equivalent of Chetty’s “moral hazard”

\(^{27}\)Chetty (2006) instead proposes using a ratio of values of consumption, but a lack of suitable data on the consumption of students and its relation to government grants, as well as the implausibility of treating consumption while in school as directly comparable to that while employed, makes such an approach unsuitable here.
effect. In the current context, the latter can more appropriately be called the “substitution effect,” as it represents the effect on enrollment of changing relative prices without providing immediate cash for students. $L$ therefore represents the ratio of the liquidity and substitution effects, and a higher value indicates a relatively larger liquidity effect of student grants. (4) is the equation I will use in my sufficient statistics analysis in the next section, as it allows me to estimate the welfare gain of a marginal change in $b$, given values of the quantities which appear in the equation.

However, before beginning the numerical analysis, I conclude this section on a theoretical point. Notice what must be true in my model if $\frac{\partial s}{\partial a_1} = 0$, i.e. if there is no causal effect of income on enrollment: this implies that $u'(c_u) = v'(c_0)$, and I expect that $v'(c_0) > v'(c_1)$, so therefore $u'(c_u) > v'(c_1)$. However, the absence of liquidity constraints requires $u'(c_u) = v'(c_1)$; therefore, a precisely-estimated zero effect of income on enrollment is in fact evidence in favour of liquidity constraints. If individuals were unconstrained, income should have a negative causal effect on enrollment, because a dollar of income would be more valuable to those who do not attend PSE, and as an illustration, when I study a case without liquidity constraints in section 5.1, I find that the model implies that each $1000 of initial assets should reduce enrollment by between 0.13 and 0.27 percentage points. This conclusion is, of course, dependent on the assumptions underlying the model; for instance, if the utility function for employed individuals was different across educated and uneducated individuals, it would not necessarily be true that $v'(c_0) > v'(c_1)$. All the same, the results here strongly suggest that previous findings indicating that income has no causal effect on PSE enrollment should not necessarily be taken as evidence against liquidity constraints.

4 Numerical Results

In this section, I will focus on providing numerical results, starting with equation (4). First of all, using estimates of the current values of each of the “sufficient statistics” in (4), I calculate an estimated value of $\frac{dW}{db}$ and thereby determine if financial aid ought to be increased or decreased. To go beyond this local derivative, I must make additional assumptions, and I follow the advice of Chetty (2009) in trying two different approaches, which take up the subsequent two subsections: I perform statistical extrapolations of the quantities in (4), predicting their values out of sample, and I use the sufficient statistics to calibrate my
model, permitting me to simulate the model to find the optimum.

## 4.1 Sufficient Statistics Method

To evaluate (4), I must specify values for a number of quantities. To begin with, I use \( S = 0.388 \), which is the estimate of the enrollment rate of 18-24-year-olds in 2007 from NCES (2011).\(^{28}\) I also specify \( b = 2 \) (defining monetary amounts as thousands of dollars per year), using data on federal aid and state grants in 2007-08 from NCES (2011) and applying the formula of Epple, Romano, and Sieg (2006) for turning loans and work-study into grant equivalents.\(^{29}\)

In choosing \( b = 2 \), I am assuming that the out-of-pocket tuition cost \( e \) prior to government aid covers the marginal cost of PSE. At public institutions, this may not be accurate if universities’ need for public funding increases with enrollment,\(^{30}\) but increased education will also lead to reductions in spending on social insurance and assistance programs, as well as lower corrections spending if PSE makes individuals “better citizens.” In fact, Trostel (2010) finds that additional expenditures on appropriations per degree are roughly offset by reductions in other government spending,\(^{31}\) supporting my assumption that the marginal cost of PSE is captured by \( e \); in that case, increased education appropriations and reduced social spending cancel out of the government budget constraint and I can ignore both to keep my analysis simple. My conclusions, however, are not sensitive to this assumption; in appendix C.2, I redo all the calculations using the most pessimistic of Trostel’s estimates, and the results are only slightly changed.

Deming and Dynarski (2009) summarize the literature on the price response of PSE

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\(^{28}\)I intend this value as a compromise. Given that the population of 18-24-year-olds includes individuals who have already completed or dropped out of PSE, this is an underestimate of the proportion of individuals ever enrolled in PSE (Lovenheim (2011) finds that 52% of his sample has completed more than 12 years of schooling). However, it is also an overestimate of the proportion actually completing a degree, which was 28.7% in 2007 according to NCES (2011).

\(^{29}\)In 2007-08, 27.6% of undergraduates received federal grants averaging $2800, 34.7% received federal loans averaging $5100, 5.6% received federal work-study averaging $2300, and 16.4% received state grants averaging $2500; Epple, Romano, and Sieg (2006) suggest using a formula of \( \text{aid} = \text{grants} + 0.25 \times \text{loans} + 0.5 \times \text{workstudy} \), which gives a per-person average of $1690. Lacking data on other government aid and tax credits, I round this total up to $2000.

\(^{30}\)The finding in Trostel (2010) that the cost to government per degree is $71400 in present value does not prove this, as this is an average cost, not a marginal cost. Additionally, Trostel emphasizes that this is likely to be an overestimate.

\(^{31}\)Trostel (2010) conservatively estimates the reduction in expenditures on such things as Medicaid, UI, and corrections per degree as $55800 in present value. Both this amount and the spending per degree, however, are dwarfed by tax revenue gains amounting to $197200 in present value.
attendance, and conclude that the general consensus is that a $1000 increase in price leads
to a 4 percentage point decline in attendance, which implies an elasticity of $\varepsilon_{Sb} \simeq 0.2$.
Most estimates of the effect of price on both enrollment and degree completion are of this
magnitude, so I will treat it as my baseline case. However, Dynarski (2008) estimates that
$2500 of financial aid leads to a 4 percentage point increase in degree completion from a base
of 27%, which suggests a value closer to 0.1, so I will present results for this case as well.

As discussed earlier, numerous papers argue that income has no causal effect on enroll-
ment, i.e. $\partial S / \partial a_1 = 0 = L$. However, several papers do claim to find a positive income effect, the
largest of which is Coelli (2011), whose results imply an effect on enrollment 25% as large as
that of $b$, suggesting $L = \frac{1}{5}$; I will therefore present results for both values, though I will
consider $L = 0$ as the preferred estimate.

To calculate a value for $\varepsilon_{Yb}$, I assume that each year of schooling increases earnings by a
constant 8%, and that the elasticity of taxable income is 0.4, as found by Gruber and Saez
(2002). Utilizing these estimates, appendix B demonstrates that I can write the elasticity
as $\varepsilon_{Yb} = \left[ \frac{\gamma_2(1.08)^4 - \gamma_1}{\gamma_2(1.08)^4 S + \gamma_1(1-S)} \right] \frac{1-r}{1-1.4r} \varepsilon_{Sb} - \frac{0.4r}{1-1.4r} \left( 1 + \frac{G}{Sb} \right)^{-1}.$

Finally, for $\frac{G}{Sb}$, I begin with my estimates of $b = 2$ and $S = 0.388$, so $Sb = 0.776$; I
then need to estimate $\tau Y$ in order to be able to compute $G$. I use a value of $\tau = 0.23$,
which incorporates a 15% federal tax rate, a 5% state tax, and 3% for the Medicare tax. For $\bar{Y}$, I turn to the CPS 2008 Annual Social and Economic Supplement, which estimates
the mean earnings of a high school graduate in 2007 to be $33609, which I round up to
$Y_{01} = 34$, meaning that $Y_{11} = 34(1.08)^4 = 46.26$. With $r = 0.12$ and $g = 0.04$, $\gamma_1 = 8.247$
and $\gamma_2 = 7.247$, so $\bar{Y} = 301.661$, and therefore $G = 68.606$ and $\frac{G}{Sb} = 88.410.$

Plugging in each of these values, I find values for $\frac{dW}{db}$ as displayed in Panel A of Table
1. They are substantial, suggesting that a one dollar per year increase in student grants

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32 This conclusion requires an assumption that the impact of parental job loss on enrollment found in Coelli
(2011) comes entirely through the channel of lost income, making it a reasonable choice for an upper bound.
33 The range of estimates in Card (1999) are described in Sianesi and Van Reenen (2003) as being from
6% to 11%, but more recent estimates are higher: Dynarski (2008) summarizes several higher estimates of
estimate “policy relevant treatment effects” of tuition subsidies that range from 9% to 25%, making 8% a
conservative estimate.
34 I adjust equilibrium earnings for changes in taxes according to this elasticity, but I do not attempt to
model the labour supply decision; in this way, I will tend to produce an underestimate of the optimal $b$, since
I overstate the cost of tax increases by ignoring increases in leisure.
35 The federal and state rates are chosen to be appropriate for the typical high school graduate; they will
be conservative for many graduates of university. In my analysis of heterogeneous returns to education in
subsection 6.2, I attempt to model the tax system in more detail.
from the baseline of $2000 would provide a welfare increase equivalent to between 18 and 54 cents per year for 4 years. A 1% increase in \( b \) to $2020, therefore, would provide an average individual with an annual welfare gain of between $0.52 and $1.57 over their lifetime, or an economy-wide gain of $101 to $305 million,\(^{36}\) a very large effect for such a small change in policy.

Table 1: Results from Sufficient Statistics and Extrapolation using (4)

<table>
<thead>
<tr>
<th>Value of ( \varepsilon_{Sb} )</th>
<th>( L )</th>
<th>( \frac{1}{3} )</th>
</tr>
</thead>
<tbody>
<tr>
<td>A. Estimate of ( \frac{dW}{db} ) at ( b = 2 )</td>
<td></td>
<td></td>
</tr>
<tr>
<td>0.1</td>
<td>0.1811</td>
<td>0.3104</td>
</tr>
<tr>
<td>0.2</td>
<td>0.4148</td>
<td>0.5442</td>
</tr>
<tr>
<td>B. Optimal Student Grants</td>
<td></td>
<td></td>
</tr>
<tr>
<td>0.1</td>
<td>$5843</td>
<td>$13718</td>
</tr>
<tr>
<td>0.2</td>
<td>$8093</td>
<td>$9075</td>
</tr>
<tr>
<td>C. Welfare Gains from Moving to Optimum</td>
<td></td>
<td></td>
</tr>
<tr>
<td>0.1</td>
<td>$947 (30.5%)</td>
<td>$3836 (123.6%)</td>
</tr>
<tr>
<td>0.2</td>
<td>$3138 (101.1%)</td>
<td>$4935 (159.0%)</td>
</tr>
</tbody>
</table>

4.2 Statistical Extrapolation

To make predictions out of sample, and thereby to produce an estimate of the optimal level of student grants, I need to make some functional form assumptions. As already mentioned, one option is to calibrate a structural model, which involves assumptions about the form of primitives such as utility functions, and I will follow this approach in the next subsection. In the current subsection, I instead make functional form assumptions about the sufficient statistics themselves; this approach is proposed in Chetty (2009), and has previously been used in sufficient statistic studies of unemployment insurance, including Baily (1978), Gruber (1997) and Lawson (2013).

First, let me now denote the baseline values of quantities using hats, i.e. \( \hat{b} = 2 \) and \( \hat{S} = 0.388 \). I assume a constant value of \( \varepsilon_{Sb} \) at either 0.1 or 0.2,\(^{37}\) implying that \( S = \phi b^{\varepsilon_{Sb}} \), where \( \phi = \frac{\hat{S}}{b^{\varepsilon_{Sb}}} \). I can then construct \( \frac{G}{Sb} = 88.41 \frac{\hat{S}}{Sb} \), and \( \varepsilon_{\bar{Y}} = \left[ \frac{\gamma_2(1.08)^4 - \gamma_1}{\gamma_2(1.08)^4 S + \gamma_1(1-S)} - \frac{0.4\tau}{1-\tau} \right] \left( 1 + \frac{G}{Sb} \right)^{-1} \) as above, but holding \( \tau \) fixed at 0.23 for simplicity.

\(^{36}\)I use the Census Bureau’s April 2010 estimate of the 18-64 population as 194,296,087.

\(^{37}\)This is generally consistent with the finding in Nielsen, Sørensen, and Taber (2010) of a considerably lower response of enrollment to price in Denmark, where financial support to students is much more generous.
This leaves only the liquidity term to be constructed. Belley, Frenette, and Lochner (2011) propose a simple calculation of the amount of aid needed to eliminate the causal income gap, by using their estimate that $1000 in grants increases enrollment by 3 to 5 percentage points, and I will follow this general approach. Belley, Frenette, and Lochner (2011) find a 16 percentage point gap in attendance at 4-year PSE institutions between the highest and lowest family income quartiles, so if I assume that the top quartile is unconstrained, then raising the enrollment rate by 16% for the lowest quartile should cause the liquidity effect to go to zero.\footnote{Of course, if enrollment responds to price in the same way for each quartile, a lump-sum grant will not eliminate the income gap, but I will simply assume that, if the lowest quartile reaches the current enrollment rate of the highest quartile, this must mean the elimination of liquidity constraints.} I assume that once $L$ reaches zero, or if it is initially zero, then it remains zero as $b$ increases further. Therefore, if the initial $L$ is $\frac{1}{3}$, I assume $\frac{\partial S}{\partial a} = \frac{1}{4} \cdot \frac{0.16 - (S - \hat{S})}{0.16} \frac{\partial S}{\partial b}$, and thus $L = \max\{\frac{\frac{25}{100}(0.16 - (S - \hat{S}))}{1 - \frac{\frac{25}{100}(0.16 - (S - \hat{S}))}{1 - \frac{25}{100}(0.16 - (S - \hat{S}))}}, 0\}.$

Putting all of this together, I get the results displayed in Panels B and C of Table 1; to get estimates of the welfare gain from moving to the optimum, I numerically integrate $\frac{dW}{db}$ from $b = 2$ to the optimum. I then express the welfare gain in two ways: I multiply by 4 to get the dollar amount of an equivalent one-year per-person consumption increase, and I also divide by $\hat{S}\hat{b}$ to express the gain as a percentage of the initial size of the student grant program; these latter values are shown in brackets in Panel C.

NCES (2011) estimates that median tuition at public 4-year universities was $5689 in 2007-08, so my results would suggest that, in the worst-case scenario, net tuition should be eliminated, and government appropriations increased accordingly. With a larger responsiveness of enrollment to tuition or more serious borrowing constraints, the optimal policy would also include a yearly stipend which reaches as high as $8000;\footnote{The optimal value of $b$ is much more sensitive to the liquidity effect when $\varepsilon_{sb}$ is small because my assumptions about how $L$ changes with $b$ mean that it takes much longer for the liquidity effect to vanish in this case.} in the baseline case of $\varepsilon_{sb} = 0.2$ and $\hat{L} = 0$, the optimal stipend is about $2400 per year. Meanwhile, the estimated welfare gains are substantial, particularly in comparison to the size of the policy change; aggregating to an economy-wide level, they indicate annual welfare improvements of between $6.6$ billion and $34.6$ billion, or as much as 0.24% of GDP.
4.3 Simulation of Structural Model

The other alternative for out-of-sample prediction is to calibrate my simple structural model. I begin by assuming CRRA utility, so \( u(c) = \frac{c^{1-\theta}}{1-\theta} \) and \( v(c) = \frac{c^{1-\rho}}{1-\rho} \). I specify \( Y_{01} = 34y \) and \( Y_{11} = 34(1.08)^4y \), where \( y = \alpha(1-\tau)^{ETI} \) and \( \alpha = \frac{1}{(1-\tau)^{ETI}} \), so \( y = 1 \) at baseline and shifts with \( \tau \) to reflect effort responses to taxation. I also assume that \( \epsilon \) follows a logistic distribution with mean \( \mu \) and scale parameter \( \sigma \).

I use \( e = 5.7 \) to represent public tuition, and \( G = 68.606 \) as described earlier. I assume that all individuals face the same debt limit \( A \), so I have to solve for 5 parameters: \( \{A, \theta, \rho, \mu, \sigma\} \). However, I only have three sufficient statistic conditions: \( \hat{S} = 0.388 \), \( \varepsilon_{sb} = \{0.1, 0.2\} \) and \( \hat{L} = \{0, \frac{1}{3}\} \), so I need to incorporate additional data.

One piece of data I can use is some comparison of \( \hat{c}_u, \hat{c}_0 \) and \( \hat{c}_1 \); any ratio of two of these, along with the equation for the debt limit and the first-order conditions, will define all three. One possibility is to use consumption values from the Consumer Expenditure Survey, where I find that college graduates consumed 73.9% of their pre-tax income and high school graduates consumed 83.4% in 2007. The NBER’s TAXSIM calculator for 2007 allows me to transform these into percentages of post-tax income (ignoring state taxes and assuming a single-earner married couple), and if I then apply those values to my estimates of \( Y_{01} \) and \( Y_{11} \), I get \( \hat{c}_1 = 1.2758\hat{c}_0 \). An alternative is to use results in Keane and Wolpin (2001) implying student consumption (not including room and board) of $8077 in 1987, and average per-equivalent-person consumption of $15816 in 1988 as estimated by Cutler and Katz (1991). Adding the estimate from NCES (2011) of average room and board expenses in 1987-88 of $3037 to student consumption, and deflating everything to 1987 dollars, I find \( \hat{c}_u = 0.7318\bar{c} \), where \( \bar{c} \) is average consumption while employed, which implies \( \hat{c}_1 = 1.2749\hat{c}_0 \).

Given the similarities of these estimates, I will use \( \hat{c}_1 = 1.275\hat{c}_0 \).

Finally, I can use external estimates of relative risk-aversion parameters to pin down one of \( \theta \) and \( \rho \). Many such estimates exist, but a CRRA parameter of 1, implying log utility, is typical; Gourinchas and Parker (2002), in particular, estimate a relative risk-aversion parameter during one’s working life of around unity. Therefore, I assume \( v(c) = \ln(c) \), but I also try a value of \( \rho = 2 \) in appendix C.1.

\( ^{40} \)Unlike with a normal distribution, the logistic allows for calibration to be done analytically.

\( ^{41} \)\( \mu \) is not normalized to zero because \( u(c) \) and \( v(c) \) both have zero intercepts, so \( \mu \) represents the difference in intercepts, the mean direct utility or disutility from schooling.
My calibration method begins by using $\hat{c}_v^1 = 1.275 \hat{c}_v^0$ to solve for $A$; the condition $u'(c_u) = (\hat{L} + 1)u'(c_u^0)$ then allows me to solve for $\theta$, and I can use these results and the conditions that $\hat{S} = 0.388$ and $\varepsilon_{sb} = \{0.1, 0.2\}$ to find $\mu$ and $\sigma$. Simulation of the model for various values of $b$ then allows me to find the optimum, and the results are displayed in Table 2.

### Table 2: Results from Calibration and Simulation

<table>
<thead>
<tr>
<th>Value of $\varepsilon_{sb}$</th>
<th>$\hat{L}$</th>
<th>$\frac{1}{3}$</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>$\frac{1}{3}$</td>
<td>$\frac{1}{3}$</td>
</tr>
<tr>
<td>A. Numerical Estimate of $\frac{dv}{db}$ at $b = 2$</td>
<td></td>
<td></td>
</tr>
<tr>
<td>0.1</td>
<td>0.1259</td>
<td>0.2550</td>
</tr>
<tr>
<td>0.2</td>
<td>0.4207</td>
<td>0.5502</td>
</tr>
<tr>
<td>B. Optimal Student Grants</td>
<td></td>
<td></td>
</tr>
<tr>
<td>0.1</td>
<td>$3876$</td>
<td>$6065$</td>
</tr>
<tr>
<td>0.2</td>
<td>$5944$</td>
<td>$7615$</td>
</tr>
<tr>
<td>C. Welfare Gains from Moving to Optimum</td>
<td></td>
<td></td>
</tr>
<tr>
<td>0.1</td>
<td>$469$ (15.1%)</td>
<td>$2035$ (65.6%)</td>
</tr>
<tr>
<td>0.2</td>
<td>$3327$ (107.2%)</td>
<td>$6222$ (200.4%)</td>
</tr>
</tbody>
</table>

The results for the optimal level of $b$ are somewhat smaller than those in Table 1, typically by around $2000$, with a much larger drop in the case where $\varepsilon_{sb} = 0.1$ and $\hat{L} = \frac{1}{3}$, in which my statistical extrapolations assume that the liquidity effect goes away very slowly. However, the qualitative conclusions are similar, in that eliminating tuition remains the optimal policy in all but one case.\(^{42}\) The estimated welfare gains, meanwhile, are actually larger when $\varepsilon_{sb} = 0.2$, reaching $23.3$ billion in the baseline case; overall, they vary from a low of $3.3$ billion to a high of $43.6$ billion, or $0.3\%$ of GDP.

Figure 1 displays the values of $S$ over the relevant range, where it can be seen that in all cases, but especially those with $\varepsilon_{sb} = 0.2$, the optimal policy (indicated by the squares) involves inducing significant fractions of the population to attend PSE. Figure 2 displays the budget-balancing tax rates, and it is striking to see that, in the cases with $\varepsilon_{sb} = 0.2$, a small increase in $b$ from the current level leads to a lower tax rate, because average income increases enough that the increased grants more than pay for themselves. This quickly ceases to be true as grants increase further, but if this standard estimate of the responsiveness of enrollment to tuition is correct, then at present we are slightly on the wrong side of the “financial

\(^{42}\)This simple policy is at least partly endorsed by Courant, McPherson, and Resch (2006): they argue that the “old tradition of making public higher education ‘free’ has much to recommend it” for reasons of simplicity and salience, and claim that this policy might be efficient if enrollment is sufficiently sensitive to tuition, but they do not attempt to evaluate the welfare implications of the policy themselves.
aid Laffer curve,” and thus there are Pareto improvements available from a small increase in tuition subsidies: taxes do not have to rise until the grant level reaches about $2550. Beyond that, the tax rate does rise, which means that there is redistribution away from high-school graduates, which is socially costly, and yet the losses of high-school graduates are more than offset by the considerable gains of PSE graduates until $b$ is well over $5000$.

Figure 1: Values of $S$

To further test the robustness of my results, I attempt a number of extensions and alterations to the model in appendix C. A higher degree of risk-aversion slightly lowers optimal grants, while uncertainty about future income raises them; an alternative specification of government spending has varying effects depending on whether the statistical extrapolation or calibration method is used. The qualitative conclusions, however, are very similar across all extensions.
5 Separating the Effects of Liquidity Constraints and Fiscal Externalities

In this section, I perform the experiment of “switching off” the liquidity constraints and the fiscal externalities one at a time, with the goal of determining which of them contributes more to the argument for more generous financial aid.

5.1 No Liquidity Constraints

I begin by assuming away liquidity constraints, and I focus on the structural approach since I have already assumed that \( L = 0 \) in the sufficient statistics approach means no liquidity constraints.\(^{43}\)

I begin by using \( c_1^\hat{v} = 1.275c_0^\hat{v} \) to solve for values of consumption, and then I use \( v'(c_1^v) = u'(c_u) \), the true no-liquidity-constraint condition, to solve for \( \theta \). The rest of the calibration

\(^{43}\)As noted earlier, an absence of liquidity constraints actually requires \( v'(c_1^v) = u'(c_u) \), which means \( \frac{\partial s}{\partial a_1} < 0 \), but there is no easy way to estimate how negative \( L \) would be if there were no liquidity constraints aside from simulating a structural model; in doing so with the model in this section, I find that \( L \) takes on a value of -0.2157 in the absence of liquidity constraints.
procedure continues as before, and the results are displayed in Table 3. The results are similar to before; the values of \( \frac{dW}{db} \) are somewhat smaller, but in the baseline case the optimal benefit level is actually higher than in the \( \hat{L} = 0 \) case, and the welfare gain is nearly identical. It therefore appears that my general conclusion, at least at the preferred estimates, is not sensitive to the existence of liquidity constraints.

Table 3: Results from Calibration and Simulation with No Liquidity Constraints

<table>
<thead>
<tr>
<th>Value of ( \varepsilon_{Sb} )</th>
<th>A. Numerical Estimate of ( \frac{dW}{db} ) at ( b = 2 )</th>
<th>B. Optimal Student Grants</th>
<th>C. Welfare Gains from Moving to Optimum</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.1</td>
<td>0.0430</td>
<td>$3263</td>
<td>$110 (3.6%)</td>
</tr>
<tr>
<td>0.2</td>
<td>0.3374</td>
<td>$6630</td>
<td>$3314 (106.8%)</td>
</tr>
</tbody>
</table>

5.2 No Fiscal Externalities

Next, I instead shut off the fiscal externality, in the sense that I ignore \( G \) and assume that \( \tau_t \) is a lump-sum tax imposed on employed workers, growing at rate \( g \) per period, so \( \tau_t = (1 + g)^{t-1} \). Re-doing my initial analysis in this context is straightforward, and the resulting equation for the welfare gain from increasing \( b \) is:

\[
\frac{dW}{db} \simeq S \left( L - \frac{\gamma_1}{\gamma_1 - S} \varepsilon_{Sb} \right). \tag{5}
\]

Implementing this formula, I get the results in Table 4. The values of \( \frac{dW}{db} \) are much smaller, as are most of the optimal values of \( b \); if \( L = 0 \) then there is no reason whatsoever to subsidize education (I set \( b = 0 \) as a lower bound). In the case of \( \varepsilon_{Sb} = 0.1 \) and \( \hat{L} = \frac{1}{3} \), the estimated optimal \( b \) is very large, but this is an anomaly resulting from the assumption that the fiscal externality is negative but small whereas the liquidity effect is significant and takes a long time to completely dissipate.

The structural approach also follows in the usual way. The results can be found in Table 5, and present conclusions that are similar to those in Table 4, with slightly larger values.

\footnote{This result also follows directly from (4) if I assume \( G = 0 \) and if I take \( \varepsilon_{\gamma_b} \) to be the elasticity of the tax base \( \gamma_1 - S \).}
Table 4: Results from Sufficient Statistics and Extrapolation with no Fiscal Externalities

<table>
<thead>
<tr>
<th>Value of $\varepsilon_{Sb}$</th>
<th>$L = 0$</th>
<th>$L = \frac{1}{3}$</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>A. Estimate of $\frac{dW}{db}$ at $b = 2$</td>
<td></td>
</tr>
<tr>
<td>0.1</td>
<td>-0.0407</td>
<td>0.0886</td>
</tr>
<tr>
<td>0.2</td>
<td>-0.0814</td>
<td>0.0479</td>
</tr>
<tr>
<td></td>
<td>B. Optimal Student Grants</td>
<td></td>
</tr>
<tr>
<td>0.1</td>
<td>$0$</td>
<td>$19218$</td>
</tr>
<tr>
<td>0.2</td>
<td>$0$</td>
<td>$3600$</td>
</tr>
<tr>
<td></td>
<td>C. Welfare Gains from Moving to Optimum</td>
<td></td>
</tr>
<tr>
<td>0.1</td>
<td>$295$ (9.5%)</td>
<td>$1859$ (59.9%)</td>
</tr>
<tr>
<td>0.2</td>
<td>$539$ (17.4%)</td>
<td>$143$ (4.6%)</td>
</tr>
</tbody>
</table>

for $\frac{dW}{db}$ and optimal grants, with the exception of a much smaller optimal $b$ in the $\varepsilon_{Sb} = 0.1$, $\hat{L} = \frac{1}{3}$ case.

Table 5: Results from Calibration and Simulation with No Fiscal Externalities

<table>
<thead>
<tr>
<th>Value of $\varepsilon_{Sb}$</th>
<th>$L = 0$</th>
<th>$L = \frac{1}{3}$</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>A. Numerical Estimate of $\frac{dW}{db}$ at $b = 2$</td>
<td></td>
</tr>
<tr>
<td>0.1</td>
<td>-0.0080</td>
<td>0.1214</td>
</tr>
<tr>
<td>0.2</td>
<td>-0.0457</td>
<td>0.0837</td>
</tr>
<tr>
<td></td>
<td>B. Optimal Student Grants</td>
<td></td>
</tr>
<tr>
<td>0.1</td>
<td>$1769$</td>
<td>$6017$</td>
</tr>
<tr>
<td>0.2</td>
<td>$1087$</td>
<td>$3961$</td>
</tr>
<tr>
<td></td>
<td>C. Welfare Gains from Moving to Optimum</td>
<td></td>
</tr>
<tr>
<td>0.1</td>
<td>$4$ (0.1%)</td>
<td>$976$ (31.5%)</td>
</tr>
<tr>
<td>0.2</td>
<td>$82$ (2.6%)</td>
<td>$333$ (10.7%)</td>
</tr>
</tbody>
</table>

To summarize my results in this section, it appears that the liquidity term makes little difference to the optimal level of $b$; eliminating the possibility of liquidity constraints does not significantly reduce the optimal $b$ or change its qualitative implications. Fiscal externalities, on the other hand, seem to be very important to establishing beneficial effects of significantly increased grants to PSE students; severe liquidity constraints would otherwise be required in order to support significant grant increases, and in several hypothetical cases without fiscal externalities, the optimal policy would involve reducing or even abolishing tuition subsidies. The logic is that, while liquidity constraints may be a motivation for financial aid, fiscal externalities on their own can justify eliminating tuition and possibly providing stipends in most cases, by which point any liquidity constraints will have ceased to be a major concern.
Thus, liquidity constraints appear to be of second-order importance when designing optimal financial aid policy for post-secondary students.

6 Heterogeneity

In this section, I undertake a structural analysis of heterogeneity along two dimensions. First, I consider heterogeneity in liquidity constraints, allowing me to calculate the optimal 2-tier grant scheme and compare it to my main results. Then I allow for a distribution of wage premiums from post-secondary education to assess how this alters my conclusions.

6.1 Heterogeneity in Liquidity Constraints and Two-Tier Grants

In appendix A, I examine how robust the sufficient statistics condition in (4) is to a distribution of debt limits. However, I have not yet examined this possibility in the structural context, so I do that now. To be precise, I allow for two groups, each representing half of the population, one of which is unconstrained while the other faces a debt limit $A$. I calibrate the model for $\{A, \theta, \mu, \sigma\}$ using the conditions described earlier as averages, and then solve for the optimal level of a lump-sum student grant, with the results displayed in Table 6. The values of $\frac{dW}{db}$ are slightly smaller than in Table 2, which is to be expected because the logistic distribution for $\epsilon$ has an increasing hazard (see appendix A), but the rest of the results are generally close to those from the baseline calculations; in most cases, including the baseline case, the optimal level of $b$ is actually higher.

With this calibrated model in hand, I can go one step further and consider what policy the government would want to set if they could observe individuals’ debt limits; with two types of individuals, the government could introduce a two-tier grant system, with one grant amount $b_1$ for the constrained group and another amount $b_2$ for unconstrained students. It is straightforward to numerically maximize welfare (still measured as equally-weighted utilitarian social welfare) over the pair $(b_1, b_2)$, and the results for this exercise can be found in Table 7. Not surprisingly, it is always optimal to provide more generous aid to the constrained group, but substantial grants to the unconstrained group are still optimal with the standard estimate of $\varepsilon_{sb} = 0.2$, as the fiscal externality motive remains strong; in the

\[45\] Brown, Scholz, and Seshadri (2009) find that approximately half of the children in their sample did not receive post-schooling cash transfers from their parents, which they claim as an indicator for student liquidity constraints.
Table 6: Results from Calibration and Simulation with Heterogeneous Liquidity Constraints

<table>
<thead>
<tr>
<th>Value of $\varepsilon_{sb}$</th>
<th>0</th>
<th>$\frac{1}{3}$</th>
</tr>
</thead>
<tbody>
<tr>
<td>A. Numerical Estimate of $\frac{dX}{db}$ at $b = 2$</td>
<td></td>
<td></td>
</tr>
<tr>
<td>0.1</td>
<td>0.1236</td>
<td>0.2333</td>
</tr>
<tr>
<td>0.2</td>
<td>0.4159</td>
<td>0.5061</td>
</tr>
<tr>
<td>B. Optimal Student Grants</td>
<td></td>
<td></td>
</tr>
<tr>
<td>0.1</td>
<td>$4227$</td>
<td>$5968$</td>
</tr>
<tr>
<td>0.2</td>
<td>$6509$</td>
<td>$7714$</td>
</tr>
<tr>
<td>C. Welfare Gains from Moving to Optimum</td>
<td></td>
<td></td>
</tr>
<tr>
<td>0.1</td>
<td>$546$ (17.6%)</td>
<td>$1808$ (58.2%)</td>
</tr>
<tr>
<td>0.2</td>
<td>$3811$ (122.8%)</td>
<td>$5978$ (192.6%)</td>
</tr>
</tbody>
</table>

In the baseline case with $L = 0$, it remains optimal to abolish tuition, plus a stipend of just over $1000 for the constrained group. The resulting welfare gains are somewhat larger than those in Table 6, of course, but in most cases the difference is not enormous.

Table 7: Results from Calibration and Simulation with Two-Tier Grants

<table>
<thead>
<tr>
<th>Value of $\varepsilon_{sb}$</th>
<th>0</th>
<th>$\frac{1}{3}$</th>
</tr>
</thead>
<tbody>
<tr>
<td>A. Optimal Two-Tier Student Grants ($b_1/b_2$)</td>
<td></td>
<td></td>
</tr>
<tr>
<td>0.1</td>
<td>$5261$/$1566$</td>
<td>$8120$/$0$</td>
</tr>
<tr>
<td>0.2</td>
<td>$6925$/$5767$</td>
<td>$9194$/$4362$</td>
</tr>
<tr>
<td>B. Welfare Gains from Moving to Optimum</td>
<td></td>
<td></td>
</tr>
<tr>
<td>0.1</td>
<td>$844$ (27.2%)</td>
<td>$3366$ (108.4%)</td>
</tr>
<tr>
<td>0.2</td>
<td>$3870$ (124.7%)</td>
<td>$6887$ (221.9%)</td>
</tr>
</tbody>
</table>

6.2 Heterogeneous Returns to Education

In this subsection, I investigate how sensitive the results are to allowing for heterogeneous returns to education.

I assume that the PSE wage premium $R$ (where $Y_{11} = RY_{01}$) follows some distribution $G(R)$, and to be precise I use a quadratic approximation to the marginal treatment effect distribution presented in Figure 4 of Carneiro, Heckman, and Vytlacil (2011). I divide the population into 100 equal masses denoted by $j = \{1, 2, ..., 100\}$, with wage premia equal to $\{G^{-1}(0.005), G^{-1}(0.015), ..., G^{-1}(0.995)\}$, and then I allow for a distribution of $\epsilon$ for each mini-population, where $\epsilon$ is allowed to be correlated with $R$. In particular, I let $\epsilon_{ij} = \bar{\epsilon}_j + \epsilon_i$, where $\epsilon_i$ is a random variable with mean 0 and variance 1.
where $\bar{\epsilon}_j$ is deterministic for each $j$ and $\epsilon_i$ comes from a logistic distribution with mean 0 and scale parameter $\sigma$. I specify $\bar{\epsilon}_j = U_0 - U_{1j} + z - \mu_s \left( \frac{j-1}{j} \right)^{1.2}$, where $U_{1j} = u(c_{uj}) + 11v(c_{1j}^1)$, as this generates a pattern of responsiveness to $b$ which is roughly consistent with that found in Carneiro, Heckman, and Vytlacil (2011).

Allowing for a distribution of wage premia makes it important to model the tax system more realistically; I assume that the state and Medicare tax rates do not vary with income, but I use an approximation to the US federal system in 2008, with a 15% marginal rate up to $41500$ and a 25% rate beyond. To account for the fact the personal exemption of $3500$ and the standard deduction of $5450$, as well as the fact that the first $8025$ of taxable income is only taxed at a 10% rate, I provide a universal tax refund of $1743.75$. To avoid discontinuities in the marginal tax schedule, I use a smoothed approximation to the tax rate between $39000$ and $44000$, specifically a sine connecting $\tau = 0.23$ at $39000$ to $\tau = 0.33$ at $44000$. I assume that the tax rate threshold moves up with wage growth, and that when taxes need to adjust to balance the budget, the base tax rate of 23% is the one that moves.

When calibrating, I select values for $\{A, \theta, \mu_s, \sigma, z\}$ in order to match five quantities, three of which are familiar: $E_1[u'(c_{ui})] = (\hat{L} + 1)v'(c_0^0)\hat{S} = 0.388$, and $\varepsilon_{Sb} = \{0.1, 0.2\}$, although in this case $\varepsilon_{Sb}$ is interpreted as a partial derivative. I also choose $z$ to generate a probability of attendance of 95% for the highest-return group, and I use the fact that college graduates consume 73.9% of their pre-tax income and high school graduates consume 83.4% to motivate setting $E_1(c_1^1) = 0.739 / 0.834$.

This leads to the results presented in Table 8. The striking finding is that the welfare derivative at baseline is significantly larger, because the average return to education among those induced to go to school is higher using the estimates from Carneiro, Heckman, and Vytlacil (2011). However, there are diminishing returns to inducing PSE attendance, because increasingly generous grants induce students with lower monetary returns to go to school; therefore, optimal grants are lower when $\varepsilon_{Sb} = 0.2$, though they are larger when $\varepsilon_{Sb} = 0.1$ because the returns to inducing PSE attendance do not decline as quickly in that case.

In the baseline case, if heterogeneous returns of this magnitude do exist, it may no longer be optimal to completely eliminate tuition, but a significant increase in the generosity of grants is still indicated, and the welfare gains are very large, amounting to $103.4$ billion per year in the baseline case. However, this can only be a first pass at considering optimal policy with heterogeneity; to be able to answer policy questions in this context, and in particular to
Table 8: Results from Calibration and Simulation with Heterogeneous Returns to Education

| Value of $\varepsilon_{sb}$ | $L$ | $
abla w / \nabla b$ at $b = 2$ |
<table>
<thead>
<tr>
<th></th>
<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td></td>
<td>A. Numerical Estimate</td>
</tr>
<tr>
<td></td>
<td></td>
<td>of $\nabla w / \nabla b$ at $b = 2$</td>
</tr>
<tr>
<td>0.1</td>
<td>1.1593</td>
<td>1.4041</td>
</tr>
<tr>
<td>0.2</td>
<td>2.9816</td>
<td>3.3447</td>
</tr>
<tr>
<td>B. Optimal Student Grants</td>
<td></td>
<td></td>
</tr>
<tr>
<td>0.1</td>
<td>$5248$</td>
<td>$6376$</td>
</tr>
<tr>
<td>0.2</td>
<td>$4574$</td>
<td>$3345$</td>
</tr>
<tr>
<td>C. Welfare Gains from Moving to Optimum</td>
<td></td>
<td></td>
</tr>
<tr>
<td>0.1</td>
<td>$7036$ (226.7%)</td>
<td>$11043$ (355.8%)</td>
</tr>
<tr>
<td>0.2</td>
<td>$14767$ (475.7%)</td>
<td>$19805$ (638.1%)</td>
</tr>
</tbody>
</table>

---

go beyond looking at lump-sum policies to consider targeting incentives at those students with high potential gains, a more realistic structural model is needed. In future work, I plan to undertake this analysis by estimating a model with observed and unobserved heterogeneity using data from the NLSY.

7 General Equilibrium Effects

In this section, I explore the controversial issue of general equilibrium effects and demonstrate how sensitive my results are to their existence and magnitude. I begin by looking at how the wage premium may shift with the supply of PSE graduates, using two different estimates of the elasticity of substitution. Then I consider the possibility of spillovers, or positive externalities of PSE onto the wages of other workers; and finally I combine both of these factors into one model.

7.1 GE Effects on PSE Wage Premium

Analysis in papers such as Katz and Murphy (1992) suggests that changes in the supply of university graduates may have significant effects on relative wages. In Heckman, Lochner, and Taber (1998a) and Heckman, Lochner, and Taber (1998b) it is shown that this has consequences for the effectiveness of tuition subsidies: if increased PSE attendance lowers the PSE wage premium, then grants to students can only induce a small increase in attendance before declines in the wage premium completely offset the increased incentives to attend. Heckman, Lochner, and Taber (1998a) estimate an elasticity of substitution between high
school and university graduates of 1.441, which means that the effect of a tuition subsidy on PSE attendance in general equilibrium is about one-tenth the size of the partial equilibrium effect that has typically been estimated.

However, this conclusion is sensitive to assumptions about the usage of skill in the economy, as Lee (2005) finds general equilibrium effects that are more than 80% as large as the partial equilibrium values. And there is reason to believe that the short-run effects of an increase in supply of PSE graduates may overstate the long-run decline in wages if increased supply of skills leads to technological change to take advantage of those skills. Acemoglu (1998), Acemoglu (1999) and Kiley (1999) all present models in which an increased supply of skilled workers leads to technological adjustment that creates more jobs designed for skilled workers, with the skill premium then increasing over time, possibly above the original level. In future work I hope to quantify this effect empirically in order to shed more light on the impact of tuition subsidies on enrollment in general equilibrium, but for now I will not take a position on what elasticity of substitution is appropriate for my calculations; I will present results corresponding to both the Heckman, Lochner, and Taber (1998a) and Lee (2005) cases.

I assume a CES production function over high school and university graduates, specifically:

\[ Y_t = \eta_t (aS^c_t + (1 - a)S^w_t)^{\frac{1}{\kappa}} \]

where \( S^c_1 = S \) and \( S^c_0 = 1 - S \). I assume that wages and the production function are specific to the generation in question, i.e. that vintage effects make the human capital of different cohorts highly non-substitutable, thereby producing an upper bound of general equilibrium effects. Therefore the wage of a PSE graduate is \( Y_{1t} = \frac{\partial Y_t}{\partial Y_{1t}} \) and the wage of a high school graduate is \( Y_{0t} = \frac{\partial Y_t}{\partial Y_{0t}} \), and \( a \) is chosen to make \( \frac{Y_{1t}}{Y_{0t}} = 1.08^4 \) at baseline. Calibration proceeds in the same way as before, since the only derivative used there is \( \frac{dS}{db} \), which I assume is evaluated at constant wages.

I produce results for two values of the elasticity of substitution, which can be written as

---

46 Additionally, Dupuy and Marey (2008) find that the elasticity of substitution has not been constant over time, rising significantly during the 1990s, and Walker and Zhu (2008) find that a significant increase in the supply of PSE graduates in the UK during 1994-2006 did not lead to a decline in the PSE premium.

47 That is, I assume that the population share of PSE graduates adjusts immediately to that of the current generation, rather than allowing for an adjustment to a new long-run equilibrium. In this I follow the approach of Heckman, Lochner, and Taber (1998b), who state that short-run general equilibrium effects on enrollment with rational expectations are also very small.
\(\frac{1}{1-\kappa}\), namely 1.441 as in Heckman, Lochner, and Taber (1998a), and 350, which generates a ratio of general equilibrium to partial equilibrium effects that is comparable to Lee (2005).\(^{48}\) These results are displayed in Tables 9 and 10.

Table 9: Results from Calibration and Simulation with Elasticity of Substitution = 1.441

<table>
<thead>
<tr>
<th>Value of (\varepsilon_{sb})</th>
<th>(\hat{L})</th>
<th>(\frac{1}{3})</th>
</tr>
</thead>
<tbody>
<tr>
<td>A. Numerical Estimate of (\frac{dW}{db}) at (b = 2)</td>
<td></td>
<td></td>
</tr>
<tr>
<td>0.1</td>
<td>-0.0507</td>
<td>0.1146</td>
</tr>
<tr>
<td>0.2</td>
<td>-0.0471</td>
<td>0.1208</td>
</tr>
<tr>
<td>B. Optimal Student Grants</td>
<td></td>
<td></td>
</tr>
<tr>
<td>0.1</td>
<td>$709</td>
<td>$5005</td>
</tr>
<tr>
<td>0.2</td>
<td>$807</td>
<td>$5145</td>
</tr>
<tr>
<td>C. Welfare Gains from Moving to Optimum</td>
<td></td>
<td></td>
</tr>
<tr>
<td>0.1</td>
<td>$133 (4.3%)</td>
<td>$661 (21.3%)</td>
</tr>
<tr>
<td>0.2</td>
<td>$114 (3.7%)</td>
<td>$728 (23.4%)</td>
</tr>
</tbody>
</table>

Table 10: Results from Calibration and Simulation with Elasticity of Substitution = 350

<table>
<thead>
<tr>
<th>Value of (\varepsilon_{sb})</th>
<th>(\hat{L})</th>
<th>(\frac{1}{3})</th>
</tr>
</thead>
<tbody>
<tr>
<td>A. Numerical Estimate of (\frac{dW}{db}) at (b = 2)</td>
<td></td>
<td></td>
</tr>
<tr>
<td>0.1</td>
<td>0.1143</td>
<td>0.2477</td>
</tr>
<tr>
<td>0.2</td>
<td>0.3645</td>
<td>0.5096</td>
</tr>
<tr>
<td>B. Optimal Student Grants</td>
<td></td>
<td></td>
</tr>
<tr>
<td>0.1</td>
<td>$3743</td>
<td>$6007</td>
</tr>
<tr>
<td>0.2</td>
<td>$5685</td>
<td>$7465</td>
</tr>
<tr>
<td>C. Welfare Gains from Moving to Optimum</td>
<td></td>
<td></td>
</tr>
<tr>
<td>0.1</td>
<td>$395 (12.7%)</td>
<td>$1946 (62.7%)</td>
</tr>
<tr>
<td>0.2</td>
<td>$2677 (86.2%)</td>
<td>$5577 (179.7%)</td>
</tr>
</tbody>
</table>

If high- and low-education workers are not good substitutes for each other, as argued by Heckman, Lochner, and Taber (1998a), then my findings confirm those of the latter paper in that the role of tuition subsidies in increasing PSE enrollment is minimal; in order to justify substantial increases in grants, significant liquidity constraints are required. However, with a much higher elasticity of substitution as in Lee (2005), the results are nearly identical to those from my baseline analysis. The magnitude of these general equilibrium effects, therefore, is

\(^{48}\)The average of the ratio for men \((\frac{1.05}{1.12})\) and for women \((\frac{1.52}{1.66})\) in Lee (2005) is 0.9266; the average ratio across the four cases displayed in Table 10 is 0.9250.
of considerable importance, clearly demonstrating the importance of future work that can shed more light into this phenomenon.

7.2 Wage Spillovers

A number of papers have sought evidence of positive wage spillovers from PSE education, i.e. a positive externality of education manifesting itself in higher wages for other workers, resulting from off-the-job interactions or some form of social capital. Moretti (2004a) and Moretti (2004b) represent two prominent examples that do find significant effects, whereas Ciccone and Peri (2006) do not; Lange and Topel (2006) survey the literature and provide their own additional findings. A number of other studies are summarized in Damon and Glewwe (2011), who state that the estimate produced by Lange and Topel (2006) implies that a one percentage point increase in the population with a bachelor’s degree increases average wages by 0.2%, and conclude that this represents a “very conservative” estimate of the effect, with other estimates often in the range of 1%. I will therefore proceed by using this estimate that each percentage point increase in PSE enrollment raises average wages by 0.2%.

This effect can easily be incorporated into the simulation to find a numerical estimate of \( \frac{dW}{db} \), but moving away from \( \hat{b} = 2 \), it does not seem plausible that this spillover would remain at the same level as \( S \) increases. Therefore, I have experimented with several assumptions about how marginal gains from spillovers decline with \( S \), and in Table 11 below I present results where the wage increase per percentage point of attendance is \( \frac{\delta}{S^2} \), where \( \delta = 0.002(0.388^2) \).

Table 11: Results from Calibration and Simulation with Spillovers

<table>
<thead>
<tr>
<th>Value of ( \varepsilon_{Sb} )</th>
<th>( \hat{L} )</th>
<th>1.6318</th>
<th>1.8020</th>
<th>3.1094</th>
<th>3.3830</th>
</tr>
</thead>
<tbody>
<tr>
<td>( b = 2 ) 0.1</td>
<td>A. Numerical Estimate of ( \frac{dW}{db} )</td>
<td>( \frac{1}{3} )</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>0.2</td>
<td>3.1094</td>
<td>3.3830</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>( b = 2 ) 0.1</td>
<td>B. Optimal Student Grants</td>
<td>$18291</td>
<td>$17179</td>
<td></td>
<td></td>
</tr>
<tr>
<td>0.2</td>
<td>$15819</td>
<td>$15027</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>( b = 2 ) 0.1</td>
<td>C. Welfare Gains from Moving to Optimum</td>
<td>$42839 (1380.1%)</td>
<td>$45899 (1478.7%)</td>
<td></td>
<td></td>
</tr>
<tr>
<td>0.2</td>
<td>$72361 (2331.2%)</td>
<td>$79748 (2569.2%)</td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>
Even with this “very conservative” assumption about wage spillovers, the welfare gain from increasing grants to PSE students is now enormous; a 1% increase in $b$ to $2020$ generates an annual economy-wide gain of $1.74$ billion in the baseline case of $\varepsilon_{Sb} = 0.2$ and $\hat{L} = 0$. We cannot have as much confidence about the results for the optimum, since they are based on an ad-hoc extrapolation, but even with spillovers that diminish at the rate of $S^2$, the optimal grants are very large, higher than the median value of tuition, room and board at public universities of $13035$ in 2007-08. The welfare gains are also very large, perhaps implausibly so, with a value of $506.6$ billion in the baseline case, or $3.6\%$ of GDP. Also, notably, because of the spillovers to uneducated individuals, there is considerable scope for Pareto improvements; in all cases, Pareto gains can be obtained from marginal increases in $b$ up to at least $10000$, and at the optimum, both high-school and PSE graduates are better off than when $b = 2$.

The results for the optimum can only be a rough approximation, given the lack of evidence on how spillovers would change with $S$, but the magnitude of the welfare derivative alone indicates that wage spillovers that might have been considered small in previous work are actually extremely important, which indicates a need for further work in this area.

### 7.3 Both GE Effects Combined

To give an indication of which of these general equilibrium effects may be expected to dominate if they occur at the same time, I can easily combine them. In Table 12, I present results with an elasticity of substitution of $1.441$, as in Heckman, Lochner, and Taber (1998a), and wage spillovers of $0.2\%$ per percentage point of PSE graduates, as in Damon and Glewwe (2011). The effects offset each other to a significant degree; optimal grants are now larger than in Table 2 if liquidity constraints are significant, and smaller with $\hat{L} = 0$ but still representing significant increases in generosity. These results are meant only to be illustrations of the considerable sensitivity of optimal policy to the existence of general equilibrium effects; future work should be focussed on providing stronger evidence about their existence, magnitude and direction.
Table 12: Results from Calibration and Simulation with Elasticity of Substitution = 1.441 and Spillovers

<table>
<thead>
<tr>
<th>Value of $\varepsilon_{SB}$</th>
<th>$L = 0$</th>
<th>$L = \frac{1}{3}$</th>
</tr>
</thead>
<tbody>
<tr>
<td>A. Numerical Estimate of $\frac{dW}{db}$ at $b = 2$</td>
<td></td>
<td></td>
</tr>
<tr>
<td>0.1</td>
<td>0.0472</td>
<td>0.2415</td>
</tr>
<tr>
<td>0.2</td>
<td>0.0537</td>
<td>0.2526</td>
</tr>
<tr>
<td>B. Optimal Student Grants</td>
<td></td>
<td></td>
</tr>
<tr>
<td>0.1</td>
<td>$3200$</td>
<td>$8239$</td>
</tr>
<tr>
<td>0.2</td>
<td>$3359$</td>
<td>$8475$</td>
</tr>
<tr>
<td>C. Welfare Gains from Moving to Optimum</td>
<td></td>
<td></td>
</tr>
<tr>
<td>0.1</td>
<td>$112 (3.6%)$</td>
<td>$2777 (89.5%)$</td>
</tr>
<tr>
<td>0.2</td>
<td>$144 (4.6%)$</td>
<td>$3007 (96.9%)$</td>
</tr>
</tbody>
</table>

8 Conclusion

In this paper, I have presented a simple model of post-secondary education, to allow for an analysis that takes seriously the Second-Best nature of the optimal tuition subsidy problem, and to permit me to derive plausible numerical results with clear policy implications. My results indicate that fiscal externalities provide justification for greater government support for students. The preferred estimates indicate as a first-order policy recommendation the elimination of tuition (at least at public schools, with perhaps a stipend of equal value for private institutions); some cases also recommend a partial stipend for basic living expenses. These results are robust to a number of sensitivity analyses and extensions of the basic model; an analysis with heterogenous liquidity constraints suggests it is generally optimal to provide more generous support to individuals who are more likely to be borrowing-constrained, but at usual parameter estimates substantial support should still be given to completely unconstrained groups. Furthermore, although the previous research on liquidity constraints has been enveloped in controversy about their magnitude and even their existence, my conclusions are driven by the fiscal externality component and are largely robust to an elimination of liquidity constraints, suggesting that the latter are of second-order importance to policy.

The one factor which can alter these conclusions is the existence of significant general equilibrium effects of tuition subsidies on wages. If effects on relative wages are as severe as estimated by Heckman, Lochner, and Taber (1998a), then the case for abolishing tuition rests entirely on the existence of significant liquidity constraints. On the other hand, even
very modest wage spillovers could make a case for large stipends on top of free tuition. Thus, further work that models and estimates wage formation in general equilibrium is called for.

Additionally, structural analysis of models with multiple dimensions of heterogeneity holds the promise of being able to answer additional policy questions of interest. In particular, policies targeting financial aid more effectively at students close to the margin of attending PSE are likely to be more efficient; in the main analysis in this paper, I focussed on a simple case of a lump-sum student grant and a proportional tax, and found that even though such an arrangement will tend to involve redistribution away from lower-income high-school graduates and towards higher-income PSE graduates, a utilitarian social planner would still prefer more generous grants. If these grants can be targeted at marginal groups, the revenue requirements will be lower and it will be more likely that Pareto gains can be obtained.

Therefore, some caution is called for in interpreting my results; there is more that we need to know. However, my analysis does suggest a baseline conclusion of eliminating public tuition, and provides a clear guide to future research by highlighting the areas where we need to know more. This paper also provides a methodological advance through a novel application of the sufficient statistics method to the area of post-secondary education, and demonstrates that empirical work on liquidity constraints among students may be of limited policy relevance, as well as relying on a potentially misleading indicator of borrowing constraints.

A Liquidity Term with Heterogeneous Constraints

To be as general as possible, let me allow for the possibility that $\epsilon_i$ and $A_i$ are jointly distributed according to some bivariate distribution function $F(\epsilon, A)$. Let me define $S_A(A)$ to be the probability of university attendance for an individual with debt limit $A$; this can be written as:

$$S_A(A) = 1 - F_{\epsilon|A}[R_1v(c_0^u) - u(c_u(A)) - R_2v(c_1^u(A))|A]$$

where $F_{\epsilon|A}$ represents the conditional cdf. Then the overall probability of university attendance is simply $S = \int_A S_A(A)f_A(A)dA$, where $f_A$ is the marginal density of $A$.

Next, observe that:

$$\frac{\partial S}{\partial b} = \int_A \frac{\partial S_A(A)}{\partial b}f_A(A)dA = \int_A f_{\epsilon|A}(\epsilon^*_A|A)f_A(A)u'(c_u(A))dA$$

$$\frac{\partial S}{\partial a_1} = \int_A \frac{\partial S_A(A)}{\partial a_1}f_A(A)dA = \int_A f_{\epsilon|A}(\epsilon^*_A|A)f_A(A)[u'(c_u(A)) - v'(c_0^u)]dA$$
where $\epsilon^*_A$ is the critical value for $A_i = A$. Therefore, using the definition of $L$ from the text:

$$L = \frac{\int_A f_{\epsilon\mid A}(\epsilon^*_A|A)f_A(A)[u'(c_u(A)) - v'(c_{u}^0)]dA}{\int_A f_{\epsilon\mid A}(\epsilon^*_A|A)f_A(A)v'(c_{u}^0)dA}. $$

Meanwhile, the term I wish to replace is $E_1[u'(c_{ui})]/\sqrt{v'(c_{u}^0)}$; this is greater or less than $L$ as:

$$E_1[u'(c_{ui})] \geq \frac{\int_A f_{\epsilon\mid A}(\epsilon^*_A|A)f_A(A)u'(c_u(A))dA}{\int_A f_{\epsilon\mid A}(\epsilon^*_A|A)f_A(A)dA}. $$

$$\frac{\int_A [1 - F_{\epsilon\mid A}(\epsilon^*_A|A)]f_A(A)u'(c_u(A))dA}{\int_A [1 - F_{\epsilon\mid A}(\epsilon^*_A|A)]f_A(A)dA} \geq \frac{\int_A f_{\epsilon\mid A}(\epsilon^*_A|A)f_A(A)u'(c_u(A))dA}{\int_A f_{\epsilon\mid A}(\epsilon^*_A|A)f_A(A)dA}. $$

If the conditional hazard rate $f_{\epsilon\mid A}(\epsilon^*_A|A)$ is constant, these two terms will be equal, and I can safely replace $E_1[u'(c_{ui})]/\sqrt{v'(c_{u}^0)}$ with $L$ in (3). More generally, let me continue by substituting $h(A)$ for the conditional hazard rate, and let me also write $k(A) = [1 - F_{\epsilon\mid A}(\epsilon^*_A|A)]f_A(A)$ to represent the measure of enrollees at a particular value of $A_i$; then the comparison becomes:

$$\frac{\int_A k(A)u'(c_u(A))dA}{\int_A k(A)dA} \geq \frac{\int_A k(A)h(A)u'(c_u(A))dA}{\int_A k(A)h(A)dA}. $$

$$\frac{\int_A k(A)h(A)dA}{\int_A k(A)dA} \geq \frac{\int_A k(A)h(A)u'(c_u(A))dA}{\int_A k(A)dA} \geq \frac{\int_A k(A)h(A)u'(c_u(A))dA}{\int_A k(A)dA}. $$

$$E_1[h(A)]E_1[u'(c_u(A))] \geq E_1[h(A)u'(c_u(A))] \geq 0 \geq Cov[h(A), u'(c_u(A))].$$

Therefore, if the covariance of the hazard and the marginal utility among those attending university is close to zero, it will be a reasonable approximation to insert $L$ into (3). Meanwhile, I will tend to underestimate the liquidity effect if the covariance is negative, which would follow, for instance, if $h(A)$ is increasing in $A$ (given that $u'(c_u(A))$ should be non-increasing in $A$).

It is hard to say if the hazard would be increasing; I expect $[1 - F_{\epsilon\mid A}(\epsilon^*_A|A)]$ to be higher for higher $A$, which means that I also need $f_{\epsilon\mid A}(\epsilon^*_A|A)$ to be increasing in $A$. Dynarski (2002) argues that there is no consistent evidence of greater responsiveness of low-income students to price, which, given that I expect $u'(c_u(A))$ is higher for low-income students, might imply $f_{\epsilon\mid A}(\epsilon^*_A|A)$ increasing in $A$; but some studies do find evidence of such greater responsiveness (for instance, Kane (1994)), which provides less encouraging evidence.

Given that $\epsilon^*_A$ is decreasing in $A_i$, I would want the hazard to be decreasing in $\epsilon$, which would be the case for distributions such as the Pareto and the $\chi^2$ for degrees of freedom less than 2 (with 2 degrees of freedom, the hazard is constant). However, many other distributions, including the logistic that I use in my calibration, feature an increasing hazard, in which case my overestimate of $L$ would tend to offset the conservative assumptions elsewhere in the model.

**B Calculation of $\varepsilon\bar{Y}_b$**

First, assuming that the only effects of $b$ on $\bar{Y}$ are from $b$’s effect on schooling and from the effect of the tax change on earnings $Y_{01}$ and $Y_{11}$, I can write:

$$\varepsilon\bar{Y}_b = \frac{b}{\bar{Y}} \frac{d\bar{Y}}{db} = \frac{b}{\bar{Y}} \left[ \frac{\partial \bar{Y}}{\partial S} \frac{dS}{db} + \frac{\partial \bar{Y}}{\partial \tau} \frac{d\tau}{db} \right].$$
It is clear that \( \frac{\partial \bar{Y}}{\partial S} = \gamma_2 Y_{11} - \gamma_1 Y_{01} = [\gamma_2(1.08)^4 - \gamma_1] Y_{01}, \) and given that I assume that the elasticity of taxable income is 0.4, I have \( \frac{\partial \bar{Y}}{\partial \tau} = -0.4 \bar{Y}. \) Using (2) for \( \frac{\partial \bar{Y}}{\partial \tau}, \) the equation for \( \varepsilon_{\bar{Y}b} \) becomes:

\[
\varepsilon_{\bar{Y}b} = \left[ \gamma_2(1.08)^4 - \gamma_1 \right] Y_{01} \frac{S}{Y} \varepsilon_{Sb} - 0.4 \frac{Sb}{(1 - \tau)Y} \left[ 1 + \varepsilon_{Sb} - \left( 1 + \frac{G}{Sb} \right) \varepsilon_{\bar{Y}b} \right]
\]

and rearranging, I arrive at:

\[
\varepsilon_{\bar{Y}b} = \left[ \frac{\gamma_2(1.08)^4 - \gamma_1}{\gamma_2(1.08)^4 S + \gamma_1(1 - S)} \right] - 0.4 \tau \left( 1 + \frac{G}{Sb} \right)^{-1} \left[ \frac{1 - \tau - \varepsilon_{Sb} - \frac{0.4\tau}{1 - 1.4\tau} \varepsilon_{\bar{Y}b}}{1 - 1.4\tau} \left( 1 + \frac{G}{Sb} \right)^{-1} \right].
\]

### C Sensitivity Analyses

This section will be devoted to an examination of the robustness of my results. I begin with an analysis of the sensitivity of my results to the coefficient of relative risk-aversion, and then I use the estimates of fiscal costs and benefits from Trostel (2010) to assess the impact on my conclusions of how these fiscal effects are modelled. I also extend the model to consider uncertainty about future incomes. The quantitative results are only slightly altered in each case, and the qualitative conclusions remain very similar.

#### C.1 Sensitivity of Results to Risk-Aversion

My first sensitivity analysis considers how the results change when I specify a coefficient of relative risk-aversion of \( \rho = 2 \) for the employed state. Since I only need to specify this parameter when using the structural method, it will only affect my simulation results. Calibration proceeds as before, and simulation yields the results displayed in Table 13. The optimal values of \( b \) and welfare effects are a bit smaller in most cases, but the conclusion of approximately abolishing tuition continues to hold in the baseline case.

<table>
<thead>
<tr>
<th>Value of ( \varepsilon_{Sb} )</th>
<th>( L )</th>
</tr>
</thead>
<tbody>
<tr>
<td>( b = 2 )</td>
<td></td>
</tr>
<tr>
<td>0.1</td>
<td>0.1505</td>
</tr>
<tr>
<td>0.2</td>
<td>0.4185</td>
</tr>
<tr>
<td>( \frac{1}{3} )</td>
<td></td>
</tr>
<tr>
<td>A. Numerical Estimate of ( \frac{dW}{db} ) at ( b = 2 )</td>
<td></td>
</tr>
<tr>
<td>0.1</td>
<td>$3660</td>
</tr>
<tr>
<td>0.2</td>
<td>$5412</td>
</tr>
<tr>
<td>B. Optimal Student Grants</td>
<td></td>
</tr>
<tr>
<td>( 0.1 )</td>
<td>( $489 \ (15.8%) )</td>
</tr>
<tr>
<td>( 0.2 )</td>
<td>( $2795 \ (90.0%) )</td>
</tr>
<tr>
<td>C. Welfare Gains from Moving to Optimum</td>
<td></td>
</tr>
<tr>
<td>( 0.1 )</td>
<td></td>
</tr>
<tr>
<td>( 0.2 )</td>
<td></td>
</tr>
</tbody>
</table>
C.2 Evidence from Trostel (2010) on Fiscal Effects of Education

In this subsection, I will test the robustness of my results to a different choice of \( \hat{b} \); specifically, I perform my analysis again using the most pessimistic estimates from Trostel (2010), in which he concludes that each year of PSE costs the government $17850 and saves expenditures amounting to $13950. Using his present-value calculations, I select \( \hat{b} = 18 \), increasing \( e \) to 21.7 to correspond, and I assume that each year of schooling also saves expenditures amounting to \( p = 14 \). This changes the government budget constraint: I now divide \( G \) into two components, one exogenous component denoted by \( G_1 \), and one component \( G_2 = (1 - S)p \) representing the expenditures which can be eliminated with increased schooling. Therefore, the derivative of the budget constraint is:

\[
\frac{d\tau}{db} = \frac{S}{Y} \left[ 1 + \left( \frac{b - p}{b} \right) \varepsilon_{sb} - \left( 1 + \frac{G}{Sb} \right) \varepsilon_{yb} \right]
\]

and inserting this into \( \frac{dV}{db} \), I derive the following variant of (4):

\[
\frac{dW}{db} \simeq S \left[ L - \left( \frac{b - p}{b} \right) \varepsilon_{sb} + \left( 1 + \frac{G}{Sb} \right) \varepsilon_{yb} \right].
\] (6)

Because the baseline value of \( b \) is 9 times larger, the earlier values of \( \varepsilon_{sb} = \{0.1, 0.2\} \) are now replaced by \( \varepsilon_{sb} = \{0.9, 1.8\} \). For the optimal grants, let me write them as \( \hat{b} = b - 16 \) to make them comparable to earlier results; evaluating (6) and using the same statistical extrapolations as before leads to the results displayed in Table 14. The values of \( \frac{dW}{db} \) are smaller now, but the optimal grants are generally larger, as are the welfare gains at the optimum, due to the assumptions involved, particularly that of a constant value of \( \varepsilon_{sb} \). The baseline result involves an optimal stipend of over $3600 and a welfare gain amounting to $36.6 billion.

Table 14: Results from Sufficient Statistics and Extrapolation using (6)

<table>
<thead>
<tr>
<th>Value of ( \varepsilon_{sb} )</th>
<th>0</th>
<th>1</th>
</tr>
</thead>
<tbody>
<tr>
<td>A. Estimate of ( \frac{dW}{db} ) at ( \hat{b} = 18 )</td>
<td>[0.1370, 0.2664]</td>
<td>[0.3267, 0.4560]</td>
</tr>
<tr>
<td>B. Optimal Student Grants ( \hat{b} = b - 16 )</td>
<td>$7795, $8883</td>
<td>$9343, $9343</td>
</tr>
<tr>
<td>C. Welfare Gains from Moving to Optimum</td>
<td>$1563 (5.6%), $3648 (13.1%)</td>
<td>$5220 (18.7%), $6252 (22.4%)</td>
</tr>
</tbody>
</table>

Calibration and simulation follows the same procedure as before, and the results are found in Table 15. In every case, the welfare derivative at baseline is smaller, as are the optimal grants and the welfare gains from moving to the optimum; the optimal grants drop by less than $1000, and the baseline result no longer involves the complete abolition of tuition, but still calls for significantly reduced out-of-pocket costs.

---

49If instead I set \( p = 16 \) to correspond to the baseline case in which I assume that government appropriations for education are exactly offset by reductions in other expenditures, all results are identical to those in section 4 except that the optimal grants and welfare gains are almost all larger using the sufficient statistics and extrapolation methods, due to functional form assumptions.
Table 15: Results from Calibration and Simulation for $\hat{b} = 18$

<table>
<thead>
<tr>
<th>Value of $\varepsilon_{Sb}$</th>
<th>0</th>
<th>$\frac{1}{2}$</th>
</tr>
</thead>
<tbody>
<tr>
<td>A. Numerical Estimate of $\frac{dV}{db}$ at $b = 18$</td>
<td></td>
<td></td>
</tr>
<tr>
<td>0.9</td>
<td>0.0700</td>
<td>0.1990</td>
</tr>
<tr>
<td>1.8</td>
<td>0.3098</td>
<td>0.4391</td>
</tr>
<tr>
<td>B. Optimal Student Grants $\tilde{b} = b - 16$</td>
<td></td>
<td></td>
</tr>
<tr>
<td>0.9</td>
<td>$3061$</td>
<td>$5219$</td>
</tr>
<tr>
<td>1.8</td>
<td>$4994$</td>
<td>$6624$</td>
</tr>
<tr>
<td>C. Welfare Gains from Moving to Optimum</td>
<td></td>
<td></td>
</tr>
<tr>
<td>0.9</td>
<td>$149$ (0.5%)</td>
<td>$1263$ (4.5%)</td>
</tr>
<tr>
<td>1.8</td>
<td>$1861$ (6.7%)</td>
<td>$4082$ (14.6%)</td>
</tr>
</tbody>
</table>

C.3 Income Uncertainty

The final extension is a case with uncertainty about future incomes. To keep the problem simple, I assume that all uncertainty is resolved after the first period. Thereafter, educated individuals receive either $Y_{1H} = (1 + g)^{t-1}Y_{11H}$ in each period or $Y_{1L} = (1 + g)^{t-1}Y_{11L}$, each with probability 0.5, where $Y_{11H} > Y_{11L}$ and $\frac{Y_{11H} + Y_{11L}}{2} = Y_{1}$. Meanwhile, uneducated workers begin with $Y_{01}$ in the first period, and thereafter receive $Y_{0H} = (1 + g)^{t-1}Y_{01H}$ or $Y_{0L} = (1 + g)^{t-1}Y_{01L}$, each with probability 0.5, where $\frac{Y_{01H} + Y_{01L}}{2} = Y_{01}$. The corresponding consumption values will be denoted as $c_{vH}$ and $c_{vL}$ for educated workers and $c_{v0}$ for uneducated workers, with $c_{v1}$ representing the consumption of first-period workers.

In deriving $\frac{dW}{db}$, the only meaningful change will come from the fact that $\frac{\partial V}{\partial \tau}$ takes a different form, specifically:

$$\frac{\partial V}{\partial \tau} = -\frac{\gamma_2}{2} S \left( v'(c_{vL})Y_{11L} + v'(c_{vH})Y_{11H} \right) - \frac{1}{2} \left( v'(c_{vL})(Y_{01} + \gamma_2 Y_{01L}) + v'(c_{vH})(Y_{01} + \gamma_2 Y_{01H}) \right).$$

However, this equation cannot be used in its current form, and the most reasonable simplification is still $\hat{Y} v'(e^*)$, where $\hat{Y}$ remains equal to $S\gamma_2 Y_{11} + (1 - S)\gamma_2 Y_{01}$, so that (4) holds in this case as well, and the results are unchanged.

I will therefore focus on the structural analysis. The calibration proceeds largely as before, except that $A$ and $\theta$ must be chosen simultaneously to generate consumption choices which match $E(c_{v}) = 1.275E(c_{v}^0)$ and $u'(c_a) = (\hat{L} + 1)u'(c_{v0})$. For the variability of of income, I collect data on the median and interquartile range of income for high school and university graduates from the CPS in the 4th quarter of 2012. Then I consider three cases: one case in which I choose the values of $\{Y_{0L}, Y_{0H}, Y_{1H}, Y_{1L}\}$ that produce the same interquartile range, specifically 74.3% for high school graduates and 81.5% for PSE graduates, one case in which I cut the high school variance in half, and one in which I cut the PSE variance in half. The results are displayed in Table 16, and the optimal grants and welfare gains are larger in every case, though there does not appear to be one unambiguous pattern of results across the three cases. The baseline result features the abolition of tuition accompanied by a stipend of about $600 to $2000 per year.
### Table 16: Results from Calibration and Simulation with Uncertain Income

<table>
<thead>
<tr>
<th>Value of $\varepsilon_{sb}$</th>
<th>CPS Variance</th>
<th>Low HS Variance</th>
<th>Low PSE Variance</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>$L$</td>
<td>$L$</td>
<td>$L$</td>
</tr>
<tr>
<td></td>
<td>0</td>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td></td>
<td>$\frac{1}{3}$</td>
<td>$\frac{1}{3}$</td>
<td>$\frac{1}{3}$</td>
</tr>
<tr>
<td>A. Numerical Estimate of $\frac{dW}{db}$ at $b = 2$</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>0.1</td>
<td>0.1567</td>
<td>0.2858</td>
<td>0.1319</td>
</tr>
<tr>
<td>0.2</td>
<td>0.4168</td>
<td>0.5464</td>
<td>0.4200</td>
</tr>
<tr>
<td></td>
<td></td>
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<td>0.4166</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td>0.5495</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td>0.5462</td>
</tr>
<tr>
<td>B. Optimal Student Grants</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>0.1</td>
<td>$4556$</td>
<td>$6904$</td>
<td>$5183$</td>
</tr>
<tr>
<td>0.2</td>
<td>$7569$</td>
<td>$8200$</td>
<td>$7839$</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td>$7669$</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td>$6354$</td>
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### References


