Optimal Monetary Policy and the Business Cycle: the Role of Multi-stage Production with Inventories

Tiantian Dai
Queen’s University
November 15, 2011

Abstract

This paper studies the role of multi-stage production for the transmission mechanism of monetary policy. I employ a monetary search model to show how multi-stage production influences both the long run and the short run effects of monetary policy. Multi-stage production provides an additional channel for monetary policy through input inventory investment between different production stages. These input inventories arise due to search frictions and amplify monetary shocks. Monetary policy has hump-shaped real effects on steady state input inventory investment. Furthermore, in steady state input inventories first increase with the growth rate of money, before falling for large growth rates. Intuitively, when the money growth rate is low, both the extensive and intensive margins in the finished goods market are positive: there are more matches and production in each market. Finally, contrary to other work, my model is able to replicate the stylized facts on inventory movements over the business cycle by solely relying on monetary shocks.

Keywords: Money; Search; Business cycles; Multi-stage production.

*I am grateful to Allen Head and Thorsten Koeppel for their enormous guidance and encouragement. I also would like to thank Jonathan Chiu, Hongfei Sun and Lealand Morin for their generous helps and comments. I also thank Oleksiy Kryvtsov and other seminar participants at the 2011 CEA meetings for their useful comments. All errors are mine.
†Email: dait@econ.queensu.ca
1 Introduction

In reality, finished goods production process involves multiple intermediate stages. Most of the macroeconomics literature usually assumes single-stage production for the sake of simplicity. But such simplicity may have to sacrifice the depth of understanding the transmission mechanism of monetary policy. In this paper, I study the role of multi-stage production for the effects of monetary policy on both the steady state and fluctuations in the economy. I find that multi-stage production has a big impact on both the steady state effects and the propagation mechanism of monetary policy. A multi-stage production model can replicate the stylized facts of inventories over business cycles by solely relying on monetary shocks.

In my model, I employ a two-stage production where input inventory investment connects the two markets and amplifies monetary shocks. Moreover, input inventories are quantitatively important in terms of matching the data. These findings are consistent with the inventory literature in which input inventories are empirically more important.1 Empirical studies show that inventories are procyclical and amplify aggregate fluctuations over business cycles. Such effects are mainly attributed to input instead of output inventories, because input inventories are the biggest and most volatile component of inventories. Humphreys, Maccini and Schuh (2001) show that manufacturing firms hold more than two times input inventories than output inventories on average, and that input inventories are three times more volatile. However, most papers in the literature do not distinguish between input and output inventories, or they only look at output inventories (e.g., [13, 15, 21, 26]).

In this paper, I employ a large household model with a monetary propagation mechanism à la Shi (1998). I extend Shi’s model from single-stage production to two-stage production. The first stage is for producing intermediate goods and the second stage is for producing finished goods. Households have to obtain intermediate goods from other households for their own finished goods production. Labor is also required for finished goods production, which can be hired from a frictional labor market. In my model, intermediate goods facilitate finished goods production, and only finished goods yield utility for households. Input inventories arise in my model because of a search friction in the finished goods market. Unmatched finished goods producers hold unused intermediate goods at the end of each period. This is in sharp contrast to the literature where a fixed delivery cost or a stockout avoidance motive is often assumed to rationalize inventories.2 Moreover, only input inventories are modeled in

1Blinder and Maccini (1991) were first to point out that input inventories deserve more attention than output inventories. By decomposing the data, they found that the effect of investment on input inventories is bigger and more volatile than that on output inventories in manufacturing. This is even true for the narrowest definition of input inventories.
this paper by assuming that finished goods producers produce in the finished goods market only when matched.

I find that the multi-stage production plays a crucial role in the transmission mechanism of monetary policy. The model predicts that the transmission mechanism is unconventional in terms of the effects of monetary policy on input inventories and quantities per match. Moreover, the multi-stage production amplifies monetary shocks through input inventories. Comparative statics show that monetary policy has hump-shaped real effects on steady state input inventory investment. Furthermore, in steady state input inventories first increase with the growth rate of money, before falling for large growth rates. Intuitively, when the money growth rate is low, both the extensive and intensive margins in the finished goods market are positive: there are more matches and production in each market. Moreover, the steady state effects of monetary policy on the quantity of finished goods per match are also not monotonic. For example, when the money growth rate is low, finished goods producers trade more in each match with higher money growth rate. These results are very different from that of a one sector large household model, in which monetary policy has monotonic real effects on both inventories and the quantity of goods per match.\(^3\)

The non-monotonicity arises through the interaction between classic search effects and a two-stage production effect. The classic search effects in a large household model are the intensive margin effect and the extensive margin effect. The intensive margin refers to the quantity of goods per match. The extensive margin refers to a buyer’s search intensity which determines the total number of matches. Varying the money growth rate affects the output along these two margins. A higher money growth rate induces a positive extensive margin effect and a negative intensive margin effect on the level of output. The overall effects on output depend on the money growth rate. If the money growth rate is low, the extensive margin effect dominates the intensive margin effect. Steady state output increases with a higher money growth rate, and verse visa.

In my model, there also exists a two-stage production effect which captures the transmission mechanism between the finished goods market and the intermediate goods market. The

\(^2\)The (S, s) model (e.g., \([8, 15, 16, 24]\)) requires fixed delivery cost to rationalize inventories. Firms hold inventories in the stockout avoidance model (e.g., \([3, 4, 7, 35]\)), because they face delivery/production lags and have to commit to production before shocks are realized. There is also a production smoothing model (e.g., \([5, 21, 32]\)) often used to rationalize inventories in the literature. Firms in this model hold inventories due to a buffer-stock motive, because they face stochastic demand and increasing marginal cost. But the production smoothing model is well known for its failure in reproducing procyclical inventory investment and more cyclical production relative to sales. See Ramey and West (1999)\(^2\) for a comprehensive review.

\(^3\)Also see Shi’s other papers \([25, 27]\). The quantity of goods per match in the Lagos and Wright \([1, 18, 22, 31]\) type of monetary search model also decreases with the money growth rate.
two-stage production effect transfers classic search effects from the finished goods market back to the intermediate goods market and amplifies classic search effects on intermediate goods sales. Because agents are randomly matched in the finished goods market, households do not observe which finished goods producers would get a match. Thus households have to obtain intermediate goods for all finished goods producers including unmatched producers in response to changes in the money growth rate. As a consequence, intermediate goods sales increase further with a higher money growth rate if it is low. And households are stuck with more input inventories even when the total number of matches increases in the finished goods market. This implies that monetary shocks are amplified by the two-stage production effect through exaggerating the classic search effects in the intermediate goods market.

The multi-stage production model also induces unconventional effects of monetary policy on the quantity of goods per match, namely the intensive margin. Similar to input inventories, the quantity of finished goods per match has a hump-shaped response across steady states. The intuition is the following. If the money growth rate is low, the intermediate goods sales are amplified by the two-stage production effect following a monetary shock. Higher intermediate goods sales imply a higher firm’s marginal profitability, and households post more vacancies. With more workers and more intermediate goods, finished goods producers produce more in each trade once they are matched. To the contrary, if the money growth rate is high, the quantity of finished goods per match decreases with a higher money growth rate as in the classical one sector search model.

The model is calibrated to match the U.S. data. The quantitative analysis shows that without technology shocks, a large household model with multi-stage production can reproduce stylized facts of inventory by solely relying on monetary shocks. Most importantly, the model predicts procyclical inventory investment, a countercyclical inventory-to-sales ratio, more volatile output relative to final sales and a positive correlation between inventory investment and final sales. Researchers in the inventory literature also use multi-stage production to model input inventories, although they use different frictions to rationalize inventories. But most papers in the inventory literature rely on real shocks to capture inventory regularities. If there were preference shocks, there would be tradeoffs between inventory investment and final sales. As a result, these models, for example the production smoothing model\(^4\) and stockout avoidance model,\(^5\) usually predict counterfactual results, in particular, countercyclical inventory investment and a negative correlation between final sales and inventory investment. Khan and Thomas [16] show that by introducing idiosyncratic shocks, the generalized stockout avoidance model without capital is able to reproduce procyclical inventory investment and a positive relationship between inventory investment and final sales under
preference shocks. But such improvements have to severely sacrifice the ability to match the long-run average inventory-to-sales ratio.

Two-stage production and input inventories are the key for matching the data, because the two-stage production effect implies that input inventories move with final sales in the same direction during a transition following a monetary shock. The model predicts hump-shaped responses of inventories to a positive shock in the money growth rate.\footnote{This result is consistent with the argument made by Jung and Yun (2006), output inventories and input inventories indeed have different behavior in response to shocks. They show hump-shaped responses of the sales-stock ratio and U-shaped responses of finished goods inventories to an expansionary monetary policy shock.} Thus, except for the initial drop, input inventories stay above the steady state levels as do output and final sales, which is the key for matching the data. This sluggish response of inventories sheds light on the "slow speed of adjustment" puzzle in the literature.\footnote{See Blinder and Maccini (1991) for details. Jung and Yun (2006) show that a sticky price model can reproduce this sluggish adjustment, but their model requires high depreciation rates and a low degree of real rigidities to match other empirical findings.}

The responses of input inventories are very different from that of output inventories. In Shi [26], households hold output inventories as a buffer stock, because whenever output increases inventories decrease. Thus, a one sector model is unable to replicate inventory regularities without technology shocks.\footnote{Also see Wang and Shi [33] and Menner [20].} Different responses of inventories imply that the policy implications in this paper are different from that in Shi’s model. In my model, monetary policy takes effect through decreased input inventories during the boom. While in Shi’s model, monetary policy takes effect through increased output inventories. The reason is that beyond the classic search effects, a multi-stage production model provides an additional channel for the monetary policy to have effects. The overall effect on input inventories are driven by the interaction between multi-stage production and search frictions, but especially the search friction in the finished goods market.

Search frictions are an important feature of this model. Theoretically, they induce a two-stage production effect which is important for the transmission mechanism. Sensitivity analysis shows that the search intensity in the finished goods market is also important quantitatively. In order to match the stylized facts of inventory investment over the cycle, the search cost

\footnote{The biggest improvement is made by Wang and Wen (2009) who tries to incorporate the production-cost-smoothing motive into the Dixit-Stiglitz RBC model. With an aggregate TFP shock and an idiosyncratic cost shock, their model is able to predict standard inventory regularities, especially procyclical inventory investment and a hump-shaped output response.}
has to be big enough in the finished goods market. The intuition is the following. If the search cost is low, households would like to consume more finished goods. The response of final sales becomes too strong which drives input inventories below the steady state during the entire transition. Thus, the correlation between final sales and inventory investment would counterfactually turn negative.

The remainder of this paper is organized as follows. Section 2 describes a search model a la Shi with multi-stage production. In section 3, I study the long run effects of monetary policy on aggregate variables. In section 4, I investigate the short run dynamics of shocks to the money growth rate and examine the role of input inventories over the business cycle. Section 5 conducts a sensitivity analysis of some parameters relative to the baseline calibration. Finally, Section 6 concludes.

2 The Model

2.1 The Environment

Time is discrete. This is a large household model with two subperiods and three markets opened in each period. The labor market opens throughout the whole period; the intermediate goods market opens in the first subperiod and the finished goods market opens in the second subperiod. The model economy consists of many types of households denoted by set \( H \). The number of households in each type is large and normalized to one. There are also many types of intermediate goods and finished goods, which are denoted by \( H^i \) and \( H^f \) respectively. Intermediate goods and finished goods are traded in the intermediate goods market which is denoted by \( i \) and the finished goods market which is denoted by \( f \) respectively. The measures of \( H, H^i \) and \( H^f \) are the same. Each household has two types of technologies which can be used to produce the household specific intermediate goods and the household specific finished goods. The type \( h \in H \) household cannot use his specific intermediate goods to produce his specific finished goods. The specific intermediate goods \( h^i \in H^i \) produced by the household \( h \) can only be utilized for the output production by some other type of households. As already discussed in the introduction, this paper focuses on input inventories instead of output inventories by assuming the intermediate goods can be stored only by the corresponding finished goods producers. Households do not consume intermediate goods. The type \( h \) household uses their other technology to produce a specific output \( h^f \in H^f \), which is desired only by some other types of households. Households have to use both intermediate goods and hired labor to produce finished goods.
Households randomly match with each other in both goods markets. Although each household neither consumes his own finished goods nor uses his intermediate goods for the output production, he has to acquire intermediate goods and finished goods from other households for his own output production and consumption. Thus an intrinsically useless object, called fiat money, can facilitate trades in both goods markets. Furthermore, assume there is no double coincidence of wants, so barter trades are excluded in this model. Since intermediate goods can be stored only by the corresponding finished goods producers, introducing input inventories do not rule out fiat money. In other words, input inventories will not circulate as a medium of exchange.

Each household consists of seven groups of agents, (with measure in parentheses): input buyers \((a^i_b)\) and intermediate goods producers \((a^i_p)\) who are active in the first subperiod; finished goods buyers \((a^f_b)\) and entrepreneurs \((a^f_p)\) who are active in the second subperiod; leisure seekers \((n_0)\), workers \((a^f_p n_t)\), and unemployed agents \((u)\). Each entrepreneur consists of an finished goods producer and an finished goods seller. Finished goods producers post vacancies on the labor market, hire workers to produce finished goods and hold intermediate goods within each period. Finished goods sellers search in the finished goods market. Once an finished goods seller matches with an finished goods buyer, he places the order for his customer. Then the corresponding finished goods producer produces. The terms of trade are determined by Nash bargaining. Since agents regard the household’s utility as a common objective and share consumption and inventories, the idiosyncratic risk generated by search friction in both goods markets and labor market is smoothed within each household. The number of agents denoted by \((a^i_b, a^f_b, a^i_p, a^f_p, u)\) are constant, while the number of effective buyers in both goods markets changes over periods, because households can choose different level of search intensity for buyers every period. The number of leisure seekers and workers \((n_0, n_t)\) varies over time.

Figure 1 depicts the timing of the model. At the beginning of each period, the household allocates a money balance between two goods markets, with the proportion \(\Delta_t\) for input buyers, and divides the money balance evenly among each type of buyers. The household also chooses the search intensity for buyers in each goods market, denoted \(s^i_t\) and \(s^f_t\). Then the intermediate goods market opens. The household sends out input buyers and intermediate goods sellers. Input buyers search for intermediate goods which can be used for output productions in the next period. Intermediate goods sellers search for buyers. Once a seller matches with a buyer, he produces intermediate goods \(\hat{q}^i_t\) for his partner on the spot with disutility \(\varphi(\hat{q}^i_t)\), where \(\hat{q}^i_t\) denotes the quantities of goods. The input buyer pays the amount of money \(\hat{m}^i_t\) to the intermediate goods seller. The terms of trade are determined by Nash
bargaining. The function \( \varphi \) satisfies \( \varphi' > 0 \) and \( \varphi'' > 0 \) for \( q > 0 \), and \( \varphi'(0) = \varphi(0) = 0 \). Variables with a hat refer to an arbitrary household. At the end of the first sub period, the intermediate goods market closes. Input buyers bring trade receipts back to the household. The household adds traded intermediate goods to the input inventories which are carried from last period, and then divides the intermediate goods evenly among entrepreneurs. Assume intermediate goods sellers will not bring their money holdings back to the household until the end of the period. This assumption simplifies the model in the sense that the equilibrium conditions do not involve the intertemporal price ratio of one good market relative to the other.

Let us describe the matching technology in the intermediate goods market. The total number of matches is determined by the following Cobb-Douglas matching function:

\[
g(s^i) = z^i (a_b^i s^i)^\alpha a_p^{1-\alpha}, \quad \alpha \in (0, 1),
\]

where \( z^i > 0 \) is a constant. Denote the ratio of buyers to sellers as \( B^i = a_b^i / a_p^i \) and normalize \( z^i \equiv z^i (B^i)^{\alpha-1} \). Then the matching rate for each unit of a buyer’s search intensity is \( g_b(s^i) \) and the matching rate for each seller is \( g_s(s^i) \), where,

\[
g_b^i \equiv z^i (s^i)^{\alpha-1}, \quad g_s^i \equiv z^i B^i (s^i)^{\alpha}.
\]

Thus the buyer and the seller get desirable matches at rates \( s^i g_b^i \) and \( g_s^i \) respectively.

The finished goods market opens in the second sub period. Buyers and sellers go out and search in the finished goods market. Each output producer carries intermediate goods and produces finished goods if his seller formed a match. The terms of trade \( (\hat{q}_f^t, \hat{m}_f^t) \) is determined by Nash bargaining. We assume the production function is a Leontief production function which means that intermediate goods and labor are not substitutable.\(^9\) Moreover, I can derive analytical results with this assumption. The Leontief production function is:

\[
q_f^t = \min\{a_t, n_t\},
\]

where \( a_t \) is the usage of intermediate goods and \( n_t \) is labor input which is hired in the last period.

Similarly to the intermediate goods market, the total number of matches in the finished goods

---

\(^9\)Because I model input inventory in this model, the Leontief production function makes more sense than the Cobb-Douglas function. For example, firms cannot produce more cars with fewer windows but more labors.
market is determined by the following matching function: \( g(\hat{s}_f) \equiv z_f (a_f \hat{a}_f)^{\alpha} a_f^{1-\alpha}, \alpha \in (0,1). \) Then the matching rate for each unit of a buyer’s search intensity is \( g_b^f \equiv z_f (\hat{s}_f)^{\alpha-1} \) and the matching rate per seller is \( g_s^f \equiv z_f B_f (\hat{s}_f)^{\alpha} \), where \( B_f = a_f^f / a_p^f \) and \( z_f \equiv z_f^1 (B_f)^{\alpha-1} \). Finally, a buyer and a seller get desirable matches at rates \( s_f g_b^f \) and \( g_s^f \) respectively.

In the labor market, each output producer posts vacancies \( v_t \). Unemployed agents search for jobs. We assume matched workers will start to work in the next period and the wage is negotiated according to Nash bargaining. Workers supply one unit of labor inelastically. Wages \( W_t \) are paid in nominal terms regardless of whether output producers formed a match or not. As in the standard labor search model [6], the total number of matches between unemployed workers and producers are: \( \bar{\mu} (a_f^p \hat{v}) \bar{A} u^{1-A} \), where \( A \in (0,1) \) and \( \bar{\mu} \) is a constant. The hat on variables refers to an arbitrary producer. The total number of matches for each firm is \( \mu(\hat{v})v \), where \( \mu(\hat{v}) \equiv \bar{\mu} (a_f^p \hat{v}/u)^{A-1} \) is the number of matches per vacancy; and the number of matches per unemployed agent is \( \mu(\hat{v})a_f^p \hat{v}/u \).

During the transaction, each household receives a lump-sum transfer \( \tau_t \) which will be added to next period’s money balance. At the end of the period, finished goods buyers bring trade receipts, and entrepreneurs bring profits and unused input inventories back to the household. Workers bring wage income back. Sellers that traded in the intermediate goods market bring their money holdings back. Finally, the household and agents share consumption. The household carries the new money balance \( M_{t+1} \) and input inventories which depreciate at rate \( \delta_i \) over period. Workers hired in the last period separate from current jobs at an exogenous rate \( \delta_n \).

Let us denote the price level in the intermediate goods market by \( P_i^t = m_i^t / q_i^t \), and \( P_f^t = m_f^t / q_f^t \) for the price level in the finished goods market. Also, the aggregate price level is defined by the weighted average of these two goods prices as follows:

\[
P_t = \xi P_i^t + (1 - \xi) P_f^t.
\]

### 2.2 The Household’s Decision Problem

At the beginning of each period, the household allocates total money balance \( M_t \) between two goods markets, \( M_i^t \) and \( M_f^t \). The household chooses search intensities for each type of buyers, \( s_i^t \) and \( s_f^t \). Assume all sellers’ search intensities and unemployed workers’ search intensities are inelastic with no cost to households. The household also chooses consumption level \( c_t \), the number of vacancies for each firm \( v_t \), and next period’s employment level \( n_{t+1} \), a new total money balance \( M_{t+1} \), and a new input inventory level \( i_{t+1} \). The household takes
the terms of trade as given when making decisions. The terms of trade are determined by Nash Bargaining and will be described later.

The representative household taking the sequence \( \{q_t^f, \hat{m}_t^i, \hat{q}_t^f, \hat{m}_t^f, \hat{W}_t\}_{t \geq 0} \) and initial conditions \( \{M_0, i_0, n_0\} \) as given, chooses \( \{C_t, s_t^i, s_t^f, \Delta_t, M_{t+1}, i_{t+1}, v_t, n_{t+1}\}_{t \geq 0} \) to maximize its expected lifetime utility:

\[
\max \sum_{t=0}^{\infty} \beta^t \mathbb{E}_t^{-1} [U(c_t) - g_{st}^i a_p^f \varphi(q_t^f) - a_p^f n_t \varphi^f - a_s^i \Phi^i(s_t^i) - a_s^f \Phi^f(s_t^f) - a_p^f K(v_t)]
\]

subject to the following constraints for all \( t \geq 0 \):

\[
\begin{align*}
(1 - \Delta_t)M_t & \leq \frac{a_f}{a_b} c_t \leq s_t^f g_{st}^i a_b^f q_t^f, & (2.1) \\
\frac{(1 - \Delta_t)M_t}{a_f} & \geq \hat{m}_t^f, & (2.2) \\
q_t^f & = \min \{a_t, n_t\}, & (2.3) \\
a_t & \leq \hat{a}_t + \frac{1}{a_p} s_t^i g_{st}^i a_b^i \hat{q}_t^i, & (2.4) \\
q_{t+1}^f & \geq \hat{q}_{t+1}^f, & (2.5) \\
\frac{\Delta_t M_t}{a_b} & \geq n_t, & (2.6) \\
M_{t+1} & \leq M_t + \tau_t - s_t^i g_{st}^i a_b^f \hat{m}_t^i + g_{st}^i a_b^f \hat{q}_t^i + a_f^f \hat{n}_t \hat{P} \hat{W}_t \]
\]

\[
- s_t^f g_{st}^i a_b^f \hat{m}_t^f + g_{st}^f a_p^f \hat{m}_t^f - \hat{P} a_f^f \hat{W}_t n_t, & (2.7) \\
0 & \leq a_p^f [(1 - \delta_n) n_t + v_t \mu_t - n_{t+1}], & (2.8) \\
a_p^f i_{t+1} & \leq (1 - \delta_t) [a_p^f i_t + s_t^i g_{st}^i a_b^i \hat{q}_t^i - g_{st}^f a_p^f a_i]. & (2.9)
\]

The household’s utility function \( U(c) \) is strictly increasing and concave, and satisfies \( \lim_{c \to 0} c U'(c) = \infty \) and \( \lim_{c \to \infty} c U'(c) = 0 \). \( \varphi(q^f) \) is the intermediate goods producer’s disutility of producing intermediate goods. \( \varphi^f \) is the worker’s disutility of working in the finished goods market. Denote a buyer’s disutility of searching in the intermediate goods market by \( \Phi^i(s_t^i) \), and \( \Phi^f(s_t^f) \) is the buyer’s disutility of searching in the finished goods market. The function \( \Phi^i \) and \( \Phi^f \) have same properties, which satisfy \( \Phi' > 0 \) and \( \Phi'' > 0 \) for \( s > 0 \), and \( \Phi(0) = \Phi'(0) = 0 \). Finally, \( K(v_t) \) is the disutility of posting vacancies, which has the same properties as \( \Phi \).

Let \( I_{bt}^* \) (with measure \( s_t^i g_{st}^i a_b^i ) \) be the set of buyers who got desired matches in the intermediate
goods market in the period \( t \), and \( F^*_{bt} \) (with measure \( s^f_t \) \( g^f_{bt} a^f_t \)) be its counterpart in the finished goods market. Similarly, \( F^*_{pt} \) (with measure \( g^f_{st} a^f_p \)) is the set of entrepreneurs who had desirable matches in the finished goods market in the current period.

Constraint (2.1), (2.2) and (2.6) are standard in the large household model. Constraint (2.1) is a budget constraint which requires that the household’s consumption does not exceed the total amount of finished goods obtained by his buyers. Constraints (2.2) and (2.6) state that in order to successfully trade with a matched seller, the buyer should have enough money. Constraint (2.3) is the Leontief production function, which implies the usage of intermediate goods and labor are equal.

Constraint (2.4) and constraint (2.5) are similar to the money constraints (2.2) and (2.6). Constraint (2.4) is constraint on the usage of intermediate goods, so for the buyer to accept the trade, the producer is required to have enough intermediate goods. Similarly, the constraint (2.5) requires that matched output producers should have enough workers and intermediate goods to produce finished goods for their buyers.

Constraint (2.7) is the law of motion of money, which states that the money balance at the beginning of next period is no larger than the money balance carried from last period plus changes in the money balance. The changes in the money balance come from the lump-sum transfer received in the second sub period, the money spent by buyers in both goods markets, the money obtained by intermediate goods sellers, wages earned by workers and profits from output producers. Output producers obtain money if and only if their sellers can find desired matches, while wages have to be paid to workers at the end of the period regardless of whether they matched or not.

Constraint (2.8) is the law of motion of employment, which states that at the beginning of next period, the number of workers in each firm is no larger than the number of workers who still stay with the current job plus newly hired workers. The last constraint is the law of motion of inventories. The first term on the right-hand side is the inventory level carried from last period, the second term is newly obtained intermediate goods, and the last term is usage of intermediate goods by matched output producers. Since it always exist unmatched sellers in the finished goods market due to search friction, the household’s next period inventory level is no larger than unused intermediate goods depreciated at rate \( \delta_i \in (0, 1) \).

Denote the multipliers of money constraint (2.2) and (2.6) by \( \Lambda^f_t \) and \( \Lambda^i_t \) respectively. Let \( \Omega^f_{at} \) be the shadow price of (2.4) at the beginning of period \( t+1 \). We are interested in the equilibrium with positive inventory level, thus inventories have positive value in each period (ex. \( \Omega^f_{at} > 0 \)). Denote the multiplier of (2.5) by \( \Omega^f_{f} \). Since entrepreneurs get positive surplus
from trading finished goods, it is optimal for them to hire enough workers. Furthermore, condition (2.5) holds with equality in the equilibrium because of the Leontief production function. Let the shadow prices of (2.7), (2.8), and (2.9) at the beginning of period \( t + 1 \) be \( \Omega_{mt}, \Omega_{nt}, \) and \( \Omega_{it} \) respectively, which are measured in terms of the household’s period \( t \) utility.

By plugging \( c_t \) into the household’s utility function, substituting \( a_t \) by \( n_t \) and holding conditions (2.5), (2.7), (2.8) and (2.9) with equality, we can derive the first-order conditions respect to \( (M_{t+1}, i_{t+1}, n_{t+1}, s_{it}, s_{ft}, \Delta_t, v_t) \) as follows:

\[
\Omega_{Mt} = \beta \mathbb{E}[\Omega_{Mt+1} + s_{t+1}^f g_{bt+1} f \Lambda_{it}^f (1 - \Delta_{t+1}) + s_{t+1}^i g_{bt+1} i \Lambda_{it+1}^i \Delta_{t+1}],
\]

\[
\Omega_{it} = \beta \mathbb{E}[(1 - \delta_i) \Omega_{it+1} + g_{st+1} f \Omega_{ft+1} - a_{ft+1} f \Omega_{at+1}],
\]

\[
\Omega_{nt} = \beta \mathbb{E}[(1 - \delta_n) \Omega_{nt+1} + g_{st+1} f \Omega_{ft+1} - a_{ft+1} f \Omega_{at+1} - (1 - \delta_i) \Omega_{it+1} - \hat{P}_{t+1} \hat{W}_{t+1} \Omega_{Mt+1}],
\]

\[
\Phi'(s_{it}) = g_{bt} f a_{ft+1} f \Omega_{at} \hat{q}_{it} - \Omega_{Mt} \hat{m}_{it} + (1 - \delta_i) \Omega_{it} \hat{q}_{it},
\]

\[
\Phi'(s_{ft}) = g_{bt} f U'(C_t) - \omega_{ft+1} f \hat{q}_{ft} + \Lambda_{ft} f s_{ft} g_{bt+1},
\]

\[
\Omega_{nt} = K'(v_t)/\mu(\hat{v}_t).
\]

Condition (2.10) equates the opportunity cost of obtaining one more unit of money and the expected benefits of carrying one more unit of money into the next period, which are the increased shadow price of money and proportionally relaxed money constraints in both goods markets. Moreover, condition (2.15) requires that there is no arbitrage opportunity between two goods markets, so the expected benefits from relaxing two goods markets’ money constraints should be the same. Similarly, condition (2.11) equates the opportunity cost of hiring an additional unit of input inventory and the expected benefits of carrying it over to the next period, which are the increased shadow value of inventory and the relaxed intermediate goods usage constraint.

Condition (2.12) equates the opportunity cost of hiring an additional worker and the expected benefits generated by this worker in the next period. This opportunity cost does not only include the shadow value of labor and the wage paid in terms of period \( t + 1 \) utilities \( (\beta P_{t+1} W_{t+1} \Omega_{Mt+1}) \), but also includes the expected cost of tightening the period \( t + 1 \) intermediate goods usage constraint and the decreased expected shadow value of inventories discounted at the proper rate.
Condition (2.13) states that the opportunity cost of increasing search intensity involves labor cost and decreased shadow value of money, which equals the expected benefits of relaxing the intermediate goods usage constraint and increasing the expected shadow value of inventory. Condition (2.14) has the similar interpretation. The last condition equates the marginal cost of posting a vacancy and the expected benefits.

2.3 Terms of Trade

Let us specify the terms of trade for each market, which are taken as given by the household. Because this is a large household model, each agent in the household is negligible and can be viewed as an identity of a small measure $\varepsilon$. Since each agent’s contribution to the household is also negligible, we compute the terms of trade brought by each agent first, then take the limit $\varepsilon \to 0$. Variables with a bar refers to the buyer in the other household and are taken as given by the representative household.

2.3.1 Goods Markets

Denote the term of trade in the intermediate goods market by $(q_i^t\varepsilon, \bar{m}_i^t\varepsilon)$, where $q_i^t\varepsilon$ is the quantity of intermediate goods and $\bar{m}_i^t\varepsilon$ is the quantity of money. Thus the trading surpluses of these two agents to their households are:

Seller’s trade surplus: $\Omega_{Mt}\bar{m}_i^t\varepsilon - [\varphi(q_i^t + q_i^t\varepsilon) - \varphi(q_i^t)]$,

Buyer’s trade surplus: $[\bar{\Omega}_{at} + (1 - \delta_i)\bar{\Omega}_{it}]q_i^t\varepsilon - (\bar{\Lambda}_i^t + \bar{\Omega}_{Mt})\bar{m}_i^t\varepsilon$.

Normalizing surpluses by $\varepsilon$, then the terms of trade is determined by Nash Bargaining between buyer and seller with equal weights as in [26]:

$$\max_{\bar{m}_i^t, q_i^t} \left[ \Omega_{Mt}\bar{m}_i^t - \frac{\varphi(q_i^t + q_i^t\varepsilon) - \varphi(q_i^t)}{\varepsilon} \right]^{1/2} \left[ [\bar{\Omega}_{at} + (1 - \delta_i)\bar{\Omega}_{it}]q_i^t - (\bar{\Lambda}_i^t + \bar{\Omega}_{Mt})\bar{m}_i^t \right]^{1/2}.$$

After substituting $\bar{m}_i^t = P_i^t q_i^t$ and taking the limit $\varepsilon \to 0$ on the first-order conditions, we can get the following two conditions:

$$P_i^t(\bar{\Omega}_{Mt} + \bar{\Lambda}_i^t) = \bar{\Omega}_{at} + (1 - \delta_i)\bar{\Omega}_{it},$$

$$\varphi'(q_i^t) = P_i^t\bar{\Omega}_{Mt}.$$
The first condition equates the opportunity cost of spending money, \( P_t(\Omega_{Mt} + \bar{\Lambda}^i_t) \), with the benefits of obtaining \( \varepsilon \) additional unit of intermediate goods, which includes relaxing the intermediate goods usage constraint and increasing the shadow value of inventories. The second condition states that the marginal cost of production equals the shadow value of real money balance in the intermediate goods market. Denote the shadow value of real money balance in the finished goods market by \( \omega^i_t = P_t'\Omega_{Mt} \).

Similarly, denote the terms of trade by \((q^f_t \varepsilon, \bar{m}^f_t \varepsilon)\) for the finished goods market. The seller’s contribution to household \( h \)’s utility is \( \Omega_{Mt}\bar{m}^f_t \varepsilon - \Omega_{ft} q^f_t \varepsilon \). Since we assumed a Leontief production function, \( a_t \varepsilon = q^f_t \varepsilon \), which allows us to derive analytical results. Thus the trading surpluses to these two agents’ households are:

\[
\begin{align*}
\text{seller’s trade surplus:} & \quad \Omega_{Mt}\bar{m}^f_t \varepsilon - \Omega_{ft} q^f_t \varepsilon, \\
\text{buyer’s trade surplus:} & \quad U(\bar{c}_t + q^f_t \varepsilon) - U(\bar{c}_t) - (\bar{\Lambda}^f_t + \bar{\Omega}_{Mt}) \bar{m}^f_t \varepsilon.
\end{align*}
\]

Normalizing surpluses by \( \varepsilon \), then the terms of trade is determined by Nash Bargaining between buyer and seller with equal weights:

\[
\max_{\bar{m}^f_t, q^f_t} \left[ \Omega_{Mt}\bar{m}^f_t - \Omega_{ft} q^f_t \right]^{1/2} \times \left[ \frac{U(\bar{c}_t + q^f_t \varepsilon) - U(\bar{c}_t)}{\varepsilon} - (\bar{\Lambda}^f_t + \bar{\Omega}_{Mt}) \bar{m}^f_t \right]^{1/2}.
\]

Substituting \( \bar{m}^f_t = P_t q^f_t \), solving for the first-order conditions and taking the limit \( \varepsilon \to 0 \):

\[
\begin{align*}
(\bar{\Omega}_{Mt} + \bar{\Lambda}^f_t) P_t &= U'(c_t), \quad (2.19) \\
P_t' \Omega_{Mt} &= \Omega_{ft}, \quad (2.20)
\end{align*}
\]

In the finished goods market, the first condition equates the marginal utility of consumption with the opportunity cost of spending money. The second condition states that the shadow value of real money balance equals the opportunity cost of obtaining money. Denote the shadow value of real money balance in the finished goods market by \( \omega^f_t = P_t'\Omega_{Mt} \).

### 2.3.2 Labor Market

The term of trade in the labor market, denoted by \((W_{t+1} \varepsilon)\), is determined by Nash Bargaining between the producer and the unemployed worker. Assuming the producer’s bargaining weight is \( \sigma \), where \( \sigma \in (0, 1) \). The producer’s surplus of hiring \( \varepsilon \) more workers is
\( \{ \Omega_{nt} - \beta \mathbb{E}[(1 - \delta_n)\Omega_{nt+1}] \} \varepsilon \). Denoting the shadow value of real money balance by \( \omega_t = P_t \Omega_{Mt} \) and rearranging condition (2.12), we can reinterpret the producer’s surplus in terms of real money balances:

\[
\beta \mathbb{E}[\Omega_{ft+1}g_{st+1}^f - \omega_{t+1}W_{t+1} - a_p^f g_{st+1}^f \Omega_{at+1} - g_{st+1}^f \Omega_{at+1}(1 - \delta_i)] \varepsilon.
\]

The unemployed agent’s contribution to his household’s utility is \( \beta (\bar{\omega}_{t+1}W_{t+1} - \varphi^f) \varepsilon \), where \( W_{t+1} \varepsilon \) is the expected wage income in terms of the real money balance. Normalizing surpluses by \( \beta \varepsilon \), the wage rate maximizes the weighted Nash product of these two agent’s surpluses:

\[
\max_{W_{t+1}} \mathbb{E} \left[ [\Omega_{ft+1}g_{st+1}^f - \omega_{t+1}W_{t+1} - a_p^f g_{st+1}^f \Omega_{at+1} - g_{st+1}^f \Omega_{at+1}(1 - \delta_i)]^\sigma \right] \times [\bar{\omega}_{t+1}W_{t+1} - \varphi^f]^{1-\sigma}.
\]

The wage rate can be obtained after taking the limit \( \varepsilon \to 0 \) on the first-order condition:

\[
W_{t+1} = \mathbb{E} \left[ \frac{(1 - \sigma) g_{st+1}^f [\Omega_{ft+1} - a_p^f \Omega_{at+1} - \Omega_{at+1}(1 - \delta_i)]}{\omega_{t+1}} + \frac{\sigma \varphi^f}{\bar{\omega}_{t+1}} \right]. \tag{2.21}
\]

The wage rate equals the weighted sum of the expected future benefit of hiring \( \varepsilon \) more workers and the opportunity cost of working.

### 3 Equilibrium

#### 3.1 Characterization

In this section, we will describe the equilibrium and study the steady state effects of monetary policy on input inventories. Although households produce and consume different types of goods, they are identical in the sense that they have same utility function and production technologies. Thus we can define the symmetric search equilibrium as the following:

**Definition 1.** A symmetric search equilibrium is a sequence of household’s choices \( \{ \Gamma_{ht} \}_{t \geq 0} \), \( \Gamma_{ht} \equiv (c_t, s_t^l, s_t^f, \Delta_t, v_t, M_{t+1}, i_{t+1}, n_{t+1}^l)_h \), expected quantities in trade \( \{ \bar{X}_t \}_{t \geq 0}, \bar{X}_t \equiv (\hat{m}_t^l, \hat{q}_t^l, \hat{m}_t^f, \hat{q}_t^f, W_t) \), and the terms of trade \( \{ X_t \}_{t \geq 0} \), such that

1. all these variables are identical across households and relevant individuals;
2. Given \( \{X_t\}_{t \geq 0} \) and the initial conditions \( (M_0, i_0, n_0), \{\Gamma_{ht}\}_{t \geq 0} \) solves the household’s maximization problem, with \( (s^i, s^f, v) = (s^i, s^f, \dot{v}) \);

3. \( X_t \) satisfies (2.17)-(2.21);

4. \( \dot{X}_t = X_t \forall t \geq 0 \).

As is standard, in order for money to play the role of a medium of exchange, we have to restrict our equilibrium to \( \lambda^i \geq 0 \) and \( \lambda^f \geq 0 \). Similarly, we assume \( \Omega_{at} \geq 0 \), which requires that output producers prefer producing to hoarding intermediate goods in the second sub period. Thus condition (2.4) and the Leontief production function imply \( i_t = q^f_t - (1/a^f_p) s^i_t g^i_t a^i_p q^i_t \). These three restrictions will be verified in the steady state. Then suppress "hat" and "bar" for a symmetric equilibrium. Condition (2.7) is reduced to \( M_t + \tau_t = M_{t+1} \) under symmetry. Define the gross rate of money growth by \( \gamma_t \equiv M_{t+1}/M_t = (M_t + \tau_t)/M_t \). By substituting conditions (2.1), (2.4), (2.15) - (2.21) and \( P^f_t = M^f_t/q^f_t \) into conditions (2.8) - (2.14), we can eliminate \( (i, n, \Omega, W, \omega, \lambda^f, c, \Omega_a, \Omega_f) \), and the dynamic system is characterized in terms of \( (q^i, q^f, v, s^i, s^f, \omega^f, \Delta, \Omega, \lambda^i) \) by the following conditions:

\[
\begin{align*}
q^f_{t+1} &= (1 - \delta_n)q^f_t + v_t \mu(v_t), \\
q^i_{t+1} &= \frac{a^f_p}{a^i_p(s^i_{t+1})^\gamma} \{q^f_{t+1} - (1 - \delta_i)[1 - z^f B^f(s^f_t) \alpha]q^f_t \}, \\
\Delta_t &= \frac{q^i \varphi(q^i) a^i_p}{q^i \varphi(q^i) a^i_p + q^f_t \omega^f a^f_p}, \\
E \left[ \frac{(1 - \Delta^i + \gamma^f) \omega^f_{t+1}}{(1 - \Delta^i)} \right] &= \beta E \left[ \frac{q^f_{t+1}}{q^f_t} \right] \left\{ \omega^f_{t+1} + z^f(s^f_{t+1})^\alpha [U'(c_{t+1} - \omega^f_{t+1})] \right\}, \\
\Omega_{it} &= \beta \{(1 - \delta_i)\Omega_{it+1} + g^f_{st+1} a^f_p [\varphi(q^i_{t+1}) + \lambda^i_{t+1} - (1 - \delta_i)\Omega_{it+1}] \}, \\
k(v_t) &= \beta E \left[ (1 - \delta_n)k(v_{t+1}) \right] + \sigma g^f_{st} [\omega^f_t - a^f_p \varphi(q^i_t)] - a^f_p \lambda^i_t \tag{3.1}
\end{align*}
\]

The second equation of the dynamic system is of particular interest. This equation shows that the quantity of future traded intermediate goods \( q^f_{t+1} \) does not only depend on the quantity of future traded finished goods \( q^f_{t+1} \), but also depends on the corresponding current...
quantities \( q^f_t \). This is because the more finished goods traded today, the less inventories are accumulated for tomorrow, which further affects the decisions for the intermediate goods market next period. Thus if a monetary shock were to hit the economy, the corresponding effect would be propagated through the inventory channel.

The steady state system can be reduced to two equations with two unknowns as follows by rewriting \((s^f_t, \Omega_t^*, v^*, s^i, q^i, \lambda^i)\) as functions of \((\omega^f_t, q^f_t)\):

\[
z^f[s^f(\omega^f_t, q^f_t)]^\alpha = \frac{\gamma - \beta}{\beta} \frac{\omega^f_t}{U'(c(\omega^f_t, q^f_t)) - w^f_t}, \tag{3.2}
\]

\[
z^i(s^i(\omega^f_t, q^f_t))^\alpha \lambda^i(\omega^f_t, q^f_t) = \frac{\gamma - \beta}{\beta} \varphi'(q^i(\omega^f_t, q^f_t)). \tag{3.3}
\]

It is clear to see from above the two equations that the money growth rate has real effects on steady state variables. The details will be described in the next section. Moreover, a unique steady state, which satisfies \( \lambda^i > 0, \lambda^f > 0, \Omega_a > 0 \), can be pinned down by these two equations (see A for the proof).

### 3.2 Comparative Statics

Now let us explore the steady state effects of money growth on real variables. I will focus on steady state effects on input inventories, final sales, net inventory investment and inventory-to-sales ratio. According to the accounting identity (\( \text{GDP} = \text{Final Sales} + \text{Inventories} \)), the effect on GDP is straightforward. I will show that monetary policy has real effect on aggregate variables. In particular, the response of inventory investment is not monotonic in the money growth rate, which has a hump-shape. Moreover, contrary to one sector search model, the intensive margin effect in the finished goods market is also not monotonic across steady states, which is due to the existence of input inventories and a two-stage production. Now let us look at details market by market.

First, in the intermediate goods market, real money balances decrease as the money growth rate increases. Because intermediate goods producers’ surpluses decrease with real money balances, the quantity of goods traded in each match also decreases. This effect is the so-called intensive margin effect. In the meantime, there is also an extensive margin effect. As real money balances decrease, buyers’ surpluses from trade increase and buyers search more intensively in order to spend their money more quickly. Overall, the extensive margin
effect dominates the intensive margin effect, and intermediate goods sales increases with a higher money growth rate. To the contrary, if the money growth is high, at first the intensive margin effect dominates the extensive margin effect with a higher money growth rate, and intermediate goods sales decrease. Then buyers’ search intensities decrease if the money growth rate is high enough, and the intensive margin and the extensive margin work together to drive intermediate goods sales further down.

Second, in the finished goods market, as the money growth rate increases, real money balances decrease and buyers search more intensively as is standard. An unconventional result in this model is that the effect on the quantity of finished goods per match is not monotonic. The quantity of finished goods per match increase with the money growth rate if it is low, but decrease with the money growth rate if it is high. The reason is the following. Households hire more labor with a higher money growth rate if it is low. This can be shown by rearranging the sixth equation of (3.1):

\[
1 - \frac{\beta (1 - \delta_n)}{\beta \sigma} k(v^*) = z^f B^f (s^f)^{\alpha} \left[ \omega^f - a_p \lambda^{i*} - a_p \varphi'(q^{i*}) - (1 - a_p)(1 - \delta_i)\Omega_i^* \right] - \varphi^f. \tag{3.4}
\]

The right-hand side of the above equation is equal to the firm’s profitability from hiring, \( g^f_i [\Omega_f - a_p \Omega_a - \Omega_i (1 - \delta_i)] - \omega W \), which can be derived from equation (2.21). As shown in Shi (1998), households post more vacancies if and only if the firm’s profitability from hiring decreases with the value of real money balances which is \( \omega_f \) in this model for given output per trade. Thus employment increases with a higher money growth rate if it is low.

Since labor and intermediate goods are shared within each household, with Leontief production function, more intermediate goods are required for each output producer. Therefore, both intermediate goods sales and output per match increase. In the opposite, when the money growth rate is high, households hire less labor as the firm’s profitability from hiring decreasing with inflation, then both intermediate goods sales and output per match decrease. The above analysis can be summarized in the following proposition:

PROPOSITION 1. The steady state quantity of finished goods traded in each match increases with the money growth rate \( \gamma \) if \( \gamma \) is low but decreases with \( \gamma \) if \( \gamma \) is high (see Appendix B for a proof).

Third, since the steady state effect on output per match is not monotonic, it is straightforward to prove that the steady state effect on final sales is also not monotonic. If the money growth rate is low, both finished goods buyers’ search intensities and output per match increase with higher money growth rate. In this case, both the extensive margin effect and
the intensive margin effect have a positive effect on final sales, so final sales increase. To the contrary, if the money growth rate is high, output per match decreases with higher money growth rate. Buyers keep searching more intensively within a certain parameter range. Then finished goods buyers’ search intensities decrease if the inflation is high enough. Overall, the intensive margin effect dominates the extensive margin effect, thus final sales decrease with higher money growth rate.

Forth, equation (2.9) implies that net inventory investment, defined as \( NII = a_f^i [i_{t+1} - (1 - \delta_i)i_t] \), depends on final sales and intermediate goods sales. Because steady state effects of monetary policy on both intermediate goods sales and final sales are non-monotonic, it is easy to verify that steady state effects on net inventory investment are not monotonic either. Since the steady state net inventory investment is \( a_f^i \delta_i i^* \), the steady state inventory level \( a_f^i i^* \) is also non-monotonic.

The intuition of the non-monotonicity is the following. First of all, the level of inventory depends on both the quantity of intermediate goods held by each of output producer and the number of unmatched output producers. When the money growth rate is low, on one hand, intermediate goods sales increase with a higher money growth rate, which drive the quantity of intermediate goods held by each output producer up. On the other hand, the number of unmatched output producers decreases as finished goods buyers search more intensively. Second, there exists a two-stage production effect, which captures the transmission mechanism between finished goods market and intermediate goods market. The two-stage production effect transfers the marginal effects on final sales from downstream market back to upstream market and amplifies standard effects on intermediate goods sales. Because buyers and sellers are random matched in the finished goods market, households do not observe which output producers would get a match. Thus households have to adjust the input levels for all output producers in response to changes in the money growth rate. As a consequence, intermediate goods sales increase further with a higher money growth rate. This can be done by amplifying both the extensive margin and the intensive margin effects on intermediate goods sales.

The exaggerated intermediate goods sales also contribute to the changes in the inventory level through higher input levels held by unmatched output producers. Therefore, in this case, the two-stage production effect has a positive effect on inventories. Overall, because the effects on intermediate goods sales are exaggerated, the intensive margin effect on inventories dominates the extensive margin effect on inventories, and inventories increase with higher money growth rate. See figure 2 for an illustration. Actually, no matter the money growth rate is low or high, the two-stage production effect always guarantee that the intensive margin
effect on inventories dominates the extensive margin effect.

If the money growth rate is high, intermediate goods sales, output per match and final sales decrease with a higher money growth rate. The reduction in final sales induces a negative two-stage production effect on intermediate goods sales. Therefore, the intermediate goods held by unmatched output producers decrease further. Overall, the intensive margin effect on inventories dominates the extensive margin effect on inventories, and inventories decrease with higher money growth rate. See figure 3 for an illustration. Thus the inventory has hump-shaped response across steady states. The intuition for the steady state net inventory investment is similar. The above analysis can be summarized by the following proposition:

**PROPOSITION 2.** Both steady state input inventory level and steady state net inventory investment increase with the money growth rate $\gamma$ if $\gamma$ is low but decrease with $\gamma$ if $\gamma$ is high (see Appendix B for a proof).

Contrary to Shi’s model, the steady state effects of monetary policy on input inventories are not monotonic. The two-stage production and input inventories are the key to make the differences. Mathematically speaking, the difference is the sign on the shadow value of inventory is negative in equation (3.4) and positive in Shi’s model. Thus the shadow value of inventory decreases and inventories increase with the real money balance if the money growth rate is low in my model. While in order for employment increasing with the money growth rate, the shadow value of inventory has to increase with the real money balance in Shi’s model. Thus his model with output inventories is like the production smoothing model, output inventories smooth production and decreases with employment and sales. My model is like the stockout avoidance model. If the money growth rate is low, households have to adjust more intermediate goods in response to higher final sales. Since intermediate goods sales are exaggerated by the two-stage production effect, input inventories increase with the money growth rate if it is low in this paper. Together with the non-monotonic effects on the quantity of finished goods per match, the transmission mechanism of monetary policy is very different from one sector model.

The long run effects of monetary policy on output share similar properties as the steady state effects on final sales and net inventory investment. This is easy to be verified by using the accounting identity GDP = Final Sales + Inventories. Thus the Friedman rule does not hold in this model. The critical money growth rate is 1.475% which is much higher than the calibrated value 1.0691%, so both GDP and net inventory investment increase with mildly higher money growth rate in this model.

Finally, GDP volatility has substantially decreased since 1984 which leads to two decades
of "great moderation". About the same time, the inventory-to-sales ratio started to show a significant downward trend. Contrary to the empirical evidence found by Iacoviello, Schiantarelli and Schuh [12], this model predicts that the long run inventory-to-sales ratio decreases with money growth rate. So this paper rejects the hypothesis that monetary policy is the reason for the decline of the inventory-to-sales ratio since 80’s. This finding can be summarized in the following proposition:

**PROPOSITION 3.** The ratio of input inventory to final sales decreases with the money growth rate (see Appendix B for a proof).

### 4 Monetary Propagation

Since we have seen that shocks to the money growth rate have a real effect on input inventories, it is natural to look at how the monetary propagation mechanism works. In this section, I study the short-run dynamic responses of the model and examine the role of input inventory over business cycles.

#### 4.1 Calibration

The model is log-linearized and calibrated to match the annual US data. The sample period is 1959:I - 2010:IV. To be comparable to the literature, the inventory data are from manufacturing and trade. The average labor participation rate and average gross money growth rate are calculated from the sample. Set the discount factor $\beta = 0.96$, which implies the annual interest rate is 4%. In order to calibrate the model, we assume the utility function, the disutility functions of searching in the goods markets, the disutility function of producing intermediate goods, and the disutility function of posting vacancies have the following functional forms:

\[
U(c_t) = \frac{c_t^{1-\eta} - 1}{1 - \eta}; \quad \Phi^i(s^i_t) = \varphi^i(\varphi_0^i s^i_t)^{1+1/\epsilon_i};
\]

\[
\Phi^f(s^f_t) = \varphi^f(\varphi_0^f s^f_t)^{1+1/\epsilon_f}; \quad \varphi(q^i_t) = \frac{b}{2}(q^i_t)^2;
\]

\[
K(v_t) = K_0 v_t^2;
\]

---


where, \( \eta, \varphi_i, \varphi_0, \epsilon_i, \varphi_f, \varphi_0, \epsilon_f, b, K_0 \) are constants. The gross rate of money growth is defined by
\[
\gamma_t = \frac{M_{t+1}}{M_t},
\]
and assumed to follow an AR(1) process with parameters \( \rho = 0.6 \) and \( \sigma_g = 0.006 \):
\[
\gamma_t = (1 - \rho)\gamma^* + \rho \gamma_{t-1} + \epsilon_t, \quad \epsilon \sim (0, \sigma^2_e)
\]
I use M2 as the money stock since it is stable within the sample period. The steady state money growth rate is calculated as the sample average. We also need to determine the following parameters: \((A, \eta, \alpha, u, \delta, z_1, b, K_0, \varphi_i, \varphi_0, \varphi_f, \varphi_0)\).

As estimated by Berentsen, Menzio and Wright [2], the elasticity of the labor market matching function \( A \) is set to be 0.28, and \( \bar{\mu} \) equals 0.364. The finished good producers’ bargaining power \( \sigma \) is set to the same value to give workers 72\% of the rent, so the Hosios condition holds.\(^{13}\) Set \( \delta_n \) to 0.42 which is consistent with the finding of an average monthly separation rate of 0.034 from employment to unemployment in [28].

Parameters \((u, a_p^f, a_p^b, z_1^f, b, K_0, \varphi_i, \varphi_0, \varphi_f, \varphi_0)\) are calibrated to match the following targets:
1. the average labor participation rate is 0.636 from 1959 to 2010.
2. The average unemployment rate is 0.052 (See [29]).
3. The average velocity of M2 money stock is 1.798 from the same sample period.
4. The average input inventory to final sales ratio is 1.16 (see [12]).
5. The shopping time of the population is 11.17\% of the working time and the working time is 30\% of agents’ discretionary time (see [26], [33]).
6. The vacancy posting cost is \(3.72 \times 10^{-4}\) (see [2]).

Since I have an intermediate goods market in my model, I make the following assumptions in order to pin down the parameters \((a_p^i, a_b^i, z_1^i)\). First, assume the intermediate goods market tightness is \(B_i = 0.5\). Second, assume the time spent on searching intermediate goods is also 11.17\% of the working time. Third, assume the number of sellers in both goods markets are equal. Forth, assume the markup of finished goods is 20\%. Finally, assume the velocity of the money stock in the intermediate goods market is 0.2. The velocity is very low in the intermediate goods market, but 0.2 is the upper bound that works for this model.

Since \((\eta, \epsilon_i, \epsilon_f, \alpha, B_i, \text{markup})\) cannot be determined, sensitivity analysis on these parameters will be discussed in section 5. Right now, I only provide the parameter values which best fit the data. Set the elasticity of goods market matching functions \( \alpha \) to be 0.6, the elasticities of the disutility functions of searching \( \epsilon_i \) and \( \epsilon_f \) to be 16 and 0.06 respectively. Set the relative risk aversion \( \eta = 0.2 \), the depreciate rate of inventories \( \delta_i = 0.0072 \) and the markup to be 20\%. I normalize the number of workers hired by each firm to \( n = 1 \). Moreover, as

\(^{12}\)See Hornstein and Sarte (1998) for details. Also see Menner (2006) for similar estimation results.
\(^{13}\)Also see Shimer (2005) and Shimer and Rogerson (2010).
discussed in Shi (1998). I modify the model to incorporate the fixed investment, $FI = 0.269$, which is a constant fraction of aggregate sales. In this case consumption equals $c_t^f = (1 - FI) a_p^f B f z_f (s^f) ^{q_f^f}$, and the characteristics of the equilibrium described before do not change. The parameter values and corresponding targets are summarized in Table 1. The strategy of calibration is described in detail in Appendix C.

4.2 Impulse Responses

Figure 4 depicts the impulse responses of the equilibrium to one positive standard deviation shock to the money growth rate. The most striking results are that both the response of inventories and the response of output per match are hump-shaped. These positive responses rely on the persistence of the shock which keep buyers’ search intensities above the steady state in both goods markets.

Now let us look at the details of the propagation mechanism. When the shock hits the economy, the money growth rate increases and the real money balance falls immediately, which stimulates buyers to search more intensively in both goods markets. Since inventories depreciate each period and households are eager to consume, households allocate proportionally more money to the finished goods market, for example $\Delta_t$ decreases. Lower $\Delta_t$ stimulates finished goods buyers to search even more intensively, drives final sales to increase and reduces inventories in the first period.

Although input buyers’ search intensities increase and the input per match decreases, the overall effect on intermediate goods sales is zero in the first period. The reason is that inventories and labor are predetermined, so the Leontief production function implies that the output per match and the level of intermediate goods are also predetermined. Since condition (2.4) is binding, households adjust to the same level of input as before; and intermediate goods sales do not response to the shock in the first period. Finally, employment jumps in the first period, because households expect higher revenue in the future.

From the second period, since households want to smooth the consumption, finished goods buyers’ search intensities stay above the steady state. The two-stage production effect implies household need more intermediate goods, so input buyers’ search intensities and intermediate goods sales jump and stay above the steady state. Both final sales and intermediate goods sales stay above the steady state for more than ten periods. As less money is allocated to the finished goods market, intermediate goods sales increase more than final sales for several

\footnote{Also see Wang and Shi (2006).}
periods and inventories are built up. Then intermediate goods sales decrease toward the steady state at a higher speed than final sales, inventories start to fall. This process forms the hump-shaped response of inventories.

Another striking result is the hump-shaped response of the output per match, which stays above the steady state after the shock occurred. Because the expected profits increasing, so households hire more labors and increase inventory level during the transition. The two-stage production model enable the quantity of finished goods per match synchronize with the employment level during the transition. The overall effect on the output per match forms the hump-shaped response. Finally, employment peaks one period before the output per match peaks since labor is predetermined.

The search friction in the labor market, inventories and persistent shocks are all necessary for persistent responses. All of these three elements are important for the propagation mechanism. As discussed in Shi [26], if the labor market were a Walrasian market, labor would response immediately with full adjustment and mute the aggregate fluctuations. (also see Menner [20])

Input inventories are also important for the persistent responses. Remember the amplification which amplifies the quantity of intermediate goods sales. The existence of the two-stage production effect also implies that inventories move with final sales in the same direction; so less inventory holding dampens such an effect and mutes all the other aggregate fluctuations. This is in sharp contrast to Khan and Thomas [16], in which there is a tradeoff between final sales and inventories and inventory accumulation dampens the response of final sales in their model. The sluggish response of inventories sheds light on the "slow speed of adjustment" puzzle in the literature.

Moreover, compared to Shi [26], inventories work differently through the propagation mechanism in this paper. Input inventories propagate the responses through the two-stage production effect in my model, while in Shi’s model inventories are important for propagation because output inventories induce a shortage of future goods supply, which keeps buyers’ search intensities above the steady state. Therefore, the responses of input inventories are very different from that of output inventories. Shi’s model is like the production smoothing model which always has a tradeoff between the inventory and output. Thus inventories decrease whenever output increases during the transition which cannot replicate the stylized facts of inventories without technology shocks.15

Different responses of inventories imply that the policy implications in this paper are different from that in Shi’s model. In my model, monetary policy takes effect through decreased input
inventories during the boom. While in Shi’s model, monetary policy takes effect through increased output inventories. The reason is that beyond the classic search effects, a multi-stage production model provides an additional channel for the monetary policy to take effect, which is missing in a one sector model. On the other hand, because of the lack of an inventory effect in Shi’s model, less persistent shocks mute the effects of final sales and inventories, so persistent shocks are necessary in this model.

4.3 Model Predictions

Now let us look at the performance of the model. Table 2 reports the cyclical behavior predicted by the model. Contrary to Khan and Thomas (2007b), under preference shocks which are shocks to the money growth rate in this paper, this model can reproduce basic stylized facts of inventory: procyclical inventories, countercyclical inventory-to-sales ratio and more volatile output relative to final sales.

This model can produce 82% of the observed correlation between output and inventory-to-sales target, 67% of the observed correlation between final sales and inventory-to-sales ratio, 92% of the observed correlation between final sales and output and 77% of the observed correlation between final sales and inventory investment. As tested in Khan and Thomas (2007b), neither the (S, s) model nor the basic stockout avoidance model can produce a positive relationship between final sales and net inventory investment under a preference shock. Even after introducing idiosyncratic shocks, the generalized stockout avoidance model still can only generate very weak positive correlation, but such an improvement has to severely sacrifice the ability to match the long-run average inventory-to-sales ratio. Moreover, as mentioned in the last section, a one sector search model is unable to replicate inventory regularities without technology shocks.

The two-stage production effect and input inventories are the key for matching the data, because the two-stage production effect implies that input inventories move with final sales in the same direction during the transition. Except for the initial drop, input inventories stay above the steady state with output and final sales, which are the key to replicate the stylized facts of inventory.

The most noticeable departure from the data is the high volatility of net inventory investment relative to output. This exaggerated volatility of net inventory investment is because the model is counterfactually assumed to be hit only by monetary shocks. By introducing

\footnote{The response of inventories and that of final sales move in the opposite directions during the transition in [20, 26, 33].}
technology shocks to the input production function, inventories will not be driven down too much and the response of net inventory investment will be smoother. The model also exaggerates the correlation between output and inventory investment, and predicts that output is 1.41 more volatile than final sales, which is higher than 1.10 predicted by Kryvtson and Midrigan (2010). Moreover, the standard deviation of the inventory-to-sales target relative to that of final sales is underestimated, which is 2.4 times lower than what Kryvtson and Midrigan (2010) predicted.

4.4 The Role of Input Inventories over Business Cycles

Finally, I examine the role of input inventories over business cycles by varying the values of $\delta_i$ and $\epsilon_i$ respectively. The idea behind varying these two values is to see how the short-run dynamic system changes with the level of inventories. The results of both cases are the same. I found that input inventories do not only propagate business cycles but also amplify business cycles. Let us examine case by case.

First, firms hold more inventories if it is easier to search for intermediate goods, for example higher $\epsilon_i$. Figure 5 depicts how the dynamic responses change by increasing $\epsilon_i$ from the benchmark value to 2000. The idea is that search cost decreases in the intermediate goods market as $\epsilon_i$ increasing. Thus firms hold more inventories. The responses of output, final sales and net inventory investment become more volatile and more persistent. Except for finished goods buyers’ search intensities becoming less volatile, the quantities of goods traded in each match are more volatile in both goods markets, and input buyers’ search intensities also respond in a higher magnitude, also see Table 3 for standard deviations.

The intuition behind these results is straightforward. Inventories are storage devices which can be used by households to protect against inflation caused by persistent monetary shocks. Thus households would like to increase inventories with a lower searching cost. Remember that there exists a two-stage production effect which amplifies the quantity of intermediate goods sales. Such a two-stage production effect implies that higher inventories require more final sales and even more intermediate goods sales. As a result, the marginal effects are amplified in both goods market, and intermediate goods sales and final sales respond in a higher magnitude, which are contributed by more volatile $s^i$, $q^i$ and $q^f$. The standard deviation of $s^f$ stays the same for a large range of $\epsilon_i$. Moreover, although $s^f$ slightly decreases with higher $\epsilon_i$, the magnitude is not significant.

Second, Figure 6 depicts how the dynamic system changes if $\delta_i$ were 10 times larger. As you
can see from the figure, output, net inventory investment, sales in both goods markets and employment become less volatile and less persistent; also see Table 3 for standard deviations. Buyers in both goods markets become less active and trade less in each match. As a result, inventories also amplify business cycles in this case.

The intuition is similar to the former case. It is more costly for firms to carry inventories over a period with a higher depreciation rate, so households choose to lower inventory levels. Lower inventory levels require a negative two-stage production effect, so both intermediate goods sales and final sales decrease. As a result, intermediate goods sales and final sales become less volatile, which are supported by less volatile $s^i$, $s^f$, $q^i$ and $q^f$.

Overall, consistent with the inventory literature, this exercise suggests that inventories not only propagate business cycles, but also amplify aggregate fluctuations. I will further discuss the sensitivity analysis on $\epsilon_i$ in the next section.

5 Sensitivity Analysis

In the preceding sections, we have seen that the search model with shocks to the money growth rate can reproduce stylized facts of inventory. The model also suggests that input inventories not only propagate but also amplify aggregate fluctuations. In section 4.1, I used parameter values of $\epsilon_i$, $\epsilon_f$, $\alpha$, $\eta$, $B_i$ and markup for the best fit to data. In this section, I will examine the sensitivity of the quantitative results to different values of these parameters. Each parameter will be analyzed separately by holding other parameters unchanged, and the model is recalibrated to the data for each analysis. The results of sensitivity analysis are reported in Table 3.

The correlations predicted by this model are sensitive to changes in $\epsilon_i$, except the correlation between output and net inventory investment. The correlation between output and the inventory-to-sales ratio, the correlation between output and final sales, and the correlation between final sales and net inventory investment match the data better with relatively large value of $\epsilon_i$. In other words, $\epsilon_i$ cannot be too low, otherwise the last two correlations would turn negative. On the other hand, the model can only match the observed correlation between final sales and the inventory-to-sales ratio well with very low $\epsilon_i$. Thus, there is a tradeoff among matching correlations to those in the data. The correlation between output and net inventory investment is not sensitive to $\epsilon_i$; and the observed correlation is slightly overshot by the model for a large range of parameter values. Finally, as discussed in the previous section, higher $\epsilon_i$ induces higher inventory levels and greater volatility of the
responses.

The intuition is the following. Higher search friction in the intermediate goods market dampens the responses of intermediate goods sales. Since the search friction in the finished goods market remains unchanged, final sales increase more than intermediate goods sales increase. Then the inventory level is lower and the corresponding response would stay below the steady state in response to a monetary shock. As a result, the correlation between final sales and output, the correlation between final sales and net inventory investment and the correlation between output and the inventory-to-sales ratio become counterfactual as searching cost increasing.

The model matches the data well with relatively low $\epsilon_f$, or when the searching cost in the finished goods market is large. If $\epsilon_f$ were high, the correlation between final sales and net inventory investment would counterfactually turn negative. The intuition is the following: households can consume more with lower search cost. Since the search intensity in the intermediate goods market remains unchanged, final sales increase much more than intermediate goods sales increase in this case, which drives inventories below the steady state for the entire transition. Finally, the dynamics become more volatile to shocks with higher search intensity.

The quantitative results are sensitive to changes in the value of relative risk aversion. The model matches the data well with low $\eta$. If $\eta$ were high, the correlation between inventory-to-sales ratio and output and the correlation between inventory-to-sales ratio and final sales would become counterfactual. For the cyclical behavior, the model becomes less volatile to shocks with higher $\eta$. The intuition is the following, the motivation for smoothing consumption is strong with high $\eta$. Since the intermediate goods market’s search friction remains unchanged, final sales respond to the shock with a higher magnitude than intermediate goods sales, which drives inventories down so much that the correspondence response stays below the steady state. As a result, all correlations respond negatively to the increases in $\eta$.

Except for the correlation between output and the inventory-to-sales ratio, all the other correlations are relatively stable in response to the changes in $\alpha$. The correlation between output and net inventory investment still overshoots the observed correlation even when $\alpha$ is as low as 0.2. As $\alpha$ decreases, the matching rates decrease for both sellers and buyers in both goods markets. The responses of final sales and intermediate goods sales weaken. But the response of final sales decreases less than the response of intermediate goods sales because of the two-stage production effect. As a result, the response of inventory investment weakens and the correlation between output and the inventory-to-sales ratio is mismatched.
Moreover, the aggregate variables are less volatile with lower $\alpha$.

Similar to Wang and Shi (2006), the inventory regularities are insensitive to changes in the intermediate goods market’s buyer/seller ratio. The intuition is the following. Increased market tightness generates negative externalities on buyers’ matching probabilities and positive externalities on sellers’ matching probabilities. Such externalities have a positive effect and an effect on aggregate variables respectively. Since positive effects cancel out negative effects as a result of the Hosios condition, the overall results are insensitive to market tightness.

Generally speaking, the results are not quite sensitive to the changes in markups, though the responses are negative with higher markups. This is also true for the cyclical behaviors. The standard deviations do not change much for a large range of values. Nevertheless, the model fits the data better with lower markups, especially for the correlation between output and the inventory-to-sales ratio.

6 Concluding Remarks

In this paper, I propose a multi-stage production model with input inventories to investigate the transmission mechanism of monetary policy. I find that the multi-stage production has big impact on both the steady state effects and the transmission mechanism of monetary policies. In particular, monetary policy has non-monotonic real effects on input inventories and the quantity of finished goods per match, because the multi-stage production provides an additional channel for monetary policy to take effect. By extending Shi [26] to two sector model, I show that there exists a two-stage production effect which transfers classic search effects from the finished goods market to the intermediate goods market and amplifies the classic search effects on intermediate goods sales. The non-monotonicity arises from the interaction between classic search effects and the two-stage production effect.

Quantitative analysis shows that without technology shocks, the model can replicate stylized facts of inventory by solely relying on monetary shocks. The two-stage production effect and input inventories are the key for matching the data. Because classic effects on intermediate goods sales are amplified by the two-stage production effect, it implies that input inventories move with final sales in the same direction during the transition, which is the key to replicate procyclical inventory investment and positive correlation between final sales and inventory investment. Input inventories are also important in this model, because the responses of input and output inventories are different. Output inventories are like a buffer stock which
decreases with output. Therefore, large household model with single-stage production cannot replicate inventory regularities without technology shocks.

In order to match the data, the model requires a large degree of search friction in the finished goods market. If the searching cost is low in the finished goods market, the response of corresponding sales would be too strong, such that input inventories are driven down below the steady state during the transition. Therefore, the correlation between final sales and inventory investment counterfactually becomes negative. Moreover, consistent with most papers in the inventory literature, my model also predicts that inventories not only propagate busyness cycles, but also amplify aggregate fluctuations. On the other hand, the search cost in the intermediate goods market should be low relative to the finished goods market. If the search cost is high, it takes a long time for households to rebuild their inventory levels after shocks. The response of inventories would be too weak to replicate procyclical inventory investment, and final sales negatively correlated with inventory investment.

Overall, this paper sheds light on the importance of multi-stage production for understanding the transmission mechanism of monetary policy. This paper also provides clues for understanding different behaviors of input and output inventories. It is also interesting to explore the effects of technology shocks to different stages of the production process. I expect different responses if shock to different sector’s technology. This would be important for better understanding the transmission mechanism of real shocks.
References


7 Appendix A

In this section, I will prove there exists a unique steady state which satisfies \((\lambda^f; \Omega^i; \lambda^i) > 0\). The dynamic system (3.1) implies the steady state as follows:

\[
\begin{align*}
\mu^* \mu(v^*) &= \delta_n q^f, \\
q^i &= \frac{a^f_p}{a^i_b(s^{i*})^\alpha} \{1 - (1 - \delta_i)[1 - z^f B^f(s^{f*})^\alpha]\} q^f, \\
z^f(s^{f*})^\alpha &= \frac{\gamma - \beta}{\beta} \frac{w^f}{U'(c^*) - w^f}, \\
\Omega^i &= \frac{\beta z^f B^f(s^{f*})^\alpha a^f_p [\lambda^i + \varphi'(q^i*)]}{1 - \beta(1 - \delta_i)(1 - z^f B^f(s^{f*})^\alpha a^f_p)} \\
(1 - \beta(1 - \delta_n)) k(v^*) &= \beta z^f B^f(s^{f*})^\alpha \sigma[\omega^f - a^f_p \lambda^i - a^i_p \varphi'(q^i*)] \\
- (1 - a^f_p)(1 - \delta_i) \Omega^i &= \beta \sigma \varphi^f, \\
q^f(s^{i*}) &= g^i_b [g^f_s a^f_p \lambda^i - (1 - g^f_s a^f_p) \varphi'(q^i*)] + (1 - g^f_s a^f_p)(1 - \delta_i) \Omega^i_q^i \frac{q^f}{q^f}, \\
q^f(s^{f*}) &= z^f(s^{f*})^\alpha \{U'(c^*) - \omega^f\} q^f, \\
\gamma \varphi'(q^i*) &= \beta [\varphi'(q^i*) + z^i(s^{i*})^\alpha \lambda^i], \\
c^* &= a^f_p B^f z^f(s^{f*})^\alpha q^f^*. 
\end{align*}
\]

The steady state system can be reduced to equations (3.2) and (3.3) which repeated here for further use:

\[
\begin{align*}
\mu^* \mu(v^*) &= \delta_n q^f, \\
z^f(s^{f*}(\omega^f, q^f)) = \frac{\gamma - \beta}{\beta} \frac{w^f}{U'(c(\omega^f, q^f)) - w^f}, \\
z^i(s^{i*}(\omega^f, q^f)) \lambda^i(\omega^f, q^f) = \frac{\gamma - \beta}{\beta} \varphi'(\omega^f, q^f). 
\end{align*}
\]

Equation(3.2) and (3.3) gives a relation between \(\omega^f\) and \(q^f\), denoted \(q^f = q^f(\omega^f)\) and \(q^f\)
\[ q^f 2(\omega^f) \]. The steady state value \( \omega^f \) is a solution to \( Q^f 1(\omega^f) = Q^f 2(\omega^f) \). To ensure \( \lambda^f > 0 \), the solution must satisfy \( U'(c^*) \geq \omega^f + \Delta \), where \( \Delta > 0 \) is an arbitrarily small number. That is, we require \( q^f \leq q^f(\omega^f, \Delta) \)

\[ U'(c(\omega_f, q^f(\omega_f, \Delta))) = \omega_f + \Delta. \]

Since equation (3.2) is identical to the steady state equation (3.4) as describe in Shi (1998) [26], we ommited the proof here. Use Lemma 3.2 from his paper, we can prove that the function \( Q_f(\omega_f, \lambda^f) \) is well defined and has the following properties for sufficiently small \( \Delta > 0 \):

\[ Q_f(\omega_f, \delta) < 0, Q_f(\infty, \Delta) = 0, \text{ and } \lim_{\Delta \to 0} Q_f(0, \Delta) = \infty. \]

The function \( q^f 1(\omega_f) \) satisfies \( q^f 1'(\omega_f) < 0 \), \( q^f 1'(0) < \infty \) and \( q^f 1'(\infty) < 0 \). Furthermore, the two curves \( q^f 1(\omega_f) \) and \( Q_f(\omega_f, \Delta) \) have a unique intersection at a level denoted \( \omega_f^1(\Delta) \) which satisfies \( \lim_{\Delta \to 0} \omega_f^1(\Delta) = 0. \)

In order to prove the uniqueness, we also need to know the properties of \( q^f 2 \). Although the properties of \( q^f 2 \) are similar to what described in Lemma 3.3 in Shi's paper, the proofs are still provided since \( q^f 2 \) has different function form here.

We are going to prove that \( q^f 2 \) has the following properties: \( q^f 2(0) = 0, q^f 2(\infty) = 0, \) and \( q^f 2'(\omega_f) < 0 \) for sufficiently large \( \omega_f \). The two curves \( q^f 2(\omega_f) \) and \( Q_f(\omega_f, \Delta) \) have a unique intersection at a level denoted \( \omega_f^2(\Delta) \) which approaches infinity when \( \Delta \) approaches zero.

First, let us use the fifth equation of the steady state system (7.1) to show \( q^f 2(0) = 0. \) Since \( (\Omega_i, \Omega_n, s^f) > 0, \)

\[
a^f_p \varphi'(q_i^*) = \omega_i^* - a^f_p \lambda_i^* - (1 - a^f_z)(1 - \delta_i)\Omega_i^* - \beta \sigma \varphi^f
\]

\[
- (1 - \beta(1 - \delta_n))k(v^*)/\beta^z B^f(s^f)^\alpha \sigma
\]

\[
\leq \omega^f.
\]

Then \( \varphi'(0) = 0 \) implies \( q^i \to 0 \) as \( \omega_f \to 0. \) Together with \( q^f'(q^f) > 0 \) which derived from the second equation of the steady state system (7.1), then equation (3.3) implies \( \lim_{\omega_f \to 0} q^f(\omega_f) = 0. \)

\[16\]First, positive nominal interest rate is enough to ensure \( \lambda^f > 0. \) Second, positive nominal interest rate implies \( \gamma > \beta. \) From equation (3.2), we can get \( U'(c^*) \geq \omega^f + \Delta. \) So, if \( q^f \leq Q_f(\omega^f, \Delta), \) then \( U'(c(\omega_f^*, q^f)) \geq U'(c(\omega_f^*, Q_f(\omega_f^*, \Delta))) = \omega^f + \Delta. \)
By using the definition of $\omega^f 2(\Delta)$, it is straightforward to show that $\omega^f 2'(c(\Delta)) < 0$. Also because $q^f 2(c, \Delta)$ is an increasing function of $c$ and $q^f 2(\omega^f) = q^f 2(\omega^f 2(c(\Delta)))$, we can show that $q^f 2'(\omega^f) < 0$.

Second, let us prove that the two curves $q^f 2(\omega^f)$ and $Q(\omega^f, \Delta)$ have a unique intersection. After plugging the equation of $c^*$ into the seventh equation of the steady state system (7.1) and rearranging, we can get a useful equation:

$$\Phi^f(s^f*)s^f = [U'(c^*) - \omega^f*] \frac{c^*}{\alpha_p B^f}, \quad (7.2)$$

As the definition of $Q^f(\omega^f, \Delta)$, set $\omega^f = u'(c) - \Delta$. Then equation (7.2) implies $s^f$ is a function of $(c, \Delta)$: $\Phi^f(s^f*)(s^f*) = \frac{\Delta c^*}{\alpha_p B^f}$. Denote the solution for $s^f$ as $s^f(c, \Delta)$. Because $\Phi^f(0) = 0$, $\Phi^f(\bullet) > 0$ and $\Phi''(\bullet) > 0$, we can get $s^f(c, 0) = 0$, $s^f(\infty, 0) = \infty$, $s^f(0, \Delta) = 0$ and $s^f(c, \Delta) > 0$.

Plug $s^f(c, \Delta)$ into the seventh equation of the steady state system (7.1), we can get:

$$q^f = \frac{\Phi^f((s^f(c, \Delta))^{a-1})}{z^f(s^f(c, \Delta))^{a-1}\Delta}. \quad (7.3)$$

Thus $q^f$ can be rewritten as a function of $(c, \Delta)$. Since $\lim_{c \to 0}(7.3) \to 0$ and $\lim_{c \to \infty}(7.3) \to \infty$, $q^f(0, \Delta) = 0$ and $q^f(\infty, \Delta) = \infty$. Moreover, $q^f(c, \Delta) > 0$ since $\Phi''(\bullet) > 0$ and $s^f(c, \Delta) > 0$.

Multiplying $q^i$ on both sides of equation (3.3) and rearranging, we can get:

$$z^i(s^i(c, \Delta))^{a} q^i(c, \Delta) = \frac{\gamma - \beta \varphi^i(q^i(c, \Delta))q^i(c, \Delta)}{\lambda^i(c, \Delta)}. \quad (7.4)$$

The left-hand side of equation (7.4) can be rewritten by the second equation of the steady state system (7.1), and the right-hand side can be rewritten by plugging equation (3.2) into equation (3.3):

$$\text{LHS(7.4)} = a^i_p \{1 - (1 - \delta_i)[1 - z^i B^i(s^i(c, \Delta))^{a}]}q^i(c, \Delta)/a^i_b$$

$$\text{RHS(7.4)} = \frac{S^f(c, \Delta)q^f(c, \Delta)[u'(c) - \Delta]}{z^f(s^f(c, \Delta))^{a} \Delta}$$

where, $S^f(c, \Delta) = a^i_p \{1 - (1 - \delta_i)[1 - z^i B^i(s^i(c, \Delta))^{a}]}/a^i_b$.
Since \( s^f(0, \Delta) = 0, q^f(0, \Delta) = 0, s^f(\infty, \Delta) = \infty \) and \( q^f(\infty, \Delta) = \infty \), it is easy to see that \( \lim_{c \to 0} \text{LHS}(7.4) \to 0 \) and \( \lim_{c \to \infty} \text{LHS}(7.4) \to \infty \). Moreover, \( \text{LHS}(7.4) \) is a decreasing function of \( c \), because we have proven that \( s^f_c(c, \Delta) > 0 \) and \( q^f_c(c, \Delta) > 0 \).

The properties of the left-hand side of equation (7.4) can be derived by rearranging the equation for one more step:

\[
\text{RHS}(7.4) = \frac{S^f(c, \Delta)}{z^f(s^f(c, \Delta))^\alpha} q^f(c, \Delta) \frac{[u'(c) - \Delta] c}{c},
\]

where, \( S^f(c, \Delta) = a_p^f \{1 - (1 - \delta_i)[1 - z^f B^f(s^f(c, \Delta))\alpha]\}/a_i^p \).

It is easy to show that the right-hand side of equation (7.4) is a decreasing function of \( c \), because \( s^f_c(c, \Delta) > 0, q^f(c, \Delta)/c = 1/[a_i^f B^f(z^f(s^f(c, \Delta))\alpha)] \) and \( u'(c)c \) is a decreasing function of \( c \). Next, we can prove that \( \lim_{c \to 0} \text{RHS}(7.4) \to \infty \) and \( \lim_{c \to \infty} \text{RHS}(7.4) \to 0 \). First, \( \lim_{c \to 0} S^f(c, \Delta)/[z^f(s^f(c, \Delta))\alpha] \to \infty \) and \( \lim_{c \to \infty} S^f(c, \Delta)/[z^f(s^f(c, \Delta))\alpha] \to 0 \), because \( s^f(0, \Delta) = 0 \) and \( s^f(\infty, \Delta) = \infty \). Second, \( q^f(c, \Delta)/c \) which has similar properties as the previous term. Third, since \( \lim_{c \to 0} u'(c)c \to \infty \) and \( \lim_{c \to \infty} u'(c)c \to 0 \), \( \lim_{c \to 0}[u'(c) - \Delta]c \to \infty \) and \( \lim_{c \to \infty}[u'(c) - \Delta]c \to -\infty \).

As we have proven that the left-hand side of (7.4) is an increasing function of \( c \) and the right-hand side of (7.4) is a decreasing function of \( c \). Moreover, since \( \lim_{c \to 0} \text{LHS}(7.4) \to 0 \), \( \lim_{c \to \infty} \text{LHS}(7.4) \to \infty \), \( \lim_{c \to 0} \text{RHS}(7.4) \to \infty \) and \( \lim_{c \to \infty} \text{RHS}(7.4) \to 0 \), there must be a unique intersection between the two curves \( q^f 2(\omega^f) \) and \( Q(\omega^f, \Delta) \).

Third, we are going to prove \( q^f 2(\infty) = 0 \). Because \( Q^f(0, \Delta) \) is a positive constant and \( q^f 2(0) = 0 \) is proved, \( q^f 2(0) < Q^f(0, \Delta) \) and the curve \( q^f 2(\omega^f) \) must cross the curve \( Q^f(\omega^f, \Delta) \) from below if the two have a unique intersection. Moreover, because \( Q^f(\infty, \Delta) = 0 \) is proved in Lemma 1 and \( 0 \leq q^f 2(\omega^f) < Q^f(\omega^f, \Delta) \) for \( \omega^f < \omega^f 2(\Delta) \), \( 0 \leq q^f 2(\infty) < 0 \) for \( \omega^f < \omega^f 2(\Delta) \). Then \( q^f 2(\infty) = 0 \) since the existence of unique intersection and \( q^f 2(\omega^f) \) is continuous.

Finally, because \( s^f(c, 0) = 0 \) and \( s^f_c(c, \Delta) \), we can prove that \( \lim_{\Delta \to 0} c(\Delta) \to 0 \), and \( \lim_{\Delta \to 0} \omega^f 2(\Delta) \to \infty \). The properties of equation (3.2) and (3.3) imply that there exists a unique steady state for the model.
8 Appendix B

B.1. Proof of proposition 1

Now we are going to prove that the long-run effect of monetary policy on $q_f$ is not monotonic. Substituting equation (3.3) into equation (3.2), we can get:

$$\frac{z^i(s^i(\omega_f^*, q_f^*))^{\alpha} \lambda^i(\omega_f^*, q_f^*)}{z^f[s^f(\omega_f^*, q_f^*)]^{\alpha}} = \phi'(q^i(\omega_f^*, q_f^*))\frac{[U'(c(\omega_f^*, q_f^*)) - w_f^*]}{w_f^*},$$  \hspace{1cm} (8.1)

which substitute equation (3.3) to pin down the steady state with equation (3.2). It is easy to prove that equation (8.1) has similar properties as equation (3.3), which we omitted here. Equation (8.1) is independent of $\gamma$, while equation (3.2) will be shifted to the right as $\gamma \to \beta$ and to the left as $\gamma \to \infty$. Since $q^f 2(0) = 0$, $q^f 2(\infty) = 0$ and $q^f 2'(\omega_f) < 0$ for sufficiently large $\omega_f$, equation (8.1) is also hump-shaped. Thus $q_f$ decreases with $\gamma$ if $\gamma$ is high, but increases with $\gamma$ if it is low.

B.2. Proof of proposition 2

Since the difference between steady state net inventory investment and steady state inventory level is just a constant multiplier $\delta_i$, I only prove the long-run response of inventory investment. Equation (2.9) implies the steady state net inventory investment is the difference between the intermediate goods sales and the final sales discounted at a proper rate, namely,

$$NII = a_f^i \delta_i^* = (1 - \delta_i)[a_p^i B^i z^i(s^i)^{\alpha} q^i - a_f^i B^f z^f(s_f)^{\alpha} q_f^*].$$  \hspace{1cm} (8.2)

Substituting the intermediate goods sales by the second equation of (7.1), we can get the steady state net inventory investment is a function of $q_f$, $NII = (1 - \delta_i)a_p^i \delta_i q_f^*$, which is not monotonic in the money growth rate.
B.3. Proof of proposition 3

By rearranging equation (8.2), we can get a expression for steady state inventory-to-sales ratio:

\[
IS = \frac{a_p^f i^*}{a_p^f B_i^f z_i^f (s_i^f)^\alpha q_i^f} = \frac{(1 - \delta_i)}{\delta_i} \left[ \frac{a_p^f B_i^f z_i^f (s_i^f)^\alpha q_i^f}{a_p^f B_i^f z_i^f (s_i^f)^\alpha q_i^f} - 1 \right],
\]

The expression for the ratio between intermediate goods sales and final sales can be derived from the second equation of (7.1),

\[
\frac{\text{intermediate goods sales}}{\text{final sales}} = \frac{\delta_i}{B_i^f z_i^f (s_i^f)^\alpha} + (1 - \delta_i) \tag{8.3}
\]

Because \(\lim_{\omega_f \to \infty} s_i^f(\omega_f, q_i^f) \to 0\), the right-hand side of equation (8.3) approaches infinity as \(\gamma \to \beta\). Also because \(\lim_{\omega_f \to 0} s_i^f(\omega_f, q_i^f) \to \infty\), the right-hand side of equation (8.3) approaches zero as \(\gamma \to \infty\). Moreover, since \(s_i^f\) is a decreasing function of \(\omega_f\), the ratio between intermediate goods sales and final sales is an increasing function of \(\omega_f\). As a result, inventory-to-sales ratio decreases with money growth rate monotonically.
9 Appendix C

Now I am going to describe the steps to pin down parameters \((u, a_p, a_{fp}, a_d, a_{dp}, z_i, z_f, b, K_0, \varphi_i, \varphi_f, \varphi_0)\). The average labor participation rate \(LP = 0.636\), the average unemployment rate \(UR=0.052\) and the assumption \(a_p = a_{fp}\) can be used to pin down \((u, a_p, a_{fp})\):

\[
u = LP * UR = 0.0331.
\]

Since the households have measure one, the labor participation rate equals \(u + a_{fp}(1+n) + a_p\). Using the assumption \(a_p = a_{fp}\), the number of sellers in the intermediate goods market \(a_p\) is equate to its counterpart in the finished goods market \(a_{fp}\), and the steady state vacancies can be calculated as the following:

\[
a_{fp} = (LP - u)/(2+n) = 0.2010,
\]
\[
a_p = LP - u - a_{fp}(1+n) = 0.2010,
\]
\[
v^* = \left[\frac{\delta_n * n}{\hat{m}(a_{fp}/u)^{A-1}}\right]^{1/A}.
\]

From the average input inventory to final sales ratio \(i_{cf} = 1.16\) and the inventory depreciation rate \(\delta_i\), I can get a relation to be used later:

\[
\frac{q_f}{c_f} = (1 - \delta_i + i_{cf})/a_p(1 - \delta_i).
\]

(9.1)

From the definition of income velocity, we can drive the relationship between extensive marge and velocity

\[
v_c = p^f c^f / m^f
\]
\[
= c^f / (a_b^f q^f)
\]
\[
= a_{bf}^f z^f (s^f)^\alpha q_f
\]
\[
= z^f (s^f)^\alpha
\]

(9.2) (9.3) (9.4) (9.5)
Using equation (5.1) and (5.3), \(a_b^f\) and \(B_f\) can be determined:

\[
a_b^f = \frac{c^f}{v_c q^f} = 0.0515, \\
B_f = \frac{a_b^f}{a_p^f} = 0.2565.
\]

Using the fifth target, the shopping time of the population is 11.17% of the working time and the working time is 30% of agents discretionary time, the buyers’ steady state search intensity in the finished goods market \(s^f\) can be calculated. Once \(s^f\) is known, \(z^f\) and \(z_1^f\) can be determined by using equation (5.5).

\[
s^f = 0.1117 \times 0.3(a_p^f(1 + n) + a_p^i)/a_b^f = 0.3919, \\
z^f = z^f (s^f)^\alpha/(s^f)^\alpha, \\
z_1^f = z^f (B^f)^{1-\alpha} = 2.8975.
\]

The measure of buyers in the intermediate goods market \(a_b^i\), the constant in the goods matching function and the buyers’ search intensity in the intermediate goods market \(s^i\) can be determined by using assumption \(B^i = 0.5, v_c^i = 0.2\) and that the time spend on searching intermediate goods is also 11.17% of the working time.

\[
a_b^i = B^i a_p^i = 0.1005, \\
s^i = 0.1117 \times 0.3(a_p^f(1 + n) + a_p^i)/a_b^i = 0.2011, \\
z^i (s^i)^\alpha = v_c^i, \\
z^i = v_c^i/(s^i)^\alpha, \\
z_1^i = z^i (B^i)^{1-\alpha} = 0.6283.
\]

Now the steady state values of \(c^f, q^i, \omega_f\) can be calculated:

\[
c^f = (1 - FI)a_p^f B^f z^f (s^f)^\alpha n, \\
q^i = \frac{a_p^f(1 - (1 - \delta_i)(1 - z^f B^f (s^f)^\alpha))n}{a_b^i s_i^\alpha z_i}, \\
\omega_f = \frac{z^f (s^f)^\alpha \beta(1 - FI) (c^f)^{-\eta}}{\gamma^* - \beta + z^f (s^f)^\alpha \beta}.
\]
The markup can be used to pin down the constant in the disutility function of producing intermediate goods and the relative price:

\[ \text{markup} = \frac{p_i^f - p_i^i}{p_i^f} = 0.2, \]
\[ \frac{p_i^i}{p_i^f} = 2/3, \]
\[ b = \frac{p_i^i \omega_f}{p_i^f q^i} = 0.2113. \]

The steady state \( \lambda_i^*, \Omega_i^*, c_i^* \) can be calculated for further use. The constant in the disutility of posting vacancies \( K_0 \) can be pinned down by using \( K = 3.72 \times 10^{-4} \):

\[ \lambda_i^* = \left( \frac{\gamma^* b q^i}{\beta} - b q^i \right)/[z^i(s^i)\alpha], \]
\[ \Omega_i^* = \frac{\beta z^f B^f s^f a_p^f \lambda_i + b q_i}{1 - \beta (1 - \delta_i) (1 - z^f B^f (s^f)\alpha a_p^f)}, \]
\[ c_i^* = a_p^i B^i z^i(s^i) q^i, \]
\[ K_0 = K/v^2 = 4.1939e - 8. \]

Finally, the parameters \( (\varphi^i, \varphi_0^i, \varphi^f, \varphi_0^f) \) can be determined by using the steady state relations:

\[ \varphi^i = \frac{b}{2}(q^i)^2 = 2.2846, \]
\[ \varphi^f = \left( [\gamma^* a_p^f(1-\delta_i)\Omega_i - b q_i] \right)/[\gamma^* a_p^f(1+1/\epsilon_i)(s^i)^{1/\epsilon_i}], \]
\[ \varphi_0^i = \left( \frac{z^f(s^f)^{\alpha-1}(z^f B^f (s^f)\alpha a_p^f \lambda_i + (1 - z^f B^f (s^f)\alpha a_p^f)((1 - \delta_i) \Omega_i - b q_i))q^i}{(\varphi^f(1 + 1/\epsilon_i)(s^f)^{1/\epsilon_i})^{\gamma_f/(1+\gamma_f)}} \right)^{\gamma_f/(1+\gamma_f)} \]
\[ = 0.1833, \]
\[ \varphi_0^f = \left( \frac{z^f(s^f)^{\alpha-1}((1 - F I)(c_f)\eta - \omega_f)\eta}{(\varphi_f(1 + 1/\epsilon_f)(s^f)^{1/\epsilon_f})^{\gamma_f/(1+\gamma_f)}} \right)^{\gamma_f/(1+\gamma_f)} \]
\[ = 2.4213. \]
Table 1: Parameter Values and Targets

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Value</th>
<th>Target</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\beta$</td>
<td>0.96</td>
<td></td>
</tr>
<tr>
<td>$\rho$</td>
<td>0.6</td>
<td></td>
</tr>
<tr>
<td>$A$</td>
<td>0.28</td>
<td></td>
</tr>
<tr>
<td>$\sigma$</td>
<td>0.28</td>
<td></td>
</tr>
<tr>
<td>$\delta_n$</td>
<td>0.42</td>
<td></td>
</tr>
<tr>
<td>$\alpha$</td>
<td>0.8</td>
<td></td>
</tr>
<tr>
<td>$\epsilon_i$</td>
<td>0.06</td>
<td></td>
</tr>
<tr>
<td>$\epsilon_f$</td>
<td>0.2</td>
<td></td>
</tr>
<tr>
<td>$\eta$</td>
<td>0.0072</td>
<td></td>
</tr>
<tr>
<td>$\delta_i$</td>
<td>0.364</td>
<td></td>
</tr>
<tr>
<td>$\mu$</td>
<td>0.269</td>
<td></td>
</tr>
<tr>
<td>$FI$</td>
<td>0.5</td>
<td></td>
</tr>
<tr>
<td>$B_t$</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

Parameters

- **Unemployment $u$**: 0.0331, Avg. LP: 0.636
- **$i$-sellers $a^i_p$**: 0.2010, Avg. UR: 0.052
- **$f$-sellers $a^f_p$**: 0.2010, Avg. Velocity of M2: 1.798
- **$i$-buyers $a^i_b$**: 0.1005, Avg. IS ratio: 1.16
- **$f$-buyers $a^f_b$**: 0.0515
- **$z^i_1$**: 0.6283, Shopping time/Working time: 11.17
- **$z^f_1$**: 2.8975, Markup: 20%
- **$b$**: 0.2113, Working time/Discretionary time: 30%
- **$K_0$**: 4.1939e-008, Vacancy posting cost: $3.72 \times 10^{-4}$
- **$\varphi^i$**: 0.1833
- **$\varphi^o$**: 0.4381
- **$\varphi^f$**: 0.0188
- **$\varphi^o_f$**: 2.4213


Table 2: Model Predictions*

<table>
<thead>
<tr>
<th>Correlation</th>
<th>Data**</th>
<th>Model</th>
<th>Standard Deviations</th>
</tr>
</thead>
<tbody>
<tr>
<td>corr(GDP, IS)</td>
<td>-0.381</td>
<td>-0.311</td>
<td>(0.036)</td>
</tr>
<tr>
<td>corr(GDP, NII)</td>
<td>0.669</td>
<td>0.744</td>
<td>(0.017)</td>
</tr>
<tr>
<td>corr(FS, IS)</td>
<td>-0.700</td>
<td>-0.466</td>
<td>(0.027)</td>
</tr>
<tr>
<td>corr(FS, GDP)</td>
<td>0.943</td>
<td>0.869</td>
<td>(0.007)</td>
</tr>
<tr>
<td>corr(FS, NII)</td>
<td>0.411</td>
<td>0.316</td>
<td>(0.015)</td>
</tr>
<tr>
<td>$\sigma(FS)/\sigma(GDP)$</td>
<td>0.710</td>
<td>0.709</td>
<td>(0.017)</td>
</tr>
<tr>
<td>$\sigma(NII)/\sigma(GDP)$</td>
<td>0.095</td>
<td>0.359</td>
<td>(1.812)</td>
</tr>
<tr>
<td>$\sigma(IS)/\sigma(GDP)$</td>
<td>0.545</td>
<td>0.280</td>
<td>(0.003)</td>
</tr>
</tbody>
</table>

* Numbers in parentheses are standard deviations over 1000 simulations. GDP: real GDP; IS: inventory-to-sales ratio; NII: net inventory investment; FS: final sales.

** corr(FS, IS) is estimated conditional on monetary shocks, see Kryvtsov and Midrigan (2010) [17] for details. The other correlations are borrowed from Khan and Thomas (2007b) [16].

*** NII are calculated as a share of GDP in order to compare the results with that in [16].

43
Table 3: Role of Inventories*

<table>
<thead>
<tr>
<th></th>
<th>$\epsilon_i = 16$**</th>
<th>$\epsilon_i = 2000$</th>
<th>$\delta_i = 0.0072$**</th>
<th>$\delta_i = 0.072$</th>
</tr>
</thead>
<tbody>
<tr>
<td>corr(GDP, IS)</td>
<td>-0.309</td>
<td>-0.328</td>
<td>-0.309</td>
<td>-0.382</td>
</tr>
<tr>
<td></td>
<td>(0.035)</td>
<td>(0.034)</td>
<td>(0.035)</td>
<td>(0.034)</td>
</tr>
<tr>
<td>corr(GDP, NII)</td>
<td>0.745</td>
<td>0.744</td>
<td>0.745</td>
<td>0.805</td>
</tr>
<tr>
<td></td>
<td>(0.017)</td>
<td>(0.016)</td>
<td>(0.017)</td>
<td>(0.012)</td>
</tr>
<tr>
<td>corr(FS, IS)</td>
<td>-0.464</td>
<td>-0.465</td>
<td>-0.464</td>
<td>-0.529</td>
</tr>
<tr>
<td></td>
<td>(0.026)</td>
<td>(0.026)</td>
<td>(0.026)</td>
<td>(0.025)</td>
</tr>
<tr>
<td>corr(FS, GDP)</td>
<td>0.868</td>
<td>0.873</td>
<td>0.868</td>
<td>0.888</td>
</tr>
<tr>
<td></td>
<td>(0.007)</td>
<td>(0.007)</td>
<td>(0.007)</td>
<td>(0.006)</td>
</tr>
<tr>
<td>corr(FS, NII)</td>
<td>0.317</td>
<td>0.324</td>
<td>0.317</td>
<td>0.441</td>
</tr>
<tr>
<td></td>
<td>(0.015)</td>
<td>(0.015)</td>
<td>(0.015)</td>
<td>(0.013)</td>
</tr>
<tr>
<td>$\sigma(GDP)$</td>
<td>0.036</td>
<td>0.039</td>
<td>0.036</td>
<td>0.033</td>
</tr>
<tr>
<td></td>
<td>(0.001)</td>
<td>(0.001)</td>
<td>(0.001)</td>
<td>(0.001)</td>
</tr>
<tr>
<td>$\sigma(NII)$</td>
<td>2.302</td>
<td>2.428</td>
<td>2.302</td>
<td>0.224</td>
</tr>
<tr>
<td></td>
<td>(0.055)</td>
<td>(0.062)</td>
<td>(0.055)</td>
<td>(0.005)</td>
</tr>
<tr>
<td>$\sigma(n)$</td>
<td>0.024</td>
<td>0.026</td>
<td>0.024</td>
<td>0.021</td>
</tr>
<tr>
<td></td>
<td>(0.001)</td>
<td>(0.001)</td>
<td>(0.001)</td>
<td>(0.001)</td>
</tr>
</tbody>
</table>

* Numbers in parentheses are standard deviations over 1000 simulations. GDP: real GDP; IS: inventory-to-sales ratio; NII: net inventory investment; FS: final sales; n: employment level of each output producer.

** Parameter values with a star are benchmark parameter values.
Table 4: Sensitivity analysis: $\epsilon_i^*$

<table>
<thead>
<tr>
<th>$\epsilon_i$</th>
<th>0.1</th>
<th>0.5</th>
<th>8</th>
<th>16**</th>
<th>100</th>
<th>Data</th>
</tr>
</thead>
<tbody>
<tr>
<td>corr(GDP, IS)</td>
<td>0.406</td>
<td>-0.179</td>
<td>-0.296</td>
<td>-0.308</td>
<td>-0.324</td>
<td>-0.381</td>
</tr>
<tr>
<td></td>
<td>(0.017)</td>
<td>(0.034)</td>
<td>(0.034)</td>
<td>(0.035)</td>
<td>(0.034)</td>
<td></td>
</tr>
<tr>
<td>corr(GDP, NII)</td>
<td>0.701</td>
<td>0.687</td>
<td>0.745</td>
<td>0.746</td>
<td>0.745</td>
<td>0.669</td>
</tr>
<tr>
<td></td>
<td>(0.015)</td>
<td>(0.015)</td>
<td>(0.017)</td>
<td>(0.016)</td>
<td>(0.016)</td>
<td></td>
</tr>
<tr>
<td>corr(FS, IS)</td>
<td>-0.853</td>
<td>-0.771</td>
<td>-0.468</td>
<td>-0.463</td>
<td>-0.465</td>
<td>-0.700</td>
</tr>
<tr>
<td></td>
<td>(0.012)</td>
<td>(0.004)</td>
<td>(0.025)</td>
<td>(0.025)</td>
<td>(0.025)</td>
<td></td>
</tr>
<tr>
<td>corr(FS, GDP)</td>
<td>-0.242</td>
<td>0.618</td>
<td>0.864</td>
<td>0.868</td>
<td>0.872</td>
<td>0.943</td>
</tr>
<tr>
<td></td>
<td>(0.043)</td>
<td>(0.029)</td>
<td>(0.007)</td>
<td>(0.007)</td>
<td>(0.007)</td>
<td></td>
</tr>
<tr>
<td>corr(FS, NII)</td>
<td>-0.862</td>
<td>-0.146</td>
<td>0.309</td>
<td>0.317</td>
<td>0.323</td>
<td>0.411</td>
</tr>
<tr>
<td></td>
<td>(0.013)</td>
<td>(0.024)</td>
<td>(0.016)</td>
<td>(0.015)</td>
<td>(0.014)</td>
<td></td>
</tr>
<tr>
<td>$\sigma(GDP)$</td>
<td>0.001</td>
<td>0.010</td>
<td>0.034</td>
<td>0.036</td>
<td>0.039</td>
<td>-</td>
</tr>
<tr>
<td></td>
<td>(0.000)</td>
<td>(0.000)</td>
<td>(0.001)</td>
<td>(0.001)</td>
<td>(0.001)</td>
<td></td>
</tr>
<tr>
<td>$\sigma(NII)$</td>
<td>0.216</td>
<td>0.964</td>
<td>2.186</td>
<td>2.305</td>
<td>2.412</td>
<td>-</td>
</tr>
<tr>
<td></td>
<td>(0.005)</td>
<td>(0.025)</td>
<td>(0.051)</td>
<td>(0.056)</td>
<td>(0.059)</td>
<td></td>
</tr>
<tr>
<td>$\sigma(n)$</td>
<td>0.001</td>
<td>0.005</td>
<td>0.022</td>
<td>0.024</td>
<td>0.025</td>
<td>-</td>
</tr>
<tr>
<td></td>
<td>(0.000)</td>
<td>(0.000)</td>
<td>(0.001)</td>
<td>(0.001)</td>
<td>(0.001)</td>
<td></td>
</tr>
</tbody>
</table>

* Numbers in parentheses are standard deviations over 1000 simulations. GDP: real GDP; IS: inventory-to-sales ratio; NII: net inventory investment; FS: final sales; n: employment level of each output producer.

** Parameter values with a star are benchmark parameter values.
Table 5: Sensitivity analysis: $\epsilon_f^*$

<table>
<thead>
<tr>
<th>$\epsilon_f$</th>
<th>0.04</th>
<th>0.06*</th>
<th>1</th>
<th>1.5</th>
<th>3</th>
<th>Data</th>
</tr>
</thead>
<tbody>
<tr>
<td>corr(GDP, IS)</td>
<td>-0.308</td>
<td>-0.308</td>
<td>-0.384</td>
<td>-0.398</td>
<td>-0.409</td>
<td>-0.381</td>
</tr>
<tr>
<td></td>
<td>(0.037)</td>
<td>(0.035)</td>
<td>(0.035)</td>
<td>(0.033)</td>
<td>(0.033)</td>
<td></td>
</tr>
<tr>
<td>corr(GDP, NII)</td>
<td>0.738</td>
<td>0.746</td>
<td>0.598</td>
<td>0.575</td>
<td>0.550</td>
<td>0.669</td>
</tr>
<tr>
<td></td>
<td>(0.017)</td>
<td>(0.016)</td>
<td>(0.015)</td>
<td>(0.015)</td>
<td>(0.015)</td>
<td></td>
</tr>
<tr>
<td>corr(FS, IS)</td>
<td>-0.407</td>
<td>-0.463</td>
<td>-0.961</td>
<td>-0.976</td>
<td>-0.989</td>
<td>-0.700</td>
</tr>
<tr>
<td></td>
<td>(0.030)</td>
<td>(0.025)</td>
<td>(0.001)</td>
<td>(0.001)</td>
<td>(0.000)</td>
<td></td>
</tr>
<tr>
<td>corr(FS, GDP)</td>
<td>0.878</td>
<td>0.868</td>
<td>0.547</td>
<td>0.513</td>
<td>0.471</td>
<td>0.943</td>
</tr>
<tr>
<td></td>
<td>(0.006)</td>
<td>(0.007)</td>
<td>(0.034)</td>
<td>(0.034)</td>
<td>(0.035)</td>
<td></td>
</tr>
<tr>
<td>corr(FS, NII)</td>
<td>0.326</td>
<td>0.317</td>
<td>-0.343</td>
<td>-0.407</td>
<td>-0.477</td>
<td>0.411</td>
</tr>
<tr>
<td></td>
<td>(0.014)</td>
<td>(0.015)</td>
<td>(0.022)</td>
<td>(0.021)</td>
<td>(0.020)</td>
<td></td>
</tr>
<tr>
<td>$\sigma(GDP)$</td>
<td>0.036</td>
<td>0.036</td>
<td>0.048</td>
<td>0.050</td>
<td>0.053</td>
<td>-</td>
</tr>
<tr>
<td></td>
<td>(0.001)</td>
<td>(0.001)</td>
<td>(0.002)</td>
<td>(0.002)</td>
<td>(0.002)</td>
<td></td>
</tr>
<tr>
<td>$\sigma(NII)$</td>
<td>2.196</td>
<td>2.305</td>
<td>5.250</td>
<td>5.789</td>
<td>6.534</td>
<td>-</td>
</tr>
<tr>
<td></td>
<td>(0.053)</td>
<td>(0.056)</td>
<td>(0.139)</td>
<td>(0.151)</td>
<td>(0.166)</td>
<td></td>
</tr>
<tr>
<td>$\sigma(n)$</td>
<td>0.025</td>
<td>0.024</td>
<td>0.012</td>
<td>0.010</td>
<td>0.008</td>
<td>-</td>
</tr>
<tr>
<td></td>
<td>(0.001)</td>
<td>(0.001)</td>
<td>(0.001)</td>
<td>(0.001)</td>
<td>(0.000)</td>
<td></td>
</tr>
</tbody>
</table>

* Numbers in parentheses are standard deviations over 1000 simulations. GDP: real GDP; IS: inventory-to-sales ratio; NII: net inventory investment; FS: final sales; n: employment level of each output producer.

** Parameter values with a star are benchmark parameter values.
Table 6: Sensitivity analysis: $\alpha^*$

<table>
<thead>
<tr>
<th>$\alpha$</th>
<th>0.2</th>
<th>0.4</th>
<th>0.5</th>
<th>0.6</th>
<th>0.8**</th>
<th>Data</th>
</tr>
</thead>
<tbody>
<tr>
<td>corr(GDP, IS)</td>
<td>-0.167</td>
<td>-0.190</td>
<td>-0.218</td>
<td>-0.247</td>
<td>-0.308</td>
<td>-0.381</td>
</tr>
<tr>
<td></td>
<td>(0.035)</td>
<td>(0.036)</td>
<td>(0.035)</td>
<td>(0.036)</td>
<td>(0.035)</td>
<td></td>
</tr>
<tr>
<td>corr(GDP, NII)</td>
<td>0.739</td>
<td>0.742</td>
<td>0.743</td>
<td>0.743</td>
<td>0.746</td>
<td>0.669</td>
</tr>
<tr>
<td></td>
<td>(0.017)</td>
<td>(0.017)</td>
<td>(0.017)</td>
<td>(0.017)</td>
<td>(0.016)</td>
<td></td>
</tr>
<tr>
<td>corr(FS, IS)</td>
<td>-0.430</td>
<td>-0.409</td>
<td>-0.418</td>
<td>-0.432</td>
<td>-0.463</td>
<td>-0.700</td>
</tr>
<tr>
<td></td>
<td>(0.020)</td>
<td>(0.023)</td>
<td>(0.024)</td>
<td>(0.025)</td>
<td>(0.025)</td>
<td></td>
</tr>
<tr>
<td>corr(FS, GDP)</td>
<td>0.830</td>
<td>0.846</td>
<td>0.853</td>
<td>0.859</td>
<td>0.868</td>
<td>0.943</td>
</tr>
<tr>
<td></td>
<td>(0.011)</td>
<td>(0.009)</td>
<td>(0.008)</td>
<td>(0.008)</td>
<td>(0.007)</td>
<td></td>
</tr>
<tr>
<td>corr(FS, NII)</td>
<td>0.238</td>
<td>0.271</td>
<td>0.284</td>
<td>0.295</td>
<td>0.317</td>
<td>0.411</td>
</tr>
<tr>
<td></td>
<td>(0.015)</td>
<td>(0.015)</td>
<td>(0.015)</td>
<td>(0.016)</td>
<td>(0.015)</td>
<td></td>
</tr>
<tr>
<td>$\sigma(GDP)$</td>
<td>0.005</td>
<td>0.016</td>
<td>0.021</td>
<td>0.026</td>
<td>0.036</td>
<td>-</td>
</tr>
<tr>
<td></td>
<td>(0.000)</td>
<td>(0.001)</td>
<td>(0.001)</td>
<td>(0.001)</td>
<td>(0.001)</td>
<td></td>
</tr>
<tr>
<td>$\sigma(NII)$</td>
<td>0.369</td>
<td>1.040</td>
<td>1.363</td>
<td>1.682</td>
<td>2.305</td>
<td>-</td>
</tr>
<tr>
<td></td>
<td>(0.009)</td>
<td>(0.025)</td>
<td>(0.033)</td>
<td>(0.043)</td>
<td>(0.056)</td>
<td></td>
</tr>
<tr>
<td>$\sigma(n)$</td>
<td>0.003</td>
<td>0.010</td>
<td>0.013</td>
<td>0.017</td>
<td>0.024</td>
<td>-</td>
</tr>
<tr>
<td></td>
<td>(0.000)</td>
<td>(0.000)</td>
<td>(0.001)</td>
<td>(0.001)</td>
<td>(0.001)</td>
<td></td>
</tr>
</tbody>
</table>

* Numbers in parentheses are standard deviations over 1000 simulations. GDP: real GDP; IS: inventory-to-sales ratio; NII: net inventory investment; FS: final sales; n: employment level of each output producer.

** Parameter values with a star are benchmark parameter values.
Table 7: Sensitivity analysis: $\eta^*$

<table>
<thead>
<tr>
<th>$\eta$</th>
<th>0.2**</th>
<th>0.4</th>
<th>0.8</th>
<th>2</th>
<th>4</th>
<th>Data</th>
</tr>
</thead>
<tbody>
<tr>
<td>corr(GDP, IS)</td>
<td>-0.308</td>
<td>-0.170</td>
<td>-0.010</td>
<td>0.198</td>
<td>0.367</td>
<td>-0.381</td>
</tr>
<tr>
<td></td>
<td>(0.035)</td>
<td>(0.033)</td>
<td>(0.028)</td>
<td>(0.022)</td>
<td>(0.019)</td>
<td></td>
</tr>
<tr>
<td>corr(GDP, NII)</td>
<td>0.746</td>
<td>0.802</td>
<td>0.848</td>
<td>0.890</td>
<td>0.922</td>
<td>0.669</td>
</tr>
<tr>
<td></td>
<td>(0.016)</td>
<td>(0.014)</td>
<td>(0.011)</td>
<td>(0.008)</td>
<td>(0.006)</td>
<td></td>
</tr>
<tr>
<td>corr(FS, IS)</td>
<td>-0.463</td>
<td>-0.377</td>
<td>-0.319</td>
<td>-0.234</td>
<td>-0.056</td>
<td>-0.700</td>
</tr>
<tr>
<td></td>
<td>(0.025)</td>
<td>(0.021)</td>
<td>(0.011)</td>
<td>(0.005)</td>
<td>(0.020)</td>
<td></td>
</tr>
<tr>
<td>corr(FS, GDP)</td>
<td>0.868</td>
<td>0.836</td>
<td>0.782</td>
<td>0.680</td>
<td>0.598</td>
<td>0.943</td>
</tr>
<tr>
<td></td>
<td>(0.007)</td>
<td>(0.008)</td>
<td>(0.010)</td>
<td>(0.015)</td>
<td>(0.018)</td>
<td></td>
</tr>
<tr>
<td>corr(FS, NII)</td>
<td>0.317</td>
<td>0.343</td>
<td>0.332</td>
<td>0.270</td>
<td>0.242</td>
<td>0.411</td>
</tr>
<tr>
<td></td>
<td>(0.015)</td>
<td>(0.016)</td>
<td>(0.017)</td>
<td>(0.017)</td>
<td>(0.019)</td>
<td></td>
</tr>
<tr>
<td>$\sigma(GDP)$</td>
<td>0.036</td>
<td>0.032</td>
<td>0.024</td>
<td>0.014</td>
<td>0.009</td>
<td>-</td>
</tr>
<tr>
<td></td>
<td>(0.001)</td>
<td>(0.001)</td>
<td>(0.001)</td>
<td>(0.000)</td>
<td>(0.000)</td>
<td></td>
</tr>
<tr>
<td>$\sigma(NII)$</td>
<td>2.305</td>
<td>2.233</td>
<td>1.887</td>
<td>1.286</td>
<td>0.930</td>
<td>-</td>
</tr>
<tr>
<td></td>
<td>(0.056)</td>
<td>(0.054)</td>
<td>(0.049)</td>
<td>(0.032)</td>
<td>(0.024)</td>
<td></td>
</tr>
<tr>
<td>$\sigma(n)$</td>
<td>0.024</td>
<td>0.019</td>
<td>0.013</td>
<td>0.007</td>
<td>0.005</td>
<td>-</td>
</tr>
<tr>
<td></td>
<td>(0.001)</td>
<td>(0.001)</td>
<td>(0.001)</td>
<td>(0.000)</td>
<td>(0.000)</td>
<td></td>
</tr>
</tbody>
</table>

* Numbers in parentheses are standard deviations over 1000 simulations. GDP: real GDP; IS: inventory-to-sales ratio; NII: net inventory investment; FS: final sales; n: employment level of each output producer.

** Parameter values with a star are benchmark parameter values.
Table 8: Sensitivity analysis: $B_i$*

<table>
<thead>
<tr>
<th>$B_i$</th>
<th>0.1</th>
<th>0.3</th>
<th>0.5**</th>
<th>0.7</th>
<th>0.9</th>
<th>Data</th>
</tr>
</thead>
<tbody>
<tr>
<td>corr(GDP, IS)</td>
<td>-0.311</td>
<td>-0.311</td>
<td>-0.308</td>
<td>-0.312</td>
<td>-0.310</td>
<td>-0.381</td>
</tr>
<tr>
<td></td>
<td>(0.036)</td>
<td>(0.036)</td>
<td>(0.035)</td>
<td>(0.034)</td>
<td>(0.037)</td>
<td></td>
</tr>
<tr>
<td>corr(GDP, NII)</td>
<td>0.744</td>
<td>0.745</td>
<td>0.746</td>
<td>0.744</td>
<td>0.745</td>
<td>0.669</td>
</tr>
<tr>
<td></td>
<td>(0.017)</td>
<td>(0.017)</td>
<td>(0.016)</td>
<td>(0.016)</td>
<td>(0.017)</td>
<td></td>
</tr>
<tr>
<td>corr(FS, IS)</td>
<td>-0.466</td>
<td>-0.465</td>
<td>-0.463</td>
<td>-0.466</td>
<td>-0.465</td>
<td>-0.700</td>
</tr>
<tr>
<td></td>
<td>(0.026)</td>
<td>(0.026)</td>
<td>(0.025)</td>
<td>(0.025)</td>
<td>(0.026)</td>
<td></td>
</tr>
<tr>
<td>corr(FS, GDP)</td>
<td>0.869</td>
<td>0.869</td>
<td>0.868</td>
<td>0.869</td>
<td>0.869</td>
<td>0.943</td>
</tr>
<tr>
<td></td>
<td>(0.007)</td>
<td>(0.007)</td>
<td>(0.007)</td>
<td>(0.007)</td>
<td>(0.007)</td>
<td></td>
</tr>
<tr>
<td>corr(FS, NII)</td>
<td>0.317</td>
<td>0.317</td>
<td>0.317</td>
<td>0.316</td>
<td>0.317</td>
<td>0.411</td>
</tr>
<tr>
<td></td>
<td>(0.016)</td>
<td>(0.015)</td>
<td>(0.015)</td>
<td>(0.015)</td>
<td>(0.015)</td>
<td></td>
</tr>
<tr>
<td>$\sigma$(GDP)</td>
<td>0.036</td>
<td>0.037</td>
<td>0.036</td>
<td>0.037</td>
<td>0.037</td>
<td>-</td>
</tr>
<tr>
<td></td>
<td>(0.001)</td>
<td>(0.001)</td>
<td>(0.001)</td>
<td>(0.001)</td>
<td>(0.001)</td>
<td></td>
</tr>
<tr>
<td>$\sigma$(NII)</td>
<td>2.300</td>
<td>2.302</td>
<td>2.305</td>
<td>2.301</td>
<td>2.303</td>
<td>-</td>
</tr>
<tr>
<td></td>
<td>(0.055)</td>
<td>(0.055)</td>
<td>(0.056)</td>
<td>(0.056)</td>
<td>(0.056)</td>
<td></td>
</tr>
<tr>
<td>$\sigma$(n)</td>
<td>0.024</td>
<td>0.024</td>
<td>0.024</td>
<td>0.024</td>
<td>0.024</td>
<td>-</td>
</tr>
<tr>
<td></td>
<td>(0.001)</td>
<td>(0.001)</td>
<td>(0.001)</td>
<td>(0.001)</td>
<td>(0.001)</td>
<td></td>
</tr>
</tbody>
</table>

* Numbers in parentheses are standard deviations over 1000 simulations. GDP: real GDP; IS: inventory-to-sales ratio; NII: net inventory investment; FS: final sales; n: employment level of each output producer.

** Parameter values with a star are benchmark parameter values.
Table 9: Sensitivity analysis: *markup*

<table>
<thead>
<tr>
<th>markup</th>
<th>0.2**</th>
<th>0.4</th>
<th>0.8</th>
<th>1</th>
<th>2</th>
<th>Data</th>
</tr>
</thead>
<tbody>
<tr>
<td>corr(GDP, IS)</td>
<td>-0.308</td>
<td>-0.300</td>
<td>-0.281</td>
<td>-0.271</td>
<td>-0.231</td>
<td>-0.381</td>
</tr>
<tr>
<td></td>
<td>(0.035)</td>
<td>(0.036)</td>
<td>(0.035)</td>
<td>(0.034)</td>
<td>(0.033)</td>
<td></td>
</tr>
<tr>
<td>corr(GDP, NII)</td>
<td>0.746</td>
<td>0.746</td>
<td>0.744</td>
<td>0.744</td>
<td>0.738</td>
<td>0.669</td>
</tr>
<tr>
<td></td>
<td>(0.016)</td>
<td>(0.017)</td>
<td>(0.017)</td>
<td>(0.016)</td>
<td>(0.017)</td>
<td></td>
</tr>
<tr>
<td>corr(FS, IS)</td>
<td>-0.463</td>
<td>-0.453</td>
<td>-0.430</td>
<td>-0.419</td>
<td>-0.364</td>
<td>-0.700</td>
</tr>
<tr>
<td></td>
<td>(0.025)</td>
<td>(0.026)</td>
<td>(0.026)</td>
<td>(0.025)</td>
<td>(0.025)</td>
<td></td>
</tr>
<tr>
<td>corr(FS, GDP)</td>
<td>0.868</td>
<td>0.868</td>
<td>0.868</td>
<td>0.868</td>
<td>0.869</td>
<td>0.943</td>
</tr>
<tr>
<td></td>
<td>(0.007)</td>
<td>(0.007)</td>
<td>(0.007)</td>
<td>(0.007)</td>
<td>(0.007)</td>
<td></td>
</tr>
<tr>
<td>corr(FS, NII)</td>
<td>0.317</td>
<td>0.317</td>
<td>0.314</td>
<td>0.313</td>
<td>0.307</td>
<td>0.411</td>
</tr>
<tr>
<td></td>
<td>(0.015)</td>
<td>(0.015)</td>
<td>(0.015)</td>
<td>(0.015)</td>
<td>(0.015)</td>
<td></td>
</tr>
<tr>
<td>σ(GDP)</td>
<td>0.036</td>
<td>0.036</td>
<td>0.035</td>
<td>0.035</td>
<td>0.033</td>
<td>-</td>
</tr>
<tr>
<td></td>
<td>(0.001)</td>
<td>(0.001)</td>
<td>(0.001)</td>
<td>(0.001)</td>
<td>(0.001)</td>
<td></td>
</tr>
<tr>
<td>σ(NII)</td>
<td>2.305</td>
<td>2.265</td>
<td>2.208</td>
<td>2.187</td>
<td>2.093</td>
<td>-</td>
</tr>
<tr>
<td></td>
<td>(0.056)</td>
<td>(0.055)</td>
<td>(0.054)</td>
<td>(0.055)</td>
<td>(0.051)</td>
<td></td>
</tr>
<tr>
<td>σ(n)</td>
<td>0.024</td>
<td>0.023</td>
<td>0.023</td>
<td>0.023</td>
<td>0.023</td>
<td>-</td>
</tr>
<tr>
<td></td>
<td>(0.001)</td>
<td>(0.001)</td>
<td>(0.001)</td>
<td>(0.001)</td>
<td>(0.001)</td>
<td></td>
</tr>
</tbody>
</table>

* Numbers in parentheses are standard deviations over 1000 simulations. GDP: real GDP; IS: inventory-to-sales ratio; NII: net inventory investment; FS: final sales; n: employment level of each output producer.
** Parameter values with a star are benchmark parameter values.
Figure 1: Timeline
Intermediate Goods Market\n\( (s=1) \)

Finished Goods Market\n\( (s=2) \)

Share \( i\)-goods

\( t \) \[ \uparrow \] \[ \uparrow \] \( (s=2) \) \[ \uparrow \] \( t+1 \)

Two-stage Production Effect

\(+\) Effect on \( i\)-goods

\( s^i: \) Extensive Margin \( (+) \)
\( q^i: \) Intensive Margin \( (-) \)

\( s^f: \) Extensive Margin \( (+) \)
\( q^f: \) Intensive Margin \( (+) \)

\( (+) \) Effect on \( i\)-goods \[ \rightarrow \]

\( (-) \) Effect on \( i\)-goods \[ \rightarrow \]

\( (+) \) Overall Effect on Input Inventories

Figure 2: Effects with a Higher Money Growth Rate: Low \( \gamma \)
Intermediate Goods Market  
(s=1) 

Finished Goods Market  
(s=2) 

Share $i$-goods 

Two-stage Production Effect 
(-) Effect on $i$-goods 

$S^i$: Extensive Margin (+)  

$q^i$: Intensive Margin (-)  

$S^f$: Extensive Margin (+)  

$q^f$: Intensive Margin (-)  

(-) Effect on $i$-goods  

(+) Effect on $i$-goods  

(-) Overall Effect on Input Inventories 

Figure 3: Effects with a Higher Money Growth Rate: High $\gamma$
Figure 4: Impulse Responses to A Positive Shock to The Money Growth Rate
Figure 5: Role of Inventories: Low intermediate goods market Searching Cost
Figure 6: Role of Inventories: High Depreciation Rate