Housing and Banking over the Business Cycle *

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Abstract

I develop a dynamic general equilibrium model that allows for the interaction between housing and banking over the business cycle. The model is used to explore two sets of issues. First, I investigate the extent to which a disruption in banks’ balance sheets affects the behavior of the housing market and the macroeconomy in an experiment that mimics the Great Recession. The model can qualitatively capture key features of the phenomenon of the Great Recession in response to a financial shock. Second, I investigate the model’s ability to more generally account for important features of the business cycle observed in the data. The model with technology shocks alone can quantitatively account for the volatility and procyclicality of consumption, business investment and house prices, the volatility of housing investment and consumer loans, and the co-movement between house prices and other quantities of interest.

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1 Introduction

During the early 1990s, U.S. house prices were relatively stable, but they began to rise sharply at the end of the decade and reached a peak in the second quarter of 2006. Between 2000 and the second quarter of 2006, house prices increased on average by 80%, causing a residential construction boom. Between the middle of 2006 and the first quarter of 2007, the housing boom quickly turned into a bust as house prices started to fall. Consequently, it led to a high level of mortgage delinquencies and defaults in the banking system, creating a vicious cycle that precipitated more and more losses on banks’ balance sheets and subsequent declines in house prices. The rapid reversal of U.S. house prices ignited a chain of events that eventually led to a credit crunch in the economy and a downturn of the housing market. Between 2007 and 2009, the United States experienced the worst financial crisis of the post-war era. Both the housing market and the financial market were brought to a halt during this period. The experience of the 2007 financial crisis has raised concerns that the movement of the housing market over the business cycle is not just driven by non-financial factors, but might also be driven by financial factors. In particular, the movements of house prices and quantities are associated with the condition of banks’ balance sheets. In order to study these issues, I construct a DSGE model that allows the housing market to interact with the financial market over the business cycle.

In this paper, the model is used to explore two sets of issues associated with recent U.S. macroeconomic history. In the first part of the paper, I want to investigate the extent to which the model can qualitatively account for the key features of the 2007 financial crisis. In particular, I consider a negative shock to capital quality as a trigger to generate a decline in capital/equity prices, causing a disruption in banks’ balance sheets, and hence a downturn of the housing market and the whole economy. In the second part of the paper, I use the baseline model with technology shocks alone to investigate whether the model fits the data. In addition, I compare the business cycle properties generated by the baseline model to those produced by previous housing literature in order to investigate whether the model improves on them in some key dimensions such as the volatility of house prices.\footnote{A notable housing paper, Davis and Heathcote (2005), performs poorly in three dimensions, and leaves them for future research. First, their model fails to account for the lead-lag pattern between residential investment and nonresidential investment. Second, they produce a counterfactual (negative) correlation between house prices and housing investment. Lastly, their model does poorly}
I find that the baseline model is successful in its ability to qualitatively capture the phenomenon of the 2007 financial crisis. More importantly, it explores the mechanism by which the financial factors affect the behavior of the housing market in response to an exogenous financial shock in the context of the financial crisis. Moreover, over the business cycle, the model with technology shocks alone replicates well several key features of housing market and macroeconomy observed in the data. For instance, the volatility and procyclicality of consumption, business investment and house prices, the volatility of housing investment and consumer loans reproduced by the model are consistent with the data. In addition, the model can reproduce the co-movements among the quantities of interest.

In this paper, I extend a version of the model of Iacoviello and Neri (2010) to include financial frictions along the lines of Gertler and Kiyotaki (2010). Specifically, my baseline model consists of four main features: (i) sectoral heterogeneity in the final goods sector and the housing sector; (ii) heterogeneity across two types of households; (iii) borrowing frictions in the household sector; and (iv) financial frictions in the banking system. These features are primarily drawn from two strands of current literature which study the role of housing or the role of banking in business cycle models. The business cycle models with housing - Greenwood and Hercowitz (1991), Benhabib, Rogerson and Wright (1991), Gervais (2002), Davis and Heathcote (2005), Iacoviello (2005), Fisher (2007), Christensen, Corrigan, Mendicino and Nishiyama (2009), Iacoviello and Neri (2010), Iacoviello and Pavan (2011), Kiyotaki, and Michaelides and Nikolov (2011) - study the behavior of the housing market over the business cycle by dealing with some combination of (i), (ii) and (iii). Business cycle models with banking such as Meh and Moran (2004), Gertler and Karadi (2009), Gerali, Neri, Sessa and Signoretti (2009), Angeloni and Faia (2009), Gertler and Kiyotaki (2010), Gertler, Kiyotaki and Queralto (2011) and Gertler and Kiyotaki (2013) study the role of banks in the transmission of financial shocks by dealing with (iv). However, none of them focus on the interaction between the housing market and financial factors in a general equilibrium context, which is the core of this paper. More importantly, none of them incorporates both (iii) and (iv) in a way that allows them to interact with each other in equilibrium, which is the main in accounting for the volatility of house prices observed in the data. The first issue has been addressed by Fisher (2007), and the second one has been addressed by Iacoviello and Neri (2010). Although the third one seems being addressed by Iacoviello and Neri (2010), their model still cannot explain the high volatility of house prices as a result of technology shocks alone.
theoretical contribution of this paper.

To my knowledge, my model is the one of the few that combines both the housing and the financial literature in order to systematically investigate the link between the housing market and financial market over the business cycle. Although others like Iacoviello (2010a) have also developed a DSGE model with housing and banking, he models the banking system in an alternative fashion. With his formulation of banks, the model cannot capture precisely the mechanism in which a disruption in banks’ balance sheet affects the housing market and the macroeconomy. Moreover, Iacoviello (2010a) is not mainly focused on the link between the financial market and housing market. Rather, he analyzes the role of financial intermediation over the business cycle. Last, the initial exogenous shock in Iacoviello (2010a) is the repayment shock - impatient households (subprimers) pay less than their obligations. Since the shock is persistent, in absence of any risk of default penalties, subprimers face a persistent positive wealth shock on the one hand, and banks face a persistent negative wealth shock on the other hand. This assumption lacks microfoundations in the current context of the mortgage market. In my baseline model, I assume that the initial disturbance that worsens the banks’ balance sheet is a capital quality shock, as in Gertler and Kiyotaki (2010). Though the 2007 financial crisis is initially triggered by a decline in housing values, I introduce the quality shock as a simple way to motivate an exogenous source that exacerbates the banks’ balance sheet. The model with a capital quality shock can still qualitatively capture the phenomenon of the crisis. More importantly, the baseline model is rich enough to capture the interactions between the housing market and financial factors during the financial crisis. Gertler and Kiyotaki (2010) is perhaps my closest antecedent since I adopt the same formulation of financial frictions in the general equilibrium context. While both housing and banking are considered in this paper, the baseline model can not only be used as a complement to its antecedents, but also provides an alternative framework to the family of business cycle models with housing and banking.

Furthermore, I am the first to quantitatively account for the main features of the business cycle in the context of a general equilibrium model with housing and banking. Compared to previous

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Some recent notable papers that incorporate the collateral constraints alone are by Iacoviello (2005), Iacoviello and Neri (2010), and Monacelli (2009); others that incorporate the banks’ incentive constraints alone are by Gertler and Kiyotaki (2010), Gertler, Kiyotaki and Queralto (2011), and Gertler and Kiyotaki (2013). None of those papers, however, consider both constraints working together in a way that allows them to interact with each other.
housing papers such as Davis and Heathcote (2005) and Iacoveillo and Neri (2010), the most attractive finding in this paper is that the baseline model replicates more closely the volatility of house prices and the correlation between house prices and consumption, conditional on technology shocks alone. For instance, the model of Davis and Heathcote (2005) understates the volatility of house prices observed in the data when using sectoral technology shocks alone. Iacoveillo and Neri (2010) can replicate the high volatility of house prices only when a variety of shocks are introduced to the model. Their model with technology shocks alone, however, can explain only half of the volatility of house prices.

That the volatility of house prices is high in my model is a product of three aspects. First, land acts as an adjustment cost on housing since it limits the extent to which the housing stock can be adjusted. Given the fixed supply of land, a larger share of land increases the volatility of house prices and decreases the volatility of housing investment. Second, the heterogeneity in discount factors across households is also responsible for a high volatility of house prices. With the heterogeneity of households, two types of households respond differently to the shocks, causing a high volatility. Third, financial factors are also at work to increase the volatility of house prices since they exacerbate the responses of households to the shocks.

Before going further, I should be up front about few apparent drawbacks of the model. First, the model understates the volatilities of commercial loans, consumer loans and net worth, and overstates the volatility of deposits. Second, it overstates the procyclicality of financial factors. Third, in the data house prices and housing investment move in the same direction. In contrast, the model generates a negative correlation between the price and the quantity of housing. Moreover, consumer loans and housing investment move in the opposite direction, which is contrary to the data. Lastly, a striking drawback of the model is that it understates the correlation between housing investment and business investment. I will return to these issues, and provide some potential approaches that may address them in the section of concluding remarks.

In what follows, Section 2 documents some of the key features of U.S. housing and financial time series from 1973 to 2011. Section 3 develops the baseline model. Section 4 characterizes the

5 In Iacoveillo and Neri (2010), they introduce nine shocks to the model. It goes without saying that using more shocks is not sensible to do. To the extent that some of shocks used in the model correlate with each other, the model, however, may bear some measurement errors.
competitive equilibrium of the model. Section 5 reports the data description and parameter calibration. In Section 6, I carry out an experiment that mimics the Great Recession to investigate the dynamic implications of the financial shocks. Section 7 reports impulse responses, cyclical properties, and robustness analysis for the model with technology shocks alone, and investigates whether the baseline model fits to the data. Section 8 concludes. All proofs and extended derivations are given in the appendix.

2 Facts

There are several interesting dimensions that matter as far as housing, banking and some of key components of GDP are concerned. In this section, I present several facts related to the topics addressed in this paper. Some of facts are not new, and have been frequently noted by other
Figure 2: Financial Variables VS GDP (1973-2011)

Note: All variables are log-transformed, detrended by using the HP filter ($\lambda = 1,600$) and expressed in quarter-on-quarter growth rates.

Authors. Others, however, are rarely noted in the housing literature.

Figure 1 plots the main components of real GDP and real house prices together with real GDP for the United States from 1973 to 2011. From Figure 1, consumption is procyclical and less volatile than GDP in the sample period. Both nonresidential investment (business investment) and residential investment (housing investment) are procyclical and much more volatile than GDP. Note that the percentage standard deviations of both nonresidential investment and residential investment are more than twice that of GDP. House prices are procyclical and more volatile than GDP. Moreover, consumption, nonresidential investment, residential investment, house prices and GDP all declined significantly between 2008 and 2010, implying that both housing market and macroeconomy suffered huge losses in the Great Recession. Although these facts have been frequently noted in the previous housing literature (see Davis and Heathcote (2005), Iacoviello (2010b), and Iacoviello and Neri (2010)), I revisit them in order to organize my discussions in the rest of this

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4 For instance, several key facts about housing have been documented in Iacoviello (2010b)

5 To my knowledge, the facts about the comovement between housing and financial factors have not been systematically documented in the context of housing literature.
Figure 3 plots several financial time series against real GDP for the United States. Commercial loans are only loosely related to GDP, but much more volatile. Consumer loans are procyclical and much more volatile than GDP. Deposits are weakly and positively related to GDP, and are slightly less volatile than the latter. Net worth is procyclical and much more volatile than GDP. The percentage standard deviation of net worth relative to GDP was high in 1970s and 1980s, and became mild after 1990. We also observe from Figure 2 that both net worth and deposits started to fall in the early 2007 and continued to fall until 2009. It was then followed by a decline in both commercial loans and consumer loans. In particular, they started to fall in the early 2008, and to the trough in 2010. These facts shed a light that the banking system has experienced a hard time in the Great Recession.

Figure 3 plots the joint behavior of house prices with the main components of GDP and consumer loans. Consumption, nonresidential investment, residential investment and consumer loans are all positively correlated with house prices. These facts can be used as an alternative evidence
Figure 4 plots the joint behavior of financial factors with the main components of GDP, and the joint behavior of residential investment and nonresident investment. From Figure 4, residential investment and nonresidential investment are positively correlated, and the former leads the latter. The pattern that peaks and troughs in residential investment precede peaks and troughs in nonresidential investment can be used as an evidence to verify the view that "housing is the business cycle". In addition, residential investment is more volatile than nonresidential investment by almost a factor of two. Commercial loans are positively correlated with nonresidential investment. Consumer loans are not only positively correlated with residential investment but also positively correlated with consumption. Combined with the observations from Figure 2, these stylized facts can lead to a new perspective that "banking is also the business cycle".

Although I might have omitted other interesting facts about housing and banking that some readers might regard as equally important, the facts that have been documented in this paper should be considered as an important yardstick to measure the success of the DSGE model with housing...
and banking in the context of the business cycle.

3 The Model

The model features multiple production sectors, heterogeneity in discount factors between two types of households, a borrowing friction faced by borrowers, and a financial friction faced by intermediaries. Following Gertler and Kiyotaki (2010), I formulate the banking system in a way that reflects a financial constraint associated with the bank’s net worth when a bank issues deposits and makes loans. Aside from the introduction of the banking system, the baseline model closely follows a modified version of Iacoviello and Neri (2010).

There are two groups of households in the economy and each group has a unit measure of households. One group consists of patient households (net savers), and the other consists of impatient households (net borrowers). The economic size of each group is measured by its wage share, which is constant due to a unit elasticity of substitution in production. Households do not hold physical capitals directly. Rather, they work, consume final goods, buy houses and deposit funds into or borrow from banks. In the equilibrium that we describe below, patient households turn out to be net savers and lend funds to impatient households and non-financial firms through the banking system. Conversely, impatient households turn out to be net borrowers in equilibrium, and in general they borrow funds from the banking system against their collateral which is tied to their housing values.

I assume that both the final goods sector and the housing sector operate under perfect competition, and that they produce consumption/investment goods and houses respectively using two different technologies. Firms in the final goods sector hire labor from the two groups of households, and borrow funds from a bank to purchase intermediate physical capital. For simplicity, it is assumed that the final good sector faces no further borrowing constraint and can commit to repay its debt obligations with its future gross profits to the creditor bank. In particular, a final goods producer obtains funds from a creditor bank by issuing state-contingent equities, and each unit of equity is a state-contingent claim to the future returns from one unit of new capital investment. Firms in the housing sector also hire labor from the two groups of households and rent land as an input from patient households in order to produce houses.
The capital producer purchases final goods to be used as inputs to produce new capital and is subject to an adjustment cost. Firms in the capital sector are assumed to be owned by patient households, and all profits are redistributed to patient households through a lump sum transfer.

Banks are assumed to operate in a national retail market only. At the beginning of each period, banks obtain deposits from patient households and make loans to impatient households (e.g. consumer loans/mortgages) and non-housing sectors (e.g. commercial loans). I rule out borrowing and lending in a wholesale market (e.g. inter-bank activities) since inter-bank activities are beyond the scope of this paper. As I mentioned earlier, banks are subject to an incentive constraint (deposit/lending constraint). Specifically, each bank with a given portfolio is constrained in its ability to issue deposits to savers and to make loans to borrowers. The incentive constraint can be motivated by government regulatory concerns or by standard moral hazard issues. Fundamentally, both incentive constraints and collateral constraints coexist and interact in the equilibrium so that banks are credit constrained in how much they can accept in the form of deposits from patient households, and impatient households are credit constrained in how much they can borrow from banks. These two frictions interact and reinforce each other to induce a credit crunch during a financial crisis, and thus lead to an economic recession.

3.1 Households

3.1.1 Patient Households (Savers)

In a similar fashion to Iacoviello and Neri (2010), I formulate the heterogeneity of households in a way that allows each group of households to differ in their discount factors and labor supply parameters. In the economy, there is a unit measure of patient households indexed by $p$. A representative patient household maximizes its lifetime utility function given by

$$E_0 \sum_{t=0}^{\infty} \beta^t \left\{ \ln c_{p,t} + j \ln h_{p,t} - \frac{1}{1 + \eta_p} ((l_{pc,t})^{1+\epsilon_p} + (l_{ph,t})^{1+\epsilon_p})^{1+\eta_p} \right\}.$$

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6A notable paper regarding the inter-bank borrowing/lending is by Gertler and Kiyotaki (2010). For the case where the interbank borrowing is frictionless, the implication of the model would be similar to the baseline model. Otherwise, the results may change since interbank rates will lie between deposit rates and loan rates. An addition of the imperfect wholesale financial market will make the model less tractable.
Here, \( c, h, l_{pc} \) and \( l_{ph} \) are consumption, housing, hours supplied to the final goods sector and hours supplied to the housing sector, respectively. The last term in the bracket is the labor disutility function where \( \eta \) and \( \epsilon \) are parameters that capture some degree of sector specificity. The formulation of the labor disutility function follows Horvath (2000) and Iacovello and Neri (2010), and with some choices of parameters it allows for imperfect labor mobility across production sectors. That is, hours are less perfect substitutes if \( \epsilon > 0 \); otherwise, they are perfect substitutes (e.g. \( \epsilon = 0 \)). The parameter \( j \) captures the degree of preference towards housing. The parameter \( \beta_p \) is denoted as the discount factor for patient households. I assume \( \beta_p > \beta_i \) in order to ensure that both impatient households and banks will be credit constrained in a neighborhood of the steady state.

Patient households supply labor to producers, consume final goods, accumulate houses, and deposit or borrow funds with banks. They do not hold physical capitals directly. Rather, they hold land and rent it to the housing sector. Banks are assumed to be owned by patient households. In each period, patient households can receive a lump sum transfer from banks. As we will discuss in the next subsection, banks divert funds only to patient households upon their exit. The representative patient household faces the following budget constraint,

\[
c_{p,t} + q_t h_{p,t} + p_{x,t} x_t + d_t = w_{pc,t} l_{pc,t} + w_{ph,t} l_{ph,t} + q_t (1 - \delta_h) h_{p,t-1} + (p_{x,t} + R^x_t) x_{t-1} + R^d_t d_{t-1} + \Pi_t.
\]

At the beginning of each period, the patient household chooses consumption \( c_{p,t} \), housing \( h_{p,t} \), land \( x_t \), deposits \( d_t \) (loans if \( d_t \) is negative), hours \( l_{pc,t} \) and \( l_{ph,t} \) to maximize his/her utility subject this budget constraint. The parameter \( \delta_h \) denotes the depreciation rate of housing. The terms \( q_t \) and \( p_{x,t} \) denote house prices and land prices, respectively. The terms \( w_{pc,t} \) and \( w_{ph,t} \) are real wages from supplying labor hours to final good and housing sectors. Deposits, \( d_t \), are set in real terms here, and will yield a riskless return of \( R^d_t \) from period \( t-1 \) to period \( t \). In addition, land is rented to the housing sector at a price of \( R^x_t \). Finally, \( \Pi_t \) is the net average transfer received by the patient household from banks upon their exit.

The patient household’s optimality conditions for consumption/deposits, houses, land, and
hours are given by:

\[ 1 = \beta_p E_t \left( \frac{c_{p,t}}{c_{p,t+1}} R_{t+1}^l \right) \]  
(1)

\[ \frac{q_t}{c_{p,t}} = \frac{j}{h_{p,t}} + \beta_p E_t \left( \frac{1 - \delta_h}{c_{p,t+1}} \right) \]  
(2)

\[ \frac{p_{x,t}}{c_{p,t}} = \beta_p E_t \left[ \frac{p_{x,t+1} + R_{t+1}^x}{c_{p,t+1}} \right] \]  
(3)

\[ \frac{w_{pc,t}}{c_{p,t}} = \left( l_{pc,t}^{1+\epsilon_p} + l_{ph,t}^{1+\epsilon_p} \right) \frac{\eta_p - \epsilon_p}{1+\eta_p} \frac{1}{1+\epsilon_p} \]  
(4)

\[ \frac{w_{ph,t}}{c_{p,t}} = \left( l_{pc,t}^{1+\epsilon_p} + l_{ph,t}^{1+\epsilon_p} \right) \frac{\eta_p - \epsilon_p}{1+\eta_p} \frac{1}{1+\epsilon_p} \]  
(5)

### 3.1.2 Impatient Households (Borrowers)

Impatient households are assumed to not hold land and physical capital.\(^7\) In addition, they do not own banks and production sectors.\(^8\) Rather, they work, consume, and are allowed to borrow funds from banks up to a fraction of the value of their houses. A representative impatient household chooses consumption \(c_{i,t}\), housing \(h_{i,t}\), hours \(l_{ic,t}\) and \(l_{ih,t}\), and loans \(b_t\) (borrowing if \(b_t\) is negative) to maximize its expected utility:

\[ E_0 \sum_{t=0}^{\infty} \beta_t \left\{ \ln c_{i,t} + j \ln h_{i,t} - \frac{1}{1+\eta_i} \left( (l_{ic,t})^{1+\epsilon_i} + (l_{ih,t})^{1+\epsilon_i} \right) \right\}, \]

subject to the following budget and collateral constraints:

\[ c_{i,t} + q_t h_{i,t} + R_t^b b_{t-1} = w_{ic,t} l_{ic,t} + w_{ih,t} l_{ih,t} + q_t (1 - \delta_h) h_{i,t-1} + b_t \]

\[ b_t \leq m E_t \left( \frac{q_{t+1} h_{i,t}}{R_{t+1}^b} \right), \]

where \(w_{ic,t}\) and \(w_{ih,t}\) are real wage rates from supplying hours to final goods sector and housing sector, respectively; and \(R_t^b\) is the rate of return on loans/borrowing incurred at date \(t\). The collateral constraint characterizes the limit on the impatient household’s ability to borrow up to the

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\(^7\)Here, impatient households can be treated as those who are poor. In general, the amount of land and physical capital held by them are relatively small so that can be ignored in the model. One may assume that this type of household also accumulate these assets, but implications of the model are similar. For simplicity, I assume that impatient households do not hold both land and capital. This assumption is consistent with Iacoviello and Neri (2010).

\(^8\)In the equilibrium, all production sectors earn zero profit so it does not matter for the results if one assume that impatient households also own production sectors. In addition, a lump-sum transfer between banks and households plays a trivial role in the model, as in Gertler and Kiyotaki (2010). In this regard, I do not assume that impatient households own banks for the sake of tractability. A similar assumption is also used in Iacoviello and Neri (2010).
discounted future value of their houses. The term \( m \) is the loan-to-value ratio (LTV) that measures the effective degree of liquidity of houses. The larger \( m \) is, the greater is the value of housing as collateral to the impatient household. Here, \( \beta_i \) denotes the discount factor to the impatient household. Recall that we have \( \beta_p > \beta_i \) in the model. This is a necessary condition that ensures impatient households are credit constrained in the neighborhood of the steady state, together with other parameters calibrated in the model.

The impatient household’s first-order conditions with respect to consumption, houses, loans, and hours can be written as:

\[
\frac{j}{h_{i,t}} + \beta_i E_t(\frac{(1 - \delta_h)q_{t+1}}{c_{i,t+1}}) = \frac{q_t}{c_{i,t}} - \lambda_{i,t} m E_t(\frac{q_{t+1}}{R_{b,t+1}}) \quad (6)
\]

\[
\frac{1}{c_{i,t}} = \beta_i E_t(\frac{R_{b,t+1}}{c_{i,t+1}}) + \lambda_{i,t} \quad (7)
\]

\[
\frac{w_{ic,t}}{c_{i,t}} = (l_{ic,t} + l_{ih,t}) \frac{\eta_{ic,i} - \epsilon_{ic}}{1 + \epsilon_{ic}} l_{ic,t} \quad (8)
\]

\[
\frac{w_{ih,t}}{c_{i,t}} = (l_{ic,t} + l_{ih,t}) \frac{\eta_{ih,i} - \epsilon_{ih}}{1 + \epsilon_{ih}} l_{ih,t} \quad (9)
\]

\[
b_t = m E_t(\frac{q_{t+1} h_{i,t}}{R_{b,t+1}}), \quad (10)
\]

where \( \lambda_{i,t} \) denotes the Lagrange multiplier on the collateral constraint. If the collateral constraint is binding, the multiplier is positive.\footnote{From equation (7), we can see that the collateral constraint is binding (\( \lambda_i > 0 \)) in the steady state if and only if \( R^d < \frac{1}{\beta_i} \). Given parameters calibrated in the model, we will observe later that this condition is satisfied.}

### 3.2 Banks

There are a large number of banks operating in a national financial market. Within the national financial market, I assume banks raise funds from patient households in a retail market rather than from inter-bank borrowing in a wholesale market. Because this paper primarily focuses on the interaction between the housing market and financial intermediation, the addition of inter-bank borrowing would go beyond the scope of this paper.

At the beginning of each period, a bank obtains deposits \( d_t \) from patient households, and pays a gross interest rate of \( R_{d,t+1} \) in the following period. In this economy, deposits are riskless
one-period securities. At the same time, a bank makes consumer loans $b_t$ to impatient households at a loan rate of $R_{t+1}^b$, and commercial loans to non-financial firms (e.g. final goods producers) in exchange for state-contingent equities from those firms at the price of $p_t$. Consumer loans are subject to a friction because impatient households can only borrow up to a fraction of their housing values. For simplicity, I assume there are no frictions associated with commercial borrowing. Following Gertler and Kiyotaki (2010), I assume banks are not only more efficient at evaluating and monitoring all activities of non-financial sectors than households, but are also more effective in enforcing contractual obligations. To motivate the logic above, I assume there are no costs for a bank performing these activities. Given these assumptions, a bank can issue frictionless loans to the final goods sector on the one hand, and a borrowing firm is able to offer the bank state-contingent equity on the other hand. In particular, each unit of equity is a state-contingent claim to the future returns from one unit of new capital investment.

Let $s_t$ be the quantity of equities held by a representative bank, and $n_t$ be the net worth of the bank in period $t$. Then the bank’s net worth is equal to the gross payoffs from its assets (e.g. commercial loans and consumer loans) incurred at period $t-1$ net of deposit costs:

$$n_t = (Z_t + (1-\delta_k)p_t)\psi_t s_{t-1} + R_{t}^b b_{t-1} - R_{t}^d d_{t-1},$$

(11)

where the term $\delta_k$ is the capital depreciation rate. For simplicity, I assume the bank receives a unit of equity every time it issues commercial loans to non-financial firms to purchase an additional unit of capital. Accordingly, the quantity of equity in the economy remains the same as the quantity of capital. The variable $Z_t$ denotes the dividends paid in period $t$ on the equities issued in period $t-1$. The variable $\psi_t$ represents a capital quality shock, and follows an AR(1) process. As we will observe later, the market price of capital in the model is determined endogenously. Accordingly, one may think of this capital quality shock as an exogenous trigger to asset price dynamics. Note that the disturbance, $\psi_t$, is different from a standard rate of physical depreciation in that it can capture some forms of economic obsolescence. In the model, this capital quality shock initially

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10 This assumption is widely used by papers like Kiyotaki, Michaelides and Nikolov (2011) and Kiyotaki and Gertler (2010). In the model, equity and capital exhibit a one-to-one relation in order to maintain tractability of the model. Thus, the price of equity equals to the price of capital in the model by this assumption.

11 Following Gertler and Kiyotaki (2010), I introduce the capital quality shock as a simple way to motivate an exogenous source of variation in the value of capital.

12 As Gertler and Kiyotaki (2010) argued, capital in general is good-specific once it is installed. Suppose that the final goods...
induces a deterioration in the banks’ balance sheet. When the losses on the balance sheet induced by the shock cannot be fully absorbed by banks, a credit crunch may arise for the whole economy.

In period $t$, the flow-of-funds constraint for a bank can be written as,

$$p_t s_t + b_t = n_t + d_t.$$  \hspace{1cm} (12)

The equation above implies that the bank’s assets (e.g. loans) must equal to its net worth plus liabilities (e.g. deposits).

In absence of some motive for paying dividends to households, banks may find it optimal to accumulate assets to the point where the financial constraint they face is no longer binding. In order to limit banks’ ability to save to overcome financial constraint, I introduce an exogenous shock that will induce the bank to leave the economy. In particular, a bank may, with probability $1 - \sigma$, exit the economy at the end of each period. Upon exiting, a bank transfers all retained earnings to patient households. At the same time, a new bank may, with probability $1 - \sigma$, be established, leaving the number of banks constant in each period. In particular, a new bank takes over the business of an exiting bank and in the process inherits the skills of the exiting bank at no costs. The new bank receives a small fraction of the total assets of an ongoing bank as a ”startup” fund from patient households. Recall that in each period a representative patient household receives an average net transfer $\Pi_t$ from banks. The net transfer then must equal the funds transferred from exiting banks minus funds transferred to patient households.

As I mentioned earlier, an endogenous financial constraint (incentive constraint) is introduced into the model. To motivate the endogenous financial constraint on the bank’s ability to obtain funds in the retail financial market, I assume that the bank may divert a fraction $\theta$ of assets to its owners (e.g. patient households) after it obtains funds (e.g. deposits) from the retail financial market. The bank’s assets comprise the total value of equities held by the bank, $p_t s_t$, and consumer loans to impatient households, $b_t$. If the bank diverts its assets to its owners, it defaults on its debts (e.g. deposits) and is then forced to shut down. The creditors may reclaim the remaining fraction are produced by a composite of intermediate goods that are in turn produced by combining capital and labor in a Cobb-Douglas production function, the capital used to produce the goods become worthless if the goods are obsolete and the capital used to produce the new goods is not fully on line. In this case, a capital quality shock can capture some form of economic obsolescence.

\footnote{The expected survival time of a bank, $\frac{1}{1-\sigma}$, is about 9 years. It is important that the survival time is finite in the model so that patient households would finally get paid with dividends from banks despite the financial constraints are still binding.}
of funds. Given the risk of banks’ default on their debts, creditors restrict the amount they lend to the bank at the beginning of each period. Accordingly, banks are constrained in their ability to obtain funds in the retail financial market, and in this way a financial constraint may arise. The bank’s decision over whether to default on its debts must be made at the end of each period after the realization of the aggregate shock, but before the realization of the shock in the next period.

Let \( V_t(s_t, b_t, d_t) \) be the value function of a bank at the end of period \( t \), given its portfolio holdings \((s_t, b_t, d_t)\). A bank will not default or divert funds, if it satisfies the incentive constraint given by,

\[
V_t(s_t, b_t, d_t) \geq \theta(p_t s_t + b_t).
\]

Let \( \Lambda_{t,t+i} \) be the stochastic discount factor, which is equal to the patient households’ marginal rate of substitution between consumption in period \( t + i \) and that in period \( t \). Since in the model banks are owned by patient households, they act on their behalf. In this regard, banks are as patient as patient households, and more patient than impatient households. That is, \( \beta_b = \beta_p > \beta_i \). In each period \( t \), a representative bank maximizes the present value of its expected future net worth,

\[
V_t(s_t, b_t, d_t) = E_t \sum_{i=1}^{\infty} (1 - \sigma)^{i-1} \Lambda_{t,t+i} n_{t+i},
\]

subject to the incentive constraint and the flow-of-funds constraint given above. Recall that banks only pay dividends when they exit. Thus, the probability that a bank exits and pays the dividends after \( i \) periods from date \( t \) is \((1 - \sigma)\sigma^{i-1}\) for \( i \in [1, \infty) \). Given the bank’s problem above, one can easily write the Bellman equation as follows,

\[
V_{t-1}(s_{t-1}, b_{t-1}, d_{t-1}) = E_{t-1} \Lambda_{t-1,t} \{(1 - \sigma)n_t + \sigma \max_{s_t,b_t,d_t} V_t(s_t, b_t, d_t)\}
\]

To solve the bank’s problem, we first guess the value function, \( V_t \), is a time-varying linear function of \((s_t, b_t, d_t)\) given by,

\[
V_t(s_t, b_t, d_t) = \nu_{st} s_t + \nu_{bt} b_t - \nu_{dt} d_t
\]

where \( \nu_{st} \) is the marginal value of equalities at the end of period \( t \); \( \nu_{bt} \) is the marginal value of consumer loans; and \( \nu_{dt} \) is the marginal cost of deposits.\(^{14}\)

\(^{14}\)One may treat these coefficients like prices. That is as being taken as given by the individual bank.
Let $\lambda^b_t$ be the Lagrangian multiplier associated with the bank’s incentive constraint. Given the conjectured form of the value function, one may use the Bellman equation together with the bank’s incentive constraint and the flow-of-funds constraint to solve the bank’s problem. Then the first order conditions for $s_t$, $d_t$, and $\lambda^b_t$ are given as:

$$\frac{\nu_{st}}{p_t} - \nu_{bt} = 0 \quad (17)$$

$$(1 + \lambda^b_t)(\nu_{bt} - \nu_{dt}) = \theta \lambda^b_t \quad (18)$$

$$\left(\theta - (\nu_{bt} - \nu_{dt})\right)b_t + \left(\theta - \left(\frac{\nu_{st}}{p_t} - \nu_{dt}\right)\right)p_t s_t \leq \nu_{dt} n_t. \quad (19)$$

Equation (17) indicates that the marginal value of equities must be equal to the marginal value of consumer loans, leading to no arbitrage opportunities across assets. That is, banks are indifferent between making commercial loans to non-financial sectors and making consumer loans to impatient households. Equation (18) implies that the marginal value of consumer loans exceeds the marginal cost of deposits if and only if the incentive constraint is binding ($\lambda^b_t > 0$). Accordingly, given equation (17) and equation (18), we will see later that in the model with financial friction ($\lambda^b_t > 0$) there are excess returns on assets over deposits. This finding is consistent with that found by previous authors in financial literature such as Gertler and Kiyotaki (2010) and Iacoviello (2010a). Equation (19) is the bank’s incentive constraint, and it states that the value of the bank’s net worth must be at least as large as a weighted average value of its assets. When equation (19) holds with equality, financial frictions may arise.

In the next section, I proceed to characterize the model for two cases: with financial frictions ($\lambda^b_t > 0$) and without financial frictions ($\lambda^b_t = 0$). Furthermore, I will subsequently compare the models, and investigate how the financial friction amplifies the effect of an exogenous shock to the banks’ balance sheet, and propagates this to the housing market and the economy as a whole.

3.2.1 Case 1: The Banking System with Financial Frictions

With financial frictions, banks are constrained in their ability to make loans to impatient households and non-financial firms. Given that the bank’s incentive constraint is binding, equation (17) requires that the marginal value of equities relative to goods must equal the marginal value of consumer loans,

$$\frac{\nu_{st}}{p_t} = \nu_{bt}. \quad (20)$$
In addition, equation (18) implies that the marginal value of consumer loans exceeds the marginal cost of deposits,

\[ \nu_{bt} - \nu_{dt} > 0. \tag{21} \]

Combining equation (20) and equation (21), we obtain

\[ \mu_t = \frac{\nu_{st}}{p_t} - \nu_{dt} > 0, \tag{22} \]

where the term \( \mu_t \) denotes the excess value of returns on assets over deposits.

Finally, given that banks are constrained by their funds to lend, one may rewrite equation (19) as,

\[ p_t s_t + b_t = \phi_t n_t \tag{23} \]

with

\[ \phi_t = \frac{\nu_{dt}}{\theta - \mu_t}. \tag{24} \]

where the time varying parameter, \( \phi_t \), represents banks’ leverage ratio. Note that the tightness of the incentive constraint depends positively on the fraction of assets that banks can divert, \( \theta \), and negatively on the excess value of assets over deposits, \( \mu_t \). Intuitively, the greater the fraction of assets that a bank can divert, the more likely it is that the bank will default on its deposits. Moreover, the greater the dispersion of returns between assets and liabilities, the less likely it is that a bank will default.

Let \( \Omega_{t+1} \) be the marginal value of net worth at period \( t + 1 \). Given the Bellman equation and the conjectured value function \( V_t \), we can derive all undetermined coefficients of the conjectured value function, which are given as follows,

\[ \nu_{bt} = R^{b}_{t+1} E_t \Lambda_{t,t+1} \Omega_{t+1} \tag{25} \]
\[ \nu_{dt} = R^{d}_{t+1} E_t \Lambda_{t,t+1} \Omega_{t+1} \tag{26} \]
\[ \nu_{st} = E_t \Lambda_{t,t+1} \Omega_{t+1} \psi_{t+1} (Z_{t+1} + (1 - \delta_k) p_{t+1}), \tag{27} \]

with

\[ \Omega_{t+1} = 1 - \sigma + \sigma (\nu_{dt+1} + \phi_{t+1} \mu_{t+1}) \tag{28} \]
\[ \mu_t = E_t \Lambda_{t,t+1} \Omega_{t+1} (R^k_{t+1} - R^d_{t+1}) \tag{29} \]
\[ R^k_{t+1} = \psi_{t+1} \frac{Z_{t+1} + (1 - \delta_k) p_{t+1}}{p_t} \tag{30} \]
Appendix B gives full details in the determination of the coefficients of the conjectured value function, and verifies that the conjectured value function is linear in \((s_t, b_t, d_t)\).

**Proposition 1:** The value function \(V_t(s_t, b_t, d_t)\) is linear in \((s_t, b_t, d_t)\) if and only if equation (25) to equation (30) are satisfied.

One may observe from equations (25) to (27), marginal values of assets and liabilities are equal to an augmented stochastic discount factor \(\Lambda_{t,t+1} \Omega_{t+1}\) multiplied by their corresponding returns. Equation (28) states that the marginal value of net worth is a weighted average of the marginal values for exiting banks and ongoing banks. If an ongoing bank has an additional unit of net worth at date \(t + 1\), not only can it save the cost of deposits \(\nu_{dt+1}\), but it can also get additional benefits from accumulating its assets by a factor equal to the leverage ratio \(\phi_{t+1}\). Equation (29) implies that the excess value of assets relative to deposits is equal to an augmented stochastic discount factor multiplied by the spread of returns between assets and liabilities. Lastly, equation (30) is an expression for the gross rate of returns on equities. If a bank holds an additional unit of equity at a price of \(p_t\) today, it will receive a dividend \(Z_{t+1}\) and the value of the equity at a price \(p_{t+1}\) tomorrow after depreciation. Here, the term \(\psi_{t+1}\) captures a persistent shock that takes place in the next period.

Given equations (20) and (22), one can easily derive a relationship between assets and liabilities in terms of their returns, which is given as

\[
R^k_{t+1} = R^b_{t+1} > R^d_{t+1}. \tag{31}
\]

During a financial crisis, the excess return on assets over liabilities (the spread) will increase. To reduce the spread, banks must deleverage or accumulate more assets. However, this deleveraging process takes a long time. So long as the spread is above its trend, financial factors act as a drag on the whole economy. Hence, the deleveraging process will slow down the recovery of the economy.

In the exposition, lower-case letters represent individual decision variables, and upper-case letters represent aggregate variables. Since all banks are homogenous in the model, given equation (23) we can obtain an expression for banks’ aggregate assets given by

\[
p_t S_t + B_t = \phi_t N_t, \tag{32}
\]

where the variable \(S_t\) denotes the aggregate equity holdings at date \(t\); \(B_t\) denotes the aggregate
consumer loans; and $N_t$ denotes the aggregate net worth of all banks. From now on, we drop all low-case letters and replace with upper-case letters to formulate the model.

Let $N_{ot}$ be the aggregate net worth of ongoing banks at date $t$, and $N_{yt}$ be the aggregate net worth of new banks. Then the aggregate net worth of all banks can be written as,

$$N_t = N_{ot} + N_{yt}. \quad (33)$$

Recall that banks may, with probability $\sigma$, survive to the next period. Thus, the aggregate net worth of ongoing banks must equal the sum of gross repayments of loans net of aggregate debt obligations, multiplied by the survival rate,

$$N_{ot} = \sigma \{(Z_t + (1 - \delta_k)p_t)\psi_t S_{t-1} + R^b B_{t-1} - R^d D_{t-1}\}. \quad (34)$$

Moreover, at the end of period $t-1$ banks may, with probability $1 - \sigma$, exit from the banking system, while new banks enter the market with a fraction of funds transferred by patient households. For simplicity, I assume that patient households transfer a fraction $\xi/(1 - \sigma)$ of the aggregate value of assets held by ongoing banks. Hence, the aggregate net worth of new banks is given by

$$N_{yt} = \xi \{(Z_t + (1 - \delta_k)p_t)\psi_t S_{t-1} + R^b B_{t-1}\}. \quad (35)$$

Finally, given the flow-of-funds constraints faced by individual banks, we can write the aggregate flow-of-funds constraint for all banks as follows,

$$p_t S_t + B_t = N_t + D_t. \quad (36)$$

Equation (36) states that the value of aggregate assets equals the sum of aggregate net worth and liabilities.

Before we proceed to the model without financial frictions in the next section, it is worth highlighting the mechanism by which an exogenous financial shock deteriorates the balance sheet of banks. During a financial crisis, a negative financial shock, directly reduces the value of assets (e.g. equities) held by banks and in turn put downward pressures on the bank’s net worth. Since banks are leveraged, the magnitude of the effect on bank’s balance sheet depends on the leverage ratio. The larger is the leverage ratio, the greater is the impact of the financial shock on the bank’s balance sheet. In addition, there will be a second round effect on the banks’ balance sheet as
their net worth worsens. A decline in a bank’s net worth reduces its ability to borrow in the retail financial market, causing a fire sale of bank’s assets that further depresses the value of assets. Hence, the bank’s balance sheet is further deteriorated. When the real economy strongly relies on the flow of funds from banks, it will suffer a huge loss during the crisis. The 2007 financial crisis is a good illustration of this mechanism.

3.2.2 Case 2: The Banking System without Financial Frictions

If there are no financial frictions, banks now convert deposits to loans without any excess returns for any time (e.g. \( R_b^t = R_k^t = R_d^t \)). Hence, we can treat the financial sector as a veil in this case. This is equivalent to thinking of banks playing no role in the model, and patient households directly leading funds to impatient households and nonfinancial firms with no financial intermediation.

In this case, the incentive constraint is not binding (\( \lambda^b_t = 0 \)), implying that banks are not constrained in how much they can lend to borrowers. Given equation (17) and equation (18), the marginal value of assets must equal the marginal cost of liabilities,

\[
\frac{\nu_{st}}{p_t} = \nu_{bt} = \nu_{dt}.
\]  

(37)

Combining equation (37) with equations (25) to (27), we can derive a perfect arbitrage condition which keeps returns on assets and liabilities equal over time:

\[
R_k^{t+1} = R_b^{t+1} = R_d^{t+1}.
\]  

(38)

As I discussed earlier, a financial crisis is associated with an increase in the excess returns on assets over liabilities. Since there is no excess return in this case, a negative exogenous shock does not generate an amplified effect on the real economy.

Because banks play no role in this case, it is equivalent to thinking of banks disappearing from the model so that \( N_t = 0 \). Moreover, all savings of patient households are converted to loans so that the aggregate flow-of-funds constraint, equation (36) is replaced by

\[
p_t S_t + B_t = D_t.
\]  

(39)

15This mechanism is first advanced by Gertler and Kiyotaki (2010). In this paper, I abstract this mechanism from their model in order to propagates the effect of a disruption in the bank’s balance sheet to the housing market and macroeconomy.

16One may solve the model with \( N_t \neq 0 \). But the implication of the model would be similar.
It is straightforward to solve the model without financial frictions. Subsequently, I will compare the dynamic paths of the key variables of interest across two cases in order to investigate the role of financial frictions in this framework.

3.3 Nonfinancial Firms

3.3.1 The Final Goods Sector

At date $t$, each final goods producer hires labors $l_{pc,t}$ and $l_{ic,t}$ from patient households and impatient households, and pays a real wage of $w_{pc,t}$ and $w_{ic,t}$ to them, respectively. As I mentioned earlier, final goods producers face no borrowing constraints. Instead, they borrow funds from banks by issuing new state-contingent equities at price of $p_t$. In particular, each unit of equity is a state-contingent claim to the future returns from one unit of capital investment. Conditional on funds borrowed from banks, they purchase new capitals as intermediate inputs from capital producers. In addition, I assume that final goods producers combine labors and capitals to produce final goods under a CRS technology in a Cobb-Douglas fashion. Due to perfect competition, the final goods producers earn zero profits state-by-state.

A representative final goods producer chooses hours $(l_{pc,t}, l_{ic,t})$ and intermediate capital $k_t$ to produce final goods $Y_t$. The producer solves the firm’s cost minimization problem as follows:

$$\begin{align*}
& \min w_{pc,t} l_{pc,t} + w_{ic,t} l_{ic,t} + Z_t k_t, \\
& \text{subject to the production function given by,} \\
& Y_t = (A_{ct}(l_{pc,t}^{\alpha} l_{ic,t}^{1-\alpha}))^{1-\mu_c} k_t^{\mu_c}. \\
& \text{(40)}
\end{align*}$$

Here $Z_t$ denotes the dividends per unit of equity paid to creditor banks, the parameter $A_{ct}$ measures labor productivity in the final goods sector, the parameter $\alpha$ denotes the labor income share of patient households, reflecting labor complementarity across different labor skills among households and $\mu_c$ is the income share of capital used in the production of final goods. Note that the capital stock $k_t$ is predetermined in period $t - 1$. The optimality conditions for the final goods producer’s
problem are given by

\[
\begin{align*}
    w_{pc,t} &= \alpha (1 - \mu_c) \frac{Y_t}{l_{pc,t}} \quad (41) \\
    w_{ic,t} &= (1 - \alpha)(1 - \mu_c) \frac{Y_t}{l_{ic,t}} \quad (42) \\
    Z_t &= \mu_c \frac{Y_t}{k_t}. \quad (43)
\end{align*}
\]

Equation (41) and equation (42) are conditional wage functions for the final goods sector. Equation (43) states that dividends per unit of equity are equal to gross profits per unit of capital.

3.3.2 The Housing Sector

Housing producers build new houses with a CRS technology under perfect competition, but using different intermediate inputs. They hire labor \((l_{ph,t}, l_{ih,t})\) from the two groups of households and rent land \(x_{t-1}\) from patient households to produce new houses \(I_{ht}\). Thus, a representative housing producer solves the firm’s cost minimization problem as follows;

\[
\begin{align*}
    \min w_{ph,t} l_{ph,t} + w_{ih,t} l_{ih,t} + R^x_t x_{t-1}.
\end{align*}
\]

subject to the production function given by,

\[
I_{ht} = \left( A_{ht} \left( \frac{\alpha}{l_{ph,t}} \right)^{1-\alpha} \right) \left( 1 - \mu_h \right) x_{t-1}. \quad (44)
\]

Here, the parameter \(A_{ht}\) measures labor productivity in the housing sector; the term \(R^x_t\) denotes the rental rate of land from period \(t - 1\) to period \(t\); the parameter \(\mu_h\) is the income share of land used to produce new houses. The optimality conditions for a representative housing producer’s problem then is given by

\[
\begin{align*}
    w_{ph,t} &= \alpha (1 - \mu_h) \frac{q_t I_{ht}}{l_{ph,t}} \quad (45) \\
    w_{ih,t} &= (1 - \alpha)(1 - \mu_c) \frac{q_t I_{ht}}{l_{ih,t}} \quad (46) \\
    R^x_t &= \mu_h \frac{q_t I_{ht}}{x_{t-1}}. \quad (47)
\end{align*}
\]
3.3.3 The Capital Goods Sector

Capital goods producers produce new capital using final goods as inputs, and incur an adjustment cost. They sell capital to the final goods sector at price $p_t$. I assume that the adjustment cost function is convex in the net growth rate of capital investment and takes the following form:

$$
f(\frac{I_{kt}}{I_{kt-1}})I_{kt} = (\ln(\frac{I_{kt}}{I_{kt-1}}))^2 I_{kt}. \tag{48}\$$

A capital producer chooses capital investment $I_{kt}$ to maximize the objective function:

$$
\max E_t \sum_{i=t}^{\infty} \Lambda_{t,i} \{p_i I_{ki} - [1 + (\ln(\frac{I_{ki}}{I_{ki-1}}))^2] I_{ki}\},
$$

where $\Lambda_{t,i}$ is the patient household’s stochastic discount factor from date $i$ to date $t$. The optimality condition then yields the price function for capital given by,

$$
p_t = 1 + [\ln(\frac{I_{kt}}{I_{kt-1}})]^2 + 2[\ln(\frac{I_{kt}}{I_{kt-1}})] - 2E_t \Lambda_{t,t+1} [\ln(\frac{I_{kt+1}}{I_{kt}})] \frac{I_{kt+1}}{I_{kt}}. \tag{49}\$$

Note profits will arise only outside of the steady state, and will be redistributed to patient households by a lump sum transfer.

3.4 Shock Process

The capital quality shock and productivity shocks all follow AR(1) process and evolve in a log-linear fashion:

$$
\ln \psi_t = \rho_k \ln \psi_{t-1} + u_{kt} \tag{50}\n$$
$$
\ln A_{ct} = \rho_c \ln A_{ct-1} + u_{ct} \tag{51}\n$$
$$
\ln A_{ht} = \rho_h \ln A_{ht-1} + u_{ht} \tag{52}\n$$

where $\rho_k$, $\rho_c$ and $\rho_h$ are persistence parameters; $u_{kt}$, $u_{ct}$ and $u_{ht}$ are innovations that are uncorrelated, with zero means and standard deviations $\sigma_k$, $\sigma_c$ and $\sigma_h$, respectively.

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17This cost function is drawn from Gertler and Kiyotaki (2010), and satisfies $f(1) = f'(1) = 0$ and $f''(\frac{I_{kt}}{I_{kt-1}}) > 0$. Therefore, the aggregate production function of capital goods producers exhibits a decreasing returns to scale in the short-run and a constant returns to scale in the long-run.

18Since patient households own capital firms, I assume that a capital producer is as patient as patient households.
In the financial crisis experiment below, I turn off the productivity disturbances and introduce the capital quality shock alone as the forcing process to explore how the deterioration of banks’ balance sheets affect the real activities during a financial crisis. I then use the baseline model with productivity shocks alone to investigate whether it fits to the data in some dimensions. Moreover, I compare the implications of the baseline model with those of Davis and Heathcote (2005) and Iacoviello and Neri (2010). In particular, I explore the extent to which my model improves on their ability to fit the data.

4. Equilibrium

To close the model, we require market clearing for goods, houses, and equities/capital as follows,

\[ Y_t = C_t + (1 + \ln\left(\frac{I_{kt}}{I_{kt-1}}\right)^2)I_{kt} \]  
\[ I_{ht} = H_t - (1 - \delta_h)H_{t-1} \]  
\[ S_t = I_{kt} + (1 - \delta_k)K_t \]

with

\[ C_t = c_{p,t} + c_{i,t} \]  
\[ H_t = h_{p,t} + h_{i,t} \]  
\[ K_{t+1} = \psi_{t+1}(I_{kt} + (1 - \delta_k)K_t). \]

Note that equation (58) is the law of motion for capital in presence of an exogenous capital quality shock. Land is fixed and normalized to one, that is \( x_t = 1 \).

Definition. A competitive equilibrium consists of a sequence of prices for \( t \in [1, +\infty] \)

\( (p_t, q_t, p_{zt}, R^d_{t+1}, R^h_{t+1}, R^r_{t+1}, w_{pc,t}, w_{ph,t}, w_{ic,t}, w_{ih,t}) \)

and the sequence of shadow prices \( (\nu_{dt}, \nu_{ht}, \nu_{st}, \lambda^i_t, \mu_t) \) such that the sequence of individual quantities

\( (c_{p,t}, c_{i,t}, h_{p,t}, h_{i,t}, l_{pc,t}, l_{ph,t}, l_{ic,t}, l_{ih,t}) \)
and the sequence of aggregate quantities
\[(Y_t, C_t, H_t, K_{t+1}, S_t, I_{ht}, I_{kt}, Z_t, N_t, B_t, D_t, \phi_t)\]
satisfy the following conditions:

1. Given prices, households maximize their expected life-time utility functions, (1)-(10).
2. Given prices, final good producers minimize their costs, (41)-(43).
3. Given prices, housing producers minimize their costs, (45)-(47).
4. Given prices, capital goods producers maximize their profits, (49).
5. Given the conjectured value function \(V_t\), banks maximize their present value of future net worth, (17)-(19).
6. The conjectured value function \(V_t\) is linear and must satisfy equations (25) to (30).
7. Banks are indifferent between making commercial loans and consumer loans, (31).
8. The banks’ leverage ratio satisfies equations (32) and (33).
9. All markets clear and satisfy equations (53) to (58).
10. The capital quality shock and the productivity shocks in the final goods sector and housing sector follows an AR(1) process given by equations (50)-(52).

Then, given eleven prices \((p_t, q_t, q_{xt}, R^d_{t+1}, R^b_{t+1}, R^k_{t+1}, R^{x}_{t+1}, w_{pc,t}, w_{ph,t}, w_{ic,t}, w_{ih,t})\) together with five shadow prices \((\nu_{dt}, \nu_{ht}, \nu_{st}, \lambda_i, \mu_t)\), eight individual quantities \((c_{p,t}, c_{i,t}, h_{p,t}, h_{i,t}, l_{pc,t}, l_{ph,t}, l_{ic,t}, l_{ih,t})\) and twelve aggregate quantities \((Y_t, C_t, H_t, K_{t+1}, S_t, I_{ht}, I_{kt}, Z_t, N_t, B_t, D_t, \phi_t)\) can be determined by a dynamic system of 36 equations. Please see full details of the dynamic system of the model in Appendix C.

5. Data and Calibration

In this section, I calibrate the parameters used in the baseline model to match the targeted ratios of key economic quantities that are observed in the data. These targeted ratios are estimated
by using nine time series from 1973: QI to 2011: QIII: real private consumption expenditure, real private residential fixed investment, real private non-residential fixed investment, real house prices, real commercial loans, real consumer loans, real deposits, real net worth and nominal interest rates. Each time series being used in the model is HP filtered with $\lambda = 1,600$ and detrended. See Appendix A for data descriptions. The model period is quarterly. In particular, depreciation rates for capital and housing, capital and land shares in the production, and the housing preference parameter are calibrated to match consumption, residential investment, nonresidential investment, the value of land and housing wealth to GDP ratios observed in the US data. In addition, I calibrate parameters used in the bank’s problem $(\theta, \xi)$ in order to match an average credit spread and an average economy-wide leverage ratio observed from the data in all US commercial banks. Following Iacoviello and Neri(2010), I do not use the time series of the labor employment to calibrate the labor supply parameters since in any multi-sector model the link between value added of the sector and available measures of total hours worked in the same sector is tenuous. For this reason, I allow for measurement error in total hours in the consumption and housing sectors.

The discount factors $\beta_p$ and $\beta_i$, depreciation rates for capital and housing in each production sector $\delta_k$ and $\delta_h$, capital and land shares in each production sector $\mu_c$ and $\mu_h$, the housing preference weight in the utility function $j$, the fraction of funds transferred from patient households $\xi$, and the fraction of assets that can be diverted from banks to patient households $\theta$ are calibrated by the model itself. Table 1 summarizes my calibrations. All calibrated parameters together with the other parameters that are set explicitly yield the steady-state ratios that are consistent with the data. A summary of steady-state ratios of the baseline model is presented in the Table 2.

The discount factor for patient households $\beta_p$ is set to 0.9925, implying that a steady-state annual real interest rate of 3%. I arbitrarily set the discount factor for impatient households to

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19The model period used in existing literature varies from one to another. For instance, in Davis and Heathcote (2005) the model period is one year since they attempt to capture the length of time between starting to plan new investment and the resulting increase in the capital stock being in place. They find that business cycle statistics, however, are virtually same for both quarterly and annual period lengths. Others like Gomme et al. (2001) and Fisher (2007) set the time to built for residential investment to one quarter, and the time to built for non-residential investment to four quarters in order to account for the fact that nonresidential investment lags residential investment. Since the lead-lag pattern between residential investment and nonresidential investment is beyond our interest in this paper, the length of the time between starting to plan both residential and nonresidential investment and the resulting increase in their corresponding stocks being in place is then set to one quarter.

20See further discussions in Iacoviello and Neri(2010).
Table 1: Calibrated Parameters

<table>
<thead>
<tr>
<th>Parameters</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\beta_p$</td>
<td>0.9925</td>
</tr>
<tr>
<td>$\beta_i$</td>
<td>0.965</td>
</tr>
<tr>
<td>$\delta_k$</td>
<td>0.0188</td>
</tr>
<tr>
<td>$\delta_h$</td>
<td>0.0106</td>
</tr>
<tr>
<td>$\mu_c$</td>
<td>0.1795</td>
</tr>
<tr>
<td>$\mu_h$</td>
<td>0.2519</td>
</tr>
<tr>
<td>$\theta$</td>
<td>0.4253</td>
</tr>
<tr>
<td>$\xi$</td>
<td>0.0038</td>
</tr>
<tr>
<td>$j$</td>
<td>0.1339</td>
</tr>
<tr>
<td>$m$</td>
<td>0.85</td>
</tr>
<tr>
<td>$\sigma$</td>
<td>0.972</td>
</tr>
</tbody>
</table>

$\beta_i = 0.965$ since the value of $\beta_i$ has little effect on the dynamics of the model. The only restriction on $\beta_i$ is that its value must be lower than $\beta_p$ to ensure that the borrowing constraint of impatient households is binding around the neighborhood of the steady state. In general, the lower is the value of $\beta_i$, the more likely will the borrowing constraint bind away from the steady state.

The survival rate of banks is arbitrarily set equal to 0.972, implying that banks survive for nine years on average. Although the expected survival time of banks may be longer than nine years, we only require that the expected time horizon is finite so that dividends paid by banks are guaranteed while the financial constraints are still binding. An alternative choice of the survival rate would not have a substantive effect on the business cycle properties reproduced by the model. A detailed discussion on this is given in the Appendix D. The fraction of funds transferred from patient households $\xi$, and the fraction of assets that can be diverted from banks to patient households $\theta$ are set equal to 0.0038 and 0.4253, respectively, in order to match two targets: an average leverage ratio of 3.32 from all US commercial banks and an average annual interest spread of 1%.

According to the Table of Assets and Liabilities of All Commercial Banks in the United States from the Fed’s, the average leverage ratio is 3.32 approximately.

---

21 The survival time $= \frac{1}{1-\sigma}$. Here, I choose the same survival rate as the one used by Gertler and Kiyotaki (2010).

22 I use time series of total assets and total liabilities of all US commercial banks from 1973 : Q1 to 2011 : Q3 to calculate the average leverage ratio, that is, LeverageRatio = $\frac{\text{Assets}}{\text{Assets} - \text{Liabilities}}$. 

---
Table 2: Steady-State Ratios

<table>
<thead>
<tr>
<th>Variables</th>
<th>Interpretation</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>4 \times (R_d - 1)</td>
<td>Annual real interest rate</td>
<td>3%</td>
</tr>
<tr>
<td>4 \times (R_k - R_d)</td>
<td>Annual interest spread</td>
<td>1%</td>
</tr>
<tr>
<td>C/GDP</td>
<td>Consumption to GDP ratio</td>
<td>83%</td>
</tr>
<tr>
<td>I_k/GDP</td>
<td>Nonresidential investment to GDP ratio</td>
<td>11%</td>
</tr>
<tr>
<td>qI_h/GDP</td>
<td>Residential investment to GDP ratio</td>
<td>6%</td>
</tr>
<tr>
<td>K/(4 \times GDP)</td>
<td>Nonresidential capital stock to annual GDP ratio</td>
<td>1.46</td>
</tr>
<tr>
<td>qH/(4 \times GDP)</td>
<td>Housing wealth to annual GDP ratio</td>
<td>1.41</td>
</tr>
<tr>
<td>p_x/(4 \times GDP)</td>
<td>Value of land to annual GDP ratio</td>
<td>0.50</td>
</tr>
<tr>
<td>\psi</td>
<td>Average leverage ratio</td>
<td>3.32</td>
</tr>
</tbody>
</table>

Note: Our definition of GDP is the sum of consumption, residential investment and nonresidential investment

the financial literature such as Gertler and Kiyotaki (2010) and Iacoveillo(2010a), I set the target for the annual interest spread to 1%, based on the pre-2007 spread as a rough average of the following three spreads: BAA corporate bond rates versus government bonds, mortgage rates versus government bonds, and commercial paper rates versus T-Bill rates.

I choose the depreciation rates for capital $\delta_k = 0.0188$ and the capital share in the consumption goods sector $\mu_c = 0.1795$ together with other estimated parameters to hit a steady-state nonresidential investment to GDP ratio of 11% and a steady-state nonresidential capital stock to annual GDP ratio of 1.46. In addition, the depreciation rate for housing is set equal to $\delta_h = 0.0106$ and the land share in the housing production function is set equal to $\mu_h = 0.2519$, implying a steady-state residential investment to GDP ratio of 6% and a steady-state land value to annual GDP ratio of 0.50. The calibrated value of $\delta_h$ is consistent with the estimation from most housing literature such as Davis and Heathcote (2005) and Iacoviello and Neri (2010). The calibrated value of $\mu_h$ seems to be also consistent with previous housing literature, see Davis and Palumbo (2008), Saiz (2010), Kiyotaki, Michaelides and Nikolov (2011), and Lloyd-Ellis, Head and Sun (2011). The

23Note that the depreciation rate for capital $\delta_k = 0.0188$ in my model is slightly less than that in Iacoviello and Neri (2010) and Iacoviello (2005) while the target of the nonresidential investment to GDP ratio in this paper is less than the one used by the latter. 24Greenwood and Hercowitz (1991) estimates a larger depreciation rate of 0.078 for residential structures; but Davis and Heathcote (2005) estimates a slightly smaller depreciation rate of 0.0157; and Iacoviello and Neri (2010) calibrates a depreciation rate of 0.01.
25Those literature calibrate a relative share of land in the price of housing within a range from 25% to 30%.
preference weight for housing in the utility function is set at \( j = 0.1339 \). Given the value of input shares in the housing production function and other parameters calibrated, the choice of housing preference weight implies a steady-state housing wealth to annual GDP ratio of 1.41.

According to the data from the Finance Board’s Monthly Survey of Rates and Terms on Conventional Single-Family Non-farm Mortgages Loans, the average loan-to-value ratio for homebuyers is about 0.76 between 1973 and 2006. Since the LTV ratio in the model refers to the fraction of housing value that can be borrowed by those who are credit constrained (impatient households), the value of the ratio for those credit constrained homebuyers might be greater than the average value for overall homebuyers. For example, in 2004 27% of new homebuyers borrowed funds from banks up to a fraction in excess of 80%, with an average ratio of 94% among them. Following present housing literature using the conventional DSGE framework, I set the LTV ratio equal to 0.85. Alternative values of the LTV ratio will be taken into considerations while conducting robustness test for the model.

The labor income share of credit-constrained households, \( 1 - \alpha \), is difficult to estimate due to data availability. More importantly, different approaches and data source generate different estimated values. Kiyotaki, Michaelides and Nikolov(2011) report that the fraction of constrained home owners in the population is 13.9%, and the fraction of tenants is 36% in the early 1990s. Suppose we treat tenants as constrained households, then the fraction of constrained households is about 50%. Jappelli (1990) uses the 1983 Survey of Consumer Finances to estimate a fraction of 20% population that are credit constrained. Iacoviello (2005) gives a wage share of credit constrained households of 36%, based on a limited information approach. In the model, I set \( \alpha = 0.36 \), following Iacoviello (2005) since it is within the range of estimates reported by most of the existing literature.

I set the disutility function parameters \( \eta_p = 0.52 \) and \( \eta_i = 0.51 \). For the inverse elasticity of substitution across hours in the consumption goods and housing sectors, I fix \( \epsilon_p = 0.66 \) and \( \epsilon_i = 0.97 \). \(^{26}\)

Finally, I choose the persistence parameter for the capital quality shock, \( \rho_k = 0.66 \), with a standard deviation of 5%. As argued by Gertler and Kiyotaki (2010), the choice of these parameters would generate a downturn of similar magnitude to that which occurred during the 2007 financial crisis.

\(^{26}\)Those parameters are estimated by the mean of posterior distribution in Iacoviello and Neri(2010).
crisis. For the experiment with positive productivity shocks in two production sectors, I choose the persistence parameter for the productivity shock in the final goods sector to be $\rho_c = 0.95$, with a standard deviation of $\sigma_c = 0.01$. The persistence parameter for the productivity shock in the housing sector is set to $\rho_h = 0.997$ and the standard deviation to $\sigma_h = 0.0193$, following the estimation of structural parameters in Iacoviello and Neri(2010).

6. Financial Crisis Experiment

I now turn to the financial crisis experiment. In the experiment, I mainly focus on the effects of a financial crisis rather than the cause itself. That is, I investigate how a deterioration in banks’ balance sheets affects real activities during the crisis. The question of what is a financial shock is not the core of this paper since the literature regarding the 2007 financial crisis offers different interpretation of the cause. Iacoviello (2010a) interprets the financial shock as a repayment shock that is purely redistributive in nature. In particular, the shock is supposed to be a negative wealth shock from the perspective of borrowers on one hand, and a positive wealth shock from the perspective of lenders on the other hand. Conversely, Gertler and Kiyotaki (2010) argue that what triggered the recent financial crisis was a decline in housing values that induces a wave of losses on mortgage backed securities (MBS) held by the banks. The true primary cause of the financial crisis still remains an open question. Although my model is not sufficient to capture precisely the main cause of the financial crisis, it can still give a full story about the consequences of a deterioration in banks’ balance sheets during a crisis. For simplicity, the initial disturbance in this financial crisis experiment is an exogenous capital quality shock as in Gertler and Kiyotaki (2010), since it allows us to capture in a parsimonious way the mechanism by which an exogenous force worsens the banks’ balance sheets, and in turn, puts downward pressure on output, investment, and asset prices in the financial crisis. Although the financial shock could also refer to a shock to the borrowing

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27 The model without financial frictions is a closed version of Iacoviello and Neri(2010). Since the time span of each series and data source used in this paper are roughly similar to that of Iacoviello and Neri(2010), I borrow the results from their estimation of these parameters.

28 The assumption used in Iacoviello (2010a) that borrowers face a persistent and positive wealth shock is not appropriate when they default on their debts. Broadly speaking, borrowers who pay less than or default their obligations today would face a penalty of delinquency or a risk of seizure of their assets tomorrow. Accordingly, the assumption that borrowers persistently face a positive wealth shock during the financial crisis is too strong in their model.
constraints (e.g. liquidity shocks), as Shi (2012) argues, this type of shocks will generate an asset price boom, which is inconsistent with what happened in the Great Recession. For asset prices to fall, a negative liquidity shock must be accompanied or caused by other shocks that induce a significant decline in investment and relax the borrowing constraints on investment. In this regard, capital quality shock is a good candidate of the financial shocks, since it can be served as a primary driving force of the business cycle that generates the dynamics of asset prices observed in the Great Recession.

6.1 The Model without Financial Frictions

I start the analysis by considering the performance of the model without financial frictions. Figure 5 reports the impulse responses of the key variables of interest following a 5% decline in capital quality. The solid line represents the dynamic paths for the baseline model with financial frictions, and the dotted line represents the model without financial frictions.

Initially, a negative capital quality shock generates a decline of the capital stock by about 3%. It is then followed by a decline in both output and employment. Since a high capital return is accompanied by the loss of capital, higher investment is induced. Recall that in the model without financial frictions, the bank acts as a veil and does not play any role in the economy. One may think of patient households lending funds to borrowers directly without any financial intermediation. In response to a negative capital quality shock, it is optimal for patient households to issue less commercial loans and consumer loans in order to smooth their consumption. Thus, both commercial loans and consumer loans fall. Note that in the model without financial frictions, the change in consumer lending is minor and can be ignored. Since deposits are equal to the sum of commercial loans and consumer loans in this case, deposits also fall. The price of equity is positively associated with capital stock/equity holdings. Thus, the equity price falls as the capital stock decreases.

Recall that capital investment (business investment) in this economy is financed using funds borrowed from patient households. Ignoring, for now, any minor response of output and consumption, an initial decline in capital investment is mainly due to the fact that the aggregate capital stock initially decreases by less than commercial loans. Moreover, the decline in capital investment is reversed when the capital stock continues to fall over time. Housing stock and house prices
Figure 5: Financial Crisis Experiment

Figure 5: Impulse Responses to A Capital Quality Shock: The Model with Banks VS The Model without Banks

Note: The y-axis measures percent deviation from the steady state.
respond to the capital quality shock in a way related to the movements in the aggregate housing demand. Because houses demanded by impatient households are primarily financed by consumer loans. When consumer loans do not fall initially, the demand for housing does not jump down immediately, and thus the housing stock remain unchanged initially. Consequently, house prices do not jump down immediately. As the aggregate housing stock eventually declines over time, both housing investment and the house prices fall. One may see from Figure 5 that the effects of a negative exogenous shock on the housing variables are minor since these effects are not magnified by the financial frictions in this case. Strictly speaking, a model without financial frictions cannot reproduce the dynamics of house prices and the quantity of the housing that were actually observed in the financial crisis. More importantly, it generates a counterfactual outcome that housing investment and business investment negatively comove in response to an exogenous financial shock.

The definition of $GDP$ in the model is the sum of output and the value of housing investment. When output, housing investment, and house prices all decrease over time, it is obvious that $GDP$ must fall over time. In addition, the excess returns on loans over deposits (interest spread) remains unchanged over time since lenders are indifferent between loans and deposits. It follows that a capital quality shock in this case does not affect the real economy through financial frictions. We will see later that in the model with financial frictions, an increase in the interest spread feeds into the real economy as a drag on its growth during the financial crisis.

Eventually, as the capital stock reverts to its steady-state level, capital investment returns to its trend, as do employment, consumption and output. Similarly, both the housing market and the financial market converge to their long-run paths. Before we progress to the model with financial frictions, for the sake of comparison, it is worth to pointing out that, in absence of the financial frictions, the model cannot generate a positive co-movement between residential investment (housing investment) and nonresidential investment (business investment), which is inconsistent with the observation from the data. Moreover, such a model is silent in its ability to reflect the amplification effects of financial constraints on real activities during the crisis. Last, it cannot capture the mechanism in which the deleveraging process of banks slows down the recovery of the housing market and the whole economy after the crisis. Therefore, the model without financial frictions cannot qualitatively account for the real story of the Great Recession.

\footnote{29 According to the data, both the housing price and quantities falls significantly over the short run in the Great Recession.}
6.2 The Model with Financial Frictions

I now consider the performance of the model with financial frictions. The dynamic paths of the key variables are presented in Figure 5. For my baseline model, I find that the model can qualitatively capture the phenomenon of the 2007 financial crisis. In particular, a disruption in the banks’ balance sheets induced by an exogenous decline in capital quality has a significant effect on the goods market, the housing market, the financial market and the economy as a whole.

With financial frictions, a 5% decline in capital quality immediately leads to a decrease in banks’ net worth by roughly 50%. This huge decline in net worth is fundamentally a product of a high bank leverage ratio and a decline in the market price of equities arising from the fire sale of the bank’s assets. In particular, an exogenous shock affects the banks’ net worth in two ways. First, an initial capital quality shock directly reduces the value of equities held by banks, and hence their net worth. Because the bank is highly leveraged, the effect on its net worth would be magnified by a factor equal to the bank’s leverage ratio. Second, the decline in the net worth then tightens the bank’s borrowing constraint, causing a fire sale of the bank’s assets (equities). Eventually, the price of the equity falls. This second round effect further depresses the value of the bank’s equity and thereby induces a huge loss in the bank’s net worth. In addition, although deposits slightly decrease initially, they eventually fall over time due to the increase in the tightness of the bank’s borrowing constraint.

In order to respond to the decline in the bank’s net worth, the bank reduces the amount of funds lent to impatient households and final goods firms. This leads to a roughly 12% decline in commercial loans and a roughly 25% decline in consumer loans. Since the movement in commercial loans is ultimately responsible for the change in business investment, the latter falls by roughly 10% as commercial loans decline. As a result, the capital stock and output fall by about 10% and 2%, respectively.

The aggregate demand for housing or final goods depends on the movement of the individual demand in each group of households. For impatient households, individual demand for housing falls by about 25% as the amount of consumer loans received from banks decreases. Individual demand for final goods, however, initially increases by about 2.5%, reflecting the fact that impatient households substitute towards final goods and away from housing due to their low patience and the
anticipated depreciation of house prices. Individual demand for final goods eventually falls as their budget constraints get tighter. For patient households, they respond to the shock in the opposite way. Specifically, individual demand for housing initially rises by 8% and continues to increase over time. This is because this group of households is patient enough that they can afford the losses arising from low anticipated house prices, and enjoy more housing service in the future. Because the losses from the temporary decline in the house prices can be compensated by the future gains from enjoying more housing, the individual demand for housing increases. In order to accumulate more housing, patient households forgo some consumption and thus the individual demand for final goods decreases. Since the decrease in demand for housing from impatient households exceeds the increase in demand for housing from patient households, aggregate demand for housing falls initially. This subsequently depresses house prices and housing investment. The depreciation of house prices and the decline in housing investment are reinforced by the continuing decline in aggregate housing demand over time. As we can see from Figure 5, both house prices and housing investment decline significantly in the short run. Note that both house prices and housing investment revert back immediately after an initial decline is a product of the interaction between the individual demand for housing from each group of households. Even if equilibrium aggregate consumption does not initially fall, it will eventually go down as the budget constraint of impatient households tightens over time.

The financial frictions at work during the crisis are reflected in the spread between the expected return on assets and the expected cost of deposits. In the model with financial frictions, the spread increases as an outcome of the decline in the bank’s net worth. In particular, when the spread rises, the cost of financing capital increases to the extent that debtor firms must downsize the amount of capital investment and reduce its volume of output. Therefore, the magnified declines in capital investment and output can be attributed to the increase in the spread during the financial crisis.

During a financial crisis, banks can restore their leverage ratio either by deleveraging or accumulating more net worth. In the process of deleveraging, banks have to increase their net worth. As we can see from Figure 5, an increase in net worth is accompanied by a decline in the spread. So long as the spread is above its trend, net worth cannot immediately revert to it’s steady-state level. Throughout the transition path to the long-run trend, the convergence process is slow and it takes a long time for the bank to restore its leverage ratio (see the panel of Leverage Ratio in
Figure 5). Strictly speaking, the change in the spread between expected returns on assets and costs on deposits induced by the financial frictions slows down the pace of recovery in the economy. In this way, our baseline model is successful in its ability to capture the mechanism through which the deleveraging process slows down the recovery of the economy during a financial crisis, which is consistent with Gertler and Kiyotaki (2010).

Within the framework with financial frictions, the effects of the financial crisis induced by an exogenous shock are amplified and propagated over time by financial factors. Compared to the alternative model without financial frictions, the decline in both GDP and output are more than doubled. Nonresidential investment (business investment) drops by nearly 10 times more, and the capital stock falls by nearly one third more than that in the frictionless model. In the housing market, residential investment (housing investment) in the short run falls by roughly 2 times more, and the housing stock falls at a larger magnitude over time, relative to its counterparts. House prices in both the short run and long run depreciates by about 2 times more than its counterpart. With financial frictions, the decline in both commercial loans and consumer loans are significantly larger than their counterparts. Deposits in the short run and medium run do not drop as much, and is only half of the decline in the frictionless model. However, it is slightly larger than its counterpart in the long run. Similarly, aggregate consumption in the short and medium run does not fall as much as in the frictionless case since impatient households are more willing to substitute towards consumption goods and away from houses. Without heterogeneity and housing, both deposits and consumption would fall by a larger amount than their counterpart, as illustrated in Gertler and Kiyotaki (2010). Moreover, in the frictionless model the spread does not change over time since the expected returns are equal across assets and deposits. Therefore, the spread does not serve as a barrier to the recovery of the economy from the financial crisis in such a model. Conversely, an increase in the spread is the key in the frictional model and generates a magnified effect, slowing down economic recovery from a financial crisis.
7. Economic Productivity Boom Experiment

7.1 Impulse Responses

To conduct an economic productivity boom experiment, I introduce a positive technology shock in the final goods sector as a trigger to boost the economy. In this experiment, a positive technology shock in the non-housing sector drives up the expected returns on capital and, hence, business investment (see the Nonresidential Investment panel in Figure 6). The response of banks’ net worth to the shock in this experiment is magnified in two ways. First, because the bank is highly leveraged, the rise in the expected return on capital raises the value of its assets and, consequently, enhances its net worth by a factor equal to the leverage ratio. Second, the rise in the net worth relaxes the bank’s borrowing constraint causing an accumulation of its assets that further raises the asset prices. Both forces reinforce each other and generate a significant rise in the banks’ net worth. With a high value of net worth, the bank can make more loans to its borrowers and accept more deposits from savers. Consequently, more funds are fed into the real economy, stimulating the capital stock, output, and house prices.

I now analyze the responses by each group of households to a positive technology shock. For impatient households, both the demand for consumption goods and housing increase over time, thanks to the relaxation of their budget constraints. For patient households, the demand for consumption goods also rises. The demand for housing, however, declines over time due to a dominant substitution effect induced by the high price of houses relative to consumption goods. Overall, aggregate consumption rises and aggregate demand for housing falls slightly. In addition, housing investment declines due to a rise in construction costs (e.g. wages and land rents) and weak demand for housing. But the magnitude of the decline in housing investment exceeds that of the aggregate demand for housing so that house prices rise over time. Note that a positive technology shock in the non-housing sector generates a negative correlation between house prices and housing investment, which contradicts the data. Even if we allow for a positive shock in the housing sector, the negative correlation cannot be reversed. Specifically, the addition of a housing technology shock is needed to account for the volatility of house prices, but this shock still cannot reproduce a positive correlation between the price and quantities of housing. The inability of the model with technology shocks alone to account for the positive comovement between house prices...
Figure 6: Impulse Responses to A Positive Productivity Shock in the Consumption Goods Sector

Note: The y-axis measures percent deviation from the steady state.
and housing quantities is in line with the findings by Davis and Heathcote (2005) and Iacoviello and Neri (2010). This is a standard problem for any supply-side shocks.

The spread decreases as an outcome of the rise in the bank’s net worth (see the Interest Spread panel in Figure 6). The magnified increase in business investment, output and GDP are primarily attributed to the decline in the cost of financing capital. In order for the spread to revert back to its trend, the bank has to re-leverage or decumulate its assets. As in the financial crisis experiment, this process takes time. So long as the spread remains below its trend, financial factors in the model act as an engine to stimulate an economic boom.

### 7.2 Cyclical Properties

In this subsection, I investigate whether the baseline model fits the business cycle properties observed in the data. In addition to a positive productivity shock in the final goods sector, from now on, I allow for a positive technology shock in the housing sector since it is needed to account for the volatility of house prices. Table 3 reports selected volatilities and correlations for the data, the baseline model and the models from previous housing literature.

The relative volatilities of the key variables of interest are presented in the Panel A, and the correlations are presented in the Panel B. From Table 3, I find that the baseline model replicates well the volatility and procyclicality of consumption, investment and house prices as well as the joint behavior between house prices and macroeconomic quantities of interest over the business cycle in the U.S. history. In particular, the model approximately reproduces the volatilities of GDP, private consumption, business investment, housing investment and house prices. Moreover, private consumption, business investment, housing investment and house prices are procyclical, as observed in the data. Last, the model replicates well the correlations of house prices with private consumption, business investment and consumer loans. Compared with the existing housing literature, Davis and Heathcote (2005) and Iacoviello and Neri (2010), the baseline model with technology shocks alone improves its ability to account for the volatility and procyclicality of house prices and the correlation between house prices and private consumption.

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30 Here, technology shocks in these sectors are different and uncorrelated with each other.
31 Statistics are averages over 500 simulations and, for each of simulated time series, the length of the main variables equals to 155 periods, that of my data sample. The business cycle component of each simulated series is obtained using the HP filter with
Table 3: Business Cycle Properties of the Baseline Model

<table>
<thead>
<tr>
<th>Variables</th>
<th>Data</th>
<th>Davis-Heathcote(DH)</th>
<th>Iacoviello-Neri(IN)</th>
<th>Model</th>
</tr>
</thead>
<tbody>
<tr>
<td>Panel A: %SD(Relative to GDP)</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>$GDP$</td>
<td>2.06</td>
<td>1.73</td>
<td>1.53</td>
<td>2.34</td>
</tr>
<tr>
<td>$C$</td>
<td>0.63</td>
<td>0.48</td>
<td>0.92</td>
<td>0.82</td>
</tr>
<tr>
<td>$I_k$</td>
<td>2.65</td>
<td>3.21</td>
<td>3.07</td>
<td>2.80</td>
</tr>
<tr>
<td>$I_h$</td>
<td>4.95</td>
<td>6.12</td>
<td>5.72</td>
<td>3.35</td>
</tr>
<tr>
<td>$q$</td>
<td>1.21(FMHP)</td>
<td>0.40</td>
<td>0.90</td>
<td>1.80</td>
</tr>
<tr>
<td></td>
<td></td>
<td>2.08(S&amp;P HPI)</td>
<td></td>
<td></td>
</tr>
<tr>
<td>$pS$</td>
<td>2.63</td>
<td>–</td>
<td>–</td>
<td>0.99</td>
</tr>
<tr>
<td>$B$</td>
<td>2.57</td>
<td>–</td>
<td>–</td>
<td>1.80</td>
</tr>
<tr>
<td>$D$</td>
<td>0.73</td>
<td>–</td>
<td>–</td>
<td>1.07</td>
</tr>
<tr>
<td>$N$</td>
<td>3.57</td>
<td>–</td>
<td>–</td>
<td>1.05</td>
</tr>
<tr>
<td>Panel B: Correlations</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>$C, GDP$</td>
<td>0.97</td>
<td>0.95</td>
<td>0.83</td>
<td>0.97</td>
</tr>
<tr>
<td>$I_k, GDP$</td>
<td>0.73</td>
<td>–</td>
<td>0.84</td>
<td>0.89</td>
</tr>
<tr>
<td>$I_h, GDP$</td>
<td>0.86</td>
<td>–</td>
<td>0.54</td>
<td>0.11</td>
</tr>
<tr>
<td>$q, GDP$</td>
<td>0.36</td>
<td>0.65</td>
<td>0.73</td>
<td>0.29</td>
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<tr>
<td>$pS, GDP$</td>
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<td>–</td>
<td>0.82</td>
</tr>
<tr>
<td>$B, GDP$</td>
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<td>–</td>
<td>–</td>
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<tr>
<td>$D, GDP$</td>
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<td>–</td>
<td>–</td>
<td>0.90</td>
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<tr>
<td>$N, GDP$</td>
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<td>–</td>
<td>–</td>
<td>0.84</td>
</tr>
<tr>
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<td>–</td>
<td>0.95</td>
<td>0.35</td>
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<tr>
<td>$q, I_k$</td>
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<td>–</td>
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<tr>
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<td>-0.20</td>
<td>0.17</td>
<td>-0.72</td>
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<td>–</td>
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<tr>
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<tr>
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<tr>
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<td>0.38</td>
<td>0.15</td>
<td>–</td>
<td>0.01</td>
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</table>

$^a$Business cycle statistics reproduced by Davis and Heathcote(2005). They estimate the model with technology shocks alone by using the data from 1948 to 2001.

$^b$Business cycle statistics reproduced by Iacoviello and Neri(2010). They estimate the model with technology shocks alone by using the data from 1965 to 2006.
To my knowledge, most of existing housing literature fail to account for the high volatility of house prices observed in the data. For instance, the Davis-Heathcote model with technology shocks alone does poorly in accounting for the swings in house prices, and only reproduces one third of that observed in the data, implying that a model without the heterogeneity of households and financial factors cannot generate a high volatility of house prices. In contrast, the Iacoviello-Neri model can be successful in accounting for high volatility of house prices after a variety of exogenous shocks are introduced into their model. Fundamentally, a high volatility of house prices is generated in their model by dealing with the fixed supply of land, the heterogeneity of households, and a variety of exogenous shocks. Their model with technology shocks alone, however, reproduces a counterfactual outcome that house prices are less volatile than GDP. These earlier works fail to reproduce the high volatility of the house prices observed in the data because they cannot create an intensive interaction between individual demand for housing from the two groups of households over the business cycle in determining the house prices. In particular, the volume of consumer loans issued by banks fundamentally affects the movement of the house prices over the business cycle. So long as the volatility of consumer loans is high, the demand for housing from the impatient household side swings at a larger magnitude than it otherwise would in the absence of financial factors. In contrast to the recent housing literature, my baseline model does a good job in accounting for the high volatility of the house prices observed in the data even with technology shocks alone. As Table 3 shows, the relative volatility of house prices is 1.80, which is significantly higher than those reproduced by previous authors. The volatility of house prices reproduced by the baseline model seems overstated, relative to the data if I use the Freddie Mac House Price Index (FMHPI) as a measure of house prices. However, it is consistent with the one using the S&P/Case-Shiller Home Price Index as a measure of house prices. Note that the volatility of house prices in the data using the Freddie Mac House Price Index (FMHPI) as a measure of house prices is 1.21, which is smaller than that using the S&P/Case-Shiller Home Price Index – a smoothing parameter equal to 1,600.

\[ \text{smoothing parameter equal to 1,600.} \]

\[ \text{The volatility of house prices using the S&P/Case-Shiller Home Price Index as a measure of house prices tends to be higher than that using the FMHPI, and is 2.08. The starting year of the S&P/Case-Shiller Home Price Index is 1985. In general, the volatility of house prices in the data differs one to another by using different source of house price indices. For instance, Davis and Heathcote (2005) calculates the volatility of house prices of 1.37 from the data, using the FMHPI from 1970 to 2001; and Iacoviello and Neri (2010) calculates the volatility of house prices of 1.87 from the data, using the U.S. Census House Price Index from 1965 to 2006.} \]
Moreover, I find that house prices are procyclical in line with the observation from the data. Thus, the model can quantitatively explain the volatility and procyclicality of house prices, implying that the baseline model fits to the data in the dimension of house prices.

For the financial quantities, the model replicates well the volatility of consumer loans. As Table 3 indicates, the volatility of consumer loans is high at 1.80, which is consistent with the data. In addition, the model approximately reproduces the procyclicality of consumer loans, deposits and net worth, though they are somewhat higher than those observed empirically. The reason that the model overstates the procyclicality of these financial quantities is that, in the baseline model, business investment is primarily financed by commercial loans, and it is the main component of GDP. With this formulation, commercial loans inevitably have a strong correlation with GDP. Since banks are indifferent between issuing commercial loans and consumer loans to their borrowers, consumer loans are also strongly correlated with GDP, and so is net worth.

Moreover, the model also replicates well the following quantitative phenomena observed in the data. First, commercial loans are positively correlated with business investment. Second, consumer loans are positively correlated with private consumption.

However, there are few aspects in which the model performs poorly. First, it understates the volatilities of commercial loans and net worth, and overstates the volatility of deposits. Second, the model overstates the procyclicality of commercial loans, consumer loans, deposits and net worth. To address this issue, we have to weaken the dependence of business investment on commercial loans. The assumption that business investment is financed by only commercial loans might be too strong in the model. Third, in the data house prices and housing investment move in the same direction. In contrast, the model generates a negative correlation between the price and investment. This is consistent with the findings of Davis and Heathcote (2005) and Iacoviello and Neri (2010) that a model with only technology shocks cannot replicate a positive correlation between the price and the quantity of housing. As in Iacoviello and Neri (2010), the baseline model can reverse this counterfactual result once other exogenous shocks (e.g. demand-side shocks) are introduced into the model. In addition, consumer loans and housing investment move in the opposite direction, which is contrary to the data. This contradiction may be altered with the similar approach above and I reserve such an exploration for future research. Last, a striking drawback of the model is that it understates the correlation between housing investment and business investment. In general,
they are positively correlated in the data, and the correlation is 0.4 roughly.

7.3. Robustness Analysis

In this section, I investigate the ability of the model in matching some certain statistics observed in the data by shutting off one or more frictions or/and changing one or more parameters which are fixed in the baseline model each time. Note that the model we are considering about here is the one with only technology shocks. I summarize the main findings as follows. Appendix D gives my results in details.

Does Full Labor Mobility Matters? - A model with perfect labor mobility only exacerbates the volatilities of housing investment, business investment, house prices and consumer loans without generating a positive correlation between housing investment and business investment. In particular, it generates a counterfactual result that two types of investment are negatively correlated. In addition, this version of the model further understates the procyclicality of housing investment. Last, the perfect labor mobility does not help to improve the model’s ability in accounting for a positive comovement between consumer loans and house prices. The correlation between consumer loans and house prices reproduced by this version of the model is even worse, relative to the baseline model.

Does the Assumption of Heterogeneous Labor Matters? - In the baseline model, I assume that two types of households differ in their labor supply parameters, reflecting the heterogenous labor supplied to a particular nonfinancial sector across households. Appendix D (Table 4) reports the results when I manipulate the labor supply parameters to be same across the two types of households. The business cycle statistics reproduced by the model with the homogenous labor (same labor supply parameters) are essentially unchanged. Accordingly, the assumption of heterogenous labor does not matter for the model.

Does the Financial Constraint Matters? - A model without financial constraints decreases the relative volatilities of consumer loans and deposits and increases the relative volatility of net worth. Moreover, it reduces the procyclicality of the two types of loans and deposits. Similar to the baseline model, it generates a counterfactual outcome that business and housing investment are negatively correlated. The counterfactual comovement between the two types of investment can
also be observed in this version of the model conditional on a financial shock (see the Financial Crisis Experiment). All other business cycle statistics reproduced by the model without financial constraints are identical to that in the baseline model.

**Effects of Lower Downpayment Requirements and Higher Banks’ Survival Rates**

Lower down-payment requirements lead to a decrease in the volatility of business investment, consumer loans, commercial loans, and the banks’ net worth, and leave the volatility of housing investment unchanged. The fact that a rise in the loan-to-value ratio leaves the volatility of housing investment unchanged in my model is the outcome of the interaction between housing purchases and labor supply. On the one hand, borrowers (impatient households) tend to mitigate aggregate volatility by working relatively less hours in response to positive technology shocks since they now can borrow more funds from banks than before, given their houses as a collateral. On the other hand, savers (patient households) tend to increase aggregate volatility by working relatively more hours in response to positive technology shocks. As long as two effects offset each other, the volatility of housing investment remains unchanged as the loan-to-value ratio rises. This result, however, is contrary to Campbell and Hercowitz (2005) and Iacoviello and Pavan (2013). In the model, consumer loans are used to finance housing purchases by indebted households, and the housing purchases of indebted households respond less to positive technology shocks as the loan-to-value ratio rises. In this regard, a rise in the loan-to-value ratio reduces the volatility of consumer loans, and therefore, the volatilities of commercial loans and net worth. Since business investment are primarily financed by commercial loans, the volatility of business investment falls as well.

A rise in the banks’ survival rates raises the volatilities of housing investment and deposits, and reduces the volatilities of business investment, commercial loans, consumer loans, and net worth. Recall that in the economy a bank only pays dividends to its owners while exiting from the banking system. When the survival time of a bank is longer, the bank has relatively less motive for

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33In Campbell and Hercowitz (2005), financial innovations account for more than half of the decrease in aggregate volatility in a model with borrowing constraints. Since they assume that labor supply of wealthier households is constant, the offsetting mechanism between the rich and the poor is silent, that is, the effect of labor supply on the aggregate volatility from the poor is not offset by that from the rich in their model. In Iacoviello and Pavan (2013), aside from modeling differences, the offsetting mechanism is at work to induce a decrease in aggregate volatility in response to positive technology shocks, which is contrary to my result. In preference of the adjustment costs of housing, the decline in aggregate volatility induced by reducing hours by indebted homeowners is not fully offset by the rise in aggregate volatility induced by increasing hours by wealthier homeowners (creditors). Therefore, the volatility of housing investment in their model is reduced as the loan-to-value ratio rises.
paying dividends. Absent some motive for paying dividends, the bank may find it more optimal to accumulate its assets to the point where the financial constraint it faces is no longer binding. In this case, both types of loans respond less to the shocks, and so does net worth. Since commercial loans and consumer loans are used to finance business investment and the housing purchases of indebted households respectively, a rise in the banks’ survival rates reduces the aggregate volatility of business investment and the aggregate volatility of housing purchases (housing investment). As I discussed in the text above, savers (patient households), however, increase the aggregate volatility of housing purchases by working relatively more hours in response to the shocks. Due to the offsetting mechanism, a rise in the banks’ survival rate slightly increases aggregate volatilities of housing investment and house prices. By learning about the bank’s increased ability to save to overcome financial constraints, depositors respond more to the shocks, and therefore the volatility of deposits increases as the loan-to-value rises.

The business cycle statistics of interest except for deposits change in the same direction in both cases. Given both lower downpayment requirement and higher banks’ survival rates, the relative volatilities of housing investment and deposits increase, and the relative volatilities of business investment, commercial loans, consumer loans, and net worth decrease. Moreover, this version of the model improves a bit the correlation between the two types of investment, but it still understates the correlation, relative to the data.

8. Concluding Remarks

This paper develops a variant of Iacoviello-Neri model by considering the financial frictions used by Gertler and Kiyotaki (2010), and creates a link between the housing market and the financial market in the context of the Great Recession. The dynamics of housing and banking over the business cycle are realistic not only at the macroeconomic level, but also at the level of individual household behavior. The baseline model works well in two dimensions. First, it can qualitatively capture the behaviors of the housing market, the financial market and the macroeconomy in the Great Recession. More importantly, it explores the mechanism by which a disruption in the banks’ balance sheet affects the housing market and macroeconomy in the crisis and the extent to which

\[34\] Here, a rise in the banks’ survival rates virtually enhances the banks’ ability to save to overcome financial constraints.
the deleveraging process of banks slows down the recovery from the crisis. Second, the baseline model with technology alone is successful in its ability to account for several key features of the U.S. business cycle. In particular, the model replicates well the volatility and procyclicality of consumption, business investment and house prices, the volatility of housing investment and consumer loans, and the co-movement of house prices with consumption, business investment and consumer loans.

The model, however, performs poorly in the following dimensions. First, it understates the volatilities of commercial loans and net worth, and overstates the volatility of deposits. Second, the model overstates the procyclicality of financial quantities. In the baseline model, business investment primarily depends on commercial loans, and is the main components of GDP. Within the framework, it inevitably strengthens the link between commercial loans and GDP, and thus the link between other financial quantities and GDP. To address this issue, we have to weaken the dependence of business investment on commercial loans. One possible approach to address this drawback might be that business investment is assumed to be partially financed by commercial loans. Third, the model generates a counterfactual correlation between house prices and housing investment, and a counterfactual correlation between consumer loans and housing investment. Last, the model understates the positive co-movement between housing investment and business investment. These counterfactual results are the general outcome of the supply-side shocks in the current context of the DSGE models. They can be addressed once we introduce some forms of demand-side shocks, such as the housing preference shocks. Iacoviello and Neri (2010) provides a detailed discussion on the role of demand-side shocks of housing over the business cycle. The approaches to address these issues above are only meant to be suggestive. It goes without saying that these approaches can successfully address the issues above without sacrificing others. Here, I leave these open issues for future explorations.

This paper does not explicitly consider the default of indebted households. One may extend this model by including the risk of default to investigate the extent to which the presence of the default affects the financial market, the housing market and the whole economy over the business cycle. Moreover, given that house prices are relatively volatile in the data, it is not clear whether the interest policies and other conventional policy instruments can successfully stabilize house prices, or provided they can do so, that they can stabilize house prices without inducing excessive
volatility in other macroeconomic quantities. Suppose that conventional policy instruments fail to stabilize house prices, it raises an open question: are there any unconventional policy instruments can be successful in stabilizing housing prices without causing excessive volatility in other macroeconomic quantities? It seems to me that addressing these issues in the DSGE models with housing and banking is a sensible thing to do and I leave them for future research.
References


Appendix A: Data and Sources

**Real Aggregate Consumption:** Real Personal Consumption Expenditures (seasonal adjusted, billions of chained 2005 dollars, table 1.1.6, line 2). Source: NIPA, Bureau of Economics Analysis (BEA).


**Real House Prices:** Freddie Mac House Price Index (FMHPI). The FMHPI series is only available from 1975. To ensure compatibility across all time series used in this paper, I extrapolate the FMHPI series backwards for the period 1973-1974 using the growth rate of the U.S. Census...
Bureau House Price Index during the same period. Source: Freddie Mac; and S&P/Case-Shiller Home Price Index (seasonally adjusted). Sources: Standard & Poor’s.

**Real Commercial Loans:** Nominal Commercial Loans (seasonally adjusted, all banks, Assets and Liabilities of Commercial Banks in the United States, Table H8) divided by Price Index for Personal Consumption Expenditures (seasonally adjusted, index numbers: 2005=100, table 1.1.4, line 2). Source: Board of Governors of Federal Reserve System (FRS) and NIPA, Bureau of Economics Analysis (BEA).

**Real Consumer Loans:** Nominal Consumer Loans (seasonally adjusted, all banks, Assets and Liabilities of Commercial Banks in the United States, Table H8) divided by Price Index for Personal Consumption Expenditures (seasonally adjusted, index numbers: 2005=100, table 1.1.4, line 2). Source: Board of Governors of Federal Reserve System (FRS) and NIPA, Bureau of Economics Analysis (BEA).

**Real Deposits:** Nominal Deposits (seasonally adjusted, all banks, Assets and Liabilities of Commercial Banks in the United States, Table H8) divided by Price Index for Personal Consumption Expenditures (seasonally adjusted, index numbers: 2005=100, table 1.1.4, line 2). Source: Board of Governors of Federal Reserve System (FRS) and NIPA, Bureau of Economics Analysis (BEA).

**Real Net Worth:** Nominal Residual: Assets less Liabilities (seasonally adjusted, all banks, Assets and Liabilities of Commercial Banks in the United States, Table H8) divided by Price Index for Personal Consumption Expenditures (seasonally adjusted, index numbers: 2005=100, table 1.1.4, line 2). Source: Board of Governors of Federal Reserve System (FRS) and NIPA, Bureau of Economics Analysis (BEA).
Appendix B: Derivation of Undetermined Coefficients for the Conjectured Value Function

In this section, I give full details in deriving undetermined coefficients for the conjectured value function. I first guess the value function $V_t(s_t, b_t, d_t)$ is linear in $(\nu_{st}, \nu_{bt}, \nu_{dt})$,

$$V_t(s_t, b_t, d_t) = \nu_{st}s_t + \nu_{bt}b_t - \nu_{dt}d_t.$$ 

Given the flow-of-fund constraint (12), the incentive constraint (13) can be rewritten as

$$V_t \geq \theta(n_t + d_t). \quad (59)$$

Then I replace $V_t$ in equation (59) with equation (16) together with the flow-of-fund constraint (12) to get an alternative expression for the incentive constraint, that is

$$[\theta - (\nu_{bt} - \nu_{dt})]d_t + (\nu_{bt} - \frac{\nu_{st}}{p_t})p_t s_t \leq (\nu_{bt} - \theta)n_t. \quad (60)$$

where (60) holds with equality if $\lambda^b_t > 0$, and with strict inequality if $\lambda^b_t = 0$. The first order condition (17) implies that

$$\frac{\nu_{st}}{p_t} = \nu_{bt}, \quad (61)$$

where the marginal value of equity is always equal to the marginal value of consumer loans no matter the incentive constraint is binding or not, implying that banks are indifferent between issuing commercial loans to non-financial firms and issuing consumer loans to impatient households. Moreover, I rewrite the first order condition (18) as follows,

$$\nu_{bt} - \nu_{dt} = \frac{\theta \lambda^b_t}{1 + \lambda^b_t}. \quad (62)$$

Substituting equations (61) and (62) into the incentive constraint (60) yields

$$d_t \leq \frac{1 + \lambda^b_t}{\theta}(\nu_{bt} - \theta)n_t. \quad (63)$$

Similarly, the conjectured value function (16) together with (61) and (62) yields

$$V_t = \frac{\theta \lambda^b_t}{1 + \lambda^b_t}d_t + \nu_{bt}n_t. \quad (64)$$

While substituting (63) into (64) to replace $d_t$, I get an new expression for the value function,

$$V_t = [\lambda^b_t(\nu_{bt} - \theta) + \nu_{bt}]n_t \quad (65)$$
where the term in the bracket is the marginal value of net worth to the ongoing bank. Intuitively, with an additional unit of net worth, the bank can issue an additional consumer loans to impatient households by obtaining a benefit of $\nu_{bt}$, and in turn, relax the incentive constraint by $\nu_{bt} - \theta$, which increases the value of the bank by a factor equal to $\lambda^b_t$.

Finally, I substitute equation (65) for date $t+1$ into the Bellman equation (15) to yield

$$V_t(s_t, b_t, d_t) = E_t \Lambda_{t,t+1} \Omega_{t+1} n_{t+1}$$

with

$$\Omega_{t+1} = 1 - \sigma + \sigma[\lambda^b_{t+1}(\nu_{bt+1} - \theta) + \nu_{bt+1}]$$

where $\Omega_{t+1}$ is the marginal value of net worth at date $t+1$.

Given equations (61) and (62), I can derive an expression for the excess value of returns on assets over deposits,

$$\mu_t = \frac{\nu_{st}}{p_t} - \nu_{dt} = \nu_{bt} - \nu_{dt} = \frac{\theta \lambda^b_t}{1 + \lambda^b_t}.$$  

Combining equation (68) for date $t+1$ with equation (67), I can obtain an new expression for the marginal value of net worth at date $t + 1$, that is

$$\Omega_{t+1} = 1 - \sigma + \sigma(\nu_{dt+1} + \phi_{t+1} \mu_{t+1})$$

with

$$\phi_{t+1} = \frac{\nu_{dt+1}}{\theta - \mu_{t+1}}$$

where $\phi_{t+1}$ is the bank’s leverage ratio at date $t + 1$.

By applying the method of the undetermined coefficients to equation (66), we can easily determine all coefficients to the conjectured value function as follows,

$$\nu_{st} = E_t \Lambda_{t,t+1} \Omega_{t+1} [Z_{t+1} + (1 - \delta_k)p_{t+1}] \psi_{t+1}$$

$$\nu_{bt} = E_t \Lambda_{t,t+1} \Omega_{t+1} R^b_{t+1}$$

$$\nu_{dt} = E_t \Lambda_{t,t+1} \Omega_{t+1} R^d_{t+1}$$

Therefore, the conjectured value function $V_t$ is linear in $(\nu_{st}, \nu_{bt}, \nu_{dt})$ if and only if the conditions (71), (72), and (73) are satisfied. **QED**
Appendix C: Dynamic System of the Model with Financial Frictions

\[ 1 \beta_p E_t \left( \frac{C_{p,t}}{C_{p,t+1}} R^d_{t+1} \right) \]

\[ \frac{q_t}{c_{p,t}} = \frac{j}{h_{p,t}} + \beta_p E_t \left( \frac{(1 - \delta_h)q_{t+1}}{c_{p,t+1}} \right) \]

\[ \frac{p_{x,t}}{c_{p,t}} = \beta_p E_t \left[ \frac{p_{x,t+1} + R^x_{t+1}}{c_{p,t+1}} \right] \]

\[ \frac{w_{pc,t}}{c_{p,t}} = \left( l^{1+\epsilon_p}_{pc,t} + l^{1+\epsilon_p}_{ph,t} \frac{\eta_p}{l^{\epsilon_p}_{pc,t}} \right) \]

\[ \frac{w_{ph,t}}{c_{p,t}} = \left( l^{1+\epsilon_p}_{pc,t} + l^{1+\epsilon_p}_{ph,t} \frac{\eta_p}{l^{\epsilon_p}_{pc,t}} \right) \]

\[ \frac{j}{h_{i,t}} + \beta_i E_t \left( \frac{(1 - \delta_h)q_{t+1}}{c_{i,t+1}} \right) = \frac{q_t}{c_{i,t}} - \lambda_{i,t} m E_t \left( \frac{q_{t+1}}{R^d_{i,t+1}} \right) \]

\[ \frac{1}{c_{i,t}} = \beta_i E_t \left( \frac{R^b_{t+1}}{c_{i,t+1}} \right) + \lambda_{i,t} \]

\[ \frac{w_{ic,t}}{c_{i,t}} = \left( l^{1+\epsilon_i}_{ic,t} + l^{1+\epsilon_i}_{ih,t} \frac{\eta_i}{l^{\epsilon_i}_{ic,t}} \right) \]

\[ \frac{w_{ih,t}}{c_{i,t}} = \left( l^{1+\epsilon_i}_{ic,t} + l^{1+\epsilon_i}_{ih,t} \frac{\eta_i}{l^{\epsilon_i}_{ic,t}} \right) \]

\[ b_t = m E_t \left( \frac{q_{t+1} h_{i,t}}{R^b_{t+1}} \right) \]

\[ c_{i,t} + q_t h_{i,t} + R^b_{b_{t-1}} = w_{ic,t} l_{ic,t} + w_{ih,t} l_{ih,t} + q_t (1 - \delta_h) h_{i,t-1} + b_t \]

\[ w_{pc,t} = \alpha (1 - \mu_c) \frac{Y_t}{l_{pc,t}} \]

\[ w_{ic,t} = (1 - \alpha) (1 - \mu_c) \frac{Y_t}{l_{ic,t}} \]

\[ Z_t = \mu_c \frac{Y_t}{K_t} \]

\[ Y_t = (Act(l^{\alpha}_{pc,t} l^{1-\alpha}_{ic,t}))^{1-\mu_c} K_t^{\mu_c} \]

\[ w_{ph,t} = \alpha (1 - \mu_h) \frac{q_t I_{ht}}{l_{ph,t}} \]

\[ w_{ih,t} = (1 - \alpha) (1 - \mu_c) \frac{q_t I_{ht}}{l_{ih,t}} \]

\[ R^x_t = \mu_h q_t I_{ht} \]

\[ I_{ht} = (Act(l^{\alpha}_{ph,t} l^{1-\alpha}_{ih,t}))^{1-\mu_h} \]
\[ p_t = 1 + \left[ \ln \left( \frac{I_{kt}}{I_{kt-1}} \right) \right]^2 + 2 \left[ \ln \left( \frac{I_{kt}}{I_{kt-1}} \right) \right] - 2 E_t \beta_p \frac{c_{p,t}}{c_{p,t+1}} \left[ \ln \left( \frac{I_{kt+1}}{I_{kt}} \right) \right] \frac{I_{kt+1}}{I_{kt}} \] (93)

\[ \nu_{bt} = E_t \beta_p \frac{c_{p,t}}{c_{p,t+1}} [1 - \sigma + \sigma (\nu_{dt+1} + \phi_{t+1} \mu_{t+1})] R^p_{t+1} \] (94)

\[ \nu_{dt} = E_t \beta_p \frac{c_{p,t}}{c_{p,t+1}} [1 - \sigma + \sigma (\nu_{dt+1} + \phi_{t+1} \mu_{t+1})] R^d_{t+1} \] (95)

\[ \nu_{st} = E_t \beta_p \frac{c_{p,t}}{c_{p,t+1}} [1 - \sigma + \sigma (\nu_{dt+1} + \phi_{t+1} \mu_{t+1})] \psi_{t+1} (Z_{t+1} + (1 - \delta_k) p_{t+1}) \] (96)

\[ \mu_t = E_t \beta_p \frac{c_{p,t}}{c_{p,t+1}} [1 - \sigma + \sigma (\nu_{dt+1} + \phi_{t+1} \mu_{t+1})] (R^k_{t+1} - R^d_{t+1}) \] (97)

\[ R^k_{t+1} = \frac{\psi_{t+1}}{p_t} Z_{t+1} + (1 - \delta_k) p_{t+1} \] (98)

\[ R^k_{t+1} = R^d_{t+1} \] (99)

\[ p_t S_t + B_t = N_t + D_t \] (100)

\[ p_t S_t + B_t = \phi_t N_t \] (101)

\[ \phi_t = \frac{\nu_{dt}}{\theta - \mu_t} \] (102)

\[ N_t = (\sigma + \xi) \{ (Z_t + (1 - \delta_k) p_t) \psi_t S_{t-1} + R^p_{t} B_{t-1} \} - \sigma R^d_{t} D_{t-1} \] (103)

\[ Y_t = C_t + (1 + (\ln \left( \frac{I_{kt}}{I_{kt-1}} \right))^2) I_{kt} \] (104)

\[ I_{ht} = H_t - (1 - \delta_h) H_{t-1} \] (105)

\[ S_t = I_{kt} + (1 - \delta_k) K_t \] (106)

\[ C_t = c_{p,t} + c_{i,t} \] (107)

\[ H_t = h_{p,t} + h_{i,t} \] (108)

\[ K_{t+1} = \psi_{t+1} (I_{kt} + (1 - \delta_k) K_t) \] (109)

\[ \ln \psi_t = \rho_k \ln \psi_{t-1} + u_{kt} \] (110)

\[ \ln A_{ct} = \rho_c \ln A_{ct-1} + u_{ct} \] (111)

\[ \ln A_{ht} = \rho_h \ln A_{ht-1} + u_{ht} \] (112)
Appendix D: Robustness Analysis

This Appendix reports the details of robustness exercises that are mentioned in this paper. The ability of the various frictions and choices of parameter values to match certain business cycle statistics observed in the data has been assessed by reestimating the model shutting off one or more frictions or/and changing one or more parameter values each time. Note that I only consider the model with technology shocks in this Appendix. Table 4 reports the business cycle properties of the alternative models shutting off some certain frictions. Table 5 reports the business cycle properties of the models with alternative choices of the parameter values.

In Table 4, column (a) reports the business cycle statistics of the data. Column (b) reports the business cycle statistics of the baseline model. Column (c) reports the business cycle statistics of the model with perfect labor mobility across production sectors. In this version of the model, I now assume that labor hours are perfect substitutes by setting the labor supply parameters $\epsilon_p$ and $\epsilon_i$ to zero. The volatilities of housing investment, business investment, house prices and consumer loans all increases, relative to that in the baseline model. With perfect labor mobility, hours respond more to sectoral wage differentials, and therefore, so do investment, prices and financial factors. Hence, the model with perfect labor mobility exacerbates the second moments of housing investment, business investment, house prices and consumer loans, respectively. Moreover, it further understates the procyclicality of housing investment. More importantly, it generates a counterfactual result that housing investment and business investment are negatively correlated. Last, the correlation between consumer loans and housing investment reproduced by this version of the model is further mismatched with the data. Column (d) reports the business cycle statistics of the model with same labor supply parameters. When I constrain $\eta$ and $\epsilon$ to be the same across two types of households, the results are essentially unchanged, relative to the baseline model. Therefore, the friction of heterogeneous labor does not matter for the model in terms of the business cycle statistics, which is consistent with the result obtained by Iacoviello and Neri (2010). Column (e) reports the business cycle statistics of the model without financial frictions. The volatilities of consumer loans and deposits reproduced by this version of the model decrease, and the volatility of net worth increases, relative to that in the baseline model. In addition, the model leads to a counterfactual result that housing investment and business investment are negatively correlated. This counterfactual result also can be observed in the financial experiment conditional on a nega-
tive financial shock. Last, this version of the model reduces the procyclicality of financial factors, implying a decline in the impact of exogenous shocks on the real activities through the financial channel. All other business cycle statistics reproduced by the model without financial constraints are similar to those reproduced by the baseline model.

In the rest of this Appendix, I investigate the roles of the downpayment requirements and banks’ survival rates over the business cycle in the model. All results are reported in the Table 5. Column (a) reports the statistics of the baseline model. Column (b) reports the statistics of the model with a lower downpayment requirement. In the baseline model, the loan-to-value ratio is fixed at 0.85. I now raise the ratio to 0.95, reflecting that impatient households make less downpayment than before while borrowing funds from banks with their houses as a collateral. Lower downpayment requirements lead to a decrease in the volatilities of business investment, commercial loans, consumer loans and the banks’ net worth, respectively. The fact that a rise in the loan-to-value ratio does not increase the volatility of housing investment in the model is the outcome of the interaction between housing purchases and labor supply across households. On the one hand, indebted households (impatient households) tend to mitigate aggregate volatility by working relatively less hours in response to positive technology shocks, thanks to the relaxation of their budget constraint induced by the lower downpayment requirements. On the other hand, savers (patient households) tend to increase aggregate volatility by working relatively more hours in response to positive technology shocks. As long as the latter offsets the former, the volatility of housing investment remains unchanged as the loan-to-value ratio rises. As a result, the volatility of house prices is similar to that in the baseline model. In this version of the model, consumer loans are primarily used to finance housing purchases by indebted households. In addition, the housing purchases of indebted households respond less to positive technology shocks as the loan-to-value ratio rises. In this regard, a rise in the loan-to-value ratio reduces the volatility of consumer loans. Recall that banks are indifferent between loans issued to impatient households and non-financial firms. A rise in the loan-to-value ratio also decrease the volatility of commercial loans. Because business investment are mainly financed by commercial loans, the volatility of business investment decreases as the loan-to-value ratio rises. When the volatility of deposits reproduced by the model with the lower downpayment requirements is similar to that in the baseline model, the volatility of net worth falls as a product of the decline in the volatilities of both types of loans. Moreover, the
procyclicality of net worth is reduced, relative to that in the baseline model. Last, the model with the lower downpayment requirements weakly improves its ability to account for the co-movement between business investment and housing investment, relative to the baseline model, but it still understates the co-movement pattern, relative to the data.

Column (c) reports the statistics of the model with higher bank’s survival rates. Recall that in the baseline model the average survival time of the banks are 9 years ($\sigma = 0.972$). Instead, I now assume that the average survival time of the banks are doubled, which is 18 years ($\sigma = 0.986$). A rise in the banks’ survival rates raises the volatilities of housing investment and deposits, and reduces the volatilities of business investment, commercial loans, consumer loans, and net worth. Recall that in the economy a bank only pays dividends to its owners while exiting from the banking system. When the survival time of a bank is longer, the bank has relatively less motive for paying dividends. Absent some motive for paying dividends, the bank may find it more optimal to accumulate its assets to the point where the financial constraint it faces is no longer binding. In another words, a rise in the banks’ survival rates virtually enhances the banks’ ability to save to overcome financial constraints. In this regard, both commercial loans and consumer loans respond less to the shocks, and so does net worth. By learning about a rise in the banks’ ability to save to overcome the financial constraints by depositors, deposits respond more to the shocks, and therefore, the volatility of deposits increases as the survival time of banks rises. Since commercial loans and consumer loans are primarily used to finance business investment and housing purchases of indebted households respectively, a rise in the banks’ survival rates reduces the aggregate volatility of business investment and the aggregate volatility of housing purchases. As I discussed in the text above, savers (patient households), however, increases the aggregate volatility of housing purchases by working relatively more hours in response to the shocks. Due to the offsetting mechanism, a rise in the banks’ survival rate increases aggregate volatility of housing investment. Similar to the case where the loan-to-value ratio rises, the procyclicality of net worth increases, relative to that in the baseline model. But, the model reproduces a counterfactual co-movement between business investment and housing investment.

Column (d) reports the statistics of the model with both lower downpayment requirements and higher banks’ survival rates. The business cycle statistics of interest, except for deposits, change in the same direction in both cases above. Combining the lower downpayment requirements and
higher banks’ survival rates, the volatilities of housing investment and deposits increase, and the volatilities of business investment, commercial loans, consumer loans, and net worth decrease, respectively, relative to that in the baseline model. Moreover, the procyclicality of net worth decreases. Last, the model generates a weakly positive co-movement between business investment and housing investment.
Table 4: Business Cycle Properties of the Alternative Models

<table>
<thead>
<tr>
<th>Variables</th>
<th>Data</th>
<th>Baseline mobility model</th>
<th>Full labor mobility supply parameters</th>
<th>Same labor supply parameters</th>
<th>No financial constraints</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td></td>
<td>(a)</td>
<td>(b)</td>
<td>(c)</td>
<td>(d)</td>
</tr>
<tr>
<td>( GDP )</td>
<td></td>
<td>2.06</td>
<td>2.34</td>
<td>2.22</td>
<td>2.34</td>
</tr>
<tr>
<td>( C )</td>
<td></td>
<td>0.63</td>
<td>0.82</td>
<td>0.84</td>
<td>0.82</td>
</tr>
<tr>
<td>( I_k )</td>
<td></td>
<td>2.65</td>
<td>2.80</td>
<td>2.93</td>
<td>2.81</td>
</tr>
<tr>
<td>( I_h )</td>
<td></td>
<td>4.95</td>
<td>3.35</td>
<td>5.08</td>
<td>3.37</td>
</tr>
<tr>
<td>( q )</td>
<td></td>
<td>1.21 (FMHPI)</td>
<td>1.80</td>
<td>2.32</td>
<td>1.77</td>
</tr>
<tr>
<td></td>
<td></td>
<td>2.08 (S&amp;P HPI)</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>( pS )</td>
<td></td>
<td>2.63</td>
<td>0.99</td>
<td>1.01</td>
<td>0.98</td>
</tr>
<tr>
<td>( B )</td>
<td></td>
<td>2.57</td>
<td>1.80</td>
<td>1.87</td>
<td>1.79</td>
</tr>
<tr>
<td>( D )</td>
<td></td>
<td>0.73</td>
<td>1.07</td>
<td>1.11</td>
<td>1.06</td>
</tr>
<tr>
<td>( N )</td>
<td></td>
<td>3.57</td>
<td>1.05</td>
<td>1.09</td>
<td>1.06</td>
</tr>
</tbody>
</table>

Panel B: Correlations

| \( C,GDP \)   | 0.97 | 0.97 | 0.95 | 0.97 | 0.97 |
| \( I_k,GDP \) | 0.73 | 0.89 | 0.87 | 0.89 | 0.91 |
| \( I_h,GDP \) | 0.86 | 0.11 | 0.03 | 0.13 | 0.08 |
| \( q,GDP \)   | 0.36 | 0.29 | 0.24 | 0.31 | 0.30 |
| \( pS,GDP \)  | 0.06 | 0.82 | 0.81 | 0.82 | 0.75 |
| \( B,GDP \)   | 0.45 | 0.86 | 0.81 | 0.85 | 0.71 |
| \( D,GDP \)   | 0.27 | 0.90 | 0.88 | 0.90 | 0.69 |
| \( N,GDP \)   | 0.33 | 0.84 | 0.82 | 0.84 | 0.85 |
| \( q,C \)     | 0.31 | 0.35 | 0.35 | 0.37 | 0.37 |
| \( q,I_k \)   | 0.46 | 0.29 | 0.27 | 0.33 | 0.32 |
| \( q,I_h \)   | 0.28 | -0.72 | -0.75 | -0.70 | -0.73 |
| \( q,B \)     | 0.18 | 0.37 | 0.31 | 0.39 | 0.41 |
| \( pS,I_k \)  | 0.46 | 0.52 | 0.49 | 0.52 | 0.47 |
| \( B,I_h \)   | 0.43 | -0.16 | -0.31 | -0.16 | -0.37 |
| \( B,C \)     | 0.41 | 0.85 | 0.84 | 0.84 | 0.77 |
| \( I_k,I_h \) | 0.38 | 0.01 | -0.13 | 0.00 | -0.04 |
Table 5: Business Cycle Properties: Changing Downpayment Requirements and Banks’ Survival Rates

<table>
<thead>
<tr>
<th>Variables</th>
<th>Data</th>
<th>(a) Baseline</th>
<th>(b)</th>
<th>(c)</th>
<th>(d)</th>
</tr>
</thead>
<tbody>
<tr>
<td>$m = 0.85$</td>
<td>$m = 0.95$</td>
<td>$m = 0.85$</td>
<td>$m = 0.95$</td>
<td></td>
<td></td>
</tr>
<tr>
<td>$\sigma = 0.972$</td>
<td>$\sigma = 0.972$</td>
<td>$\sigma = 0.986$</td>
<td>$\sigma = 0.986$</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

**Panel A:** %SD (Relative to GDP)

<table>
<thead>
<tr>
<th>Variables</th>
<th>Data</th>
<th>(a) Baseline</th>
<th>(b)</th>
<th>(c)</th>
<th>(d)</th>
</tr>
</thead>
<tbody>
<tr>
<td>GDP</td>
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<td>2.34</td>
<td>2.31</td>
<td>2.29</td>
<td>2.21</td>
</tr>
<tr>
<td>C</td>
<td>0.63</td>
<td>0.82</td>
<td>0.83</td>
<td>0.83</td>
<td>0.83</td>
</tr>
<tr>
<td>$I_k$</td>
<td>2.65</td>
<td>2.80</td>
<td>2.72</td>
<td>2.53</td>
<td>2.47</td>
</tr>
<tr>
<td>$I_h$</td>
<td>4.95</td>
<td>3.35</td>
<td>3.36</td>
<td>3.42</td>
<td>3.45</td>
</tr>
<tr>
<td>$q$</td>
<td>1.21 (FMHPI)</td>
<td>1.80</td>
<td>1.75</td>
<td>1.82</td>
<td>1.82</td>
</tr>
<tr>
<td>$pS$</td>
<td>2.63</td>
<td>0.99</td>
<td>0.93</td>
<td>0.88</td>
<td>0.85</td>
</tr>
<tr>
<td>B</td>
<td>2.57</td>
<td>1.80</td>
<td>1.33</td>
<td>1.74</td>
<td>1.28</td>
</tr>
<tr>
<td>D</td>
<td>0.73</td>
<td>1.07</td>
<td>1.05</td>
<td>1.20</td>
<td>1.20</td>
</tr>
<tr>
<td>N</td>
<td>3.57</td>
<td>1.05</td>
<td>0.88</td>
<td>0.69</td>
<td>0.55</td>
</tr>
</tbody>
</table>

**Panel B:** Correlations

<table>
<thead>
<tr>
<th>Variables</th>
<th>Data</th>
<th>(a) Baseline</th>
<th>(b)</th>
<th>(c)</th>
<th>(d)</th>
</tr>
</thead>
<tbody>
<tr>
<td>$C,GDP$</td>
<td>0.97</td>
<td>0.97</td>
<td>0.97</td>
<td>0.97</td>
<td>0.97</td>
</tr>
<tr>
<td>$I_k,GDP$</td>
<td>0.73</td>
<td>0.89</td>
<td>0.90</td>
<td>0.91</td>
<td>0.90</td>
</tr>
<tr>
<td>$I_h,GDP$</td>
<td>0.86</td>
<td>0.11</td>
<td>0.13</td>
<td>0.07</td>
<td>0.14</td>
</tr>
<tr>
<td>$q,GDP$</td>
<td>0.36</td>
<td>0.29</td>
<td>0.30</td>
<td>0.32</td>
<td>0.28</td>
</tr>
<tr>
<td>$pS,GDP$</td>
<td>0.06</td>
<td>0.82</td>
<td>0.80</td>
<td>0.80</td>
<td>0.80</td>
</tr>
<tr>
<td>$B,GDP$</td>
<td>0.45</td>
<td>0.86</td>
<td>0.84</td>
<td>0.79</td>
<td>0.73</td>
</tr>
<tr>
<td>$D,GDP$</td>
<td>0.27</td>
<td>0.90</td>
<td>0.91</td>
<td>0.91</td>
<td>0.90</td>
</tr>
<tr>
<td>$N,GDP$</td>
<td>0.33</td>
<td>0.84</td>
<td>0.58</td>
<td>0.77</td>
<td>0.59</td>
</tr>
<tr>
<td>$q,C$</td>
<td>0.31</td>
<td>0.35</td>
<td>0.37</td>
<td>0.39</td>
<td>0.36</td>
</tr>
<tr>
<td>$q,I_k$</td>
<td>0.46</td>
<td>0.29</td>
<td>0.31</td>
<td>0.33</td>
<td>0.29</td>
</tr>
<tr>
<td>$q,I_h$</td>
<td>0.28</td>
<td>-0.72</td>
<td>-0.70</td>
<td>-0.72</td>
<td>-0.72</td>
</tr>
<tr>
<td>$q,B$</td>
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<td>0.37</td>
<td>0.38</td>
<td>0.40</td>
<td>0.36</td>
</tr>
<tr>
<td>$pS,I_k$</td>
<td>0.46</td>
<td>0.52</td>
<td>0.50</td>
<td>0.52</td>
<td>0.50</td>
</tr>
<tr>
<td>$B,I_h$</td>
<td>0.43</td>
<td>-0.16</td>
<td>-0.15</td>
<td>-0.28</td>
<td>-0.25</td>
</tr>
<tr>
<td>$B,C$</td>
<td>0.41</td>
<td>0.85</td>
<td>0.82</td>
<td>0.80</td>
<td>0.73</td>
</tr>
<tr>
<td>$I_k,I_h$</td>
<td>0.38</td>
<td>0.01</td>
<td>0.04</td>
<td>-0.03</td>
<td>0.05</td>
</tr>
</tbody>
</table>

Notes: Baseline model and robustness analysis. (a) is the baseline model. (b) is the model with the loan-to-value ratio raising from 0.85 to 0.95. (c) is the model with the banks’ survival rates raising from 0.972 to 0.986, reflecting that the average survival time of the banks increases from 9 to 18 years. (d) is the model with both the loan-to-value ratio and the banks’ survival rates that are increased.