Economics 250 — Midterm 2 **16 November 2017**

Instructions: You may use a hand calculator. Do not hand in the question and formula sheets. Answer all **Four** questions in the answer booklet provided. Show your work: incorrect answers without any work shown cannot be given partial marks; write down formulas even when using a calculator. Formulas and tables are provided at the end of the question pages; you may wish to detach these from the question pages for easier reference.

1. [6 points] In the game of roulette, a metal ball is spun around a rotating wheel containing 18 red-numbered slots, 18 black-numbered slots, and 2 green slots.

i) Find the probability that the ball falls into a green slot two or more times in 20 spins. [2 points]

Answer:

Let X be the number of the ball falling into a green slot in 20 spins. Note that the probability of the ball falling into a green slot in a single spin is $\frac{2}{18+18+2} = \frac{2}{38}$.

$$\begin{aligned} P(X \geq 2) &= 1 - P(X < 2) \\ &= 1 - P(X = 0) - P(X = 1) \\ &= 1 - \frac{20!}{0!20!}(\frac{2}{38})^0(\frac{36}{38})^{20} - \frac{20!}{1!19!}(\frac{2}{38})^1(\frac{36}{38})^{19} \\ &= 0.2840 \end{aligned}$$

ii) Find the probability that the ball falls into a red slot exactly 14 times in 20 spins. [2 points]

Answer:

Let X be the number of the ball falling into a red slot in 20 spins. Note that the probability of the ball falling into a red slot in a single spin is $\frac{18}{18+18+2} = \frac{18}{38}$.

$$P(X = 14) = \frac{20!}{14!6!}(\frac{18}{38})^{14}(\frac{20}{38})^6 = 0.0236$$

iii) Find the probability that the ball does not fall into a red slot on the next seven spins. [2 points]

Answer:

Let X be the number of the ball falling into a red slot in 7 spins. Note that the probability of the ball falling into a red slot in a single spin is $\frac{18}{18+18+2} = \frac{18}{38}$.

$$P(X=0) = \frac{7!}{0!7!}(\frac{18}{38})^0(\frac{20}{38})^7 = 0.0112$$

2. [8 points] A survey asks a random sample of adults in Ohio if they support an increase in the state sales tax from 6% to 7%, with the additional revenue going to education. Let X denote the number in the sample that say they support the increase. Suppose that 40% of all adults in Ohio support the increase.

i) If the survey asks a random sample of 10 adults in Ohio, what are the exact distribution of X, the mean of X, and the standard deviation of X? [2 points]

Answer:

$X \sim B(n,p) = B(10, 0.4)$ with mean of $\mu_X = np = 4$ and standard deviation of $\sigma_X = \sqrt{np(1-p)} = \sqrt{2.4} = 1.5492$

ii) If the survey asks a random sample of 1500 adults in Ohio, what are the exact and approximate distribution of X, the mean of X, and the standard deviation of X? [2 points]

Answer:

Exact distribution: $X \sim B(n,p) = B(1500, 0.4)$ with mean of $\mu_X = np = 600$ and standard deviation of $\sigma_X = \sqrt{np(1-p)} = \sqrt{1500(0.4)(0.6)} = 18.9737$

Approximate distribution: $X \sim N(np, \sqrt{np(1-p)}) = N(600, 18.9737)$

iii) If the survey asks a random sample of 1500 adults in Ohio, what is the probability that less than 610 adults support the increase? [4 points]

$$
\begin{aligned}
P(X \leq 610) &= P(\frac{X - \mu_X}{\sigma_X} \leq \frac{610 - 600}{18.9737}) \\
&= P(Z \leq 0.53) \\
&= 0.7019
\end{aligned}
$$

3. [15 points] You measure the weights of a random sample of 400 male workers in the automotive industry. The sample mean is $\bar{x} = 176.2$ lbs. Suppose that the weights of male workers in the automotive industry follow a Normal distribution with unknown mean μ and standard deviation $\sigma = 11.1$ lbs. A confidence interval for μ is (175.11 , 177.29) at confidence level C.

i) Find the value of confidence level C (in %). [3 points]

Answer:

CI is $(\bar{x} - m, \bar{x} + m) = (175.11, 177.29)$, this implies margin of error is $m = 1.09$. We know that $m = z^* \frac{\sigma}{\sqrt{n}}$ and z^* depends on the confidence level C. $m = z^* \frac{\sigma}{\sqrt{n}}$ implies $z^* = \frac{m\sqrt{n}}{\sigma} = \frac{1.09\sqrt{400}}{11.1} = 1.96$. From Table A or Table D in textbook, the corresponding confidence level for $z^* = 1.96$ is $C = 95\%$.

ii) Given the confidence level C calculated from part i), what is the interpretation of the level C confidence interval (175.11 , 177.29)? [3 points]

Answer:

Interpretation: If the population is repeatedly sampled and intervals are calculated in this fashion (this means: if we took many, many additional random samples with the same sample size and from each computed a 95% confidence interval for μ), then in the long run approximately 95% of the intervals would contain the true value of the unknown population mean, μ.

iii) If we want the margin of error for the level C (takes the value from part i)) confidence interval to be 0.9, what is the sample size needed? [3 points]

Answer:

Margin of error is $m = z^* \frac{\sigma}{\sqrt{n}}$ this implies $n = \left(z^* \frac{\sigma}{m}\right)^2$. So, the required sample size is

$$n = \left(z^* \frac{\sigma}{m}\right)^2 = \left(1.96 \left(\frac{11.1}{0.9}\right)\right)^2 = 585$$

iv) Suppose we can not change the sample size (400) and the population standard deviation, but we still want the margin of error to be 0.9. What can you do? Show your result. [3 points]

Answer:

Margin of error is $m = z^* \frac{\sigma}{\sqrt{n}}$, if $m = 0.9$, $n = 400$, and $\sigma = 11.1$ are all fixed. We can choose different confidence level to make z^* to satisfy $m = z^* \frac{\sigma}{\sqrt{n}}$ this implies the required z^* is $z^* = \frac{m\sqrt{n}}{\sigma} = \frac{0.9(\sqrt{400})}{11.1} = 1.62$. From Table A, the corresponding confidence level is around 90%.

v) Use the sample size calculated from part iii) to construct a 99% confidence interval for the population mean, μ . [3 points]

Answer:

Given information: sample size $n = 585$; sample mean $\bar{x} = 176.2$; population standard deviation $\sigma = 11.1$. From Table A or Table D in textbook, we can find $z^* = 2.576$ for 99% confidence interval. Then, we can compute the margin of error $m = z^* \frac{\sigma}{\sqrt{n}} = 2.576 \left(\frac{11.1}{\sqrt{585}}\right) = 1.18$. So, the 99% confidence interval for the unknown population mean is

$$(\bar{x} - m, \bar{x} + m) = (175.02, 177.38)$$

4. [11 points] The level of calcium in the blood of healthy young adults follows a Normal distribution with mean $\mu = 10$ milligrams per deciliter and standard deviation $\sigma = 0.4$ milligrams. A clinic measures the blood calcium of 100 healthy young adults. The mean of these 100 measurements is $\bar{x} = 9.9$. Is this evidence that the mean calcium level in the population is less than 10?

i) Use hypothesis test to answer the above question. How would you set up the appropriate null hypothesis and alternative hypothesis? [2 points]

Answer:

This is a one-tailed test:

$$H_0 : \mu = 10$$
$$H_a : \mu < 10$$

ii) Perform the hypothesis test from part i) at significance level of $\alpha = 0.01$. What is your conclusion? [4 points]

Answer:

For the one-tailed hypothesis test:

$$H_0 : \mu = 10$$
$$H_a : \mu < 10$$

Calculate the test statistic z:

$$z = \frac{\bar{x} - \mu_0}{\sigma/\sqrt{n}} = \frac{9.9 - 10}{0.4/\sqrt{100}} = -2.5$$

The p-value is

$$\text{p-value} = P(Z < z) = P(Z < -2.5) = 0.0062$$

You will reject the null hypothesis at this significance level $\alpha = 0.01$, since p-value $= 0.0062 < \alpha = 0.01$. The conclusion is that there is evidence that the mean calcium level in the population is less than 10.

iii) For this test, what is the probability that you will commit a type I error? Suppose you designs your study to have a power of 0.90 at a particular alternative value of μ, what is the probability that you will commit a type II error? [2 points]

Answer:

$$P(\text{ type I error }) = \alpha = 0.01$$
$$P(\text{ type II error}) = \beta = 1 - \text{ power } = 0.1$$

iv) For the test you set up at part i), calculate the power of a test (at significance level $\alpha = 0.01$) against a specific alternative $\mu = 9.79$. [3 points]

Answer:

For $\alpha = 0.01$ one-sided test as in part i), to reject H_o we need the test statistic to be less than the critical value $z^* = -2.33$ (this can be found from Table A, this is also a rounded number, in Table D, it gives 2.326). First, we will find the value of sample statistic, $\bar{x}$ can lead to reject the H_o.

$$z = \frac{\bar{x} - \mu_0}{\sigma/\sqrt{n}} = \frac{\bar{x} - 10}{0.4/\sqrt{100}} \leq -2.33$$

this gives $\bar{x} \leq 9.9068$. The power of this test is the probability computed assuming that the alternative $\mu = 9.79$ is true.

$$\begin{aligned}
\text{power} &= P(\text{observing } \bar{x} \text{ leading to reject the } H_o \text{ when specific } H_a \text{ is true}) \\
&= P(\bar{x} \leq 9.9068 \quad \text{when} \quad \mu = 9.79) \\
&= P(Z \leq \frac{9.9068 - 9.79}{0.4/\sqrt{100}}) \\
&= P(Z \leq 2.92) \\
&= 0.9982 \\
&= 99.82\%
\end{aligned}$$