

Instructions: You may use a hand calculator. Do not hand in the question and formula sheets. Answer all **Four** questions in the answer booklet provided. Show your work: incorrect answers without any work shown cannot be given partial marks; write down formulas even when using a calculator. Formulas and tables are provided at the end of the question pages; you may wish to detach these from the question pages for easier reference.

1. [6 points] In the game of roulette, a metal ball is spun around a rotating wheel containing 18 red-numbered slots, 18 black-numbered slots, and 2 green slots.

- i) Find the probability that the ball falls into a green slot two or more times in 20 spins. [2 points]

- ii) Find the probability that the ball falls into a red slot exactly 14 times in 20 spins. [2 points]

- iii) Find the probability that the ball does not fall into a red slot on the next seven spins. [2 points]

2. [8 points] A survey asks a random sample of adults in Ohio if they support an increase in the state sales tax from 6% to 7%, with the additional revenue going to education. Let X denote the number in the sample that say they support the increase. Suppose that 40% of all adults in Ohio support the increase.

- i) If the survey asks a random sample of 10 adults in Ohio, what are the exact distribution of X , the mean of X , and the standard deviation of X ? [2 points]

- ii) If the survey asks a random sample of 1500 adults in Ohio, what are the exact and approximate distribution of X , the mean of X , and the standard deviation of X ? [2 points]

- iii) If the survey asks a random sample of 1500 adults in Ohio, what is the probability that less than 610 adults support the increase? [4 points]

3. [15 points] You measure the weights of a random sample of 400 male workers in the automotive industry. The sample mean is $\bar{x} = 176.2$ lbs. Suppose that the weights of male workers in the automotive industry follow a Normal distribution with unknown mean μ and standard deviation $\sigma = 11.1$ lbs. A confidence interval for μ is (175.11 , 177.29) at confidence level C .

- i) Find the value of confidence level C (in %). [3 points]
- ii) Given the confidence level C calculated from part i), what is the interpretation of the level C confidence interval (175.11 , 177.29)? [3 points]
- iii) If we want the margin of error for the level C (takes the value from part i)) confidence interval to be 0.9, what is the sample size needed? [3 points]
- iv) Suppose we can not change the sample size (400) and the population standard deviation, but we still want the margin of error to be 0.9. What can you do? Show your result. [3 points]
- v) Use the sample size calculated from part iii) to construct a 99% confidence interval for the population mean, μ . [3 points]

4. [11 points] The level of calcium in the blood of healthy young adults follows a Normal distribution with mean $\mu = 10$ milligrams per deciliter and standard deviation $\sigma = 0.4$ milligrams. A clinic measures the blood calcium of 100 healthy young adults. The mean of these 100 measurements is $\bar{x} = 9.9$. Is this evidence that the mean calcium level in the population is less than 10?

- i) Use hypothesis test to answer the above question. How would you set up the appropriate null hypothesis and alternative hypothesis? [2 points]
- ii) Perform the hypothesis test from part i) at significance level of $\alpha = 0.01$. What is your conclusion? [4 points]
- iii) For this test, what is the probability that you will commit a type I error? Suppose you designs your study to have a power of 0.90 at a particular alternative value of μ , what is the probability that you will commit a type II error? [2 points]
- iv) For the test you set up at part i), calculate the power of a test (at significance level $\alpha = 0.01$) against a specific alternative $\mu = 9.79$. [3 points]