

Instructions: You may use a hand calculator.

Do not hand in the question and formula sheets.

Answer all **three** questions in the answer booklet provided.

The grade weight of each question is shown. **Allocate your time accordingly.**

There are 80 points in total and 80 minutes to complete the exam.

Show your work: incorrect answers without any work shown cannot be given partial marks. If using your calculator, it is acceptable to only write down the first few terms of a calculation.

Formulas and tables are provided at the end of the question pages; you may wish to detach these from the question pages for easier reference.

Best of Luck!

1. [25] Over a long period of time, it has been observed that a basketball player on Queen's varsity team can score on a free throw with probability equal to 0.8, it is also observed that her free throws are independent. Suppose she shoots 20 free throws.
  - a) [2] Find the expected number and standard deviation of throws on which she scores.
  - b) [3] Find the probability that she will score on *exactly* 17 free throws?
  - c) [5] Find the probability that she will score on *at least* 17 throws?

Suppose over an entire season she makes 300 free throws.

- d) [6] Find the probability that she will score on *at least* 230 free throws over the entire season?
- e) [6] Find the probability that she will score on *more than* 230 but *less than* 245 free throws over the entire season?

Suppose you have two coins with the following probability distribution over each toss.

	Heads	Tails
Coin 1	0.5	0.5
Coin 2	0.6	0.4

Consider the following *two* coin toss experiment. You toss Coin 1 on the first toss. If the outcome is heads you toss Coin 2 on the second toss. However, if the outcome on the first toss is tails, you toss Coin 1 again on the second toss. Let  $\mathbf{X}$  be the number of heads observed.

- f) [3] Is a binomial distribution an appropriate model for  $\mathbf{X}$ ? **Briefly** state the reasoning to your conclusion.
2. [15] A Maryland highway study found that in 77% of all vehicle accidents, the driver was wearing a seatbelt. Accident reports indicated that 92% of those drivers wearing a seatbelt escaped serious injury (defined as hospitalization or death), but only 63% of the non-belted drivers escaped serious injury.
    - a) [3] Based on the probabilities reported in the Maryland highway study, does wearing a seatbelt reduce the chances of serious injury?
    - b) [6] Find the probability of a serious injury.
    - c) [6] Find the probability that a driver who was seriously injured was not wearing a seat belt.

3. [40] Sports that involve a significant amount of running, jumping, or hopping put participants at risk for Achillestendinopathy (AT), an inflammation and thickening of the Achilles tendon. Suppose that the Achilles tendon diameters in the general population of individuals diagnosed with AT have a standard deviation of 1.95 mm but the mean is unknown. A study in The American Journal of Sports Medicine measured the diameters of the affected Achilles tendons for a sample of 31 patients diagnosed with AT. The average sample diameter of the affected tendons was 9.80 mm. The researchers of the study want to test if the population mean of Achilles tendon of individuals diagnosed with AT is *at most* 9.2 mm.
- a) [2] Write down a null and alternative hypothesis which will allow the researcher to test their hypothesis.
  - b) [2] Calculate a test statistic that will allow the researcher to test their hypothesis from part a).
  - c) [4] Find the p-value associated with your test statistic.
  - d) [4] Can you reject the null at the 95% confidence level? At the 99% confidence level?
  - e) [5] Starting with a theorem discussed in class derive the distribution of the test statistic in part b), under the null hypothesis.
  - f) [8] Assuming a significance level of 5% find the power and probability of Type II error for the fixed alternative that the mean tendon diameter for the general population of individuals diagnosed with AT is *exactly* 10 mm.

Now suppose one of the researchers of the study is *not* convinced that the population standard deviation of Achilles tendon diameters in the general population of individuals diagnosed with AT is known. She observes that the *sample* standard deviation of Achilles tendon for the sample of 31 individuals diagnosed with AT is 1.89 mm. She wants to test the hypothesis that the Achilles tendon of individuals diagnosed with AT is *exactly* 9.20 mm.

- g) [2] Write down a null and alternative hypothesis which will allow her to test her hypothesis .
- h) [3] Calculate a test statistic which will allow the researcher to test the hypothesis in part g). State the distribution of this test statistic under the null hypothesis.
- i) [6] Can you reject the null at the 10% significance level? At the 5% significance level?
- j) [4] Find a 95% two sided confidence interval for your test in part g).