Term Test 2 (Solutions)

(1) a) Let X be the number of scoring throws in 20 free throws.

$X \sim Bin(20, 0.8)$

$p = 0.8$ is the probability of success which in this question is the event that the player scores on a free throw

(a) $E[X] = np = 20(0.8) = 16$

$\sigma_X^2 = 20(0.8)(1-0.8) = np(1-p)$
$= 3.2$

$\sigma_X = \sqrt{3.2} = 1.79$

(b) $P(X = 17) = \binom{20}{17}(0.8)^{17}(0.2)^3$
$= 0.205 \approx 0.21$

(c) $P(X \geq 17) = P(X=17) + P(X=18) + P(X=19) + P(X=20)$

$= 0.21 + [P(X=18) + P(X=19) + P(X=20)]$

$= 0.21 + \left[\binom{20}{18}(0.8)^{18}(0.2)^{2} + \binom{20}{19}(0.8)^{19}(0.2) + \binom{20}{20}(0.8)^{20}(0.2)^{0} \right]$

$= 0.21 + [0.1369 + 0.0576 + 0.01153]$

$= 0.41603 \approx 0.42$

(d) Since $np = 300 \times 0.8 = 240 > 10$

$n(1-p) = 300 \times 0.2 = 60 > 10$

$X \sim Bin(300, 0.8)$ can be approximated by the normal distribution.

$\mu = np = 300 \times 0.8 = 240$

$\sigma = \sqrt{np(1-p)} = 6.928$

$$X \overset{approx}{\sim} N(240, 6.928)$$

$$P(X \geq 230)$$

$$= P\left(\frac{X-240}{6.928} \geq \frac{230-240}{6.928}\right)$$

$$= P(z \geq -1.4434 \approx -1.44)$$

$$= P(z \geq -1.44)$$

$$= 1 - P(z < -1.44)$$

$$= 1 - 0.0749$$

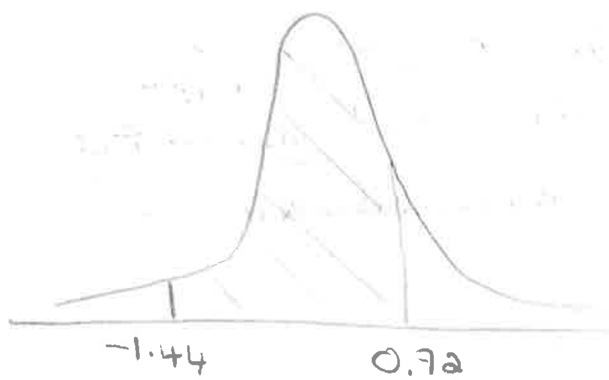
$$= 0.9251 \approx 0.93$$

(e) $P(230 < X < 245)$

$$= P\left(\frac{230-240}{6.928} < \frac{X-240}{6.928} < \frac{245-240}{6.928}\right)$$

$$= P(-1.44 < z < 0.72)$$

$z \sim N(0,1)$



$$P(-1.44 < z < 0.72)$$

$$= P(z < 0.72) - P(z < -1.44)$$

$$= 0.7642 - 0.0749$$

$$= 0.6893 \approx 0.69$$

(C) Binomial model is not appropiate for X. This is because the two trials are not independent. In parbiculer the probability distribution of the second toss depends on the outcome of the first toss.

Question 2)

(a) Denote the following event

B = Driver was wearing seatbelt.

I = Driver was seriously injured.

$$P(I \mid B) = 0.08 < P(I \mid B^c) = 0.37$$

or equivalently $P(I^c \mid B) = 0.92 > P(I^c \mid B^c) = 0.63$

Therefore wearing a seatbelt does reduce chances of serious injury.

(b)

$P(B) = 0.77$

$P(B^c) = 0.23$

$P(I^c \mid B) = 0.92$

$P(I \mid B) = 0.08$

$P(I^c \mid B^c) = 0.63$

$P(I \mid B^c) = 0.37$

By Law of total probability

$$\begin{aligned} P(I) &= P(I \mid B^c)P(B^c) + P(I \mid B)P(B) \\ &= (0.37)(0.23) + (0.08)(0.77) \\ &= 0.1467 \end{aligned}$$

(c) By Bayes' Rule

$$P(B^c \mid I) = \frac{P(I \mid B^c)\, P(B^c)}{P(I)}$$

$$= \frac{(0.37)(0.23)}{0.1467}$$

$$= 0.58$$

Question 3

8

(a) Denote μ as the population mean Achilles tendon diameter for individual with AT.

$H_0: \mu = 9.2$
$H_a: \mu > 9.2$

or equivalently

$H_0: \mu \le 9.2$
$H_a: \mu > 9.2$

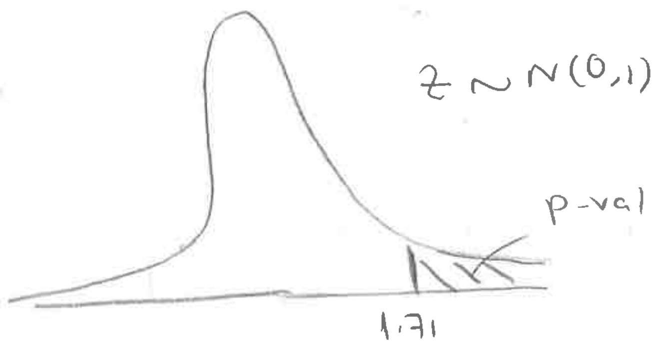
(b)
$$z = \frac{\bar{x} - \mu_0}{\sigma/\sqrt{n}}$$

$$= \frac{9.8 - 9.2}{1.95/\sqrt{31}}$$

$$= \frac{0.6}{0.3502}$$

$$= 1.713 \approx 1.71$$

(c)



$$p\text{-val} = P(z > 1.71) = P(z < -1.71)$$
$$= 0.0436$$

(d) $0.0436 < 0.05 = \alpha$

$$\Rightarrow \quad p\text{-val} < \alpha = 0.05$$

Therefore we can reject the null at 5% significance level or equivulenbly at 95% confidence level.

$0.0446 > 0.01 = \alpha$

$$\Rightarrow \quad p\text{-val} > \alpha = 0.01$$

Therefore we cannot reject the null at 1% significance level or equivulenbly at 99% confidence level.

(e) by the Central limit Theorem (CLT)

$$\bar{X} \sim N\left(\mu, \frac{\sigma}{\sqrt{n}}\right)$$

under H_0: $\mu = \mu_0$

Therefore under H_0:

which implies $\bar{X} \sim N\left(\mu_0, \frac{\sigma}{\sqrt{n}}\right)$

$$z = \frac{\bar{X} - \mu_0}{\sigma/\sqrt{n}} \sim N(0,1)$$

Therefore using CLT and assuming H_0, the z test statistic is $N(0,1)$

(f) Rejection rule

$$z > 1.64$$

$$\frac{\bar{X} - \mu_0}{\sigma/\sqrt{n}} > 1.64$$

$$\bar{X} > 0.574 + 9.20 = 9.774$$

Under the fixed alternative hypothesis

$$\mu_A = 10$$

$$\bar{X} \sim N(10, 0.3502)$$

$$P(\bar{X} > 9.774) = \text{power}$$

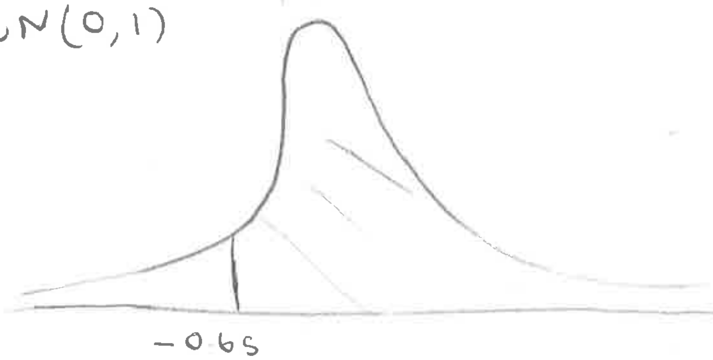
where $X \sim N(10, 0.3502)$

$$P\left(\frac{\bar{X} - 10}{0.3502} > \frac{9.774 - 10}{0.3502}\right)$$

$$= P(z > -0.645 \approx 0.65)$$

$$\text{Power} = P(z > -0.65)$$

$z \sim N(0,1)$



$$P(z > -0.65) = 1 - P(z < -0.65)$$
$$= 1 - 0.2578$$

$$\text{Power} = 0.742$$

Probability of Type II error

$$= 1 - \text{power}$$
$$= 1 - 0.742$$
$$= 0.2578$$

(g) $H_0: \mu = 9.2$

$H_a: \mu \neq 9.2$

(h) $t = \dfrac{\bar{x} - \mu_0}{s/\sqrt{n}} = \dfrac{9.8 - 9.2}{1.89/\sqrt{31}}$

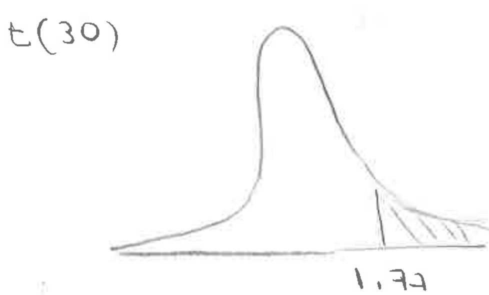
$= \dfrac{9.8 - 9.2}{0.339}$

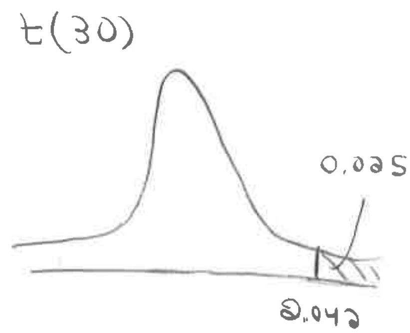
$= 1.769 \approx 1.77$

Under H_0

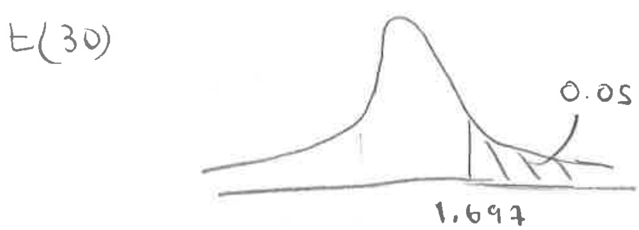
$t \sim t(30)$

(i)





$P(t > 2.042) = 0.025$



$P(t > 1.697) = 0.05$

$0.025 < P(t(30) > 1.77) < 0.05$

$0.025 < \frac{p\text{-val}}{2} < 0.05$

$0.05 < p\text{-val} < 0.1$

Since $p\text{-val} < 0.1 = \alpha$

we can reject the null hypothesis

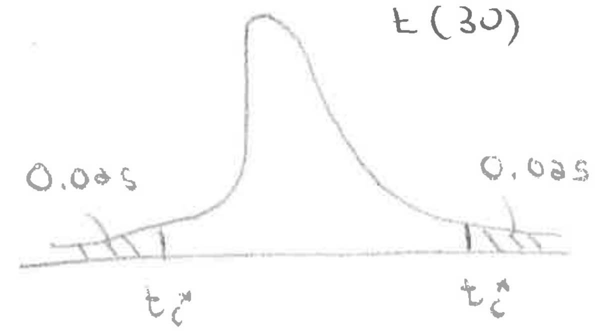
10% significance level.

Since $p\text{-val} > 0.5 = \alpha$

we cannot reject the null hypothesis

at 5% significance level.

(j)



$$P(t(30) > 2.042) = 0.025$$

Therefore $t_c^* = 2.042$

$$m = 2.042\left(\frac{s}{\sqrt{n}}\right)$$

$$= 2.042 \times 0.339$$

$$= 0.692$$

95% C.I

$$[9.80 - 0.692 \quad 9.80 + 0.692]$$

$$= [9.108 \quad 10.492]$$