

Skewed Business Cycles*

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Abstract

Using firm-level panel data from the US Census Bureau and more than forty other countries, we show that the skewness of the growth rate of employment and sales is procyclical. In particular, during recessions, they display a large left tail of negative growth rates (and during booms, a large right tail of positive growth rates). These results are robust to different selection criteria, across countries, industries, and measures. We find similar results at the industry level: industries with falling growth rates see more left-skewed growth rates of firm sales and productivity. We then build a heterogeneous-agent model in which entrepreneurs face shocks with time-varying skewness that matches the firm-level distributions we document for the United States. Our quantitative results show that a negative shock to the skewness of firms' productivity growth (keeping the mean and variance constant) generates a significant and persistent drop in output, investment, hiring, and consumption.

JEL Codes: D22, D25, E23, E32.

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1 Introduction

This paper studies the cyclicity of the distribution of the growth rate of firm-level outcomes. In the previous literature, recessions have been characterized as a combination of a negative first-moment (mean) shock and a positive second-moment (uncertainty) shock (Bloom, 2014). In this paper we argue that recessions are also accompanied by negative third-moment (skewness) shocks implying that, during economic downturns, a subset of firms does extremely badly, leading to a left tail of large negative outcomes. This is often accompanied by a deceleration of growth for a subset of firms at the top end, leading to a compression of the right tail of positive outcomes. The opposite patterns happen during expansions, with the left tail shrinking and the right tail expanding. Consequently, the skewness of the growth rates is procyclical.

Using panel data on US publicly traded firms from Compustat, panel data on US firms including privately held ones from the Census Bureau, and panel data on firms from thirty-nine other countries, we show that the cross-sectional skewness of the distribution of several firm-level outcomes, such as sales growth, employment growth, productivity growth and stock returns, is strongly procyclical, declining sharply during recessions. As an illustration of our main empirical result, the top panel of Figure 1 displays the distribution of firms' employment growth from the US Census Bureau's Longitudinal Business Database (LBD). The solid line shows the empirical density of annual employment growth pooling observations from the most recent two recession years, 2001–02 and 2008–09. The dashed line instead shows the density for the subsequent expansion years, in this case, years 2003 to 2006 and 2010 to 2014. One can clearly see that, relative to expansion periods, the distribution of employment growth during recessions has a thicker left tail, whereas the right tail exhibits little change, indicating an increase in dispersion that is mostly due to a widening left tail.

This asymmetric change in the distribution of employment growth from expansion to recession years can be quantified using the Kelley skewness (Kelley, 1947), an outlier robust measure of skewness. This is defined as the difference between the log 90th-to-50th percentiles spread (a measure of dispersion in the right tail) and the log 50th-to-10th percentiles spread (a measure of dispersion in the left tail) divided by the log 90th-to-10th percentiles spread (a measure of the total dispersion of the distribution). For a distribution with a compressed upper half and a dispersed lower half (i.e., a left skew), the Kelley skewness is negative. In the case of the top panel of Figure 1, we find a

decline in the dispersion of employment growth above the median from 0.22 to 0.20 from expansion to recession years, whereas the dispersion below the median increases from 0.17 to 0.25. This asymmetric change in the tails generates a decline in the Kelley skewness from 0.10 to -0.12 during recessions.¹ The bottom panel of Figure 1 shows a similar pattern for the distribution of sales growth for Compustat firms for recessions years (2001-02 and 2008-09) and the corresponding density for expansion years (2003 to 2006, and 2010 to 2014). As in the case of employment growth, here we also find that recessions are characterized by a widening left tail, which gives rise to both an increase in dispersion and a decline in the skewness of the sales growth distribution.

We find the same empirical pattern at the two-digit NAICS industry level: the within-industry skewness of firm-level employment growth, sales growth, productivity growth and stock returns is positively correlated with the industry economic cycle. Moreover, the same pattern is also seen globally—using a panel of firms spanning thirty-nine countries that are both geographically and economically diverse, we show that the skewness of the same firm-level variables within each country is robustly procyclical with respect to that country’s business cycle.

Motivated by this empirical evidence, in the second part of the paper we build a heterogeneous-agents model where the key feature is the presence of a large number of entrepreneurs that face shocks with time-varying risk featuring both time-varying variance and time-varying skewness. To capture the potentially non-linear response of firms to shocks, we assume that entrepreneurs are risk-averse, face a combination of convex and non-convex adjustment costs to capital, and can invest in capital and a risk-free asset. We numerically solve the model and choose the parameters of the firm’s productivity process so that our modeled economy matches the average skewness of the sales growth distribution we observe among US firms during expansionary periods and the large decline in skewness observed during a typical recession. Our results suggest that first-moment shocks combined with risk-aversion and capital adjustment costs, both of which generate asymmetries in the response of firms to shocks, are not sufficient to generate the large swings in the skewness of firm outcomes we document. Hence, to match the changes in skewness we observe in the data, we consider time-varying skewness in the firms’ productivity process.

¹Put differently, a Kelley skewness of 0.10 indicates that during expansion, 45% of all the dispersion is accounted for by firms with employment growth below the median, whereas during recessions, this share increases to 56%.

In our main quantitative exercise, we study the aggregate effects of a pure skewness shock—that is, a decline in the skewness of firms’ productivity shocks while keeping the mean and variance constant. Our model predicts that a change in the skewness of the distribution of firm-level shocks that matches the decline in the skewness of sales growth we observe among US firms would reduce GDP by 1.7%. The decline in aggregate economic activity is quite persistent as GDP stays below its pre-shock level several quarters after the shock. This is in contrast to the standard uncertainty shock analyzed in the literature that typically generates a sharp drop and rapid rebound of GDP. This significant and persistent drop in output is driven by a decline in capital investment, which is the result of three forces. First, the presence of a fixed cost to capital adjustment creates a real options effect that reduces the incentives of firms to invest when skewness declines. This is a reflection of the [Bernanke \(1983\)](#) “*Bad News Principle*”, that only outcomes about the bad state of the world matter for option value to delay investment. Second, the drop in skewness makes capital riskier, inducing an increase in investment in the risk-free asset. Finally, relative to the standard uncertainty shock (a symmetric increase in dispersion), in our model a decline in skewness results in a widening left tail of the firm productivity distribution without a corresponding widening of the right tail (an asymmetric increase in dispersion). This ameliorates the Oi-Hartman-Abel effect from uncertainty shocks², suppressing the overshoot from pure uncertainty shocks. Hence, our results indicate that a negative shock to the skewness of firms’ productivity (that keeps the mean and variance constant) can generate a recession by itself.

This paper is related to several strands of literature. First and foremost, our paper relates directly to the study of the effects of uncertainty on firms’ decisions. Several papers have shown that an increase in uncertainty can have important macroeconomic implications in the presence of adjustment costs or financial frictions.³

Second, several authors have suggested that rare disasters—presumably arising from an asymmetric distribution of shocks—can generate large fluctuations in economic activity, such as the Great Recession. Reviving the ideas introduced first by [Rietz \(1988\)](#), [Barro \(2006\)](#) uses a panel of countries to estimate the probability of large disasters and argue that these low-probability events can have substantial implications for aggregate

²See the Oi-Hartman-Abel discussion in the survey article [Bloom \(2014\)](#).

³See, for example, [Arellano et al. \(2010\)](#), [Fernandez-Villaverde et al. \(2011\)](#), [Bachmann and Bayer \(2013\)](#), [Bachmann and Bayer \(2014\)](#), [Gilchrist et al. \(2014\)](#), [Jurado et al. \(2015\)](#), [Leduc and Liu \(2016\)](#), [Basu and Bundick \(2017\)](#), [Berger et al. \(2017\)](#), [Alfaro et al. \(2018\)](#) and [Bloom et al. \(2018\)](#).

economic activity and asset pricing.⁴ The results of our paper can be seen as evidence that rare disasters also occur at the microeconomic level.

Finally, our paper contributes to a growing literature that focuses directly on the skewness of firm and worker outcomes such as firm productivity (Kehrig (2011)), employment growth (e.g. Ilut *et al.* (2018) and Decker *et al.* (2015)), stock returns (e.g. Harvey and Siddique (2000), Kapadia (2006), Oh and Wachter (2018), and Ferreira (2018)), and labor earnings (e.g. Guvenen *et al.* (2014)).

The rest of the paper is organized as follows. Section 2 describes the data we use and the statistics discussed in the empirical section. Section 3 shows the main empirical results of our paper, that is, that the skewness of several firm-level outcomes is procyclical. Section 4 describes the model and Section 5 presents our quantitative results. Section 6 concludes.

2 Data and Measurement

2.1 Data and Sample Selection

Our analysis is based on four large datasets.⁵ First, we extract panel data on employment at the firm level from the Census Bureau’s Longitudinal Business Database (LBD). The LBD provides high-quality measures of employment, wage bill, industry, and firm age for the entire US nonfarm private sector linked over time at the establishment level from 1976 to 2015. From the LBD we construct employment at the firm and establishment levels and use it to calculate cross-sectional moments of the distribution of employment growth at narrow firm population groups. The LBD has over 6 million firms, which for measuring higher-order moments like skewness is a major advantage.

Second, we draw panel data information of publicly traded firms from Compustat, which contains information on sales, employment, stock prices, and other firm-level outcomes. We use data on quarterly sales, daily stock prices, annual sales, and annual employment from 1970 to 2017, and we restrict attention to a sample of firms with more than ten years of data to minimize the types of compositional issues identified in Davis *et al.* (2006).

⁴See for instance Gabaix (2008, 2012), Gourio (2008, 2012, 2013), Wachter (2013), and Kilic and Wachter (2015), among others.

⁵Replication files for the empirical section can be downloaded from <https://bit.ly/2ZfaMVR>.

Third, we study whether the patterns we document for the United States are also observed in other countries, both developed and developing. To that end, we use cross-country publicly traded firm-level panel data containing sales and employment information between 1986 and 2016 from Osiris dataset collected by the Bureau van Dijk (BvD). To ensure that changes in the sample of firms do not bias our results, we focus on firms that are present in the sample for ten years or more. Additionally, we restrict our sample to country-year bins with more than one hundred firms, countries with at least ten years of data, and years with five countries or more. Our main results are based on an unbalanced panel of firms spanning thirty nine countries from 1991 to 2015. We complement this dataset with information on firm-level stock prices obtained from the Global Compustat dataset. Applying similar selection criteria, we obtain a sample of daily stock price information for firms in twenty-nine countries from 1985 to 2017.

Finally, we obtain additional firm-level data from the Amadeus dataset also collected by BvD. This dataset comprises a smaller sample of countries, for a shorter timespan, but with rich firm-level information for small and large firms, both publicly traded and privately held. In particular, Amadeus provides information on sales, employment, value added, capital, and labor input cost so that we can estimate TFP at the firm level. Our sample contains information for seven European countries starting in the mid 2000s for most of the countries.

Table I summarizes the data sources and the main sample characteristics.⁶ Additional details on data construction, selection criteria, and moment calculation for each dataset used in our analysis can be found in Appendix A.⁷

2.2 Measuring Skewness

For most of our results, we measure the growth rate of a firm-level outcome as the log-difference between period t and $t+k$ where t is a quarter for stock returns and a year in the case of employment and sales. For both dispersion and skewness, we use quantile-based measures that are robust to outliers, which are common in micro datasets. As we shall see, they also have magnitudes that are easy to interpret. Our measure of dispersion is the differential between the 90th and 10th percentiles, denoted by $P9010_t$, where t is a

⁶Appendix Table A.1 shows a list of the countries we consider in our analysis and the data available for each of them.

⁷The online appendix—available at the authors’ websites—provides further details about the underlying data and provides the material to replicate the results presented in the empirical section of the paper.

quarter or a year depending on the dataset. Additionally, we use the differentials between the 90th and 50th percentiles, $P90_{50,t}$, and between the 50th and 10th percentiles, as measures of dispersion in the right and left tails, respectively. Finally, our preferred measure of skewness is the Kelley skewness (Kelley, 1947), which is defined as

$$KSK_t = \underbrace{\frac{P90_t - P50_t}{P90_t - P10_t}}_{\text{Right Tail Share}} - \underbrace{\frac{P50_t - P10_t}{P90_t - P10_t}}_{\text{Left Tail Share}} \in [-1, 1]. \quad (1)$$

As seen here, the Kelley measure provides a simple decomposition of the share of total dispersion that is accounted for by the left and the right tails of a distribution.⁸ A negative value of Kelley skewness indicates that the left tail accounts for more than one-half of the total dispersion and the distribution is negatively skewed, and vice versa for a positive value.

3 Skewness over the Business Cycle

In this section, we show that the distribution of firm-level growth rates has a longer left tail in recessions in both the United States (Section 3.1) and across countries (Section 3.2), and then confirm that our results hold within industries (Section 3.3) and for firms' productivity shocks (Section 3.4).

3.1 US Evidence

The first contribution of our paper is to show that the skewness of the growth rates of firm-level outcomes varies over time and is strongly procyclical, declining substantially during recessions and rising in booms. We start by considering the evolution of the Kelley skewness of the distribution of the growth rate of firm's employment from the US Census Bureau's LBD, which is displayed in the top panel of Figure 2. To calculate Kelley skewness, we weight observations by firms' employment so that our measure reflects the underlying firm-size distribution.⁹ Figure 2 shows, first, that the skewness of employment

⁸An important drawback of this measure of skewness is that it is invariant to 20% of the observations in the sample (the top and bottom 10% of the distribution). In principle, Kelley skewness can be computed using any two symmetric percentiles, such as the 95th and 5th or 98th and 2nd. We have explored some of these alternative choices and did not find them to matter for our results (see Section 3.4.1). Additional measures of skewness can be found in Kim and White (2004).

⁹In particular, we weight the employment growth of firm i in period t by the average employment in periods t and $t + 1$, that is, $\bar{E}_{i,t} = 0.5 \times (E_{i,t} + E_{i,t+1})$. The results for publicly traded firms are unweighted since most of the firms are large.

growth, on average, is positive and around 10% for most of the sample period. Second, the skewness of employment growth is strongly procyclical, declining from an average of 11% at the peak of the typical recession to around -10% at the trough, that is, a drop of 21 percentage points. Similarly, the bottom of Figure 2 shows the cross-sectional skewness of annual sales growth for Compustat firms. Relative to the LBD, this is a more selective set of mostly large firms. Nevertheless, we find that the skewness of the distribution of sales growth is positive on average and declines around 20 percentage points during a recession.

The decline in the skewness of firm growth occurring during recessions is driven by a rapid change in the relative weight of the tails of the distribution. This can be observed in the top panel of Figure 3, where we plot $P5010_t$ (black line with circles) and $P9050_t$ (blue line with squares) of employment growth. The bottom panel of Figure 3 shows the same set of statistics for sales growth. Two important aspects are worth noticing. First, during expansionary periods, the right tail outweighs the left tail (most of the time, $P9050_t$ is above $P5010_t$), generating a distribution of firms' outcomes that is positively skewed. Second, for both employment and sales growth, recessions are episodes in which the $P5010_t$ expands, indicating a left tail that stretches out, whereas the right tail contracts. This asymmetric change in the tails drives the drop in the skewness of firms' employment and sales growth.¹⁰

To have a better sense of the magnitude of the change in skewness and its relation with the cycle, the left panel of Table II shows a set of time series regressions of the form

$$KSK_t = \alpha + \beta \Delta GDP_t + \delta t + \epsilon_t, \quad (2)$$

where the dependent variable is the Kelley skewness of the cross-sectional distribution of different firm-level outcomes. In all regressions, the independent variable is the growth rate of real GDP per capita, which we have normalized to have unitary variance so the coefficients are comparable across columns, and t is a linear trend. The estimated coefficients are positive and large for all three variables—employment and sales growth, and stock returns—and also strongly statistically significant (at the 1% level for the first two and the 5% level for the third). For example, the estimated coefficient of 4.6 in column (1) implies that a three standard deviation—or about a 6%—drop in GDP per

¹⁰This asymmetric change is also observed at higher frequencies. For instance, Figure A.2 shows a similar set of results for the annual change of quarterly sales.

capita growth is associated with a 0.14 fall in the Kelley skewness of firm employment growth distribution. Column (2) shows a similar result for sales growth with a larger coefficient (5.37). Column (3) shows a smaller coefficient for stock returns (2.1) that is still highly significant.¹¹ The change in skewness of stock-returns over the cycle also suggests the increase in skewness is driven, at least in part, by a rise in skewed external shocks (e.g. productivity or demand shocks) rather than skewed firm control variable (like investment and employment).¹²

3.2 Cross-Country Evidence

Is the procyclical skewness we have documented so far a pattern specific to the United States, or is it also observed in other countries? The second contribution of our paper is to shed light on this question using firm-level panel data covering almost forty countries that are both geographically and economically diverse, spreading over five continents including developing countries (such as the United States, Germany, Japan, and others) and developed countries (such as Peru, Egypt, Thailand, and others).

The top panel of Figure 4 displays the empirical density of the distribution of the growth rate of annual real sales (in US dollars as of 2005) for a panel of firms spanning thirty-nine countries from 1991 to 2015. The solid red line is the density of the growth rate of sales during recession periods, where a recession is defined as a year in which the growth rate of GDP is in the first decile of the country-specific GDP growth distribution. The dashed black line is the density of sales growth during expansion periods defined as years in which GDP growth is above the first decile of the country-specific distribution of GDP growth. Similar to the results presented in Figure 1, the dispersion in sales growth increases little during recession years, with $P9010_t$ rising slightly from 0.82 to 0.85. However, this modest increase masks larger changes in each tail: the left tail stretches out, with $P5010_t$ rising from 0.36 to 0.43, and the right tail shrinks, although by a smaller amount, with $P9050_t$ falling from 0.46 to 0.43. The opposite moves of each tail dispersion partially cancel out each other, leading to the smaller rise in $P9010_t$

¹¹Table A.6 in Appendix C shows that the skewness of firm-level outcomes remains strongly procyclical if we residualize the firms' outcomes by the firms' observable characteristics and fixed unobserved heterogeneity, if we consider the growth rate of sales per worker (more closely related to firm productivity), or if we look at the three-year growth rate of firms' outcomes (Appendix Table A.4). We also confirm that the dispersion in firms' outcomes is countercyclical (Appendix Table A.5). We do not find significant business cycle variation in the kurtosis of firms' outcomes (right panel of Table A.6).

¹²The value of optimizing firms with continuous control variables should only jump in response to external shocks (an implication of the envelope theorem).

just mentioned. In contrast, for skewness, the contraction of the upper tail and the expansion of the lower tail inequality reinforce each other to generate a larger decline in Kelley skewness, which falls from 0.12 to 0.0.

To have a clearer picture of how skewness changes over the business cycle, the bottom left panel of Figure 4 shows a binned scatter plot in which the x -axis is the average of firm employment growth within a country-year bin, and the y -axis is the Kelley skewness of the same firm-level outcome. The data points align nicely along a straight line over a wide range of average employment growth rates (ranging from -15% to 20%), confirming the strong positive relationship between skewness and the business cycle. Further, the slope of the relationship is equal to 1.64 and is both statistically and economically significant. For example, when average firm employment growth is -15% (typically during a big recession) the Kelley skewness is -30% , implying that two-thirds of the mass of the distribution of employment growth is accounted for by the left tail. In contrast, when the average employment growth is 10% , skewness is 30% , indicating the opposite split, with two-thirds of total dispersion now being accounted for by the right tail. The bottom right panel of Figure 4 shows a similar result for sales growth. Importantly, to construct these figures we have controlled for country- and time-fixed effects, so these results are not driven by fixed characteristics of the countries considered in the sample or by global shocks—such as the Great Recession—that can affect all countries at the same time.¹³

The center panel of Table II repeats the cyclicity regression discussed above for the United States but this time exploiting the panel dimension of the cross-country dataset to assess the cyclicity of skewness in international data. This time, the dependent variable is the skewness of employment growth, sales growth, or stock returns within a given country each year. The business cycle is captured by the growth rate of GDP per capita in the respective country. The regressions also include a full set of time- and country-fixed effects to control for aggregate economic conditions that might affect all countries simultaneously or fixed differences across countries. The regression results confirm our previous findings of procyclical skewness for all three variables with similar levels of statistical significance. Compared with the United States, the estimated coefficient is slightly higher for employment (5.39 across countries versus 4.64 for the United States),

¹³One important concern is that our cross-country results are based exclusively on publicly traded firms. Interestingly, we also find remarkably similar results if we consider an unbalanced panel of firms (private and publicly traded) drawn from the BvD Amadeus dataset, as Figure A.3 in the Appendix shows. The BvD Amadeus dataset covers a shorter period of time (2000 to 2015 for most countries) over a smaller sample of European countries.

somewhat lower for sales (3.19 versus 5.37), and nearly identical for stock returns. These results further confirm the procyclical nature of skewness in firm-level outcomes.

3.3 Industry-Level Evidence

We finally turn to industry-level data from the United States and investigate the extent to which the skewness results are found in different industries. To this end, using LBD data, the top panel of Figure 5 shows a binned scatter plot of the skewness of employment growth within each industry against the average employment growth for the same industry in that year.¹⁴ In this case, a positive correlation indicates that periods of low economic activity at the *industry* level are associated with a negative shift in skewness within that industry, and vice versa for periods of high economic activity. In terms of magnitudes, the top panel of Figure 5 shows that when the industry employment growth is -8% , the Kelley skewness is around 20% , indicating that 60% of the total dispersion in employment growth within an industry is accounted for by the left tail of the distribution. When the average employment growth is 8% instead, the Kelley is skewness is 20% , indicating that the right tail accounts for 60% of the total dispersion. Similarly, the bottom panel of Figure 5 shows that the within-industry skewness of sales growth is higher when the average sales growth for that industry is higher.¹⁵ Hence, sectors that grow faster are also sectors in which the skewness of firm-level outcomes is higher.¹⁶

We then use firm-level data from Compustat to examine the relation of the industry cycle and the skewness of sales growth, employment growth, and quarterly returns within NAIC two-digit industry-period bins. Columns (7) to (9) of Table II display a series of industry panel regressions in which the dependent variable is the Kelley skewness of the growth rate of different firm-level outcomes across all firms within an industry-period bin. In this case, we capture the within-industry business conditions by the average

¹⁴Throughout the paper, an industry is defined at the NAICS two-digit industry level for a total of twenty-two industries.

¹⁵The change in the skewness of the distribution of firms; growth is driven by an asymmetric response of the right and left tails to the industry business conditions. This is clearly seen in Figure A.4 in Appendix C that shows that the share of dispersion accounted for the right tail of the distribution of sales growth is positively correlated with the within industry cycle, expanding during periods in high economic activity. In contrast, the share of total dispersion accounted for by the left tail of the distribution is counter cyclical, expanding during periods of low economic activity. This uneven within industry pattern is what drives the positive correlation between skewness and economic conditions depicted in figure 5.

¹⁶Appendix Figure A.5 shows remarkably similar results for other firms' outcomes, such as three-year sales growth, three-year employment growth, and stock returns.

growth rate in an industry-year bin, and we have rescaled the real sales growth within each sector to have a variance of one so that the regression coefficient can be interpreted as the effect of a change in the within-industry sales growth of one standard deviation and can be easily compared to the coefficients of columns (1) to (2) in Table II. Importantly, we also include a full set of time and industry fixed effects, so that the results are driven by within-industry changes rather than aggregate changes in growth rates.¹⁷

Column (7) of Table II shows that the skewness of employment growth is significantly lower during industry slowdowns. Specifically, a one standard deviation decline in the within-industry average sales growth is associated with a decline in the skewness of employment growth of 7 percentage points. This is almost two times larger than the effect of a change of one standard deviation in GDP growth on the skewness of employment growth across all firms in the economy. Similarly, a one standard deviation decline in average sales growth is correlated with a decline of 13 percentage points in the skewness of sales growth and a decline of 1.6 percentage points in the skewness of stock returns.

3.4 The Skewness of Firms' Productivity Shocks

The evidence we have provided so far strongly indicates that the skewness of the distribution of firm-level outcomes is procyclical, declining during periods of low economic activity. Such pattern, however, can be the result of shocks with time-varying higher order moments (i.e. time-varying skewness), the endogenous responses of firms to symmetric shocks that generate asymmetric changes in firm's outcomes, or a combination of both. In order to study whether the procyclicality in the skewness of firm's outcomes also reflects changes in the distribution of shocks affecting firms, in this section we use firm-level data for a subsample of European countries for which we have enough information to calculate TFP at the firm level.¹⁸ In particular, we estimate firm-level log productivity, z_i , as

$$z_{i,t} = \log Y_{i,t} - \alpha_K \log K_{i,t} - \alpha_L \log L_{i,t},$$

¹⁷We find a similar positive and statistically significant relationship between industry cycles and skewness when we consider each industry separately. Appendix Figure A.6 shows the coefficient of a set of within-industry time series regressions of the Kelley skewness of firms' growth on the within-industry average firm growth. Notice that, although there is substantial heterogeneity across industries, for all of them the coefficient on the average firm growth is economically and statistically significant.

¹⁸Our firm-level data comprises information of small and large firms, both publicly traded and privately held from seven European countries, namely, Germany, Spain, Italy, France, Portugal, Finland, and Sweden. Appendix B describes in full detail the sample selection and estimation procedure.

where $Y_{i,t}$ is the deflated value added of firm i in year t , $K_{i,t}$ is a measure of the capital stock, and $L_{i,t}$ is a measure of labor input. As is standard in the productivity literature (e.g. Syverson (2011)) we assume constant returns to scale at the firm-level (so $\alpha_K = 1 - \alpha_L$) and measure α_L as the industry-country labor share (the ratio of the total wage bill to total value added).¹⁹

Once we have calculated $z_{i,t}$, we obtain a measure of firms' productivity shock, denoted by $\epsilon_{i,t}$, from the residual of the following firm-level panel regression,

$$z_{i,t} = \beta_0 + \beta_1 z_{i,t-1} + \mu_i + \delta_t + \epsilon_{i,t},$$

where μ_i is a firm fixed effect and δ_t is a year fixed effect. We then calculate different moments of the distribution of $\epsilon_{i,t}$ within a country-industry-year cell.

Figure 6 displays a scatter plot pooling industry-level data across all countries and years in our sample. Similarly to our results for sales growth and employment growth, we find that the skewness of the distribution of firms' productivity shocks is negative in industries in which the average productivity shock is also negative.²⁰ In this figure, we have controlled for country, industry, and year fixed effects and therefore, our results are neither driven by fixed differences across countries and industries, nor by aggregate economic fluctuations that can affect all firms at the same time. In terms of magnitudes, a decline of productivity of 5 percent is associated to a similar decline in the skewness of the distribution. As we show in Appendix B, the procyclicality of the skewness of firm's shocks is robust and independent of the estimation method we use to calculate productivity, how we measure industry growth (sales or productivity), and holds for each individual country in our sample.²¹

Taken together, these results indicate—along with the procyclical skewness of firms

¹⁹Note that we measure TFPR rather than TFPQ since our dataset measures firm revenue rather than physical output.

²⁰We find similar results hold in the United States. Using US Census data for a panel of manufacturing firms from the Annual Survey of Manufacturing and the Census of Manufacturing firms we find that the skewness of the distribution of firm-level productivity growth is strongly procyclical. These results are under review for disclosure.

²¹In particular, in B we use three additional measures of productivity. In the first, we reestimate the productivity residuals, $z_{i,t}$, by running an OLS panel regression using firm-level data at the country level; Second, we estimate $z_{i,t}$ using the method developed by Olley and Pakes (1996); third, we estimate labor productivity by regressing firm value added on employment and a firm industry fixed effect. All these methods deliver similar results, qualitatively and quantitatively: the skewness of firm-level productivity shocks is strongly procyclical.

stock-returns reported in Section 3.1—that the shocks driving firm growth have procyclical skewness. One reason could be, for example, rising bankruptcy during recessions, which would generate left-skewed demand shocks (e.g. if a major customer goes bankrupt this will generate a large left-tail shock). Another could be the underlying driving process itself heterogeneously impacts firms—that is, a few firms lose badly in recessions and a few firm gain heavily in booms—which is similar in spirit to the granularity work in Gabaix (2011).

3.4.1 Robustness

In this section, we perform several robustness checks using the Census LBD, which for evaluating skewness has a major advantage having over 6 million firms. First, we examine the robustness of procyclical skewness for firms in different age or size categories, as well as for establishments as opposed to firms. We then consider alternative measures of skewness and also allow for firm entry and exit.

Figure 7 reports the results of these different analyses. The two figures (A, B) in the top panel plot the skewness of employment growth for firms within various size categories, ranging from firms with 1 to 19 employees at the low end to firms with more than 1,000+ employees at the high end. The skewness of employment growth is procyclical for all groups. Second, panel C splits the sample by age and shows that despite level differences in skewness across groups (in particular, younger firms have more positive skewness than average, as could be expected; see Haltiwanger *et al.* (2016)), fluctuations in skewness are procyclical for all firm age categories. Panel D shows that skewness fluctuations are very similar for establishments and firms, showing that our baseline results are not driven by a small number of large firms.²²

Second, we explore the effect of the particular percentiles the Kelley skewness is based on because by using the 90th and 10th percentiles, we are effectively dropping 20% of the distribution, which is all in the tails. Because our results hinge on the differential response of the tails to business cycles, truncating these tails could matter. As noted earlier, the Kelley measure can be constructed for any two symmetric quantiles, so we compute two additional versions of the Kelley skewness: one that uses the 95th and 5th percentiles and another that uses the 97.5th and 2.5th percentiles. The bottom left panel (E) of Figure 7 shows that these measures behave qualitatively similar to the standard

²²Furthermore, Appendix Figure A.7 shows that the skewness of employment growth is also procyclical within even finer categories—establishment groups defined by size and age.

Kelley measure.

Third, our main results are based on log growth rates which required us to exclude firms that either enter or exit in one of the two years for which the growth rate is calculated. Since entry and exit have a clear cyclical nature, this could potentially matter for the cyclical nature of the skewness of the distribution of employment or sales growth. In particular, if a firm exits the market because of a change in aggregate economic conditions or a new firm enters, our measure of the growth rate, and consequently, the skewness of the distribution, will not take them into account. To address this issue, we calculate the skewness of the employment growth distribution measured as the arc-percent change in employment, which is defined as $2(x_{i,t+k} - x_{i,t}) / (x_{i,t+k} + x_{i,t})$. This measure has been popularized in the firm dynamics literature by [Davis and Haltiwanger \(1992\)](#) and has the advantage that, while it is similar to a percentage change, it allows for entry and exit by including both time t and $t+k$ measures in the denominator, one of which is allowed to be zero.²³ The bottom right panel of [Figure 7](#) shows that the cyclical properties of the skewness of employment growth do not change substantially when accounting for the entry and exit of firms.

In summary, we have shown that the skewness of firm-level outcomes declines sharply during recessions, both at the aggregate and at the industry level. Motivated by this robust evidence, in the next section we study a heterogeneous agents model that we use to evaluate the macroeconomic importance of the large swings in the skewness we observe in the data.

4 Model

Given the evidence in the previous section, a natural question is to ask whether fluctuations in skewness at the firm level have aggregate implications. To answer this question, we build a heterogeneous-agent model populated by a large number of infinitely lived households/entrepreneurs who combine capital and labor using a technology subject to idiosyncratic productivity shocks to produce a homogeneous good. We make two important modeling choices which are important in generating large impacts of skewness shocks, but which we also think are empirically reasonable.

²³Notice that, for a firm with a positive value of $x_{i,t}$ which is inactive in period $t+k$, and henceforth has a value of $x_{i,t+k}$ equal to 0, the arc-percent change takes the value of -2 . Similarly, for an entering firm (that is, $x_{i,t}$ is equal to 0 but $x_{i,t+k}$ is positive) the arc-percent change takes the value of 2.

First, entrepreneurs are not able to insure against idiosyncratic shocks, so they are exposed to idiosyncratic risk. This seems plausible—very few businesses are able to insure fully (or even partially) against the risks they face, and since the managers of most firms have significant equity stakes in their businesses they are exposed to business risk.²⁴ Second, entrepreneurs are able to save for tomorrow in the form of both the capital good and a one-period bond with a risk-free return. The risk-free return can be thought of as a government of foreign sector, which at least from the perspective of the entrepreneur, provides a return independent of idiosyncratic risk.

We now describe each component of the model in more detail.

4.1 Entrepreneurs

4.1.1 Production Technology

The production function of entrepreneur j is given by

$$y_{j,t} = A_t e_{j,t} k_{j,t}^\alpha n_{j,t}^\nu, \text{ with } \alpha + \nu < 1,$$

so it displays decreasing returns to scale. The aggregate productivity shock, A_t , follows a first-order autoregressive process:

$$\log A_t = \rho_A \log A_{t-1} + \sigma_\eta \eta_t,$$

where η_t is a Gaussian innovation with zero mean and unitary variance. The idiosyncratic productivity process $e_{j,t}$ is given by

$$e_{j,t} = \rho e_{j,t-1} + \epsilon_{j,t}, \tag{3}$$

where $\epsilon_{j,t}$ has zero mean, time-varying variance, denoted by $\sigma_{\epsilon,t-1}$, and time-varying skewness, denoted by $\gamma_{\epsilon,t-1}$. Notice that we have assumed that the distribution of innovations in period t depends on the values of the variance and skewness observed in period $t - 1$. This timing captures the “news shock” aspect of firm-level risks in the

²⁴Even in the very large publicly US top executives own substantial equity stakes, and in most private firms the businesses is often owned by the manager or their family. In fact, the vast majority of firms in the United States are passthrough entities which resemble an entrepreneurial firm. As shown by [Smith et al. \(2019\)](#), the average passthrough entity employs 13 workers and had 2.3 owners. Furthermore, even individuals at the top of the income distribution—who own more profitable mid-size firms—maintain a very concentrated portfolio deriving most of their income from one mid-market firm.

model: an increase in dispersion or left-skewness of firms' shocks represents news about the characteristics of the distribution of innovations in the future but not a change in the distribution from which the current realizations of $\epsilon_{j,t}$ are drawn.

4.1.2 Capital Adjustment Costs

We consider a flexible combination of convex and non-convex adjustment costs to capital²⁵. To this end, let $i_{j,t}$ denote *net* investment in capital:

$$i_{j,t} = k_{j,t+1} - (1 - \delta) k_{j,t}, \quad (4)$$

where δ is the depreciation rate of capital. Capital adjustment costs are given by the sum of a fixed *disruption cost*, ϕ_1 , paid by the entrepreneur for any net investment or disinvestment, a quadratic adjustment cost, ϕ_2 , and a *resale cost* for net disinvestment (partial irreversibility), ϕ_3 . Therefore, the total adjustment cost function for capital input is

$$\phi(k_{j,t+1}, k_{j,t}) = \phi_1 \mathbb{I}_{|i_{j,t}| > 0} y_{j,t} + \frac{\phi_2}{2} \left(\frac{i_{j,t}}{k_{j,t-1}} \right)^2 + (1 - \phi_3) |i_{j,t}| \mathbb{I}_{i_{j,t} < 0}, \quad (5)$$

where \mathbb{I} is an indicator function.

4.1.3 The Problem of the Entrepreneur

Entrepreneurs do not value leisure and value consumption streams according to an Epstein-Zin utility function as specified below. Entrepreneurs supply labor to their own firm (they cannot work for someone else's firm). They can save in capital and in a risk-free asset that pays an interest rate r_t . Denote the entrepreneur's value function by $V(k_{j,t}, a_{j,t}, e_{j,t}; \Omega_t)$ where $k_{j,t}$ is the entrepreneur's stock of physical capital, $a_{j,t}$ is the beginning-of-the-period holdings in the risk-free asset, and $e_{j,t}$ is her idiosyncratic productivity. For notational simplicity, define the vector of aggregate states as $\Omega_t \equiv (A_t, \sigma_{\epsilon,t-1}, \gamma_{\epsilon,t-1}, \mu_t)$ where A_t is the aggregate productivity level, $\sigma_{\epsilon,t-1}$ and $\gamma_{\epsilon,t-1}$ are the variance and the skewness of the distribution of idiosyncratic shock, respectively, and μ_t is the distribution of entrepreneurs over idiosyncratic states. Then, we can write the

²⁵It is important to allow for general form of adjustment costs, as non-convex (convex) adjustment costs can increase (decrease) the skewness of control variables. For example, [Ilut *et al.* \(2018\)](#) have a model where non-convex labor adjustment costs generate left-skewed employment growth distributions.

dynamic problem of the entrepreneur with Epstein-Zin preferences as

$$V(k_{j,t}, a_{j,t}, e_{j,t}; \Omega_t) = \max_{\{c_{j,t}, k_{j,t+1}, a_{j,t+1}, n_{j,t}\}} \left(c_{j,t}^{1-\lambda} + \beta \mathbb{E} \left[V(k_{j,t+1}, a_{j,t+1}, e_{j,t+1}; \Omega_{t+1})^{1-\xi} \right]^{\frac{1-\lambda}{1-\xi}} \right)^{\frac{1}{1-\lambda}}, \quad (6)$$

$$\begin{aligned} \text{s.t. } c_{j,t} + i_{j,t} + a_{j,t+1} &\leq y_{j,t} - w_t n_{j,t} - \phi(k_{j,t+1}, k_{j,t}) + (1 + r_t) a_{i,t}, \\ i_{j,t} &= k_{j,t+1} - (1 - \delta) k_{j,t}, \\ \mu_{t+1}(k_{j,t+1}, a_{j,t+1}, e_{j,t+1}) &= \Gamma(\Omega_t), \\ k_{j,t} &> 0, a_{j,t} \geq 0, n_{j,t} > 0, \end{aligned}$$

given the laws of motion for A_t , $\sigma_{\epsilon,t}$, and $\gamma_{\epsilon,t}$. In this specification, ξ is risk aversion, and λ is inversely related to the elasticity of intertemporal substitution. The term $w_t \equiv w(\Omega_t)$ denotes the wage rate in the economy. In what follows, we assume the interest rate on the risk-free asset is fixed, that is $r_t = r(\Omega_t) = r$.²⁶ Let $C^e(k_{j,t}, a_{j,t}, e_{j,t}; \Omega_t)$, $K^e(k_{j,t}, a_{j,t}, e_{j,t}; \Omega_t)$, $N^e(k_{j,t}, a_{j,t}, e_{j,t}; \Omega_t)$, and $A^e(k_{j,t}, a_{j,t}, e_{j,t}; \Omega_t)$ denote the policy rules for consumption, next period capital, current period labor, and the risk-free asset for the entrepreneurs.

4.2 Non-Entrepreneurial Households

The economy is populated by a large number of identical hand-to-mouth households that consume C_t units of the homogeneous good and supply labor elastically which we denote by N_t . More concretely, the non-entrepreneurial households solve the static problem

$$U(C_t, N_t) = \max_{C_t, N_t} \left\{ \frac{C_t^{1-\sigma}}{1-\sigma} - \psi \frac{N_t^{1-\gamma}}{1-\gamma} \right\}, \quad (7)$$

$$C_t \leq w_t N_t,$$

given the law of motion of the aggregate state, Ω_t . Denote by $C(\Omega_t)$ and $N(\Omega_t)$ the optimal choices of consumption and labor for the non-entrepreneurial household.

²⁶This implies that we will not solve the interest rate in equilibrium. The wage rate, however, is such that the labor market clears.

4.3 Recursive Competitive Equilibrium

Given the exogenous process for aggregate productivity, A , the exogenous process of the variance and skewness of e_j , the interest rate of the risk-free asset, r , and the evolution of the idiosyncratic productivity processes for the entrepreneurs, $\{e_j\}_{j \in J}$, a recursive competitive equilibrium for this economy is a set of policy functions

$\left\{ \left\{ C_j^e, K_j^e, N_j^e, A_j^e \right\}_{j \in J}, C, N \right\}_{t=0}^{\infty}$, a wage function $\{w\}$, and value functions $\{V, U\}$ such that i) the policy and value functions solve (6) and (7), respectively; ii) the labor market clears, that is,

$$\int N^e(k_j, a_j, e_j; \Omega) d\mu(k_j, a_j, e_j) = N(\Omega),$$

and iii) the mapping $\Gamma(\omega)$ that determines the evolution of the joint distribution of e_j , k_j , and a_j is consistent with the policy functions, the evolution of the aggregate productivity process, and the evolution of the process of σ_ϵ and γ_ϵ .

4.4 Parameters, Estimation, and Model Fit

In this section, we describe the quantitative specification of our modeled economy. To solve the entrepreneurs' problem, we employ non-linear methods similar to [Krusell and Smith \(1998\)](#). Most of our parameters are standard in the macro literature, and we take them from the existing estimates when possible. However, the parameters governing the stochastic process of productivity are novel to our analysis, and we use a simulated method of moments approach to estimate them.

Frequency and Preferences

We set the time period to be a quarter. For the entrepreneurs, we set ξ , the risk aversion coefficient, equal to 6.0 and $1/\lambda$, the elasticity of substitution, to $1/\lambda = 0.2$, which are at the midpoint of the values used in [Güvönen \(2009\)](#). The household's discount rate, β , is set to $0.95^{0.25}$, whereas the interest rate on the risk-free asset is set to match an annual return of 2%. For the non-entrepreneurial sector, we set σ to 2. For the labor supply of the non-entrepreneurial households, we fix a value of γ to 1.5, and we choose ψ so that they spend an average of 33% of their time working.

The exponents of the capital and labor inputs in the entrepreneur's technology are set to $\alpha = 0.25$ and $\nu = 0.5$. The capital depreciation rate, δ , is set to match 14% of annual depreciation. As for the adjustment cost parameters, we set the fixed adjustment

cost of capital, ϕ_1 , equal to 1.5%, a quadratic adjustment cost, ϕ_2 , equal to 7.0, and a resale cost, ϕ_3 , equal to 34.0%.

Productivity

We assume that aggregate productivity follows a standard first-order autoregressive process with an autocorrelation of 0.95 and normally distributed innovations with mean 0 and a standard deviation of 0.75%, similar to the quarterly values used in other papers in the literature. Table III summarizes the set of calibrated parameters.

To capture time-varying risk, we assume that the economy transitions between two states. The first, which we denote as the low-risk state, corresponds to periods in which the variance of the innovations of the idiosyncratic shocks is low, $\sigma_{\epsilon,t} = \sigma_L$, and the skewness is positive, $\gamma_{\epsilon,t} = \gamma_H$, as we observe in non-recession periods. The second state, or high-risk state, corresponds to periods of high dispersion, $\sigma_{\epsilon,t} = \sigma_H$, and negative skewness, $\gamma_{\epsilon,t} = \gamma_L$, as we observe during a typical recession. Low- and high-risk states alternate following a first-order Markov process. To capture the potential non-Gaussian nature of the idiosyncratic shocks, we assume that, conditional on the values of $\sigma_{\epsilon,t}$ and $\gamma_{\epsilon,t}$, the innovations in (3) are drawn from a mixture of two normally distributed random variables, that is,

$$\epsilon_{j,t} \sim \begin{cases} N(\mu^s, \sigma_1^s) & \text{with prob } p^s, \\ N\left(-\frac{p^s}{1-p^s}\mu^s, \sigma_2^s\right) & \text{with prob } 1 - p^s, \end{cases} \quad (8)$$

where s can be a high- or low-risk state. Hence, to fully characterize the stochastic process faced by firms, we need to find ten parameters, namely, $\{\mu^s, \sigma_1^s, \sigma_2^s, p^s\}$ with $s \in \{H, S\}$, and the parameters governing the transition probabilities between low- and high-risk periods, denoted by π_L and π_H , respectively.

Since we do not directly observe the productivity process faced by the firms, we choose the parameters of the stochastic process of firms' productivity to match the features of the US data described in the empirical section of the paper. In particular, we take the data of quarterly sales growth from Compustat, and we search for parameters of the stochastic process so that the cross-sectional distribution of sales growth derived from the model reproduces the observed average values of the 90th-to-50th percentiles spread, the 50th-to-10th percentiles spread, the Kelley skewness, and the 90th-to-10th percentiles spread during expansion periods and the same set of moments for recession periods for

a total of eight moments of the quarterly sales growth distribution.²⁷ The probability of being in the high-risk state in the next period conditional on being in the high-risk state in this period, π_H , is set to be equal to the fraction of recession quarters that are followed from another recession quarter in the data, $\pi_H = 0.84$, whereas the transition probability of the low risk state, π_L , is set so that the share of expansion quarters following another expansion quarter is 0.95. Recession and expansion periods in the data correspond to the recession quarters defined by the NBER from 1970 to 2014.

Based on our estimations, we find that in periods of low risk, the variance of the idiosyncratic productivity shocks, η , is equal to 4.85%, whereas the skewness is equal to 0.85. In contrast, in periods of high-risk, the variance of the productivity shocks is equal to 6.85%, and the skewness is negative and equal to -1.14. Table VI displays our estimates for the different parameters of the idiosyncratic productivity process, whereas Table IV shows the targeted and model-simulated moments.²⁸

5 Quantitative Results

Table VII shows a set of standard business cycle statistics generated from our modeled economy. To obtain these statistics, we simulate our economy for 5,000 periods and discard the first 500. We then calculate the standard deviation and correlation with aggregate output for several aggregate time series. All statistics are in the neighborhood of what is observed in the data: investment is more volatile than output, whereas consumption is less volatile. Additionally, our model generates an average annual risk premium of 5.3%, which is in line with the empirical estimates based on US data. We conclude that our model is consistent with the standard business cycle statistics found in the literature.

²⁷Appendix Figure A.2 displays the evolution of the cross-sectional dispersion and skewness of the sales growth distribution for our sample of publicly traded firms from Compustat at the quarterly frequency.

²⁸The variance of a random variable η , which is distributed as a mixture of two normally distributed random variables, is given by $Var(\eta) = \mathbb{E}(\eta^2) - \mathbb{E}(\eta)^2$, whereas the skewness is given by $Skew(\eta) = (\mathbb{E}(\eta^3) - 3\mathbb{E}(\eta)Var(\eta) - \mathbb{E}(\eta)^3) / Var(\eta)^{3/2}$. Here $\mathbb{E}(\eta)$ is the first moment of the η given by $\mathbb{E}(\eta) = p_1\mu_1 + p_2\mu_2$. Similarly, $\mathbb{E}(\eta)^2 = p_1(\mu_1^2 + \sigma_1^2) + p_2(\mu_2^2 + \sigma_2^2)$ and $\mathbb{E}(\eta^3) = p_1(\mu_1^3 + 3\mu_1\sigma_1^2) + p_2(\mu_2^3 + 3\mu_2\sigma_2^2)$ are the second and third moments.

5.1 Idiosyncratic Shocks and Model Fit

To evaluate the effects of a decrease in the skewness of firm-level shocks, we independently simulate 1,000 economies, each of 300 quarters' length. For the first 150 periods, the economy remains in the low-risk state, and then all economies are hit by a change in the level of risk (i.e. a decrease in the skewness of firm-level shocks, an increase in the dispersion of firm-level shocks, or both at the same time). From that period on, all economies evolve normally. We then average different macroeconomic outcomes across all simulated economies and calculate the impact of the change in risk as the percentage deviation of a given macro variable relative to its value in the period previous to the shock.

The left panels of Figure 8 display moments of the distribution of firms' idiosyncratic productivity growth, $\Delta e_{j,t} = e_{j,t} - e_{j,t-4}$, for three cases. In the first, the economy moves from the low-risk state to the high-risk state, leading to an increase in the variance and a decrease in the skewness of idiosyncratic shocks (blue line with circles), which corresponds to what is observed during a typical recession. In the second case, the increase in risk leads only to a decrease in the skewness of idiosyncratic shocks (black line with diamonds), and finally, in the third case, the increase in risk leads to an increase in the variance of idiosyncratic shocks only, which is the typical uncertainty shock studied in the literature (red line with triangles).²⁹ The top left panel of Figure 8 shows that the average firm in our model does not experience a change in ~~firm-level~~ productivity when risk changes. This ensures that our results are not driven by a change in average firm productivity. Then, comparing the black line in the middle and bottom left panels, one can see that our model is able to generate a pure change in the skewness, that is, a change in the productivity distribution that reflects only a decrease in the skewness but a muted change in the mean and the variance of the firm-level productivity distribution.³⁰ Similarly, our model can generate a pure uncertainty shock (the red line with triangles in the middle panels of Figure 8).

It is also important to analyze the impact of the change in risk on the sales growth distribution. The right panels of Figure 8 show the average, the dispersion, and the

²⁹To make this comparison, we reestimate the parameters of the stochastic process in (8) to separate the changes in dispersion (a symmetric increase in risk) from changes in dispersion and skewness (an asymmetric increase in risk). Table V shows the estimation targets for each case.

³⁰The median firm, however, experiences an increase in productivity after a decline in the skewness that keeps the mean and variance constant. This increase in productivity goes against our results as our model predicts a negative aggregate response of the economy to a drop in skewness.

skewness of the annual change in quarterly sales implied by the model calculated as $\Delta y_{j,t} = \log y_{j,t} - \log y_{j,t-4}$. It is not surprising that a change in risk that combines a simultaneous increase in the variance and a decrease in the skewness of firm-level productivity shocks generates an increase in the cross-sectional dispersion of sales growth and a large decrease in skewness (blue line with circles in the middle and bottom right panels). Comparing the case in which only dispersion changes—which is the typical uncertainty shock—with the case in which only the skewness changes—the baseline case we discuss in the following section—one can see that by considering a shock with time-varying skewness, the model is able to capture the asymmetric response of the tails of the sales growth distribution (compare the red line with triangles to the blue line with circles in the bottom right panel). Moreover, the model generates a drop in Kelley skewness of around 20 percentage points which is in line with the drop observed during recession periods in the United States. These are the first results of our quantitative analysis: in the context of a model with adjustment costs to capital and risk-averse entrepreneurs, a pure uncertainty shock does not generate the large asymmetric changes in the sales growth distribution that we document in Section 3.³¹ Notice also that the average sales growth greatly responds to a change in the risk conditions in the economy (the bottom left panel) but this response is only driven by the endogenous capital and hiring response of firms to a change in the risk conditions as the average productivity growth is unaltered.

5.2 The Macroeconomic Effect of a Skewness Shock

We now study the macroeconomic effect of a decrease in the skewness of firm-level productivity. For doing that, we shock the economy with a change in the skewness of the innovations of $e_{j,t}$ and calculate the response of different macroeconomic aggregates as the percentage change relative to their value prior to the shock. In our exercise, when the economy receives a skewness shock that drives the skewness from γ_H to γ_L , we keep the mean and variance of the idiosyncratic productivity constant at their low-risk level, so our results reflect a pure change in the skewness of the distribution.

Figure 9 shows that output declines by 1.4% four quarters after a skewness shock and 1.7% after eight quarters. This is a significant decline in aggregate economic activity considering that only the shape of the distribution of firm-level shocks has changed.

³¹Figure A.9c in the appendix shows that the dispersion and skewness of sales growth do not respond to a shock to aggregate productivity, A_t , either. Furthermore, as shown in Figure A.9a, a change in the skewness of firms' shocks generates a persistent decline in the skewness of employment growth and a decline in the skewness of three-year sales growth.

Moreover, the decline in output is quite persistent, staying below its pre-shock level even after twelve periods after the shock. This is in contrast with the typical uncertainty shock that generates a decrease in output and a rapid rebound a few quarters after the shock. In our model, the drop in output is generated by the rapid and persistent decline in capital investment after a change in skewness. The top right panel of Figure 10 shows that capital investment drops around 15% during the first quarter after the shock and stays below its pre-shock level for several quarters. Labor does not drop in the first period after the shock because labor is fully flexible and news about the future conditions of risk does not change firms' hiring decisions.³² In contrast, consumption declines rapidly in response to the decrease in the skewness of firm-level shocks, dropping by around 1% relative to its pre-shock level, whereas the accumulation of the risk-free asset increases because capital is now riskier.

Importantly, in the first quarter after the shock, the response of investment and consumption is not driven by a change in the skewness of the realizations of $e_{j,t}$ received by the firms—recall our timing assumption in equation (3)—but by a change in the perception about the risk in the economy: at the moment of the shock, entrepreneurs receive news that in the future the distribution of $e_{j,t}$ will be left-skewed, and their endogenous responses drive a decline in investment and consumption. A decrease in skewness triggers a precautionary increase in entrepreneurs' savings, but since capital is riskier, investment in the risk-free asset surges, as shown in the bottom right panel of Figure 10. We conclude that a decline in the skewness of the distribution of idiosyncratic shocks can by itself generate a persistent drop in aggregate economic activity.

5.3 Robustness

In this section, we discuss the robustness of our findings to different parameterizations. Recall that in our baseline results, in the period in which a change in risk occurs, firms do not experience a change in the actual realizations of shocks but only receive news that in the next period, the skewness of productivity shocks, for instance, will be lower. In the next period, however, the firms' productivity distribution changes as the shocks are drawn from a left-skewed distribution. We compare this baseline case to one in which we keep the underlying distribution of firms shocks fixed so that we can evaluate

³²Adding labor adjustment costs will trigger an automatic response of labor to changes in risk, increasing the aggregate impact of a change in variance and skewness.

the pure effect of a change in news about the future risk conditions.³³ This exercise is similar to an increase in the probability of a *disaster* (Gourio (2008); Barro and Ursua (2011)), although in our case it represents an increase in *disasters* at the microeconomic level. The blue line with circles in Figure 11 shows that the overall effect of a skewness shock combines the impact of a change in the perceptions about future risk conditions and the actual change in the realizations of idiosyncratic shocks. In fact, a shock that only represents news about the future risk generates a decline in output of about 0.5%, which is around one-third of the overall decline in our baseline results.

We then study how our results change with the degree of risk aversion of the entrepreneurs and their elasticity of intertemporal substitution while keeping the rest of the parameters at their values in Table VI. The red line with triangles in Figure 11 shows that decreasing entrepreneurs' risk aversion, ξ , from 6 to 2, does not have a substantial impact on our main results in terms of aggregate output and consumption, although it alters the effect of skewness on the accumulation of capital and the risk-free asset. An increase in the elasticity of intertemporal substitution, $1/\lambda$, from 0.2 to 0.5, does reduce the impact of skewness shocks on output and consumption (green line with squares), although the overall effect is still significant. Investment, in this case, changes less relative to the benchmark. These differences highlight the importance of separating the effect of risk aversion from intertemporal substitution when evaluating the impact of risk shocks.

6 Conclusions

This paper studies how the distribution of the growth rate of firm-level variables changes over the business cycle. Using firm-level panel for the United States from Census and non-Census datasets and firm-level panel data for over thirty other countries, we reach three main conclusions. First, recessions are characterized by a large drop in the skewness of firm-level outcomes such as employment growth, sales growth, productivity growth and stock returns. Hence, the skewness of firms' outcomes is strongly procyclical. Second, the decline in the skewness of firms' outcomes is a phenomenon observed not only in the United States but also in other countries, both developed and developing. Finally, we find strong procyclicality of the skewness at the industry level, in that industries

³³In particular, we simulate our model using the same realizations of the aggregate risk process used in our baseline analysis. In period T all economies receive a skewness shock, however, in this case, we keep the parameters determining the underlying idiosyncratic productivity process fixed at their pre-shock low-risk level.

with low and high growth rates experience a negative and positive skewness of shocks respectively.

In the second part of our paper, we analyze the impact of a change in the skewness of firms' idiosyncratic productivity in the context of a heterogeneous-agent model. We assume that the exogenous idiosyncratic productivity process faced by entrepreneurs is subject to time-varying skewness, and we choose the parameters of this model to match the evolution of the dispersion and skewness of the sales growth distribution in the United States. Our results suggest that a change in the skewness of the firm-level productivity distribution can by itself generate a significant decline in aggregate economic activity even though the mean and variance of firms' shocks are held constant. In fact, in our modeled economy, a decline in the skewness of firms' shocks of the magnitude observed in a typical US recession generates a drop in GDP of 1.7%. The combined impact of a variance and skewness shock generates an even large decline in output (2.0%), consumption (2.0%), and investment (40.0%).

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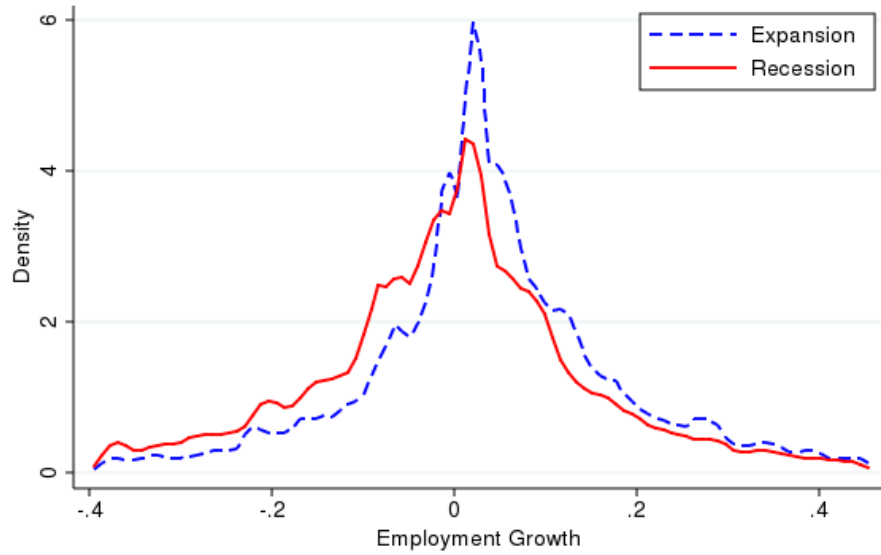
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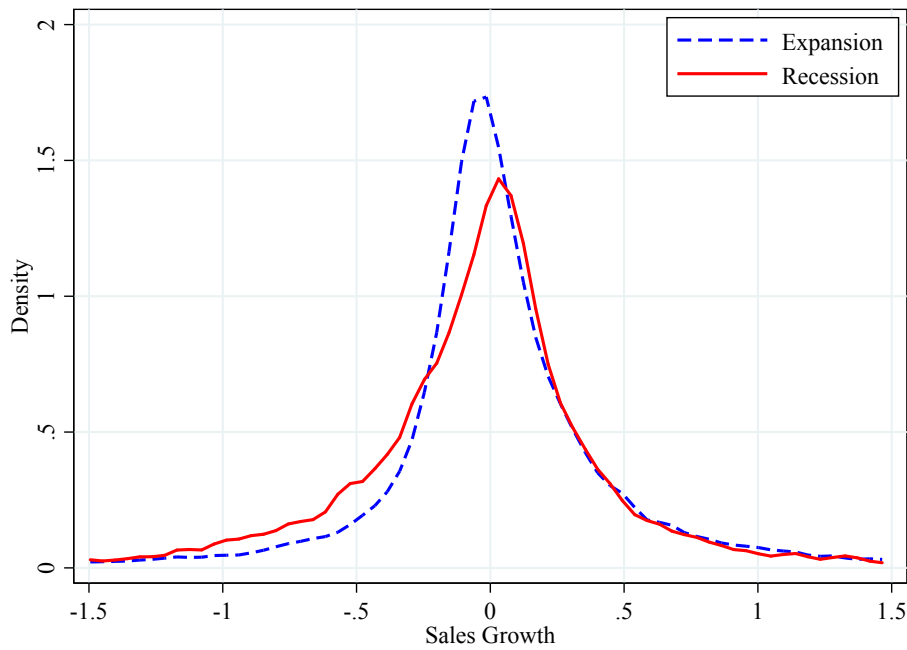
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FIGURE 1 – THE SKEWNESS OF FIRM OUTCOMES IS LOWER DURING RECESSIONS

(A) Census LBD: Employment Growth Distribution



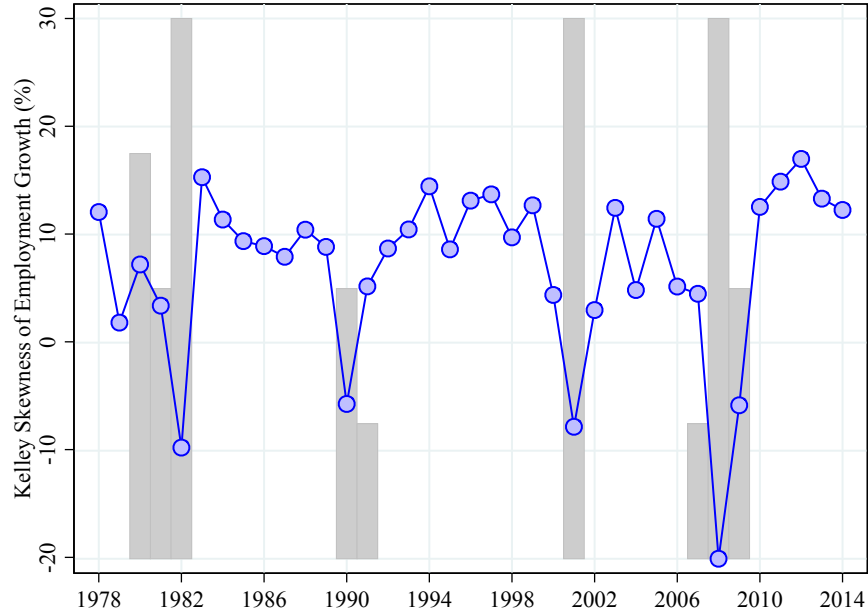
(B) Compustat: Sales Growth Distribution



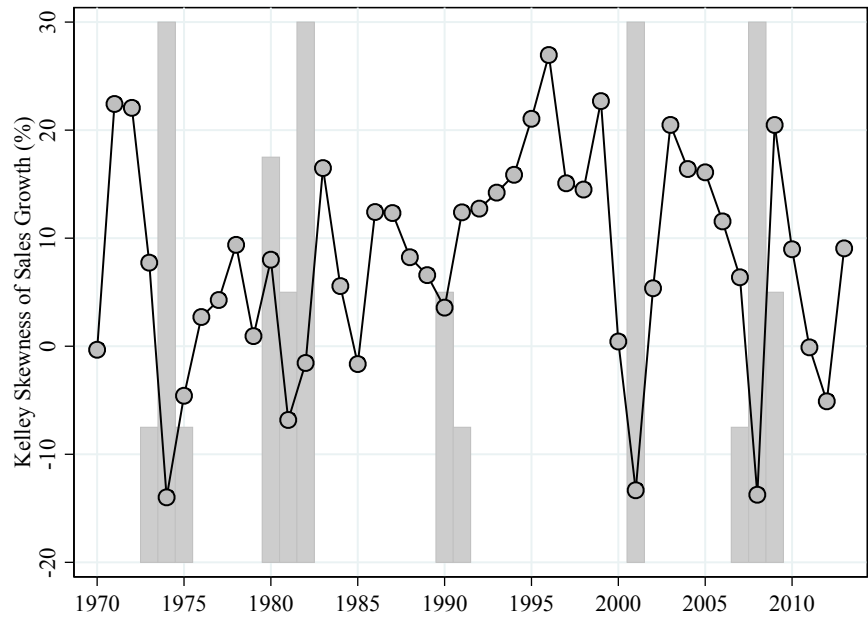
Note: The top panel of Figure 1 shows the employment-weighted empirical density of the distribution of firms' employment growth constructed from the LBD. The lower panel shows the empirical density of the distribution of firms' sales growth constructed from Compustat. Each density has been rescaled to have a median of zero and unitary variance. The blue-dashed line shows the density of a pooled sample of Expansion years (2003 to 2006 and 2010 to 2014) whereas the red-solid line shows the density of a pooled sample of Recession years (2001 and 2008). In the top panel, the unscaled 10th percentile of the employment growth distribution during expansion (recession) periods is -16.5% (-26.9%), the 50th is 1.3% (-1.8%), and the 90th is 23.3% (18.0%). In the bottom panel, the corresponding moments are -21.7% (-47.4%), 5.3% (-3.0%), and 44.6% (33.0%). See Appendix A for additional details on the sample construction and moment calculations in the LBD and Compustat.

FIGURE 2 – THE SKEWNESS OF FIRM OUTCOMES IS STRONGLY PROCYCLICAL

(A) Census LBD: Skewness of Employment Growth Distribution



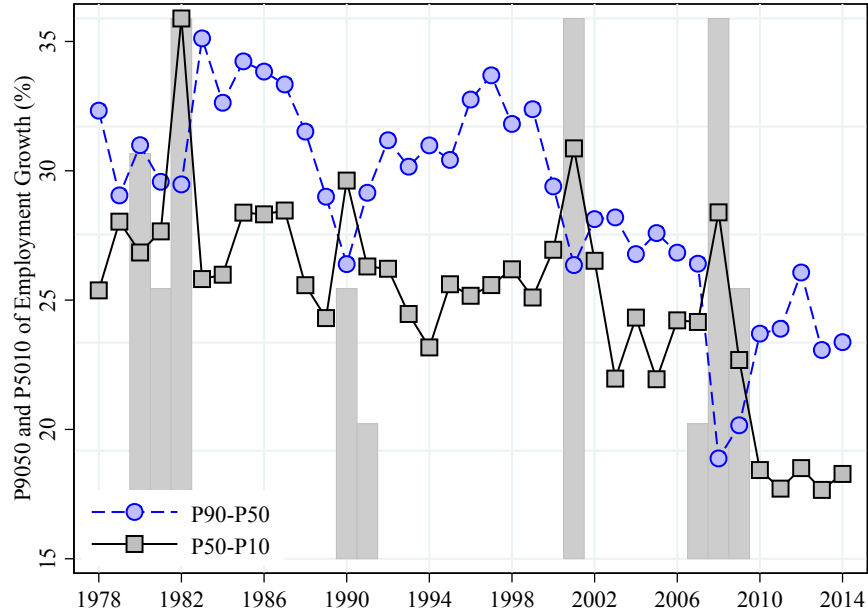
(B) Compustat: Skewness of Sales Growth Distribution



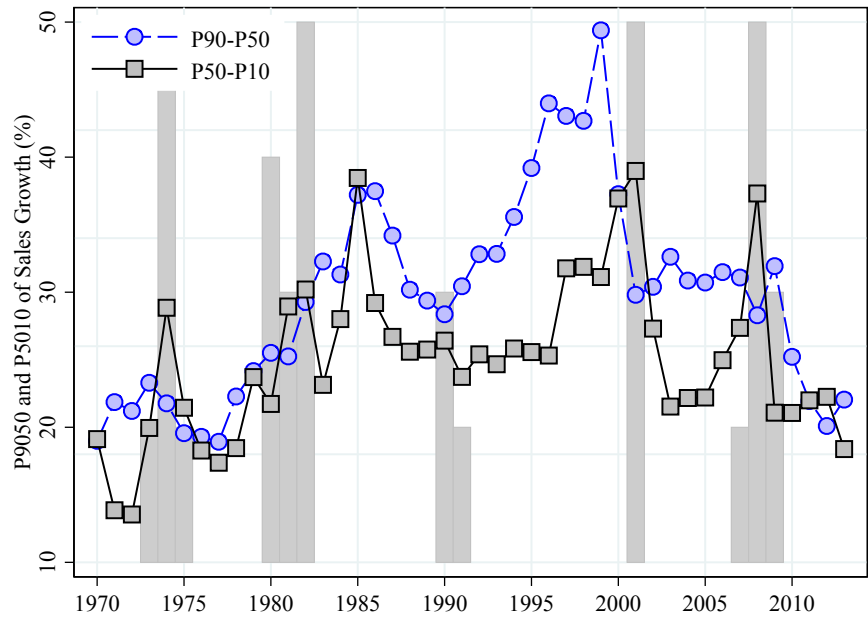
Note: The top panel of Figure 2 shows the time series of the cross-sectional Kelley skewness of the distribution of firms' employment growth constructed from the LBD. Moments are weighted by the average firm employment between years t and $t + 1$. The bottom panel shows the time series of the cross-sectional Kelley skewness of the distribution of firms' sales growth constructed from Compustat. The shaded bars represent NBER recession periods. See Appendix A for details on the sample construction and moment calculations in the LBD and Compustat.

FIGURE 3 – DISPERSION IN LEFT TAIL OF FIRM OUTCOMES IS COUNTERCYCLICAL

(A) Census LBD: Dispersion in Employment Growth



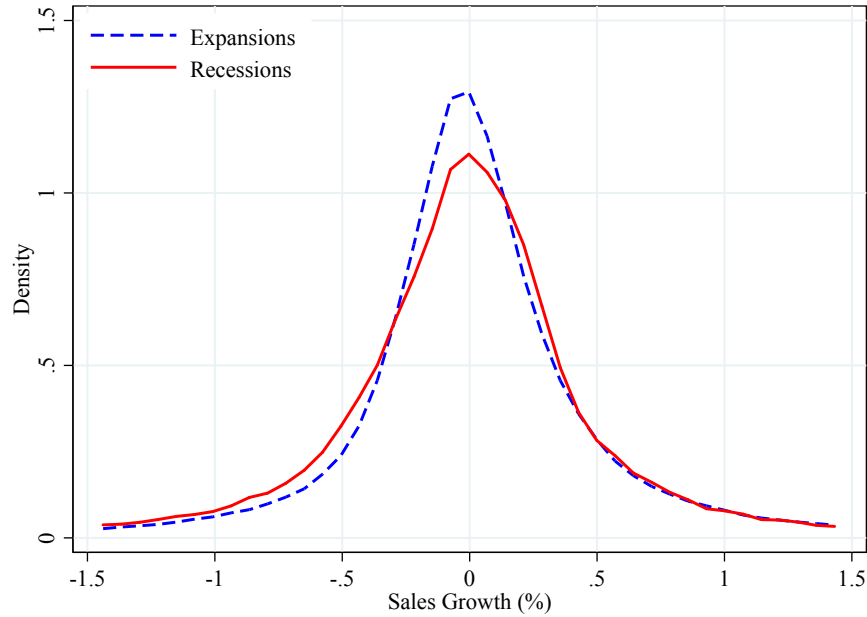
(B) Compustat: Dispersion in Sales Growth



Note: The top panel of Figure 3 shows the time series of the cross-sectional dispersion of the distribution of firms' employment growth constructed from the LBD. Moments are weighted by the average firm employment between years t and $t + 1$. The bottom panel shows the time series of the cross-sectional dispersion of the distribution of firms' sales growth constructed from Compustat. The shaded bars represent NBER recession periods. See Appendix A for details on the sample construction and moment calculations in the LBD and Compustat.

FIGURE 4 – THE SKEWNESS OF FIRM OUTCOMES IS CYCLICAL: CROSS-COUNTRY EVIDENCE

(A) Cross-Country: Sales Growth Distribution



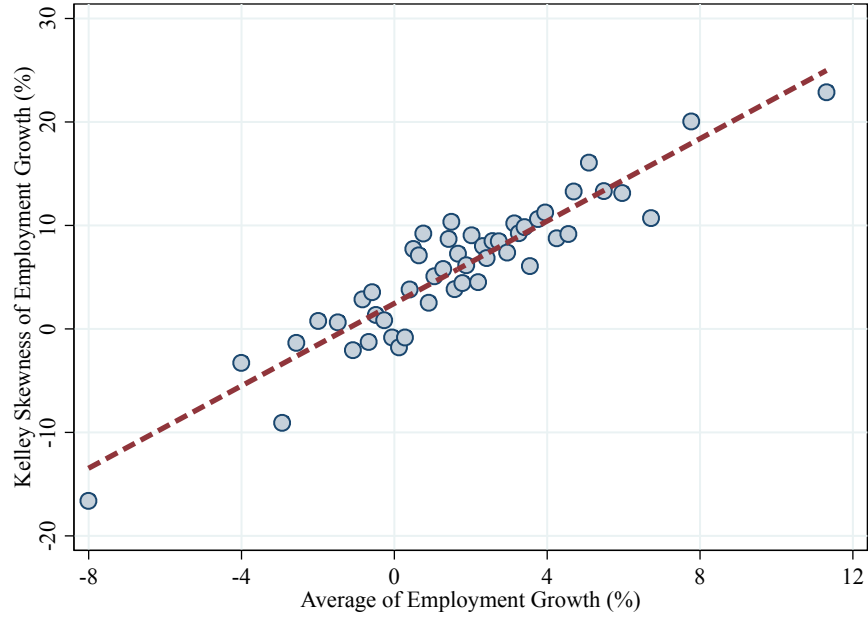
(B) Cross-Country: Firm-Level Employment and Sales Growth



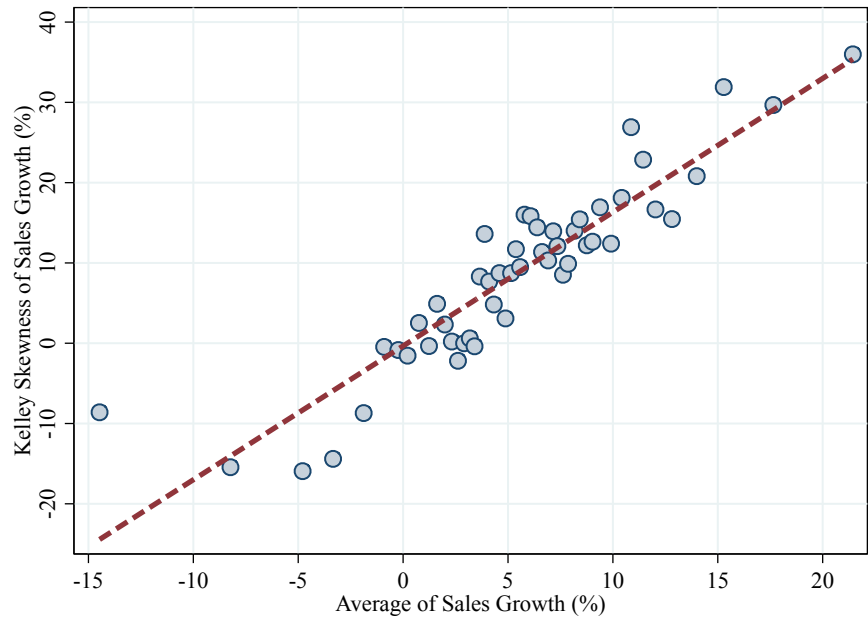
Note: The top panel of Figure 4 shows the empirical density of the growth rate of annual sales in US dollars constructed from the BvD Osiris dataset. Each density has been rescaled to have a median of zero and unitary variance. The red solid line is the empirical density over all the observations of firms during recession years, defined as years in which the country is in the first decile of the country-specific distribution of the growth rate of GDP per capita (74,009 observations). The blue dashed line is the empirical density over all the observations of firms during expansion periods (523,655 observations) which are years not classified as recessions. The unscaled 10th percentile of the sales growth distribution during expansion (recession) periods is -30.5% (-42.4%), the 50th percentile is 5.6% (0.0%), and the 90th percentile is 52.5% (43.6%). The bottom left (right) panel displays a binned scatter plot showing the relation between the within-country average firm employment (sales) growth and the within-country Kelley skewness of firm employment (sales) growth constructed from the BvD Osiris dataset. The regression slope is equal to 1.64 (0.50). Binned scatter plots controlling for time and country fixed effects. See Appendix A for details on the sample construction and moment calculations.

FIGURE 5 – THE SKEWNESS OF FIRM OUTCOMES IS CYCLICAL AT THE INDUSTRY LEVEL

(A) Census LBD: Industry Employment Growth Distribution

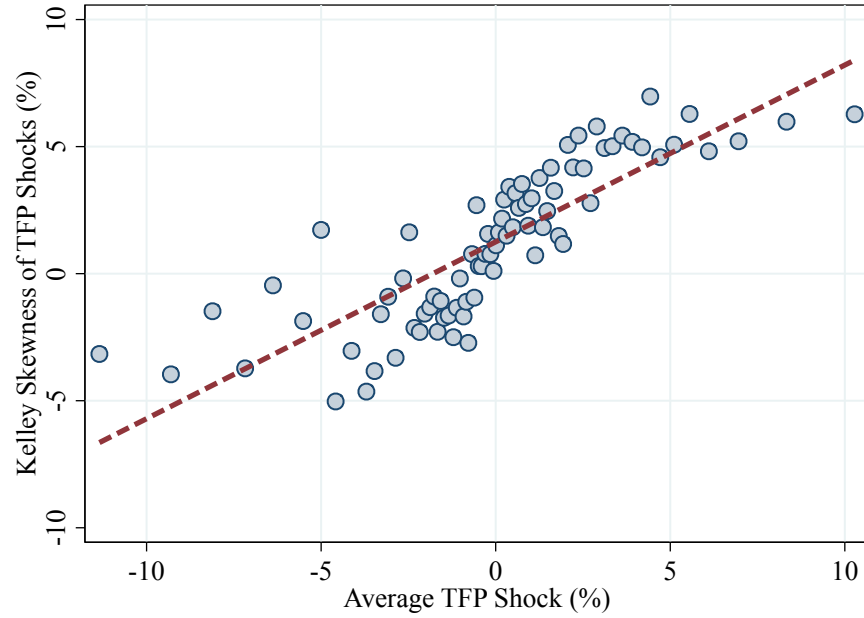


(B) Compustat: Industry Sales Growth Distribution



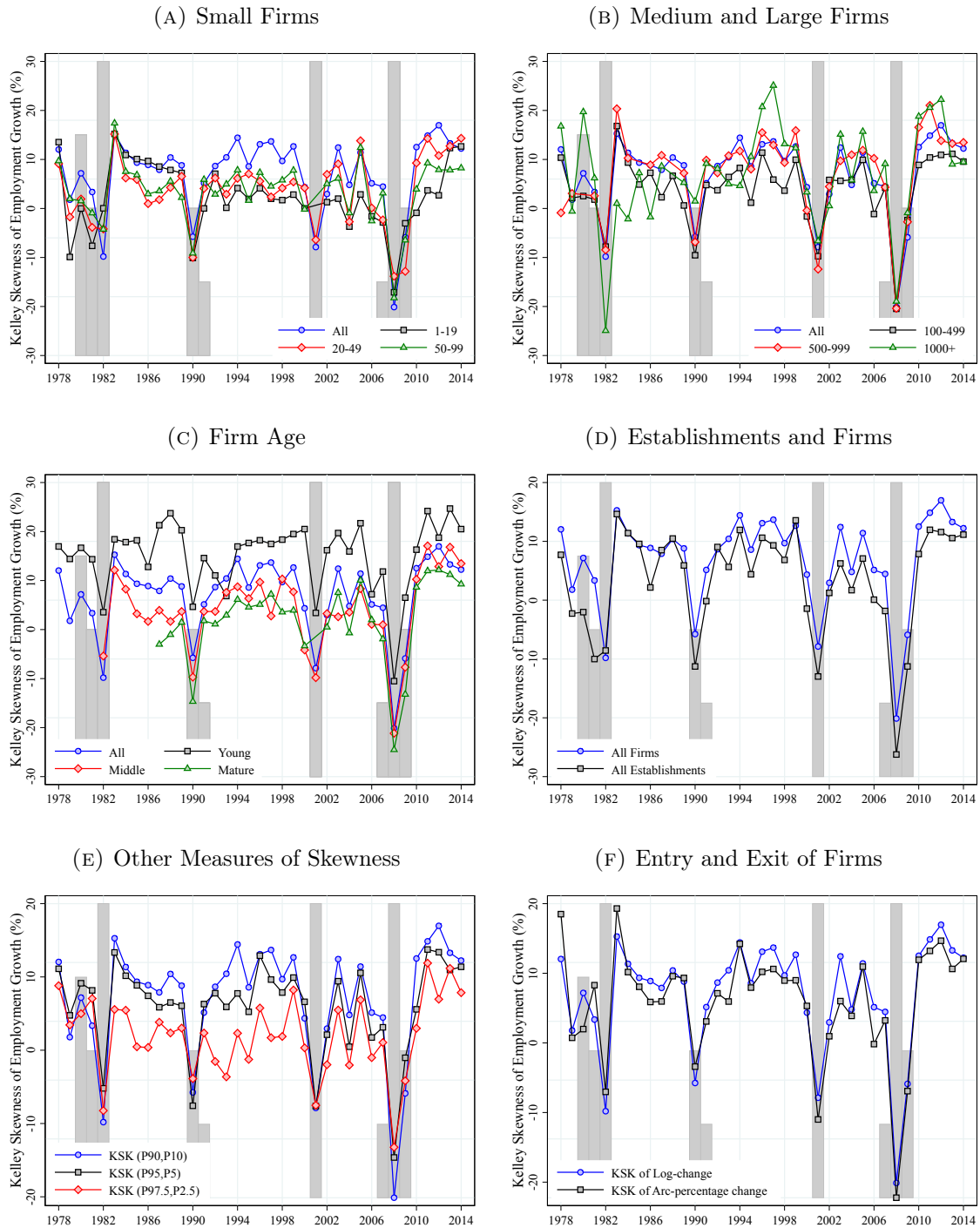
Note: The top panel of Figure 5 displays a binned scatter plot showing the relation between the within-industry business cycle, measured by the average growth rate of employment, and the within-industry skewness, measured by the Kelley skewness of firms' employment growth constructed from the LBD. Each dot is a quantile of the industry-year distribution of average employment growth, where an industry is defined by a two-digit NAICS group. Moments are weighted by the average firm employment. Binned scatter plots controlling for industry and time fixed effects. The slope coefficient is equal to 1.99 and is statistically significant at 1%. The bottom panel shows the same statistics for sales growth distribution from Compustat. The slope coefficient is 1.33. See Appendix A for details on the sample construction and moment calculations in the LBD and Compustat.

FIGURE 6 – AMADEUS: THE SKEWNESS OF FIRM’S PRODUCTIVITY SHOCKS IS CYCLICAL



Note: Figure 6 displays a binned scatter plot showing the relation between the skewness of firms’ productivity shocks within a country-industry-year cell, and the average productivity shock within the same cell constructed from the BvD Amadeus dataset. Each dot is a quantile of the industry-year distribution of average TFP growth, where an industry is defined by a two-digit NAICS group. To create this figure, we winsorize the distribution of Kelley skewness and average growth at the top and bottom 1%. Binned scatter controlling for industry, country, and time fixed effects. See appendix B for additional details on the sample selection and the TFP estimation procedure.

FIGURE 7 – ROBUSTNESS USING CENSUS DATA



Note: Figure 7 is based on aLBD firm-level data. The top panels show the Kelley skewness of the distribution of firms' employment growth within different firm size groups. The center left panel shows the skewness of the distribution of firm-employment growth within different firm age groups. Young firms are those less than five years, middle-aged firms are those between six and ten years old, and mature firms are those of more than ten years old. Firms already in the sample in 1976 are not considered in any of these groups. Shaded bars represent the share of the year (in quarters) declared as recession years by the NBER. All moments weighted by average employment at the firm or establishment level. See Appendix A for details on the sample construction and moment calculations in the LBD.

TABLE I – DATA AND SAMPLE CHARACTERISTICS

Source	Country	Sample Period	Frequency	Comments
Census Longitudinal Business Database	United States	1978-2015	Annual	Employment for entire nonfarm private sector
Compustat	United States	1970-2017	Quarterly	Employment, Sales, and Stock Prices for publicly traded firms
BvD Osiris	Several countries	1986-2015	Annual	Employment and Sales for publicly traded firms across 39 countries
BvD Amadeus	Several countries	1999-2018	Annual	Sales, Employment, and TFP for large and small firms across 7 European countries
Global Compustat	Several Countries	1970-2017	Daily	Stock Prices for publicly traded firms across 29 countries

Note: Table I shows the list of datasets and time-frame used in the analysis. Data availability varies country by country. See Table A.1 in the Appendix for a complete list of countries and available data.

TABLE II – THE SKEWNESS OF FIRMS OUTCOMES IS LOWER DURING RECESSIONS

Dependent Variable:	Kelley Skewness of the Growth Rate of Firms' Outcomes								
	United States			Cross-Country			Cross-Industry		
Sample:	(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)	(9)
Outcome:	Emp.	Sales	Stock Price	Emp.	Sales	Stock Price	Emp.	Sales	Stock Price
$\Delta GDP_{i,t}$	4.64*** (1.45)	5.47*** (1.07)	2.09** (1.03)	5.39*** (1.47)	3.19*** (1.05)	2.11** (0.88)	6.62*** (1.18)	13.24*** (1.39)	1.35** (0.51)
$\Delta S_{j,t}$									
R^2	0.32	0.23	0.07	0.27	0.38	0.41	0.28	0.40	0.24
N	39	47	184	701	720	2,428	1,045	1,046	4,133
Period	1976-2014	1970-2017	1970-2016	1991-2015	1991-2015	1970-2017	1970-2017	1970-2017	1970-2016
Freq.	Yr	Yr	Qtr	Yr	Yr	Qtr	Yr	Yr	Qtr
F.E.	-	-	-	Yr/Ctry	Yr/Ctry	Qtr/Ctry	Yr/Ind	Yr/Ind	Qtr/Ind
Source	LBD	CSTAT	CSTAT	BvD	BvD	GCSTAT	CSTAT	CSTAT	CSTAT
Sample	-	231K	650K	357K	633K	5,800K	231K	231K	733K

Note: The left panel of Table II shows a set of time series regressions for the United States in which the dependent variable is the Kelley skewness of the distribution of one-year firms' employment growth from the LBD (column 1), one-year sales growth distribution (column 2), and one-year stock returns (column 3), from Compustat (CSTAT). In each regression, the independent variable is the annual growth rate of GDP per capita. LBD moments are weighted by firm size measured by the average employment of the firm between years t and $t+1$. All regressions include a linear trend. Newey-West standard errors in parentheses below the point estimates. Columns (4) to (6) show a set of country-panel regressions in which the dependent variable is the Kelley skewness of firm-level outcomes. Employment and sales data come from BvD Osiris dataset, and stock returns are from Global Compustat (GCSTAT). In each regression, the independent variable is the growth rate of annual GDP per capita. Regressions include year and country fixed effects. Columns (7) to (9) show a series of industry panel regressions in which the dependent variable is the Kelley skewness of the within-industry distribution of firms' outcomes. In each regression, the independent variable is the average sales growth within the industry. The row labeled Sample shows the underlying sample of firms used to calculate the cross-sectional moments. Sample size in LBD not disclosed. * $p < 0.1$, ** $p < 0.05$, *** $p < 0.01$.

TABLE III – CALIBRATED PARAMETERS

Preferences and Technology		
γ	0.45	Frisch elasticity of labor supply
ψ	2.5	Leisure preference, non-entrepreneurs spend 1/3 time working
σ	2.0	Risk aversion, non-entrepreneurial sector
$1/\lambda$	1/5	Elasticity of intertemporal substitution
ξ	6.0	Risk aversion
β	$0.95^{0.25}$	Annual discount factor of 95%
r	0.005	Annual return of risk-free asset of 2%
α	0.25	CRS production, markup of 33%
ν	0.50	CRS labor share of 2/3, capital share of 1/3
δ	3.8%	Annual depreciation of capital stock of 14.4%
ρ_a	0.95	Quarterly persistence of aggregate productivity
σ_a	0.75%	Standard deviation of innovation of aggregate productivity
ρ	0.95	Quarterly persistence of idiosyncratic productivity
Adjustment costs		
ϕ_1	1.50%	Fixed cost of changing capital stock
ϕ_2	7.0	Quadratic cost of changing capital stock
ϕ_3	34.0%	Resale loss of capital

Note: Table III shows the calibrated parameters referring to preferences, technology, and adjustment costs.

TABLE IV – RISK PROCESS MOMENTS

	$P90 - P10$	$P90 - P50$	$P50 - P10$	KSK	Yrs
Data					
Low-Risk	0.54	0.30	0.24	0.10	03-06;10-14
High-Risk	0.70	0.31	0.39	-0.11	01,08
$\Delta(H - L)$	0.16	0.01	0.15	-0.20	-
Model					
Low-Risk	0.48	0.27	0.20	0.15	-
High-Risk	0.58	0.26	0.32	-0.10	-
$\Delta(H - L)$	0.10	-0.01	0.12	-0.25	-

Note: The top panel of Table IV shows cross-sectional moments of the annual growth rate of quarterly sales from Compustat for low-risk periods—quarters in the years 2003 to 2006 and quarters in the years 2010 to 2014—and high-risk periods—quarters in years 2001 and 2008. Quarters in years 2002 and 2009 are discarded for not representing full recession years. The model moments, shown in the lower panel of Table IV, are calculated from a 5,000-quarter simulation with the first 500 periods discarded.

TABLE V – TARGETED MOMENTS FOR NUMERICAL COMPARISON

	$P9010$	$P9050$	$P5010$	KSK
Low-Risk	0.54	0.30	0.24	0.10
High-Risk	0.70	0.31	0.39	-0.10
Only Skewness	0.54	0.243	0.297	-0.10
Only Variance	0.70	0.39	0.31	0.10

Note: Table V shows the target used in the estimation of the firm-level productivity process. Rows labeled “Low-Risk” and “High-Risk” are used in the baseline estimation. The values for “Only Skewness” are used to estimate the parameters when the economy is shocked with a change in the skewness only. Similarly, the values for “Only Variance” are used to estimate the parameters when the economy is assumed to be shocked only by a change in the variance of firms’ shocks while keeping the skewness constant.

TABLE VI – PARAMETERS OF THE STOCHASTIC PROCESS

Parameter of Idiosyncratic Stochastic Process		
σ_1^L	1.45	Standard deviation of first mixture in low-risk periods (%)
σ_2^L	7.55	Standard deviation of second mixture in low -risk periods (%)
μ^L	-0.92	Mean of first mixture in low-risk periods (%)
p^L	63.67	Probability of first mixture in low-risk periods (%)
σ_1^H	4.37	Standard deviation of first mixture in high-risk periods (%)
σ_2^H	9.06	Standard deviation of second mixture in high-risk periods (%)
μ^H	1.98	Mean of first mixture in high-risk periods (%)
p^H	78.28	Probability of first mixture in high-risk periods (%)
Transition Probabilities of Risk States		
π_L	0.97	Quarterly probability of remaining in low risk state
π_H	0.84	Quarterly probability of remaining in high-risk state

Note: The top panel of Table VI shows the parameters of the stochastic process of firm-level productivity. We target moments of the annual change of quarterly sales in Compustat. The parameters for low-risk periods (denoted by an upper script L) are obtained by targeting the P90-P10, P90-P50, P50-P10, and Kelley skewness of the sales growth distribution for all the expansion years between 2000 and 2014. The parameters for high-risk periods (denoted by an upper script H) are obtained by targeting the same set of moments for years 2001 and 2008 (full recession years). The transition probability π_L is calculated as the share of expansion quarters that were followed by another expansion quarter, whereas π_H is calculated as the share of recession quarters that were followed by another recession quarter using data from 1970 to 2014.

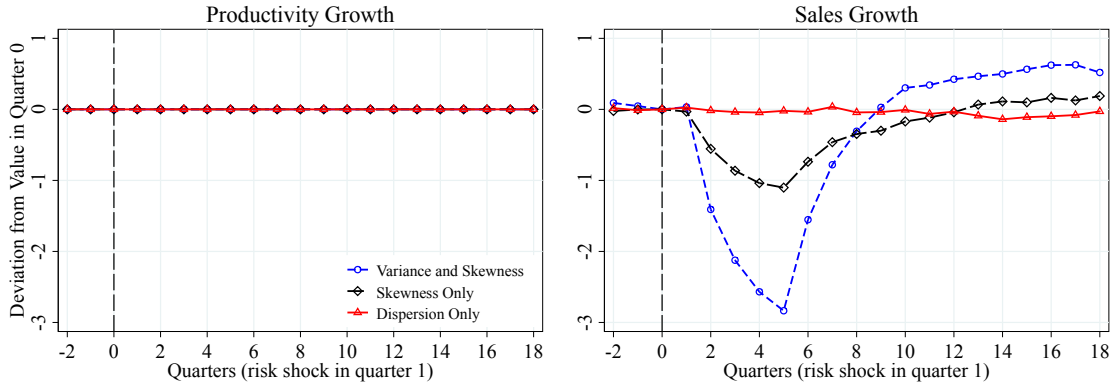
TABLE VII – BUSINESS CYCLE STATISTICS

	Data			Model		
	$\sigma(x)$	$\sigma(y)/\sigma(x)$	$\rho(x,y)$	$\sigma(x)$	$\sigma(y)/\sigma(x)$	$\rho(x,y)$
Output	1.47	1.00	1.00	2.00	1.00	1.00
Capital Investment	6.86	4.64	0.91	9.38	4.69	0.30
Consumption	1.21	0.82	0.87	1.81	0.91	0.65
Hours	1.89	1.28	0.87	2.00	1.00	1.00

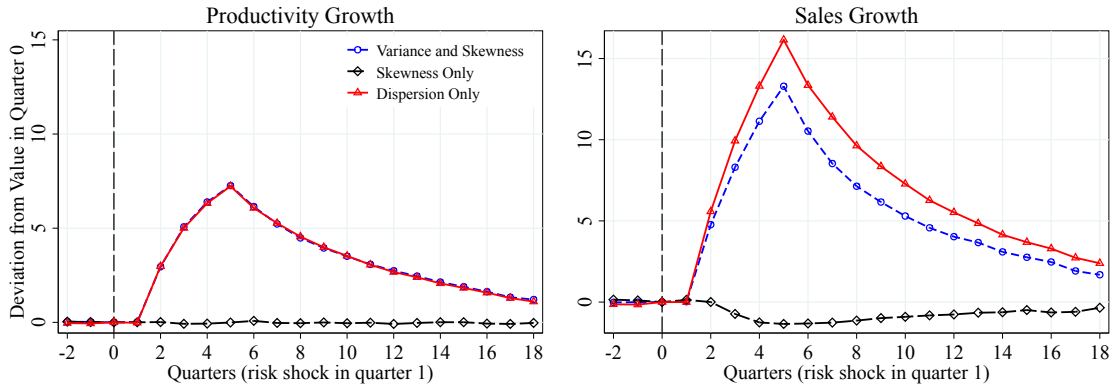
Note: The left panel of Table VIII displays business cycle statistics for quarterly US data covering 1970Q1 to 2017Q4. The column $\sigma(x)$ is the standard deviation of the log variable in the first column. The column $\sigma(y)/\sigma(x)$ is the standard deviation of the variable relative to the standard deviation of log output. All business cycle data are current as of February 3, 2019. Output is real gross domestic product (FRED GDPC1), investment is real gross private domestic investment (FRED GPDIC1), consumption is real personal consumption expenditures (FRED PCECC96), and hours is total non-farm business sector hours (FRED HOANBS). The second panel contains business cycle statistics computed from a simulation of the model of 5,000 quarters with the first 500 periods discarded. All series are HP-filtered with smoothing parameter 1,600, in logs expressed as percentages.

FIGURE 8 – PRODUCTIVITY AND SALES GROWTH AFTER AN INCREASE IN RISK

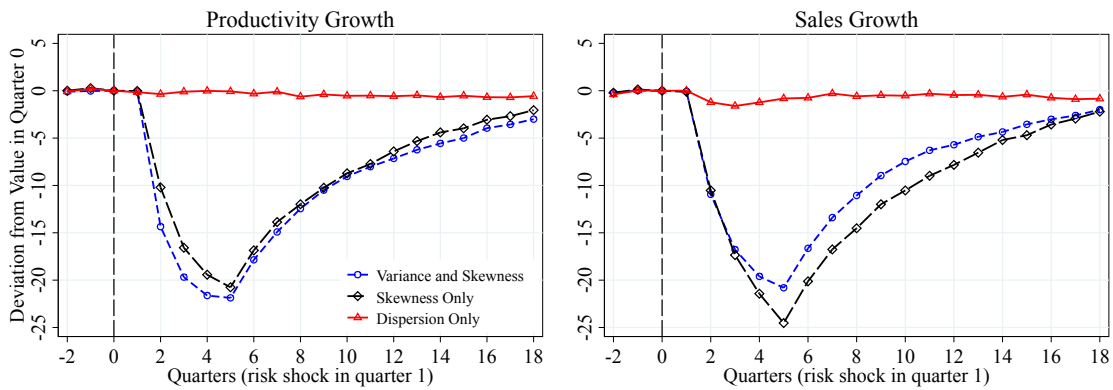
(A) Average



(B) P90-P10

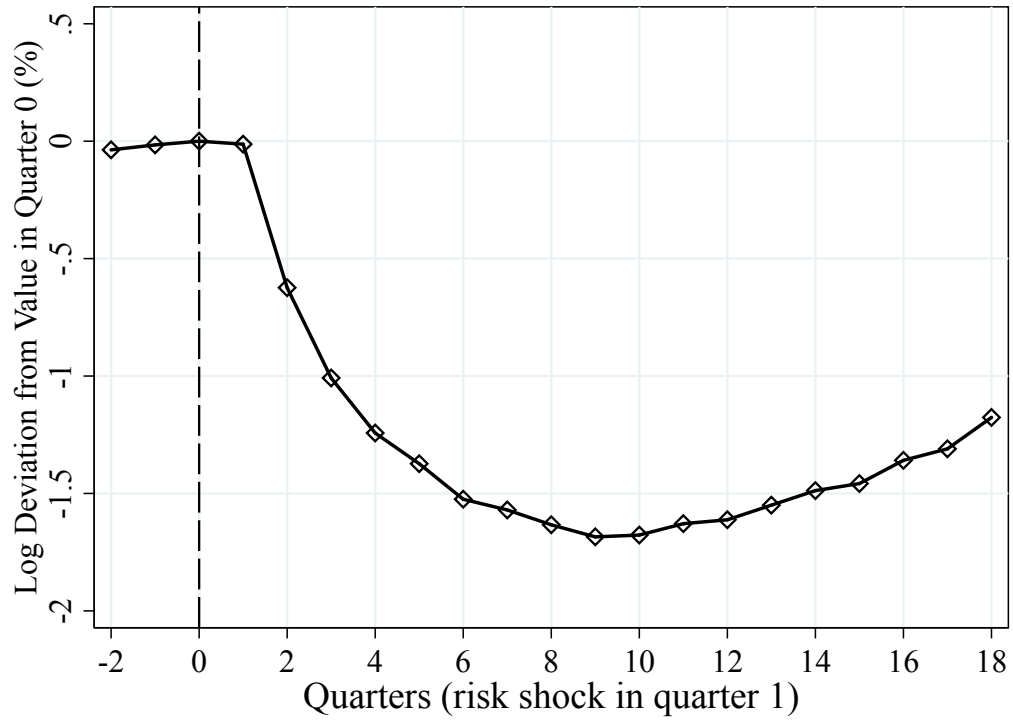


(C) Kelley skewness



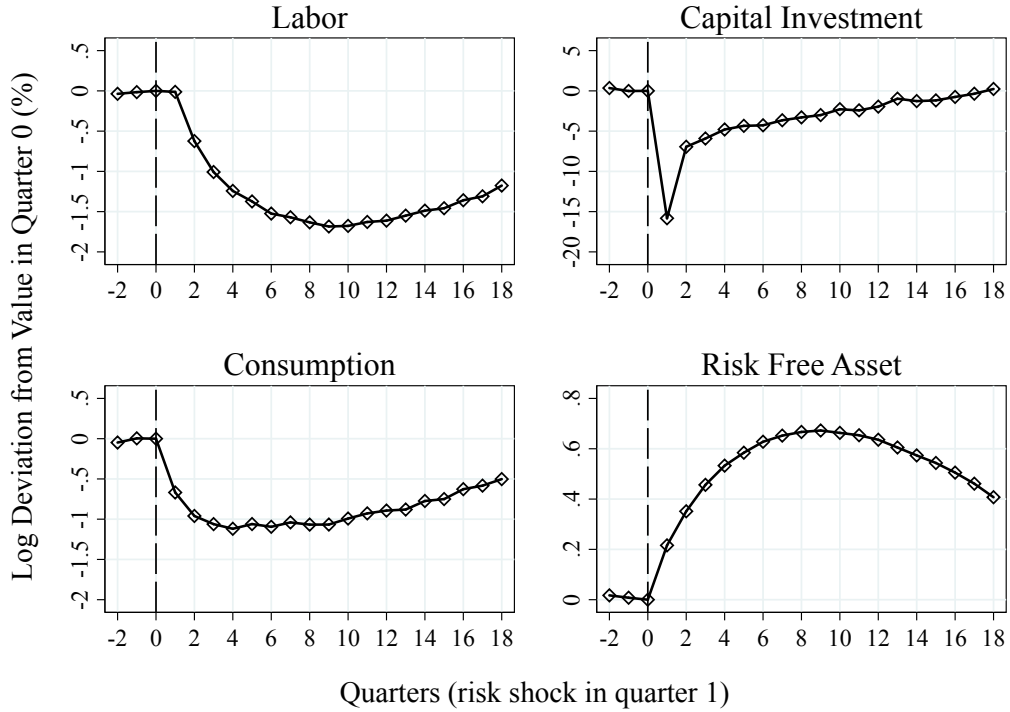
Note: The top left panel of Figure 8 shows the average of the one-year productivity growth distribution ($\Delta e_{j,t} = e_{j,t} - e_{j,t-4}$), whereas the top right shows the average of the one-year sales growth distribution ($\Delta y_{j,t} = \log y_{j,t} - \log y_{j,t-4}$) for different risk shocks. The middle and bottom panels show the dispersion and skewness. Each plot is based on independent simulations of 1,000 economies of 300-quarter length. We impose a decline in the skewness in quarter 1, allowing normal evolution of the economy afterward. We plot the deviation relative to the moment value in quarter 0.

FIGURE 9 – EFFECT OF SKEWNESS SHOCK ON OUTPUT



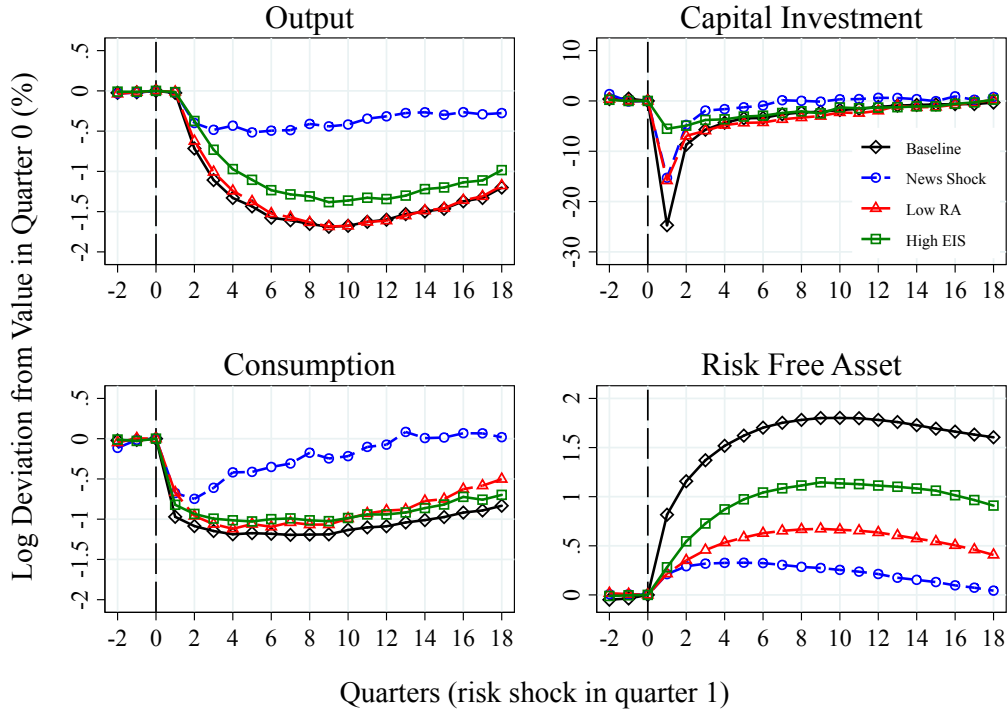
Note: Figure 9 shows the effect of a decline in the skewness of firm idiosyncratic productivity. The plot is based on independent simulations of 1,000 economies of 300-quarter length. We impose a decline in the skewness in quarter 1, allowing normal evolution of the economy afterward. We plot the percentage deviation of output from its value in quarter 0.

FIGURE 10 – EFFECT OF SKEWNESS SHOCK ON MACRO AGGREGATES



Note: Figure 10 shows the effect of a decline in the skewness of firm idiosyncratic productivity. Each plot is based on independent simulations of 1,000 economies of 300-quarter length. We impose a decline in the skewness in quarter 1, allowing normal evolution of the economy afterward. We plot the percentage deviation of each macroeconomic aggregate from its value in quarter 0.

FIGURE 11 – EFFECT OF SKEWNESS SHOCKS UNDER DIFFERENT PARAMETERIZATION



Note: Figure 11 shows the effect of a skewness shock (black line with diamonds) and the effect of a shock that only represents news about the future conditions of skewness without a change in the realizations of the idiosyncratic shocks. Each plot is based on independent simulations of 1,000 economies of 300-quarter length. We impose a drop in the skewness in quarter 1, allowing normal evolution of the economy afterwards. We plot the percentage deviation of each macroeconomic aggregate from its value in quarter 0. Labor is omitted since it follows the same pattern of output.

A Appendix: Data Sources and Variable Construction

This appendix describes the data sources and sample selection. Firm-level data for the United States comes from the Census Bureau’s Longitudinal Business Database (LBD) and Compustat. For the cross-country comparisons, we use firm-level data available in the Bureau van Dijk’s Osiris database and Global Compustat. The online appendix and replication packet—available on the authors’ websites—contains further details of the construction of the sample and moments calculation.

A.1 United States: Longitudinal Business Database

We construct measures of employment growth at the firm-level using the Census Bureau’s LBD. The LBD covers the universe of establishment in the nonfarm private sector in the United States from 1976 to 2015. It provides detailed establishment and firm-level information on employment, payroll, location, firm age, industry, legal form of organization, and so on. Crucially, firm and establishment identifiers in the LBD allow us to construct measures of employment growth at different time horizons. From the LBD, we select a sample of establishments that, in a given year, have nonnegative, non-missing employment and payroll and have valid industry data. We then sum up the employment within the same firm to construct an annual measure of employment. We measure the growth rate of employment of firm j in period t as the log-difference between periods t and $t+k$, $g_{j,t}^e = \log E_{j,t+k} - \log E_{j,t}$ where $k \in \{1, 3, 5\}$ and by the arc percentage change between the same periods.

Calculating the Kelley skewness requires the computation of specific percentiles of the distribution of employment growth. Notice that a percentile provides information about a particular firm, which violates the disclosure criteria imposed by the Census Bureau. Hence, to avoid the disclosure of any sensitive information, we calculate the p th percentile of the employment growth distribution as the employment-weighted average on a band of +1 and -1 percent centered on p th. For instance, the 90th percentile of the distribution is the weighted average of the employment growth across all observations between the 89th and 91st percentiles of the distribution, both ends included. We proceed in the same way to construct the 10th and 50th percentiles of the distribution and use these values to calculate the Kelley skewness. All measures are weighted by the average employment of the firm between periods t and $t+k$, that is, $\bar{E}_{j,t} = 0.5 \times (E_{j,t+k} + E_{j,t})$. The massive sample size of the LBD ensures that the sample used to calculate each of the percentiles is large enough to have an accurate approximation to the actual quantiles of the distribution.

We also use the LBD to compare the distribution of employment growth between recession and expansion years using a kernel density estimation. The sample selection is the same used in the rest of our results; however, the Census Bureau requires to drop the bottom and top 5% of the distribution. The kernel densities presented in Figure 3 were calculated over the remaining sample.

A.2 United States: Compustat

For the United States, we construct time series of the cross-sectional dispersion and skewness of the sales growth distribution and the distribution of stock returns. To construct the time series of the sales growth distribution we proceed as follows. We begin by retrieving firm-level

data of net sales and stock prices at the annual and quarterly frequency, and employment at the annual frequency, from Compustat from 1964q1 to 2017q4 accessed through the Wharton Research Data Services (WRDS).

The raw dataset of sales (Compustat variable `saleq`) and stock prices (Compustat variable `prccq`) contains more than 1.7 million quarter-firm observations with an average of approximately 4,660 firms per quarter. We drop all observations with negative sales, repeated observations, and firms incorporated outside the United States (we keep observations with Compustat variable `fic` equal to “USA”). We also drop all observations that do not have a SIC classification or with a classification above 90. Then, we deflate nominal sales by the CPI (FRED series `CPIAUCSL`), and we calculate the growth rate of sales as the log-difference and the arc percentage change between quarter t and $t + k$ with $k \in \{4, 12, 20\}$. This leaves us with around 1 million sales growth (log-difference) observations. For our main results, we consider firms with at least 10 years of data on quarterly sales (40 quarters, not necessarily continuous), which further reduces the sample to 819,977 observations between 1970q4 and 2017q2, with an average of 5,359 firms per quarter. Finally, in each quarter we calculate different cross-sectional moments discussed in the main body of this document. Our main sample considers firms with at least 10 years of data (40 quarters), although our results remain robust if we drop this restriction or if we consider firms with 25 years of data. When accounting for entry and exit of firms using the arc-percentage change, for each firm we add an observation upon entry (equal to 2) and one additional observation upon exit (equal to -2) under the assumption that before entering and after exit, the firm has a value of sales or employment equal to 0. We consider entry firms as newly listed firms, while exiting firms are those delisted in a particular period, independent of the reason (M&A, bankruptcy, or any other).

For our result at the annual frequency, we follow a similar sample selection. The raw annual dataset contains 500,004 year/firm observations. We drop all observations with negative sales and duplicated entries, with missing SIC classification or two digit SIC above 90. We deflate nominal variables using CPI (FRED series `CPIAUCSL`) and we calculate the growth rate of sales (Compustat variable `sale`) and employment (Compustat variable `emp`) as the log change between year t and $t + k$ with $k \in \{1, 3, 5\}$. This leaves us with 266,192 firm/year observation (sales growth) between 1970 and 2016, with an average of 5,663 firms per year. Our main sample considers only firms with at least 10 years of data (not necessarily continuous) but our results remain robust if we drop this restriction or if we consider firms with at least 25 years of data. When accounting for entry and exit of firms using the arc-percentage change, for each firm we add an observation upon entry (equal to 2) and one additional observation upon exit (equal -2) under the assumption that before and after exit, the firm has a value of sales or employment equal to 0. We consider entry firms as newly listed firms, while exiting firms are those delisted in a particular period, independent of the reason (M&A, bankruptcy, or any other). We complement this data with macroeconomic series from FRED (real gross domestic product per capita, FRED series `A939RX0Q048SBEA`).

A.3 Cross-Country: BvD Osiris and Global Compustat

Cross country firm-level panel data on sales and employment come from the Bureau van Dijk’s Osiris database. Osiris is a database of listed public companies, commodity-producing firms, banks, and insurance companies from over 190 countries. The combined industrial com-

pany dataset contains financial information for up to 20 years and 80,000 companies. In our analysis, we focus on the industrial dataset.

The raw dataset contains 977,412 country/firm/year observations from 1982 to 2018. We then drop all observations with missing or negative sales, all duplicated entries, and all firms with missing NAIC classification. We transform all observations into US dollars using the exchange rate reported in the same database. Then, we deflate nominal sales using US annual CPI and calculate the growth rate of real sales as the log change and arc percentage change between years t and $t+k$ with $k \in \{1, 3\}$. This leaves us with 748,574 observations (log change of sales). We further restrict the sample to firms with more than 10 years of data; country/year cells with more than 100 observations; countries with more than 10 years of data; and years with more than 5 countries. This sample selection reduces the dataset to an unbalanced panel of 678,563 observations in 45 countries between 1989 and 2015. We complement this data with real GDP in US dollars from the World Bank's World Development Indicators database.

The data on daily stock prices come from the Global Compustat database (GCSTAT), which provides standardized information on publicly traded firms for several countries at annual, quarterly, and daily frequencies. The raw data contain firm-level observations of daily stock prices between 1985 and 2018 for 48 countries. We drop all duplicated observations and drop all firms with less than 2000 observations (firms with approximately 10 years of data). Then we calculate daily price returns as the log-difference of the stock price between two consecutive trading days. We apply a similar sample selection, keeping firms with at least 10 years of daily price data. The total sample contains an unbalanced panel of 44 countries from 1985 to 2017 from which we drop all country quarters with less than 100 firms. The final data contains a total of 29 countries from 1985 to 2017. Then, within each quarter, we calculate the cross-sectional moments of the daily stock price distribution. We complement this dataset with per capita GDP growth from the World Bank's World Development Indicators and quarterly GDP growth from the OECD Stats. Table A.1 shows the list of countries available in our dataset and the data available for each country.

TABLE A.1 – DATA AVAILABILITY BY COUNTRY

Source:	Osiris		Global Compustat		Amadeus		Osiris		Global Compustat		Amadeus	
	Sales	Emp	Stock	Emp	Sales	Emp	Sales	Emp	Stock	Emp	Sales	Emp
ARG	x	x			IRN		x	x				
AUS	x	x	x		ISL						x	x
AUT				x	ISR	x	x	x	x			
BEL	x	x	x	x	ITA**	x	x	x	x	x	x	x
BLR				x	JPN	x	x	x	x			
BMU	x	x			KOR	x	x	x	x			
BRA	x	x	x		MEX	x	x	x				
CAN	x	x			MYS	x	x	x				
CHE	x	x	x	x	NLD	x	x	x	x	x	x	x
CHL	x	x	x		NOR	x	x	x	x	x	x	x
CHN	x	x			NZL				x			
DEU**	x	x	x		PAK	x	x	x				
DNK	x	x	x	x	PER	x	x	x				
EGY	x	x			PHL	x	x	x				
ESP**	x	x	x	x	POL				x		x	x
FIN**	x	x	x	x	PRT**						x	x
FRA**	x	x	x	x	RUS	x	x	x	x			
GBR	x	x	x	x	SGP	x	x	x				
GRC	x	x	x	x	SWE**	x	x	x	x		x	x
HKG	x	x			THA	x	x	x				
HUN					TUR	x	x	x	x			
IDN	x	x	x		UKR						x	x
IND	x	x	x		USA*	x	x	x	x			
IRL			x		ZAF	x	x	x	x			

Note: Table A.1 shows data available for each country (identified by its isocode). GCSTAT refers to Global Compustat. Notice the data of Global Compustat contains stock price data for almost 100 countries. We keep only those countries with at least 100 firm-level observations per year with data of quarterly GDP growth, which reduces the number of countries to 28. *We obtain data for the United States from Compustat and the LBD. ** Countries for which we measure TFP at the firm-level.

B Calculating TFP from BvD Amadeus

In this appendix, we describe in detail the construction of our measure of firm-level TFP using data from Amadeus. We consider a small set of countries, namely, Portugal, France, Italy, Germany, Spain, Finland, and Sweden, for which firm-level information is available for enough industries and sectors.³⁴

B.1 Variables and Sample Selection

For each country in the sample, we retrieve firm-level panel data from Amadeus through WRDS. Our data contains a large range of firms, from small to very large firms (V+L+M+S: plus Small Companies dataset), both publicly traded and privately held. The main variables we use in our analysis are the following (Amadeus names of variables in parenthesis):

- Sales (TURN),
- Operating revenues (OPRE),
- Employment (EMPL),
- Cost of Employees (STAF),
- Cost of Material (MATE),
- Total Fixed Assets (FIAS),
- Industry (NAICS and SIC codes),
- Exchange rate from local currency to Euros (EXCHANGE2).

In order to estimate firm-level productivity for a large number of firms within each country, we perform a simple sample selection. For each country, we drop duplicates, observations without information on industry (NAICS) and with discrepancies between the country identifier and the firm identifier (INDR).³⁵ We also drop all observations with missing, zero or negative values in either of the following variables: OPRE, MATE, FIAS, and STAF. We also drop all observations with zero or negative value of $VA = OPRE - MATE$ which is our measure of value added.

We deflate all monetary values by the country-specific CPI (obtained from the World Bank). Firms in Sweden report information in their local currency, which we transform to Euros using the exchange rate also reported by Amadeus.

B.2 Estimating TFP

The literature has used several different methods to measure TFP at the firm-level. Assuming the production function at the firm level is Cobb-Douglas, the goal is to obtain firm-level productivity, $z_{i,j,t}$, as the residual of the following equation,

$$\log y_{i,j,k,t} = \alpha_K \log K_{i,j,k,t} + \alpha_L \log E_{i,j,k,t} + z_{i,j,k,t}, \quad (9)$$

³⁴See [Kalemli-Ozcan et al. \(2015\)](#) for additional details on the Amadeus dataset.

³⁵The first two characters in the firm identifier in Amadeus refer to the country.

where $y_{i,j,k,t}$ is the value added of firm i , in industry j , in country k , in year t , $K_{i,j,k,t}$ is the deflated measure of fixed asset, and $E_{i,j,k,t}$ is a measure of labor input (employees or wage bill).

We use four different methods to estimate $z_{i,j,t}$. The first method, described in section uses country-industry factor shares to estimate α_L and α_K . In particular, we calculate the total wage bill and total value added at the country-industry-year level. Industries are defined by two-digit NAICS. To ensure our measure of factor shares is calculated with enough firms, we restrict our estimates to years in which the country-industry-cell contains more than one hundred observations and periods with more than five sectors within a country-year. We then obtain the labor as

$$\alpha_{j,k,t} = \frac{\sum_{i \in I_{j,k,t}} w_{i,j,k,t}}{\sum_{i \in I_{j,k,t}} y_{i,j,k,t}},$$

where $I_{j,k,t}$ is the set of firms in the industry-sector-year cell and $w_{i,j,k,t}$ is the cost of employees at the firm-level (STAF). Then, we calculate the capital share as $\alpha_{K,j,k,t} = 1 - \alpha_{L,j,k,t}$.³⁶ We then apply these factor shares in equation 9 to obtain our first measure of productivity as a residual.

In the second method, we obtain $z_{i,j,t}$ as the residuals of a firm-level OLS panel regression. In order to control for firm differences in labor quality, we replace the labor input measure for a measure of the wage bill (STAF) at the firm level. We then run, an OLS panel regression to obtain $\hat{z}_{i,j,k,t}$ for each country.

The third approach uses the methodology developed by [Olley and Pakes \(1996\)](#) to estimate $\hat{z}_{i,j,k,t}$. This method has stricter data requirements and therefore, we further restrict our within-country sample to firms with information about investment expenditure (change in the value of total fixed assets, FIAS), and firms with at least 5 years of data. Furthermore, because the data available in BvD Amadeus was increasingly populated until 2005, we consider information only after that year.³⁷

The fourth method abstracts from capital differences across firms and proxies a measure of labor productivity. In particular, we obtain labor productivity as the residual of the following equation estimated using OLS within each country

$$\log y_{i,j,k,t} = \tilde{\alpha}_L \log E_{i,j,k,t} + \mu_i + \tilde{z}_{i,j,k,t}. \quad (10)$$

Then, for each of the fourth productivity measures and within each country, we estimate firm level productivity shocks as the residual of the following OLS panel regression

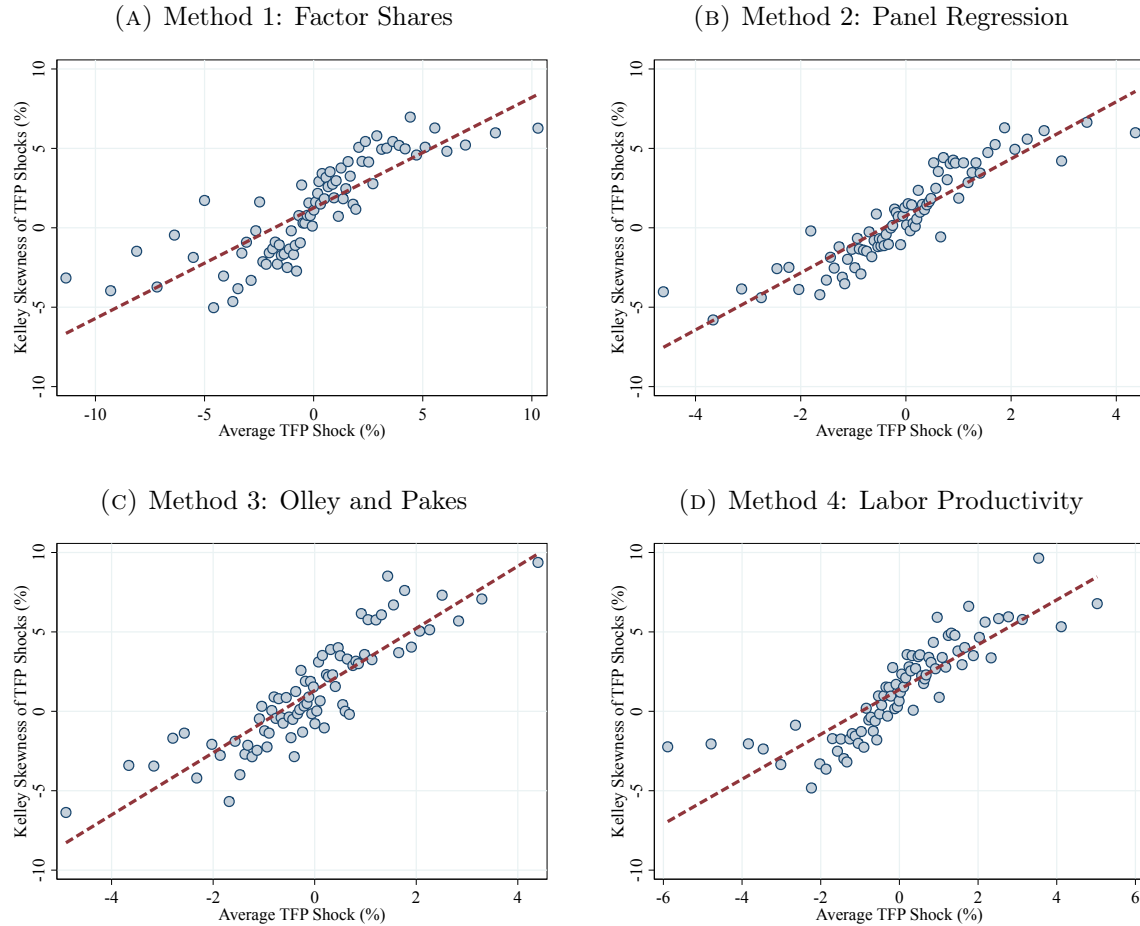
$$\hat{z}_{i,j,k,t} = \beta_{0,k} + \beta_{1,k} \hat{z}_{i,j,k,t-1} + \mu_i + \delta_t + \epsilon_{i,j,k,t}, \quad (11)$$

where μ_i and δ_t are firm and year fixed effects respectively. For each country, we winsorize each measure of productivity shock at the top and bottom 1% and we calculate different moments of the distribution of productivity shocks. Additionally, we use the average of the real sales growth within a bin as a measure of business condition at the country-industry-year.

³⁶In this calculation, we use the nominal values of value added and cost of employees.

³⁷To obtain the [Olley and Pakes \(1996\)](#) estimates we use the Stata command OPREG implemented by [Yasar et al. \(2008\)](#).

FIGURE A.1 – NEGATIVE SKEWNESS IN COUNTRY-SECTORS WITH BELOW AVERAGE SHOCK



Note: Figure 7 is based on a sample of firms from BvD Amadeus. Each plot shows a binned scatter plot polling information across all countries, industries, and years in the sample. In each plot, the y-axis is the Kelley skewness of the within country-industry-year distribution of firm productivity shocks whereas the x-axis is the average productivity shock with the same cell. Productivity shocks are calculated using four different methods: Factor shares (top left plot), panel regression (top right plot), Olley and Pakes (bottom left), and labor productivity (bottom right). The slopes (standard errors) sorted from top left to bottom right are the following: 0.69 (0.04), 1.79 (0.07), 1.97 (0.11), and 1.41 (0.07). To create this figure, we winsorize the distribution of Kelley skewness and average growth at the top and bottom 1%. Binned scatter plots controlling for country, industry, and time fixed effects.

For each measure of productivity shock we calculate the average shock within a country-industry-year bin and different percentiles of the distribution. To ensure our results are not driven by outliers, after we have obtained these percentiles, we further trim the measures of Kelley skewness and the average productivity shocks at the top and bottom 1% and we restrict our sample to country-industry-year bins with more than 100 firms. Dropping this additional sample restriction does not change our results substantially.

B.3 Additional Evidence on the Skewness of Productivity Shocks

As we discussed in Section 3.4, the skewness of productivity shocks is robustly negative during periods of low economic activity at the country-industry level. Here we show some

TABLE A.2 – POSITIVE CORRELATION OF SALES GROWTH AND SKEWNESS OF FIRM’S SHOCKS

	(1)	(2)	(3)	(4)
Estimation Method:	Factor Shares	Panel Regression	Olley and Pakes	Labor Productivity
Ave. Sales Growth	0.23*** (0.05)	0.28*** (0.04)	0.31*** (0.05)	0.22*** (0.06)
R^2	0.12	0.16	0.11	0.11
N	1,773	1,773	1,652	1,773

Note: Table A.2 shows a set of country-industry panel regressions in which the dependent variable is the Kelley skewness of firm productivity shocks calculated using the four different methods described in Section B.2. In all regressions, the explanatory variable is the average sales growth within the same bin. All regressions control for country, industry, and year fixed effects. Standard errors (below the point estimates) are clustered at the country level. * $p < 0.1$, ** $p < 0.05$, *** $p < 0.01$.

additional robustness results. Figure A.1 shows that the positive relation between the skewness of the productivity shocks and the business conditions is robustly positive, independently of the estimation method one uses to calculate firm-level productivity. For comparison, the top left panel repeats the results shown in Figure 6.

Comparing the slopes in the plots in Figure A.1 is difficult to appreciate whether some measures of productivity lead to more cyclical measures of skewness of TFP shocks since each plot has a different x-axis. In order to have a more direct comparison between the skewness across different estimation methods, Table A.2 shows a series of panel regressions in which the independent variable is the skewness of TFP shocks for the four methods described in Section B.2 and the main regressor is the average of the real sales growth (log change of operative revenues) within a country-year-industry cell. The coefficient associated to the average sales growth is positive and statistically significant in all cases and of the same order of magnitude. This indicates that in periods in which industries experience a decline in sales, the skewness of the productivity shocks affecting the firms in that industry is negative as well. In unreported results, we further shows that this positive correlation is independent of the estimation method used to calculate firm-level productivity.

Finally, Table A.3 shows that the skewness of firm’s shocks is procyclical at the country level. In particular, we show the results of an industry-panel regression for each country in our sample using the average TFP shock as our measure of business condition. The results are indicative that the procyclicality of the skewness of firm shocks is not driven by any particular country in our sample. And it is stronger in countries such as Germany and France.³⁸

³⁸Similar to the results presented in Table A.2, our replication materials show that for each country, sales growth is positively correlated with the skewness of firm’s TFP shocks.

TABLE A.3 – SKEWNESS OF FIRM’S SHOCKS IS PROCYCLICAL AT THE COUNTRY LEVEL

	(1)	(2)	(3)	(4)	(5)	(6)	(7)
	Germany	Spain	Finland	France	Italy	Portugal	Sweden
Ave. TFP Shock	4.96*** (1.05)	1.88*** (0.20)	2.49*** (0.33)	3.29*** (0.54)	1.26*** (0.21)	1.86*** (0.22)	2.80*** (0.40)
R^2	0.53	0.66	0.58	0.59	0.59	0.56	0.54
N	174	345	211	316	272	210	217

Note: Table A.3 shows a set of industry panel regressions in which the dependent variable is the Kelley skewness of firm productivity shocks. Firm-level productivity was calculated as the residuals of a firm-level panel regression (the second method described in section B.2). In each column, the independent variable is the average TFP shock within an industry. Each regression includes a set of industry and time fixed effects. Standard errors (below the point estimates) are clustered at the industry level. * $p < 0.1$, ** $p < 0.05$, *** $p < 0.01$.

C Appendix: Additional Robustness Results

TABLE A.4 – SKEWNESS IS LOWER DURING INDUSTRY RECESSIONS

Kelley Skewness of Three-Year Growth Rate of Firm's Outcomes						
Sample:	United States			Cross-Industry		
	(1)	(2)	(3)	(7)	(8)	(9)
Source	CSTAT	LBD	CSTAT	CSTAT	CSTAT	CSTAT
	Sales	Emp.	Stock Price	Sales	Emp.	Stock Price
$\Delta GDP_{i,t}$	2.89** (1.32)	1.14** (0.50)	3.96*** (1.12)			
Industry Growth				5.92*** (1.60)	6.02*** (1.67)	0.70 (1.17)
R^2	0.01	0.15	0.10	0.12	0.11	0.09
N	182	35	180	3,652	930	3,602
Freq.	Qtr	Yr	Qtr	Qtr/ Yr	Yr	Qtr
F.E.	N	N	N	Qtr/Ind	Yr/Ind	Qtr/Ind
Sample	640K	-	650K	780K	193K	651K

Note: Table A.4 shows a series of industry-level panel regressions. In each column, the dependent variable is the cross-sectional Kelley skewness of the growth rates of real quarterly sales, annual employment growth, and quarterly stock returns distribution within period-industry cells defined by two-digit NAICS (total of 22 industries) for a sample of publicly traded firms from the Compustat dataset. The independent variable, $\Delta \bar{S}_{j,t}$, is the average of the sales growth distribution within the period-industry cell. LBD moments were calculated weighting by firm-size. In all regressions, the sample period is 1970 to 2017, and we consider a full set of period and industry fixed effects. The row labeled Sample corresponds to the total firm-period observations used to calculate the cross sectional moments. N corresponds to the number of period-industry observations used in the regressions. Standard errors in parentheses below the point estimates are clustered at the NAIC-2 industry level. * $p < 0.1$, ** $p < 0.05$, *** $p < 0.01$.

TABLE A.5 – DISPERSION OF FIRM'S OUTCOMES IS HIGHER DURING RECESSIONS

		90th-to-10th Percentiles Spread of the Growth Rate of Firms' Outcomes							
		United States			Cross-Country				
		(1)	(2)	(3)	(4)	(5)	(7)	(8)	(9)
		Firm Sales		Stock Returns		Firm Emp.		Firm Stock	
		One Year	Three Year	One Year	Three Year	One Year	Growth	Returns	Firm Emp. Growth
$\Delta GDP_{i,t}$		-3.91*** (1.14)	2.55** (0.99)	-3.93** (1.62)	-4.78*** (1.78)	0.93* (0.50)	-0.79 (0.59)	-1.84 (1.79)	-0.76 (0.74)
N		184	182	180	180	39	838	4,306	824
Freq.		Qtr	Qtr	Qtr	Qtr	Yr	Yr	Qtr	Yr
F.E.		N	N	N	N	N	Yr/Ctry	Qtr/Ctry	Yr/Ctry
Source		CSTAT	CSTAT	CSTAT	CSTAT	LBD	BvD	GCSTAT	BvB

Note: The left panel of Table A.5 shows a series of time series regressions in which the dependent variables are the 90th-to-10th percentiles spread of the one-year and three-year growth rate of sales growth (columns 1 and 2), stock returns (columns 3 and 4), and employment growth (columns 5 and 6) for a sample of firms from Compustat (columns 1 to 4) and the LBD (columns 5 and 6). Compustat data cover the period 1970 to 2017 whereas LBD data covers the period 1976 to 2015. In each regression, the independent variable is the annual growth rate of quarterly GDP per capita. All regressions include a linear trend. Newey-West standard errors in parentheses below the point estimates. The right panel of Table A.5 shows a series of country-panel regressions where the dependent variable is the within-country P90-P10 spread of firm-level sales growth, stock returns, or employment growth. The independent variable is the growth rate of GDP per capita at the country level. Sales and employment data are obtained from the BvD Osiris database, whereas stocks returns are obtained from Global Compustat. All cross-sectional moments were calculated weighting growth rate observations by firm size. All regressions consider a full set of time and country fixed effects. The row labeled Sample shows the underlying sample of firms used to calculate the cross-sectional moments. LBD sample size is not disclosed. * $p < 0.1$, ** $p < 0.05$, *** $p < 0.01$.

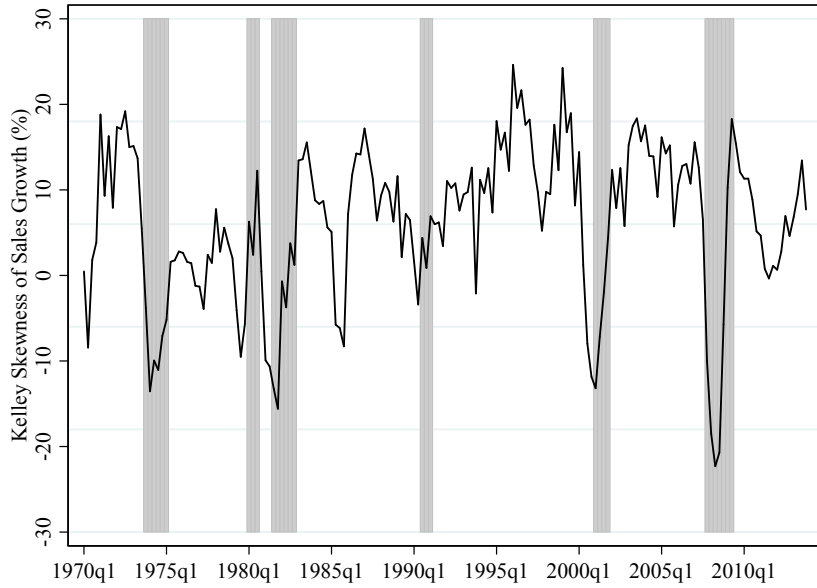
TABLE A.6 – HIGHER ORDER MOMENTS OF FIRM’S OUTCOMES

	Kelley Skewness				Crow-Siddiqui Kurtosis				
	(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)	(9)
	Residual Sales One Year	Three Years	Sales per Employee One Year	Three Years	Sales Deviation	Firm Sales One Year	Three Years	One Year	Three Years
$\Delta GDP_{i,t}$	3.80*** (1.42)	1.480 (0.91)	3.01** (1.23)	4.17*** (0.97)	1.46** (0.58)	0.36*** (0.08)	-0.19 (0.12)	0.16*** (0.06)	-0.21* (0.13)
N	178	178	174	166	47	184	182	180	180
Freq.	Qtr	Qtr	Qtr	Qtr	Yr	Qtr	Qtr	Qtr	Qtr
Sample	500K	500K	500K	500K	113K	640K	640K	650K	650K
Source	CSTAT	CSTAT	CSTAT	CSTAT	CSTAT	CSTAT	CSTAT	CSTAT	CSTAT

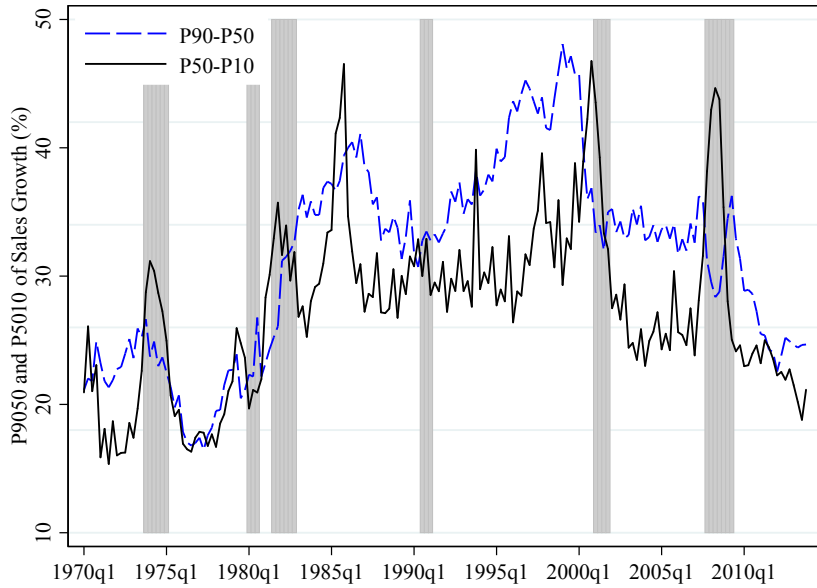
Note: The left panel of Table A.6 shows a series of time series regressions for the United States in which the dependent variable is the Kelley Skewness of the one-year and three-year growth rate of residualized sales growth (columns 1 and 2) and the growth rate of sales per employee (columns 3 and 4) for a sample of firms from Compustat. In columns (1) and (2), we have orthogonalized the growth rates of sales from time fixed-effects, firm-fixed effect, size, and other firm-level observable characteristics. Column (5) shows the correlation of GDP growth and the cross-sectional skewness of the deviation of annual firms’ sales from an HP trend. Compustat data cover the period 1970 to 2017. The dependent variable in columns (6) to (9) is the Crow-Siddiqui measure of kurtosis defined as $CKU_t = \frac{P_{97.5_t} - P_{02.5_t}}{P_{75_t} - P_{25_t}}$. In each regression, the independent variable is the annual growth rate of quarterly GDP per capita. All firm-level moments were calculated weighting the growth rate observations by firm size measured by the average sales of the firm between periods t and $t+k$. All regressions include a linear trend. Newey-West standard errors in parentheses below the point estimates. * $p < 0.1$, ** $p < 0.05$, *** $p < 0.01$.

FIGURE A.2 – THE SKEWNESS OF FIRM-LEVEL QUARTERLY SALES GROWTH IS PROCYCLICAL

(A) Compustat: Skewness of Sales Growth Distribution



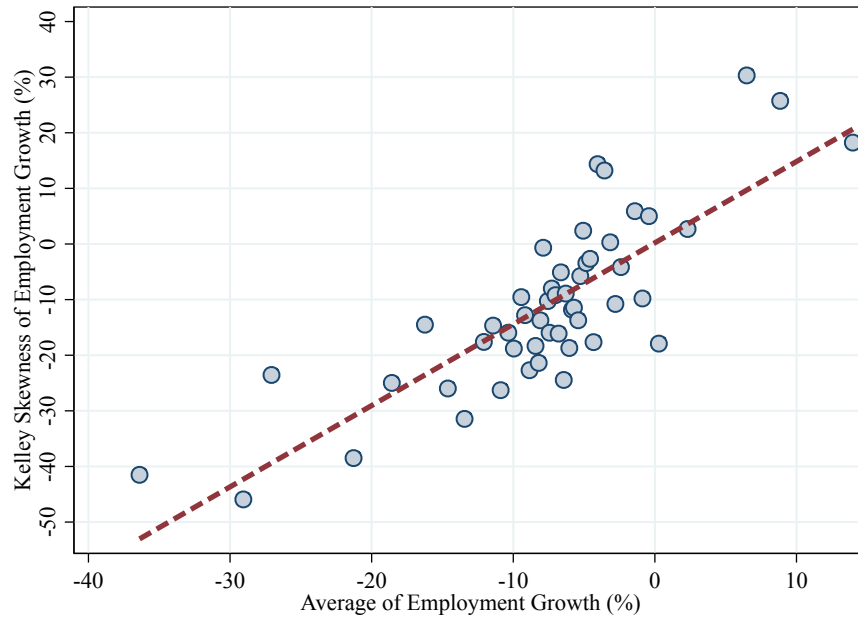
(B) Compustat: Upper and Lower Tail Dispersion of Sales Growth



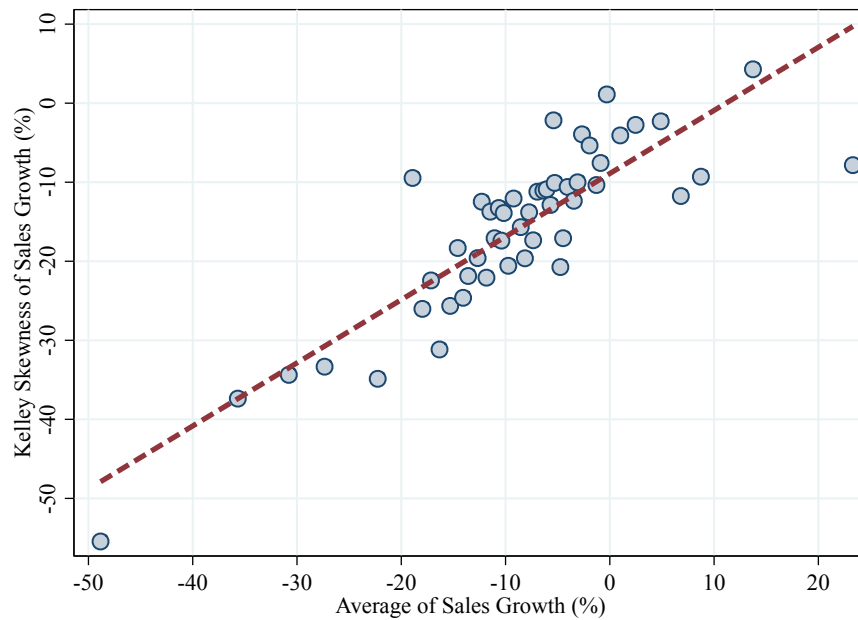
Note: The top panel of Figure A.2 shows the time series of the cross-sectional Kelley skewness of the distribution of firm quarterly sales growth for a sample of firms from LBD. Moments are weighted by the average firm employment size between years t and $t + 1$. The bottom panel of Figure A.2 shows the time series of the cross-sectional Kelley skewness of the distribution of firm quarterly sales growth for a sample of publicly traded firms from Compustat. The shaded bars represent NBER recession periods. See Appendix A.1 for additional details on the sample construction and moment calculations in the LBD and Compustat.

FIGURE A.3 – PROCYCLICAL SKEWNESS IS ROBUST TO THE INCLUSION OF PRIVATE FIRMS

(A) Amadeus: Firm-Level Employment Growth



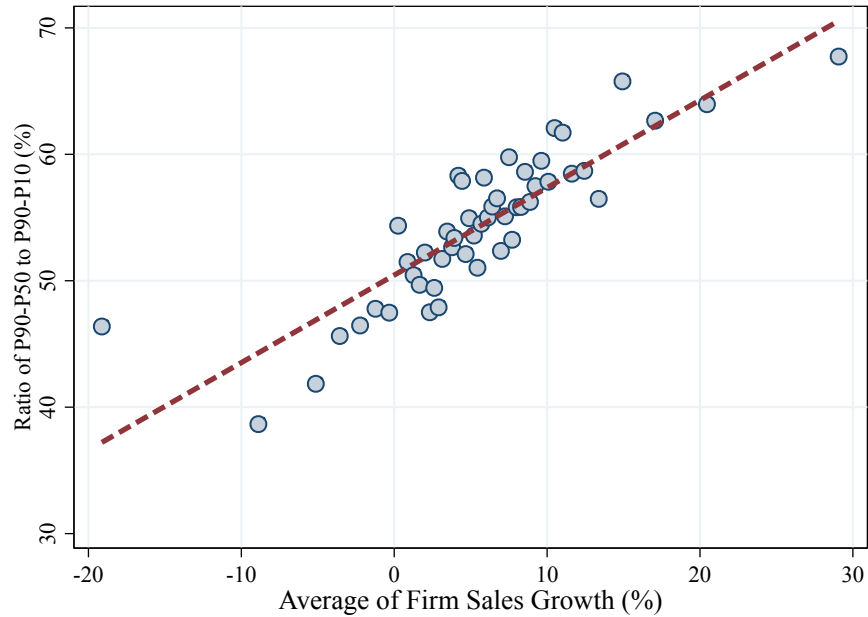
(B) Amadeus: Firm-Level Sales Growth



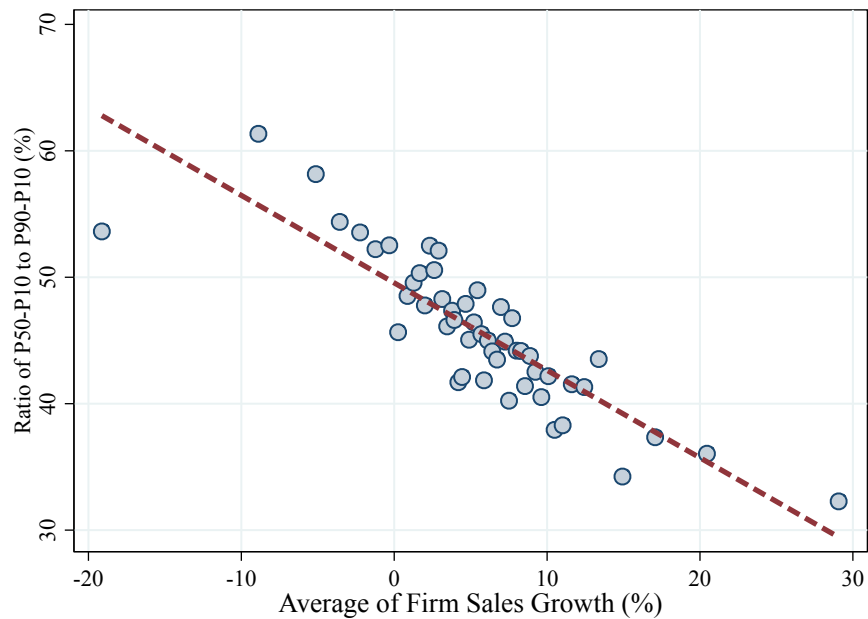
Note: Figure A.3 shows binned scatter plots of the Kelley skewness and average employment and sales growth. The figure is based on an unbalanced panel of firms from the BvD Amadeus database in the following European countries: AUT, BEL, BLR, CHE, DEU, DNK, ESP, FIN, FRA, GBR, GRC, HUN, IRL, ISL, ITA, NLD, NOR, POL, PRT, SWE, UKR. The data cover years 2000 to 2015. BvD Amadeus contains private and publicly traded firms. We apply the same selection criteria we use for the rest of the data.

FIGURE A.4 – RIGHT AND LEFT TAIL DISPERSION AND INDUSTRY CYCLE

(A) Share of Right Tail on Total Dispersion

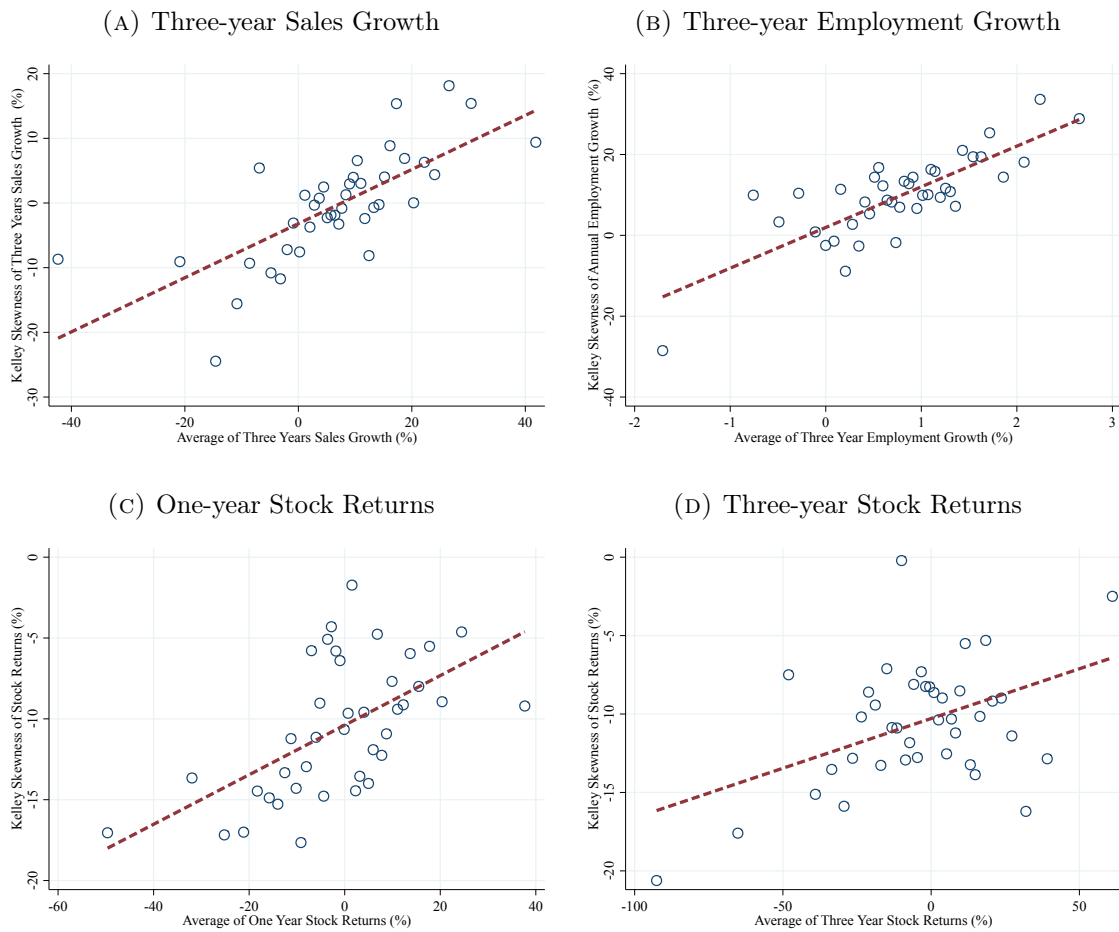


(B) Share of Left Tail on Total Dispersion



Note: The top panel of Figure A.4 displays a binned scatter plot showing the relation between the within-industry business cycle, measured by the average growth rate of sales growth, and the within industry dispersion of sales growth constructed from Compustat data. The upper panel shows the share of total dispersion (measured by the 90th to 10th percentiles differential) accounted for by the dispersion above the median (measured by the 90th to 50th percentiles differential). The bottom panel shows the share of dispersion accounted for by the left tail (measured by the 50th to 10th percentiles differential).

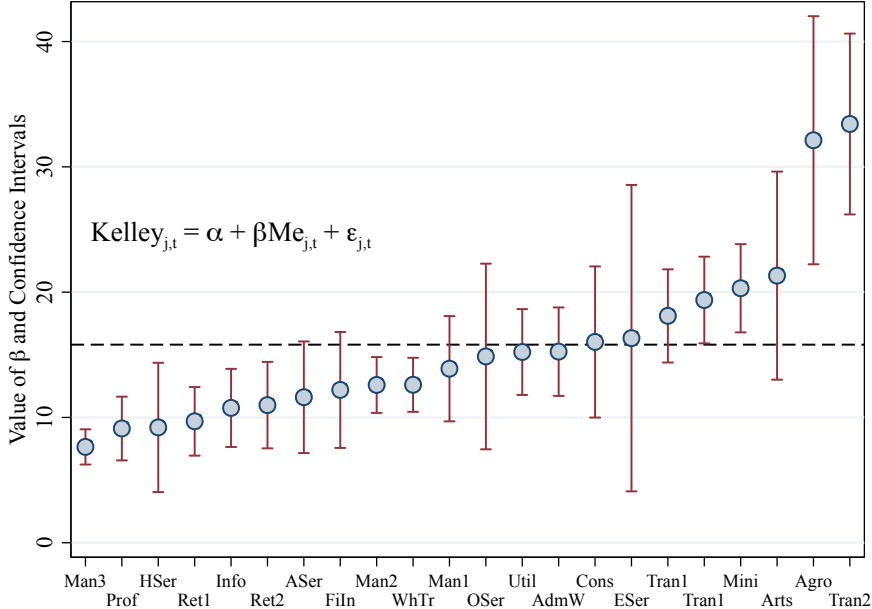
FIGURE A.5 – THE SKEWNESS OF FIRM’S OUTCOMES IS LOWER DURING INDUSTRY CYCLES



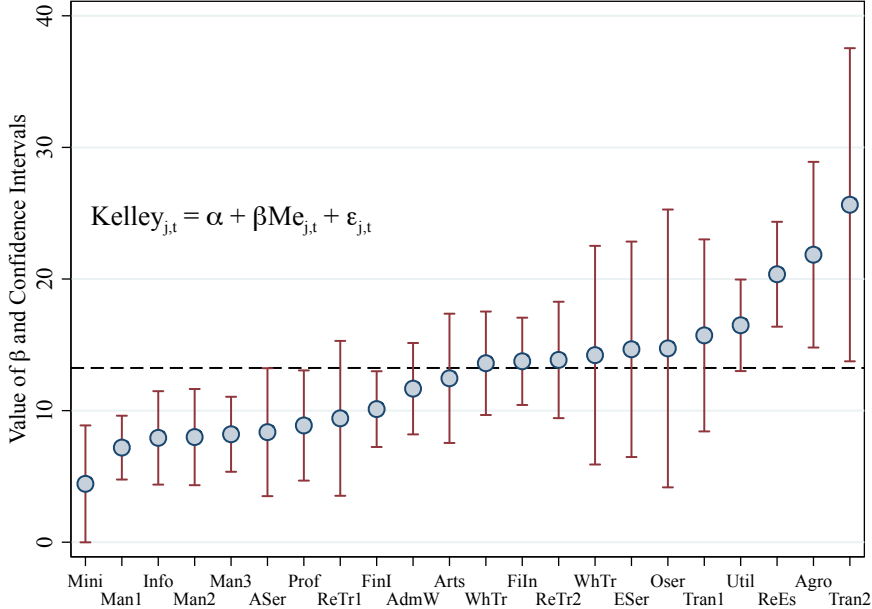
Note: The top left panel of Figure A.5 displays a binned scatter plot showing the relation between the within-industry business cycle, measured by the average growth rate of sales, and the within-industry skewness, measured by the Kelley skewness of sales growth for a sample of Compustat firms. Each dot is a quantile of the industry-year distribution of average sales growth. The rest of the plots show similar statistics for employment growth and stock returns.

FIGURE A.6 – THE SKEWNESS FIRMS’ OUTCOMES IS LOWER DURING WITHIN-INDUSTRY CYCLES

(A) Industry: Firm-Level Employment Growth

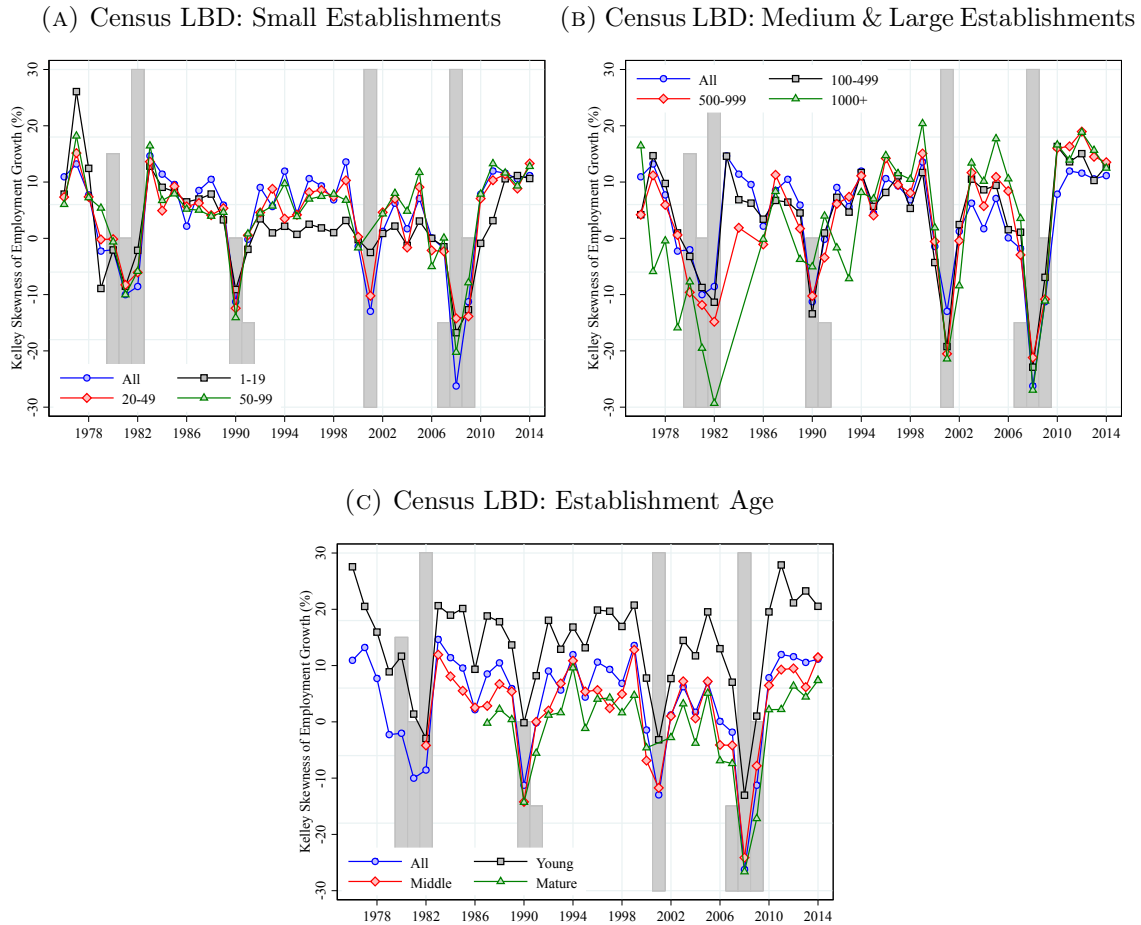


(B) Industry: Firm-Level Sales Growth



Note: Figure A.6 shows the coefficients and confidence intervals for within-industry regression of the cross-sectional Kelley skewness on the average growth of employment (top panel) and sales (bottom panel) for a sample of publicly traded firms from Compustat. Each industry regression includes a linear trend. Confidence intervals are calculated at 95% of significance. Industries are defined as two-digit NAIC. In each plot, the dashed line is the coefficient of a panel regression of the within industry skewness and average firm growth controlling for time and fixed effect. See Appendix A.1 for additional details on the sample construction and moment calculations in Compustat.

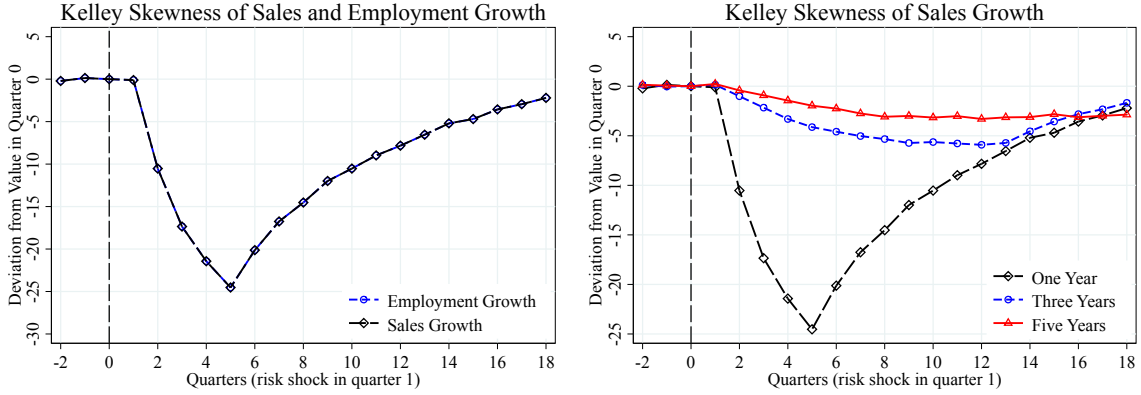
FIGURE A.7 – SKEWNESS OF EMPLOYMENT GROWTH DISTRIBUTION ESTABLISHMENT GROUPS



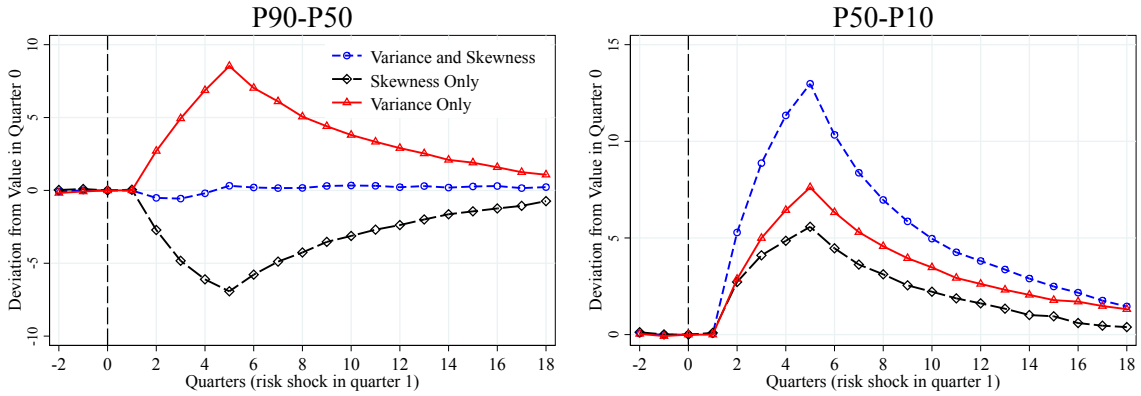
Note: Figure A.7 is based on the LBD. The top left and right panels show the skewness of the distribution of establishment-employment growth within different establishment size groups defined by establishment average employment calculated for each establishment i as $\bar{E}_{i,t} = 0.5 \times (E_{i,t} + E_{i,t+1})$; The bottom panel shows the skewness of the distribution of establishment-employment growth within different establishment age groups. Young establishments are those of less than five years, middle-aged establishments are those between six and ten years, and mature establishments are those of more than ten years old. Establishment already in the sample in 1976 were not considered in any of these groups. All moments weighted by establishment size defined by $\bar{E}_{i,t}$. In all plots, the blue line with circles is the skewness of employment growth for all establishments in the sample.

FIGURE A.8 – MODEL-GENERATED MOMENTS

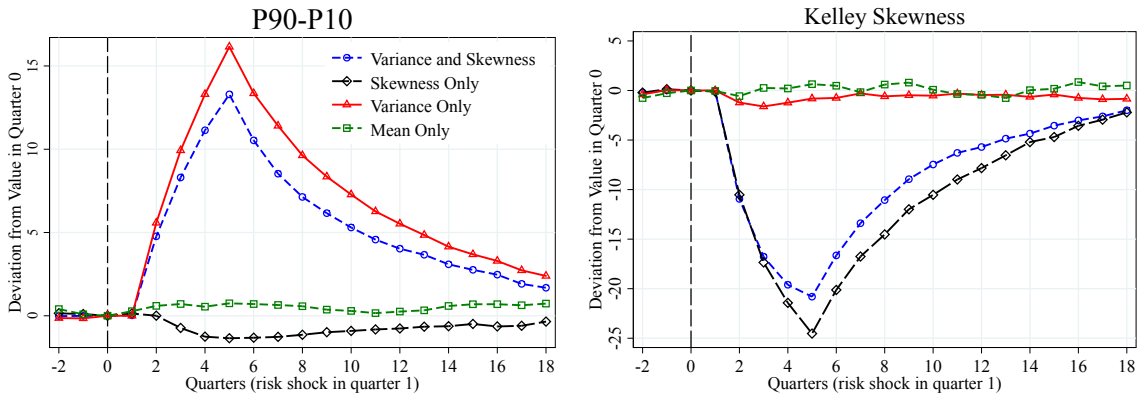
(A) Skewness of Employment and Sales Growth



(B) Right and Left Tail Dispersion of Sales Growth



(C) Aggregate Productivity Shock does not Affect Dispersion or Skewness of Sales Growth



Note: Figure A.8 shows different model-generated moments of the sales growth and employment growth distribution. Each plot is based on independent simulations of 1,000 economies of 300-quarter length. We impose a drop in the skewness, increase in variance, or both, in quarter 1, allowing normal evolution of the economy afterward. We plot the deviation of each macroeconomic aggregate from its value in quarter 0.