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Finite Sample Comparison of Parametric, Semiparametric, and Wavelet Estimators of Fractional Integration

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Sunnier, Denser and More Productive Cities

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Abstract

We set out an open, monocentric city with residential structures and reflect on how changes to an amenity index affects the city. On the consumption side an amenity shock is represented by an exogenous boost to the utility of a resident's current commodity bundle. The city's population, land rent and footprint expand and its density rises. We test for an amenity effect in local wages with household data for the US in 1990 and discover that city density is statistically much stronger in explaining local wage premia than is city population. We test for amenity effects in local house prices with the same data set.

- key words: climatic amenities, density, wages
- JEL Classification: R140, J610

1 Introduction

The resident of a sunny city is receiving an unmarketed consumption flow which should get capitalized in her inelastically supplied factor, here the land her house is on. This in turn should imply *ceteris paribus* that the resident is consuming marginally less land than her counterpart in a city with poorer climatic amenities and lower average land rents.[1] Hence the prediction that sunnier cities should be denser cities, other things being the same. But we find matters to be more complicated than this: we observe that density turns out to be, from an econometric standpoint, a very effective explanatory variable for the well-known, "big-city" wage premium. Thus a first-blush inference is that sunnier cities tend to be denser cities and since density drives a local labor productivity premium, sunnier cities are super-productive cities since they are also high wage cities. This line of argument may be relevant for cities such as Phoenix, Atlanta, Austin and others but leaves one begging for an explanation of high wages in say New York, Chicago and Seattle. Clearly New York's natural deep water port and location at the mouth of the Hudson River had much to do with its early growth and development. We would argue that New York had a special geographic amenity on the production side, namely its good quality port, which led to high worker productivity near the port and high wages in the city. High local wages led to high local land rents and relatively high densities of settlement which in turn drove an additional bit of high labor productivity. The extra productivity of a worker in a place of relatively high density has a variety of explanations including good fits of people with specialized human capital in appropriate jobs, interactions among workers for spreading useful knowledge about doing jobs and finding good niche employment for their skills, interactions among R&D people

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[1] Shapiro [2006; p. 325] takes these ideas as uncontroversial, though back in 1979, Rosen [1979] argued for consumption amenities to be negatively capitalized in local wages as in sunnier cities having lower wages. Shapiro addresses how higher levels of human capital in certain cities feeds into higher rates of growth of local land rent, housing and wages. When local human capital is measured by the number of local college graduates, Shapiro concludes: "evidence from wages and rents implies that, although the majority of the employment growth effect of college graduates operates through changes in productivity, roughly one-third of the effect seems to come from more rapid improvement in the quality of life." A large dimension of "quality of life" are "consumer city" amenities such as bars and restaurants.

that lead to useful inventions, and so on.

Here we analyze capitalization of superior climate variables in local land rent as well as superior "geography" in local wages and then to local land rent in an abstract, textbook model of a monocentric city. We then turn to data on US workers for 1990 and consider evidence for capitalization of local utility-based amenities in local wages. Worker characteristics are controlled for in a Mincer-type hedonic formulation and our attention is on how variables at the city level "filter through" or fail to "filter through" to local wages. We find evidence for no direct "filtering through" of superior utility-based amenities such as more local days of sunshine per year into local wages but we do find that local "higher" density of residents "filters through" to local wage premia. Hence the claim that city $i's$ wage premium can be explained by city $i's$ relatively high density of residents. Recall that Hall and Ciccione [1996] attributed higher local labor productivity to local "high" density in their analysis of labor productivity in US counties. Our work with a wage equation rather than a labor productivity equation seems to be yielding good support for the Hall-Ciccione view. For some cities in our sample of 94 US cities with over 100,000 people, the "extra" density could then be driven by utility-based amenities like above-average days of sunshine (many municipalities around Los Angeles come to mind) whereas for other cities the "extra" density could be driven by a geographic "amenity" on the production side, such as a port. Our data set, defined at the level of the worker, allowed us to carry out a rough test for "more sunshine" locally capitalized in local house prices and we found good support for this view. We proceeded to fine tune this test by dropping some large cities that were notable ports from our sample (eg. New York, Philadelphia, Los Angeles) to fine-tune this test somewhat.

Our wage equation work is obviously in the tradition of Rosen [1979], Roback[2] [1982] and Glaeser-Mare [2001] but our results come out rather differently. We work with a much larger sample (820,000 workers) than did Rosen and Roback and our specification of the Mincer part of our wage equation draws on the work of Lemieux [2000] and ends up somewhat different from the formulation in Glaeser-Mare. There are in addition differences in our econometric

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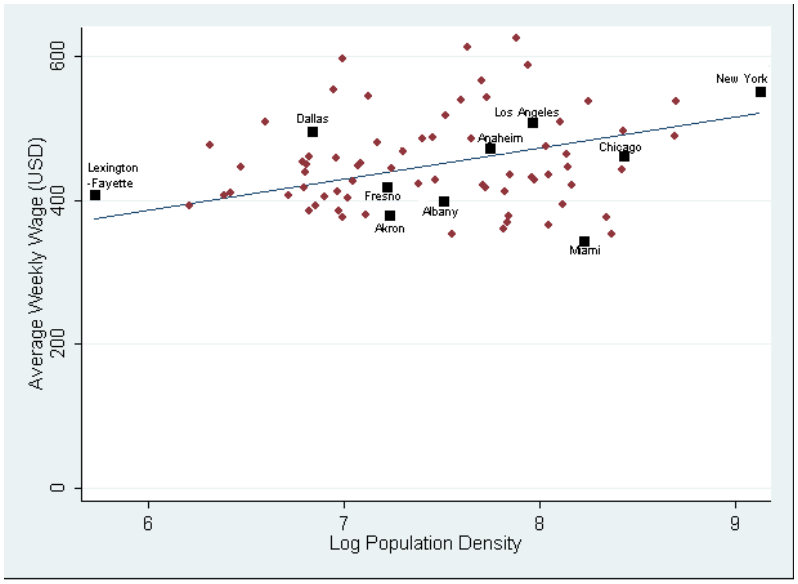
[2] Roback [1982; p.1275]: "This study has proven that the conventional wisdom which holds that only land prices are affected by local amenities is incorrect. The theory demonstrated that the value of the amenity is reflected in both the wage and the rent gradient. The precise decomposition depends on the influence of the amenity on production and the strength of consumer preferences."

approach to the empirical work, specifically the use of cluster analysis, a technique described in Wooldridge [2003]. The "cluster issue" involves tranches of our workers "slotted in" to specific cities and thus variation in say sunshine per city "hits" groups of workers the same. We are able to control for this cluster problem. We have then quite precise differences in our estimation strategy from those of Roback [1982] and Glaeser-Mare [2001] and these seem to account for our results being different from those reported in earlier studies.

We explore capitalization issues first in what we believe is the standard textbook model of the monocentric city. Our abstract city is assumed to be one of a large group of cities and thus we invoke "the open city" assumption, namely that the utility level of a household cannot change when say local wages change, or local sunshine hours, or local costs of commuting. The open city assumption makes the analysis of capitalization not too complicated to work through for our various hypothetical shocks, including an amenity shock such as more hours of sunshine. This theory-work sets the stage for our empirical work on the amenity content or lack thereof in local wages. Our finding on local density being an effective explanatory variable for local wage premia (sunnier cities are denser and denser cities exhibit wage premia) conforms in part with the interesting recent arguments of Ross DeVol, Armen Bedroussian, Kevin Klowden and Soojung Kim [2008] of the Milken Institute. We depict this finding in the figure below in a relationship between average weekly wage and local density.



Average Weekly Wage (USD = 1990) = 38.065(Log Population Density) + 165.91; $R^2 = 0.240$. Number of Observations = 94. Data from 1990 U.S Census, 5% Sample, IPUMS Database. Each observation is at the city level.

Researchers of the institute remark that in their set of the twenty-five best or most economically dynamic US cities, sixteen are in the south and nine are in the west, with California's one representative being Bakersfield. This suggests to us that natural amenities such a "more sunshine" can be driving growth. Growth driven by amenities is in our framework a disequilibrium phenomenon. We view the end of adjustment for a city to "more amenities" is a denser settled city. Brian Knudson, Richard Florida and Kevin Stolarick [2005] argue that density is a driver of the process of technical innovation in cities and present a statistical analysis to support their view.

Our basic model has the wage for a city emerge from the unit cost of its export good, given an infinitely elastic demand schedule facing the city for its export. The level of the wage is influenced in turn by a location-specific parameter in the production function. Davis and Weinstein [2002] refer to production amenities, such as a good port, as ”locational fundamentals” and argue that the spatial distribution of cities is strongly influenced by such amenities while city size can in addition be a positive function of scale economies in production. We buy into the Davis-Weinstein view but in addition interpret our location-specific parameter as a standard technical progress index number, a premium turning on say earlier R&D activity. Matters become complicated when the value of the parameter in question is considered to reflect current city size or current city density and we return to this issue later. We do argue that a city can experience a once-over shock of technical progress and in turn experience a jump in the local wage for a representative worker-resident.

2 The Model

The wage can be explained to emerge from the export sector where export good price $p^e = \zeta w^B i^{1-B}$, p^e and i parameters. "Amenity advantage" (eg. superior port) is a lower ζ and a larger w. Associated with a particular w could be a large city, given good climate, or a smaller city, given a poorer climate. ζ can also be viewed as a "level of technology". Then an decrease in ζ can be interpreted as technical progress.[3] Once again a decline in ζ (technical progress) corresponds to a rise in w and an increase in the size of the city.

We take as given each worker's utility function and utility level, $\overline{U}$ in a city. Migration

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[3] A more complicated production amenity would not be simply multiplicative.

arbitrage of similar workers insures that each household-worker achieves the same utility in a representative city and between two cities. A shock such as a local wage increase leads to a local jump in the utility being achieved and induces in-migration from surrounding cities. The in-flow of new residents pushes up the local rent schedule (house-price schedule) and this chokes off in-migration. The local jump in utility gets "washed out" by the in-migration. We work with a monocentric city with all production activity at a point in the center and commuting cost is t per unit distance. The rental rate on capital i is exogenous to the city as well. City size emerges from a city edge rent equal to the agricultural land rent, ρ, at the edge. That is, given values for w, t, i, ρ and $\overline{U}$, equilibrium is essentially an endogenous edge distance, $\overline{x}$ such that $r(\overline{x}; w, t, i, \overline{U}) = \rho$. Given $\overline{x}$, we can solve for the population (labor force), $N(\overline{x})$ which is the supply of labor to production, given an infinitely elastic demand at the center at wage, w.

Given our representative city with its wage and population in place, we consider the impact of amenity differences, such as changes in sunshine, traffic congestion, crime, etc. An amenity shift shows up in the local level of utility shifting up for say more sunshine on average. This draws in workers from surrounding cities with their utility levels unchanged and ultimately expands the city with the amenity fillip, pushes local rents upward and brings utility back to the initial level.[4] We move on to consider a shift in the local wage, via a shift in the location parameter in the production function for exports, and a shift in the local commuting cost parameter, t.

We summarize. A city in our system of cities is defined by its export sector

$$p^e = \zeta w^B i^{1-B},$$

with B, i, ζ and p^e as parameters and by its equilibrium size in

$$r(\overline{x}; w, t, i, \overline{U}) = \rho$$

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[4] Rosen and Roback [1982] developed the view that the amenity differences we are focusing on get capitalized in wage differences between cities. For example a location with a very good climate would have relatively low wages in the Rosen-Roback view. Our view is clearly quite different. In our view, wage levels are determined almost entirely in the production side of the economy while consumption amenity differences show up first in changes in the size of a household's commodity bundle currently consumed. Places with a better climate imply a contraction in the household's commodity bundle, *ceteris paribus.*

given a utility function for the representative household. The first equation defines the equilibrium wage and the second the equilibrium edge. We will proceed to consider comparative statics effects on a representative city in this framework. We turn to filling in some detail on the internal structure of a representative city.

In the Mills-Brueckner monocentric city a worker commutes from his home at radial distance x to the workplace at the center at roundtrip cost, tx. The wage at the center is w, leaving $w - tx$ to be spent on housing, $h(x)$ and the other consumer good, $c(x)$. That is, the budget constraint for a worker-household is $w - tx = pc(x) + q(x)h(x)$. A home comprises some capital (structure) and some land. That is, we have $h(x) = L(x)^A k(x)^{1-A}$ with i the fixed rental price of a unit of capital and $r(x)$ current land rent. Here $L(x)$ is the land for a house at radial distance x and $k(x)$ is the capital in the house at x. For $q(x)$ the price of a unit of house, we know that $q(x) = Mr(x)^A i^{1-A}$ for $r(x)$ land rent at x and i the rental of a unit of capital at x, with $M = A^{-A}(1-A)^{-(1-A)}$.

Each household has Cobb-Douglas utility, $U = c(x)^\alpha h(x)^{1-\alpha}$ where $c(x)$ is an amount of composite of other goods available at a constant unit price, p. The residential area is an annulus surrounding the CBD, with x radial distance from the center. Hence at $\widehat{x}$, a household spends income net of roundtrip commuting cost, $w - t\widehat{x}$ on "other", $pc(\widehat{x})$ and housing, $h(\widehat{x})q(\widehat{x})$.[5] Given the Cobb-Douglas form for the utility function, we have

$$\begin{aligned}(1-\alpha)[w - t\widehat{x}] &= h(\widehat{x})q(\widehat{x}),\\ \text{and } \alpha[w - t\widehat{x}] &= pc(\widehat{x}).\end{aligned}$$

We assume that our city in question belongs to a system of cities with equilibrium utility level $\overline{U}$ prevailing for like workers. All households are thus achieving for the moment the same utility level, $\overline{U}$. Hence $h(\widehat{x}) = \left[\frac{\overline{U}}{c(\widehat{x})^\alpha}\right]^{\frac{1}{1-\alpha}}$, with $c(\widehat{x}) = \frac{\alpha}{p}[w - t\widehat{x}]$. This gives us, at an

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[5] We treat each household a commuting to the exact center of the city for technical convenience. The alternative is to have each household commute to the edge of the CBD and this latter approach is slightly more complicated to deal with. A cleaner, more abstract approach is to have a CBD with zero area (eg. Black and Henderson [1999]).

arbitrary distance x,

$$\begin{aligned} q(x) &= \frac{(1-\alpha)[w-tx]}{h(x)} \\ &= \frac{(1-\alpha)}{\overline{U}^{\frac{1}{1-\alpha}}}[w-tx]c(\widehat{x})^{\alpha/(1-\alpha)} \\ &= \frac{(1-\alpha)}{\overline{U}^{\frac{1}{1-\alpha}}}\left[\frac{\alpha}{p}\right]^{\frac{\alpha}{1-\alpha}}[w-tx]^{\frac{1}{1-\alpha}}. \end{aligned}$$

This is the distance profile of unit floorspace price. A household's housing expenditure, as a flow, is $h(x)q(x)$. House-price in colloquial parlance would be this flow divided by the market rate of interest.

We can solve for the land rent function by substituting in $q(x)$ above from our expression for $q(x)$ earlier. That is, $Mr(x)^{A}i^{1-A} = (1-\alpha)\left[\frac{\alpha}{p}\right]^{\frac{\alpha}{1-\alpha}}\{[w-tx]/\overline{U}\}^{\frac{1}{1-\alpha}}$ and we obtain the rent-distance function

$$r(x) = \xi \times [w-tx]^{\frac{1}{(1-\alpha)A}} \text{ for } \xi = \left[\frac{(1-\alpha)}{Mi^{1-A}\overline{U}^{\frac{1}{1-\alpha}}}\left[\frac{\alpha}{p}\right]^{\frac{\alpha}{1-\alpha}}\right]^{\frac{1}{A}}.$$

Given rent ρ prevailing at the city edge, $\overline{x}$, we can solve for size $\overline{x}$ in $r(\overline{x};\overline{U}) = \rho$. We can then solve for city size in $N(\overline{x};\overline{U}) = \int_0^{\overline{x}} \frac{2\pi x}{L(x)}dx$ for

$$\begin{aligned} \frac{1}{L(x)} &= \frac{r(x)}{Aq(x)h(x)} \\ &= \frac{r(x)}{A(1-\alpha)[w-tx]}. \end{aligned}$$

Total rent is $R(\overline{x};\overline{U}) = \int_0^{\overline{x}} r(x)2\pi x dx$.

3 Comparative Statics

We are interested in three particular comparative statics results.

(1) Amenity improvement (more sunshine, lower crime, less traffic congestion, etc.) Formally this is an increase in parameter ψ in the utility function. Since $\overline{U}$ is the open city "opportunity cost" utility level and $\psi U(c(x), h(x)) = \overline{U}$, an increase in ψ (as with more

sunshine for our city), $U(c(x), h(x))$ must decline for each distance, x. (In our model above ψ was unity.[6] To obtain a new equilibrium (post amenity shock) we simply redo our calculations above with $\overline{U}$ set at a smaller value.) This yields the revised Rosen-Roback rule: cities with more amenities will "offer" workers there smaller commodity bundles $(c(x), h(x))$. A family is in a sense substituting more sunshine for a smaller home h and less c. Since w is unchanged, the smaller commodity bundles will result in a denser city with increased land rent (higher priced homes). Since a household at the edge experiences no change in prices for c and h, their bundle will have c and h shrink proportionately, given a homothetic utility function like the Cobb-Douglas, and their "extra" income will be directed to marginally more commuting ($\overline{x}$ increases). Every "interior" household will have its $\frac{c(x)}{h(x)}$ ratio rise. Extra income available from the contraction in each $(c(x), h(x))$ bundle for an interior household will, roughly speaking, end up in higher land rents. Hence the result: marginally more amenity on the utility side yields a larger city (larger $\overline{x}$), a larger population with on average a higher density, a rise everywhere except at the edge in the $\frac{c(x)}{h(x)}$ ratio, and a shift out in the rent function at every interior location. Calculations confirm these results. For parameter values, $\alpha = 0.7$, $A = 0.35$, $\rho = 0.1$, $t = 0.1$, $i = 0.06$ and $w = 12.5$, we solved for two cities with distinct values for $\overline{U}$ and have recorded outputs in Table 1.

Table 1

$\overline{U}$	$\overline{x}$	$pop'n$	$r - intercept$
5.5	69.185	1.6086×10^5	215.0
5.4	70.200	1.917×10^5	258.0

The results confirm that with a decline in $\overline{U}$, the edge expands, rent shifts upward and population increases considerably. Density rises. Hence an amenity increase leaves us with a larger, denser city. O'Sullivan [2007; Figure 7-6] reports on population density for thirty-five large cities of the world. Of the densist fifteen, only two have what might be termed "cold climates", namely Moscow and Seoul. A website[7] lists 125 of large, dense cities of the

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[6] A more complicated formulation would not have the amenity index Ψ simply multiplicative.

[7] http://www.citymayors.com/statistics/largest
-cities-density-125.htm. This was consulted on April 4, 2008.

world by density. US cities Los Angeles, San Francisco-Oakland, San Jose, New York, New Orleans, Las Vegas, Denver appear on the list, starting a place 90. Only New York would have what we might consider a non-sunny climate. If we believe that warm climate is a location amenity then we are seeing denser cities with "more sunshine".

For this Cobb-Douglas case above we have

$$\frac{dR(\overline{x})}{d\overline{U}} = -\frac{R(\overline{x})}{(1-\alpha)A\overline{U}} + \rho 2\pi\overline{x}\frac{d\overline{x}}{d\overline{U}}$$

with $\frac{d\overline{U}}{d\overline{x}} = -t \times \left[\frac{\rho}{\varphi}\right]$ for $\varphi = \left[\frac{(1-\alpha)}{Mi^{1-A}}\left[\frac{\alpha}{p}\right]^{\frac{\alpha}{1-\alpha}}\right]^{\frac{1}{A}}$. Hence with $\frac{dR(\overline{x})}{d\overline{U}} < 0$. With an increase in the local amenity indicator, $d\overline{U} < 0$ and net aggregate rent for the city rises.

(2) "Amenity" improvement on the production side.

On the production side we postulate production for export from the city at the center with a constant returns to scale function, $Q = \gamma F(K,N)$. Here γ is the "amenity" index. Given the rental on capital fixed at i and the price of output fixed "out there" at p^e, the wage is determined via the unit cost function. An increase in γ implies a higher wage, w via the unit cost function, $p^e = \zeta w^B i^{1-B}$. An increase in γ can be interpreted as a reduction in ζ.

Consider now a decrease in ζ in $p^e = \zeta w^B i^{1-B}$ which induces an increase in w. Consider the impact of this wage increase. Observe that if one differentiates $r(x)$ above with respect to w, one gets $\frac{dr(x)}{dw} = \frac{1}{(1-\alpha)A}\xi \times [w - tx]^{[\frac{1}{(1-\alpha)A}]-1}$, which is the same as $1/L(x)$. Hence

$$\frac{dR}{dw} = \int_0^{\overline{x}} \frac{1}{L(x)} 2\pi x dx + \rho 2\pi\overline{x}\frac{d\overline{x}}{dw}.$$

Hence we have the basic result $dR - \rho 2\pi\overline{x}d\overline{x} = N(\overline{x})dw$.[8] The wage increase aggregated over all workers is precisely capitalized in aggregate net land rent increase, where net refers to the netting out of the new rent ascribable to the edge expansion (and population expansion).

A numerical illustration with our above functions (Cobb-Douglas for both utility and production of housing) allows us to get a feel for the impact of not just a marginal increase

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[8] This result holds for any homogeneous utility function and homogeneous production function for housing. See the Appendix.

in the local wage but also a non-marginal increase. Our parameter values for the runs reported in Table 2 are $\alpha = 0.7$, $A = 0.35$, $\rho = 0.1$, $i = 0.06$ and $\overline{U} = 5.7$.

Table 2

	w	t	$\overline{x}$	Pop'n	M	Agg. Rent
base	12.5	0.1	67.1552	114,350.1	2,457,320.0	124,282.6
	12.6	0.1	68.1552	124,386.1	2,695,296.0	136,255.6
	13.6	0.1	78.1552	278,370.35	6,538,746.2	328,856.03

For the first experiment, we increase the wage from 12.5 to 12.6. $\Delta R = 11973.0$ and $N\Delta w$ is 11936.8 (with N the average of the two values, 114,350.1 and 124,386.1). $\rho 2\pi \overline{x} d\overline{x} = 42.49$ (with $\overline{x}$ the average of 67.1552 and 68.1552). Thus $\Delta R - \rho 2\pi \overline{x} d\overline{x} = 11930.51$ which is quite close to $N\Delta w$ at 11936.8.

Our second experiment was a repeat of the first one, except for a "large" wage change, namely 1.1. Our main result is that net rent, $\Delta R - \rho 2\pi \overline{x} d\overline{x}$, falls short of change $N\Delta w$ for the large wage change experiment (204,071.53 compared with 215,996.25). The formula which we derived was of course for small changes (infinitesimals) and our "large change" experiment is a rough test of the robustness of our capitalization formula.

(3) Commuting cost reduction

We can repeat the derivation above for the case of a small reduction in commuting cost, t and obtain the capitalization formula, $dR - \rho 2\pi \overline{x} d\overline{x} = -M dt$, for $-dt$ the result of say an infrastructure improvement. M is total roundtrip commuting miles in the city for all households. That is $M = \int_0^{\overline{x}} x \frac{2\pi x}{L(x)} dx$.We checked on this formula with a numerical example and the same parameters above.

Table 3

	w	t	$\overline{x}$	Pop'n	M	Agg. Rent
base	12.5	0.1	67.1552	114,350.1	2,457,320.0	124,282.6
	12.5	0.096	69.9534	124,077.8	2,777,461.46	134,855.3
	12.5	0.08	83.9440	178,670.0	4,799,453.75	194,191.6

Our first experiment involved a small reduction in t from 0.1 to 0.096. For this $dR - \rho 2\pi\overline{x}d\overline{x} = 10572.7 - 120.47 = 10452.23$ for the case of $\overline{x}$ the average of 67.1552 and 69.9534. $-Mdt = 10469.56$ for the case of M the average of 2,457,320.0 and 2,777,461.46. Hence our formula checks out reasonably well. We proceeded to an experiment with a "large" change in t. That is, $\Delta t = -.02$ for this large-change case. Here we observed that $dR - \rho 2\pi\overline{x}d\overline{x} = 69,112.45$ and $-Mdt = 72,567.74$. Clearly the rent-change under-estimates the value of the change in t for this large-change case. This is qualitatively the same as what we observed for the case of the large change in w above.

This commuting cost capitalization result also holds for general functional forms for utility and housing production. See the steps in the Appendix A for the case of a wage increase.

4 Discussion

First, we should note that though our model resembles one for textbook small, open economy, it could be brought closer to the standard case if (a) land rent were returned as an equal lump sum to each household and (b) if the local price of say the composite good, c were endogenized so that the value of aggregate exports from the city equalled the value of the aggregate composite good consumed in the city. The "fix" for land rent return can be interpreted as the local provision for a public good consumed in equal amount by each household. We see no reason for the qualitative nature of our comparative statics results to change, though details would. Thus though what we have above is a rough version of a small, open economy, it lends itself to manipulation leading to useful comparative statics "predictions".

Two familiar shortcomings of our model are (a) only a single income group is present and

(b) all jobs are located at the center. It is pretty well understood how to extend the model to accommodate multiple income groups when all workers continue to work at the center. The introduction of multiple groups should not change our qualitative results, though the analysis would become complicated when various boundaries between income groups shift under a comparative statics "shock". Secondly it is fairly straightforward to extend the textbook model to accommodate a fringe of suburban job centers (an annulus) if one accepts the idea that workers employed in lower densities are somewhat less productive (Ciccione and Hall [1996]) than others and thus command somewhat lower wages. One can then set out a suburban ring of employment centers with the workers who work there commuting radially from farther out. Each of these outer households will achieve the same equilibrium $\overline{U}$ for the city but will have a lower wage than the counterpart worker who works in the center and commutes from inside the suburban job ring. There will be separate land rent functions for the two groups, center workers (inside commuters) and suburban workers (outer commuters) and the difference in wages allows us to join the two rent functions with no jump. If the suburban jobs are at distance $\widetilde{x}$ from the center then inside workers will command wage w and suburban workers wage w^o with $w - t\widetilde{x} = w^o$. The arrangement of wages will assure that the equilibrium $\overline{U}$ is the same for both types of workers and that the land rent function exhibits no jump at $\widetilde{x}$. Thus we argue that one can extend the monocentric city model to accommodate an outer ring of suburban job places without invoking a host of new economic arguments. Given the extended model, we see no reason for our comparative statics results not to remain valid in a qualitative sense.[9]

A fairly large extension of our model would have the location parameter in the production function, γ (in $Q = \gamma F(K, N)$) depend on the size of the city or perhaps on the density of the city. That is, $\gamma(N, \phi)$ might increase with N and say ϕ, where ϕ is both location and technology specific. Such a formulation would incorporate the current view that labor productivity is city-size specific and hence that local wages reflect a city-size productivity

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[9] McGrath [2005] and Spivey [2008] argue that their econometric results support traditional comparative statics predictions from the textbook Mills-Muth model first reported on by Brueckner and Fansler [1983]. They fail to emphasize that a model with multiple employment locations can exhibit qualitative behavior much like one with a single employment (the textbook Mills-Muth model). On the other hand their line of thought supports the idea that there is a basic textbook model of a modern city with agreed upon characteristics and thus we are not off base in appealling to "the standard textbook model" for our analysis.

indicator. In this formulation, larger cities can offer similar workers higher wages because workers in larger cities are more productive simply because they are working "along side" many more workers. This seems like the proper extension of our formulation, in view of the work of Glaeser and Mare [2001] and others on the subject of higher wages in larger cities. We have not pursued this extension to our model because it complicates our analysis considerably to have a city size feedback onto our local wage in our comparative statics investigation. A positive feedback of city size to the current wage in the city becomes in our view a second order step from our perspective. First the increase in a utility-amenity boosts city size and density and then the increase in city size and density boosts local wages somewhat. This line of thinking runs counter to that of Rosen and Roback. They wanted an increase in an amenity impinging directly on households to lead to lower local wages as in a compensating response.

5 Empirical Work

Our analysis above is based on workers with similar skills and experience "earning" the same utility in any city in a system. Zero cost migration eliminates any spatial arbitrage possibilities. We buy into the notion that a particular worker will earn more if she works in a larger city in our system of cities. Our analysis predicts that extra amenities (as with more sunny days per year) will substitute for "more housing" in a particular city and thus that city j with more amenities will be on average denser. This density will in turn make city j somewhat larger than average and this size premium will show up as an indirect wage premium for city j. We test these contentions empirically in two models. We begin in this section with a model examining the effect of local utility-based amenities (specifically climatological amenities,) on wages, with our model building on the exercise of Roback [1982]. We then proceed, in section 7, to test the hypothesis that the in-migration induced by local consumption amenities in a city produces an increase in population density. In a previous section we considered a local amenity shift and the subsequent increase in utility, which led to an increase in local land rents caused by the rise in density. In this second exercise, we will examine the effect of our local utility-based amenities (as in the first exercise,) on the housing price schedule.

5.1 Utility-amenities in the Wage Equation

We consider a Mincerian construction with an amenities vector, in the vein of Roback [1982].[10] We make modifications to the Mincerian base model in light of findings by Lemieux [2000],[11] adding degree attainment cohorts, and quartic years of potential labour market experience variables. In selecting amenity variables for our amenities vector, we differ from the approach of Roback [1982] in two ways, (1) clear and cloudy days are measured by a single, continuous variable that measures the average percentage of daily sunshine from sunrise to sunset, and (2) we choose a more extreme measurement of aversion to cold climates by replacing average annual heating degree days with average annual days below freezing. To improve model fit, we alter the functional form of the population size and density variables used in Roback (1982) by using their logarithmic forms. These considerations form the regression model,

$$\log(w_k) = X_k'\beta + C_k'\delta + \Lambda_k'\xi + \varepsilon_k$$

where w_k is weekly earnings for individual k; X_k is an individual's vector of personal characteristic variables, and β is the vector of labour market prices for those characteristics; C_k is a vector reflecting population size and density of an individual's city of residence, and δ represents the productivity increase (decrease) an individual experiences from living in a city with a certain population size and density; Λ_k represents our vector of climatological amenities that are included in individual k's consumption bundle by virtue of living in a

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[10] Our exercise attempts to replicate the model of Roback (1982), with necessary modifications. Glaeser and Mare (2001) and others use log hourly earnings as their regressand, but we choose to stay true to Roback (1982) and thus use log weekly earnings. Weekly earnings are also nominal values.

[11] The 1990 U.S Census data used in our exercise does not provide data for years of schooling, so we do not include variables for years of schooling, and quadratic years of schooling. Data is available for degree attainment cohorts, and so we use the cohort vector as our education variable. This vector of categorical education variables is similar to that of Glaeser and Mare [2001], where they use the 1990 U.S Census data to create arbitrary cohorts for an education variable vector. Furthermore, Glaeser and Mare [2001] create arbitrary years of potential labour market experience cohorts from their years of education cohorts, and the convention, $\max\{age - years\ of\ schooling - 6, 0\}$. Our exercise stops short of creating experience cohorts, and instead uses the approximate continuous years of potential labour market experience variable that is generated as an intermediate in the Glaeser and Mare [2001] exercise. The continuous nature of the years of potential market experience variable allows us to generate and include its quadratic and quartic versions. Our reasoning behind using approximate continuous years of potential labour market experience variables is that it improves the model fit considerably (5% increase in adjusted-R^2).

particular city, and ξ describes the market price (compensation) of each amenity. (For a detailed description of the variable contents of the model, see Table 1.1, Appendix B.)

5.2 Data Description and Results

The source for wage data used in our exercise is the 1990 U.S Census 5% Integrated Public Use Microdata Series (IPUMS). The census provides us with wage data for all U.S cities with populations of 100,000 or greater. Our data for city population, land area and density is obtained from the Demographia demographic brief on U.S cities with populations over 100,000 in 1990. [Wendel Cox Consultancy, 2006][12] We use climatological data from the National Climatic Data Centre (NCDC), and as a result, our investigation is restricted to a sample of 94 U.S cities for which the NCDC collects climatic data.[13] We estimate the regression model in (1) using the OLS estimation method, but compensate for what would (likely) be underestimated standard errors under standard OLS estimation by taking into account the clustering created by our city variable and all other variables at the city level.[14]

Table 2.1 of Appendix C presents a table of the regression results of our model in (1), which can be found in Appendix C.[15] We begin by noting the well-behaved nature of the traditional Mincer portion of our model (as in Roback [1982] and Glaeser and Mare [2001])[16]. The vector of personal characteristics variables, consisting of gender, marital status and visible minority status, exhibit signs that are consistent with the historical gender and visible minority wage gap, as females, visible minorities and married females are shown to be earning less, *ceteris paribus*, then their counterparts. Further, degree attainment cohort variables

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[12] This data can be found outlined in Table 3.1 of Appendix D.

[13] Restricting our data to the 94 cities for which we have climatic data reduces our data set to 820442 observations, a respectable size for the purpose of our investigation. The NCDC directly collects data for a wide variety of cities throughout the United States and provides detailed climatological topography maps for our most limited variable, average daily percentage of sunshine (from sunrise to sunset). These maps allow us to estimate (with reasonable accuracy) the values of this variable for some cities for which we are missing data.

[14] We use the method of hierarchal cluster analysis included in STATA statistical software. We cite the use of this correction from the discussion on large numbers of large groups (in our case, cities) as discussed in section II of Wooldridge (2003).

[15] Contrary to the method used by Roback (1982), we include all amenity variables in our regression to test for joint insignificance in addition to their individual behaviour.

[16] We use the term 'well-behaved' in the sense that, in addition to expected coefficient signs and relative magnitudes, all coefficients for our mincer equation (with the exception of the martial status and visible minority cross-effect,) are statistically significant at the 1%, 5% and 10% levels.

verify the view that there is increasing return to higher education, as noted in the findings of Lemieux [2000]. Potential labour market experience also exhibits behaviour consistent with Lemieux [2000], that is, a positive, diminishing return.

While our Mincer variables perform well, our results support the view that city $i's$ density rather than its population is driving a local wage premium. This is in accord with the results of Hall and Ciccione [1996] who, with US county data, identified density as the driver of local labor productivity premia in denser counties. Our results indicate a ten percent wage increase for a doubling of city density, for individuals fifteen years or older, regardless of employment status. Hall and Ciccione (p. 55) report a six percent increase in productivity for a doubling of employment density at the county level.

Our coefficients on the climatic amenity variables are not statistically significant. Hence the case for local climate amenities being capitalized in local land rent, not in local wages DIRECTLY. All three amenities variables prove to be strongly statistically insignificant at the 1%, 5% and 10% significance levels.[17] Moreover, the values of the coefficients for our amenity variables are simply very small. But our results raise the question of an indirect capitalization in local wages via our density variable. Our prediction from the theoretical model was that "sunny" cities would have on average higher housing (land rent) prices and thus be denser on average. But density we find in the data to be a significant driver of local wage premia. The direct density effect we have in mind is of course, "more urbanness" or a place with more "productive interactions" among workers linked to a higher local density of workers. But the density effect observed in our abstract model was simply the effect of a sunny city being more attractive and ultimately having higher land rent and more expensive "housing" as a result.[18] These two density effects are clearly convolved in our estimated wage equation. This convolvement is itself very interesting but leaves us unable to

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[17] In fact, the smallest p-value amongst the amenity variables belongs to average annual snowfall (in inches), at 0.343. That is, all amenity variables are strongly statistically insignificant, even at the 10% level. Moreover, these results are relatively robust in the sample size, as a regression on the 1990 U.S Census 1% IPUMS (201198 observations,) yields similar results. Our p-values in general are smaller, but of the three amenity variables included in the previous examination of the 5% sample, only 'average annual days below freezing' is significant at the 10% level.

[18] According to the regression results of Table 2.1, a 1 percent increase in average daily sunshine (from sunrise to sunset) decreases wages by 0.013% (mean = 57.42, range = {0,85}); an additional day below freezing per year (on average) increases wages by 0.024% (mean = 59.05, range = {0,166}); an additional inch of snowfall per year (on average) increases wages by 0.049% (mean = 19.81, range = {0,120}).

disentangle the two "forces" at play in our single coefficient on the density variable. Sunny cities are both "high" wage cities as well as "high" land rent cities. This suggests a theory of urban development in which good climate leads to high density agglomeration and high density results in highly productive local workers via a "scale" or positive worker interaction effect. We see no problem with this theory of urban development but we believe it applies only to some cities (eg. Pheonix, Atlanta, and Austin perhaps) and not others (eg. New York, Chicago and Los Angeles perhaps). It is a good theory but has in our view obvious limited scope. Thus the argument that New York developed as a world class, large city because of its attractive climate seems somewhat silly. Clearly it emerged as a place with a good deep water port and a place where industrial and residential activity could spread out relatively easily. It's geography also allowed it to evolve as a principal gateway to the interior of North America. But cities such as Phoenix, Atlanta, Dallas and Austin might be considered places that developed because they were situated where sunshine was abundant and winters were not severe. The "free" consumption of "good climate" ended up capitalized in local land rent and led to these cities developing as relatively high house-price, high density cities.

The bottom line is that our estimated wage equation is not telling us that a local climate amenity is not being capitalized in local "higher" wages because we obtain "climate" working through our density variable and density is operating to "soak up" local wage premia in our regression.

We summarize here why we believe that our regression results for our wage equation come out differently than those obtained in somewhat similar regressions by Roback and Glaeser-Mare. Our results show that the positive effect of city size (population) common to Roback [1982] and Glaeser and Mare [2001] is small and statistically insignificant when tested against our data. We note here a number of departures on the Roback and Glaeser-Mare models which may have led to the discrepancy in results. While our model closely mirrors the work of Roback [1982], our data sets differ, as do some of our statistical analysis techniques. To begin, while we use roughly the same number of cities, our data set is almost seventy-times greater than the data set used in Roback [1982]. Furthermore, and perhaps the more significant difference, is the use of cluster analysis, as in Wooldridge [2003]. The use of this statistical analysis technique produces robust estimates for our standard errors by adjusting for the 'clustering' of our city variable.[19]

We depart from the work of Glaeser and Mare [2001] by modifying our hedonic Mincer equation to reflect the recommendations of Lemieux [2000]. Both our work and that of Glaeser and Mare do not represent years of schooling as a continuous variable, as that data is not available for the 1990 U.S Census, but instead represent years of schooling as intervals. We deviate from the Glaeser and Mare model, following the recommendation of Lemieux [2000] to represent the categories as degree cohorts. Furthermore, we differ from Glaeser and Mare in that we adopt the Lemieux potential labour market experience vector, experience, experience quadratic, and experience quartic, whereas Glaeser and Mare construct a vector of categorical years of potential labour market experience. Our final variation on the Glaeser and Mare [2001] model is in our selection of city size and density variables. Glaeser and Mare represent their population variables by place-of-residence indicator variables, namely dense metropolitan area (population over 500,000), non-dense metropolitan area, with the remainder living in non-metropolitan areas. Our study considers only metropolitan areas with a population of over 100,000. Further, our city size variable is represented as the natural logarithm of city size, closely mirroring the model of Roback [1982]. We choose the Roback formulation of the city size variable because our interest lies in city size effect between cities at the individual level, whereas Glaeser and Mare concentrate on the wage premia between dense metropolitan areas and non-dense metropolitan areas in comparison to non-metropolitan areas. In terms of city density, Glaeser and Mare did not apparently attempt to control for density.

5.3 Utility-amenities in Local House Prices

We turned to a test of whether local climate amenities are showing up in local house-price premia. The data set we drew on for the estimation of our wage equation allowed us to estimate a rough[20] house-price per worker equation along hedonic lines. We ask then whether

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[19] We use cluster analysis due to the fact that while we have over eight-hundred thousand observations, we only have 118 unique observations for city size, density, and amenity variables. In adjusting the standard errors of these variables to control for clustering, the standard errors of our city-related variables are re-adjusted upward, which produces a t-statistic nearer to zero.

[20]

local climatic amenities are showing up in local house-price premia, as our theoretical model predicts.

We begin with choosing our dependent variable as the natural logarithm of house price. We control for a list of basic hedonic housing price regression variables: number of bedrooms, number of rooms, house age, city size and city area.[21] Lastly, our climatological amenities vector is as in the model from (1). Therefore, we have the regression model,

$$\log(H_k) = \Pi_k' \eta + \Gamma_k' \alpha + \Lambda_k' \xi + \varepsilon_k$$

where H_k is the house price (in dollars) owned by individual k; Π_k is the vector of house and owner characteristic variables for an individual (and their house), k,[22] and η is the vector of housing market prices for those characteristics; Γ_k is a vector reflecting population size and land area of the city in which the house (and therefore, individual,) resides, and α represents the vector of housing market prices for city size and area; Λ_k represents our vector of climatological amenities as in our first exercise, and ξ represents the housing market price (compensation) of each amenity. (For a detailed description of the variable contents of the model, see Table 1.1, Appendix B.)

5.4 Data Description and Results

We now proceed with tests of the house-price model. Again, we use the 1990 U.S Census 5% Integrated Public Use Microdata Series (IPUMS) for our wage data, and in addition, our housing data, for the same 94 U.S cities, as well as the Demographia demographic brief that we used in the first exercise.[23] We estimate our equation by the OLS method, and continue to compensate for clustering in our city variable, as in Wooldridge [2003]

We present our regression results in Table 2.2 of Appendix C. [24] We note that both city

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We say rough because we do not have fine details of the quality of each house and its location characteristic within its city.

[21] As a matter of functional form, we use the natural logarithm of city size and city area.

[22] The variables in this vector include: number of bedrooms, number of rooms, and house age.

[23] As housing value data is more scarce within the IPUMS sample, our second exercise is carried on fewer observations (418312) than our first exercise.

[24] The coefficients of our hedonic housing price regression variables for number of rooms (negative), number of bedrooms (positive), and age of house (negative), are also significant at the 1%, 5% and 10% significance

size and city area are statistically significant at the 10% level. The sign for density is positive and for city area is negative. This latter is not surprising given that we are controlling for density. Each of these variables has a coefficient of similar magnitude. The local climate amenities affect housing pricing, in accord with our theoretical model. While the variables for average daily sunshine (of percent possible, from sunrise to sunset) and average annual snowfall (in inches) are statistically insignificant at the 10% level, we find that average annual days below freezing is statistically significant at the 10% level. The disamenity exhibits the expected negative effect on housing prices, implying that colder cities are less dense, and that people accept :"less housing" for a warmer climate.[25]

We were surprised to see that of the densest cities in the United States with populations over 100,000, approximately 40% are cities in California, typically smaller cities (less than 300,000 people,) surrounding Los Angeles, San Francisco and San Diego. We are certainly observing sunnier cities more dense for those in this subset.[26] We decided to control for size by re-estimating with the five largest cities (New York, Los Angeles, Chicago, Houston, and Philadelphia) removed from our sample.[27] These cities have over 25 percent of the workers in our data set. One might argue that our omission of disamenities normally associated with large cities (crime, pollution and congestion) may be offsetting the amenity effect of sunshine in these large cities. These five cities can be considered outliers since approximately 75% of the cities included in our original sample have a populations under 500,000, and this size is at least three times less than population of Philadelphia, the smallest of the big five. We

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levels. The data of our sample describe newer, open-concept houses with a large number of bedrooms, as those which retain the highest market value.

[25] The average annual days below freezing variable is (weakly) statistically significant at the 1% level. It is also of practical significance, describing a 0.58% decrease in housing price for each additional average annual day below freezing. This is 20 times the magnitude of the effect of this variable on weekly wages in the first exercise, which was found to be statistically insignificant at the 10% level.

[26] These essentially suburban cities are naturally creatures of the main core city and exist as independent cities for historico-legal reasons rather than textbook economics reasons. We are of course doing urban economics and our interest is in urban economies. Ideally the relevant cities would be geographically separated and would be considered to function as small, open economies. Since the data are based on legally defined entities, we are no doubt dealing with some units that might best not be considered as the independent economic entities that we would like to be working with. However, we work with the data as they are published and remark that some of the data has shortcomings for the analyses that we are pursuing.

[27] Each of these bid five cities is a significant port with Chicago on Lake Michigan being the only one far interior to North America. We argued earlier that cities with good natural ports can be expected to develop as relatively high wage cities since labor productivity should be somewhat higher in such places.

proceed to re-estimate our house-price equation with the big five cities and their associated workers excised from the sample.[28]

Our results of the second estimation of the house-price model are tabulated in Table 2.3 of Appendix C. The results show that city size is strongly insignificant at the 10% level, while city area continues to command the same effect magnitude and statistical significance.[29] We see that average daily sunshine and average annual days below freezing reflect the expected positive and negative signs, respectively, and are both statistically significant at the 5% level.[30] As an illustration, the regression of average daily sunshine on average housing value is displayed in the figure below.

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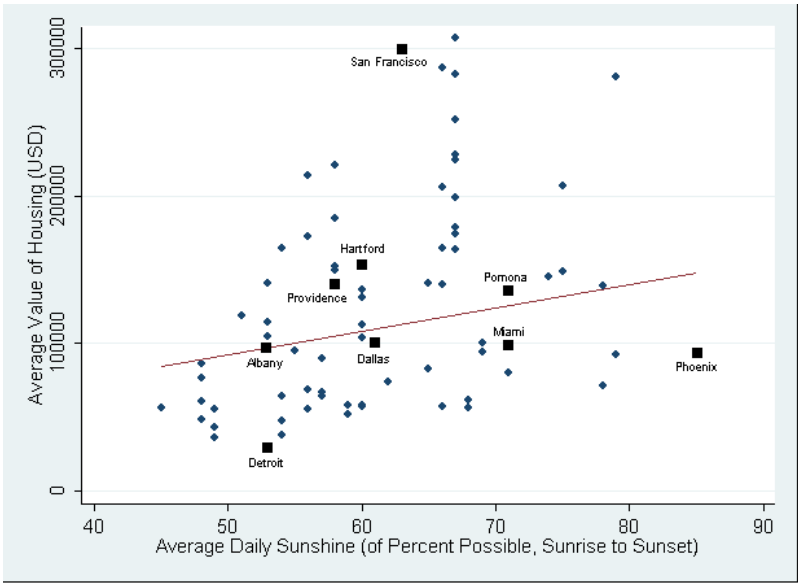
[28] Removing these 5 cities shrinks our data set to 288132 observations.

[29] The correlation coefficient matrix of (the natural logarithms of) city size, area and density for the second estimation of the model in (2) show a remarkable difference from the first estimation, which included observations from New York, Chicago, Los Angeles, Houston and Philadelphia. The first estimation demonstrated a high positive correlation (0.661) between city size and density, but a low positive correlation (0.0925) between city area and density. The second estimation produced radically different results, where the correlation coefficient of city size and density was 0.0615, and the correlation coefficient for city area and density was -0.6165. Moreover, in the second estimation, we noted that city size is only weakly significant at the 10% level, with a p-value of 0.219. That is, omitting the five most populated cities as 'outliers' produces the result that city size has a relatively weak correlation with city density, for the majority of highly populated cities in the United States, and that it is city area has a relatively high correlation with city density. This brings light to a theory that the effect of climatological amenity on city density exists more evidently in suburban 'cities' outside major 'central business district' cities because individuals in these cities substitute the disamenities of big city pollution and crime for more sunshine. In fact, approximately 30 of the top 55 densest cities in the United States have populations below 200,000, and 15 of the top 55 densest cities can be considered 'suburban' to Los Angeles, San Francisco, and San Diego.

[30] We note that average annual snowfall is still insignificant at the 10%.



Average Value of Housing (USD = 1990) = 1569.842(Sunshine) + 13156.99; $R^2 = 0.051$. Number of observations = 94. Data from 1990 U.S Census, 5% Sample (five most populated cities removed), IPUMS Database. Each observation is at the city level.

The coefficient of average daily sunshine indicates a 0.84% increase in housing price for a 1% increase in daily sunshine (of percent possible, from sunrise to sunset), while the coefficient for days below freezing indicates a 0.39% decrease in housing price for an additional average annual day below freezing. The results of our exploratory exercise demonstrate that when the five largest US cities are removed as city size 'outliers', two of our three local climate amenities are statistically significant at the 5% level. That is, for our reduced sample of cities with populations over 100,000 in the United States, a 'shock' to a local climate amenity (disamenity) within a city produces an increase (decrease) in local house-prices.

6 Comments

Our point of departure is that costless migration will put similar workers on the same indifference curve, regardless of location. This view implies that a "free" local climate amenity will show up in "higher" local land rent. The wedge in local land rent premia represent the implicit price a resident pays to enjoy living in a better climate. Higher local land rent implies relatively higher local density of households. We pursued this line of thinking in an abstract model of a city and then pursued some tests with the fairly detailed data defined at the level of the household for the US in 1990. Since our theory is one of each household being in equilibrium, one might ask why do we observe intercity migration in the US? Our model abstracts from shocks to local employment in the form of technical change and/or business cycle impacts. Each household is also subject to life-cycle changes as with more children or older children moving out and so on. These latter "shocks" also induce re-location by a household. We abstracted from these causes of location re-adjustment. For simplicity we worked with each household being in equilibrium. It is not clear what the impact of this approach is when we turned to estimation. Reality involves a fraction of households relocating in a process of adjustment to new exogenous variables and reality is more complicated than our model suggests but our position is that reasonable theory implies that a representative household in the US, given its tastes, demographics, skills and work experience of adults, cannot gain in utility by re-locating. This implies local implicit prices for local climate amenities which we have attempted to analyse.

We did not get a clear signal of climate amenity effects in local wages in our estimation of a wage equation because a local amenity contributes to a local density premium and also serves to "soak up" local wage premia very effectively. The path of local climate amenity effects into local wage premia is not direct but we did end up with strong support for the view that local density is a variable that moves significantly with local wage premia. We found support for a local climate amenity being capitalized in local house prices.

Our empirical work supports the view that some cities develop because they offer workers benign climates in addition to jobs and the good climate results in a local house price premium, via a land rent premium, which in turn implies a local "higher" density. Density appears to correlate significantly with local wage premia in our work and thus a local "good climate" can be a basic driver of the development of a "high" wage (and presumably high productivity) city. This inference about climate driving the development of a certain class of cities certainly deserves more analysis than we have been able to give it in this somewhat preliminary paper. We see two other thoeries of urban development that have been touched on above. The production amenity such as a good port can be thought of as a positive technical advantage say tied to a city's production function. This advantage leads to a worker productivity premium and a relative wage premium. The wage premium feeds directly into a local land rent premium as we argued in our theoretical model and this induces the city to be a "high density" place. High density we associate with positive worker and entrepreneur interaction effects which we argue drive a local productivity premium, a premium on top of that induced by the presence of a port. Port cities become high wage, high density, high productivity places.

A third theory of urban development has high density driving local technical change. A port city can become an innovation city and the presence of "extra" technical progress, locally can drive up local wages. Cities with "extra" technical progress become high wage cities and high density cities. The density can then drive up local labor productivity and add to the wage premium in the city in question. Our empirical work supports local density as a driver of local wage premia. We also elicited from the data support for the view that utility amenities are capitalized in local house prices. We hope to be able to link different cities to different theories of development, perhaps along the lines of Davis and Weinstein [2002], in the future.

7 Appendix A: The General Case

Once again, we can obtain our capitalization results for this model with housing stuctures for a general utility function and general production function for housing. At radial distance x, consumer and house ”production” equilibrium satisfy the following six equations in $h(x), c(x), q(x), L(x), k(x)$ and $r(x)$:

$$\begin{aligned}
\frac{U_c}{U_h} &= \frac{1}{q(x)}, \text{ with the price of } c \text{ at unity,} && (1)\\
w - tx &= c + q(x)h(x) && (2)\\
U(c(x), h(x)) &= \overline{U} && (3)\\
h &= h(L(x), k(x)), \text{ with function } h \text{ homogeneous of degree unity,} && (4)\\
\frac{h_L}{h_k} &= \frac{r(x)}{i} && (5)\\
\text{and } h(x)q(x) &= L(x)r(x) + ik(x). && (6)
\end{aligned}$$

Given constant returns to scale in housing ”production”, we have also that

$$q(x)h_k = i. \tag{7}$$

Using equations (1), (2) and (3), we can repeat our steps for the general case earlier and obtain

$$\frac{dq(x)}{dw} = \frac{1}{h(x)}. \tag{8}$$

Now from (4) and (5), we have

$$\begin{aligned} dh &= h_L dL + h_k dk \\ &= \{\frac{r(x)}{i} dL + dk\} h_k. \end{aligned} \tag{9}$$

From (6), we get

$$\begin{aligned} hdq + qdh &= rdL + Ldr + idk \\ &= Ldr + \left[\frac{rdL}{i} + dk\right] i \\ &= Ldr + \frac{idh}{h_k} \end{aligned}$$

using (9). This latter becomes

$$hdq - Ldr = \left[\frac{i}{h_k} - q\right] dh.$$

The term in square brackets is zero, given (7). Hence $h(x)dq(x) = L(x)dr(x)$. This in (8) yields

$$\frac{dr(x)}{dw} = \frac{1}{L(x)}.$$

Hence

$$\begin{aligned} \frac{dR}{dw} &= \int_0^{\overline{x}} \frac{dr(x)}{dw} 2\pi x dx + \rho 2\pi \overline{x} \frac{d\overline{x}}{dw} \\ &= N + \rho 2\pi \overline{x} \frac{d\overline{x}}{dw} \end{aligned}$$

and our capitalization result, $dR - \rho 2\pi \overline{x} d\overline{x} = N dw$ is established.

And the same steps yields our result on transportation improvements for the case of a general utility function and a general production function for housing, namely, $dR - \rho 2\pi \overline{x} d\overline{x} = -M dt$ for $M = \int_0^{\overline{x}} \frac{x}{L(x)} 2\pi x dx$. M is total commuting miles of all residents.

8 Appendix B: Description of Variables

Table 1.1 – Description of Variables

Variable Name	Variable Contents
Log weekly earnings	Continuous variable representing annual earnings divided by weeks worked for the year 1990.
Female and visible minority	Binary variable indicating an individual has both visible minority status and is of the female gender
Married and female	Binary variable indicating an individual both married and of the female gender
Married and visible minority	Binary variable indicating an individual has both visible minority status and is married
Female	Binary variable indicating an individual is of the female gender
Married	Binary variable indicating an individual is married
Visible minority	Binary variable indicating an individual has visible minority status
High School diploma	Binary variable indicating a high school diploma as an individual's highest education attainment.
Associate degree (occupational)	Binary variable indicating an associate degree (occupational) as an individual's highest education attainment.
Associate degree (academic)	Binary variable indicating an associate degree (academic) as an individual's highest education attainment.
Bachelor's degree	Binary variable indicating a bachelor's degree as an individual's highest education attainment.
Master's degree	Binary variable indicating a master's degree as an individual's highest education attainment.
Doctorate degree	Binary variable indicating a doctorate degree as an individual's highest education attainment.
Professional degree	Binary variable indicating a professional degree as an individual's highest education attainment.
Experience	Continuous variable representing years of potential labour market experience, max{age – years of schooling – 6, 0}
Experience, quadratic	Continuous variable representing years of potential labour market experience, squared.
Experience, quartic	Continuous variable representing years of potential labour market experience, to the fourth power.
Log population	Continuous variable representing the natural logarithm of the population of an individual's city of residence
Log population density	Continuous variable representing the natural logarithm of the population density of an individual's city of residence
Log city area	Continuous variable representing the natural logarithm of the land area of an individual's city of residence (measured in persons per square kilometer)
Average daily sunshine	Continuous variable representing average daily sunshine as a percentage of total possible, from sunrise to sunset.
Average annual days below freezing	Continuous variable representing average annual days below freezing (measured in Fahrenheit).
Average annual snowfall	Continuous variable measuring the average annual amount of snowfall (measured in inches).

Table 1.1 – Description of Variables (continued)

Variable Name	Variable Contents
Log house value	Continuous variable representing the natural logarithm of the house value of an individual's city of residence (measured in dollars)
Bedrooms	Continuous variable representing the number of bedrooms in the house owned by the observed individual
Rooms	Continuous variable representing the number of rooms in the house owned by the observed individual
House age, 2 to 5 years	Binary variable indicating the age of the house owned by the observed individual as between 2 and 5 years
House age, 6 to 10 years	Binary variable indicating the age of the house owned by the observed individual as between 6 and 10 years
House age, 11 to 20 years	Binary variable indicating the age of the house owned by the observed individual as between 11 and 20 years
House age, 21 to 30 years	Binary variable indicating the age of the house owned by the observed individual as between 21 and 30 years
House age, 31 to 40 years	Binary variable indicating the age of the house owned by the observed individual as between 31 and 40 years
House age, 41 to 50 years	Binary variable indicating the age of the house owned by the observed individual as between 21 and 50 years
House age, 51 years or more	Binary variable indicating the age of the house owned by the observed individual as 51 years or more.

9 Appendix C: Statistical Tables

Table 2.1
Regression Results - Log Weekly Earnings

Variable Name	Coefficient Value	t-statistic	p-value
Female and Visible Minority	.1442975	12.59	0.000
Married and Female	-.3256973	-36.81	0.000
Married and Visible minority	-.0209809	-1.22	0.226
Female	-.1890866	-30.66	0.000
Married	.2829063	31.79	0.000
Visible Minority	-.2230124	-15.99	0.000
High School Diploma	.0675679	7.10	0.000
Associate Degree, Occupational Program	.317356	27.46	0.000
Associate Degree, Academic Program	.3100981	26.38	0.000
Bachelor's Degree	.5091497	33.92	0.000
Master's Degree	.6837259	43.78	0.000
Doctorate Degree	.6837662	29.42	0.000
Professional Degree	.839789	39.96	0.000
Experience	.0712868	35.47	0.000
Experience, Quadratic	-.0016268	-31.02	0.000
Experience, Quartic	1.35e-07	25.22	0.000
Log population	.0020281	0.20	0.839
Log population density	.1004476	5.48	0.000
Average daily sunshine	-.0001106	-0.10	0.920
Average annual days below freezing (32 degrees Fahrenheit)	-.000247	-0.93	0.356
Average annual snowfall (inches)	-.0005021	-0.95	0.343
Constant	4.408585	36.74	0.000
Number of Observations: 820442	F-Ratio: 2039.36	R^2: 0.2712	

Table 2.1a
Summary Statistics - Log Weekly Earnings

Variable Name	Mean	Standard Deviation
Log weekly earnings	5.814975	.8926328
Female and Visible Minority	.2064692	.4047713
Married and Female	.1868948	.3898272
Married and Visible minority	.1136165	.317345
Female	.528483	.4991882
Married	.3716731	.4832518
Visible Minority	.3872922	.4871315
High School Diploma	.2032248	.4023985
Associate Degree (Occupational)	.0205321	.1418115
Associate Degree (Academic)	.0213365	.1445035
Bachelor's Degree	.0963289	.2950418
Master's Degree	.0340638	.1813933
Doctorate Degree	.0129724	.0746031
Professional Degree	.0055969	.1131553
Experience	18.58774	20.09585
Experience, Quadratic	749.3471	1177.309
Experience, Quartic	1947576	4875104
Log population	13.68422	1.395877
Log population density	7.904122	.8087636
Average daily sunshine	57.42835	16.47424
Average annual days below freezing (32 degrees Fahrenheit)	59.05627	52.23977
Average annual snowfall (inches)	19.81623	24.37618

Table 2.2
Regression Results - Log House Price (full sample)

Variable Name	Coefficient Value	t-statistic	p-value
Log population	.4811585	8.12	0.000
Log city area	-.4981643	-5.49	0.000
Bedrooms	.1726887	10.54	0.000
Rooms	-.0417013	-5.74	0.000
House age, 2 to 5 years	-.004463	-0.17	0.868
House age, 6 to 10 years	-.1390027	-3.22	0.002
House age, 11 to 20 years	-.2469176	-5.22	0.000
House age, 21 to 30 years	-.2979049	-5.26	0.000
House age, 31 to 40 years	-.3679058	-5.50	0.000
House age, 41 to 50 years	-.4825874	-5.74	0.000
House age, 51 years or more	-.4856942	-5.36	0.000
Average daily sunshine	.0081233	1.33	0.187
Average annual days below freezing (32 degrees Fahrenheit)	-.0062386	-2.66	0.009
Average annual snowfall (inches)	.0018998	0.42	0.673
Constant	7.635688	11.87	0.000
Number of observations: 670073 F-ratio: 43.82	R^2: 0.3269		

Table 2.2a
Summary Statistics - Log House Price (full sample)

Variable Name	Mean	Standard Deviation
Log house value	11.46536	.8545999
Log population	13.53199	1.337889
Log city area	5.737446	1.008205
Bedrooms	2.996502	.9387021
Rooms	2.514285	2.144344
House age, 2 to 5 years	.0421426	.2009146
House age, 6 to 10 years	.0490979	.2160725
House age, 11 to 20 years	.1168988	.3212999
House age, 21 to 30 years	.1527327	.3597299
House age, 31 to 40 years	.2044688	.4033132
House age, 41 to 50 years	.1366847	.3435144
House age, 51 years or more	.2890149	.4533052
Average daily sunshine	57.48643	16.8202
Average annual days below freezing (32 degrees Fahrenheit)	59.56165	53.4271
Average annual snowfall (inches)	19.60833	24.88945

Table 2.3
Regression Results - Log House Price (five most populated cities removed)

Variable Name	Coefficient Value	t-statistic	p-value
Log population	.1725574	1.24	0.219
Log city area	-.4401545	-4.69	0.000
Bedrooms	.2077848	12.76	0.000
Rooms	-.0447236	-8.71	0.000
House age, 2 to 5 years	.0129864	0.54	0.588
House age, 6 to 10 years	-.1094802	-2.85	0.005
House age, 11 to 20 years	-.2406455	-5.53	0.000
House age, 21 to 30 years	-.2935977	-5.69	0.000
House age, 31 to 40 years	-.3621208	-7.02	0.000
House age, 41 to 50 years	-.4500309	-7.66	0.000
House age, 51 years or more	-.411346	-5.94	0.000
Average daily sunshine	.0084933	2.00	0.048
Average annual days below freezing (32 degrees Fahrenheit)	-.0041995	-2.41	0.018
Average annual snowfall (inches)	-.0011546	-0.44	0.664
Constant	10.80332	8.19	0.000

Number of observations: 455932 F-ratio: 52.39 R^2: 0.3464

Table 2.3a
Summary Statistics - Log House Price (five most populated cities removed)

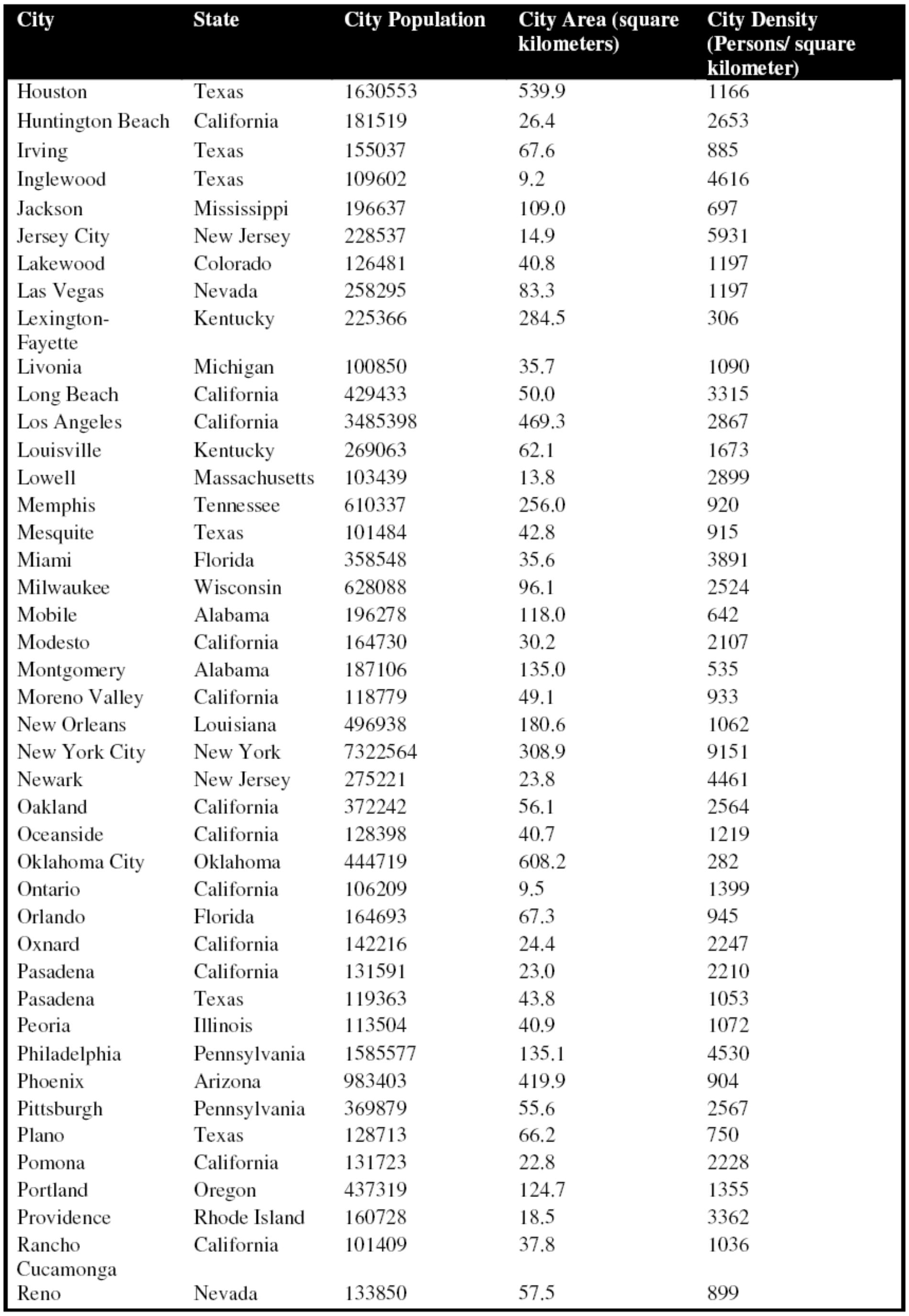
Variable Name	Mean	Standard Deviation
Log house value	11.31409	.8136467
Log population	12.74138	.7358795
Log city area	5.316831	.9328481
Bedrooms	3.03017	.90063
Rooms	2.487115	2.15854
House age, 2 to 5 years	.0515112	.2210382
House age, 6 to 10 years	.0601692	.2378004
House age, 11 to 20 years	.1401663	.3471599
House age, 21 to 30 years	.1593668	.3660181
House age, 31 to 40 years	.20742	.4054593
House age, 41 to 50 years	.1258332	.3316616
House age, 51 years or more	.2442893	.4296655
Average daily sunshine	56.67308	19.79608
Average annual days below freezing (32 degrees Fahrenheit)	55.28968	55.48119
Average annual snowfall (inches)	18.61126	28.18461

10 Appendix D: U.S Population, Land Area and Density Data

Table 3.1 – United States Population, Land Area and Density Data

City	State	City Population	City Area (square kilometerss)	City Density (Persons / square kilometer)
Akron	Ohio	223019	62.2	1385
Albany	New York	101082	21.4	1823
Allentown	Pennsylvania	105090	17.7	2291
Anaheim	California	266406	44.3	2323
Ann Arbor	Michigan	109592	25.9	1634
Arlington	Texas	261721	93.0	1087
Atlanta	Georgia	394017	131.8	1166
Aurora	Colorado	222103	132.5	647
Austin	Texas	465622	217.8	826
Bakersfield	California	174820	91.8	735
Baltimore	Maryland	736014	80.8	3517
Baton Rouge	Louisiana	219531	73.9	1146
Beaumont	Texas	114323	80.1	551
Bridgeport	Connecticut	141686	16.0	3418
Buffalo	New York	328123	40.6	3119
Chattanooga	Tennessee	152466	118.4	497
Chicago	Illinois	2783726	227.2	4730
Chula Vista	California	135163	29.0	1800
Cleveland	Ohio	505616	77.0	2535
Colorado Springs	Colorado	281140	183.2	593
Dallas	Texas	1006877	342.4	1135
Denver	Colorado	467610	153.3	1178
Des Moines	Iowa	193187	75.3	991
Detroit	Michigan	1027974	138.7	2861
El Monte	California	106209	9.5	4316
Elizabeth	New Jersey	110002	12.3	3447
Erie	Pennsylvania	108718	22.0	1909
Escondido	California	108635	35.6	1177
Eugene	Oregon	112669	38.0	1144
Flint	Michigan	140761	33.8	1606
Fort Lauderdale	Florida	149377	31.4	1839
Fort Wayne	Indiana	173072	62.7	1067
Fort Worth	Texas	447619	281.1	615
Fresno	California	354202	99.1	1379
Fullerton	California	114144	22.1	1992
Garden Grove	California	143050	17.9	3079
Garland	Texas	180650	57.3	1216
Gary	Indiana	116646	50.2	896
Glendale	California	180038	30.6	2271
Grand Rapids	Michigan	189126	44.3	1650
Hartford	Connecticut	139739	17.3	3118
Hialeah	Florida	188004	19.2	3772
Hollywood	Florida	121697	27.3	1723

Table 3.1 – United States Population, Land Area and Density Data (continued)



City	State	City Population	City Area (square kilometers)	City Density (Persons/ square kilometer)
Houston	Texas	1630553	539.9	1166
Huntington Beach	California	181519	26.4	2653
Irving	Texas	155037	67.6	885
Inglewood	Texas	109602	9.2	4616
Jackson	Mississippi	196637	109.0	697
Jersey City	New Jersey	228537	14.9	5931
Lakewood	Colorado	126481	40.8	1197
Las Vegas	Nevada	258295	83.3	1197
Lexington-Fayette	Kentucky	225366	284.5	306
Livonia	Michigan	100850	35.7	1090
Long Beach	California	429433	50.0	3315
Los Angeles	California	3485398	469.3	2867
Louisville	Kentucky	269063	62.1	1673
Lowell	Massachusetts	103439	13.8	2899
Memphis	Tennessee	610337	256.0	920
Mesquite	Texas	101484	42.8	915
Miami	Florida	358548	35.6	3891
Milwaukee	Wisconsin	628088	96.1	2524
Mobile	Alabama	196278	118.0	642
Modesto	California	164730	30.2	2107
Montgomery	Alabama	187106	135.0	535
Moreno Valley	California	118779	49.1	933
New Orleans	Louisiana	496938	180.6	1062
New York City	New York	7322564	308.9	9151
Newark	New Jersey	275221	23.8	4461
Oakland	California	372242	56.1	2564
Oceanside	California	128398	40.7	1219
Oklahoma City	Oklahoma	444719	608.2	282
Ontario	California	106209	9.5	1399
Orlando	Florida	164693	67.3	945
Oxnard	California	142216	24.4	2247
Pasadena	California	131591	23.0	2210
Pasadena	Texas	119363	43.8	1053
Peoria	Illinois	113504	40.9	1072
Philadelphia	Pennsylvania	1585577	135.1	4530
Phoenix	Arizona	983403	419.9	904
Pittsburgh	Pennsylvania	369879	55.6	2567
Plano	Texas	128713	66.2	750
Pomona	California	131723	22.8	2228
Portland	Oregon	437319	124.7	1355
Providence	Rhode Island	160728	18.5	3362
Rancho Cucamonga	California	101409	37.8	1036
Reno	Nevada	133850	57.5	899

Table 3.1 – United States Population, Land Area and Density Data (continued)

City	State	City Population	City Area (square kilometer)	City Density (Persons / square kilometer)
Riverside	California	226505	77.7	1126
Rochester	New York	231636	35.8	2499
St. Louis	Missouri	396685	61.9	2473
St. Petersburg	Florida	238629	59.2	1557
Sacramento	California	369365	96.3	1481
Salem	Oregon	107786	41.5	1002
Salinas	California	108777	18.6	2255
San Antonio	Texas	935933	333.0	1085
San Bernardino	California	164164	55.1	1151
San Diego	California	1110549	324.0	1323
San Francisco	California	723959	46.7	5985
San Jose	California	782248	171.3	1764
Santa Ana	California	293742	27.1	4187
Santa Rosa	California	113313	33.7	1298
Seattle	Washington	516259	83.9	2376
Shreveport	Louisiana	198525	98.6	777
South Bend	Indiana	105511	36.4	1119
Spokane	Washington	177196	55.9	1224
Springfield	Massachusetts	156983	32.1	1888
Springfield	Missouri	140494	68.0	798
Sterling Heights	Michigan	117810	36.6	1241
Stockton	California	210943	52.6	1549
Sunnyvale	California	117229	21.9	2067
Syracuse	New York	163860	25.1	2520
Tampa	Florida	280015	108.7	995
Tulsa	Oklahoma	367302	183.5	773
Vallejo	California	109199	30.2	1395
Washington	District of Columbia	606900	61.4	3816
Waterbury	Connecticut	108961	28.6	1473
Wichita	Kansas	304011	115.1	1020
Worchester	Massachusetts	169759	37.6	1745
Yonkers	New York	188082	18.1	4016

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