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Understanding the DeFi Network Through the Lens of a Production-Network Model

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Abstract

Decentralized Finance (DeFi) is composed of a variety of heterogeneous sectors that are interconnected through an input-output network of its tokens. We use a panel data set to empirically document the evolution of the DeFi network across its different sectors. We then employ a standard, theoretical production-network model to measure the value added and service outputs of different DeFi sectors which is fundamentally different from the commonly used metric of Total Value Locked (TVL). Our calibrated model is then used to study DeFi token prices and to predict the equilibrium effects of increasing network interconnectedness.

Keywords: Blockchain, Crypto, Decentralized Finance, Production Network

JEL Classification: G2, L14

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1 Introduction

Decentralized Finance (DeFi) is an umbrella term for various applications that provide financial services with the goal to replace traditional intermediaries by running smart contracts on a blockchain. Since the summer of 2020, DeFi has grown substantially in both scale and scope. Central banks and regulators have raised concerns about the vulnerabilities of DeFi (e.g., leverage and pro-cyclicality), its growing linkages with traditional financial markets and institutions, and the implied spillover effects. (*BIS_{defi}end* : 22, *BIS_{defi}report* : 21, *board2022assessment*,

Despite these policy concerns, the functioning of the DeFi system remains opaque and not well understood for three reasons. First, the DeFi ecosystem is complex and composed of many heterogeneous but interconnected sectors. Second, there is no proper metric to measure DeFi activities. Total value locked (TVL) – a commonly used metric – is an incomplete and misleading measure subject to double counting and manipulation.¹ Third, existing studies focus on individual DeFi sectors instead on the entire network, since there is no unified analytical framework for modeling the DeFi system as a whole. As a result, the guidance provided by empirical and theoretical economic research is limited.

Against this backdrop, we contribute to the literature by studying the entire DeFi network both, empirically and theoretically. First, we use a panel data set to document empirical facts how different DeFi sectors rely on an input-output network of its tokens. Second, we adjust the standard general equilibrium production-network model to capture the input-output structure of DeFi tokens across individual sectors.

Our approach yields four insights. First, data limitation make it hard to measure the economic contribution of DeFi. Following a basic NIPA approach, we therefore define the “value-added” of each sector as the difference between the sectoral output and its interme-

¹For example, according to a 2022 CoinDesk report, the Macalinao brothers admitted that they had inflated the total value locked of Saber on Solana: “I devised a scheme to maximize Solana’s TVL: I would build platforms that stack on top of each other, such that a dollar could be counted several times,” Ian Macalinao wrote in an unpublished blog post (source).

mediate inputs. In our model, this difference is identified as labour input for which we proxy by the implied flow value of governance tokens. We find that only about 1.6% of TVL can be seen as the total annual value-added of DeFi, making it apparent that TVL is flawed measure to assess the importance of DeFi.

Second, DeFi applications produce both, tokens and services. Assuming that only tokens serve as intermediate inputs, we can then infer again through accounting the share of services provided. Only about 30% of the total measured value of output is attributed to services.

Third, the general equilibrium approach allows us to interpret data on DeFi to be driven by underlying factors such as technology, preferences, and other external factors. We therefore use equilibrium conditions from our model to identify the role of productivity shocks, preference shocks and interconnectedness from different crypto data sets. Most of the fall in token prices across our sample period is driven by shocks to DeFi productivity. Interestingly, the network structure has dampened the fall in prices, as the sectors have become more interconnected.

Finally, we use our estimated parameters from the model to simulate the impact of an increase in interconnectedness. Our results show that relying 1% more on token inputs at the expense of labour increases DeFi output by about 0.34% with a similar feedback to Ethereum prices, but much smaller ones for other tokens.

A distinct feature of our paper is its focus on the DeFi ecosystem rather than individual sectors or applications. Existing economic papers focus on individual types of platform. For example, *aoyagi;toy : dex, capponi;21, parour : 21andpark;amm : 21studydecentralizedexchangesintheform*

The paper is organized as follows. We provide a brief overview of DeFi in Section 2. Section 3 discusses the main data sets and some descriptive statistics about the DeFi network on Ethereum. Section 4 presents the model and characterizes the equilibrium. Section 5 presents some quantitative exercises before the last section concludes.

²chiu;22, harvey2021defiandschar : 21provideageneraloverviewofDeFiarchitectureandapplications.

2 What is DeFi?

DeFi uses immutable computer programs called smart contracts to replace costly, third-party intermediaries. In order to offer financial services it relies on a multi-layered architecture. As shown in Figure 2.1, the bottom layer offers record-keeping and settlement of transaction commonly in the form of a blockchain. The middle layer is comprised of crypto tokens that enable the exchange of financial services and whose ownership and transactions are recorded on the blockchain. The top layer contains the applications, smart contracts deployed by DeFi platforms that provide financial services such as payments, lending, trading and asset management.

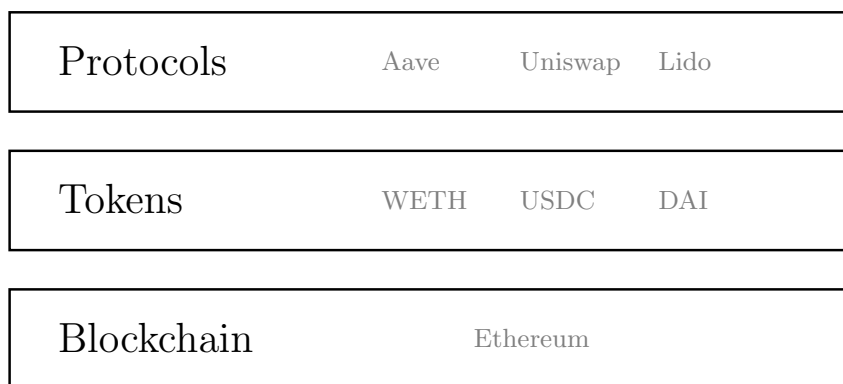


Figure 2.1: Basic DeFi Layers and Examples

Ethereum was the first smart contract blockchain and is now the leading blockchain layer for DeFi platforms. It offers two popular standards, the ERC20 one for creating fungible tokens and the ERC721 one for creating non-fungible tokens (NFTs). One particular feature of smart contracts is composability. Smart contracts deployed on Ethereum can be pieced together like lego blocks to create more complicated contracts that are called protocols. In DeFi, these protocols are designed to offer financial services with prominent examples being Aave, a lending platform, Uniswap, an person-to-person exchange for cryptos and MakerDAO that offers the stablecoin DAI.³

³See Chiu, Kahn and Koepl (2022) for details and a discussion of DeFi's value proposition and limita-

The central idea of DeFi is to enable financial exchange without trust through smart contract that guarantee the autonomous execution of state-contingent contracts. There are no intermediaries that explicitly enforce contracts and, since users are anonymous, reputation does not provide for implicit enforcement. Without enforcement through intermediaries or reputation, DeFi requires assets to be posted into smart contracts. Platforms therefore issue tokens (for example, currencies, stablecoins or governance tokens) that can be used in other applications and require that users “lock” tokens into the smart contracts to access the financial services:

- Borrowers lock crypto assets in a lending platform as collateral to secure a loan.
- Users lock crypto assets in a collateralized debt position to mint decentralized stablecoins.
- Liquidity providers lock assets in an automatic market maker’s liquidity pool to earn rewards.
- Risk assessors stake assets in an insurance contract to earn rewards by providing a risk assessment.

Composability together with the linkages from the locking of tokens give rise to a complex network structure that we study in this paper. DeFiLlama, a crypto data aggregator, provides comprehensive Total-Value-Locked (TVL) data about DeFi platforms broken down by token for each day starting from November 3, 2018. As of October 26, 2022, DeFiLlama listed 151 different blockchains and over 2,000 platforms, around 550 of which existed on the Ethereum blockchain. Table 2.1 reports the TVL and the total number of platforms in the main DeFi sectors operating on Ethereum in January 2023.⁴

tions.

⁴Appendix C provides more details on the data sets used for the analysis.

Table 2.1: Main DeFi Sectors on Ethereum (January 2023)

Sector	TVL (Billion)	No. of platforms	Main platforms
DEXES	15.46	100	Curve, Uniswap, Balancer
Lending	11.24	65	Aave, Compound, Morpho
Dec. Stablecoins	10.54	22	MakerDAO, Liquity, Abracadabra
Yield & Asset Mgmt	8.84	30	Yearn Finance, Idle Finance, LUSD ChickenBonds
Liquid Staking	7.18	16	Lido, Coinbase Wrapped Staked ETH, Rocket Pool
Bridge & Cross Chain	3.15	37	WBTC, Poly Network, hBTC
Staking	2.62	22	Stafi, Redacted platform, GoodDollar
Derivatives	0.84	21	dYdX, Keep3r Network, NestFi
Insurance	0.44	20	Nexus Mutual, Unslashed, Sherlock
Privacy	0.19	4	Tornado Cash, Aztec, Railgun
NFT Marketplace	0.13	7	Blur, NFTX, Solv platform
Oracle	0.00	4	Nest platform Staking, WitSwap, Umbrella Network

TVL is a popular metric used as an indicator for how popular and valuable a DeFi platform’s services are. It is the total value of crypto assets locked into the smart contracts that comprise the platform or application. It is calculated by multiplying the quantity of the crypto assets locked by their prices and summing them up. Most of our analysis is based on data for this measure.

One needs to be careful, however, when interpreting TVL for two main reasons. First, it does not measure the flow of inputs, but is the stock of assets being used as inputs for DeFi applications. Second, the measure cannot be directly used for the value added of services, since DeFi produces two outputs, services and tokens.

3 Some Stylized Facts

3.1 Aggregate TVL

Figure 3.1 shows the TVL of Defi platforms on Ethereum over our sample period from mid 2019 to late 2022. There are several events that are important. First, the introduction of a few decentralized apps in Summer 2020 kick-started the so-called “DeFi Summer”. Second, despite a crash associated with the crackdown on crypto in China (May 19, 2021), TVL reached its peak in late 2021 at 180bn USD. Third, there was a sharp decline throughout 2022 driven by three important events, the Fed tightening (Jan 21, 2022), the Terra Luna collapse (May 7, 2022) and the Celsius collapse (June 12, 2022).

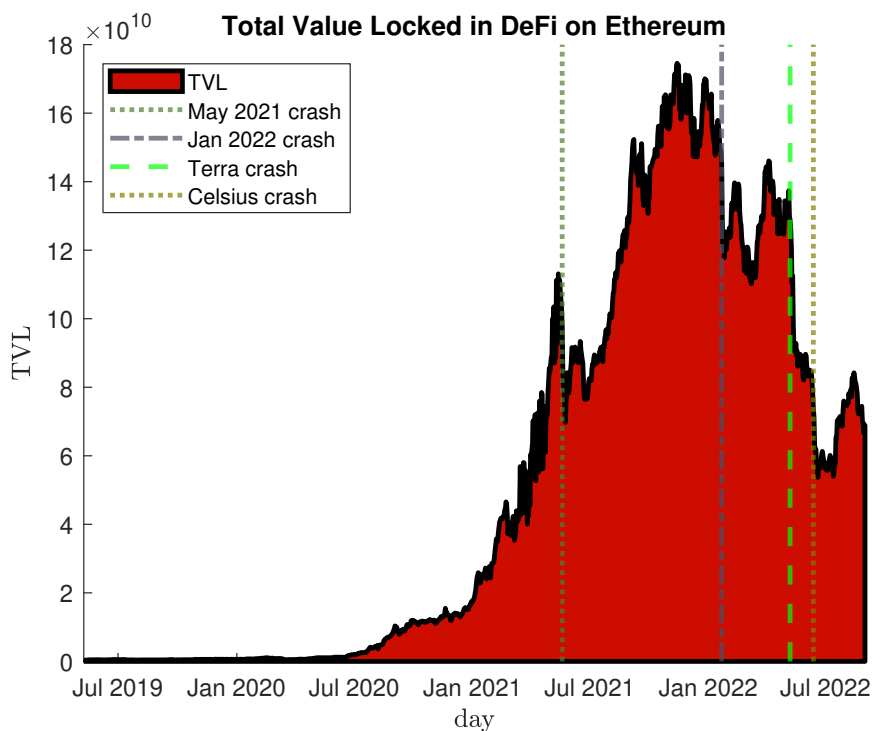


Figure 3.1: Total Value Locked in DeFi on Ethereum

Table 3.1: Destination of Token Inputs

	All	CV	1	2	3	4	5
Bridge & Cross Chain	4.78	0.59	3.77	5.88	6.91	6.30	5.26
Dec. Stablecoins	25.71	0.62	33.55	15.16	17.61	15.55	16.78
Derivatives	7.36	1.03	11.98	1.78	1.52	1.52	1.48
DEXES	12.79	0.58	14.15	10.22	9.58	10.62	16.03
Insurance	0.65	0.35	0.53	0.87	0.76	0.71	0.68
Lending	23.84	0.32	22.77	29.28	21.62	22.39	18.49
Liquid Staking	2.54	1.35	0.17	2.91	6.42	10.11	10.03
NFT Marketplace	0.07	2.11	0.00	0.01	0.42	0.25	0.22
Oracle	0.00	-	0.00	0.00	0.00	0.00	0.00
Other	7.87	0.73	4.97	11.94	9.62	10.59	12.83
Privacy	0.37	0.70	0.22	0.62	0.42	0.52	0.50
Staking	6.18	0.92	4.52	9.85	7.93	6.96	4.28
Yield & Asset Mgmt	7.82	0.80	3.38	11.49	17.19	14.48	13.41

Note: The first column is the mean share of TVL over the entire sample. The second column is the coefficient of variation. The other columns are the mean share of TVL over the five subsamples: (1) May 9, 2019 to May 18, 2021; (2) May 19, 2021 (China crackdown) to Jan 20, 2022; (3) Jan 21, 2022 (Fed tightening) to May 6, 2022; (4) May 7, 2022 (Terra-Luna collapse) to June 11, 2022; (5) June 12, 2022 (Celsius collapse) to August 31, 2022.

3.2 Destination of Token Inputs

To examine the input-output structure of tokens, we group tokens issued by DeFi platforms into 13 different sectors corresponding to different financial services. Looking at where tokens are locked into or used as inputs, Table 3.1 summarizes the destination of locked tokens with respect to average TVL over different sample periods. The first data column gives the mean share of total TVL per sector over the whole sample period, while the second data column is the corresponding coefficient of variation. Decentralized Stablecoins, Lending and Decentralized Exchanges (DEXES) are the top three sectors, accounting for over 60% of TVL.

The remaining columns give the mean shares per sector across our five subperiods defined by the events described in the previous graph. Over time, Decentralized Stablecoins, Derivatives and Lending have lost their shares over time, while DEXES, Liquid Staking, and Yield &

Asset Management have significantly gained in share of TVL. Currently, with the exception of Derivatives, these five sectors comprise about 3/4 in TVL where DeFi tokens are used as inputs.

3.3 Source of Token Inputs

Table 3.2 reports the composition of where tokens locked originate from in terms of TVL for the 13 different sectors. Again, Decentralized Stablecoins are a main source for token inputs, and towards the end of the sample period also tokens from Liquid Staking. Lending plays much less of a role, while Bridge & Cross Chain play a very important role throughout the sample.

Table 3.2: Source of Tokens Locked

	All	CV	1	2	3	4	5
Bridge & Cross Chain	13.75	0.63	12.35	16.54	15.44	15.04	12.61
Dec. Stablecoins	12.58	0.63	6.78	19.83	20.42	14.94	20.84
Derivatives	18.93	1.11	32.61	2.70	1.12	0.91	1.46
DEXES	2.95	0.90	1.13	5.49	4.77	4.65	5.21
Insurance	0.08	1.16	0.04	0.17	0.09	0.08	0.05
Lending	5.40	0.98	4.85	7.45	4.78	4.45	4.24
Liquid Staking	2.61	1.73	0.00	1.95	6.20	14.49	13.35
NFT Marketplace	0.17	2.05	0.00	0.02	1.00	0.67	0.58
Oracle	3.44	0.99	4.70	2.67	1.22	0.97	0.78
Other	22.81	0.42	25.00	16.37	22.63	22.03	27.26
Privacy	0.00	1.41	0.00	0.01	0.01	0.01	0.01
Staking	15.51	0.81	11.30	24.92	18.59	18.76	11.54
Yield & Asset Mgmt	1.76	1.04	1.24	1.88	3.74	3.00	2.06

Note: The first column is the mean share of TVL over the entire sample. The second column is the coefficient of variation. The other columns are the mean share of TVL over the five subsamples: (1) May 9, 2019 to May 18, 2021; (2) May 19, 2021 (China crackdown) to Jan 20, 2022; (3) Jan 21, 2022 (Fed tightening) to May 6, 2022; (4) May 7, 2022 (Terra-Luna collapse) to June 11, 2022; (5) June 12, 2022 (Celsius collapse) to August 31, 2022.

Note that tokens used as inputs can originate from other DeFi sectors, but also from non-DeFi, external sectors. The external sectors include native blockchain tokens (e.g., BTC) and

centralized stablecoins (e.g., USDT). Figure 3.2 partitions the sources of all locked tokens into two subsets with internal (DeFi) sectors shown in red and external sectors in blue. The top panel shows dollar values, with the bottom panel showing their shares. The plot suggests that initially all tokens locked were initially issued by external sources. Over time this share has declined significantly to about 60%.

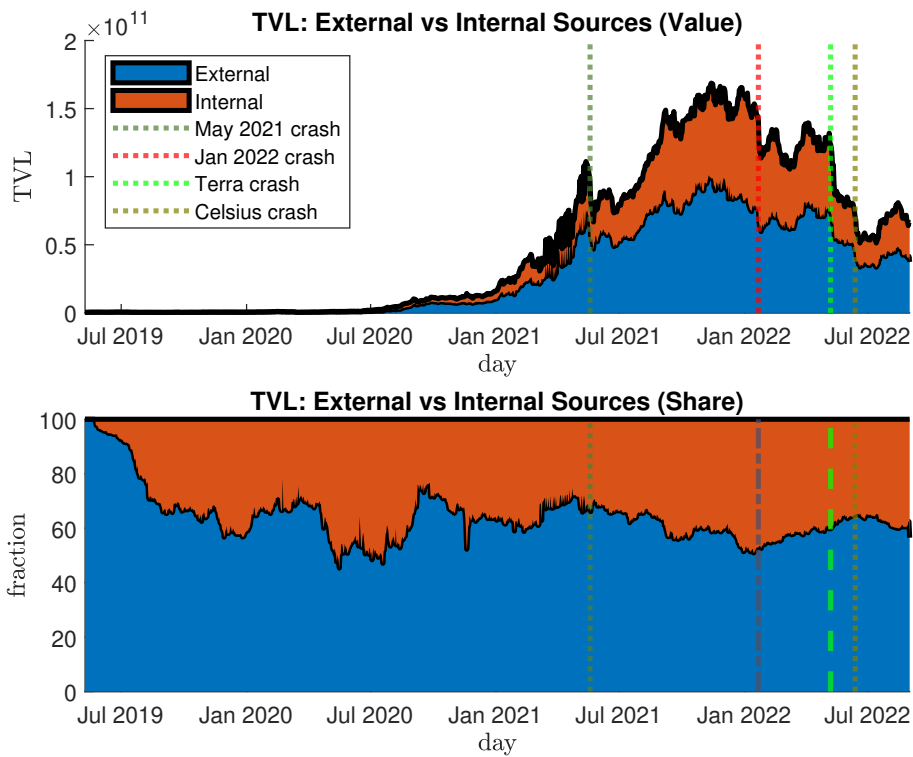


Figure 3.2: Origin: External vs. Internal Tokens

Figure 3.3 shows the composition of external tokens locked into DeFi: Bitcoin (BTC), Wrapped Ether (WETH), centralized stablecoins (e.g., USDT and USDC), and Terra (i.e., Luna and Terra USD). Initially, WETH accounted for over 95% of these external TVL, and has converged to a stable 40% share. Centralized stablecoins gained a substantial share over time and account for over 50% at the end of the sample period. BTC's share is less than 10%, and Terra has not really played a role as a token input.

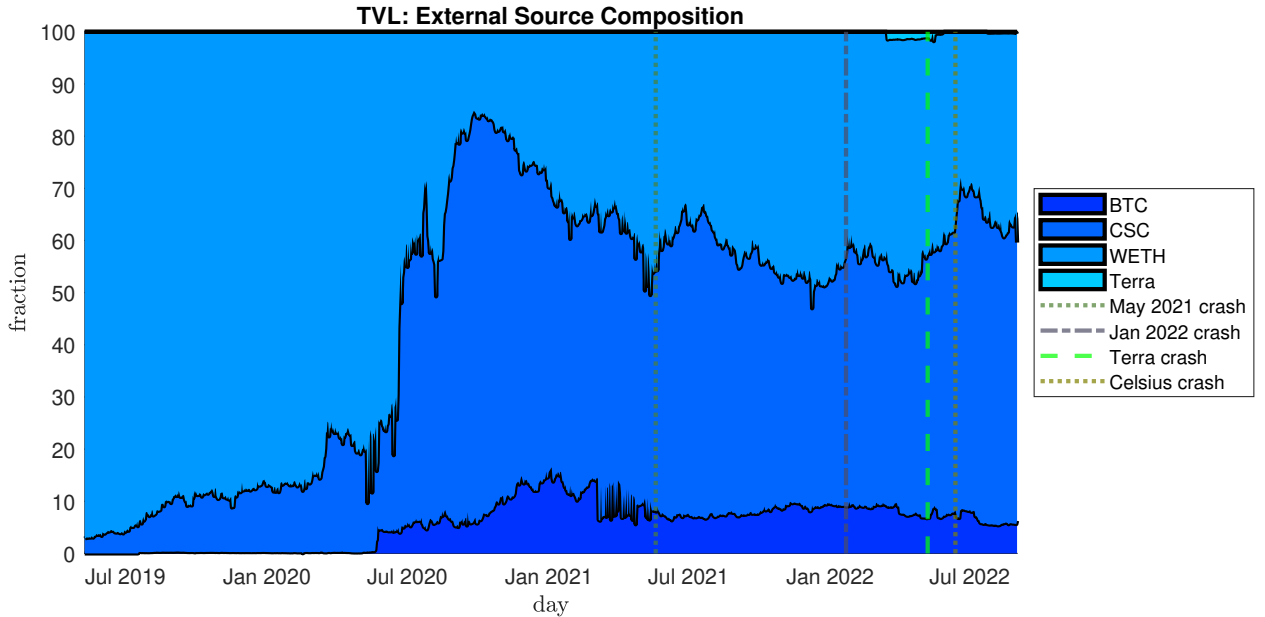


Figure 3.3: TVL: External Source Composition

3.4 Network Nodes and Edges

Figure 3.4 visualizes the network structure that arises from the usage of tokens as inputs into DeFi services. Blue nodes denote internal (DeFi) sectors, while orange nodes in the center denote external sectors. Each directed edge indicates tokens issued by a source node that are locked into a destination node, with the width representing the amount of value locked. Note that a sector can hold tokens issued by platforms from the same sector, leading to a “reflexive” edge shown as a circular edge.⁵

There are altogether 17 nodes, for the 13 DeFi sectors and the four external tokens, Bitcoin, Ethereum, Terra and centralized stablecoins. Figure 3.5 shows a summary of how links between these nodes have evolved over time. About a third of all 221 potential links between individual nodes were active in September 2022. Among these links, about one-third were unilateral edges connecting one internal node to another, but not vice versa. Bilateral

⁵The DeFi network graph for each day of the sample period can be found at https://hannayu.shinyapps.io/TVL_Network/.

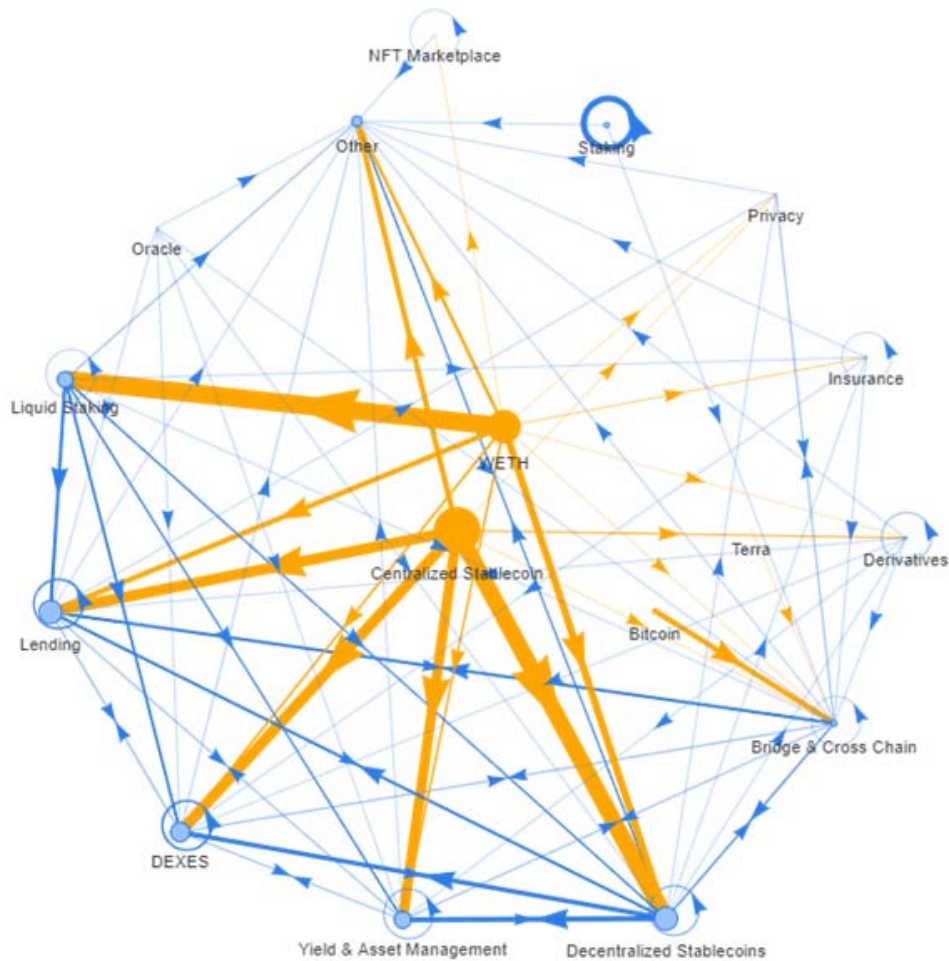


Figure 3.4: DeFi Network (September 21, 2022)

connections increased substantially in mid-2021, accounting now for about one-fifth of the connections.

3.5 Input-Output Matrix of Tokens

As discussed, a sector can both lock tokens that are issued by other sectors and issue tokens that are locked by others. The Defi network can thus be summarized by an input-output matrix as shown in Figure 3.6 where entry (i, j) gives the fraction of sector i 's TVL that is

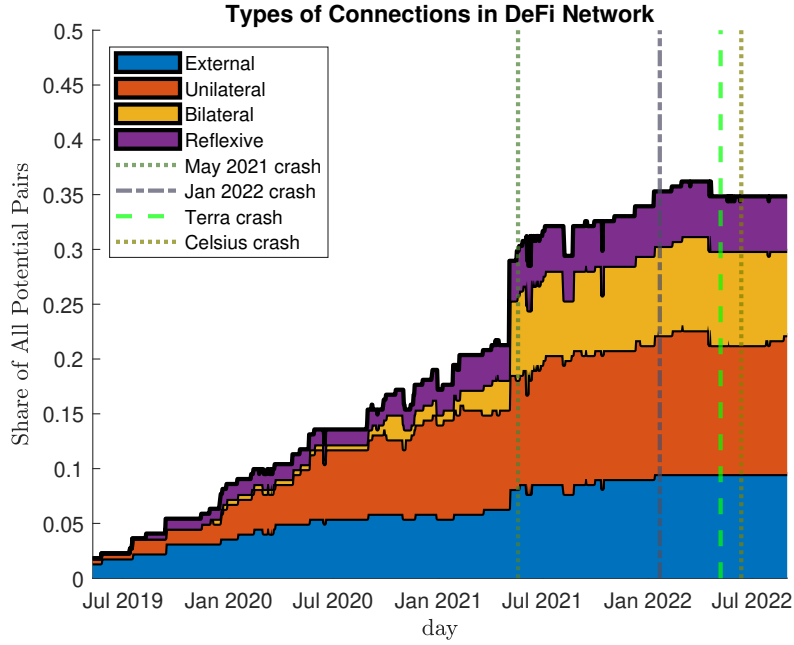


Figure 3.5: Types of Edges in DeFi Network

locked into sector j . Note that external tokens by definition do not lock other tokens. We will use the empirical information from this input-output matrix extensively to calibrate our production-network model developed in the next section.

4 A General Equilibrium Model of DeFi Network

4.1 Environment

In our environment, all production and consumption decisions will be static. Hence, we assume for ease of exposition that there is only a single period and surpress until our quantitative exercises that there are many periods. Hence, we assume that tokens are short-lived, being created in a period and destroyed at the end of the period.

There is a measure one of infinitely lived households that inelastically supplies one unit of labour. The economy is comprised of a non-DeFi sector and N DeFi sectors that produce

	BTC	Con. SC	WETH	Terra	Bridge & CC	Dec. SC	Derivatives	DEXES	Insurance	Lending	L. Staking	NFT MP	Oracle	Other	Privacy	Staking	Y & A. Mgmt
BTC	0.0	0.0	0.0	0.0	100.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0
Con. Stablecoins	0.0	0.0	0.0	0.0	0.4	14.2	1.7	29.4	0.0	25.7	0.0	0.0	0.0	13.3	0.0	0.0	15.3
WETH	0.0	0.0	0.0	0.0	0.1	33.4	1.2	1.6	2.6	26.9	17.3	0.0	0.0	9.8	1.9	0.0	5.3
Terra	0.0	0.0	0.0	0.0	100.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0
Bridge & Cross Chain	0.0	0.0	0.0	0.0	8.9	20.9	0.0	3.6	0.0	45.0	0.0	0.0	0.0	9.1	0.8	0.0	11.7
Dec. Stablecoins	0.0	0.0	0.0	0.0	0.1	20.5	0.1	3.4	0.0	51.1	0.0	0.0	0.0	8.8	0.3	0.0	15.6
Derivatives	0.0	0.0	0.0	0.0	0.0	0.0	97.1	0.4	0.0	1.4	0.0	0.0	0.0	1.1	0.0	0.0	0.1
DEXES	0.0	0.0	0.0	0.0	0.0	1.6	0.0	58.2	0.0	21.7	0.0	0.0	0.0	14.5	0.0	0.0	4.1
Insurance	0.0	0.0	0.0	0.0	0.0	0.0	0.0	13.3	69.3	0.0	0.0	0.0	0.0	17.4	0.0	0.0	0.0
Lending	0.0	0.0	0.0	0.0	1.1	9.2	0.0	0.9	0.0	72.2	0.0	0.0	0.0	11.5	0.0	0.0	5.0
Liquid Staking	0.0	0.0	0.0	0.0	0.0	12.5	0.0	4.4	6.3	30.6	3.0	0.0	0.0	5.4	0.0	0.0	37.8
NFT Marketplace	0.0	0.0	0.0	0.0	0.0	0.0	0.0	1.2	0.0	0.0	0.0	98.8	0.0	0.0	0.0	0.0	0.0
Oracle	0.0	0.0	0.0	0.0	0.1	15.1	0.0	9.1	0.0	68.4	0.0	0.0	0.0	5.8	0.0	0.0	1.6
Other	0.0	0.0	0.0	0.0	2.4	1.5	0.0	4.5	0.0	11.7	1.3	0.0	0.0	39.2	0.0	0.0	39.4
Privacy	0.0	0.0	0.0	0.0	0.1	0.0	0.0	38.3	0.0	0.0	0.0	0.0	0.0	61.6	0.0	0.0	0.0
Staking	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	100.0	0.0
Yield & Asset Mgmt	0.0	0.0	0.0	0.0	10.7	12.2	0.0	1.2	0.0	11.1	0.0	0.0	0.0	3.3	0.0	0.0	61.5

Figure 3.6: Input-output Matrix

both tokens and DeFi services. There is also one external token available. Production of DeFi services and tokens use labour, other services and tokens and external tokens as inputs. The non-DeFi sector only uses labour as input to produce a single good. Figure 4.1 illustrates our setup.

Production On the production side our set-up is an extension of Carvalho and Tahbaz-Salehi (2019) that sets up a production network of N industries. Each platform creates tokens (denoted by $k = c$) and services (denoted by $k = s$) by employing labor and other tokens as inputs. The production function for each DeFi sector n is given by

$$y_{nk} = \epsilon_{nk} \zeta_{nk} (l_{nk})^{\alpha_{nk}} \prod_{j=1}^N (x_{nk,jc})^{a_{nk,jc}} (x_{nk,e})^{a_{nk,e}}, \quad (1)$$

where $\alpha_{nk} + \sum_{j=1}^N a_{nk,jc} + a_{nk,e} = 1$ for $k = c, s$. Here, y_{nc} and y_{ns} are the output of sector n of tokens and services, while $x_{nk,j}$ and $x_{nk,e}$ are the inputs of tokens from sector j and external tokens respectively for the production of $k = c, s$. Labour input is denoted by l_{nk} .

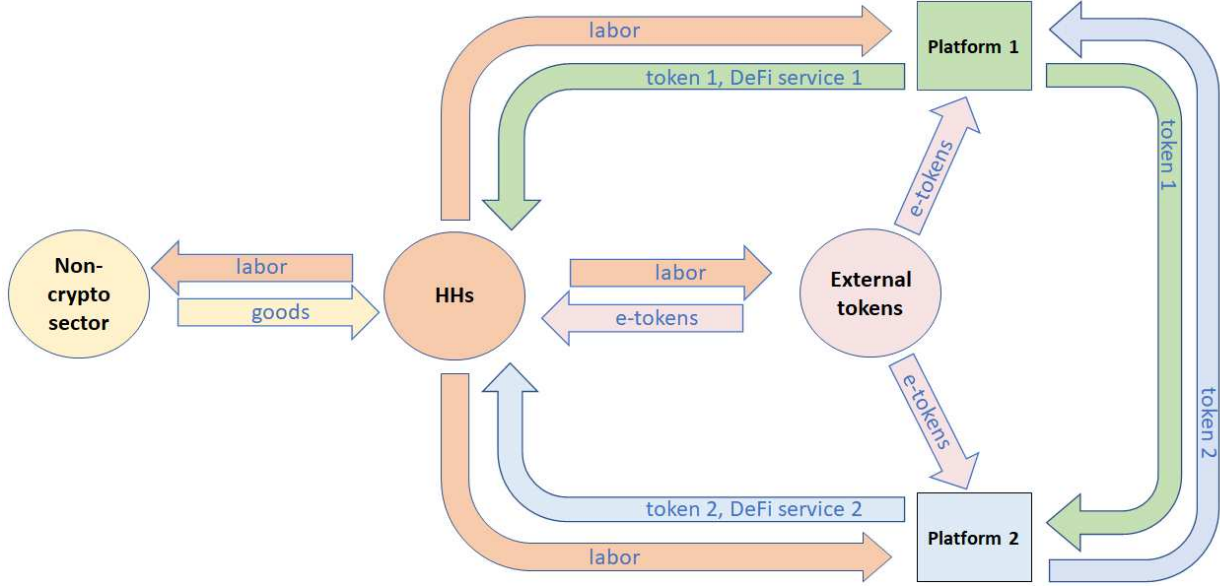


Figure 4.1: Setup of the Model

For convenience, we normalize $\zeta_{nk} = \alpha_{nk}^{-\alpha_{nk}} \prod_{j=1}^N a_{nk,jc}^{-a_{nj,jc}}$.

Denote the wage rate by w and the prices of services and tokens by (p_{ns}, p_{nc}, p_e) . The sector then maximizes profits by solving

$$\max_{\{x_{nk,jc}, x_{nk,e}, l_{nk}\}} \sum_{k=c,s} \left[p_{nk} y_{nk} - \sum_{j=1}^N p_{jk} x_{nk,jc} - p_e x_{nk,e} - w l_{nk} \right] \quad (2)$$

with the first-order conditions given by

$$l_{nk} = \alpha_{nk} p_{nk} y_{nk} / w, \quad (3)$$

$$x_{nk,jc} = a_{nk,jc} p_{nk} y_{nk} / p_{jc}, \quad (4)$$

$$x_{nk,e} = a_{nk,e} p_{nk} y_{nk} / p_e. \quad (5)$$

Finally, there are two other sectors that produce external tokens y_e and a non-DeFi good y_o with linear technology

$$y_k = \epsilon_k l_k. \quad (6)$$

We denote p_e and p_o as the respective prices. Profit maximization implies that

$$p_k = \frac{w}{\epsilon_k}. \quad (7)$$

Household Problem The representative household has an endowment of H units of labour which it inelastically supplies. Preferences are defined over tokens, services and goods $(z_{nc}, z_e, z_{ns}, z_o)$ and given by

$$\sum_{k=c,s} \sum_{n=1}^N \delta_{nk} \log(z_{nk}/\delta_{nk}) + \delta_e \log(z_e/\delta_e) + \delta_o \log(z_o/\delta_o), \quad (8)$$

where $\sum_{k=c,s} \sum_{n=1}^N \delta_{nk} + \delta_e + \delta_o = 1$. Using the budget constraint

$$wH \geq \sum_{k=c,s} \sum_{j=1}^N p_{nk} z_{nk} + p_e z_e + p_o z_o, \quad (9)$$

demand is given by

$$z_{nk} = \frac{\delta_{nk} w H}{p_{nk}}, \quad (10)$$

$$z_e = \frac{\delta_e w H}{p_e}, \quad (11)$$

$$z_o = \frac{\delta_o w H}{p_o}. \quad (12)$$

4.2 Equilibrium Characterization

The economy can be reformulated as a standard input-output network with $I = 2(N + 1)$ industries. To do so, we only have to slightly adjust the indexation of goods. The matrix

$$\hat{\mathbf{A}} = \begin{bmatrix} a_{11} & \cdots & a_{1,I} \\ \vdots & \ddots & \vdots \\ a_{1,I} & \cdots & a_{I,I} \end{bmatrix} \quad (13)$$

expresses the coefficients of production where

- $i = 1, \dots, N$ represents N DeFi tokens nc
- index $i = N + 1, \dots, 2N$ represents N DeFi services ns
- index $i = 2N + 1$ represents the external token e
- index $i = 2(N + 1)$ represents the non-DeFi good o .

Since we have constant returns to scale, the production function can be summarized by this matrix and the vector of productivities $\boldsymbol{\epsilon}$.

Analogously, we can define an $I \times I$ matrix of inputs, \mathbf{X} , where entry x_{ij} expresses the quantity of j used in the production of i . Similarly, \mathbf{Y} and \mathbf{p} are vectors of length I expressing total output and prices of the different tokens and services.

The equilibrium is then described by the first-order conditions of the $N + 2$ sectors, the households optimal demands and the market clearing conditions for output $j = 1, \dots, 2(N+1)$

$$y_j = z_j + \sum_{i=1}^I x_{ij}.$$

Denote $\hat{\mathbf{p}} = (\log(p_1/w), \dots, \log(p_{2N+2}/w))$. Similarly, denote $\hat{\mathbf{y}} = \log \mathbf{y}$ and $\hat{\boldsymbol{\epsilon}} = \log \boldsymbol{\epsilon}$. The following result follows immediately from Carvalho and Tahbaz-Salehi (2019).

Proposition 1. *Equilibrium quantities and prices are given by*

$$\hat{\mathbf{p}} = -(\mathbf{I} - \mathbf{A})^{-1}\hat{\boldsymbol{\epsilon}} \quad (14)$$

$$\hat{\mathbf{y}} = (\mathbf{I} - \mathbf{A})^{-1}\hat{\boldsymbol{\epsilon}} + \log((\mathbf{I} - \mathbf{A}')^{-1}\boldsymbol{\delta}) \quad (15)$$

In equilibrium, prices are only driven by productivity shocks $\boldsymbol{\epsilon}$ and the network structure, \mathbf{A} , while quantities also depend on preferences as expressed by the parameter $\boldsymbol{\delta}$. The structure is familiar from the macroeconomics literature with the expression $\mathbf{L} \equiv (\mathbf{I} - \mathbf{A})^{-1} = \sum_{k=0}^{\infty} \mathbf{A}^k$ being the Leontief inverse, while $\boldsymbol{\lambda} \equiv (\mathbf{I} - \mathbf{A}')^{-1}\boldsymbol{\delta}$ expresses the Domar weights.

4.3 Identification of Parameters from Data

We slightly abuse notation and denote the submatrices of \mathbf{A} by A_{ij} where $i, j \in c, s, e, o$. By the functional form of the production function, we have assumed that services and the non-DeFi good are never used as inputs so that

$$A_{co} = A_{so} = A_{cs} = A_{ss} = 0 \quad (16)$$

Furthermore, there are no inputs other than labour used to produce external goods and tokens; i.e., $A_{ej} = A_{oj} = 0$ for $j \in c, s, e, o$. We make one additional assumption to identify the parameters of the model from the data.

Assumption 2. *For each DeFi sector, token inputs are allocated equally to the production of its tokens and services; i.e. $A_{sc} = A_{cc} \equiv A_c$ and $A_{se} = A_{ce} \equiv A_e$. The input-output matrix*

is thus given by

$$\mathbf{A} = \begin{bmatrix} A_c & 0 & A_e & 0 \\ A_c & 0 & A_e & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix}$$

We have from Proposition 1, that the output of tokens is given by

$$\log(y_{nc}) = \sum_{i=1}^I \ell_{n,i} \epsilon_i + \log(\lambda_n) \quad (17)$$

$$\log(y_e) = \sum_{i=1}^I \ell_{2N+1,i} \epsilon_i + \log(\lambda_{2N+1}), \quad (18)$$

where l_{ij} are entries in the Leontief matrix \mathbf{L} and λ_i are entries in the vector of Domar weights $\boldsymbol{\lambda}$. Cobb-Douglas preferences and technologies together with constant-returns-to-scale imply that productivity shocks only propagate downstream; i.e., shocks to inputs influence output, but not vice versa. Equation (16) then implies that productivity shocks to services and external goods have no influence on token production. Define then the $(N + 1) \times (N + 1)$ matrix

$$\tilde{\mathbf{A}} = \begin{bmatrix} A_c & A_e \\ 0 & 0 \end{bmatrix}. \quad (19)$$

The following proposition establishes that one can directly back out the productivity shocks for DeFi and external tokens, without relying on information about DeFi services production.

Proposition 3. *Productivity and output of tokens are given by the relationship*

$$\hat{\boldsymbol{\epsilon}} = (\mathbf{I} - \tilde{\mathbf{A}})^{-1} \left[\hat{\mathbf{y}} - \log \left((\mathbf{I} - \tilde{\mathbf{A}}')^{-1} \tilde{\boldsymbol{\delta}} \right) \right]$$

where $(\hat{\boldsymbol{\epsilon}}, \hat{\mathbf{y}}, \tilde{\boldsymbol{\delta}})$ are $N + 1$ dimensional vectors associated with DeFi tokens and the external token.

This allows us to calibrate all the parameters of the model from the data. First, we use the empirical input-output matrix together with the assumed production function and Assumption 2 to derive $\tilde{\mathbf{A}}$. Second, we can use data on the market value of tokens and data on disposable household income to obtain $\tilde{\delta}$ from equation (10). Finally, Proposition 3 together with price data on tokens will pin down productivity $\tilde{\epsilon}$.

5 Quantitative Exercise

5.1 Data and Model Calibration

We reduce the complexity of the model by consolidating DeFi individual sectors into $N_c = 6$ main Defi sectors: Bridge, DSC, Derivatives, DEXES, Lending, and Others. Similarly, we will focus only on $N_e = 4$ external non-Defi tokens: BTC, CSC, ETH, Terra and a consolidated sector of centralized stablecoins (CSC). This implies that the DeFi network will be summarized by a 10×10 input-output matrix $\tilde{\mathbf{A}}$.

We calibrate the model using one year of DeFi data from September 2021 to August 2022. First, we use data on TVL from DeFiLlama discussed earlier to find the input-output matrix. Second, token price and quantity data from CoinMarketCap, Dune Analytics and Etherscan. Since each sector provides multiple tokens, we follow the standard NIPA approach to derive a price and quantity index for each sector (see the appendix for details). Since tokens are long-lived in the data, we convert them into one period prices according to

$$p_{nc}(1 - \beta) \text{ and } p_e(1 - \beta)$$

where β is a discount factor derived from the annual discount rate of 4%. Finally, we use Disposable Personal Income (DSPI) data from the FRED database to match the total labor income wH in the model. We can then use price and quantity data on tokens to determine

how many tokens z_c and z_e are held by households to determine the preference parameters.

5.2 Measurement – Total Value Added and Service Output of DeFi

While TVL is a popular metric used to measure the scale and growth of DeFi, it is subject to two problems. First, it ignores the input-output structure of the DeFi system, leading to a double-counting problem. Second, TVL measures a stock and, hence, does not capture the flow of DeFi output, especially services.

Guided by national income accounting, we can provide more meaningful measures for the value added of DeFi and for services provided. Zero profits imply that the value of total output – from tokens and services – has to be equal to the value of total inputs – from tokens and labour. Hence, the value of labour input directly measures the value added from DeFi.

Developers and people that maintain application on DeFi applications are often remunerated in the form of governance tokens. Our approach is thus to use the value of such tokens – adjusted to a flow value by using a 4% discount rate – as a proxy to measure the spending on labor input for each DeFi sector.

Figure 5.1 shows TVL on Ethereum from Sep 2021 to Aug 2022 is represented by the black line. A back-of-envelope calculation suggests that, during our sample period, life-time value added of a sector is about 40% of TVL and annual value added is about 1.6% of TVL as shown by the other lines in Figure 5.1). While our calculation is limited by modeling assumptions and various data restrictions, the main takeaway is that we need to be cautious when we use TVL to measure DeFi activities due to the problems pointed out above.

Next, we can decompose total revenue into its two components, revenue from tokens and services. Using the convention from national income accounting, total revenue is equal to

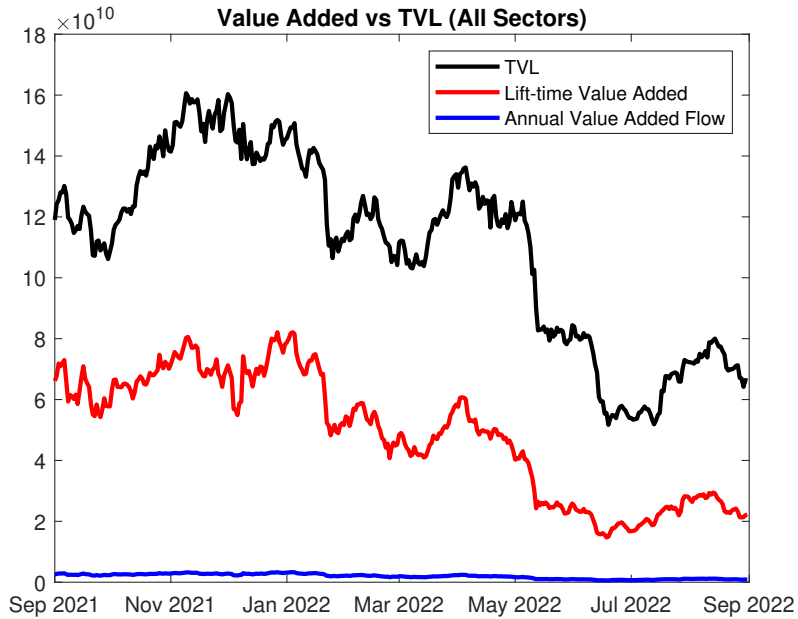


Figure 5.1: Value Added vs. TVL

the value of inputs used, which is the sum of TVL and the value of governance tokens that proxies for the cost of labour inputs. The value of DeFi services produced is then simply total revenue minus the value of tokens produced. Figure 5.2, we find that overall, services account for 30% of DeFi outputs. The share of services is highest in the lending sector and lowest in the Bridge & Cross-chain sector.

5.3 Identification – Sources of DeFi Token Price Changes

After relying simply on accounting principles, we now make use the calibrated production-network model. We first explain the total change in a price index of all DeFi tokens across our sample period. To do so, we can use Proposition 1 to decompose the variations of the DeFi token prices p into three factors. We first hold the input-output matrix and the productivity of DeFi tokens fixed at their initial value and allow only the productivity of external token production to vary. Then, we also include the variation in DeFi productivity,

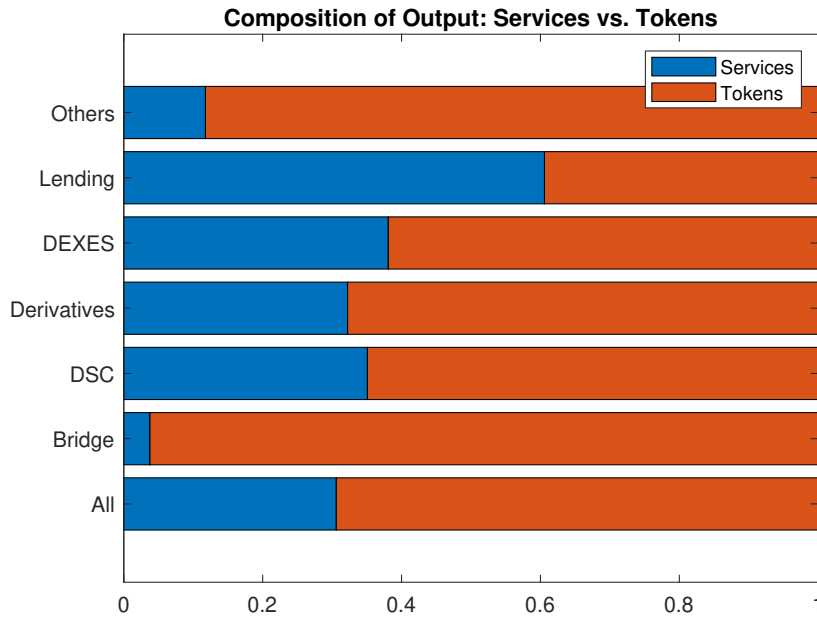


Figure 5.2: Composition of Output; Services vs. Tokens

before finally incorporating also the variation in the input-output matrix. Figure ?? shows the result of this exercise with the three percentages adding up to the total price change across the sample period.

Overall, DeFi token prices declined during the sample period. Our decomposition exercise suggests that productivity shocks from external tokens alone account for about 43 % of the price decline. Productivity shocks within DeFi sectors contribute to an additional 86% of the decline. Finally, changes in the network structure partially offset some of the price decline.

5.4 Simulation – More Connected DeFi Network

We can also use our model to perform a simulation exercise. Empirical evidence suggests that the DeFi system has become increasingly connected over time. What is the impact on nominal outputs of different sectors when the share of all tokens as inputs increases by 1% at the expense of labour input. Figure ?? shows large positive responses of both, the aggregate

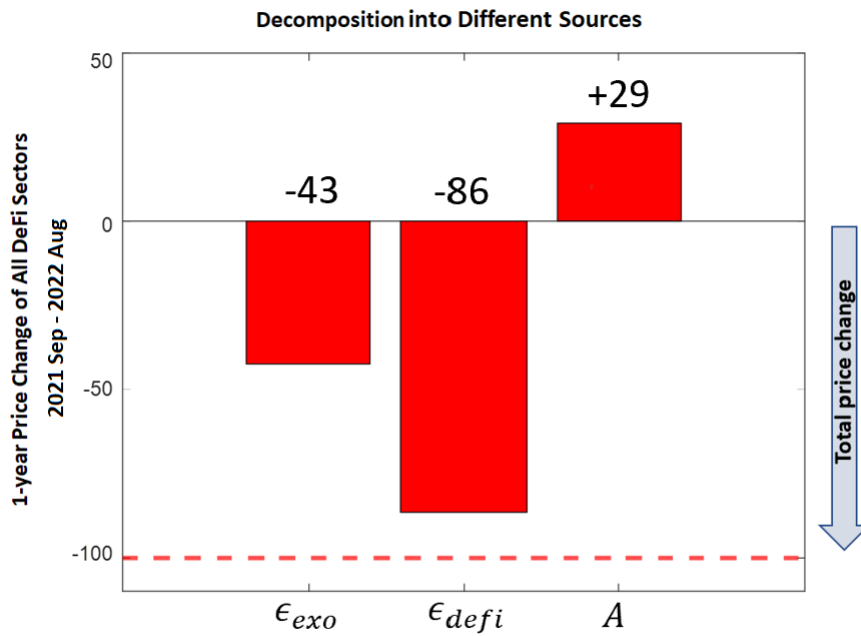


Figure 5.3: Decomposition of DeFi Token Price Changes

nominal output of DeFi sectors (0.34%) as well as the market cap of Ether (0.35%).

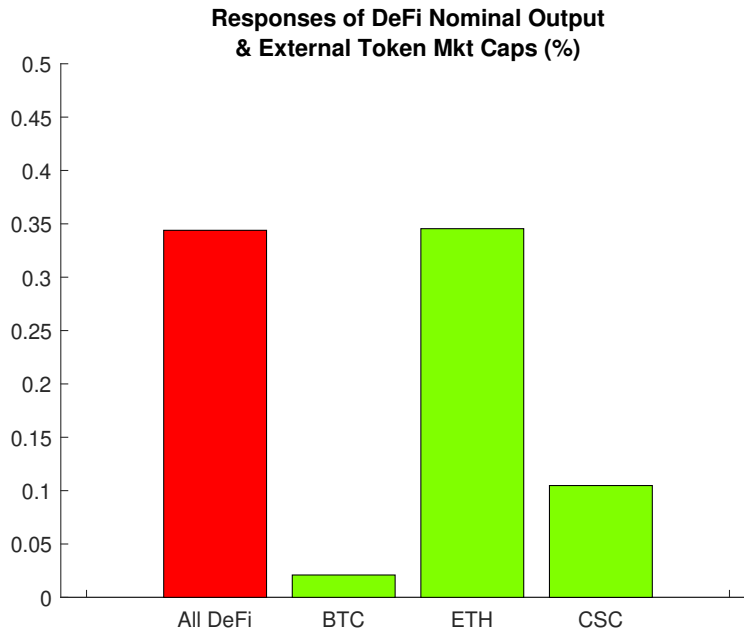


Figure 5.4: Responses of DeFi Nominal Output and External Token Market Caps

6 Conclusion

We have demonstrated that the output of DeFi and its token prices depend on a complex structure of interconnectedness. Hence, it is imperative to look at DeFi from the perspective of an input-output network that primarily operates via tokens produced by some DeFi sectors being used as inputs by other sectors. This simple insight has allowed us to make progress to properly measure the value-added of DeFi, estimate the service outputs of DeFi sectors, quantify the impact of different forces driving DeFi token prices and predict the effects of increasing network interconnectedness. Our work shows that a network approach is promising for studying DeFi. Future work will have to incorporate network structure, but needs to take the particular nature of tokens as assets more seriously.

A Proofs

A.1 Proof of Proposition 1

The price equation can be derived by plugging the sector's first-order conditions into the production function, taking logs and stacking the $2N + 2$ equations to arrive at a matrix formulation.

For deriving quantities, we first define a sector's Domar weights as the fraction of aggregate nominal output, $\mathcal{Y} = \sum_i \hat{p}_i \hat{y}_i$, or

$$\lambda_i = \frac{p_i y_i}{\mathcal{Y}}.$$

The quantity equation is then obtained in two steps.

Lemma 4. *The log output of industry i is given by*

$$\log(y_i) = \sum_{j=1}^I \ell_{ij} \log \epsilon_j + \log(\lambda_i).$$

Proof. Divide the Domar weight by y_i and taking the log gives us

$$\log\left(\frac{p_i}{\mathcal{Y}}\right) = \log(\lambda_i) - \log(y_i).$$

Using the equilibrium vector of relative prices $\hat{\mathbf{p}} = -\mathbf{L}\hat{\boldsymbol{\epsilon}}$ we obtain for each i

$$\log\left(\frac{p_i}{\mathcal{Y}}\right) = -\sum_{j=1}^I \ell_{ij} \log \epsilon_j$$

or

$$\log(y_i) = \sum_{j=1}^I \ell_{ij} \log \epsilon_j + \log(\lambda_i),$$

which is the log output of industry i .

Lemma 5. *The Domar weights can be calculated as*

$$\lambda_i = \sum_{j=1}^I \delta_j \ell_{ji},$$

where ℓ_{ji} is the (j, i) element of the Leontief inverse \mathbf{L} .

Proof. Plug the platforms' and households' first-order conditions into the market clearing conditions and divide by wH to obtain

$$p_j y_j / wH = \delta_j wH / wH + \sum_{i=1}^I a_{ij} p_i y_i / wH.$$

Note here that the value added in this economy is equal to the household's labor income, since households supply one unit of labor while firms earn zero profits. Using the definition of Domar weights, we can then rewrite this equation as

$$\lambda_j = \delta_j + \sum_{i=1}^I a_{ij} \lambda_i.$$

Stacking these equations and solving for the vector of Domar weights, we have

$$\begin{aligned} \lambda &= \delta + \mathbf{A}' \lambda \\ &= (\mathbf{I} - \mathbf{A}')^{-1} \delta. \end{aligned}$$

Recall that $\mathbf{L} = (\mathbf{I} - \mathbf{A})^{-1}$. Then for each industry i , we have

$$\lambda_i = \sum_{j=1}^I \delta_j \ell_{ji}.$$

□

Combining the results from the two lemmas, we obtain the quantity equation which completes

the proof.

A.2 Proof of Proposition 3

We first prove the following auxiliary lemma.

Lemma 6. $\ell_{n,N+k} = 0$ for $n \in \{1, \dots, N, 2N + 1\}$ and for $k \in \{1, \dots, N, N + 2\}$.

Proof. Recall that the inverse of a matrix \mathbf{M} is $\mathbf{M}^{-1} = \frac{adj\mathbf{M}}{|\mathbf{M}|}$, where the (i, j) 'th element of $adj\mathbf{M}$ is given by the cofactor $|C_{ji}|$.

For the Leontief matrix $\mathbf{L} = \mathbf{I} - \mathbf{A}$, the entries representing DeFi services are given by the columns $N + 1, \dots, 2N$. Since services are not inputs into token production, we have for the following structure for these columns

$$\begin{bmatrix} \mathbf{0}_{N \times N} \\ \mathbf{I}_{N \times N} \\ \mathbf{0}_{2 \times N} \end{bmatrix}.$$

This implies that, for $n, k = 1, \dots, N$, the matrix for calculating cofactor $|C_{N+k,n}|$ has a null column after deleting the $N + k$ 'th row. Hence, $|C_{N+k,n}| = 0$ and $\ell_{n,N+k} = 0$ for $n, k = 1, \dots, N$. A similar argument applies for $\ell_{n,2N+2} = 0$ expressing the goods sector. \square

The Lemma directly implies that productivity shocks of DeFi services and the regular good have no influence on the output of tokens. Hence, we can rewrite our input-output network only in tokens which completes the proof.

B Constructing Price and Quantity Indices for Tokens

B.1 Basic Idea

We follow a basic NIPA approach to decompose nominal output (Y_t) into a quantity (y_t) and a price (p_t) for each DeFi sector in our data work. For most tokens – except for stablecoins –, prices change more frequently than the quantities of tokens. Hence, we adopt the following approach:

1. Break the sample into small time intervals (e.g., every ten days and every time the set of tokens is updated).
2. Use a fixed quantity basket to measure the price change within the time interval.
3. Determine the change in quantity as a residual.
4. Update the quantity basket for the new interval.

B.2 Decomposition Procedure

Consider a sector with “firms” (aka different DeFi applications) indexed by j . We follow the following notation:

- output of firm j : $y_t(j)$ [i.e., number of tokens],
- price of firm j : $p_t(j)$ [i.e., price of tokens],
- set of firms J_t [i.e., set of tokens],
- sectoral nominal output: $Y_t = \sum_{J_t} y_t(j)p_t(j)$ [i.e., market cap of tokens].

Step 1: Given a sample covering periods $t = 1, 2, 3, \dots$, partition the sample into time intervals $T(i) = [t_0(i), \dots, t_1(i)]$ indexed by i such that the set of firms is fixed within each interval:

$$J_t = J(i) \text{ for all } t \in T(i)$$

Step 2: Let $i = 1$. Given any arbitrary y_1 , decompose Y_t into y_t and $p_t = Y_t/y_t$ for all $t \in T(1)$. The calculations are as follows:

(i) obtain period 1 price level

$$p_1 = \frac{Y_1}{y_1}$$

(ii) compute price change using the fixed basket $\{y_1(j)\}$ at $t = 1$

$$\pi_2 = \frac{\sum_{J_t} y_1(j)p_2(j)}{\sum_{J_t} y_1(j)p_1(j)}$$

(iii) derive price and real output for period 2

$$\begin{aligned} p_2 &= \pi_2 p_1 \\ y_2 &= \frac{Y_2}{p_2} \end{aligned}$$

(iv) repeat (ii) and (iii) for $t \in [t_0(1) + 1, \dots, t_1(1)]$

$$\begin{aligned} \pi_t &= \frac{\sum_{J_t} y_1(j)p_t(j)}{\sum_{J_t} y_1(j)p_{t-1}(j)} \\ p_t &= \pi_t p_{t-1} \\ y_t &= \frac{Y_t}{p_t} \end{aligned}$$

Step 3: Update the firm sample at $t = t_1(i) + 1 = t_0(i + 1)$ to $J(i + 1)$. The detailed

calculations are as follows:

(i) obtain the set of firms that belong to both intervals $\bar{J}(i) = J(i) \cap J(i+1)$

(ii) compute price change between the two intervals using the fixed basket $\{y_{t_1(i)}(j)\}$ at $t = t_1(i)$

$$\pi_{t_1(i)+1} = \frac{\sum_{\bar{J}(i)} y_{t_1(i)}(j) p_{t_1(i)+1}(j)}{\sum_{\bar{J}(i)} y_{t_1(i)}(j) p_{t_1(i)}(j)}$$

(iii) derive price and real output for the first date of the new interval

$$\begin{aligned} p_{t_0(i+1)} &= p_{t_1(i)+1} = \pi_{t_1(i)+1} p_{t_1(i)} \\ y_{t_0(i+1)} &= y_{t_1(i)+1} = \frac{Y_{t_1(i)+1}}{p_{t_1(i)+1}} \end{aligned}$$

(iv) Repeat steps 2 and 3

B.3 Centralized stablecoins

For centralized stablecoins, since their prices are always close to 1, we use a fixed price basket and allow quantities to change.

Step 1: partition the sample into time intervals $T(i)$

Step 2: For $i = 1$, we have $p_1 = 1$:

(i) obtain period 1 output level

$$y_1 = \frac{Y_1}{p_1}$$

(ii) compute output change using the fixed basket $\{p_1(j)\}$ at $t = 1$

$$g_2 = \frac{\sum_{J_t} p_1(j) y_2(j)}{\sum_{J_t} p_1(j) y_1(j)}$$

(iii) derive price and real output for period 2

$$\begin{aligned}y_2 &= g_2 y_1 \\ p_2 &= \frac{Y_2}{y_2}\end{aligned}$$

(iv) repeat (ii) and (iii) for $t \in [t_0(1) + 1, \dots, t_1(1)]$:

$$\begin{aligned}g_t &= \frac{\sum_{j_t} p_1(j) y_t(j)}{\sum_{j_t} p_1(j) y_{t-1}(j)} \\ y_t &= g_t y_{t-1} \\ p_t &= \frac{Y_t}{y_t}\end{aligned}$$

Step 3: Update the firm sample at $t = t_1(i) + 1 = t_0(i + 1)$ to $J(i + 1)$:

(i) derive price and real output for the first date of the new interval

$$\begin{aligned}p_{t_0(i+1)} &= p_{t_1(i)+1} = p_{t_1(i)} \\ y_{t_0(i+1)} &= y_{t_1(i)+1} = \frac{Y_{t_1(i)+1}}{p_{t_1(i)+1}}\end{aligned}$$

(ii) Repeat steps 2 and 3

C Data Appendix

C.1 TVL Network

TVL Data Our main data source is DeFiLlama. We downloaded the complete dataset in a CSV file from the website on September 21, 2022. While this download option is no longer available, it can be reconstructed using their APIs.

The TVL on DeFiLlama does not include tokens that are not circulating or are yet to be issued. DeFiLlama also does not count token issuance within the same platform: if users deposit a token, obtain a receipt token in return, and deposit the receipt token back into the same platform, then the value is only counted once.

The majority of TVL feeds from DeFi platforms into DeFiLlama are contributed and maintained by the developers of those platforms. These feeds make calls to the blockchain or some API endpoint to return the balances of all assets locked in a platform. The code for these feeds is open source and can be examined on DeFiLlama’s GitHub repository. Almost all the prices used to calculate the USD value of TVL are from CoinGecko’s API. When tokens are not listed on CoinGecko, DeFiLlama uses on-chain methods to estimate their value, most commonly by comparing the pool weights of a very liquid Uniswap V2 market.

Data for the WBTC platform was missing at certain dates on DeFiLlama. We replaced the WBTC data by using CoinMarketCap, with the assumption that the total supply of WBTC is equivalent to the amount of BTC locked into the WBTC wrapping platform.

Token-Platform-Sector Mapping. In order to analyze the network at the sectoral level, we put together a mapping of the main tokens on Ethereum, the platforms that issue them, and the sector in which we categorize those platforms. The tokens and platforms identified in this mapping include the top 30 platforms from May 11, 2022, which account for 90% of the TVL on Ethereum, and the top 30 tokens which contribute to 95% of the TVL on

Ethereum. The remaining tokens and platforms are grouped into a category called “Other”. We created our list of sectors starting with categories provided by DeFiLlama but combine some of them for simplicity.

C.2 Token Market Cap

DeFi Tokens We obtain token prices from Dune Analytics, which in turn obtains prices from Coinpaprika. The price on Dune is the volume-weighted price based on real-time market data, translated to USD.

For token quantities, we use the total supply provided by Etherscan’s API, which is based on the “totalSupply” function available on token contracts as part of the ERC20 standard. This does not take into account other mint/burn events, if the contract itself does not record it.

External Tokens We retrieve price and quantity data for bitcoin, ether, Terra, and UST from CoinMarketCap.