Sequentially Mixed Search and Equilibrium Price Dispersion*

Shouyong Shi
Pennsylvania State University
(sus67@psu.edu)

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Abstract

Many markets feature sequentially mixed search (SMS), which has directed search for an offer distribution followed by noisy matching with multiple offers. I construct a tractable model of SMS, establish existence of a unique equilibrium, and analyze the novel implications of the equilibrium on quantities and price dispersion. Moreover, I show that an increase in the meeting efficiency widens price dispersion. An extension endogenizing search effort shows that the equilibrium is constrained inefficient, where visitors' search effort is inefficiently high and can be a strategic complement. Under a mild condition, policies that restore efficiency should lean against the wind by increasing the tax on the joint value of trade in an economic boom and reducing this tax in a recession.

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1. Introduction

A real estate agent often asks a house buyer what price range and neighborhood the buyer is searching for. The agent gives a list of houses on the market that fit into the buyer's description and the buyer can visit more than one house on the list before making an offer. Similarly, when searching for a job, a worker has a desired range of wages and restricts the applications to such wages. The worker may be interviewed for several jobs that fit into the desired range before choosing whether to accept one of them. In the market for goods and services, a consumer narrows down search to a specific range of prices and conditions and, after getting a number of quotes in the range, chooses whether to trade. An example is online booking of airline tickets or hotel rooms.

The common feature of all these markets is *sequentially mixed search* (SMS) – directed search for a price distribution followed by noisy matching with multiple offers. In the *meeting* stage, individuals choose a price distribution to search, with the expectation that the meeting rate and the price distribution are related. For example, searching in a distribution that has a higher maximum house price increases the probability of finding such a house. In this stage, searchers cannot target any particular offer because they do not know who offers what in the targeted distribution. The second stage – noisy matching – is as in Burdett and Judd (1983, henceforth BJ). A searcher can receive multiple meetings drawn from the targeted distribution and chooses which one to accept.

Given the ubiquitous nature of SMS, it is surprising that most papers in the literature have, instead, modelled search to be either purely directed or purely undirected.[1] Purely directed search assumes that searchers know what is offered by each individual on the other side of the market. This requirement on the information is too strong to be realistic in most markets and often makes the offer distribution degenerate in the equilibrium. For example, even though a large fraction of workers "have a pretty good idea" about what a firm offers before being interviewed (Hall and Krueger, 2012), this information should be interpreted

[1] The literature on undirected search goes back at least to Diamond (1971). For directed search models, well-known examples are Peters (1991), Montgomery (1991), Moen (1997), Julien et al. (2000), Burdett et al. (2001), and Shi (2001). See additional references later in this Introduction.

as a narrow range of wages. On the other extreme, purely noisy matching assumes that search is undirected. This assumption prevents searchers from making the tradeoff between prices and the meeting rate, and results in generic inefficiency in the equilibrium (Hosios, 1990). Moreover, it is questionable whether price dispersion under purely noisy matching, such as in BJ, can sustain some extent of directed search.

I construct a tractable model of SMS to study the positive and normative implications. The positive analysis will examine how directed search in the first stage regulates price dispersion and search effort, and how the equilibrium responds to changes in the meeting efficiency. The normative analysis will investigate when the equilibrium is socially inefficient, what policies can correct the inefficiency, and how these efficient policies should respond to economic conditions.

Section 2 constructs the baseline model of SMS. There is a unit measure of homogeneous visitors, each wanting to consume one indivisible unit of a good. Homogeneous stayers enter the market competitively under a fixed cost of entry and commit to posted prices to sell the good.[2] Trade takes place in submarkets, each of which is described by a visitor's meeting rate, x, and a distribution of posted prices, $F(.|x)$. Stayers and visitors choose the submarkets to participate, with rational expectations that a submarket with a higher x will have higher prices. Once in submarket $(x, F(.|x))$, a stayer chooses a price in the support of $F(.|x)$ to post. In each submarket, a meeting function with constant returns to scale determines the amount of meetings. Meetings are *rivalrous* in the sense that the meeting rates on both sides of a submarket vary with the tightness. A visitor receives meetings, potentially multiple ones, at the rate x according to the Poisson distribution. For simplicity, I assume that a stayer receives at most one meeting in the period, and so the meeting process is one-to-many. At the end of a period, each visitor chooses the lowest price among the received meetings to trade.

In the meeting stage, visitors cannot direct search toward any particular stayer because they do not observe which stayer posts what price. Instead, visitors direct search to a

[2] Some SMS markets, such as the housing market, may have bargaining or bidding in the second stage. I abstract from bargaining or bidding in order to focus on the role of posted prices in directing search.

submarket with a distribution of posted prices. They can choose a submarket with a degenerate distribution of prices if doing so is optimal. The upper bound on the support of the price distribution in a submarket, p_H, plays a special role. As explained at the end of section 2.1, commitment to not posting prices higher than p_H is necessary for a meaningful tradeoff in directed search. In section 2.1, I use the labor market to illustrate realism of SMS and, especially, of the commitment to not posting prices higher than p_H.

The sequentially mixed search equilibrium (SMSE) exists, is unique and has one active submarket. In the SMSE, offers are distributed continuously in a connected interval. Choosing a submarket with no price dispersion is feasible, but not optimal for the tradeoff between the meeting rate and the price distribution. The SMSE also differs importantly from purely noisy matching, as detailed in section 3. First, the choice of the submarket regulates the price distribution. Second, the reserve surpluses on both sides are strictly positive. In particular, p_H is lower than the monopoly price y that sets a visitor's reserve surplus to zero, in contrast to $p_H = y$ in BJ. Both directed search and rivalrous meetings are necessary for $p_H < y$. Third, the SMSE is constrained efficient in the baseline model, despite the existence of noisy matching in SMS.

An increase in the meeting efficiency widens price dispersion in the SMSE (see section 4). This effect arises because visitors respond to the higher meeting efficiency by choosing a submarket that has a higher meeting rate x. By increasing the expected number of meetings for each visitor, a higher x reduces the probability of trade for all stayers. This reduction is more severe at high prices than at low prices, because a stayer posting a high price is more likely to lose a visitor to a competitor. Since all prices must yield the same expected surplus to a stayer, prices fall by less at high levels than at low levels, resulting in wider dispersion. This result helps explain the empirical puzzle that the Internet has not reduced price dispersion, e.g., Baye et al. (2004), Ellison and Ellison (2005).

Section 5 endogenizes search effort by letting each visitor choose search effort after entering a submarket. In most models, visitors' search effort is a strategic substitute, because one's high search effort increases congestion and reduces the return on search to other visitors. In the current model, visitors' search effort can be a strategic complement because of the effect of x on price dispersion. When visitors increase search effort, their meeting rate x increases. This widens price dispersion, which increases the return on search to other visitors. Despite the possible complementarity in search effort, the SMSE is unique, because the usual congestion from high search effort makes search effort a weak complement. In the SMSE, a reduction in the search cost widens price dispersion, similar to an increase in the meeting efficiency.

The SMSE with endogenous search effort is socially inefficient. Relative to the constrained social optimum, the SMSE has excessive search effort, an inefficiently high meeting rate for a visitor, a deficient meeting rate per search effort for a visitor, and excessive aggregate output. Also, the SMSE has inefficiently wide dispersion in prices, although price levels are lower in the SMSE than in the social optimum. The government can restore social efficiency of the equilibrium by a proportional tax on the joint value of a trade and a proportional subsidy on stayers' entry cost, with a lump-sum rebate to visitors to balance the budget. Under a mild condition, the corrective policies should lean against the wind by increasing the tax on the joint value of trade in an economic boom and decreasing the tax in a recession. Note that this policy recommendation has nothing to do with increasing returns to scale in the matching function that Diamond (1982) emphasized. Price dispersion is necessary for the inefficiency in the equilibrium, because the desire to find lower prices is the cause of inefficiently high search effort.

This paper makes several contributions to the literature. First, it constructs a tractable framework for modeling an important phenomenon, SMS. The framework can serve as a basis for incorporating features such as heterogeneity, private information, and many-to-many meetings (see section 6).[3] Second, the paper explains the puzzling effect of the meeting efficiency on price dispersion. Third, the paper shows that visitors' search effort can be a strategic complement, proves social inefficiency of the SMSE with endogenous

[3] Bethune et al. (2018) introduce noisy search into the directed search submarket, but they take the meeting rate as exogenous instead of modeling it as part of the choice of directed search. Kennes et al. (2018) examine three directed search models, one of which (buyer posting) has the flavor of SMS. However, they do not focus on SMS and they abstract from visitors' rivalry in meetings (see section 3.2).

search effort, and prescribes corrective policies.

Despite the simplicity, the model overcomes a major difficulty in integrating directed search with noisy matching. Simply grafting directed search onto the BJ model of noisy matching would not work. In BJ, stayers' indifference among posted prices generates $p_H = y$ and pins down all aspects of the price distribution. There is nothing left for directing search![4] What then makes SMS possible in this paper? It is the rivalry in meetings derived from a general meeting function. Because stayers' meetings are rivalrous, a visitor is able to obtain a strictly positive surplus even at the highest price, as explained in section 3.2. In turn, the interior value of $p_H < y$ enables the price distribution to vary across submarkets to direct search.[5]

The result $y - p_H > 0$ leads to a general insight on the importance of rivalrous meetings in all models where directed search is followed by competition ex post (i.e., after meetings). For individuals who do not face ex post competition, such as visitors in the current model, their reserve surplus in the equilibrium is positive if and only if meetings are rivalrous for individuals on the other side who face ex post competition (see section 3.2). This insight clarifies the role of the reserve price in competing auctions. Peters and Severinov (1997) construct a model of competing auctions in which sellers (auctioneers) set the reserve price above their outside option to direct bidders' search. Albrecht et al. (2012) show that the reserve price does not direct search because an auctioneer's reserve surplus must be zero in the equilibrium. This is consistent with the general insight here, since auctioneers do not face ex post competition and bidders' meetings are non-rivalrous. The reserve price can serve the role of directing search if bidders face rivalrous meetings.

The label SMS brings up two strands of the literature. One is on mixed search, e.g., Lester (2011), Godoy and Moen (2013), and Delacroix and Shi (2013). In these papers, the mixing is simultaneous across markets rather than sequential in the same market.

[4] In addition, grafting directed search onto BJ would face an intractable integer choice problem because the number of meetings is the primary characteristic there. I model the choice as the continuous variable x, which also enables me to incorporate a general meeting function.

[5] Some papers distinguish meetings from matching, e.g., Lester et al. (2015) and Cai et al. (2017), but they do not analyze the importance of this distinction for directing search in SMS.

That is, some markets are open for purely directed search while other markets for random search simultaneously, but no searcher uses both modes of search in any market or receives multiple offers. There is no dispersion in the offers in the directed search market.

The other strand of the literature is purely directed search with a two-stage process. The first stage is purely directed search and the second stage is a selection process. This literature includes competing auctions (e.g., Peters and Severinov, 1997, Julien et al., 2000, Albrecht et al., 2012), auctions with cheap talk (Kim and Kircher, 2015), directed search with vague messages followed by bargaining (Menzio, 2007), list prices with bargaining (Stacey, 2015) or with sequential inspection (Lester et al., 2017). Search in these models is purely directed because visitors know exactly the pricing mechanism each stayer offers. Visitors can target a particular stayer. As an implication, all stayers offers the same mechanism in the equilibrium when individuals are homogeneous on each side of the market, which contrasts to the non-degenerate distribution of offers in the SMSE.

The above literature with a two-stage process caps the number of a visitor's meetings by one. By increasing this cap to be more than one, some papers have examined purely directed search with multiple applications, e.g., Albrecht et al. (2006) and Galenianos and Kircher (2009). In both papers, each worker can apply to more than one firm and receive more than one offer. Albrecht et al. (2006) assume that firms are able to bid for the received applicants, and so their model is essentially a model of competing auctions where each visitor can participate in multiple auctions. Galenianos and Kircher (2009) assume that firms must commit to the posted wage. This creates an interesting portfolio choice; that is, a worker chooses a portfolio of wages to apply to. Again, search in the model is purely directed, because a worker knows the tradeoff between *each* firm's wage offer and the meeting probability. The number of equilibrium wages is discrete and cannot exceed the maximum number of applications a worker can make. In contrast, posted prices in the SMSE lie in a connected interval even though each visitor receives a discrete number of meetings. Moreover, the equilibrium in Galenianos and Kircher (2009) is not constrained efficient, but the SMSE is constrained efficient in the baseline model here. Inefficiency of the SMSE arises, instead, in the extension with endogenous search effort.

2. Baseline Model

2.1. The model environment

Consider a one-period economy with a continuum of homogeneous individuals on each side of the market. On one side are *stayers* whose measure is elastically determined by competitive entry with the entry cost $k > 0$. The cost of producing a good is normalized to 0. On the other side are *visitors* whose measure is relatively inelastic and, for simplicity, fixed at unity. A visitor wants to consume one unit of an indivisible good and a stayer can produce one unit. Stayers post prices and visitors search. In a trade, the joint value is y, the stayer obtains a surplus p, and the visitor gets $(y - p)$, where p is the posted price. If an individual fails to trade, the value is 0.[6]

The trading process is sequentially mixed search (SMS). In the first stage, stayers and visitors choose a submarket to enter, and each stayer chooses a price to post. There is a continuum of potential submarkets differentiated by a visitor's Poisson meeting rate, x, and the distribution of posted prices, $F(.|x)$. Search is directed in the sense that individuals take into account the dependence of $F(.|x)$ on x but, in contrast to purely directed search, visitors do not observe the price posted by any particular stayer before a meeting. By entering submarket $(x, F(.|x))$, a stayer can only post a price in the support of $F(.|x)$ and must commit to the posted price. The second stage is noisy matching. Given the densities of stayers and visitors in a submarket, a meeting function, described below, generates meetings. A visitor receives meetings according to the Poisson distribution at the rate x and, at the end of the period, chooses to trade at the lowest price among the received meetings. An accepted meeting is called a *match*. For simplicity, I focus on one-to-many meetings by assuming that a stayer receives at most one meeting in the period. See section 6 for a discussion on many-to-many meetings.

[6] In a directed search environment, Shi and Delacroix (2018) address the question which side should incur the cost to organize trade in order to maximize social welfare in an economy with search frictions. In the current model, the entry cost k is the elastic side's participation cost in the market, while the participation cost of the inelastic side and the cost of creating a trading post are zero. Under these assumptions, the analysis in Shi and Delacroix (2018) implies that social welfare is the same regardless of whether the elastic side or the inelastic side posts prices to organize trade.

In submarket $(x, F(.|x))$, let n_s be the density of stayers and n_v the density of visitors. If there is a mass of individuals in the submarket, then these symbols denote the measures of individuals there. The meeting function is $M(n_s, n_v)$ $(< n_s)$, which has constant returns to scale. For a visitor, the probability of receiving a number j of meetings in the period is $\frac{x^j e^{-x}}{j!}$, and so the expected number of meetings is x. Since xN_v and $M(n_s, n_v)$ are both equal to the amount of meetings, the equality yields:

$$x = \frac{M(n_s, n_v)}{n_v} = M(\theta, 1), \tag{2.1}$$

where $\theta \equiv \frac{n_s}{n_v}$ is the tightness of the submarket. For any given x, define $\theta(x)$ as the solution for θ to (2.1).[7] This is the tightness needed to deliver the meeting rate x for each visitor in the submarket. A stayer receives at most one meeting in the period and the meeting probability is $\lambda(x) \equiv \frac{M(n_s,n_v)}{n_s} = \frac{x}{\theta(x)}$.

Two features are worth noting. First, there are rivalries in meetings. Assumption 1 later will capture the rivalries by assuming that x increases in θ and λ decreases in θ. The rivalries make the tradeoff between x and $F(.|x)$ non-trivial. Second, a visitor chooses a trade at the end of the period, rather than whenever a meeting arrives. Thus, as in BJ, matching is noisy rather than sequential. By creating the possibility of multiple meetings for a visitor, noisy matching generates competition among stayers ex post (after meetings). Noisy matching may reflect a fixed cost of deciding on an offer or simply the inability to decide on an offer on the spot. In reality, there may also be a cost of waiting for offers. The validity of noisy matching is likely to depend on the length of a period. The shorter is a period, the lower is the waiting cost relative to the fixed cost of deciding on an offer, and the more reasonable is the assumption of noisy matching.[8]

To illustrate realism of SMS, consider the labor market. A submarket is a group of job openings that lie in a range of wages, together with workers applying to these jobs.

[7] The case where $M(\theta, 1)$ is constant over θ is uninteresting because it does not generate a tradeoff for visitors between the meeting rate and prices.

[8] Menzio and Trachter (2015) construct a model to generate equilibrium price dispersion with sequential matching instead of noisy matching. The important ingredient of their model is the existence of a large player who has positive mass and receives meetings at a different rate from all other stayers who have zero mass and lie in a continuum.

For example, jobs may be listed in different sections of a job search site according to the offers, and different sections are expected to have different meeting rates. Choosing a submarket is nothing more than choosing a section of the job site to search or to post a job. Commitment to posting wages in the specified range is guaranteed by the design of the job site. If the posted wage does not lie in the specified range, the job will not be placed in the corresponding section and, hence, it will not show up to workers who search for jobs in that range. The jobs in the specified range appear in a random order for each worker's search. A worker may apply to some of these jobs. The proper interpretation of a meeting is that an applicant is interviewed by a firm. Meetings are rivalrous for workers because, if more workers apply to the jobs in the same range, a particular applicant will be less likely to be interviewed. Similarly, meetings are rivalrous for firms as the probability of receiving an application decreases when more firms post jobs in the same range. As a worker may receive more than one interview, matching is noisy.

Consider a stayer S who posts a price p in submarket $(x, F(.|x))$. The following lemma is useful for computing the trading probabilities:

Lemma 2.1. *If Z_1 is Poisson with the rate ζ and $Z_2|(Z_1 = j)$ is binomial with (j, α), then Z_2 is Poisson with the rate $\zeta\alpha$ (see Haight, 1967, p.46). This implies that, conditional on having a meeting, the trading probability for stayer S is $e^{-xF(p|x)}$. For a visitor, the probability of trading at a price no higher than p is $\left[1 - e^{-xF(p|x)}\right]$.*

Let me explain the lemma briefly. Suppose that stayer S meets a visitor V. Let j be the number of visitor V's meetings with stayers other than S, and q the number of such meetings with prices no higher than p. Because meetings arrive to V independently from different stayers, j is Poisson distributed with the rate x. In each of those meetings, the price is not higher than p with the probability $F(p|x)$. Conditional on j, q is binomial with $(j, F(p|x))$, and so $\Pr(q = 0|j) = [1 - F(p|x)]^j$. Unconditional on j, stayer S expects visitor V to accept the match with the following probability:

$$\Pr(q = 0) = \sum\nolimits_{j=0}^{\infty} \frac{x^j e^{-x}}{j!} [1 - F(p|x)]^j = e^{-xF(p|x)}.$$

This verifies the result for stayer S in Lemma 2.1.[9] Similarly, for a visitor, conditional on the number of meetings, j, the number q of meetings with prices no higher than p is binomial with $(j, F(p|x))$. Unconditional on j, q is Poisson with the rate $xF(p|x)$. Hence, the visitor trades at a price no higher than p with the probability $\Pr(q \geq 1) = 1 - e^{-xF(p|x)}$.[10]

Conditional on having a meeting, a stayer's expected surplus is

$$\Pi(p, x) \equiv pe^{-xF(p|x)}. \tag{2.2}$$

Let p_L be the lower bound on the support of $F(.|x)$ and p_H the upper bound. These bounds depend on x but the dependence is suppressed. Denote the maximum of a stayer's expected surplus in a meeting as

$$\pi(x) = \max_{p \leq p_H} \Pi(p, x). \tag{2.3}$$

A stayer's expected profit of entering submarket $(x, F(.|x))$ is $\lambda(x)\pi(x) - k$, where λ is the stayer's meeting probability in the period and k the entry cost. For a visitor, the expected surplus in the submarket is:

$$D(x) \equiv \int_{p_L}^{p_H} (y - p)\, \mathrm{d}\left[1 - e^{-xF(p|x)}\right], \tag{2.4}$$

where d is with respect to p. $D(x)$ has incorporated the dependence of $F(.|x)$ on x.

The trading process endogenously generates a matching function and a distribution of transaction prices. A visitor trades with the probability $1 - e^{-x}$. The amount of matches in the submarket is $n_v(1 - e^{-x})$. This endogenous *matching function* has constant returns

[9] If a visitor receives two or more identical prices that are the lowest among the received offers, the visitor chooses one of them with equal probability. It will become clear later that the offer distribution contains no mass point. Thus, the probability that stayer S faces a competitor who offers the same price and has met the same visitor is zero.

[10] Generally, for any $q' \in \{0, 1, ...\}$, the probability of $q = q'$ is

$$\sum\nolimits_{j=q'}^{\infty} \frac{x^j e^{-x}}{j!} C_{q'}^{j} \left[F(p|x)\right]^{q'} \left[1 - F(p|x)\right]^{j-q'} = \frac{\left[xF(p|x)\right]^{q'} e^{-xF(p|x)}}{(q')!}.$$

That is, q is Poisson distributed with the rate $xF(p|x)$. The expected value of q, which is equal to $xF(p|x)$, is a sufficient statistic for the distribution of q. Lester et al. (2015) and Cai et al. (2017) refer to this property of the meeting technology as invariance.

to scale. Conditional on having at least one meeting, a visitor in the submarket succeeds in trading at a price no higher than p with the probability:

$$G(p|x) \equiv \frac{1-e^{-xF(p|x)}}{1-e^{-x}}. \tag{2.5}$$

This is the cumulative distribution of transaction prices in the submarket. It is clear that $G(p|x) \geq F(p|x)$ for all p, with strict inequality if $p_L < p < p_H$. Search enables visitors to obtain prices lower than posted prices in the first-order stochastic dominance.

Denote $U = \max_x \lambda(x)\pi(x)$ as a stayer's market value. The maximum of a stayer's expected profit of entering the market is $U - k$. Let I denote the set of active submarkets, defined as the submarkets that receive positive densities of visitors.[11]

Definition 2.2. *A* sequentially mixed search equilibrium (SMSE) *consists of a stayer's market value, U, the beliefs on all submarkets $(x, F(.|x))$, stayers' choices of the submarket to enter and the price to post, visitors' choice of the submarket to enter, and a set I of active submarkets, $(x_i, F(.|x_i))_{i\in I}$, that satisfy (i)-(iv) below:*

(i) Optimality of stayers' entry and pricing decisions: Given $F(.|x)$ and U, a stayer chooses a submarket to enter to maximize expected profit of entry. For each $i \in I$, a stayer in submarket i optimally chooses the price p to post to maximize $\Pi(p, x_i)$ under the constraint $p \leq p_{Hi}$. All posted prices generate $\pi(x_i)$ as the maximized Π and they together constitute the distribution $F(.|x_i)$.

(ii) Optimality of visitors' search: Given $F(.|x)$, a visitor chooses the submarket to enter to maximize $D(x)$. The submarkets with positive densities of visitors constitute I.

(iii) The beliefs on all submarkets $(x, F(.|x))$: (a) $\lambda(x)\pi(x) = U$ for all submarkets with $\pi(x) \geq U$, and $\theta(x) = 0$ for all submarkets with $\pi(x) < U$; (b) In every submarket with $\pi(x) \geq U$, the support of $F(.|x)$ is such that $\Pi(p, x) = \pi(x)$ for all $p \in$supp$(F(.|x))$, and $\Pi(p, x) < \pi(x)$ for all $p \notin$supp$(F(.|x))$ with $p \leq p_H$.

(iv) Competitive entry of stayers: $U = k$.

[11] This definition allows for the possibility that I has a positive measure in the space of x, although the possibility does not materialize in the equilibrium (see Proposition 2.4 later).

Requirements (i) and (ii) are for submarkets active in the SMSE, but requirement (iii) is for all submarkets as explained below. Requirement (iv) determines a stayer's market value U. I separate (iv) from (iii) to facilitate the comparison with BJ where the measures of individuals are fixed on both sides of the market (see section 3.1).

Requirement (iii) restricts the beliefs on both active and inactive submarkets. Part (a) is common in the literature (see Menzio and Shi, 2010, and Guerrieri, et al., 2010). For every submarket that satisfies $\pi(x) \geq U$, regardless of whether it is active, x and $F(.|x)$ must be such that a stayer's expected surplus is equal to the market value. For every submarket that satisfies $\pi(x) < U$, the tightness is zero. By entering such a submarket, a stayer makes negative profit even if the stayer is certain to meet a visitor. This requirement "completes" the submarkets by eliminating the case where a submarket is inactive just because individuals expect no one to enter on the other side of the submarket. It *defines* the dependence of $F(.|x)$ on x across submarkets.

Part (b) of requirement (iii) completes the support of $F(.|x)$ in every submarket with $\pi(x) \geq U$. The requirement is omitted for submarkets with $\pi(x) < U$, because a stayer makes less than U by entering such submarkets. In every submarket with $\pi(x) \geq U$, a stayer's expected surplus conditional on having a meeting is equal to $\pi(x) = U/\lambda(x)$. All prices in the support of $F(.|x)$ must yield the same expected surplus to a stayer as $\pi(x)$. Conversely, all prices no higher than p_H that yield $\pi(x)$ to a stayer must be in the support of $F(.|x)$. This requirement eliminates the case where a price $p \leq p_H$ is not in the support of $F(.|x)$ just because no stayer tries to post it.

Part (b) of requirement (iii) is not imposed for $p > p_H$, which indicates a special role of p_H. Commitment to only posting $p \leq p_H$ is necessary for directing search. To see this, suppose that the commitment is not required and consider any submarket with $p_H < y$ and $\pi(x) > U$. A stayer can profit by deviating to p_H^+ slightly higher than p_H after entering the submarket. A stayer's meeting probability in the submarket is independent of the posted price. Moreover, at p_H^+ and p_H, a stayer has the same probability to trade conditional on having a meeting, which occurs when the visitor in the meeting has no other meeting.

Since a trade at p_H^+ yields higher profit to a stayer than at p_H, expected profit is higher at p_H^+. Precisely, $\Pi_1\left(p_H^+, x\right) > 0$ if $p_H < y$. Thus, if stayers are not required to commit to only posting $p \leq p_H$, then $p_H = y$ and $\pi(x) = ye^{-x}$ in all submarkets where $\pi(x) \geq U$. In this case, there is only one value of x that satisfies $\lambda(x)\pi(x) = U$, and so part (a) of requirement (iii) fails to generate the variations in $F(.|x)$ across submarkets. In contrast to p_H, there is no need to commit to posting $p \geq p_L$. Relative to p_L, all prices lower than p_L have the same probability of trade but yield lower profit to a stayer in a trade.

2.2. Optimal choices and the SMSE

To characterize optimal decisions, define $\bar{x} \leq \infty$ by $\theta(\bar{x}) = \infty$ as the natural upper bound on x and restrict the domain of x to $[0, \bar{x}]$. Define the elasticity of θ as $\varepsilon(x) \equiv \frac{x\theta'(x)}{\theta(x)}$.

Assumption 1. *The meeting function is such that* $0 \leq \lambda(x) \leq 1$, $\theta'(x) \geq 0$, *and* $\theta''(x) \geq 0$ *for all* $x \in [0, \bar{x}]$, *where all of the inequalities are strict if* $x \in (0, \bar{x})$. *Also,* $\theta(0) = 0$, $\theta'(0) > 0$, *and* $\theta'(\bar{x}) = \infty$. *Moreover,* $\lambda'(x) < 0$ *and* $\varepsilon(x) > 1$ *for all* $x > 0$; $\lambda(\bar{x}) = 0$, *and* $\theta'(0)k < y$.

Since I have restricted the number of meetings received by a stayer to be either zero or one, λ is the probability that a stayer receives one meeting. The above assumption requires $\lambda(x) < 1$ for all interior values of x. However, $\lambda = 1$ can occur at the lower bound $x = 0$, as illustrated in Example 2.3 below. The assumptions on $\theta(x)$, including the properties on the boundaries of the domain $[0, \bar{x}]$, are equivalent to the standard properties: M is strictly increasing and concave in each of the two arguments, $M(0,1) = 0$, $M_1(0,1) < \infty$ and $M_1(\infty, 1) = 0$.[12] The assumption $\theta''(x) > 0$ and the properties of $\theta(x)$ on the boundaries imply $\lambda'(x) < 0$ for all $x > 0$. In turn, $\lambda'(x) < 0$ implies $\varepsilon(x) > 1$, because $\varepsilon = 1 - \theta\lambda'$. Also, $\theta'(\bar{x}) = \infty$ implies $\lambda(\bar{x}) = 0$. These implied properties of $(\lambda, \lambda', \varepsilon)$ are listed above for references. The assumption $\theta'(0)k < y$ requires the joint surplus of a trade to exceed the cost of a stayer's entry when the density of stayers in a submarket is arbitrarily small.

[12]Because $M(n_s, n_v) = n_v M(\theta, 1)$, then $M_2(n_s, n_v) = x - \theta M_1(\theta, 1)$. The property $M_2(n_s, n_v) > 0$ and the fact $M_1(\theta, 1) = 1/\theta'(x)$ imply $\varepsilon(x) > 1$.

If this assumption is violated, it is socially efficient to shut down the market. The following well-known meeting functions satisfy Assumption 1 with $k/A < y$:

Example 2.3. *The Dagum (1975) function is* $M(\theta, 1) = A(\theta^{-\rho} + 1)^{-1/\rho}$ *with* $\rho \in (0, \infty)$ *and* $A \in (0, 1]$. *This function yields* $\theta(x) = [(A/x)^{\rho} - 1]^{-1/\rho}$ *and* $\varepsilon(x) = [1 - (x/A)^{\rho}]^{-1}$. *Another function is the urn-ball function* $M(\theta, 1) = A\theta\left(1 - e^{-1/\theta}\right)$ *with* $A \in (0, 1]$.[13] *This function yields* $\varepsilon(x) = \frac{e^{1/\theta(x)} - 1}{e^{1/\theta(x)} - 1 - \frac{1}{\theta(x)}}$. *With both functions,* $\lambda(0) = \bar{x} = A$, $\lambda(\bar{x}) = 0$, $\theta'(0) = 1/A$, $\varepsilon(0) = 1$, *and* $\varepsilon'(x) > 0$.

The assumptions on the function $\theta(x)$ capture the rivalry in meetings on the two sides of the market, as introduced by Eeckhout and Kircher (2010) and used by Lester et al. (2015). The rivalry refers to the feature that x increases, and λ decreases, in the market tightness θ. That is, an increase in θ alleviates congestion of visitors but exacerbates congestion of stayers in the meeting stage. It is important to emphasize that the defining feature of rivalrous meetings is that the meeting rates vary with the market tightness, not that the meeting probabilities are less than one. To see this, consider the following two examples where meetings are non-rivalrous on at least one side of the market. In both examples, the constant $A > 0$ can be less than one. In the first example, $x = A\min\{\theta, 1\}$ and $\lambda = A\min\{1, \frac{1}{\theta}\}$. Individuals on the short side of the market do not face the rivalry in meetings, because their meeting rate is equal to the constant A. The second example is $M(n_s, n_v) = An_s$. Because $\lambda(x) = A$ is a constant, meetings are non-rivalrous for stayers.

Consider first a stayer, who chooses which submarket to enter and what price to post. Suppose that the stayer has already chosen to enter submarket $(x, F(.|x))$. Without loss of generality, suppose that this submarket has $\pi(x) \geq U$. By part (a) of requirement (iii) in Definition 2.2, a stayer's expected surplus in the submarket is equal to U, i.e.,

$$\pi(x) = \frac{U}{\lambda(x)}. \tag{2.6}$$

In submarket $(x, F(.|x))$, the stayer is required to only post prices no higher than p_H. The posted price maximizes the expected surplus in a meeting, $\Pi(p, x)$. Since all prices in the

[13] The urn-ball function often represents the number of matches in an urn-ball process, see Peters (1991) and Burdett et al. (2001). I use this term for the number of meetings, instead.

support of $F(.|x)$ must yield the same expected surplus as $\pi(x)$, then

$$\Pi(p,x) = \pi(x) \text{ for all } p \in \text{ supp}(F(.|x)).$$

Conversely, if any $p \leq p_H$ satisfies the above equation, it must be in the support of $F(.|x)$, as required by part (b) of requirement (iii) in Definition 2.2. Because $\Pi(p,x)$ is continuous in p, the values of $p \leq p_H$ that satisfy the above equation form an interval $[p_L, p_H]$. This is the support of $F(.|x)$. The above equation determines:

$$F(p|x) = \frac{1}{x} \ln\left(\frac{p}{\pi(x)}\right) \text{ for all } p \in [p_L, p_H]. \tag{2.7}$$

It is convenient to invert $F(.|x)$ as

$$p = \pi(x)\, e^{xF(p|x|)} \text{ for all } p \in [p_L, p_H]. \tag{2.8}$$

Because the bounds on the support must satisfy $F(p_L|x) = 0$ and $F(p_H|x) = 1$, then

$$p_L = \pi(x), \quad p_H = \pi(x)\, e^x. \tag{2.9}$$

Equation (2.7) is the relationship between $F(.|x)$ and x that directs search. Note that $p \geq \pi(x)$ is needed for $F(p|x) \geq 0$. Also, $\lambda(x)$ is intuitively a decreasing function of x, and so $\pi'(x) > 0$ by (2.6). Then, (2.7) shows that $F(p|x)$ is a decreasing function of x for any given $p \geq \pi(x)$. That is, a submarket with a higher meeting rate for a visitor has higher posted prices in the first-order stochastic dominance in $F(.|x)$. Note from (2.9) that the submarket with no price dispersion has either $x = 0$ or $\pi(x) = 0$.

Now consider a visitor. If the visitor participates in submarket $(x, F(.|x))$, the expected surplus is $D(x)$ given by (2.4). The visitor chooses x to maximize $D(x)$. Substituting p from (2.8) into (2.4), changing the integration variable from p to $F(p|x)$ and then substituting π from (2.6), I have:

$$D(x) = y\left(1 - e^{-x}\right) - \theta(x)\, U. \tag{2.10}$$

Because $\theta(x)$ is a strictly convex function, $D(x)$ is strictly concave and, hence, is maximized at a unique x. Denote the unique maximizer as x^*. The function $\theta'(x)\, e^x$ is increasing in

x. Recall that $\bar{x}$ is defined by $\theta(\bar{x}) = \infty$. It is easy to verify $D'(\bar{x}) < 0$ so that $x^* < \bar{x}$.[14] The maintained assumption $y > \theta'(0)k$ and the equilibrium requirement $U = k$ imply $D'(0) > 0$, and so $x^* > 0$. This shows that x^* is in the interior of $(0, \bar{x})$ and is the unique solution to the first-order condition:

$$\theta'(x^*) e^{x^*} = \frac{y}{U}. \tag{2.11}$$

In the equilibrium, $U = k$. The above analysis has established the following proposition:

Proposition 2.4. *A unique SMSE exists and has only one active submarket* $(x^*, F(.|x^*))$, *where* $x^* \in (0, \bar{x})$ *solves (2.11) with* $U = k$ *and* $F(.|x^*)$ *is given by (2.7) with* $x = x^*$. *The distribution* $F(.|x^*)$ *is continuous on the support* $[p_L, p_H]$, *where* p_L *and* p_H *are given by (2.9) with* $x = x^*$. *Transaction prices are distributed according to* G *in (2.5).*

To explain the above Proposition, let me examine the benefit and cost of a higher x to a visitor. A submarket with a higher x generates a higher trading probability for a visitor, $1 - e^{-x}$. The benefit is a higher expected value of trade, $y(1 - e^{-x})$. The cost to a visitor is a higher expected payment to a stayer, given by the term $\theta(x)U$ in (2.10). This term can be explained by simple accounting. Because the sum of payments received by all stayers in the submarket is $n_s U$, the expected payment made by each visitor in the submarket is $n_s U/n_v$, which is equal to $\theta(x)U$. Intuitively, submarket $(x, F(.|x))$ requires $\theta(x)$ stayers per visitor to deliver the meeting rate x for a visitor, and each stayer needs to be compensated with an expected value U in the market. The expected payment increases in x. The higher is x, the more should each visitor pay to stayers to incentivize stayers to enter the submarket to deliver the higher x.

There is a unique submarket that features the optimal tradeoff for a visitor between the price distribution and the meeting rate.[15] The uniqueness is similar to that in models of purely directed search, e.g., Peters (1991) and Burdett et al. (2001). Specifically, a

[14]Because $\varepsilon(x) > 1$ for all $x > 0$ by Assumption 1, then $\theta'(\bar{x}) e^{\bar{x}} > \theta(\bar{x}) e^{\bar{x}}/\bar{x} > \theta(\bar{x}) = \infty > \frac{y}{k}$.

[15]The uniqueness of the active submarket in the SMSE is subject to the usual qualification arising from constant returns to scale in the meeting function. Dividing visitors into many submarkets that have the same (x^*, F^*) but different scales does not change the equilibrium. Similarly, the Supplementary Appendix F describes a way to divide all visitors' effort into two submarkets without affecting the equilibrium.

unique x^* equates the marginal benefit of x to a visitor, ye^{-x}, and the marginal cost, $\theta'(x)U$. The marginal benefit is diminishing in x since a visitor only needs one meeting to trade. The marginal cost of x is increasing since the marginal productivity of stayers in the meeting function is diminishing. As x continues to increase, the tightness must increase progressively to deliver it, and so the visitor's expected payment should increase progressively to incentivize more stayers to enter the submarket.

For future references, it is useful to note that $\theta(x)U = \pi(x)x$ and that a visitor's expected number of meetings is equal to x. Although a visitor chooses one of the received meetings to trade and pays only the stayer in such a meeting, the expected payment is $\pi(x)x$, as if a visitor pays the amount $\pi(x)$ to every stayer he/she meets. Stayers "price in" the risk of losing a visitor in a meeting to a competitor. The higher is a visitor's meeting rate, the more likely will a stayer in a meeting lose the visitor to a competitor, and the higher is the price that a stayer charges in order to earn the market value U.

3. Equilibrium Properties, Efficiency and Interpretations

3.1. Directed search, noisy matching and price dispersion

In purely directed search, a visitor knows what *each* stayer offers before a meeting. The offer can be a posted price, as in Peters (1991) and Burdett et al. (2001), or a pricing mechanism, as in Peters and Severinov (1997) and Julien et al. (2000). With homogeneous individuals on each side of the market, the equilibrium generates no dispersion in the posted price or mechanism. In contrast, in SMS, a visitor only knows the distribution of prices in each submarket and cannot target any particular stayer. The equilibrium generates a continuum of posted prices. Similar to BJ, noisy matching with multiple offers is responsible for such dispersion in posted prices. A visitor receives two or more meetings with a probability $Q \in (0,1)$. In BJ, Q is assumed to be exogenous. In the current model, Q is endogenous and equal to $Q = 1-(1+x^*)e^{-x^*}$. Because $x^* \in (0,\bar{x})$, then $Q \in (0,1)$.

As in BJ, the intuition for why $Q \in (0,1)$ induces price dispersion is as follows. With $Q \in (0,1)$, stayers face competition ex post. A stayer who meets a visitor may lose the visitor to a competitor with a positive probability. This competition implies that the price distribution cannot have a mass point anywhere above the lowest price p_L. To see this, note first that $p_L > 0$. If $p_L = 0$, then posting a price $p > p_L$ would yield a higher (positive) expected profit than posting p_L. Next, if there were a mass point at a price $p_1 \geq p_L$, posting a price slightly lower p_1 would increase a visitor's acceptance probability by a discrete amount and, hence, would increase the stayer's expected surplus. In the equilibrium, prices are continuously distributed. In addition, the support of the price distribution must be a connected interval. If there were a "hole" in the support, a stayer posting a price inside the hole could increase the ex post gain without reducing the acceptance probability, relative to posting the price at the lower boundary of the hole.

The price distribution becomes degenerate when Q reaches the bounds, 0 or 1. If $Q = 0$, every visitor in a meeting has no other meeting. Anticipating this absence of ex post competition, all stayers will post the "monopoly" price $p = y$. That is, $p_L = p_H = y$. If $Q = 1$, every visitor in a meeting has at least one other meeting. Anticipating this certainty of ex post competition, all stayers will post the "monopsony" price $p = 0$. In the current model, $Q \to 0$ if and only if $x^* \to 0$, which occurs when the entry cost of stayers k reaches the upper bound $\frac{y}{\theta'(0)}$. On the other hand, $Q \to 1$ if and only if $x^* \to \bar{x}$, where $\bar{x}$ is defined by $\theta(\bar{x}) = \infty$. This case occurs in the limit as $k \to 0$, which implies $\pi(x^*) \to 0$.

The current model differs from BJ primarily in the presence of directed search in SMS. To emphasize this difference, I use Appendix C to eliminate another difference between the two models: The measure of stayers is endogenous in the model here but fixed in BJ. When the measure of stayers is fixed, the tightness in the equilibrium submarket is fixed, say, at θ_0, which fixes the meeting rate for a visitor as $x_0 = M(\theta_0, 1)$. In lieu of the competitive entry requirement (iv) in Definition 2.2, the equilibrium requires that stayers over all active submarkets should sum up to the fixed measure. This new requirement determines a stayer's market value U. Despite the exogenous x_0, directed search continues to exert its force because individuals can explore other submarkets outside of the equilibrium. A visitor's optimal choice of search must be consistent with x_0. That is, the optimality condition, (2.11), must be satisfied by $x^* = x_0$, which is achieved in the equilibrium by the adjustment in U. Such a condition epitomizes the tradeoff under directed search between the price distribution in a submarket and the meeting rate. The tradeoff regulates the entire distribution of prices in the equilibrium. A particular implication is $p_H < y$, which contrasts to $p_H = y$ in BJ (see section 3.2).

Let me return to the current model with competitive entry of stayers and examine price dispersion in the equilibrium. Denote the *price spread* as $\Delta p \equiv p_H - p_L$ and the coefficient of variation in posted prices as $cv_F \equiv \frac{[var_F]^{1/2}}{\mathbb{E}_F}$, where $\mathbb{E}_F$ is the mean and var_F the variance of posted prices under a distribution $F(.|x)$. The coefficient of variation in transaction prices is cv_G, where the distribution $G(.|x)$ in (2.5) is used instead of $F(.|x)$. Computation yields:

$$cv_F(x) = \left[\frac{x(e^x+1)}{2(e^x-1)} - 1\right]^{\frac{1}{2}}, \quad cv_G(x) = \left[e^x\left(\frac{1-e^{-x}}{x}\right)^2 - 1\right]^{1/2}. \tag{3.1}$$

These coefficients of variation are functions of only x, with no parameters. Any change in a parameter of the model can change cv_F and cv_G only if it affects x. For this reason, it is useful to examine this effect of x on price dispersion, even though x is endogenous. The proof of the following corollary is straightforward and omitted:

Corollary 3.1. *An increase in x increases posted and transaction prices in the first-order stochastic dominance in F and G. Also, an increase in x increases the bounds (p_L, p_H), the price spread Δp, and the coefficients of variation (cv_F, cv_G).*

For a stayer, an increase in x reduces both the meeting probability and the matching probability, leading to lower expected profit. To compensate for a stayer's entry cost, posted and transaction prices must be higher in the sense of first-order stochastic dominance. However, for x to affect price dispersion, it must affect stayers differently depending on their posted prices. This non-uniform effect arises from the effect of x on a stayer's matching instead of meeting probability. Conditional on having a meeting, a stayer posting a high price has a lower probability of trade than a stayer posting a low price. An increase in x widens this difference in the trading probability by increasing the probability that a visitor to a stayer has other meetings. That is, an increase in x reduces a stayer's trading probability by more if the stayer posts a higher price. To yield the same expected surplus conditional on having a meeting, π, the price must increase by more if the price is initially higher. This is why an increase in x increases the price bounds, the price spread and the coefficient of variation in prices.

3.2. Rivalrous meetings and reserve surpluses in the equilibrium

In the equilibrium, the lowest surplus for a visitor is $(y - p_H)$ and the lowest surplus for a stayer's entry is $(p_L - k)$. These are the reserve surpluses for the two sides of the market. The following Corollary describes the reserve surpluses:

Corollary 3.2. *$p_H < y$ and $p_L > k$, and so the reserve surpluses are strictly positive in the equilibrium for both sides of the market. If $\lambda(x)$ is constant for all x, then $p_H = y$.*

Proof. The bounds on equilibrium prices are given by (2.9). Since competitive entry of stayers yields $\pi(x^*) = \frac{k}{\lambda(x^*)}$, then $p_L = \frac{k}{\lambda(x^*)}$ and $p_H = \frac{ke^{x^*}}{\lambda(x^*)}$. Because $x^* > 0$, then $\lambda(x^*) < 1$ and $p_L > k$. In addition, a visitor's optimal choice x^* obeys $ye^{-x^*} = k\theta'(x^*)$ (see (2.11) with $U = k$). Dividing the expression for p_H by this condition for x^* and substituting $\lambda(x^*) = x^*/\theta(x^*)$, one gets $\frac{p_H}{y} = \frac{1}{\varepsilon(x^*)}$. Since $\varepsilon(x) > 1$ for all $x > 0$ by Assumption 1 and the SMSE has $x^* > 0$, then $p_H < y$. Moreover, $\varepsilon(x) = 1 - \lambda'(x)\theta > 1$ if and only if $\lambda'(x^*) < 0$. That is, $p_H < y$ if and only $\lambda'(x^*) < 0$. **QED**

The result $p_L > k$ is easy to explain. A stayer gets zero net profit from entering the market at every equilibrium price. Because a stayer's meeting probability is less than one, the lowest price must exceed the entry cost k for a stayer's expected surplus in a meeting to cover the cost k. For the result $p_H < y$, directed search in the first stage of SMS is crucial, as shown by the use of a visitor's optimal choice of x in the above proof. If a submarket has $p_H = y$, prices are too high to be optimal for a visitor's tradeoff between the trading probability and the trade surplus. Catering to this tradeoff, stayers optimally set $p_H < y$. In contrast, when search is purely noisy as in BJ, this tradeoff does not exist, which creates the profitable deviation to $p_H = y$ (see the end of section 2.1). Although directed search is necessary for $p_H < y$, it is *not sufficient*. The above proof shows that $p_H < y$ if and only if $\varepsilon(x^*) > 1$ or, equivalently, $\lambda'(x^*) < 0$. That is, a visitor's reserve surplus is positive if and only if meetings are rivalrous for stayers.

For a general insight on the result $p_H < y$, let $x(\theta)$ denote the inverse of the function $\theta(x)$ and note that $\lambda'(x) < 0$ is equivalent to $\frac{\theta x'(\theta)}{x(\theta)} < 1$. Stayers face potential ex post competition but visitors do not. Since a visitor gains from competing offers, he/she has an incentive to choose a submarket with a high density of stayers. To incentivize stayers' entry, a visitor can reduce his/her reserve surplus. How low should this reserve surplus be depends on how much a visitor benefits from stayers' entry into the market. If stayers are rivalrous in meetings, an increase in stayers' entry into the market increases a visitor's meeting rate by less than one for one, as captured by $\frac{\theta x'(\theta)}{x(\theta)} < 1$. In this case, a visitor is not willing to reduce his/her reserve surplus to zero. In contrast, if stayers' meetings are non-rivalrous, a visitor should reduce his/her reserve surplus to zero to increase stayers' participation. Let me summarize:

> An insight: For the side of the market that does not face ex post competition, the reserve surplus is strictly positive if and only if meetings are rivalrous on the other side that faces ex post competition.

This insight is also valid for competing auctions. In Julien et al. (2000), workers post auctions to direct firms' search. In one of the models in Kennes et al. (2018), firms post auctions to direct workers' search. Trading follows the urn-ball process: each ball (bidder) is thrown randomly at an urn (auctioneer), and then each urn selects one ball from the received ones. Since the number of meetings (not matches) is equal to the number of bidders, bidders' meetings are non-rivalrous. Note carefully that bidders face ex post competition, and so they behave like stayers in the current model. Auctioneers do not face ex post competition. Because bidders' meetings are non-rivalrous, the above insight suggests that an auctioneer should set the reserve surplus to zero in order to increase bidders' participation in the auction. This is indeed the case: a worker's reserve wage is zero in Julien et al. (2000) and a firm's reserve profit is zero in Kennes et al. (2018).[16]

[16] In the baseline setup in Julien et al. (2000), the market is finite. As a result, the matching function has decreasing returns to scale, and an auctioneer's surplus at the reserve price is positive. This surplus

Interestingly, in one of the first models of competing auctions with directed search, Peters and Severinov (1997) state that an auctioneer's reserve surplus is strictly positive in the equilibrium so that the reserve price can direct search. Albrecht et al. (2012) prove that this is a mistake. The above insight indicates that for the reserve price to serve the role of directing search in competing auctions, bidders must face rivalrous meetings.

3.3. Social efficiency of the SMSE

Another main difference of the SMSE from an equilibrium with purely noisy search is constrained social efficiency. To analyze efficiency, consider a social planner who maximizes social welfare measured by the sum of expected surpluses in the economy. The planner chooses the measure of stayers to enter the market, but is constrained by the meeting frictions embodied in the meeting function M. Because the measure of visitors is fixed at 1, choosing the measure of stayers is equivalent to choosing the market tightness and to choosing a visitor's meeting rate x. Anticipating the result that only one active submarket will be active, the choice x requires the measure of stayers entering the submarket to be $\theta(x)$. If a visitor receives multiple meetings, the planner randomly chooses one of them for the visitor to trade. Total cost of stayers' entry is $\theta(x)k$ and the measure of trades is $(1-e^{-x})$. Since a trade generates the joint surplus y, social welfare is:

$$y\left(1-e^{-x}\right)-\theta(x)k.$$

This welfare measure is the same as a visitor's expected surplus in the equilibrium, given by $D(x)$ in (2.10) after substituting $U=k$. Thus, the planner's choice of x is identical to a visitor's choice in the SMSE. This proves the following proposition:

Proposition 3.3. *The SMSE is constrained efficient.*

Directed search in the first stage of SMS and competitive entry of stayers are important for the equilibrium to be socially efficient, as emphasized in the directed search literature (e.g., Moen, 1997, Shi, 2001). The two ingredients ensure the social marginal benefit of a

declines to 0 when the market becomes infinitely large.

higher x to be the same as the private marginal benefit, which is to increase the trading probability $(1-e^{-x})$. They also ensure the social marginal cost of a higher x to be the same as the private marginal cost, which is the marginal cost to induce stayers to enter the market, $\theta'(x)\,k$. The above proposition adds to the literature by showing that the equilibrium is efficient even if the search process has noisy matching in the second stage. In contrast, if search is purely noisy, then the equilibrium is socially inefficient. In this case, $p_H = y$, which is inefficiently higher and induces excessive entry of stayers.

4. Effects of the Meeting Efficiency

The Internet has increased the convenience of trading, but it has not reduced price dispersion significantly. Baye et al. (2004) compared the online market with the offline market for consumer electronics and found similar price dispersion in the two markets. Ellison and Ellison (2005) found a similar puzzle in other online markets. To resolve this puzzle, I model the increasing use of the Internet as an increase in the meeting efficiency. Section 5.1 will re-examine the issue as a reduction in the search cost.

Let the meeting function be $AM(n_s, n_v)$ where A is the meeting efficiency. Denote $r = \frac{x}{A}$. The definition of x implies $r = M(\theta, 1)$, which solves θ as $\theta(r)$. The elasticity of $\theta(r)$ is $\varepsilon(r) = \frac{r\theta'(r)}{\theta(r)}$. The meeting probability for a stayer is $\lambda(r) = \frac{Ar}{\theta(r)}$. Except the change from x to r in the argument of $(\theta, \varepsilon, \lambda)$, the SMSE is characterized as before. Similar to (2.11), the first-order condition for the optimal x^* is:

$$\frac{1}{A}\theta'(\frac{x^*}{A})e^{x^*} = \frac{y}{k}. \tag{4.1}$$

Define $A_0 \in (0, \infty)$ as the unique solution to:

$$\frac{e}{A_0}\theta'(\frac{1}{A_0}) = \frac{y}{k}. \tag{4.2}$$

Then, $x^* < 1$ if and only if $A < A_0$. The following proposition is proven in the Supplementary Appendix D:

Proposition 4.1. *(i)* $\frac{dx^*}{dA} > 0$ *and* $\frac{d}{dA}cv_i(x) > 0$ *for* $i = F, G$. *(ii)* $\frac{d\theta}{dA} > 0$ *iff* $A < A_0$. *(iii)* $\frac{dp_H}{dA} \leq 0$ *iff* $(A_0 - A)\,\varepsilon' > 0$; *If* $\varepsilon' \geq 0$ *then* $\frac{dp_L}{dA} < 0$ *and* $\frac{d\Delta p}{dA} > 0$. *(iv) If* $\varepsilon' \geq 0$ *and*

$A < A_0$, an increase in A reduces posted and transaction prices in the first-order stochastic dominance.

An increase in the meeting efficiency unambiguously widens price dispersion measured by the coefficient of variation, although price levels may change ambiguously. By increasing stayers' meeting rate, an increase in A has direct effects of increasing entry of stayers and depressing price levels. However, an increase in A also increases a visitor's meeting rate x, which increases prices and widens price dispersion (see Corollary 3.1). Specifically, an increase in x reduces a stayer's matching probability by more at high prices than at low prices. To keep a stayer indifferent between posting different prices, a stayer's match surplus conditional on having a trade must fall by less at high prices than at low prices. If prices fall after the increase in A, they must fall by less at high levels than at low levels. If prices rise after the increase in A, they must rise by more at high levels than at low levels. In both cases, price dispersion widens.

In the empirical evidence cited above, the increased use of the Internet did not widen price dispersion significantly, although it did not compress price dispersion either. To reconcile with this evidence, one may note that the market concentration has also increased in many sectors, which can be captured by an increase in k. It is easy to verify that an increase in k reduces x and compresses price dispersion. Thus, the combined effect of an increase in A and an increase in k may leave price dispersion unchanged.

Proposition 4.1 contains additional information about the effects of an increase in A. First, an increase in A may increase or decrease the market tightness. For any given x, an increase in A increases the meeting rate for stayers, which induces more stayers to enter the market to increase the market tightness. However, an increases in A also induces visitors to choose a submarket with a higher x, which reduces the matching rate for a stayer and, hence, reduces stayers' entry. The direct effect of A on a stayer's matching rate dominates if $x < 1$, which is equivalent to $A < A_0$. In this case, an increase in A increases stayers' entry to increase the market tightness.

Second, an increase in A reduces the lowest price and widens the price spread if and only if $\varepsilon' \geq 0$. Because $\varepsilon(r) = \frac{r\theta'(r)}{\theta(r)}$ is the elasticity of θ, the derivative ε' measures the sensitivity of the market tightness to x. The condition $\varepsilon' \geq 0$ is satisfied by well-known meeting functions in Example 2.3. If $\varepsilon' \geq 0$, then an increase in x causes the market tightness to increase sharply as more stayers enter the market, which depresses prices. This negative effect on prices is stronger at low prices than at high prices, as explained for Corollary 3.1 and repeated above. Thus, if $\varepsilon' \geq 0$, the lowest price falls and the price spread widens. The highest price also falls if the market tightness indeed increases, i.e., if $A < A_0$. Conversely, if ε' is sufficiently negative, the market tightness is insensitive to changes in x. In this case, prices must rise in order to induce more stayers to enter the market to deliver the increase in x, and the price spread narrows.

Third, when $A < A_0$ and $\varepsilon' \geq 0$, an increase in the meeting efficiency reduces all prices according to the distribution. This is the case where the market tightness increases in A and is sensitive to x. In this case, an increase in A induces a large increase in stayers' entry into the market and, hence, reduces prices. Despite lower prices, the increase in A compensates stayers' entry into the market by increasing a stayer's meeting probability.

One may relate and contrast the effects of A to those of of Q in BJ, where Q is the probability that a visitor receives two or more meetings (see section 3.1). In BJ, price dispersion has a hump-shaped dependence on Q. At $Q = 0$, there is no price dispersion, as all stayers post the monopoly price $p = y$. At $Q = 1$, there is no price dispersion either, as all stayers post the monopsony price. At all $Q \in (0, 1)$, prices are continuously distributed in an interval. Thus, when Q increases from 0 to 1, price dispersion first widens and eventually narrows. For two related reasons, this hump-shaped dependence does not arise here for the effects of A. First, the meeting efficiency is not the same as an increase in Q, although the two are related by x^* as $Q = 1 - (1 + x^*) e^{-x^*}$. An increase in A increases not only a visitor's meeting rate x^*, but also a stayer's meeting probability which affects stayers' entry decisions. Second, with directed search in SMS, visitors make the optimal tradeoff between the meeting rate x^* and price dispersion. When an increase in A makes a higher x^* optimal, visitors need to tolerate wider dispersion in prices in order to induce stayers to enter the market to deliver the higher x^*.

5. Endogenous Search Effort and Efficient Policies

This section endogenizes visitors' search effort. I show that price dispersion can induce visitors' search effort to be a strategic complement and the equilibrium to be socially inefficient. I analyze the effects of a reduction in the search cost and examine policies for efficiently managing aggregate activities.

5.1. Equilibrium with endogenous search effort

A visitor chooses search effort, $\tilde{s}$, after entering a submarket. Let $\psi(\tilde{s})$ be the cost of search effort, with $\psi' > 0$ and $\psi'' > 0$ for all $\tilde{s} > 0$, $\psi'(0) = 0$ and $\psi'(\infty) = \infty$. In a submarket, let s be the average search effort per visitor and z the meeting rate per search effort. The meeting rate for a visitor with search effort $\tilde{s}$ is $\tilde{x} = (\tilde{s} + s_0)\, z$. The constant $s_0 > 0$ is arbitrarily small and is used to rule out the uninteresting case where the equilibrium can be stuck at $z = s = 0$.[17] Denote $x = (s + s_0)\, z$. The total amount of visitors' search effort in a submarket is $(s + s_0)\, n_v$, and the amount of meetings in the submarket is $M(n_s, (s + s_0)\, n_v)$. Since the total amount of meetings in the submarket is also equal to $n_v x$, then

$$z = \frac{M(n_s, (s + s_0)\, n_v)}{(s + s_0)\, n_v} = M(\theta, 1),$$

where $\theta = \frac{n_s}{(s+s_0)n_v}$ is the effective tightness of the submarket. The above equation solves $\theta = \theta(z)$, which is a function of z instead of x. The meeting probability for a stayer in a period is $\lambda(z) = \frac{z}{\theta(z)}$. Define $\bar{z} \leq \infty$ by $\theta(\bar{z}) = \infty$. Submarkets are now described by $(z, F(.|z))$ instead of $(x, F(.|x))$. As a result, competitive entry of stayers into the market yields a stayer's expected surplus conditional on having a meeting as $\pi(z) = \frac{k}{\lambda(z)}$, which is a function of z instead of x.

[17] If $s_0 = 0$, a visitor's meeting rate is sz, in which case the marginal gain to increasing s is zero if $z = 0$ and the marginal gain to increasing z is also zero if $s = 0$. Even a small $s_0 > 0$ can prevent this uninteresting case from happening in the equilibrium.

For stayers, the formulas in the baseline model remain valid after replacing $\pi(x)$ by $\pi(z)$ and $F(p|x)$ by $F(p|z)$. In a meeting, a stayer posting p succeeds in trade with the probability $e^{-xF(p|z)}$. The stayer's expected surplus in the meeting is $pe^{-xF(p|z)}$. Equating this expected surplus to $\pi(z)$ for all prices on the support of $F(.|z)$ yields:

$$F(p|z) = \frac{1}{x}\ln\frac{p}{\pi(z)} \text{ for all } p \in [p_L, p_H]. \tag{5.1}$$

The inverse of this distribution function is:

$$p = \pi(z)\, e^{xF(p|z)} \text{ for all } p \in [p_L, p_H]. \tag{5.2}$$

The bounds on the support of $F(.|z)$ are:

$$p_L = \pi(z), \quad p_H = \pi(z)\, e^{x}. \tag{5.3}$$

For a deviating visitor who searches with effort $\tilde{s}$ in submarket $(z, F(.|z))$, the meeting rate is $\tilde{x} = (\tilde{s} + s_0)\, z$, and the probability of trading at a price no higher than p is $1 - e^{-\tilde{x}F(p|z)}$. This visitor's expected surplus in the submarket is:

$$D(z, \tilde{s}) \equiv -\psi(\tilde{s}) + \int_{p_L}^{p_H} (y - p)\, \mathrm{d}\left[1 - e^{-\tilde{x}F(p|z)}\right]. \tag{5.4}$$

The dependence of D on the average search effort s through p_H is suppressed. (5.4) modifies (2.4) by subtracting the search cost and changing the meeting rate to $\tilde{x}$. Note that stayers make their decisions based on the expectation that the average search effort of visitors is s. That is, p is given by (5.2) where search effort is s instead of $\tilde{s}$. Substituting such p, integrating, substituting π from (2.6) and substituting $U = k$, I have:

$$D(z, \tilde{s}) = -\psi(\tilde{s}) + y\left[1 - e^{-\tilde{x}}\right] - \theta(z)(\tilde{s} + s_0)\, L((\tilde{s} - s)\, z)\, k, \tag{5.5}$$

where $\tilde{x} = (\tilde{s} + s_0)\, z$ and

$$L(t) \equiv \frac{1 - e^{-t}}{t} \text{ for all } |t| < \infty. \tag{5.6}$$

The visitor chooses $(z, \tilde{s})$ to maximize $D(z, \tilde{s})$.

The term $-L((\tilde{s} - s)\, z)$ captures the visitor's incentive to search more intensively than others in the submarket in order to find a lower price. To see this, note that the last term in (5.5) is equal to $\tilde{x}L\pi$, which is the visitor's expected payment. Similar to the explanation at the end of section 2.2, the expected payment is so as if the visitor pays $L\pi$ to every stayer he/she meets even though the visitor trades in only one of the meetings. If the visitor chooses the same search effort as the average effort, the expected payment per meeting is $L(0)\pi = \pi$. However, $L \neq 1$ if $\tilde{s} \neq s$. The following lemma lists the properties of L (see the Supplementary Appendix D for a proof):

Lemma 5.1. *$L(t)$ has the following properties: (i) $L(0) = 1$, $L'(0) = -\frac{1}{2}$, $L''(0) = \frac{1}{3}$, and $L(\infty) = L'(\infty) = \lim_{t\to\infty} L'(t)\, t = 0$; (ii) $L(t) > 0$, $L'(t) < 0$, $L''(t) > 0$ for for all $|t| < \infty$; (iii) $L + L' > 0$, and $\frac{1}{t}(L + 2L') = -(L' + L'') > 0$ for all $|t| < \infty$.*

The properties $L'(t) < 0$ and $L''(t) > 0$ are the most important ones. The property $L' < 0$ is equivalent to $\frac{\partial}{\partial \tilde{s}}(-L) > 0$, which captures the benefit of higher search effort in reducing a visitor's expected payment. If a visitor searches more intensively than the average effort, the visitor expects to be more likely to encounter a meeting with a lower price and, hence, to pay less than π to a stayer in a meeting. Conversely, if the visitor searches less intensively than the average effort, the visitor expects to pay more than π to a stayer in a meeting. The property $L'' > 0$ is equivalent to $\frac{\partial^2}{\partial \tilde{s} \partial s}(-L) > 0$, which is the supermodularity of $-L$ in $(\tilde{s}, s)$. The intuition is that a visitor's expected payment depends on the difference between the visitor's effort and the average effort in the submarket. When other visitors search with higher effort, a deviating visitor faces a higher risk of not being able to trade at relatively low prices. In this case, the cost savings are greater for the visitor to increase search effort to find a low price. Note that the strict inequalities $L' < 0$ and $L'' > 0$ hold even when $\tilde{s}$ is equal to s.

An SMSE can be defined similarly to that in section 2.1 by adding search effort $\tilde{s}$ to a visitor's choice. As shown in the proof of Proposition 5.2 below, the optimal $(z, \tilde{s})$ are interior and satisfy the first-order conditions:

$$0 = ye^{-\tilde{x}} - \left[\theta'(z)\, L + (\tilde{s} - s)\, \theta L'\right] k \tag{5.7}$$

$$0 = -\psi'(\tilde{s}) + yze^{-\tilde{x}} - \theta(z)\left[L + \tilde{x}L'\right] k. \tag{5.8}$$

The argument in L and L' is $(\tilde{s}-s)\,z$. For any given (s,z), denote $B(s,z)$ as the solution for $\tilde{s}$ to (5.8), which is a visitor's best response to the average search effort s, given z. Optimal search effort is symmetric in the equilibrium if, for every optimal z, $B(s,z)$ is single valued. Denote the optimal choice of z as z^* and denote $x^* = (s+s_0)\,z^*$. The following proposition holds (see Appendix A for a proof):

Proposition 5.2. *The optimal choices of $(z,\tilde{s})$ are interior and satisfy (5.7)-(5.8). A unique SMSE exists, is symmetric, and solves:*

$$\begin{array}{l} \theta'(z^*) = \frac{y}{k}e^{-x^*}, \\ \psi'(s^*) = \left[\varepsilon(z^*) - 1 + \frac{x^*}{2}\right]\theta(z^*)\,k. \end{array} \tag{5.9}$$

$B_1(s,z^*)\,|_{\tilde{s}=s} > 0$ *iff* $x^* > \frac{3}{2}$*. Also,* $B_1(s,z^*)\,|_{\tilde{s}=s} < 1$*. In the SMSE, an exogenous reduction in ψ' reduces z, increases (s,x), and increases (cv_F, cv_G). If $\varepsilon' \geq 0$, this reduction in the search cost reduces posted and transaction prices in the first-order stochastic dominance.*

For any given z, $B(s,z)$ is the best response of a visitor to other visitors' choice s. The equilibrium can feature $B_1(s,z^*)\,|_{\tilde{s}=s} > 0$. That is, visitors' search effort can be a "strategic complement" in and near the equilibrium. Specifically, by widening price dispersion, higher search effort by other visitors can increase the return on search to an individual visitor. This feature may sound puzzling at first glance. Why does not higher search effort of other visitors compress price dispersion, instead? The answer lies again in the effect of x on price dispersion. For any given z, other visitors' higher search effort increases their meeting rates, x, and pushes prices down. This negative effect on prices is stronger at high prices than at low prices, as explained for Corollary 3.1. To keep the expected surplus to be equal at all posted prices, prices fall by less at high levels than at low levels. Thus, price dispersion widens, which motivates an individual visitor to increase search effort in order to find lower prices. The countering force comes from the fact that prices are lower everywhere than before, which reduces visitors' incentive to search. The effect of price dispersion dominates if and only if prices are sufficiently dispersed. This occurs when search effort is higher than a threshold (3/2).[18]

[18] This condition can be expressed as a restriction on the parameters. Since x^* is an increasing function of y, x^* is above the threshold 3/2 if y is above some threshold.

To support the above explanation, note that the interaction between a deviating visitor's search effort and other visitors' effort appears in the visitor's expected surplus through the term $-(\tilde{s}+s_0)\,L(\tilde{s}-s)$. The multiplier $(\tilde{s}+s_0)$ captures the effect on the "extensive margin" that a visitor's high search effort increases the expected payment by increasing the number of meetings. As explained after Lemma 5.1, $(-L)$ is supermodular in $(\tilde{s}, s)$. The supermodularity creates the possibility that visitors' search effort can be a strategic complement. However, the term $-(\tilde{s}+s_0)\,L(\tilde{s}-s)$ is not necessarily supermodular in $(\tilde{s}, s)$, because of the effect of $\tilde{s}$ on the extensive margin. As a result, the condition $x^* > \frac{3}{2}$ is needed for the supermodularity of $(-L)$ to be the dominating effect.

Although visitors' search effort can be a strategic complement, the complementarity is not sufficiently strong to generate multiple equilibria. Precisely, $B_1(s, z^*)\,|_{\tilde{s}=s} < 1$, which leads to the uniqueness of the equilibrium. Complementarity is weak because search creates rivalries in meetings. When other visitors search with higher effort, the aggregate amount of search effort increases, which reduces a visitor's meeting rate per search effort, z. The reduction in z reduces a visitor's meeting rate for any given search effort $\tilde{s}$ and, hence, weakens the response of $\tilde{s}$ to other visitors' search effort.

The equilibrium is depicted in Figure 1. Given any s, $z_a(s)$ is the solution for z to the first equation in (5.9) and $z_b(s)$ the solution for z to the second equation. Figure 1 depicts $z_a(s)$ and two possibilities of $z_b(s)$. The SMSE is unique, which is depicted as points E1 and E2, respectively, for the two possibilities of $z_b(s)$.

Proposition 5.2 shows that a reduction in the search cost has similar effects on price dispersion as an increase in the meeting efficiency studied by Corollary 4.1. Figure 1 illustrates these effects of an exogenous reduction in ψ'. Before the reduction in the search cost, let $z_b(s)$ be the solid curve in Figure 1. A reduction in the search cost shifts the curve $z_b(s)$ to the dashed curve, which results in a higher s and a lower z. It is intuitive that a reduction is the search cost induces a visitor to search with higher effort and leads to a higher meeting rate for a visitor. As the matching probability for a stayer falls, a smaller measure of stayers enter the market, resulting in a fall in z. Because x increases, dispersion widens in both posted and transaction prices. Under the mild condition $\varepsilon' \geq 0$, posted and transaction prices fall in the first-order stochastic dominance.[19]

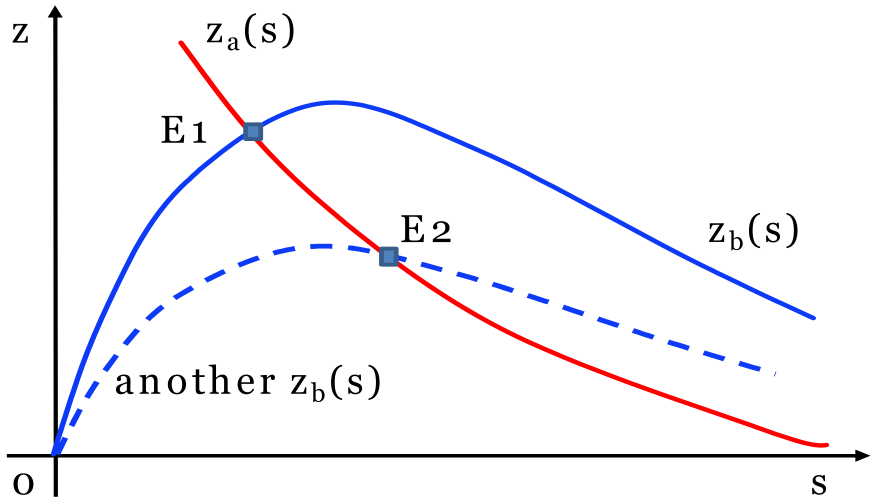

Figure 1. The unique equilibrium with endogenous search effort

5.2. Social inefficiency and corrective policies

The possible complementarity in search effort raises the question whether the SMSE is socially efficient. To address this question, I incorporate visitors' search effort into the social planner's problem in section 3.3. Since the total measure of visitors is 1, the total measure of stayers is $(s + s_0)\,\theta(z)$, and the sum of stayers' entry costs is $(s + s_0)\,\theta(z)\,k$. Social welfare is measured by the following sum of surpluses in the economy:

$$-\psi(s) + y\left(1 - e^{-x}\right) - (s + s_0)\,\theta(z)\,k.$$

Maximizing social welfare, the planner's choices (z, s) satisfy the first-order conditions:

$$\theta'(z) = \frac{y}{k} e^{-x}, \quad \psi'(s) = [\varepsilon(z) - 1]\,\theta(z)\,k. \tag{5.10}$$

Compare these conditions with the counterparts in the SMSE, (5.9). The first condition is the same as in the SMSE. The second condition, which characterizes the socially efficient s, differs from that in the SMSE in the absence of the term $\frac{x}{2}$ on the right-hand side. This difference implies the following proposition (see Appendix A for a proof):

[19] In a different model, Bethune et al. (2018) show that a reduction in the cost of becoming informed of prices can increase price dispersion.

Proposition 5.3. *Relative to the social optimum, the SMSE has excessive search effort s, a deficient z, and an excessive meeting rate for a visitor, x.*

The inefficiency arises from an externality created by visitors' search effort. There is a private benefit for a visitor to increase search effort $\tilde{s}$ above other visitors'. At $\tilde{s} = s$, this private benefit is: $-\pi x L'(0) = \pi x/2$. But if all visitors increase search effort, visitors face stronger rivalry in meetings, which is a negative externality. In the SMSE, search effort and the meeting rate for a visitor are inefficiently high. To maintain the optimal tradeoff between the meeting rate and the surplus of a trade, visitors choose an inefficiently low z. This low z mitigates, but does not correct completely, the excessive x.

Two elements are important for the inefficiency – the inability to commit to search effort and the presence of noisy matching. To see the importance, first suppose that visitors can commit to search effort before entering the market and so submarkets are indexed by $(z, s, F(.|z, s))$ instead of only $(z, F(.|z))$. In this case, the optimal tradeoff between search effort and the trading probability will internalize the externality by "pricing" search effort correctly. Next, if search is purely directed, as in Burdett et al. (2001), endogenizing search effort does not generate inefficiency. This should be true more generally if search is directed by mechanisms instead of posted prices. The purpose for a visitor to increase search effort is to find a better mechanism. If there is no ex ante dispersion among mechanisms in the equilibrium, then endogenizing visitors' search effort should not lead to inefficiency even though there is ex post dispersion in transaction prices within a mechanism.

To restore efficiency and maintain a balanced budget, the government can consider the following policies: a subsidy rate σ_e on the entry cost, a proportional tax rate τ_y on the joint value of a trade, and a lump-sum rebate σ_v to a visitor's participation in the market.[20] With the subsidy σ_e, competitive entry of stayers into the submarket pushes a stayer's expected surplus conditional on having a meeting to:

$$\pi(z) = \frac{(1 - \sigma_e) k}{\lambda(z)}. \tag{5.11}$$

[20] A tax on visitors' search effort can also restore efficiency, but it requires the tax authority to have the unrealistic ability of observing visitors' search effort.

With this modified formula of $\pi(z)$, the inverse of the distribution of posted prices is still given by (5.2). A visitor's expected surplus in the market is given by (5.4), with y being replaced by $(1-\tau_y)\,y$ and ψ by $\psi-\sigma_v$. This expected surplus is:

$$\begin{aligned} D(z,\tilde{s}) = \;& -\psi(\tilde{s})+\sigma_v+[(1-\tau_y)\,y]\left[1-e^{-\tilde{x}}\right] \\ & -(1-\sigma_e)(\tilde{s}+s_0)\,L((\tilde{s}-s)\,z)\,\theta(z)\,k. \end{aligned}$$

In the symmetric SMSE, the first-order conditions of $(z,\tilde{s})$ become:

$$\begin{aligned} \theta'(z) &= \tfrac{(1-\tau_y)y}{(1-\sigma_e)k}e^{-x}, \\ \psi'(s) &= (1-\sigma_e)\left[\varepsilon(z)-1+\tfrac{x}{2}\right]\theta(z)\,k. \end{aligned} \tag{5.12}$$

Comparing these conditions with (5.10) for the social optimum, it is easy to verify that the two sets of conditions coincide if and only if

$$\tau_y=\sigma_e=\left[\frac{2\left[\varepsilon(z)-1\right]}{x}+1\right]^{-1}, \tag{5.13}$$

where (z,s) are the quantities in the social optimum. Because the measure of matches is $1-e^{-x}$, total tax revenue on the joint output across matches is $y\tau_y\left[1-e^{-x}\right]$. Total subsidy to the entry of stayers is $\sigma_e(s+s_0)\,\theta k$. The difference between the two is the lump-sum rebate to visitors that balances the government budget. That is,

$$\sigma_v=\sigma_e\left[(e^x-1)\,\varepsilon-x\right]\frac{\theta k}{z}. \tag{5.14}$$

With $(\sigma_e,\tau_y,\sigma_v)$ above, one can verify that D in the SMSE is equal to social welfare. The following proposition holds (see Appendix B for a proof):

Proposition 5.4. *Let $(\sigma_e,\sigma_v,\tau_y)$ satisfy (5.13)-(5.14). (i) The policies induce the equilibrium to be socially efficient and maintain a balanced budget. (ii) The efficient policies increase z and reduce s, x, aggregate output $Y=y\left[1-e^{-x}\right]$, and (cv_F,cv_G). (iii) If $\varepsilon'(z)\geq 0$, the efficient policies reduce $\pi(z)$, reduce $(p_L,p_H,\Delta p)$, and reduce prices in the first-order stochastic dominance in $F(.|z)$ and $G(.|z)$.*

The policies (σ_e,τ_y) restore social efficiency of the equilibrium while the rebate σ_v balances the government budget. With the subsidy to a stayer's entry, σ_e, a stayer can break-even on entry with a smaller expected payment from a visitor in a meeting. A visitor's incentive to search weakens, which improves efficiency. However, σ_e distorts a visitor's tradeoff between the match surplus and the meeting rate by inducing a visitor to enter a submarket with an inefficiently high z. A tax on the joint output in a match, τ_y, eliminates this distortion by increasing the importance of the match surplus in a visitor's tradeoff relative to the meeting rate. The two policies (σ_e, τ_y) generate a budget surplus that is rebated to visitors through σ_v.

The corrective policies also affect price levels and price dispersion. In the absence of the policies, search effort is excessive and price dispersion is inefficiently wide. The policies tame search effort by reducing price dispersion. Price levels respond to the policies ambiguously in general. However, if $\varepsilon' \geq 0$, then posted and transaction prices fall in the first-order stochastic dominance. The derivative ε' measures the sensitivity of θ', as discussed for Proposition 4.1, and the condition $\varepsilon' \geq 0$ is satisfied by well-known meeting functions (see Example 2.3). When $\varepsilon' \geq 0$, the responses of z and $\lambda(z)$ to the policies are relatively weak. In this case, the policies affect a stayer's expected surplus conditional on having a meeting, $\pi(z)$, primarily through the effective cost of entry, $(1 - \sigma_e)k$. Because σ_e reduces this effective cost, it reduces the level of π needed for a stayer to break-even on entry. The fall in π is delivered by lower prices.

5.3. Management of aggregate activities

I focus on how the corrective policies depend on y. A higher y is a better economic condition, arising from aggregate supply (e.g., an increase in the quality of a good) or demand (e.g., an increase in the valuation of a good). In Figure 1, an increase in y shifts up the curve $z_a(s)$ and leaves the curve $z_b(s)$ intact. Equilibrium s increases. Equilibrium z increases if the SMSE is one like point E1, but decreases if the SMSE is one like point E2. The following proposition states how the SMSE and the corrective policies respond to y (see the Supplementary Appendix E for a proof):

Proposition 5.5. *Without the corrective policies, an increase in y has the following effects on the SMSE: $\frac{ds}{dy} > 0$, $\frac{dx}{dy} > 0$, $\frac{dp_H}{dy} > 0$, $\frac{d\Delta p}{dy} > 0$, $\frac{d}{dy}cv_F > 0$, and $\frac{d}{dy}cv_G > 0$. If $\psi''' > 0$,* *then* $\frac{dz}{dy} > 0$*. If* $\psi''' < 0$*, then there exists* $s_a \in (0, \infty]$ *such that* $\frac{dz}{dy} > 0$ *iff* $s < s_a$*. With the efficient policies in (5.13), a sufficient condition for* $\frac{d\sigma_e}{dy} > 0$ *and* $\frac{d\tau_y}{dy} > 0$ *is* $\varepsilon'(z) \leq (\varepsilon - 1)/z$*.*

An increase in y increases the market tightness, increases the meeting rate for a visitor, increases the two bounds on prices and shifts the price distributions to higher prices. These effects are intuitive. When the joint value of a trade increases, more stayers enter the market, which increases the market tightness. Equilibrium prices increase to be consistent with the optimal choices of both sides of the market. For visitors, the benefit of a higher y induces them to enter a submarket that has a higher meeting rate, which necessarily comes with higher prices. For stayers, the meeting probability falls because of the higher entry of stayers. However, the expected surplus for a stayer in the market (unconditional on a meeting) must remain the same as the fixed cost of entry. This implies that a stayer's expected surplus conditional on a meeting must rise. For this to happen despite the decrease in the stayer's trading probability, prices must increase to raise the stayer's surplus conditional on a trade. Moreover, as explained for Corollary 3.1, the increase in a visitor's meeting rate x widens price dispersion by reducing the trading probability for a stayer by more at high prices than at low prices.[21]

Without the corrective policies, an improvement in the economic condition results in an over-heated economy. Search effort and a visitor's meeting rate increase by an excessive amount, resulting in inflated prices and inefficiently wide dispersion in prices. The policies described by Proposition 5.4 are automatic stabilizers for the economy even if the policies do not respond to the increase in y. These policies tame visitors' search effort, moderate the increase in aggregate output and prevent price dispersion from widening excessively. Moreover, Proposition 5.5 shows that if $\varepsilon' \leq (\varepsilon - 1)/z$, the tax on the joint value of a trade should increase in an economic boom and decrease in a recession. To accompany this procyclical tax, the subsidy to stayers' entry should also be procyclical.

[21] The variable z responds to a higher y ambiguously. Because a higher y induces search effort to increase, the surplus that a visitor expects to concede to a stayer in a meeting increases. To restrict the increase in this concession, a visitor may reduce z. This happens when search effort is high. When search effort is low, the direct effect of y on z dominates, in which case z increases.

The corrective policies contrast with those in Diamond (1982), who assumes the matching function to have increasing returns to scale. In the current model, the meeting and the matching functions have constant returns to scale. Instead, the inefficiency arises from an externality in visitors' choice of search effort. Also, in contrast to Diamond's recommendation to manage aggregate demand, the corrective policies in Proposition 5.5 call for a policy mix that manages both aggregate demand and supply.

6. Conclusion

This paper constructs a tractable model of sequentially mixed search, which has directed search for a price distribution followed by noisy matching with multiple offers. I establish existence of a unique equilibrium and analyze the novel implications of the equilibrium on quantities and price dispersion. Moreover, I show that an increase in the meeting efficiency widens price dispersion. An extension endogenizing search effort shows that the equilibrium is constrained inefficient, where visitors' search effort is inefficiently high and can be a strategic complement. Under a mild condition, policies that restore efficiency should lean against the wind by increasing the tax on the joint value of trade in an economic boom and reducing this tax in a recession.

The simple framework of SMS allows for several extensions. One is to introduce heterogeneity among visitors, say, in their valuation of the good y. This valuation can be either public or private information. In both cases, the modifications to the SMSE are straightforward. Visitors of different types will self-select into different submarkets. For each type y, the optimal x^* obeys (2.11) and, hence, is increasing in y. That is, higher-valuation visitors choose to participate in a submarket with a higher meeting rate for them and a distribution of higher prices. As in this paper, the SMSE is constrained efficient when search effort is exogenous and inefficient when search effort is endogenous.

A more substantial extension is to allow for many-to-many meetings. That is, each stayer and each visitor can both have multiple meetings. This extension needs to specify a procedure by which individuals select trading partners from the received meetings. In a sequel (Shi, 2018), I have studied many-to-many meetings with two selection mechanisms. In one, stayers select among received visitors, where stayers posting lower prices select earlier. In the other, visitors selected among received stayers according to a predetermined order of visitors. Both selection mechanisms lead to the same expected payoffs to the market participants and constrained efficiency of the equilibrium.

The third extension is to incorporate trading relationships and business cycles. In the goods market, past trades create customer relationships, as analyzed in Gourio and Rudanko (2014) and Shi (2016). In the labor market, employment is a lasting relationship between a firm and a worker, as analyzed by Burdett and Mortensen (1998). In both markets, individuals in a relationship can continue to search. By embedding search on the relationship into a business cycle model, the analysis can shed light on how the turnover of relationships and the price distribution fluctuate in the business cycle. Directed search in the first stage of SMS will be important for such an analysis to be tractable by making the SMSE block recursive, as formulated by Shi (2009) and Menzio and Shi (2010).

Appendix

A. Proofs for Propositions 5.2 and 5.3

Proof of Proposition 5.2

First, I prove that a visitor's optimal choices $(z, \tilde{s})$ are interior and satisfy the first-order conditions, (5.7) and (5.8). A visitor's objective function is $D(z, \tilde{s})$ given by (5.5). The derivative $D_1(z, \tilde{s})$ is equal to the right-hand side of (5.7) multiplied by $(\tilde{s} + s_0)$. That is,

$$D_1(z, \tilde{s}) = (\tilde{s} + s_0) \left\{ ye^{-\tilde{x}} - \left[\theta'(z) L + (\tilde{s} - s) \theta L'\right] k \right\}.$$

Since $\theta(0) = 0$, $y > \theta'(0) k$ (see Assumption 1), $L(0) = 1$ and $L'(0) = -\frac{1}{2}$, then

$$D_1(0, \tilde{s}) = (\tilde{s} + s_0) \left[y - \theta'(0) k\right] > 0 \text{ for all } \tilde{s} \geq 0.$$

This implies that the optimal z is strictly positive. Also, $D_1(\infty, \tilde{s}) < 0$, and so the optimal z is finite. Thus, the optimal z is interior. Since interior optima satisfy the first-order condition, then the optimal z satisfies (5.7). Similarly, for the choice $\tilde{s}$, the derivative $D_2(z, \tilde{s})$ is given by the right-hand side of (5.8), i.e.,

$$D_2(z, \tilde{s}) = -\psi'(\tilde{s}) + yze^{-\tilde{x}} - \theta(z) \left[L + \tilde{x}L'\right] k, \tag{A.1}$$

where the argument in L and L' is $(\tilde{s} - s) z$. Substituting $ye^{-\tilde{x}}$ from (5.7) yields:

$$D_2(z^*, \tilde{s}) = -\psi'(\tilde{s}) + \left[(z^*\theta' - \theta) L - (s + s_0) z^*\theta L'\right] k, \tag{A.2}$$

Since $L(\infty) = L'(\infty) = \lim_{t\to\infty} L'(t) t = 0$, then $D_2(z^*, \infty) < 0$, and so the optimal $\tilde{s}$ is finite. Because $z\theta' > \theta$, $L > 0$ and $L' < 0$ (see Lemma 5.1), then $D_2(z^*, 0) > -\psi'(0) = 0$, which implies that the optimal $\tilde{s}$ is strictly positive. Thus, the optimal $\tilde{s}$ is interior and satisfies the first-order condition (5.8).

Second, I prove that all equilibria are symmetric. Recall that $B(s, z)$ denotes the solution for $\tilde{s}$ to (5.8), which is a visitor's best response to the average search effort s, given z. I show that $B(s, z^*)$ is single-valued for each s, given the optimal z^*, and so all visitors choose the same search effort in the equilibrium. Use (A.1) to compute:

$$D_{22}(z, \tilde{s}) = -\psi''(\tilde{s}) - z^2 ye^{-\tilde{x}} - \left[2L' + \tilde{x}L''\right] \theta kz.$$

Substitute $ye^{-\tilde{x}}$ from (5.7) and use the fact $L + 2L' = -t\left(L' + L''\right)$ (see Lemma 5.1):

$$D_{22}\left(z^*, \tilde{s}\right) = -\psi''\left(\tilde{s}\right) - \left[\left(z^*\theta' - \theta\right)L + z^*\left(s + s_0\right)\theta L''\right]z^*k. \tag{A.3}$$

The use of z^* in (A.3) emphasizes that I have substituted $ye^{-\tilde{x}}$ from (5.7). Because $\psi'' > 0$, $z\theta' > \theta$ and $L'' > 0$, then $D_{22}\left(z^*, \tilde{s}\right) < 0$. This implies that the first-order condition for $\tilde{s}$, $D_2\left(z^*, \tilde{s}\right) = 0$, has at most one solution for $\tilde{s}$ if z is optimal. Thus, all visitors in the submarket with an optimal z search with the same effort $\tilde{s} = s$ in the equilibrium.

Third, I prove that a unique SMSE exists and solves (5.9). Setting $\tilde{s} = s$ in (5.7) yields the first equation in (5.9). The second equation in (5.9) comes from setting $\tilde{s} = s$ in (A.2) and then setting $D_2 = 0$. The equilibrium is unique if the solution to (5.9) is unique. Given any s, $z_a\left(s\right)$ denotes the solution for z to the first equation in (5.9) and $z_b\left(s\right)$ the solution to the second equation. The equilibrium s solves $z_b\left(s\right) - z_a\left(s\right) = 0$. The assumption $y > k\theta'\left(0\right)$ implies $z_a\left(0\right) > 0$. Also, $z_a\left(\infty\right) = 0$. Since $\psi'\left(0\right) = 0$ and $\theta\left(0\right) = 0$, then $z_b\left(0\right) = 0 < z_a\left(0\right)$ and $z_b\left(\infty\right) > 0 = z_a\left(\infty\right)$. Thus, the equation $z_b\left(s\right) = z_a\left(s\right)$ has at least one solution in $(0, \infty)$. To prove that the solution is unique, compute:

$$z_a'\left(s\right) = \frac{-z}{\frac{\theta''}{\theta'} + s + s_0} < 0, \quad z_b'\left(s\right) = \frac{\psi'' - kz\theta/2}{k\left[\left(\varepsilon + 1\right)\frac{s+s_0}{2} + z\theta''\right]}.$$

Here I substituted $\varepsilon' = \frac{z\theta''}{\theta} - \frac{\varepsilon(\varepsilon-1)}{z}$. The difference $\left[z_b'\left(s_0\right) - z_a'\left(s_0\right)\right]$ has the same sign as

$$\left(\frac{\theta''}{\theta'} + s + s_0\right)\psi'' + k\theta\left[\frac{z\theta''}{\theta'}\left(\varepsilon - \frac{1}{2}\right) + \varepsilon\frac{x}{2}\right].$$

Since this is positive, then $z_b'\left(s\right) > z_a'\left(s\right)$ for all $s \geq 0$. Thus, the solution to $z_a\left(s\right) = z_b\left(s\right)$ is unique, as multiple solutions necessarily have alternating signs of $\left[z_b'\left(s\right) - z_a'\left(s\right)\right]$.

Fourth, I verify $B_1\left(s, z^*\right)|_{\tilde{s}=s} < 1$ and find the condition for $B_1\left(s, z^*\right)|_{\tilde{s}=s} > 0$. $B\left(s, z\right)$ is the solution for $\tilde{s}$ to $D_2\left(z, \tilde{s}\right) = 0$, where D_2 is given by (A.1). Computing B_1 from this equation and then setting $z = z^*$, one obtains:

$$B_1\left(s, z^*\right) = \frac{z^*k\theta}{\left[-D_{22}\left(z^*, \tilde{s}\right)\right]}\left[L' + \left(\tilde{s} + s_0\right)z^*L''\right],$$

where $D_{22}\left(z^*, \tilde{s}\right) < 0$ is given by (A.3). Since $L\left(0\right) = 1$, $L'\left(0\right) = \frac{-1}{2}$ and $L''\left(0\right) = \frac{1}{3}$, then $B_1\left(s, z^*\right)|_{\tilde{s}=s} > 0$ if and only if $x^* = \left(s + s_0\right)z^* > \frac{3}{2}$. Also, $B_1\left(s, z^*\right) < 1$ if and only if

$$\psi'' + z^*k\left\{z^*\theta'L + \left[1 + z^*\left(\tilde{s} - s\right)\right]\theta L'\right\} > 0.$$

Because $[z^*\theta' L + [1 + z^*(\tilde{s} - s)]\theta L']_{\tilde{s}=s} = z^*\theta' - \frac{\theta}{2} > 0$, then $B_1(s, z^*)|_{\tilde{s}=s} < 1$.

Finally, I establish the effects of the search cost. Suppose $\psi(s) = \psi_0 \hat{\psi}(s)$ for some constant $\psi_0 > 0$ and a function $\hat{\psi}(s)$. An increase in ψ_0 is an exogenous increase in the marginal cost of search effort. A reduction in ψ_0 is an exogenous reduction in the marginal cost of search effort. Differentiating (5.9) with respect to ψ_0 shows $\frac{\mathrm{d}z}{\mathrm{d}\psi_0} > 0$, $\frac{\mathrm{d}s}{\mathrm{d}\psi_0} < 0$, and $\frac{\mathrm{d}x}{\mathrm{d}\psi_0} < 0$. Since $cv_F'(x) > 0$ and $cv_G'(x) > 0$ by Corollary 3.1, then $\frac{\mathrm{d}}{\mathrm{d}\psi_0} cv_F < 0$ and $\frac{\mathrm{d}}{\mathrm{d}\psi_0} cv_G < 0$. Similar to the proof of the effects of σ_e later in Proposition 5.4, the increase in z and the fall in x imply that, if $\varepsilon' \geq 0$, then $\frac{\mathrm{d}F}{\mathrm{d}\psi_0} < 0$ and $\frac{\mathrm{d}G}{\mathrm{d}\psi_0} < 0$. That is, an increase in ψ_0 increase prices in the first-order stochastic dominance in F and G. **QED**

Proof of Proposition 5.3

The first equation in (5.10) is identical to the first equation in (5.9). For any given s, the common solution for z to the two equations is $z_a(s)$. Let $z_b^e(s)$ denote the solution for the equilibrium z to the second equation in (5.9), and $z_b^o(s)$ the solution for the socially optimal z to the second equation in (5.10). Denote $\Delta z^e(s) = z_b^e(s) - z_a(s)$ and $\Delta z^o(s) = z_b^o(s) - z_a(s)$. Then, the equilibrium s solves $\Delta z^e(s) = 0$, and the socially optimal s solves $\Delta z^o(s) = 0$. As shown in the proof of Proposition 5.2, $\Delta z^{e\,\prime}(s) > 0$. Similarly, $\Delta z^{o\,\prime}(s) > 0$. For any $s \geq 0$ and $z > 0$, the right-hand side of the second condition in (5.9) is strictly greater than that of the second condition in (5.10). For any given $s \geq 0$, a smaller z is required to satisfy the second equation in (5.9) than to satisfy the second equation in (5.10). That is, $z_b^e(s) < z_b^o(s)$ for all $s \geq 0$. This implies $\Delta z^e(s) < \Delta z^o(s)$ for all $s \geq 0$. Since $\Delta z^{e\,\prime}(s) > 0$ and $\Delta z^{o\,\prime}(s) > 0$, the solution for s to $\Delta z^e(s) = 0$ must be strictly larger than the solution for s to $\Delta z^o(s) = 0$. Because z satisfies the common equation $z = z_a(s)$ in the SMSE and the social optimum, and because $z_a'(s) < 0$, then z is lower in the equilibrium z than in the social optimum. Backing out x from the common equation for $z_a(s)$ in the SMSE and in the social optimum, I conclude that x is higher in the SMSE than in the social optimum. This completes the proof of Proposition 5.3. **QED**

B. Proof of Proposition 5.4

(i) The text preceding the proposition has established the result that the policies $(\sigma_e, \tau_y, \sigma_v)$ induce the SMSE to be socially efficient.

(ii) By Proposition 5.3, the SMSE without the policies has an excessive s, a deficient z and an excessive x, relative to the social optimum. Because the efficient policies restore the social optimum, they reduce s, increase z and reduce x.

(iii) I analyze the effects of the policies on prices in three steps. First, I reduce the dimension of the policies and variables. To do so, let τ_y depend on σ_e as in (5.13). Then, for all σ_e, the first equation in (5.12) is the same as the first equation in (5.9), and the common solution for z to the two equations is $z = z_a(s)$. For any given s, $z_a(s)$ does not depend on the policies directly. Also, $z_a'(s) < 0$, as shown in the proof of Proposition 5.2. Substituting $z = z_a(s)$ into the second equation of (5.12), I get:

$$\psi'(s) = (1 - \sigma_e)\,\theta(z)\,k\left[\varepsilon(z) - 1 + \frac{x}{2}\right]_{z=z_a(s)}. \tag{B.1}$$

Second, I compute the effects of σ_e on (s, z), Y and (cv_F, cv_G), taking into account the dependence of τ_y on σ_e in (5.13). In this computation, σ_e is arbitrary instead of the efficient one. However, if the derivative of a variable with respect to σ_e is positive, then the variable increases under the efficient policies, because the efficient policies have $\sigma_e > 0$. Differentiating (B.1) with respect to σ_e, I get:

$$\frac{\mathrm{d}s}{\mathrm{d}\sigma_e} = \frac{-\psi'}{1-\sigma_e}\left[\psi'' + \frac{(2\varepsilon - 1)\,z\theta'' + x\theta'\varepsilon}{\theta'' + (s + s_0)\,\theta'}\frac{1-\sigma_e}{2}k\theta\right]^{-1} < 0.$$

This implies $\frac{\mathrm{d}z}{\mathrm{d}\sigma_e} = z_a'(s)\,\frac{\mathrm{d}s}{\mathrm{d}\sigma_e} > 0$ and

$$\frac{\mathrm{d}x}{\mathrm{d}\sigma_e} = \frac{z\theta''}{\theta'' + (s + s_0)\,\theta'}\frac{\mathrm{d}s}{\mathrm{d}\sigma_e} < 0.$$

Since aggregate output is $Y = y\left[1 - e^{-x}\right]$, it is clear that $\frac{\mathrm{d}Y}{\mathrm{d}\sigma_e} < 0$. The coefficients of variation in prices, $cv_F(x)$ and $cv_G(x)$, are given by (3.1). Since $cv_F'(x) > 0$ and $cv_G'(x) > 0$ by Corollary 3.1, and since $\frac{\mathrm{d}x}{\mathrm{d}\sigma_e} < 0$, then $\frac{\mathrm{d}}{\mathrm{d}\sigma_e}cv_F < 0$ and $\frac{\mathrm{d}}{\mathrm{d}\sigma_e}cv_G < 0$.

Third, I compute the effects of σ_e on $\pi(z)$ and prices, again taking into account the dependence of τ_y on σ_e. With the policies, $\pi(z) = \frac{(1-\sigma_e)k}{\lambda(z)}$. Substituting $\frac{\mathrm{d}s}{\mathrm{d}\sigma_e}$ and $\frac{\mathrm{d}z}{\mathrm{d}\sigma_e}$, I compute:

$$\begin{aligned}&-\tfrac{\lambda}{k}\left\{\left[\theta''+(s+s_0)\,\theta'\right]\psi''+\left[(2\varepsilon-1)\,z\theta''+x\theta'\varepsilon\right]\tfrac{1-\sigma_e}{2}\theta k\right\}\tfrac{\mathrm{d}\pi(z)}{\mathrm{d}\sigma_e}\\&=\left[(2\varepsilon-1)\,z\theta''+x\theta'\varepsilon\right]\tfrac{1-\sigma_e}{2}\theta k-(\varepsilon-1)\,\theta'\psi'+\left[\theta''+(s+s_0)\,\theta'\right]\psi''.\end{aligned}$$

For the right-hand side to be strictly positive, a sufficient condition is that the difference between the first two terms is non-negative. Substituting ψ' from (B.1) and substituting $z\theta''=\theta\varepsilon'+(\varepsilon-1)\,\theta'$, I rewrite this difference as:

$$\frac{1-\sigma_e}{2}\theta k\left\{(2\varepsilon-1)\,\theta\varepsilon'+\left[\varepsilon-1+x\right]\theta'\right\}.$$

Since $\varepsilon>1$, a sufficient condition for the above expression to be strictly positive is $\varepsilon'\geq 0$. Thus, $\varepsilon'\geq 0$ is sufficient for $\frac{\mathrm{d}\pi(z)}{\mathrm{d}\sigma_e}<0$.

Assume $\varepsilon'\geq 0$. For any given p, (5.1) implies:

$$x\frac{\mathrm{d}F}{\mathrm{d}\sigma_e}=-F\frac{\mathrm{d}x}{\mathrm{d}\sigma_e}-\frac{\mathrm{d}\pi\,(z)}{\mathrm{d}\sigma_e}>0.$$

Thus, the policies reduce posted prices in the first-order stochastic dominance. Similar to (2.5), the distribution of transaction prices is:

$$G\,(p|z)\equiv\frac{1-e^{-xF(p|z)}}{1-e^{-x}}=\frac{1-\frac{\pi(z)}{p}}{1-e^{-x}},$$

where the second equality comes from substituting $F\,(p|x)$ from (5.1). Because σ_e reduces $\pi\,(z)$ and x, then $\frac{\mathrm{d}G}{\mathrm{d}\sigma_e}>0$ for any given p. That is, the policies reduce transaction prices in the first-order stochastic dominance. Moreover, (5.2) implies:

$$\frac{\mathrm{d}p_L}{\mathrm{d}\sigma_e}=\frac{\mathrm{d}\pi\,(z)}{\mathrm{d}\sigma_e}<0,\qquad\frac{\mathrm{d}p_H}{\mathrm{d}\sigma_e}=e^{xF}\left[\frac{\mathrm{d}\pi\,(z)}{\mathrm{d}\sigma_e}+\pi\,(z)\,F\frac{\mathrm{d}x}{\mathrm{d}\sigma_e}\right]<0.$$

The policies affect the price spread, $\Delta p=p_H-p_L$, as follows:

$$\frac{\mathrm{d}\Delta p}{\mathrm{d}\sigma_e}=\left(e^{xF}-1\right)\frac{\mathrm{d}\pi\,(z)}{\mathrm{d}\sigma_e}+\pi\,(z)\,Fe^{xF}\frac{\mathrm{d}x}{\mathrm{d}\sigma_e}<0.$$

This completes the proof of Proposition 5.4. **QED**

C. The Model with a Fixed Measure of Stayers

Consider an economy where the measure of stayers is fixed at $\theta_0 \in (0, \infty)$. If the equilibrium yields only one active submarket, then a visitor's meeting rate in the submarket is $x_0 = M(\theta_0, 1)$, i.e., $\theta_0 = \theta(x_0)$. However, individuals can still direct search in the first stage of SMS by exploring other submarkets outside of the equilibrium. Requirements (i) -(iii) in Definition 2.2 continue to hold. Requirement (iv) on the equilibrium, which sets $U = k$, is no longer valid. Rather, a stayer's market value U adjusts endogenously for the optimal x to be equal to x_0. Recall that $(x_i, F(.|x_i))_{i \in I}$ denotes the active submarkets in the equilibrium. Let N_{vi} be the distribution of visitors across submarkets. Requirement (iv) is replaced by:

(iv') A stayer's market value U is such that

$$\int_{i \in I} \theta(x_i) \ \mathrm{d}N_{vi} = \theta_0.$$

The equilibrium can be determined as follows. As in section 2.2, a visitor's optimal choice of the submarket maximizes the expected surplus $D(x)$ given by (2.10). Because $D(x)$ is strictly concave on the domain $[0, \bar{x}]$, the optimal choice of x is unique. This implies that at most one submarket is active in the equilibrium. In this case, the equilibrium requirement (iv') requires the tightness in the active submarket to be equal to θ_0. The optimal choice of x must be equal to $x_0 = M(\theta_0, 1)$. For any given $\theta_0 \in (0, \infty)$, $x_0 = M(\theta_0, 1)$ lies in the interior of $(0, \bar{x})$. Because an interior optimal x must satisfy the first-order condition, (2.11), the condition determines $U = \frac{y}{\theta'(x_0)e^{x_0}}$. Clearly, U lies in $(0, \frac{y}{\theta'(0)})$, and the probability $Q = [1 - (1 + x_0) e^{-x_0}]$ lies in $(0, 1)$. Moreover, since $\pi(x_0) = \frac{U}{\lambda(x_0)}$, then $p_H = \frac{Ue^{x_0}}{\lambda(x_0)} = \frac{y}{\varepsilon(x_0)} < y$. I have proven the following proposition:

Proposition C.1. *Suppose that the measures of individuals on both sides of the market are fixed, with a tightness $\theta_0 \in (0, \infty)$. There exists a unique SMSE, where $U = \frac{y}{\theta'(x_0)e^{x_0}}$ and $x_0 = M(\theta_0, 1) \in (0, \bar{x})$. The probability that a visitor receives two or more meetings is equal to $Q = [1 - (1 + x_0) e^{-x_0}]$, which lies in $(0, 1)$. Moreover, $p_H = \frac{y}{\varepsilon(x_0)} < y$.*

References

[1] Albrecht, J., Gautier, P. and S. Vroman, 2006, "Equilibrium Directed Search with Multiple Applications," Review of Economic Studies 73, 869-891.

[2] Albrecht, J., Gautier, P. and S. Vroman, 2012, "A Note on Peters and Severinov, 'Competition Among Sellers Who Offer Auctions Instead of Prices'," Journal of Economic Theory 147, 389-392.

[3] Baye, M.R., Morgan, J. and P. Scholten, 2004, "Price Dispersion in the Small and in the Large: Evidence from an Internet Price Comparison Site," Journal of Industrial Economics 52, 463-496.

[4] Bethune, Z., Choi, M. and R. Wright, 2018, "Frictional Goods Markets: Theory and Applications," manuscript, University of Virginia.

[5] Burdett, K. and K.L. Judd, 1983, "Equilibrium Price Dispersion," Econometrica 51, 955-969.

[6] Burdett, K. and D. Mortensen, 1998, "Wage Differentials, Employer Size, and Unemployment," International Economic Review 39, 257-273.

[7] Burdett, K. Shi, S. and R. Wright, 2001, "Pricing and Matching with Frictions," Journal of Political Economy 109, 1060-1085.

[8] Cai, X., Gautier, P. and R. Wolthoff, 2017, "Search Frictions, Competing Mechanisms and Optimal Market Segmentation," Journal of Economic Theory 169, 453-473.

[9] Dagum, C., 1975, "A Model of Income Distribution and the Conditions of Existence of Moments of Finite Order," Bulletin of the International Statistical Institute 46, 199-205.

[10] Delacroix, A. and S. Shi, 2013, "Pricing and Signaling with Frictions," Journal of Economic Theory 148, 1301-1332.

[11] Diamond, P.A., 1971, "A Model of Price Adjustment," Journal of Economic Theory 3, 156-168.

[12] Diamond, P.A., 1982, "Aggregate Demand Management in Search Equilibrium," Journal of Political Economy 90, 881-894.

[13] Eeckhout, J. and P. Kircher, 2010, "Sorting vs Screening - Search Frictions and Competing Mechanisms," Journal of Economic Theory 145, 1354-1385.

[14] Ellison, G. and S. Ellison, 2005, "Lessons about Markets from the Internet," Journal of Economic Perspectives 19 (2), 139-158.

[15] Galenianos, M. and P. Kircher, 2009, "Directed Search with Multiple Job Applications," Journal of Economic Theory 144, 445-471.

[16] Godoy, A. and E. Moen, 2013, "Mixed Search," manuscript, University of Oslo.

[17] Gourio, F. and L. Rudanko, 2014, "Customer Capital," Review of Economic Studies 81, 1102-1136.

[18] Guerrieri, G., Shimer, R. and R. Wright, 2010, "Adverse Selection in Competitive Search Equilibrium," Econometrica 78, 1823-1862.

[19] Haight, Frank A., 1967, Handbook of the Poisson Distribution. New York: John Wiley & Sons.

[20] Hall, R.E. and A.B. Krueger, 2012, "Evidence on the Incidence of Wage Posting, Wage Bargaining, and On-the-Job Search," American Economic Journal: Macroeconomics 4, 56-67.

[21] Hosios, A., 1990, "On the Efficiency of Matching and Related Models of Search and Unemployment," Review of Economic Studies 57, 279-298.

[22] Julien, B., Kennes, J. and I. King, 2000, "Bidding for Labor," Review of Economic Dynamics 3, 619-649.

[23] Kennes, J., le Maire, D. and S. Roelsgaard, 2018, "Equivalence of Canonical Matching Models," manuscript, Aarhus University.

[24] Kim, K. and P. Kircher, 2015, "Efficient Competition through Cheap Talk: The Case of Competing Auctions," Econometrica 83, 1849-1875.

[25] Lester, B., 2011, "Information and Prices with Capacity Constraints," American Economic Review 101, 1591-1600.

[26] Lester, B., Visschers, L., and R. Wolthoff, 2015, "Meeting Technologies and Optimal Trading Mechanisms in Competitive Search Markets," Journal of Economic Theory 155, 1-15.

[27] Lester, B., Visschers, L. and R. Wolthoff, 2017, "Competing with Asking Prices," Theoretical Economics 12, 731-770.

[28] Menzio, G., 2007, "A Theory of Partially Directed Search," Journal of Political Economy 115, 748-769.

[29] Menzio, G. and S. Shi, 2010, "Block Recursive Equilibria for Stochastic Models of Search on the Job," Journal of Economic Theory 145, 1453-1494.

[30] Menzio, G. and N. Trachter, 2015, "Equilibrium Price Dispersion with Sequential Search," Journal of Economic Theory 160, 188-215.

[31] Moen, E.R., 1997, "Competitive Search Equilibrium," Journal of Political Economy 105, 385-411.

[32] Montgomery, J.D., 1991, "Equilibrium Wage Dispersion and Interindustry Wage Differentials," Quarterly Journal of Economics 106, 163-179.

[33] Peters, M., 1991, "Ex Ante Price Offers in Matching Games: Non-Steady State," Econometrica 59, 1425-1454.

[34] Peters, M. and S. Severinov, 1997, "Competition among Sellers Who Offer Auctions Instead of Prices," Journal of Economic Theory 75, 141-179.

[35] Shi, S., 2001, "Frictional Assignment, I: Efficiency," Journal of Economic Theory 98, 232-260.

[36] Shi, S., 2009, "Directed Search for Equilibrium Wage-Tenure Contracts," Econometrica 77, 561-584.

[37] Shi, S., 2016, "Customer Relationship and Sales," Journal of Economic Theory 166, 483-516.

[38] Shi, S., 2018, "Sequentially Mixed Search Equilibrium with Many-to-Many Meetings," manuscript, Pennsylvania State University.

[39] Shi, S. and A. Delacroix, 2018, "Should Buyers or Sellers Organize Trade in a Frictional Market?", Quarterly Journal of Economics 133, 2171-2214.

[40] Stacey, D., 2015, "Posted Prices, Search and Bargaining," manuscript, Ryerson University.

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Supplementary Appendix for

"**Sequentially Mixed Search and Equilibrium Price Dispersion**"

Shouyong Shi

D. Proofs of Proposition 4.1 and Lemma 5.1

Proof of Proposition 4.1:

I prove the following proposition that includes the effects of y, in addition to the effects of A stated in Proposition 4.1:

Proposition D.1. *(i)* $\frac{dx^*}{dy} > 0$, $\frac{d\theta}{dy} > 0$, $\frac{dp_L}{dy} > 0$, $\frac{dp_H}{dy} > 0$, $\frac{d\Delta p}{dy} > 0$, *and prices increase in* y *in the first-order stochastic dominance in* F *and* G. *(ii)* $\frac{dx^*}{dA} > 0$, $\frac{d}{dy} cv_i(x) > 0$ *and* $\frac{d}{dA} cv_i(x) > 0$ *for* $i = F, G$. *(iii)* $\frac{d\theta}{dA} > 0$ *iff* $A < A_0$, *where* A_0 *is defined by (4.2). (iv)* $\frac{dp_H}{dA} \leq 0$ *iff* $(A_0 - A)\varepsilon' > 0$*; If* $\varepsilon' \geq 0$ *then* $\frac{dp_L}{dA} < 0$ *and* $\frac{d\Delta p}{dA} > 0$. *(v) If* $\varepsilon' \geq 0$ *and* $A < A_0$, *an increase in* A *reduces posted and transaction prices in the first-order stochastic dominance.*

Proof. Suppress the asterisk on x. Differentiating (4.1) yields:

$$\left(\frac{r\theta''}{\theta'} + x\right)\frac{\mathrm{d}x}{x} = \frac{\mathrm{d}y}{y} + \left(\frac{r\theta''}{\theta'} + 1\right)\frac{\mathrm{d}A}{A}, \tag{D.1}$$

where the argument of θ is $r = x/A$. Since $\theta' > 0$ and $\theta'' > 0$, then $\frac{\mathrm{d}x}{\mathrm{d}y} > 0$ and $\frac{\mathrm{d}x}{\mathrm{d}A} > 0$. The coefficients of variation given by (3.1) are only functions of x and do not depend on (y, A) directly. Moreover, $cv_F'(x) > 0$ and $cv_G'(x) > 0$ (see Corollary 3.1). Because $\frac{\mathrm{d}x}{\mathrm{d}y} > 0$ and $\frac{\mathrm{d}x}{\mathrm{d}A} > 0$, then $\frac{\mathrm{d}}{\mathrm{d}y} cv_i(x) > 0$ and $\frac{\mathrm{d}}{\mathrm{d}A} cv_i(x) > 0$ for $i = F, G$.

Differentiating $\theta(r)$ and substituting $(\mathrm{d}x)$ yields:

$$\left(\frac{r\theta''}{\theta'} + x\right)\frac{\mathrm{d}\theta}{r\theta'} = \frac{\mathrm{d}y}{y} + (1 - x)\frac{\mathrm{d}A}{A}.$$

Clearly, $\frac{\mathrm{d}\theta}{\mathrm{d}y} > 0$. The definition of A_0 in (4.2) implies that $x < 1$ is equivalent to $A < A_0$. Then, $\frac{\mathrm{d}\theta}{\mathrm{d}A} > 0$ iff $x < 1$ and, hence, iff $A < A_0$. Substituting π from (2.6) and $U = k$ into

(2.9) and differentiating, I get:

$$\frac{x}{k\theta}\left(\frac{r\theta''}{\theta'}+x\right)\mathrm{d}p_L=\frac{\varepsilon-1}{y}\mathrm{d}y-\left(\frac{\theta}{\theta'}\varepsilon'+x\varepsilon\right)\frac{\mathrm{d}A}{A} \tag{D.2}$$

$$\frac{xe^{-x}}{k\theta}\left(\frac{r\theta''}{\theta'}+x\right)\mathrm{d}p_H=\frac{\varepsilon-1+x}{y}\mathrm{d}y-\frac{\theta\varepsilon'}{\theta'}\left(1-x\right)\frac{\mathrm{d}A}{A} \tag{D.3}$$

$$\begin{aligned}\tfrac{xe^{-x}}{k\theta}\left(\tfrac{r\theta''}{\theta'}+x\right)\mathrm{d}\Delta p= \quad & \tfrac{\left(1-e^{-x}\right)(\varepsilon-1)+x}{y}\mathrm{d}y \\ & +\left[\left(x-1+e^{-x}\right)\tfrac{\theta}{\theta'}\varepsilon'+x\varepsilon e^{-x}\right]\tfrac{\mathrm{d}A}{A}\end{aligned} \tag{D.4}$$

I have used $\varepsilon'(r)=\frac{1}{\theta}\left[r\theta''-(\varepsilon-1)\theta'\right]$, where the argument of θ and ε is $r=x/A$. Because $\varepsilon>1$, the above equations show that $\frac{\mathrm{d}p_L}{\mathrm{d}y}>0$, $\frac{\mathrm{d}p_H}{\mathrm{d}y}>0$ and $\frac{\mathrm{d}\Delta p}{\mathrm{d}y}>0$. The effects of A on $(p_L,p_H,\Delta p)$ depend on ε'. Clearly, $\frac{\mathrm{d}p_H}{\mathrm{d}A}\leq 0$ iff $(1-x)\,\varepsilon'>0$ and, hence, iff $(A_0-A)\,\varepsilon'>0$. Note that $x-1+e^{-x}>0$ for all $x>0$. If $\varepsilon'\geq 0$, then $\frac{\mathrm{d}p_L}{\mathrm{d}A}<0$ and $\frac{\mathrm{d}\Delta p}{\mathrm{d}A}>0$.

The cumulative distribution function of posted prices is $F(.|x)$ given by (2.7) and the cumulative distribution function of transaction prices is $G(.|x)$ given by (2.5). For any given p, differentiating these functions with respect to (y,A) yields:

$$x\left(\frac{r\theta''}{\theta'}+x\right)\mathrm{d}F=-\frac{\varepsilon-1+xF}{y}\mathrm{d}y+\left[\left(1-xF\right)\frac{\theta}{\theta'}\varepsilon'+x\varepsilon\left(1-F\right)\right]\frac{\mathrm{d}A}{A} \tag{D.5}$$

$$\begin{aligned}x\left(\tfrac{r\theta''}{\theta'}+x\right)\tfrac{\mathrm{d}G}{G}= \quad & -\left(\tfrac{\varepsilon-1}{e^{xF}-1}+\tfrac{x}{e^{x}-1}\right)\tfrac{\mathrm{d}y}{y} \\ & +\left[\tfrac{\theta\varepsilon'}{\theta'}\left(\tfrac{1}{e^{xF}-1}-\tfrac{x}{e^{x}-1}\right)+x\varepsilon\left(\tfrac{1}{e^{xF}-1}-\tfrac{1}{e^{x}-1}\right)\right]\tfrac{\mathrm{d}A}{A}.\end{aligned} \tag{D.6}$$

Because $\varepsilon>1$, the coefficients of $(\mathrm{d}y)$ in both equations are strictly negative. Thus, an increase in y increases both posted and transaction prices in the first-order stochastic dominance. Note that $xF\leq x$. If $\varepsilon'\geq 0$ and $A<A_0$ (i.e., $x<1$), then the coefficients of $(\mathrm{d}A)$ in (D.5) and (D.6) are strictly positive for all interior p. Thus, an increase in A reduces both posted and transaction prices. **QED**

Proof of Lemma 5.1:

It is easy to verify that $L(t)$ defined by (5.6) satisfies $L(\infty)=L'(\infty)=\lim_{t\to\infty}L'(t)\,t=0$. Using L'Hopital's rule, one can confirm the values of $L(0)$, $L'(0)$ and $L''(0)$ stated in

(i) of Lemma 5.1. To verify $L(t) > 0$ in (ii) of Lemma 5.1, note that $1 > e^{-t}$ if and only if $t > 0$. Thus, $L(t) > 0$ for all $t \neq 0$. In addition, $L(0) = 1 > 0$. To prove $L'(t) < 0$ for all t, it suffices to prove $L'(t) < 0$ for all $t \neq 0$, since $L'(0) = -\frac{1}{2} < 0$ by (i). Compute

$$L'(t) = \frac{e^{-t}}{t^2}\left[t + 1 - e^t\right].$$

Examine the expression in [.] on the right-hand side for $t \neq 0$. The expression is strictly concave in t. The unique maximum is achieved at $t = 0$, and it is equal to 0. Thus, the expression is negative for all $t \neq 0$, which implies $L'(t) < 0$. Similarly, compute:

$$L''(t) = \frac{e^{-t}}{t^3}\left[2e^t - \left(t^2 + 2t + 2\right)\right].$$

Since $L''(0) = \frac{1}{3} > 0$, as proven above, focus on $t \neq 0$. Of the expression in [.] on the right-hand side of L'', the third-order derivative is $2e^t$, which is strictly positive. This implies that the first-order derivative, which is equal to $2(e^t - t - 1)$, has a unique minimum. This minimum occurs at $t = 0$ and is equal to 0. That is, $[2e^t - (t^2 + 2t + 2)]$ is strictly increasing for all $t \neq 0$. Since this expression is equal to 0 at $t = 0$, it is strictly positive if and only if $t > 0$. Clearly, $t^3 > 0$ if and only if $t > 0$. Then $L''(t) > 0$ for all $t \neq 0$. For (iii) of Lemma 5.1, compute:

$$\begin{array}{l} L + L' = \frac{1}{t^2}\left[t - 1 + e^{-t}\right] \\ \frac{1}{t}\left(L + 2L'\right) = -\left(L' + L''\right) = \frac{1}{t^3}\left[t - 2 + (2 + t)\, e^{-t}\right]. \end{array}$$

Using the same method of proving $L' < 0$ and $L'' > 0$ above, one can prove $L + L' > 0$ and $\frac{1}{t}(L + 2L') > 0$ for all $|t| < \infty$. **QED**

E. Proof of Proposition 5.5

Comparative statics with respect to y without the policies: In the absence of the policies, the SMSE is characterized by (5.9). For any given s, the solution for z is $z_a(s)$ to the first equation in (5.9) and $z_b(s)$ to the second equation. To prove $\frac{\mathrm{d}s}{\mathrm{d}y} > 0$, note that $\frac{\partial z_a(s)}{\partial y} > 0$ and $\frac{\partial z_b(s)}{\partial y} = 0$, where the partial derivatives are taken for any given s. Since $z_b'(s) > z_a'(s)$ with the optimal s, then

$$\frac{\mathrm{d}s}{\mathrm{d}y} = \frac{\partial z_a(s)/\partial y}{z_b'(s) - z_a'(s)} > 0.$$

Because $\frac{\partial z_b(s)}{\partial y} = 0$, then

$$\frac{\mathrm{d}z}{\mathrm{d}y} = z_b'(s)\frac{\mathrm{d}s}{\mathrm{d}y}.$$

Therefore, the sign of $\frac{\mathrm{d}z}{\mathrm{d}y}$ is the same as $z_b'(s)$, which is ambiguous and examined below.

Similarly, using the notation $x = (s + s_0)z$ to express $s = \frac{x}{z} - s_0$, I can use the two equations in (5.9) to solve for $z = z_a(x)$ and $z = z_b(x)$. Then,

$$z_a'(x) = \frac{-\theta'}{\theta''}, \quad z_b{}'(x) = \frac{\psi'' - \frac{k\theta}{2}z}{kz\left[z\theta'' + \frac{x}{2}\theta'\right] + \psi''\frac{x}{z}}.$$

Again, it can be verified that $z_b'(x) > z_a'(x)$. Thus, there is a unique solution x to $z_b(x) = z_a(x)$, and the solution satisfies $\frac{\mathrm{d}x}{\mathrm{d}y} > 0$. Furthermore, $\frac{\mathrm{d}(\pi e^x)}{\mathrm{d}y} > 0$, and so $\frac{\mathrm{d}p_H}{\mathrm{d}y} > 0$. If $\frac{\mathrm{d}z}{\mathrm{d}y} \leq 0$, then $\frac{\mathrm{d}p_L}{\mathrm{d}y} \leq 0$ and $\frac{\mathrm{d}\Delta p}{\mathrm{d}y} > 0$. If $\frac{\mathrm{d}z}{\mathrm{d}y} > 0$, then $\frac{\mathrm{d}\pi}{\mathrm{d}y} > 0$ and

$$\frac{\mathrm{d}\Delta p}{\mathrm{d}y} = (e^x - 1)\frac{\mathrm{d}\pi}{\mathrm{d}y} + \pi e^x\frac{\mathrm{d}x}{\mathrm{d}y} > 0.$$

In all cases, $\frac{\mathrm{d}\Delta p}{\mathrm{d}y} > 0$. Since $cv_F'(x) > 0$ and $cv_G'(x) > 0$, the result $\frac{\mathrm{d}x}{\mathrm{d}y} > 0$ implies $\frac{\mathrm{d}}{\mathrm{d}y}cv_F > 0$ and $\frac{\mathrm{d}}{\mathrm{d}y}cv_G > 0$.

Finally, I establish the sign of $z_b'(s)$. Return to the use of s instead of x as the variable. Denote $z_c(s)$ as the solution to $\theta(z_c)z_c = 2\psi''(s)/k$. Then, $z_b'(s) > 0$ if and only if $z < z_c(s)$. Because $\psi'' > 0$, then $z_c(s_0) > 0$. Consider the following cases:

Case (i): $\psi''' > 0$. In this case, $z_c'(s) > 0$ for all s. I prove that $z_b'(s) > 0$ for all $s \geq s_0$, which implies $\frac{\mathrm{d}z}{\mathrm{d}y} > 0$ by the above proof. A sufficient condition for this result is $z_b(s) < z_c(s)$ for all $s \geq 0$. To prove that this sufficient condition holds, suppose, to the contrary, that $z_b(s_a) = z_c(s_a)$ for some $s_a \in [0, \infty)$. Clearly, $s_a > 0$ and $z_b'(s_a) = 0$. Without loss of generality, let s_a be the smallest solution to $z_b(s) = z_c(s)$. Because $z_b(s) < z_c(s)$ for all $s < s_a$, and $z_b(s_a) = z_c(s_a)$, then $z_b'(s_a) \geq z_c'(s_a) > 0$. This contradicts the fact that $z_b'(s_a) = 0$. Thus, $s_a > 0$ does not exist, and so $z_b(s) < z_c(s)$ for all $s > s_0$. In this case, the SMSE is depicted by point E1 in Figure 1.

Case (ii): $\psi''' < 0$, but $z_b(s) < z_c(s)$ for all s. As in case (i), this case has $z_b'(s) > 0$ for all s, and so $\frac{\mathrm{d}z}{\mathrm{d}y} > 0$.

Case (iii): $\psi''' < 0$, and there exists $s_a > 0$ such that $z_b(s_a) = z_c(s_a)$. Let s_a be the smallest solution to $z_b(s) = z_c(s)$. I prove that s_a is the only solution to $z_b(s) = z_c(s)$.

Suppose, to the contrary, that there is another solution s_1 ($> s_a$) to $z_b(s) = z_c(s)$. Without loss of generality, let s_1 be the smallest solution among all $s > s_a$. Then $z_b(s_1 - \varepsilon) > z_c(s_1 - \varepsilon)$ for sufficiently small $\varepsilon > 0$. This fact implies $z_b'(s_1) \le z_c'(s_1) < 0$. This contradicts the fact $z_b'(s_1) = 0$. Thus, s_1 does not exist; i.e., $z_b(s) > z_c(s)$ for all $s > s_a$. Therefore, $z_b'(s) < 0$ if and only if $s > s_a$. That is, $\frac{\mathrm{d}z}{\mathrm{d}y} > 0$ if and only if the optimal s satisfies $s < s_a$. In this case, the SMSE is depicted by point E2 in Figure 1.

Case (iv): $\psi''' = 0$. In this case, $z_c(s)$ is constant over s. If $z_b(s) < z_c(s)$ for all $s > 0$, the case is qualitatively the same as case (i). If there is a solution $s_a > 0$ to $z_b(s) = z_c(s)$, then $z_b(s) = z_c(s)$ for all $s > s_a$. In this case, $\frac{\mathrm{d}z}{\mathrm{d}y} > 0$ if $s < s_a$, and $\frac{\mathrm{d}z}{\mathrm{d}y} = 0$ if $s > s_a$.

The effect of y on the efficient policies: Under the efficient policies in (5.13), the equilibrium allocation coincides with the social optimum given by (5.10). Differentiating (5.10) with respect to y, I get:

$$\frac{\mathrm{d}z}{\mathrm{d}y} = \frac{e^{-x}}{k\left[(s+s_0)\,\theta' + \left(\frac{z^2 k\theta'}{\psi''} + 1\right)\theta''\right]} > 0, \qquad \frac{\mathrm{d}s}{\mathrm{d}y} = \frac{kz\theta''}{\psi''}\frac{\mathrm{d}z}{\mathrm{d}y} > 0.$$

I have substituted $\varepsilon' = \frac{1}{\theta}\left[z\theta'' - (\varepsilon - 1)\,\theta'\right]$. All variables in the above expressions, as in the remainder of this proof, are the ones in the social optimum. Differentiating the expression for σ_e in (5.13) with respect to y, I have:

$$\frac{\mathrm{d}\sigma_e}{\mathrm{d}y} = \left(\frac{2\mathrm{d}z}{\mathrm{d}y}\right)\frac{\sigma_e^2}{x}\left[\frac{(\varepsilon-1)\,kz}{s+s_0}\frac{\theta''}{\psi''} + \frac{\varepsilon-1}{z} - \varepsilon'\right].$$

A sufficient condition for $\frac{\mathrm{d}\sigma_e}{\mathrm{d}y} > 0$ is $\varepsilon' \le \frac{\varepsilon-1}{z}$. The expression for τ_y in (5.13) shows that this condition is also sufficient for $\frac{\mathrm{d}\tau_y}{\mathrm{d}\theta} > 0$. **QED**

F. Splitting the Participation in Two Submarkets

The baseline model assumes that a visitor chooses one submarket to participate and establishes the result that only one submarket is active in the equilibrium (see Proposition 2.4). Because the meeting function has constant returns to scale, dividing visitors into many miniatures of this equilibrium submarket does not change the equilibrium.

In this appendix I examine another way to split visitors' participation into different submarkets. Suppose that each visitor has one unit of effort and can divide the effort between two optimally chosen submarkets. In order to compare the results with those in the baseline model, let me assume that the division of effort is exogenous. (An endogenous division of effort will generate the externality analyzed in section 5, which makes the latter section the proper reference to compare, instead.) In submarket $i \in \{1, 2\}$, let α_i be the amount of effort per visitor, n_{vi} the density of visitors, n_{si} the density of stayers, and $\theta_i = \frac{n_{si}}{\alpha_i n_{vi}}$ the effective tightness. By assumption, $\alpha_1 + \alpha_2 = 1$ and $\alpha_i \in (0, 1)$. In submarket i, the amount of meetings is $M(n_{si}, \alpha_i n_{vi})$, and the meeting rate per search effort is $z_i = M(\theta_i, 1)$, which solves $\theta_i = \theta(z_i)$. In submarket i, a visitor's meeting rate is $x_i = \alpha_i z_i$, the meeting probability for a stayer is $\lambda(z_i) = \frac{z_i}{\theta(z_i)}$, and the distribution of posted prices is $F(.|z_i)$. Let p_{Li} be the lower bound and p_{Hi} the upper bound on the support of $F(.|z_i)$. Refer to a submarket by $(z, F(.|z))$ instead of $(x, F(.|x))$.

In the baseline equilibrium in Proposition 2.4, the active submarket is $(x^*, F(.|x^*))$, the tightness of the submarket is $\theta^* = \theta(x^*)$, and the support of $F(.|x^*)$ is $[p_L^*, p_H^*]$. The following proposition states the equivalence between the equilibrium with two active submarkets and the baseline equilibrium:

Proposition F.1. *Let p_{H1} be an arbitrary number in $(p_L^*, p_H^*]$ and maintain $U = k$. The two submarkets $(z_1, F(.|z_1))$ and $(z_2, F(.|z_2))$ form an equilibrium if and only if the following conditions are met: (1) $z_1 = z_2 = x^*$; (2) $F(.|z_1)$ and $F(.|z_2)$ are continuous, with the support $[p_L^*, p_{H1}]$ and $[p_L^*, p_H^*]$, respectively; (3) $\Sigma_{i=1,2}\alpha_i F(p|z_i) = F(p|x^*)$ for all $p \in [p_L^*, p_H^*]$. This equilibrium with the two active submarkets and the baseline equilibrium deliver the same probabilities of meeting and matching for a visitor, the same market tightness, and the same overall distribution of prices in the economy.*

The two active submarkets have the same tightness which is equal to the tightness in the active submarket in the baseline equilibrium. As a result, the meeting rate per effort in the two submarkets is the same as in the baseline equilibrium. The distributions of posted prices in the two submarkets are indeterminate. However, the weighted sum of the two distributions is determinate and equal to the distribution in the baseline equilibrium, as stated in condition (3) above. Because a visitor receives meetings from both submarkets, it is the weighted sum of the two distributions that matters for trade. Viewing the two submarkets together, a visitor faces the same distribution of posted prices in the economy as in the baseline equilibrium.

There is a clarification on the "only if" statement in Proposition F.1: The upper bounds on the supports of $F(.|z_1)$ and $F(.|z_2)$ can be switched. Because of the assumption that stayers who enter submarket i must commit to not posting prices higher than p_{Hi}, it may be possible that $p_{Hi} < p_H^*$ for one value of i. However, one of the upper bounds must be equal to p_H^*. I set $p_{H2} = p_H^*$ above. If $p_{H1} = p_H^*$, instead, then $p_{H2} \le p_H^*$ is possible.

Proof of Proposition F.1:

Consider a stayer in submarket 1 and let p be the price posted by the stayer. Suppose that the stayer has a meeting. The visitor in the meeting can receive other meetings in submarket $i \in \{1, 2\}$ at the rate $\alpha_i z_i$, and the price in such a meeting is no higher than p with the probability $F(p|z_i)$. The rate at which the visitor receives a competing meeting with a price no higher than p is $\Sigma_{i=1,2}\alpha_i z_i F(p|z_i)$. The stayer's expected surplus is:

$$\Pi_1(p, z_1, z_2) = pe^{-\Sigma_{i=1,2}\alpha_i z_i F(p|z_i)}.$$

Denote $\pi_i = \max_{p \le p_{Hi}} \Pi_i(p, z_1, z_2)$. For submarket i to have $\theta(z_i) > 0$, it must have $\pi_i \ge U$. Then, part (a) of requirement (iii) in Definition 2.2 requires $\lambda(z_i)\pi_i = U$. All prices in the support of $F(.|z_1)$ make the same expected surplus to the stayer, which is equal to $\pi_1 = \frac{U}{\lambda(z_1)}$. Thus,

$$p = \frac{U}{\lambda(z_1)} e^{\Sigma_{i=1,2}\alpha_i z_i F(p|z_i)} \text{ for all } p \in \operatorname{supp}(F(.|z_1)). \tag{F.1}$$

Similarly,

$$p = \frac{U}{\lambda(z_2)} e^{\Sigma_{i=1,2}\alpha_i z_i F(p|z_i)} \text{ for all } p \in \operatorname{supp}(F(.|z_2)). \tag{F.2}$$

Now examine a visitor. Since the visitor's meeting rate in submarket i is $x_i = \alpha_i z_i$, the visitor's probability of trading at a price no higher than p is $1 - e^{-\Sigma_{i=1,2}\alpha_i z_i F(p|z_i)}$. The visitor's expected surplus in the market is:

$$D = \int (y - p)\, \mathrm{d}\left[1 - e^{-\Sigma_{i=1,2}\alpha_i z_i F(p|z_i)}\right].$$

The integration is over the union of the supports of $F(.|z_1)$ and $F(.|z_2)$, which is suppressed as in all integrals below. Separating the parts with y and p, I have:

$$D = y\left[1 - e^{-\Sigma_{i=1,2}\alpha_i z_i}\right] - \int p e^{-\Sigma_{i=1,2}\alpha_i z_i F(p|z_i)} \Sigma_{i=1,2}\left[\alpha_i z_i \mathrm{d}F(p|z_i)\right].$$

Substitute p from (F.1) for the integration with respect to $F(p|z_1)$ and from (F.2) for the integration with respect to $F(p|z_2)$. Since $\theta(z_i) = \frac{z_i}{\lambda(z_i)}$, I can compute:

$$D = y\left[1 - e^{-\Sigma_{i=1,2}\alpha_i z_i}\right] - U\Sigma_{i=1,2}\alpha_i\theta(z_i).$$

The visitor chooses (z_1, z_2) to maximize D. Under Assumption 1, D is strictly concave in (z_1, z_2), and the optimal choices are interior. Thus, the optimal z_i is uniquely given by the first-order condition:

$$y e^{-\Sigma_{i=1,2}\alpha_i z_i} = U\theta'(z_i), \; i = 1, 2.$$

The solution to this equation is independent of i and is equal to x^*, where x^* is defined by (2.11). That is, $z_1 = z_2 = x^*$. This result implies $\lambda(z_1) = \lambda(z_2) = \lambda(x^*)$ and $\pi_1 = \pi_2 = \pi^* \equiv \frac{U}{\lambda(x^*)}$, where x^* and π^* are the values in the baseline equilibrium. Moreover, (F.1) and (F.2) become the same except possibly the supports of the two distributions.

Part (b) of requirement (iii) in Definition 2.2 requires $\Pi_i(p, z_1, z_2) = \pi^*$ for all $p \in$supp$(F(.|z_i))$ and $\Pi_i(p, z_1, z_2) < \pi^*$ for all $p \notin$supp$(F(.|z_i))$ with $p \le p_{Hi}$. This requirement implies that the supports of $F(.|z_1)$ and $F(.|z_2)$ have the features (a)-(d) below:

(a) The support of $F(.|z_i)$ is an interval $[p_{Li}, p_{Hi}]$. If there were any hole in the support, say, (p_a, p_b), with p_a and p_b in the support of $F(.|z_i)$, then posting a price in the hole would yield a higher expected surplus to a stayer than posting p_a.

(b) $F(p|z_i)$ is continuous in p on $[p_{Li}, p_{Hi}]$. If there were a mass point at any p_a in the support, posting a price slightly lower than p_a would increase expected surplus.

(c) The union of the two supports is connected, and so $\min\{p_{H1}, p_{H2}\} \ge \max\{p_{L1}, p_{L2}\}$. This is similar to (a) but the proof compares the two submarkets. Suppose $p_{H1} < p_{L2}$, on the contrary. (A similar proof applies if $p_{H2} < p_{L1}$.) Compare a stayer posting $p_a \in (p_{H1}, p_{L2})$ in submarket 2 and a stayer posting p_{H1} in submarket 1. The two stayers have the same meeting probability. They also have the same probability to trade, which occurs when the visitor in the meeting has no other meeting with stayers in submarket 1. However, posting p_a in submarket 2 yields higher profit in a trade than posting p_{H1} in submarket 1 and, hence, higher than π^*. This contradicts the requirement that all prices outside the support of $F(.|z_2)$ should yield an expected surplus strictly lower than π^*.

(d) $p_{L1} = p_{L2} = p_L^*$ and $\max\{p_{H1}, p_{H2}\} = p_H^*$, where $[p_L^*, p_H^*]$ is the support of $F(.|x^*)$. It is clear that $\min\{p_{L1}, p_{L2}\} = \pi^* = p_L^*$ and $\max\{p_{H1}, p_{H2}\} = \pi^* e^{x^*} = p_H^*$. To prove $p_{L1} = p_{L2}$, suppose $p_{L1} < p_{L2}$, on the contrary. Posting any $p_a \in [p_{L1}, p_{L2})$ in submarket 1 yields the expected surplus π^* to a stayer. If a stayer posts p_a in submarket 2, the stayer has the same probability of trade and same profit in a trade as posting p_a in submarket 1. Thus, posting p_a in submarket 2 yields the expected surplus π^*. This contradicts part (b) of requirement (iii) in Definition 2.2 that all such prices yielding π^* and not exceeding p_{H2} should be posted by some stayers in submarket 2.

I assume $p_{H1} \leq p_H^*$, without loss of generality. Then, result (4) above implies $p_{H2} = p_H^*$. With $z_1 = z_2 = x^*$, (F.1) and (F.2) yield:

$$p = \pi^* e^{-x^* \Sigma_{i=1,2} \alpha_i F(p|z_i)} \text{ for all } p \in [p_L^*, p_H^*].$$

Comparing this equation with (2.7) in the baseline equilibrium yields $\Sigma_{i=1,2} \alpha_i F(p|z_i) = F(p|x^*)$ for all $p \in [p_L^*, p_H^*]$. Thus, if the two submarkets constitute an equilibrium, they must satisfy conditions (1)-(3) in Proposition F.1.

Conversely, if the two submarkets satisfy conditions (1)-(3) in Proposition F.1, they satisfy all requirements (i)-(iii) in Definition 2.2. With the maintained condition $U = k$, the two submarkets constitute an equilibrium.

By the construction above, it is evident that the equilibrium with two active submarkets is equivalent to the baseline equilibrium in the sense that the two equilibria deliver the same probabilities of meeting and matching for a visitor, the same market tightness, and the same overall distribution of prices in the economy. **QED**