QUEEN'S UNIVERSITY
Economics Department
ECON 811 Advanced Microeconomic Theory I Fall 2018

Instructor: Ruqu Wang

# FINAL EXAMINATION

Monday, 9:00am – 12:00noon, December 10, 2018

*Instructions:*

There are five questions in this exam.

You must answer ALL questions.

Put your name and student I.D. number on each answer booklet.

Please read carefully and ASK if you have any questions!

Good Luck!

**1. (20 points)** For the following $3 \times 3$ game, find all pure-strategy and mixed-strategy Nash equilibria.

| | $D$ | $E$ | $F$ |
|---|---|---|---|
| $A$ | $1, 2$ | $3, 5$ | $5, 3$ |
| $B$ | $5, 5$ | $3, 2$ | $1, 3$ |
| $C$ | $2, 1$ | $2, 2$ | $2, 8$ |

**2. (15 points)** Consider an infinitely repeated game of the following stage game:

| | $C$ | $D$ |
|---|---|---|
| $A$ | $4, 3$ | $0, 4$ |
| $B$ | $6, 0$ | $1, 1$ |

Let $\delta$ be the discount factor for both players. Determine the condition on $\delta$ under which (4,3) can be sustained as the per-period payoffs for the players in a subgame-perfect equilibrium.

**3. (20 points)** (a) **(5 points)** Consider the following multi-period, alternating-offer bargaining game. Two risk-neutral players want to share a pie. There is no discounting, but the size of the pie is reduced by 0.2 every period. That is, the size of the pie is equal to 1 in period 1, 0.8 in period 2, ..., 0 in period 6. Therefore, if these two players cannot reach an agreement by the end of period 5, both players will get 0.

The timing of this alternating-offer bargaining game is as follows. In period 1, player 1 makes an offer, and player 2 accepts or rejects. If the offer is rejected, in period 2, player 2 makes an offer and player 1 accepts or rejects. etc. Characterize all pure-strategy subgame-perfect Nash equilibria in this game.

(b) **(15 points)** Now suppose that the size of the pie is reduced by 10% every period, and there is still no discounting. Characterize the stationary pure-strategy subgame-perfect Nash equilibria in this game.

**4. (15 points)** Consider a public project that needs \$100 to complete. The project is a common good; Once completed, it has a value of \$200 to every player in the game. There are three players who make contributions to this project sequentially. The timing of this dynamic game is as follows. Player 1 contributes first (amount denoted by $x_1$). Observing $x_1$, player 2 contributes next (amount denoted by $x_2$). Observing $x_1$ and $x_2$, player 3 contributes last (amount denoted by $x_3$). If $x_1 + x_2 + x_3 \geq 100$, then the project will be completed and any extra money will be thrown away. If $x_1 + x_2 + x_3 < 100$, then the project will not be completed, and all contributions will be thrown away. Find the subgame perfect equilibrium in this game. Are there any Nash equilibria which are not subgame perfect?

**5. (30 points)**

Consider the following **First-Price, Common-Value** auction. There are $N$ bidders. Each bidder receives a private signal, $\theta$, which is a random and independent draw from a distribution with c.d.f. $F(\theta)$ and p.d.f. $f(\theta)$, where $\theta \in [\underline{\theta}, \bar{\theta}]$. Given $\theta_i$, $1 = 1, 2, ..., N$, each bidder values the object at

$$V = \theta_1 \theta_2 \cdots \theta_N.$$

In the auction, each bidder submits a sealed bid. The bidder with the highest bid wins the object and pays his bid to the seller. The losers (i.e., all other bidders) pay nothing.

(a) **(15 points)** Set up bidder $i$'s maximization problem and derive the first-order condition assuming a symmetric strictly increasing Bayesian equilibrium bidding function $b_i = B(\theta_i)$.

(b) **(15 points)** Determine the initial value $B(\underline{\theta})$ and solve for the equilibrium bidding function.