# Flighty Liquidity* 

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#### Abstract

We study how the risks to future liquidity flow across corporate bond, Treasury, and stock markets. We document distribution "flight-to-safety" effects: a deterioration in the liquidity of high yield corporate bonds forecasts an increase in the average liquidity of Treasuries and a decrease in uncertainty about the liquidity of investment grade corporate bonds. While the liquidity of Treasuries securities both affects and is affected by the liquidity in the other two markets, corporate bond and equity market liquidity appear to be largely divorced from each other. Finally, we show that measures of market-wide volatility and market-maker constraints do not contain information useful for predicting the distribution of future liquidity over and above that contained in the recent history of bid-ask spreads.


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## 1 Introduction

"If you're looking for a potential black swan, one area you could look at is what happens when people realize they don?t have the underlying liquidity that they thought they had."
(Mohamed El-Erian, chief economic adviser at Allianz, as reported by The Wall Street Journal, "Wild Markets Pinch Trading, Vexing Investors", December 7, 2018.)

Traditional measures of liquidity focus on the cost, either in terms of money or in terms of time, of buying or selling an asset given contemporaneous market conditions. By their nature, such measures capture the contemporaneous state of market liquidity but remain silent on whether market liquidity will change in the future. In this paper, we model empirically the full distribution of future market liquidity across corporate bond, Treasury and equity markets as a function of current liquidity and recent history of liquidity.

We estimate a quantile vector autoregression in the spirit of Koenker and Xiao (2006), allowing the future distribution of liquidity for each of the five securities portfolios we consider - investment grade and high yield corporate bonds, on-the-run Treasuries, stocks in the S\&P 500 index (large stocks) and small stocks - to depend on current and lagged bid-ask spreads for all portfolios. We document cross-market effects of current market liquidity conditions on the predicted distribution of future liquidity, summarized in Table 1. Three features of the estimated quantile autoregression are striking. First, there is a distributional flight-tosafety: a decrease in the current liquidity of high yield corporate bonds is associated with an increase in expected future liquidity of Treasuries and a decrease in uncertainty about the future liquidity of investment grade bonds. Similarly, a deterioration in the liquidity of small stocks is associated with a decrease in uncertainty about the future liquidity of Treasuries and a decrease in uncertainty about the future liquidity of large stocks. Second, current Treasury market liquidity affects the distribution of future liquidity for all five portfolios,

Table 1. Summary of cross-market liquidity effects. This table summarizes the effect of current bid-ask spreads on the distribution of one-week-ahead (negative) bid-ask spreads across markets. Diagonal entries correspond to the persistence estimated for the median of the distribution. "Right shift" denotes an increase in the median liquidity, without a corresponding change in the shape of the distribution; "left shift" denotes a decrease in the median liquidity, without a corresponding change in the shape of the distribution; "increase upside" denotes an increase in the upside risk to liquidity; "decrease downside" denotes a decrease in the downside risk to liquidity; "spread" denotes an increase in both downside and upside risks to liquidity; "compression" denotes a decrease in both downside and upside risks to liquidity. Empty cells correspond to effects not statistically significantly different from 0 .

|  | IG | HY | Treasury | S\&P 500 | Small stock |
| :---: | :---: | :---: | :---: | :---: | :---: |
| IG | 0.9 | Right shift | Right shift | - | - |
| HY | Compression | 0.7 | Left shift | - | - |
| Treasury | Increase upside | Spread | 0.9 | Decrease upside | Compression |
| S\&P 500 | - | Spread | Spread | 1 | Right shift |
| Small stock | - | Compression | Compression | Compression | 0.95 |

and the current liquidity of all five portfolios affects the distribution of future Treasury market liquidity. That is, Treasury market liquidity both affects and is affected by the liquidity of all five portfolios. Third, the distribution of future liquidity in the corporate bond market is largely unaffected by the current liquidity of the equity market, and the distribution of future stock market liquidity is largely unaffected by the current liquidity of the corporate bond market. Thus, after we account for the effect on Treasury market liquidity, there appear to be little spillovers in liquidity between corporate bond and equity markets.

We summarize the downside and upside risks to the median liquidity forecast using two metrics: (1) the upside and downside entropy of the unconditional distribution of bid-ask spreads relatively to the empirical conditional distribution; (2) the five percent expected shortfall and its upper tail counterpart, the five percent expected longrise. While downside relative entropy captures the conditional risks of liquidity deteriorating in excess of the downside risks predicted by the unconditional distribution, expected shortfall measures the average level of liquidity conditional on the bottom five percent tail outcomes realizing.

Thus, downside relative entropy and the expected shortfall measure two complementary but distinct features of upside risk to liquidity. Downside relative entropy captures the probability of a negative liquidity shock occurring, relative to the probability predicted by the unconditional distribution, while expected shortfall captures the expected extreme effects of a negative liquidity shock. Similarly, upside relative entropy captures the probability of a positive liquidity shock occurring, relative to the probability predicted by the unconditional distribution; the expected longrise captures the expected extreme effects of a positive liquidity shock. We find that, on average, upside relative entropy is more volatile than downside relative entropy, so that there is more uncertainty about whether liquidity will improve than about whether liquidity will deteriorate, relative to what would be predicted by the unconditional distribution. In contrast, the expected shortfall is more volatile than the expected longrise, so that how adverse are the bottom five percent of liquidity outcomes is more volatile than how beneficial are the top five percent of liquidity outcomes.

We argue that these relationships are a robust and stable feature of the data, and, thus, that our approach can be used to monitor potential risks to liquidity in real-time. We begin by showing that out-of-sample estimates of the conditional distributions of the future bid-ask spreads are very similar to the in-sample distributions. We further document our strong out-of-sample performance by analyzing predictive scores and probability integral transforms. We show that the conditional distribution is well-calibrated and performs better out-of-sample than the unconditional distribution for both investment grade and high yield bonds. This suggests that the recent history of bid-ask spreads for both credit ratings robustly reflects information relevant for the future evolution of liquidity.

Finally, we show that including proxies of demand-side and supply-side funding liquidity pressures in the market do not lead to consistent improvements in the in-sample accuracy of the predicted distribution and, in most cases, lead to decreases in the out-of-sample accuracy of the predictive distribution. That is, market-wide proxies for uncertainty, risk premia, overall financial conditions, and measures of dealer activity in the corporate bond
market do not contain information useful for predicting future bond market liquidity above what is contained in the recent history of bid-ask spreads for both credit ratings.

Our paper deviates from the prior literature in focusing on predicting the future evolution of liquidity, forecasting both the expected future liquidity and the downside risks to liquidity. In this aspect, our paper is related to prior literature that has investigated the time series properties of liquidity in other markets. Chordia et al. (2004) estimate a vector autoregression for stock and Treasury bond liquidity, and find cross-market dynamics from volatility to liquidity in both markets. More recently, Nagel (2012) argues that market liquidity declines during the financial crisis is partially explained by demands for higher expected returns by liquidity providers. Similarly, Comerton-Forde et al. (2010) show that when the revenues of NYSE specialists are low, liquidity on the NYSE is low as well. Relatedly, Baele et al. (2018) find increases in the VIX and the TED spread are associated with decreases in Treasury bond liquidity.

The relationship between VIX and liquidity is further investigated by Chung and Chuwonganant (2014), who show that VIX has a pervasive impact on liquidity. Karolyi et al. (2012) examine additional proxies for demand-side and supply-side pressures and find that liquidity in several countries varies across time because of demand-side reasons and not with proxies for funding liquidity. In contrast, Karnaukh et al. (2015) document that FX liquidity declines with both funding constraints and global risk, with stronger comovement of FX liquidity when funding is constrained, global volatility is high, and FX speculators incur losses. Similarly, Mancini et al. (2013) find commonality across liquidity measures for FX, U. S. stock, U. S. Treasury, and U. S. corporate bond markets. While our paper also finds common variation in liquidity across different markets, unlike this prior literature, we find that market-wide volatility and proxies for market-maker constraints do not help predict future liquidity once we control for the recent history of market liquidity.

The 2007-2009 financial crisis highlighted the need to better understand corporate bond market liquidity. Friewald et al. (2012) document that liquidity explains about one third of
the variation in the aggregate market corporate yield spread in the time-series, and about half during the crisis. Direct measures of trading activity, such as trade volume, and other commonly-used liquidity measures, do not show significant explanatory power. In the crosssection, they find that the overall liquidity of bonds issued by financial firms is higher on average, than those of industrial firms. Dick-Nielsen et al. (2012) document that liquidity deteriorated for both investment grade and high yield bonds, but it was slow and persistent for the first and short-lived for the latter. Moreover, they find consistent evidence with flight-to-quality only for AAA-rated bonds. Bao et al. (2011) calculate the Roll liquidity measure at the bond-level and then aggregate the liquidity measure across individual bonds. Using the aggregate measure they find that the aggregate illiquidity doubled relative to its pre-crisis average when the credit problem first broke out in August 2007, and subsequently tripled in March 2008 when Bear Stearns collapsed. Their measure peaks in October 2008, after Lehman's default and the bailout of AIG, and slowly declines thereafter. Adrian et al. (2017a) show that the relationship between bond-level liquidity and dealer-level constraints changes with the introduction of post-crisis regulation, with bonds traded by more levered institutions and institutions with investment bank like characteristics less liquid after the financial crisis. Our paper contributes to this literature by studying the evolution of the entire distribution of corporate bond liquidity as a function of current market conditions.

From a technical perspective, our paper contributes to the growing literature that has uncovered interesting patterns by analyzing the entire predictive density. We use the methodology from Adrian et al. (2019), who find that financial conditions are an important driver of macroeconomic vulnerabilities, measured as downside risk of GDP growth. Relatedly, Smith and Vahey (2016) show substantial asymmetries in that the forecast densities of GDP growth and inflation during the great recession. In financial markets, Ghysels (2014) documents that there are substantial and time-varying asymmetries of the predictive distribution of returns. Similarly, Schmidt and Zhu (2016) show that, while the tails of the predictive distribution of stock returns vary over time, the median of the distribution is essentially time invariant.

Using the quantile regression approach of Adrian et al. (2019), Crump et al. (2018) find that current realized volatility of stock returns has strong predictive content for the uncertainty of future returns and, thus, for the overall future distribution of market returns. Our paper is complementary to this prior literature as it studies the entire predictive density in a novel setting, market liquidity across five different portfolios.

The rest of the paper is organized as follows. Section 2 describes the construction of measures of liquidity studied in this paper. Section 3 lays out the empirical methodology, and Section 4 documents the basic features of the conditional distribution of illiquidity. We present the out-of-sample evidence and investigate the information contained in alternative explanatory variables in Section 5. Section 6 concludes.

## 2 Data Description and Sample Construction

### 2.1 Measures of market liquidity

Corporate bond market liquidity We use corporate bond transaction data from a supervisory version of TRACE, which contains the uncapped trade size, price, buyer and seller identities. FINRA members are identified by a designated Market Participant Identifier, MPID, and non-FINRA members are identified either as $C$ (for client), or as $A$ (for a nonmember affiliate). Our trades dataset spans from July 2002, when TRACE was introduced, to December 2017. Real-time, public dissemination of trades was staggered, and its full implementation was completed on February 7, 2005, when all U. S. corporate bonds, except the TRACE-eligible Rule 144A bonds, were subject to dissemination. Therefore, we limit our sample to start on January 2005. We address the data issues in TRACE and clean the data as described in Adrian et al. (2017a).

Using the traded prices in TRACE, we calculate the weekly effective bid-ask spread at the bond-level. The effective bid-ask is the difference between the dollar weighted average price of the buy trades and the dollar weighted average price of the sell trades (see Hong
and Warga 2000 and Chakravarty and Sarkar 2003):

$$
\mathrm{BAS}_{b, t}=\sum_{n=1}^{N} P_{n}^{B} W_{n}^{B}-\sum_{m=1}^{M} P_{m}^{S} W_{m}^{S}
$$

The measure is calculated using only client-dealer trades, and requires at least one client buy trade and one client sell trade each day.

We merge the weekly measure of bond-level liquidity with Mergent FISD to get the characteristics of the bonds. We exclude bonds denominated in foreign currency, which are agency backed, or issued as private placements, unit deals, perpetual, and preferred. We also drop bonds with a maturity of less than one year, and unrated bonds. We exclude trades of bonds 30 days prior to default, and, if the bond is reinstated, then we exclude the first 30 days after it was reinstated.

Using the credit rating information from Mergent FISD, we construct aggregate liquidity measures for the portfolio of AAA-rated bonds, investment-grade (excluding AAA) rated bonds, and high yield rated bonds as the gross-trading-volume-weighted average of bid-ask spreads for the corporate bonds with the corresponding trading volume. Figure 1a plots the time series of bid-ask spreads for these portfolios. Three features are worth noting about these time series. First, bid-ask spreads increase dramatically during periods of market stress, such as the 2007-2009 financial crisis. Second, during these stress periods, bonds with higher credit ratings have higher bid-ask spreads than bonds with lower credit ratings, suggesting that the market anticipates the eventual downgrade of these bonds. Finally, after August 2011, the bid-ask spread for the AAA category is extremely volatile. This is due to the fact that, after August 2011, very few corporate bonds actually have AAA credit rating, and the fraction of gross trading volume accounted for by trades in AAA-rated bonds drops dramatically. Because of this dramatic decrease in trading volume, we exclude AAA bonds from the results reported in the main body of the paper.

Additionally, the bid-ask spread series for high-yield bonds exhibits a year-end seasonality
when trading in the corporate bond market is thin. We correct for this seasonality by regressing the bid-ask spread of high-yield bonds on a year-end indicator, and work with seasonality-adjusted bid-ask spreads for the rest of our analysis.

Treasury market liquidity We use daily bid-ask spreads from Adrian et al. (2017b). Adrian et al. calculate bid-ask spread at the security level based on the average spread between the best bid and the best offer in the limit order book from GovPX and BrokerTec, averaged daily within the New York trading hours. Figure 1b plots the time series of the bid-ask spreads for 2-, 5-, and 10-year on-the-run Treasuries, as well as the average bid-ask spread for the market. The bid-ask spreads for all three maturities co-move together, spiking during periods of market stress, such as the financial crisis and the Taper Tantrum episode in June 2013. In our analysis, we focus on the bid-ask spread for the aggregate market.

Stock market liquidity We use daily bid and ask data from CRSP to calculate the daily effective bid-ask spread at the stock level. We then compute a volume-weighted average of the daily effective bid-ask spreads within the week to obtain a weekly time series of stocklevel effective spreads. Finally, we aggregate these weekly time series of stock-level effective bid-ask spreads into weekly times series of S\&P 500 stocks and small stocks by computing the market-capitalization weighted average of bid-ask spreads of stocks in the S\&P 500 and the stocks not in the S\&P 500 as of a given week, respectively. Figure 1c plots the time series of effective bid-ask spreads for these portfolios as well as for the equity market as whole.

### 2.2 Market-wide variables

In some empirical specifications, we control also for market-wide proxies of liquidity demand and supply. We proxy for liquidity demand using measures of option-implied equity volatility (VIX), Treasury volatility (MOVE 1M), and interest rate swap volatility (SMOVE 1M), as well as the Baa-Aaa spread (which proxies for credit risk premia), the Treasury slope (difference between yields on a 10 year and a 3 month Treasury, which proxies for term
premia), and the Chicago Fed National Financial Conditions Index (NFCI, which proxies for economy-wide financial conditions). On the supply-side, we use data from FR 2004 on Treasury and corporate securities transactions and repo market activity by primary dealers, as well as delivery fails into Treasury and corporate securities borrowing agreements. While the first two measures proxy for funding liquidity in the Treasury and corporate bond markets as they capture the willingness and ability of the traditional market makers to trade in and provide financing against Treasury and corporate securities, the third measure captures the scarcity of desirable bonds. For VIX, MOVE 1M, SMOVE 1M, Baa-Aaa spread and the Treasury slope, we aggregate the daily market prices into weekly measures by averaging within the week. The rest of the variables are available at a weekly frequency only.

## 3 Empirical Methodology

In this section, we describe how we apply the methodology in Adrian et al. (2019) to construct conditional distributions of market liquidity (rather than of real GDP growth). We refer the interested reader to Adrian et al. (2019) for more details on the quantile-regression methodology itself.

We characterize the relationship between future bid-ask spreads and current bid-ask spreads using quantile regressions. In particular, let $y_{i, t+h}$ be the log bid-ask spread for portfolio $i$ in future week $t+h$, and denote by $x_{t}$ the vector of conditioning variables, including a constant. In a quantile regression of $y_{i, t+h}$ on $x_{t}$, the regression slope $\beta_{i, \tau, h}$ is chosen to minimize the quantile-weighted absolute value of prediction errors

$$
\begin{equation*}
\hat{\beta}_{i, \tau, h}=\underset{\beta_{i, \tau, h} \in \mathbb{R}^{k}}{\operatorname{argmin}} \sum_{t=1}^{T-h}\left(\tau \cdot \mathbb{1}_{\left(y_{i, t+h} \geq x_{t} \beta_{i, \tau, h}\right)}+(1-\tau) \cdot \mathbb{1}_{\left(y_{i, t+h}<x_{t} \beta_{i, \tau, h}\right)}\right)\left|y_{i, t+h}-x_{t} \beta_{i, \tau, h}\right|, \tag{1}
\end{equation*}
$$

where $\mathbb{1}_{(\cdot)}$ denotes the indicator function. Unlike ordinary least squares, which predicts the average realization of $y_{i, t+h}$ conditional on $x_{t}$, the predicted value from the regression above
is the quantile of $y_{i, t+h}$ conditional on $x_{t}$

$$
\hat{Q}_{y_{i, t+h} \mid x_{t}}\left(\tau \mid x_{t}\right)=x_{t} \hat{\beta}_{i, t+h, \tau} .
$$

To reduce the influence of outliers in bid-ask spreads on the estimated coefficients, we estimate the quantile regression (1) for the natural logarithm of the bid-ask spread for a particular portfolio. We include four lags of bid-ask spreads in our regressions to capture the dependence on the whole pattern of liquidity over the previous month. That is, we parametrize the quantile function of the negative $\log$ bid-ask spread of portfolio $i$ in week $t$, $y_{i, t}$, as

$$
\begin{equation*}
Q_{y_{i, t+h} \mid x_{t}}\left(\tau \mid x_{t}\right)=\alpha_{i, h, \tau}+\sum_{k=1}^{5} \sum_{l=1}^{4} \varphi_{i, k, l, h, \tau} y_{k, t-l+1}+\epsilon_{i, h, t, \tau} \tag{2}
\end{equation*}
$$

We focus on the negative logarithm of the bid-ask spread to have a measure of liquidity: higher bid-ask spreads correspond to higher illiquidity of the bond, while higher negative (log) bid-ask spreads correspond to higher liquidity of the bond. Including the lagged bidask spreads of all portfolios into the specification (2) allows us to study the differential persistence of bid-ask spreads at various quantiles (through the coefficients $\left\{\varphi_{i, i, l, h, \tau}\right\}$ ), as well as the differential correlation of bid-ask spreads across portfolios at various quantiles (through the coefficients $\left\{\varphi_{i, k, l, h, \tau}\right\}$ ).

In the following, we report the cumulative effect of a change in either own or other portfolios $\log$ bid-ask spreads on the quantile function. That is, when we report regression coefficients, we are reporting

$$
\varphi_{i, k, h, \tau} \equiv \sum_{l=1}^{4} \varphi_{i, k, l, h, \tau}
$$

Figure 2 shows the scatter plot of one-week-ahead negative log bid-ask spreads for investment grade and high yield bonds, Treasuries, S\&P 500 and small stocks against the current
realization of negative log bid-ask spreads for the five portfolios, as well as the univariate quantile regression lines for the fifth, fiftieth and ninety-fifth quantiles and the OLS regression line. Consider first the relationship between future bid-ask spreads and own current bid-ask spreads. For investment grade bonds and Treasuries, the slopes of the three quantile regression lines are similar to each other and, moreover, similar to the linear regression slope, suggesting a linear relationship between current and future bid-ask spreads. Instead, for high yield bonds and the two equity portfolios, the slope of the ninety-fifth percentile is noticeably different from the slopes of the other two quantile regression lines and the OLS regression line, suggesting that bid-ask spreads for these markets have different persistence across different quantiles. Turning next to the cross-credit-rating relationship between future and current bid-ask spreads, we can see that there is a non-linear relationship between one-week-ahead bid-ask spreads on investment grade bonds and current bid-ask spreads on high yield bonds, but a potentially linear relationship between one-week-ahead bid-ask spreads on high yield bonds and current bid-ask spreads on investment grade bonds.

We test formally the marginal effects of including the history of bid-ask spreads for all portfolios in a multivariate regression setting in Figure $3 .{ }^{1}$ Consider first the estimated coefficients from the quantile regression of one-week-ahead negative log bid-ask spreads for investment grade bonds, plotted in the first column of Figure 3. Bid-ask spreads for investment-grade bonds are extremely persistent, with the estimated autoregressive coefficient of around 0.9. This persistence is mostly flat across quantiles but increases slightly for the right-most quantiles (most liquid) and decreases slightly for the left-most quantiles (least liquid). Turning next to the loading on current bid-ask spreads for the high yield bonds, we see that there is a positive relationship between future bid-ask spreads on investment

[^1]grade bonds and current high yield bid-ask spreads in the left tail of the bid-ask spread distribution. We also observe a negative relationship between the right tail of the future bid-ask spreads on investment grade bonds and current high yield bid-ask spreads. That is, when high yield bonds are relatively more liquid, both downside and upside risks to liquidity of investment grade bonds are lower and the distribution is more concentrated around the mean. In contrast, there is a positive relationship between the right tail of future bid-ask spreads on investment grade bonds and current Treasury bid-ask spreads, so that an improvement in liquidity of Treasuries corresponds to an increase in the upside risk to liquidity of investment grade bonds, and little relationship between the liquidity of investment grade bonds and both equity portfolios.

The second column of Figure 3 plots the estimated coefficients from the quantile regression of one-week-ahead negative log bid-ask spreads for high yield bonds. Liquidity of high yield bonds is much less persistent than the liquidity of investment grade bonds, with the estimated autoregressive coefficient at the median of around 0.7. In addition, persistence increases for the leftmost quantiles and decreases for the rightmost quantiles for high yield rated bonds, with both of these extremes different from the median estimate at the 5 percent confidence level. Turning to the loadings on current bid-ask spreads for other markets, we see that higher upside risks to the liquidity of high yield bonds is associated with higher liquidity of investment grade bonds, higher liquidity of stocks in the S\&P 500 index, and lower liquidity of small stocks. Higher liquidity of investment grade bonds also lowers the downside risk to liquidity of high yield bonds, while higher liquidity of Treasuries, higher liquidity of stocks in the S\&P 500 index, and lower liquidity of small stocks increases downside risk to liquidity of high yield bonds.

Turning next to the relationship between future Treasury market liquidity and current bid-ask spreads (middle column of Figure 3), we see that the liquidity of Treasury securities is most persistent in the middle of the distribution, with the estimated autoregressive coefficient at the median of around 0.9 , and the estimated autoregressive coefficients in either tail of
around 0.8 . Increases in liquidity of investment grade bonds and decreased in liquidity of high yield bonds are associated with a right shift in the distribution of liquidity of Treasuries, while increases in liquidity of stocks in the S\&P 500 index and decreases in liquidity of small stocks increase both upside and downside risks to Treasury market liquidity.

Finally, the results for the two stock portfolios mirror the results for the two corporate bond portfolios: increases in liquidity of the small stock portfolio lead to a compression of the one-week-ahead distribution of liquidity of stocks in the S\&P 500 index, while increases in the liquidity of stocks in the S\&P 500 index lead to a right shift of the overall one-week-ahead distribution of liquidity of small stocks. As with the corporate bond portfolios, the liquidity of the small stock portfolio is less persistent than the liquidity of the S\&P 500 portfolio, with persistence of the rightmost quantiles lower than persistence of the median. Current corporate bond market liquidity conditions do not affect significantly the distribution of future liquidity of either stock portfolio, while increases in current Treasury market liquidity are associated with decreased upside to the liquidity of stocks in the S\&P 500 index and a compression of the one-week-ahead distribution of small stock liquidity around the median.

Taken together, the results (summarized in Table 1) suggest that increases in current Treasury market liquidity have opposite effects on the distribution of future liquidity of corporate bond and equity markets; that increases in liquidity of large stocks lead to a spread of the distribution of future liquidity of high yield bonds, while increases in the liquidity of small stocks lead to a compression of the distribution of future liquidity of high yield bonds around the median; and that current corporate bond market liquidity conditions do not contain information about the future distribution of stock market liquidity beyond the information contained in the current liquidity conditions in the Treasury and stock markets. Figure A. 2 in the Appendix shows that these patterns also hold for the estimated coefficients for the four-weeks-ahead distribution.

Turning to the implications of these relationships for the dynamic evolution of risks to liquidity, Figure 4 shows realized liquidity together with the conditional median and the
conditional 5th, 25 th, 75 th, and 95 th percentile quantiles of the one-week-ahead and four-weeks-ahead predicted distribution across markets. ${ }^{2}$ This figure demonstrates one of the key results of the paper: while the distribution around the median for Treasuries is largely symmetric, there is significant asymmetry between the upper and lower conditional quantiles of liquidity for other markets. That is, for corporate bonds and stocks, the lower quantiles vary significantly over time but the upper quantiles are stable. Figure 5 shows that this leads to a strong negative correlation between the median and the interquartile range, and between the interquartile range and the bottom fifth percentile of the distribution of liquidity for these markets: decreases in median liquidity are associated with an increase in uncertainty around the median and a corresponding shift to the left of the left tail of the liquidity distribution. In contrast, in the Treasury market, there is little relationship between the median of the conditional distribution of liquidity and the uncertainty around the median, leading to more overall volatility in both tails of the distribution.

## 4 Conditional Distribution of Liquidity

The quantile regression (1) provides us with estimates of the quantile function, a representation of the inverse cumulative distribution function (ICDF). In this section, we describe how to translate the estimated quantile function into the associated conditional probability distribution function, describe the evolution of the conditional distribution of liquidity around the Taper Tantrum, and study two summary measures of liquidity flightiness.

### 4.1 Recovering the conditional distribution

Prior literature has struggled with inverting the empirical ICDF produced from quantile regressions to obtain a conditional probability distribution function. Instead, we follow Adrian et al. (2019) and smooth the quantile distribution function using the skewed- $t$ distribution

[^2]developed by Azzalini and Capitanio (2003): ${ }^{3}$
\[

$$
\begin{equation*}
f(y ; \mu, \sigma, \alpha, \nu)=\frac{2}{\sigma} t\left(\frac{y-\mu}{\sigma} ; \nu\right) T\left(\alpha \frac{y-\mu}{\sigma} \sqrt{\frac{\nu+1}{\nu+\left(\frac{y-\mu}{\sigma}\right)^{2}}} ; \nu+1\right) \tag{3}
\end{equation*}
$$

\]

where $t(\cdot)$ and $T(\cdot)$ respectively denote the PDF and CDF of the Student $t$-distribution. The four parameters of the distribution pin down the location $\mu$, scale $\sigma$, fatness $\nu$, and shape $\alpha$. Relative to the $t$-distribution, the skewed $t$-distribution adds the shape parameter which regulates the skewing effect of the CDF on the PDF. The skewed $t$-distribution is part of a general class of mixed distributions proposed by Azzalini (1985) and further developed by Azzalini and Dalla Valle (1996). The intuition for the derivation is that a base probability distribution - in this case $t\left(\frac{y-\mu}{\sigma} ; \nu\right)$ - gets shaped by its cumulative distribution function, and rescaled by a shape parameter $\alpha$. The notable special case is the traditional $t$-distribution when $\alpha=0$. In the case of both $\alpha=0$ and $\nu=\infty$, the distribution reduces to a Gaussian with mean $\mu$ and standard deviation $\sigma$. When $\nu=\infty$ and $\alpha \neq 0$, the distribution is a skewed normal.

Besides its flexibility, an advantage of using the skewed- $t$ distribution is that it has closed-form expressions for both the PDF and the ICDF. This allows us to fit the skewed- $t$ distribution $f$ in week $t$ by minimizing the distance between the estimated quantile function $\hat{Q}_{y_{i, t+h} \mid x_{t}}(\tau)$ and the $\operatorname{ICDF} F^{-1}\left(\tau ; \mu_{i, t, h}, \sigma_{i, t, h}, \alpha_{i, t, h}, \nu_{i, t, h}\right)$ of the skewed- $t$ distribution. More specifically, for each week and each credit rating, we choose the four parameters $\left\{\mu_{i, t, h}, \sigma_{i, t, h}, \alpha_{i, t, h}, \nu_{i, t, h}\right\}$ to match the fifth, twenty-fifth, fiftieth, seventy-fifth and ninetyfifth percent conditional quantiles

$$
\left\{\hat{\mu}_{i, t, h}, \hat{\sigma}_{i, t, h}, \hat{\alpha}_{i, t, h}, \hat{\nu}_{i, t, h}\right\}=\underset{\mu, \sigma, \alpha, \nu}{\operatorname{argmin}} \sum_{\tau}\left(\hat{Q}_{y_{i, t+h} \mid x_{t}}(\tau)-F^{-1}\left(\tau ; \mu_{i, t, h}, \sigma_{i, t, h}, \alpha_{i, t, h}, \nu_{i, t, h}\right)\right)^{2}
$$

[^3]where $\tau \in\{0.05,0.25,0.5,0.75,0.95\}, \hat{\mu}_{i, t, h} \in \mathbb{R}, \hat{\sigma}_{i, t, h} \in \mathbb{R}^{+}, \hat{\alpha}_{i, t, h} \in \mathbb{R}$, and $\hat{\nu}_{i, t, h} \in \mathbb{N}^{4}{ }^{4}$ This can be viewed as an exactly identified nonlinear cross-sectional regression of the predicted quantiles on the quantiles of the skewed- $t$ distribution. ${ }^{5}$

Figure 6 plots the time series evolution of the conditional distribution function for each market over time. Two features of the estimated conditional distributions are striking. First, the entire predictive distribution for each market evolves over time: during periods of market stress, high bid-ask spreads are more likely not just because the average liquidity is lower but also because there is more uncertainty and more downside risk to market liquidity. Second, markets with higher average liquidity during normal times can be more vulnerable to liquidity evaporating during periods of stress: although on average investment grade corporate bonds are more liquid than high yield corporate bonds, average future liquidity is lower and uncertainty about future liquidity is higher for investment grade than high yield bonds during periods of market stress, such as the recent financial crisis. That is, during the financial crisis, the downside risk to liquidity of investment grade bonds was greater than the downside risk to liquidity of high yield bonds, potentially reflecting market expectation of future downgrades of investment grade bonds.

The estimated skewed- $t$ distribution allows us to formally test the in-sample differences in the conditional and unconditional distributions, with the average $\log$ scores for both distributions by market and horizon reported in Table 2. For all markets and both forecast horizons, the conditional model significantly outperforms the unconditional model. Thus, lagged bid-ask spreads contain information that is crucial for predicting the distribution of future liquidity outcomes.

[^4]
### 4.2 Taper Tantrum

We now study how the conditional distribution of liquidity evolves in response to market stress in more detail. We focus on the evolution of the one-week-ahead density around the "Taper Tantrum" on June 19, 2013. The Taper Tantrum episode was characterized by a selloff of longer maturity Treasuries on fears - in response to Chair Bernanke's Congressional testimony - of faster-than-anticipated tapering of asset purchases by the Federal Reserve, and a resulting decline in liquidity across fixed-income markets.

Figure 7 plots the one-week-ahead distributions of liquidity estimated in (pseudo-)real time ${ }^{6}$ for four dates: June 14, 2013 (week before the Taper Tantrum), June 21, 2013 (week of Taper Tantrum), June 28, 2013 (week after Taper Tantrum), and July 18, 2013 (four weeks after Taper Tantrum). Consider first the evolutions of the one-week-ahead distribution of Treasury market liquidity, Figure 7c. Relative to the distribution on June 14, 2013, the realized Treasury market bid-ask spread the week of June 19, 2013 was in the extreme left tail of the conditional distribution. The conditional distribution adjusted to this change by shifting to the left: the mean predicted bid-ask spread increased, reflecting the overall selling pressure in the market, but so did the right tail of the liquidity distribution. Reflecting the high persistence of shocks to Treasury market liquidity, the conditional distribution of Treasury market liquidity slowly adjusted in the subsequent week to a more "normal" shape but, even four weeks after the event, the conditional mean of liquidity was lower than the week before the event.

The remaining four panels in Figure 7 illustrate how these dynamics played out in the corporate bond (Figures 7 a and 7 b ) and equity markets (Figures 7 d and 7 e ). In the corporate bond market, the Taper Tantrum coincided with both a downward revision in the mean of and an increase in uncertainty around the mean of the conditional distribution of liquidity,

[^5]with the investment grade corporate bond market experiencing greater deteriorations in liquidity outlook than the high yield corporate bond market. Intuitively, investment grade bonds represent the closest substitute to Treasuries for fixed income investors, and thus react more strongly to liquidity deteriorations in the Treasury market. For both credit ratings, the deterioration in the liquidity outlook was short-lived, with the one-week-ahead distribution of bid-ask spreads in the high yield market on July 18, 2013 more favorable than the distribution one week before the Taper Tantrum. For stocks in the S\&P 500 index, the Taper Tantrum coincided with a short-term increase in uncertainty around the mean of the conditional distribution of liquidity without a corresponding decrease in the mean of the distribution. Interestingly, there appears to be no effect on the distribution of liquidity of smaller stocks.

### 4.3 Summarizing liquidity flightiness

The Taper Tantrum episode illustrates that market stress can affect the shape of the distribution of future liquidity in different ways. We summarize the risks encoded in the conditional distribution of liquidity using two complementary measures proposed by Adrian et al. (2019): upside and downside relative entropy, and expected longrise and shortfall. Downside entropy answers the question: "Relative to the unconditional distribution, how much more likely is liquidity to deteriorate given current market liquidity?", while expected shortfall answers the question: "If liquidity does deteriorate, how adverse can the realized bid-asks spreads be?"

Upside and downside relative entropy. We start with upside and downside relative entropy, which measures the "extra" probability assigned by the conditional model to outcomes above and below the median of the distribution, respectively, relative to the probability assigned to the same outcomes by the unconditional distribution. Put simply, upside relative entropy measures to what extent "good" outcomes are more likely to happen under the conditional distribution than under the unconditional distribution. Simi-
larly, downside relative entropy measures to what extent "bad" outcomes are more likely to happen under the conditional distribution than under the unconditional distribution. Formally, we denote by $\hat{g}_{y_{i, t+h}}$ the unconditional density computed by matching the unconditional empirical distribution of the log bid-ask spread on bonds with credit rating $i$ and by $\hat{f}_{y_{i, t+h} \mid x_{t}}\left(y \mid x_{t}\right)=f\left(y_{i} ; \hat{\mu}_{i, t+h}, \hat{\sigma}_{i, t+h}, \hat{\alpha}_{i, t+h}, \hat{\nu}_{i, t+h}\right)$ the estimated skewed $t$-distribution. Then the upside, $\mathcal{L}_{i, t}^{U}$, and downside, $\mathcal{L}_{i, t}^{D}$, entropy of $\hat{g}_{y_{i, t+h}}\left(y_{i}\right)$ relative to $\hat{f}_{y_{i, t+h} \mid x_{t}}\left(y_{i} \mid x_{t}\right)$ are defined as
$\mathcal{L}_{i, t}^{D}\left(\hat{f}_{y_{i, t+h} \mid x_{t}} ; \hat{g}_{y_{i, t+h}}\right)=-\int_{-\infty}^{\hat{F}_{y_{i, t+h} \mid x_{t}}^{-1}\left(0.5 \mid x_{t}\right)}\left(\log \hat{g}_{y_{i, t+h}}\left(y_{i}\right)-\log \hat{f}_{y_{i, t+h} \mid x_{t}}\left(y_{i} \mid x_{t}\right)\right) \hat{f}_{y_{i, t+h} \mid x_{t}}\left(y_{i} \mid x_{t}\right) d y_{i}$,
$\mathcal{L}_{i, t}^{U}\left(\hat{f}_{y_{i, t+h} \mid x_{t}} ; \hat{g}_{y_{i, t+h}}\right)=-\int_{\hat{F}_{y_{i, t+h} \mid x_{t}}^{-1}\left(0.5 \mid x_{t}\right)}^{+\infty}\left(\log \hat{g}_{y_{i, t+h}}\left(y_{i}\right)-\log \hat{f}_{y_{i, t+h} \mid x_{t}}\left(y_{i} \mid x_{t}\right)\right) \hat{f}_{y_{i, t+h} \mid x_{t}}\left(y_{i} \mid x_{t}\right) d y_{i}$,
where $\hat{F}_{y_{i, t+h} \mid x_{t}}\left(y_{i} \mid x_{t}\right)$ is the cumulative distribution associated with $\hat{f}_{y_{i, t+h} \mid x_{t}}\left(y_{i} \mid x_{t}\right)$ and $\hat{F}_{y_{i, t+h} \mid x_{t}}^{-1}\left(0.5 \mid x_{t}\right)$ is the conditional median.

Figures 8a and 8b illustrate the downside entropy calculation for the one-week-ahead distribution of liquidity of investment grade bonds on two dates at different points in the liquidity cycle: September 19, 2008, the week after the liquidation of Lehman Brothers; and January 13, 2006, which represents normal liquidity conditions. On September 19, 2008, the conditional distribution is much more pessimistic than the unconditional distribution so that the conditional distribution at the median is above the unconditional distribution, and downside relative entropy is the area between the conditional and unconditional distribution, shaded in grey. Thus, in periods of time when the conditional distribution is more pessimistic than the unconditional distribution, the downside relative entropy is positive. On January 13, 2006, instead, the conditional distribution is somewhat more optimistic than the unconditional distribution, and the median of the conditional distribution is above
the median of the unconditional distribution. The downside relative entropy is then the area between the conditional and unconditional distribution below the conditional median and above the unconditional median (in grey) less the area between the conditional and unconditional distribution below the unconditional median (shaded in red). Thus, in periods when the conditional distribution is more optimistic than the unconditional distribution, the downside relative entropy is negative, and the upside relative entropy is positive.

Figures 9a - 9e plot the time series of one-week-ahead upside and downside relative entropy for investment grade and high yield bonds, Treasuries, and large and small stocks, respectively. ${ }^{7}$ On average, upside relative entropy is more volatile than downside relative entropy, indicating that, on average, there is more uncertainty about whether liquidity will improve than whether uncertainty will deteriorate, relative to what would be predicted by the unconditional distribution. For both investment grade and high yield bonds, upside and downside entropy co-move positively during periods of market stress, so that there is greater overall uncertainty about corporate bond liquidity during market downturns.

Expected longrise and shortfall. In addition to upside and downside entropy, we also study the expected shortfall encoded in the conditional distribution of liquidity together with its right tail counterpart, the expected longrise. The expected longrise measures how high the average liquidity in the conditional top fifth percentile is, while the expected shortfall captures how low the average liquidity in the bottom fifth percentile of the conditional distribution is. Formally, for a chosen target probability $\pi$, the expected shortfall and longrise are defined, respectively, as

$$
S F_{i, t+h}(\pi)=\frac{1}{\pi} \int_{0}^{\pi} \hat{F}_{y_{i, t+h} \mid x_{t}}^{-1}\left(\tau \mid x_{t}\right) d \tau ; \quad L R_{i, t+h}(\pi)=\frac{1}{\pi} \int_{1-\pi}^{1} \hat{F}_{y_{i, t+h} \mid x_{t}}^{-1}\left(\tau \mid x_{t}\right) d \tau .
$$

We illustrate the expected five percent shortfall calculation for the one-week-ahead distribution of investment grade bonds in Figures 8c and 8d. For both dates, the expected

[^6]shortfall is measured as the grey area between the zero line and the conditional ICDF for quantiles between 0 and $0.05 .{ }^{8}$ On September 19, 2008 (8c), that area is substantially larger than the area on January 13, 2006 (8d), reflecting the greater illiquidity of the corporate bond market during the financial crisis. Unlike the relative upside and downside entropy calculation described above, expected longrise and shortfall do not compare the conditional and unconditional distributions. Thus, the fact that the unconditional ICDF is uniformly above the conditional ICDF during periods of market stress is not reflected in the expected shortfall.

Figures $9 \mathrm{f}-9 \mathrm{~g}$ plot the time series of one-week-ahead expected longrise and shortfall for investment grade and high yield bonds, Treasuries, and large and small stocks, respectively. ${ }^{9}$ Across all portfolios, expected longrise and shortfall co-move positively during the entire sample, with the expected shortfall more volatile than the expected longrise. Thus, although the probability of liquidity increases is more volatile than the probability of liquidity decreases, how adverse are the bottom five percent of liquidity outcomes is more volatile than how beneficial are the top five percent of liquidity outcomes.

## 5 Robustness

The previous section demonstrates that the predictive model which includes lagged bid-ask spreads for all markets outperforms the unconditional model in-sample. We now turn to evaluating the performance of the conditional and unconditional models out-of-sample, and study whether including additional predictors improves the statistical performance.

[^7]
### 5.1 Out-of-sample performance

We backtest the model by replicating the analysis that an economist would have done using the proposed methodology in real time. We produce predictive distributions recursively for two horizons ( 1 week and 4 weeks), starting with the estimation sample that ranges from January 1, 2003 to August 1, 2007. The first out-of-sample estimates are thus for the average liquidity in the week ending on August 8, 2007 (one-week-ahead) and the average liquidity in the week ending on September 1, 2007 (4 weeks ahead). We then iterate the same procedure, expanding the estimation sample one week at a time, until the end of the sample (December 31, 2017). At each iteration, we repeat the estimation steps above, estimating quantile regressions and matching the skewed $t$-distribution. The outcome of this procedure is a ten year time series of out-of-sample density forecasts for each of the two forecast horizons and each of the five portfolios.

We perform two types of out-of-sample analyses. First, we study the robustness of our predicted distributions by comparing the in-sample predicted distributions with their real time counterparts. Second, we evaluate the out-of-sample accuracy and calibration of the density forecasts by analyzing the predictive score and the probability integral transform (PIT); that is, the predictive density and cumulative distribution evaluated at the outturn, respectively.

We begin by comparing the in-sample and out-of-sample predicted distribution, presented in Figure 10. The figure illustrates that the in-sample and out-of-sample estimates of the quantiles are virtually indistinguishable for both horizons (Figure A.8) and all five portfolios. The only case in which the in-sample and out-of-sample quantiles deviate noticeably is for the bottom fifth percentiles of liquidity during the financial crisis, with the out-of-sample more negative than the in-sample estimate. The full sample estimate incorporates the reversion of bond liquidity to more normal levels in the post-crisis period, while the real time procedure estimates a somewhat lower worst case outcomes. The similarities are more striking as
the financial crisis of 2007-2009 is a significant tail event that is not in the data when estimating the out-of-sample distributions. The similarity between in-sample and out-ofsample estimates suggests that our methodology can be used to detect liquidity risks in real time.

Next, we assess the reliability of the predictive distribution using the predictive score, computed as the predictive distribution generated by a model (either the conditional or the unconditional model) and evaluated at the realized value of the time series. Higher predictive scores indicate more accurate predictions on average as higher predictive scores indicate that outcomes that the model considers more likely are closer to the ex-post realization. Figures 11a - 11e plot the time series of the scores of the conditional and unconditional one-week-ahead predictive distribution for investment grade and high yield bonds, Treasuries, stocks in the S\&P 500 index and small stocks, respectively. For all of these markets, the predictive score for the conditional distribution is almost always above the predictive score for the unconditional model, indicating that the conditional model is almost always more accurate than the unconditional model. We test the predictive scores differences formally in Table 3. The conditional distribution outperforms the unconditional distribution across both horizons (one-week-ahead and four-weeks-ahead) and all markets.

We conclude the out-of-sample evaluation by analyzing the calibration of the predictive distributions. We compute the empirical cumulative distribution of the PITs, which measures the percentage of observations that are below any given quantile. A model is said to be better calibrated the closer the empirical cumulative distribution of the PITs is to the 45 degree line. In a perfectly calibrated model, the cumulative distribution of the PITs is exactly the 45 degree line, so that the fraction of realizations falling below any given quantile $Q_{y_{i, t+h} \mid x_{t}}(\tau)$ of the predictive distribution is exactly equal to $\tau$. We plot the PITs for the conditional and unconditional one-week-ahead distribution for investment grade and high yield bonds, Treasuries, stocks in the S\&P 500 index and small stocks, together with the corresponding
confidence bounds, ${ }^{10}$ in Figures $11 \mathrm{f}-11 \mathrm{j}$. Strikingly, across all five markets, the empirical distribution of the PITs for the conditional model is well within the confidence bands across all quantiles, while the empirical distribution of the PITs for the unconditional model falls outside the confidence bands - and, for the two equity portfolios, dramatically so - for the bottom half of the distribution.

Overall, the results in Figure 11, Figure A. 9 in the Appendix, and Table 3 suggest that the quantile regression approach generates robust predictive distributions, across multiple predictive horizons and across multiple markets, and is able to capture the downside vulnerability of liquidity particularly well. We turn next to quantifying the amount of upside and downside risks present in the conditional predictive distributions of liquidity.

### 5.2 Other predictors

Prior literature (see e.g. Nagel, 2012; Chung and Chuwonganant, 2014) has shown that measures of market-wide volatility are correlated with measures of market liquidity. We now examine whether such proxies for demand-side pressures as well as proxies for funding liquidity supply contain additional information about the future distribution of market liquidity. In particular, we augment the quantile regression specification (2) to include observations of market-wide variables $z_{t}$ as predictors

$$
\begin{equation*}
Q_{y_{i, t+h} \mid x_{t}}\left(\tau \mid x_{t}\right)=\alpha_{i, h, \tau}+\sum_{k=1}^{5} \sum_{l=1}^{4} \varphi_{i, k, l, h, \tau} y_{k, t-l+1}+\sum_{n=1}^{N} \eta_{i, n, h} z_{n, t-1+1}+\epsilon_{i, h, t, \tau}, \tag{6}
\end{equation*}
$$

where $z_{n, t}$ is the observation of the $n^{\text {th }}$ external predictor in week $t$. The market-wide variables that we consider here are defined in Section 2.2. We compare the in-sample and out-of-sample performance of the distribution conditional on lagged bid-ask spreads and market-wide variables to the performance of the distribution conditional on lagged bid-ask

[^8]spreads only by conducting log-likelihood ratio tests for both credit ratings and both horizons.

In-sample performance Consider first the results of the in-sample log-likelihood ratio comparisons, reported in Table 4. Each column (except for the first column, which reports the log-likelihood ratio between the unconditional and the conditional models) in Table 4 corresponds to the log-likelihood ratio between the augmented conditional distribution (6) and the baseline conditional distribution (2) for different market-wide variables, with positive numbers indicating better performance of the augmented model than the baseline model insample. Table 4 shows the striking result that, even in-sample, market-wide predictors do not consistently contain information about the future distribution of liquidity over and above the information contained in lagged bid-ask spreads. For investment grade bonds, the augmented model outperforms the baseline conditional model at the four week horizon only if the VIX or the NFCI or all the dealer condition variables or all the market-conditions variables (or both) are included. For high yield bonds, the four-week-ahead performance is improved only if all the market-conditions variables are included; for Treasuries, the one-week-ahead performance is improved only if the first principal component of the dealer conditions variables is included and the four-week-ahead performance is improved only if either dealers' corporate bond transactions or all the dealer- and market-conditions variables are included; for both stock portfolios, the one-week-ahead performance is improved only if all the dealer- and marketconditions variables are included. All other specifications do not generate an improvement in in-sample predictability over and above the predictability generated by lagged bid-ask spreads. Thus, although proxies for demand-side pressures may help predict future average liquidity, augmenting the conditional model (2) with these variables does not consistently improve the ability of the model to predict the full distribution of future liquidity.

Out-of-sample performance Turn now to the out-of-sample performance of the augmented models reported in Table 5. Strikingly, for almost all specifications, including the additional predictor either does not improve the out-of-sample performance of the condi-
tional model or harms the out-of-sample performance. The only two exceptions are the one-week-ahead distribution of Treasury market liquidity, for which the inclusion of (log) Treasuries lent in repo improves the out-of-sample performance, and the one-week-ahead distribution of the liquidity of stocks in the S\&P 500 index, for which the inclusion of (log) fails in Treasuries borrowed agreements improves the out-of-sample performance.

Overall, the results of this Section suggest that market-wide measures do not consistently improve the predictive performance of the conditional model in-sample and, for most specifications, either do not improve or even detriment the out-of-sample performance. Thus, market-wide measures of demand-side and supply-side pressures do not seem to contain information about future market liquidity beyond the information contained in the history of bid-ask spreads.

## 6 Conclusion

This paper studies the predictability of liquidity and downside risk to market liquidity of five key U. S. markets: investment grade and high yield bonds, Treasuries, and large and small stocks. We find evidence of liquidity spillovers across markets: lower current liquidity of high yield bonds is associated with greater expected future Treasury market liquidity and lower uncertainty about future liquidity of investment grade bonds, while greater current Treasury market liquidity is associated with greater upside risk to the liquidity of investment grade bonds and lower downside risk of large stocks. We argue that augmenting the baseline model with proxies for demand-side and supply-side pressures in the market does not improve the predictive performance of the model. This suggests that, although dealer balance sheet constraints affect liquidity at the individual security level, the current state of dealer balance sheets does not contain additional information about the future evolution of market-level liquidity beyond that contained in the current level of bid-ask spreads.

The global financial crisis highlighted the importance of understanding risks to liquidity
for both individual institutions and the financial system as a whole. As a result, a number of jurisdictions have introduced liquidity stress tests, arguing that liquidity stress tests generate valuable information on institutions' liquidity profile beyond that captured by standardized liquidity metrics, such as the Liquidity Coverage Ratio and the Net Stable Funding Ratio. ${ }^{11}$ In this paper, we show that the predictive model that conditions on recent history of market liquidity performs well out-of-sample, both on average and around stress events in the market, suggesting that the model could be used to produce plausible, date- and horizon-dependent liquidity stress scenarios.

[^9]
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Table 2. In-sample log-likelihoods. This table reports the in-sample log-likelihoods for the unconditional and conditional models for log bid-ask spreads. The conditional model includes four lags of log bid-ask spreads for all markets. Numbers reported below the diagonal correspond to the log-likelihood ratio between the unconditional and conditional model. HAC standard errors reported in parentheses below the estimates. Bolded estimates indicate point estimates at least two standard deviations away from 0 .

(e) Small stock; 1 wk ahead |  | Uncond | BAS |
| :---: | :---: | :---: |
| Uncond | $\mathbf{- 0 . 5 3}$ |  |


Small stock; 4 wks ahead

|  | Uncond | BAS |
| :--- | :---: | :---: |
| Uncond | $\mathbf{- 0 . 5 2}$ |  |
| (s.e.) | $\mathbf{( 0 . 1 3 )}$ |  |
| BAS | 1.31 | $\mathbf{0 . 7 9}$ |
| (s.e.) | $(0.08)$ | $\mathbf{( 0 . 0 7 )}$ |


| (d) S\&P 500; 1 wk ahead |  |  |
| :--- | :---: | :---: |
|  | Uncond | BAS |
| Uncond | $\mathbf{- 0 . 7 8}$ |  |
| (s.e.) | $\mathbf{( 0 . 2 1 )}$ |  |
| BAS | 1.77 | $\mathbf{0 . 9 9}$ |

 |  | Uncond | BAS |
| :---: | :---: | :---: |
| Uncond | $\mathbf{- 0 . 7 7}$ |  |

| BAS | 1.46 | $\mathbf{0 . 6 9}$ |
| :---: | :---: | :---: |
| (s.e.) | $(0.08)$ | $\mathbf{( 0 . 1 1 )}$ |

реәче ум I ! K..nser.лL (د) |  | Uncond | BAS |
| :---: | :---: | :---: |
| Uncond | 1.27 |  |

| (s.e.) | $(0.27)$ |  |
| :--- | :---: | :---: |
| BAS | 1.06 | $\mathbf{2 . 3 2}$ |


(h) Treasury; 4 wks ahead

|  | Uncond | BAS |
| :--- | :---: | :---: |
| Uncond | $\mathbf{1 . 2 8}$ |  |
| (s.e.) | $(\mathbf{0 . 2 8})$ |  |
| BAS | 0.57 | $\mathbf{1 . 8 5}$ |
| (s.e.) | $(0.17)$ | $\mathbf{( 0 . 1 6 )}$ |



 |  | Uncond | BAS |
| :--- | :---: | :---: |
| Uncond | $\mathbf{- 0 . 3 5}$ |  |
| (s.e.) | $\mathbf{( 0 . 1 0 )}$ |  |
| BAS | 0.80 | $\mathbf{0 . 4 5}$ |
| (s.e.) | $(0.09)$ | $\mathbf{( 0 . 0 9 )}$ |

(f) IG; 4 wks ahead

|  | Uncond | BAS |
| :---: | :---: | :---: |
| Uncond | $\mathbf{- 0 . 3 5}$ |  |


|  | Uncond | $\mathbf{- 0 . 3 5}$ |
| :--- | :---: | :---: |
| (s.e.) | $\mathbf{( 0 . 1 0 )}$ |  |
| BAS | 0.70 | $\mathbf{0 . 3 5}$ |
| (s.e.) | $(0.08)$ | $\mathbf{( 0 . 0 7 )}$ |

Table 3. Out-of-sample log-likelihoods. This table reports the out-of-sample log-likelihoods for the unconditional and conditional models for $\log$ bid-ask spreads. The conditional model includes four lags of log bid-ask spreads. Numbers reported below the diagonal correspond to the log-likelihood ratio between the unconditional and conditional model. HAC standard errors reported in parentheses below the estimates. Bolded estimates indicate point estimates at least two standard deviations away from 0 .

(e) Small stock; 1 wk ahead |  | Uncond | BAS |
| :--- | :---: | :---: |
| Uncond | -0.69 |  |

| (s.e.) | $(0.17)$ |  |
| :---: | :---: | :---: |
| BAS | 1.41 | 0.72 |



| (j) Small stock; 4 wks ahead |  |  |
| :--- | :--- | :--- |
|  | Uncond | BAS |
| Uncond | $-\mathbf{- 0 . 7 2}$ |  | |  | Uncond | BAS |
| :--- | :---: | :---: |
| Uncond | -1.15 |  |
| (s.e.) | $(0.13)$ |  |
| BAS | 1.99 | 0.84 |

 |  | Uncond | BAS |
| :--- | :---: | :---: |
| Uncond | $\mathbf{1 . 1 9}$ |  |
| (s.e.). | $(\mathbf{0 . 1 4 )}$ |  |
| BAS | 1.74 | 0.56 |
| (s.e.) | $(0.18)$ | $\mathbf{( 0 . 1 4 )}$ |

| (c) Treasury; 1 wk ahead |  |  |  |  |  |
| :--- | :---: | :---: | :---: | :---: | :---: |
|  |  |  |  | Uncond | BAS |
| Uncond | 1.53 |  |  |  |  |
| (s.e.) | $(0.15)$ |  |  |  |  |
| BAS | 0.74 | 2.27 |  |  |  |
| (s.e.) | $(0.10)$ | $(0.11)$ |  |  |  |

(h) Treasury; 4 wks ahead


(b) HY; 1 wk ahead |  | Uncond | BAS |
| :--- | :---: | :---: |
| Uncond | -0.08 |  | (s.e.) (0.13)

| (s.e.) | $(0.57$ | 0.50 |
| :---: | :---: | :---: |
| BAS | 0.57 |  |
| (s.e.) | $(0.09)$ | $(0.05)$ |



 | (a) IG; 1 wk ahead |  |  |
| :--- | :---: | :---: |
|  | Uncond | BAS |
| Uncond | -0.53 |  |
| (s.e.) | $(0.22)$ |  |
| BAS | 1.00 | 0.46 |
| (s.e.) | $(0.08)$ | $(0.07)$ |

| (f) IG; 4 wks ahead |  |  |
| :--- | :---: | :---: |
|  | Uncond | BAS |
| Uncond | -0.55 |  |
| (s.e.) | $\mathbf{( 0 . 2 4 )}$ |  |
| BAS | 0.81 | 0.26 |
| (s.e.) | $(0.12)$ | $(\mathbf{0 . 1 0 )}$ |

Table 4. In-sample comparisons to alternative models. This table reports the in-sample log-likelihood ratios between the conditional model and models that also include additional predictors. All models include four lags of log bid-ask spreads. "Slope" is the difference between the yield on a 10 year Treasury and the yield on a 3 month Treasury. In the "All Predictors" category, the " 1 st PCs" column includes the first principle component of the market conditions variables and the first principle component of the dealers conditions variables. HAC standard errors reported in parentheses below the estimates. Bolded estimates indicate point estimates at least two standard deviations away from 0 .

$H Y$


| 8 |
| :---: |
| 0 |
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| 0 |
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| $\vdots$ |



| 0 |
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| SESP 500 |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| $h=1$ | $-1.770$ <br> (0.059) | $\begin{aligned} & -0.000 \\ & (0.009) \end{aligned}$ | $-0.008$ | $0.002$ | $0.032$ | $0.003$ | $0.026$ | $0.004$ | $0.028$ | $-0.002$ <br> (0.030) | $\begin{aligned} & -0.024 \\ & (0.018) \end{aligned}$ | $\begin{gathered} 0.001 \\ (0.012) \end{gathered}$ | $0.026$ | $0.021$ | $0.028$ | $0.010$ | $0.013$ | $0.086$ | $0.033$ <br> (0.030) |
| $h=4$ | -1.457 | 0.025 | 0.003 | -0.017 | 0.020 | -0.009 | 0.002 | 0.028 | 0.001 | 0.008 | -0.024 | -0.053 | -0.011 | 0.011 | 0.028 | 0.058 | -0.028 | 0.066 | -0.067 |
|  | (0.078) | (0.021) | (0.012) | (0.027) | (0.014) | (0.010) | (0.027) | (0.029) | (0.009) | (0.010) | (0.023) | (0.066) | (0.019) | (0.015) | (0.028) | (0.031) | (0.048) | (0.036) | (0.077) |


| 2 |
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 $\stackrel{\emptyset}{0}$
Table 5. Out-of-sample-sample comparisons to alternative models. This table reports the out-of-sample log-likelihood ratios between the conditional model and models that also include additional predictors. All models include four lags of log bid-ask spreads. "Slope" is the difference between the yield on a 10 year Treasury and the yield on a 3 month Treasury. In the "All Predictors" category, the "1st PCs" column includes the first principle component of the market conditions variables and the first principle component of the dealers conditions variables. HAC standard errors reported in parentheses below the estimates. Bolded estimates indicate point estimates at least two standard deviations away from 0 .

|  | Unconditional | Market conditions predictors |  |  |  |  |  |  |  | Dealer conditions predictors |  |  |  |  |  |  |  | All predictors |  |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
|  |  | VIX | MOVE 1M | SMOVE 1M | BAA-AAA | Slope | NFCI | All | 1st PC | TSY Fails | $\begin{gathered} \text { TSY } \\ \text { Transactions } \end{gathered}$ | TSY Repo | Corp Fails | $\begin{gathered} \text { Corp } \\ \text { Transactions } \end{gathered}$ | Corp Repo | All | 1st PC | All | 1st PCs |
| IG |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |
| $\begin{aligned} & h=1 \\ & h=4 \end{aligned}$ | $\begin{gathered} -0.996 \\ (0.080) \\ -0.806 \\ (0.115) \end{gathered}$ | $\begin{gathered} -0.051 \\ (0.064) \\ 0.088 \\ (0.047) \end{gathered}$ | $\begin{gathered} 0.007 \\ (0.035) \\ -0.077 \\ (0.051) \end{gathered}$ | $\begin{gathered} -0.075 \\ (0.081) \\ -0.032 \\ (0.030) \end{gathered}$ | $\begin{gathered} 0.024 \\ (0.044) \\ -0.133 \\ (0.153) \end{gathered}$ | $\begin{gathered} 0.001 \\ (0.035) \\ -0.053 \\ (0.055) \\ \hline \end{gathered}$ | $\begin{aligned} & -0.089 \\ & (0.070) \\ & -0.202 \\ & (0.143) \end{aligned}$ | $\begin{aligned} & -0.187 \\ & (0.106) \\ & -0.292 \\ & (0.157) \end{aligned}$ | $\begin{aligned} & -0.145 \\ & (0.145) \\ & -0.008 \\ & (0.077) \end{aligned}$ | $\begin{gathered} -0.055 \\ (0.055) \\ 0.051 \\ (0.037) \\ \hline \end{gathered}$ | $\begin{gathered} 0.023 \\ (0.024) \\ -0.039 \\ (0.085) \end{gathered}$ | $\begin{gathered} -0.124 \\ (0.072) \\ -0.062 \\ (0.065) \end{gathered}$ | $\begin{aligned} & -0.085 \\ & (0.057) \\ & -0.200 \\ & (0.163) \end{aligned}$ | $\begin{aligned} & -0.041 \\ & (0.034) \\ & -0.125 \\ & (0.121) \end{aligned}$ | $\begin{aligned} & -0.111 \\ & (0.090) \\ & -0.147 \\ & (0.055) \end{aligned}$ | $\begin{gathered} -0.136 \\ (0.082) \\ -0.393 \\ (0.261) \end{gathered}$ | $\begin{gathered} -0.104 \\ (0.068) \\ -0.082 \\ (0.084) \end{gathered}$ | $\begin{gathered} -0.120 \\ (0.086) \\ -0.592 \\ (0.308) \end{gathered}$ | $\begin{gathered} -0.127 \\ (0.093) \\ 0.048 \\ (0.065) \end{gathered}$ |
| HY |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |
| $\begin{aligned} & h=1 \\ & h=4 \end{aligned}$ | $\begin{aligned} & -0.571 \\ & (0.090) \\ & -0.344 \\ & (0.086) \end{aligned}$ | $\begin{gathered} -0.132 \\ (0.069) \\ -0.000 \\ (0.039) \end{gathered}$ | $\begin{gathered} -0.053 \\ (0.034) \\ 0.005 \\ (0.042) \end{gathered}$ | $\begin{gathered} -0.064 \\ (0.037) \\ -0.034 \\ (0.038) \end{gathered}$ | $\begin{gathered} -0.080 \\ (0.052) \\ -0.064 \\ (0.060) \end{gathered}$ | $\begin{gathered} -0.096 \\ (0.049) \\ 0.032 \\ (0.032) \end{gathered}$ | $\begin{gathered} -0.076 \\ (0.033) \\ 0.002 \\ (0.076) \end{gathered}$ | $\begin{aligned} & -0.195 \\ & (0.108) \\ & -0.184 \\ & (0.089) \end{aligned}$ | $\begin{aligned} & -0.300 \\ & (0.165) \\ & -0.003 \\ & (0.054) \end{aligned}$ | $\begin{gathered} 0.006 \\ (0.020) \\ 0.024 \\ (0.038) \end{gathered}$ | $\begin{gathered} -0.115 \\ (0.089) \\ -0.006 \\ (0.028) \end{gathered}$ | $\begin{gathered} -0.144 \\ (0.076) \\ 0.010 \\ (0.048) \end{gathered}$ | $\begin{gathered} -0.190 \\ (0.095) \\ -0.110 \\ (0.056) \end{gathered}$ | $\begin{gathered} -0.089 \\ (0.053) \\ -0.077 \\ (0.066) \end{gathered}$ | $\begin{gathered} -0.083 \\ (0.041) \\ 0.039 \\ (0.037) \end{gathered}$ | $\begin{aligned} & -0.222 \\ & (0.072) \\ & -0.311 \\ & (0.114) \end{aligned}$ | $\begin{gathered} -0.067 \\ (0.036) \\ 0.015 \\ (0.030) \end{gathered}$ | $\begin{aligned} & -0.522 \\ & (0.178) \\ & -0.332 \\ & (0.106) \end{aligned}$ | $\begin{gathered} -0.088 \\ (0.036) \\ -0.073 \\ (0.058) \end{gathered}$ |
| Treasury |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |
| $\begin{aligned} & h=1 \\ & h=4 \end{aligned}$ | $\begin{gathered} -0.738 \\ (0.096) \\ -0.147 \\ (0.179) \end{gathered}$ | $\begin{aligned} & -0.116 \\ & (0.107) \\ & -0.098 \\ & (0.058) \end{aligned}$ | $\begin{gathered} -0.129 \\ (0.084) \\ -0.093 \\ (0.115) \end{gathered}$ | $\begin{gathered} -\mathbf{0 . 1 2 0} \\ (0.059) \\ -0.013 \\ (0.068) \end{gathered}$ | $\begin{gathered} -\mathbf{0 . 1 6 2} \\ (\mathbf{0 . 0 8 0 )} \\ -0.123 \\ (0.101) \end{gathered}$ | $\begin{gathered} 0.008 \\ (0.088) \\ -0.040 \\ (0.129) \end{gathered}$ | $\begin{gathered} 0.073 \\ (0.064) \\ -0.101 \\ (0.104) \end{gathered}$ | $\begin{gathered} -0.160 \\ (0.118) \\ -0.180 \\ (0.157) \\ \hline \end{gathered}$ | $\begin{aligned} & -0.256 \\ & (0.131) \\ & -0.112 \\ & (0.111) \end{aligned}$ | $\begin{gathered} -0.032 \\ (0.086) \\ 0.102 \\ (0.120) \end{gathered}$ | $\begin{gathered} 0.012 \\ (0.062) \\ -0.033 \\ (0.035) \end{gathered}$ | $\begin{gathered} \mathbf{0 . 1 1 6} \\ (\mathbf{0 . 0 5 6}) \\ -0.145 \\ (0.113) \end{gathered}$ | $\begin{gathered} -0.188 \\ (0.143) \\ -0.319 \\ (0.186) \end{gathered}$ | $\begin{gathered} 0.068 \\ (0.060) \\ -0.019 \\ (0.060) \end{gathered}$ | $\begin{gathered} -0.077 \\ (0.152) \\ 0.124 \\ (0.113) \end{gathered}$ | $\begin{gathered} 0.057 \\ (0.085) \\ -0.092 \\ (0.138) \end{gathered}$ | $\begin{gathered} 0.050 \\ (0.062) \\ -0.250 \\ (0.192) \end{gathered}$ | $\begin{aligned} & -0.112 \\ & (0.143) \\ & -0.035 \\ & (0.132) \end{aligned}$ | $\begin{gathered} -0.011 \\ (0.076) \\ -0.192 \\ (0.104) \end{gathered}$ |
| S8P 500 |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |
| $\begin{aligned} & h=1 \\ & h=4 \end{aligned}$ | $\begin{gathered} -1.990 \\ (0.163) \\ -1.745 \\ (0.176) \end{gathered}$ | $\begin{aligned} & -0.157 \\ & (0.144) \\ & -0.027 \\ & (0.069) \end{aligned}$ | $\begin{gathered} -0.165 \\ (0.133) \\ -0.044 \\ (0.045) \end{gathered}$ | $\begin{gathered} -0.229 \\ (0.226) \\ -0.164 \\ (0.133) \end{gathered}$ | $\begin{gathered} 0.001 \\ (0.089) \\ -0.053 \\ (0.079) \end{gathered}$ | $\begin{gathered} -0.212 \\ (0.154) \\ -0.134 \\ (0.092) \end{gathered}$ | $\begin{aligned} & -0.032 \\ & (0.071) \\ & -0.094 \\ & (0.094) \end{aligned}$ | $\begin{gathered} -0.033 \\ (0.133) \\ -0.362 \\ (0.112) \end{gathered}$ | $\begin{aligned} & -0.095 \\ & (0.112) \\ & -0.133 \\ & (0.067) \end{aligned}$ | $\begin{gathered} 0.105 \\ (0.049) \\ -0.055 \\ (0.082) \end{gathered}$ | $\begin{gathered} -0.000 \\ (0.044) \\ -0.089 \\ (0.047) \end{gathered}$ | $\begin{gathered} -0.072 \\ (0.049) \\ -0.217 \\ (0.123) \end{gathered}$ | $\begin{gathered} -0.055 \\ (0.165) \\ -0.110 \\ (0.086) \end{gathered}$ | $\begin{gathered} -0.209 \\ (0.151) \\ -0.149 \\ (0.084) \end{gathered}$ | $\begin{gathered} -0.106 \\ (0.080) \\ -0.224 \\ (0.134) \end{gathered}$ | $\begin{aligned} & -0.015 \\ & (0.088) \\ & -0.435 \\ & (0.207) \end{aligned}$ | $\begin{gathered} -0.313 \\ (0.148) \\ -0.154 \\ (0.078) \end{gathered}$ | $\begin{gathered} -0.157 \\ (0.198) \\ -0.797 \\ (0.231) \end{gathered}$ | $\begin{gathered} 0.001 \\ (0.111) \\ -0.699 \\ (0.413) \end{gathered}$ |
| Small stocks |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |
| $h=1$ $h=4$ | $\begin{gathered} -1.408 \\ (0.146) \\ -1.013 \\ (0.261) \end{gathered}$ | $\begin{gathered} -0.064 \\ (0.051) \\ 0.017 \\ (0.057) \end{gathered}$ | $\begin{gathered} -0.054 \\ (0.059) \\ -0.088 \\ (0.100) \end{gathered}$ | $\begin{gathered} -0.061 \\ (0.044) \\ -0.028 \\ (0.021) \end{gathered}$ | $\begin{gathered} -0.122 \\ (0.079) \\ -0.000 \\ (0.065) \\ \hline \end{gathered}$ | $\begin{aligned} & -0.096 \\ & (0.066) \\ & -0.298 \\ & (0.234) \end{aligned}$ | $\begin{aligned} & -0.163 \\ & (0.124) \\ & -0.123 \\ & (0.124) \\ & \hline \end{aligned}$ | $\begin{aligned} & -0.324 \\ & (\mathbf{0 . 1 3 2 )} \\ & -0.133 \\ & (0.102) \end{aligned}$ | $\begin{gathered} 0.014 \\ (0.094) \\ -0.176 \\ (0.175) \end{gathered}$ | $\begin{gathered} -0.097 \\ (0.077) \\ 0.019 \\ (0.045) \end{gathered}$ | $\begin{aligned} & -0.071 \\ & (0.062) \\ & -0.069 \\ & (0.046) \end{aligned}$ | $\begin{gathered} -0.145 \\ (0.083) \\ 0.077 \\ (0.055) \\ \hline \end{gathered}$ | $\begin{gathered} -0.120 \\ (0.067) \\ -0.332 \\ (0.225) \\ \hline \end{gathered}$ | $\begin{gathered} -0.089 \\ (0.061) \\ 0.068 \\ (0.082) \end{gathered}$ | $\begin{gathered} -\mathbf{0 . 1 5 1} \\ (0.056) \\ -0.152 \\ (0.102) \end{gathered}$ | $\begin{gathered} -\mathbf{0 . 1 9 4} \\ (\mathbf{0 . 0 9 2 )} \\ -0.729 \\ (0.369) \end{gathered}$ | $\begin{gathered} -\mathbf{0 . 1 4 6} \\ (0.055) \\ -0.061 \\ (0.118) \end{gathered}$ | $\begin{gathered} -0.204 \\ (0.093) \\ -0.357 \\ (0.134) \end{gathered}$ | $\begin{aligned} & -0.078 \\ & (0.097) \\ & -0.139 \\ & (0.168) \\ & \hline \end{aligned}$ |

Figure 1. Illiquidity Over Time. This figure plots the time series of volume-weighted average corporate bid-ask spread by credit rating category (1a), volume-weighted average Treasury bid-ask spread by maturity (1b), and the volume-weighted average equity bid-ask spread by size (1c).
(a) Corporate bid-ask spreads

(b) Treasury bid-ask spreads

-2 year -5 year -10 year - All
(c) Equity bid-ask spreads

Figure 2. Quantile Regressions. This figure shows the univariate quantile regressions of one-week-ahead bid-ask spreads for investment grade and high yield bonds, Treasuries, S\&P 500 stocks and small stocks on current bid-ask spread for investment grade and high yield bonds, Treasuries, S\&P 500 stocks and small stocks. Each row corresponds to an explanatory variable, each column to a market, so that e.g. the plot in row 2 , column 1 shows the relationship between one-week-ahead bid-ask spread on investment grade bonds and current bid-ask spread on high yield bonds. Treasury $\left\lvert\, \begin{aligned} & \text { S\&P } 500\end{aligned}\right.$

| IG |  |  |  |  |  |
| :---: | :---: | :---: | :---: | :---: | :---: |
| HY |  |  |  |  |  |
| Treasury |  |  |  |  |  |
| S\&P 500 |  |  |  |  |  |
| Small stock |  |  |  |  |  |

Figure 3. Estimated Quantile Regression Coefficients. This figure shows the estimated coefficients in quantile regressions of one-week-ahead bid-ask spreads for investment grade and high yield bonds, Treasuries, S\&P 500 stocks and small stocks on four lags of bid-ask spread for investment grade and high yield bonds, Treasuries, S\&P 500 stocks and small stocks. Each row corresponds to an explanatory variable, each column to a market, so that e.g. the plot in row 2 , column 1 shows the relationship between one-week-ahead bid-ask spread on investment grade bonds and four lags of bid-ask spread on high yield bonds. Regression coefficients reported as the sum of the coefficients on the four lag of the respective variable. We report confidence bounds for the null hypothesis that the true data-generating process is a general, flexible linear model for bid-ask spreads (VAR with 4 lags); bounds are computed using 1000 bootstrapped samples.

> | S\&P 500 | Small stock |
| :--- | :--- |


Treasury
 HY

1





Figure 4. Predicted Distributions. This figure shows the time series evolution of the predicted distribution one-week-ahead of volume-weighted average (negative) bid-ask spread by asset market. Shaded areas correspond to the $(5 \%, 95 \%),(10 \%, 90 \%)$ and $(25 \%, 75 \%)$ interquantile ranges, respectively.
(a) IG
(b) HY

(c) Treasury

(d) S\&P 500

(e) Small stock

Figure 5. Median, Interquartile Range and Tail Outcomes. This figure shows relationship between the interquartile range and the median top row) and the interquartile range and the 5 th percentile (bottom row) of the one-week-ahead conditional distribution of volume-weighted average (negative) bid-ask spread by asset market.









Figure 6. Distribution of liquidity over time. This figure plots the time series of one-week-ahead predictive distribution of volume-weighted average (negative) bid-ask spread by asset market, based on quantile regressions with four lags of bid-ask spreads for all asset markets as conditioning variables.


Figure 7. Conditional Distribution around the Taper Tantrum. This figure illustrates the evolution of the one-week-ahead distribution of liquidity around the Taper Tantrum episode on June 19, 2013. Each distribution is estimated using only information up to the date reported.
(a) IG
(b) HY


Figure 8. Information Captured by Downside Entropy and Expected Shortfall. This figure illustrates the information captured by downside relative entropy and the five percent expected shortfall for the one-week-ahead out-of-sample predicted distribution of investment grade (excluding AAA) bid-ask spread on September 19, 2008 ("stressed market"; left column) and January 13, 2006 ("calm" market; right column). Downside entropy is the area below the conditional median between the conditional and the unconditional distribution. Expected shortfall is the expected bid-ask spread in the worst (bottom) five percent outcomes.
(a) Relative entropy: 9/19/2008

(c) Expected shortfall: 9/19/2008

(b) Relative entropy: 1/13/2006

(d) Expected shortfall: $1 / 13 / 2006$



Figure 10. Out-of-Sample Predictions. This figure compares the in-sample and out-of-sample predicted distribution of one-week-ahead volume-weighted average (negative) bid-ask spread by asset market. The quantiles plotted are the 5 th, 50 th and 95 th percentile.
(a) IG
(b) HY

(c) Treasury

(d) $\mathrm{S} \& \mathrm{P} 500$

(e) Small stock

Figure 11. Out-of-Sample Accuracy. This figure reports the predictive scores (upper row) and the cumulative distribution of the probability integral transform (PITs; bottom row) for the one-week-ahead predictive distribution of volume-weighted average bid-ask spread by market. Predictive distribution conditions on lagged bid-ask spreads for all markets. Scores and PITs for the unconditional distribution included for comparison. Critical





 values obtained as in Rossi and


A Four-Week-Ahead Results
Figure A.1. Quantile Regressions. This figure shows the univariate quantile regressions of four-weeks-ahead bid-ask spreads for investment grade and high yield bonds, Treasuries, S\&P 500 stocks and small stocks on current bid-ask spread for investment grade and high yield bonds, Treasuries, S\&P 500 stocks and small stocks. Each row corresponds to an explanatory variable, each column to a market, so that e.g. the plot in row 2 , column 1 shows the relationship between four-weeks-ahead bid-ask spread on investment grade bonds and current bid-ask spread on high yield bonds.

| Curr |  |  | Treasury |  |  |
| :---: | :---: | :---: | :---: | :---: | :---: |
| IG |  |  |  |  |  |
| HY |  |  |  |  |  |
| Treasury |  |  |  |  |  |
| S\&P 500 |  |  |  |  |  |

 $\underbrace{\begin{array}{r}\text { ahead } \\ \text { 4wks }\end{array}}_{\text {Current }}$
нY
Figure A.2. Estimated Quantile Regression Coefficients. This figure shows the estimated coefficients in quantile regressions of four-weeksahead bid-ask spreads for investment grade and high yield bonds, Treasuries, S\&P 500 stocks and small stocks on four lags of bid-ask spread for investment grade and high yield bonds, Treasuries, S\&P 500 stocks and small stocks. Each row corresponds to an explanatory variable, each column to a market, so that e.g. the plot in row 2 , column 1 shows the relationship between four-weeks-ahead bid-ask spread on investment grade bonds and four lags of bid-ask spread on high yield bonds. Regression coefficients reported as the sum of the coefficients on the four lag of the respective variable. We report confidence bounds for the null hypothesis that the true data-generating process is a general, flexible linear model for bid-ask spreads (VAR
Small stock
 trapped samples
HY

$\left.\underbrace{}_{\text {Current }} \begin{array}{r}\text { a wks } \\ \text { akd }\end{array} \right\rvert\,$


S\&P 500


Figure A.3. Predicted Distributions. This figure shows the time series evolution of the predicted distribution four-week-ahead of volume-weighted average bid-ask spread by asset market. Shaded areas correspond to the $(5 \%, 95 \%),(10 \%, 90 \%)$ and $(25 \%, 75 \%)$ interquantile ranges, respectively.
(a) IG
(b) HY

(c) Treasury

(d) S\&P 500

(e) Small stock

Figure A.4. Median, Interquartile Range and Tail Outcomes. This figure shows relationship between the interquartile range and the median (upper row) and the interquartile range and the 5th percentile (bottom row) of the four-week-ahead conditional distribution of volume-weighted average (negative) bid-ask spread by asset market.











Figure A.5. Distribution of liquidity over time. This figure plots the time series of four-weeksahead predictive distribution of volume-weighted average (negative) bid-ask spread by asset market, based on quantile regressions with four lags of bid-ask spreads for all asset markets as conditioning variables.


Figure A.6. Conditional Distribution around the Taper Tantrum. This figure illustrates the evolution of the four-week-ahead distribution of liquidity around the Taper Tantrum episode on June 19, 2013. Each distribution is estimated using only information up to the date reported.



Figure A.8. Out-of-Sample Predictions. This figure compares the in-sample and out-of-sample predicted distribution of four-week-ahead volume-weighted average bid-ask spread by asset market. The quantiles plotted are the 5 th, 50 th and 95 th percentile.
(a) IG
(b) HY

(c) Treasury

(d) S\&P 500

(e) Small stock
Figure A.9. Out-of-Sample Accuracy. This figure reports the predictive scores (upper row) and the cumulative distribution of the probability integral transform (PITs; bottom row) for the four-week-ahead predictive distribution of volume-weighted average bid-ask spread by market. Predictive distribution conditions on lagged bid-ask spreads for all asset markets. Scores and PITs for the unconditional distribution included for comparison Critical values obtained as in Rossi and Sekhposyan (2017).

Critical values obtained as in Rossi and Sekhposyan (2017)



[^0]:    *The views expressed here are the authors' and are not representative of the views of the Federal Reserve Bank of New York, the Federal Reserve System, or the Financial Industry Regulatory Authority. Patrick Adams, Daniel Roberts and Francisco Ruela provided excellent research assistance. The authors also thank Michael Fleming for sharing Treasury bid-ask spread data and Richard Berner for providing thoughtful comments on previous drafts of the paper. The authors thank FINRA for providing corporate bond data. Emails: nina.boyarchenko@ny.frb.org; domenico.giannone@ny.frb.org; or.shachar@ny.frb.org.

[^1]:    ${ }^{1}$ The confidence bounds plotted in Figure 3 are the 95 percent confidence bounds for the null hypothesis that the true data-generating process is a flexible and general linear model for liquidity. In particular, we estimate a vector autoregression (VAR) with four lags, Gaussian innovations, and a constant using the full-sample evolution of $\log$ bid-ask spreads, and bootstrap 1000 samples to compute bounds at different confidence levels for the OLS relationship. Quantile coefficient estimates that fall outside of this confidence bound thus indicate that the relationship between log bid-ask spreads and the predictive variable is nonlinear.

[^2]:    ${ }^{2}$ We transform the conditional distribution for log bid-ask spread from the quantile regression (2) to the conditional distribution for the negative bid-ask spread using the change of variables formula for distributions.

[^3]:    ${ }^{3}$ An alternative approach to smoothing the quantile densities is to interpolate the quantile function using splines. Imposing monotonicity and smoothness requires additional modeling choices, as in for example Schmidt and Zhu (2016).

[^4]:    ${ }^{4}$ Notice that these parameters are functions of the conditioning variables in week $t$.
    ${ }^{5}$ We fit the skewed- $t$ distribution to the quantile function of $\log$ bid-ask spreads, and then use change of variables formula for distributions to convert that to the distribution for bid-ask spreads.

[^5]:    ${ }^{6}$ That is, for each date, we re-estimate the parameters of the model using only information available up to that date. Thus, week-over-week changes in the distribution arise both because the history of bid-ask spreads included as conditioning information changes and because the parameters of the model are re-estimated with additional observations.

[^6]:    ${ }^{7}$ Figures A.7a - A.7e plot the time series of four-weeks-ahead upside and downside relative entropy.

[^7]:    ${ }^{8} \mathrm{~A}$ further measure of downside risk is "liquidity-at-risk", or, in our notation, $\hat{F}_{y_{i, t+h} \mid x_{t}}^{-1}\left(\tau \mid x_{t}\right)$ for a given level of $\tau$, which corresponds to, e.g., the fifth percentile worst outcome of liquidity. Expected shortfall, instead, averages across all percentiles below (and including) the target quantile and thus provides a more comprehensive metric of the severity of worst-case outcomes.
    ${ }^{9}$ Figures A.7f - A.7g plot the time series of four-weeks-ahead expected longrise and shortfall.

[^8]:    ${ }^{10}$ We follow Rossi and Sekhposyan (2017) in computing the bounds. The confidence bounds should be taken as general guidance since they are derived for forecasts computed using a rolling, rather than expanding, sample. For the one-week-ahead and the four-weeks-ahead, the bands are based on critical values derived under the null of uniformity and independence of the PIT.

[^9]:    ${ }^{11}$ See the overviews in e.g. BCBS (2013) and Jobst et al. (2017).

