Foreign Reserves Management and Original Sin*

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This version: 10/17/2023

Abstract

Foreign reserves management changes the risk profile of a currency, therefore influencing the pricing of sovereign debt, and the sovereign debt currency portfolio. Empirically, inflation-targeters in emerging countries with higher foreign reserves feature an "original sin" dissipation: high local currency share in the sovereign debt portfolio. We propose a quantitative model of optimal reserves management and sovereign currency portfolios. The optimal reserves policy leans against the global wind so the exchange rate depreciates less in global bad times, resulting in a lower premium charged by global investors and more local currency sovereign debt. We confirm these features empirically and via data-simulated regressions.

JEL Codes F30, F40

Keywords: Foreign Exchange Intervention, Real Exchange Rate, Local Currency Sovereign Debt

*We appreciate comments from Paul Bergin, Javier Bianchi, David Cook, Jens Christensen, Wenxin Du, Charles Engel, Tarek Hassan, Sylvain Leduc, Zheng Liu, Hidehiko Matsumoto, Thomas Mertens, JungJae Park, Diego Perez, Carolin Pflueger, Jesse Schreger, Jing Zhang, Tony Zhang and seminar participants at U Wisconsin, HKUST, UC Davis, UC Irvine, UC Santa Barbara, WEAI 2021, the NBER IFM Spring meeting 2022, the Reserve Bank of Chicago, the Reserve Bank of San Francisco, the Board of Governors of the Federal Reserve, VEAMS, the SED 2022, the CICF 2022 and 10th BFI IMF conference. Devereux gratefully acknowledges financial support from the Social Sciences and Humanities Council of Canada.

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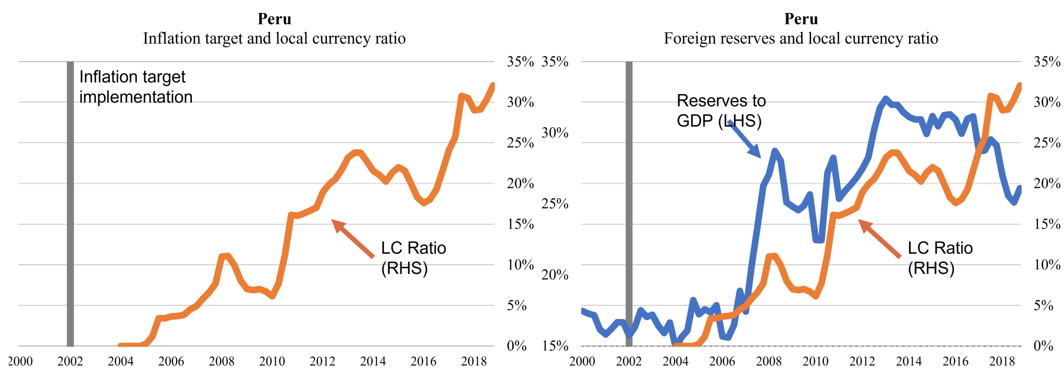
1 Introduction

It is widely acknowledged that government borrowing in local currency is advantageous due to the state contingent nature of domestically denominated debt. However, for long periods many emerging country (EM) sovereigns could only borrow externally in foreign currency (FC), a phenomenon dubbed as "original sin" (Eichengreen and Hausmann (1999), Eichengreen et al. (2005)). The constraints of "original sin" have been associated with many painful economic crises, such as the Latin American crisis in the 1980s and the Asian Financial crisis in 1997-1998. Surprisingly however, this pattern has changed more recently since the early 2000s. The local currency (LC) share of the external EM sovereign debt has been increased from a median of 5% in 2005 to 38% in 2018 in our data sample.

Figure 1: Peru Local Currency Share and Reserves Management



Understanding how some EMs managed to escape from "original sin" is a key question for shaping emerging market economic policy. Recent studies have pointed to the importance of a credible monetary policy, such as the adoption of an inflation targeting regime, as contributing to the LC rise. The logic is that if a country commits to forego inflating away the LC denominated debt, it helps to attract external investors, since their investment is more protected from inflation-induced depreciation. The left panel of Figure 1 plots the LC share of Peru's sovereign debt. The Central Bank of Peru has implemented inflation targeting based monetary policy since 2002 and the LC share of sovereign borrowing started to pick up in 2004 and eventually reached about 30% at the end of 2019.

We take a complementary tack in this paper. We argue that the enormous foreign exchange reserves accumulation and foreign exchange management during the same period also contributed to the rise in LC share. Some evidence for this can be seen in the right panel of Figure 1, which plots the empirical association between reserves accumulation and LC share for Peru. There is a strong positive relationship between the two, especially since the large ramp up of reserves in 2006.

Why would the presence of foreign reserves be important in fostering local currency sovereign borrowing? The reason is that reserves can optimally be used to change the exchange rate cyclicality of global shocks. Currencies that depreciate in international investors' bad times are risky from investors point of view. For example, an emerging country that enters a "sudden stop" crisis with a sharp currency depreciation every time a global downturn occurs would result in a high risk premium being charged on LC debt. But if a country uses reserves to smooth out the exchange rate response to global fluctuations, referred to as "leaning against the global wind", this can reduce the size of currency depreciation associated with a global downturns. It therefore should lower the risk premium embedded in the LC interest rate. From the sovereign's standpoint, this makes the LC debt more attractive and tilts the currency portfolio to LC.

To be more concrete, suppose a risk-averse international investor is indifferent between lending to the same EM sovereign in FC and LC, then no-arbitrage would imply the following log return condition:

$$i_t = i_t^* + E_t(S_{t+1} - S_t) + cov_t(m_{t+1}^*, S_{t+1} - S_t)$$

where i_t is the LC interest rate for an EM sovereign, i_t^* is the FC interest rate for the same EM sovereign,[1] S_t is the (log) nominal exchange rate, $E_t(S_{t+1} - S_t)$ is the expected nominal exchange rate change, m_{t+1}^* is the international investor's stochastic discount factor. The LC interest rate (i_t) is a markup over the FC interest rate (i_t^*), which consists of the expected exchange rate change ($E_t(S_{t+1} - S_t)$) and the exchange rate risk premium ($cov_t(m_{t+1}^*, S_{t+1} - S_t)$). In the extreme case where the EM sovereign faces no cost of ex-post inflation, $E_t(S_{t+1} - S_t) \to \infty$ as the sovereign is expected to inflate everything away. This rationalizes the case of "original sin" as $i_t \to \infty$. Engel and Park (2022) and Ottonello and Perez (2019) provide accounts of how EM countries reduced this expectation term by establishing inflation commitment. While this mechanism is present in our model, our analysis focuses more on the $cov_t(m_{t+1}^*, S_{t+1} - S_t)$ term, which is measured to be empirically large in various studies. A "lean against the global wind" reserves management policy that dampens the exchange rate depreciation in states when m_{t+1}^* is high can reduce

[1] These are interest rates for the same government. This is not the same as the uncovered interest party.

the size of $cov_t(m^*_{t+1}, S_{t+1} - S_t)$ and therefore the risk premium being charged in the LC rate.

We begin by documenting some novel empirical facts and then complement the empirical section with a quantitative model of optimal sovereign currency portfolio and reserves management. Empirically, we study the interrelationship of the sovereign currency portfolio, inflation targeting and reserves management. First, exclusively among inflation targeters, an increase in foreign reserves to GDP is associated with an increase in the LC share. Second, inflation targeting EMs that hold substantial reserves tend to have exchange rates which are less sensitive to global factors. Third, inflation targeters decumulate (accumulate) reserves upon a globally driven exchange rate depreciation (appreciation). Finally, an increase in reserves to GDP is associated with lower local currency sovereign spreads and moreover, sovereign spreads are lower not due to lower credit risk but rather due to a lower exchange rate risk component. Interestingly, these findings do not hold for non-inflation targeters. In sum, these empirical patterns are consistent with our hypothesis: inflation targeting emerging economies who actively manage their reserves tend to "lean against the global wind", experience a lower exchange rate risk premium in sovereign spreads and display a higher LC share.

To account for the empirical findings, we build a small open economy model with tradable and non-tradable goods. The model features an endogenous currency composition of sovereign debt, endogenous foreign reserves management, and risk averse international investors. The small open economy consists of a public sector and a private sector. The public sector includes a Central Bank that conducts foreign exchange management and monetary policy, and a sovereign that finances public goods by issuing both foreign currency and local currency debt. To motivate reserves management, we introduce representative households who are subject to an occasionally binding collateral constraint and a pecuniary externality (Bianchi (2011)) and a banking sector that is constrained by moral hazard (Gabaix and Maggiori (2015)). The pecuniary externality provides a rationale for foreign reserves management. The constrained banking sector allows for effective foreign reserves management.[2]

The economy is subjected to a domestic tradable endowment shock and a global risk premium shock. Ceteris paribus, the Sovereign prefers LC debt over FC debt because it is a good hedge for domestic endowment shocks. Upon a bad endowment shock, the

[2]This approach to foreign reserves management is similar to that in Davis et al. (2023). It ensures that the Central Bank has two separate policy instruments in FX managements and monetary policy.

exchange rate endogenously depreciates because of a lower tradable consumption and also due to debt inflation directly pursued by the sovereign. However, the LC interest rate is higher because international investors charge a premium on LC debt, due to exchange rate risk. In response to negative global shocks, the exchange rate depreciates due to the effect of endogenous private sector deleveraging on the real exchange rate. Therefore, LC debt is risky for risk averse international investors.

We show that the optimal foreign exchange management that maximizes domestic welfare is to "lean against the global wind". Because of the collateral constraint on borrowing and a pecuniary externality, households overborrow in normal times and underborrow upon negative shocks (as in Schmitt-Grohe and Uribe (2020)). In particular, in response to a global negative shock, households cut back on borrowing, which can result in real exchange rate depreciation that tightens the collateral constraint too much. This can precipitate a "sudden stop" (a binding collateral constraint) and results in even sharper exchange rate depreciation. Ex-ante, the central bank accumulates reserves to counter the overborrowing. The economy is less leveraged and it lowers the probability of a sudden stop. Ex-post, upon a global negative shock, the central bank decumulates reserves to avoid a sudden stop. The optimal foreign exchange intervention policy is to "lean against the global wind" and by doing so mitigates the impact of global shocks on the real exchange rate.

The optimal "lean against the global wind" foreign reserves policy explains the second and third empirical facts that reserves holding countries have a lower sensitivity to global factors. The lower sensitivity to global factors results in less exchange rate premium on LC interest rate, which explains the fourth empirical fact. The lower LC interest rate leads the sovereign to endogenously issue more debt in LC, which explains the first empirical fact. Intuitively, by providing insurance to global investors via a more stable exchange rate during global bad times, the Sovereign enjoys a lower insurance premium (risk premium) in the issuance of instruments (local currency debt) that insure against domestic shocks.

We calibrate the quantitative model to Brazil, one of the success stories in overcoming the original sin. The model is able to accurately match the average LC share in Brazil (50% in the data vs 53% in model). Our model also predict a realistic average reserves to GDP (14% in the data vs 13.4% in model). The literature on reserves accumulation has started from a "mercantilist" view that reserves are used as a buffer for trade disturbance (see for example Frenkel and Jovanovic (1981), Flood et al. (2001) and Aizenman and Lee (2007)). Since the Asian financial crisis in late 1990s, the accumulation of reserves has been substantial, surpassing the predictions of many models and policymakers and often viewed as "excessive" (see for example Bird and Rajan (2003), Edison (2003), Jeanne (2007) and Jeanne and Rancière (2011)). The core mechanism in our FXI model, which builds on Bianchi (2011), Gabaix and Maggiori (2015) and Davis et al. (2023), is consistent with empirical observations from Obstfeld et al. (2010) and Gourinchas and Obstfeld (2012), who documents that rapid build-up of leverage and appreciation are predictors of crisis and higher FX reserves reduces the chance of a crisis through managing domestic financial instability. Our model indicates there is no excess reserves accumulation and highlight the role of reserves accumulation and global shocks toward currency composition of sovereign portfolio.

We then shut down different channels. We find that inflation targeting explains roughly two thirds of the LC share and reserves management explains the other one third. However, we show that the gain in the LC share by further inflation commitment (full price stabilization) is very limited. Therefore, reserves management makes possible a level of LC share that is not attainable solely by strict inflation targeting, and allows rooms for inflation flexibility. We validate our model by model simulated regressions. Our model simulation can generate regression estimates that are quantitatively close to the empirical counterpart.

Our quantitative model allows us to compute the welfare benefits from optimal foreign reserves intervention (FXI) separately for private households and for the sovereign. For private households, the welfare benefits are positive but modest, in line with the benefits of macroprudential policy as in Bianchi (2011), for instance. But for the sovereign, we find large welfare benefits. The sovereign benefits in two ways from optimal FXI. First, there is the benefit of a better smoothing of public goods spending from the use of local currency debt. But also there is the benefit of being able to borrow at a lower world interest rate. The combination of these two effects of optimal FXI imply big welfare effects for the sovereign in the production of public goods.

Literature review. Our paper is related to recent literature on overcoming "original sin", including Du et al. (2020), Engel and Park (2022) and Ottonello and Perez (2019). These papers study the optimal currency composition and its interplay with monetary credibility (inflation policy).[3] Sunder-Plassmann (2020) and Hurtado et al. (2022) also study the interplay of inflation/monetary policy and local currency debt while taking the

[3] Drenik et al. (2022) and Liu et al. (2021) study the liability currency choice for the private sector.

currency portfolio as given. Ogrokhina and Rodriguez (2018) provides empirical evidence of the relationship between inflation targeting and sovereign debt denomination. Our paper advances the literature by linking up the relationship between currency composition and foreign reserves.[4]

The paper is also related to the literature of reserves management. Papers by Bianchi et al. (2018), Bianchi and Sosa-Padilla (2020) and Hur and Kondo (2016) focus on the role of foreign reserves in reducing sovereign default risk. We show both empirically and theoretically the relevance of reserves on the currency composition of sovereign debt. A closely related paper is Alfaro and Kanczuk (2019), which studies local currency debt and reserves management to hedge income risk in a model of sovereign default with risk neutral foreign lenders. Their paper recognizes that it may be optimal for an emerging market to jointly issue local currency debt as well as hold foreign exchange reserves. In their model, reserves are useful due to valuation effects that become positive in a bad state due to currency depreciation, but reserves may also affect the incentive for sovereign default. By contrast, in our paper, reserves are actively deployed to prevent depreciation in global bad times, and we focus on the way in which this affects the currency risk premium charged by risk averse lenders.[5] Reserves management is also motivated as a tool to correct for different types of externalities, as studied in Arce et al. (2019), Benigno et al. (2022), Davis et al. (2023), Fanelli and Straub (2021) and Kim and Zhang (2020). Finally, reserves management is also viewed as a form of risk management. Hassan et al. (2022) proposes a risk-based theory of stabilizing currency movements, in which the currency that stabilizes its return to the anchor country enjoy a lower interest rate and receives a larger share of global investment.[6] Bocola and Lorenzoni (2020) argues that foreign reserves can stabilize the exchange rate in a crisis and reduce liability dollarization. Amador et al. (2020) studies exchange rate managment at the zero lower bound. Itskhoki and Mukhin (2022) argue that in the presence of financial friction and nominal rigidities, FXI should be used to completely eliminate UIP deviation. In our model, the primary friction is pecuniary externality and the optimal FXI focuses on reducing risk of sudden stops which reduces UIP deviation rather than completely eliminating it.

[4]In addition, Engel and Park (2022), Ottonello and Perez (2019) and Sunder-Plassmann (2020) considers only a risk-neutral investor.

[5]In addition Alfaro and Kanczuk (2019) do not consider inflation and and endogenous currency choice in their framework, both of which represent an important feature of our paper.

[6]Their model also involves a covariance between the exchange rate and the marginal utility of global investors but does not explore the implications for the currency composition of sovereign debt.

The model of sudden stops in the paper is closely related to Schmitt-Grohe and Uribe (2020), who show the existence of "under-borrowing" equilibria in Bianchi (2011) framework, where investor's deleveraging can generate large real exchange rate depreciation and current account reversals. We show that the use of foreign reserves to lean against the wind, following the reserves management policy as in Davis et al. (2023), can improve interest rate terms of LC interest rate faced by a sovereign government.

This paper also contributes to the literature on "original sin redux" (Carstens and Shin (2019), Bertaut et al. (2021), Hofmann et al. (2020) and Hofmann et al. (2022)), which concerned about international investors' currency mismatch issue. We provide a theoretical setup and study the role of reserves management in dampening the response to global factors (Rey (2015), Kalemli-Özcan (2019) and Bruno et al. (2021)).

The paper is organized as follows. Section 2 presents the empirical analysis of LC share, foreign reserves and inflation targeting. The model is presented in Section 3. Section 4 discusses the results from the quantitative model, including policy functions, simulated moments, simulated regressions and a welfare analysis. Section 5 concludes.

2 Empirical findings

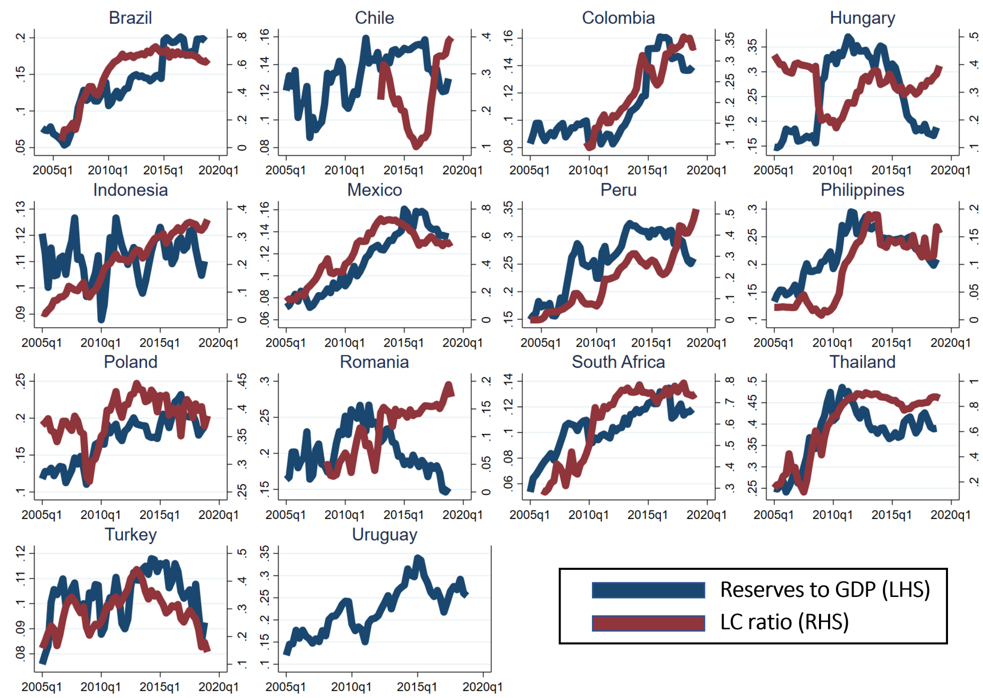
In this section, we show that among inflation targeting EMs, when reserves are high, the sovereign's LC debt share increases, the countries' exhcange rate is less sensitive to global factors and the LC sovereign spread is low due to a low exchange rate premium.

Data description. We focus on 24 emerging countries as in Arslanalp and Tsuda (2014). Arslanalp and Tsuda (2014) maintain a panel dataset of currency composition of EM sovereign debt using various data sources. Our sample covers from 2004Q1 to 2019Q2. The country list and the rest of the datasources are listed in Appendix Table 11.

2.1 Foreign reserves and local currency debt

Figure 2 illustrates the unconditional relationship between the local currency share of sovereign debt and reserves to GDP for a sub-sample of our countries who adopted inflation targeting before 2010. It is apparent that there is a positive relationship for most countries. In order to explore this in more detail however, we estimate a panel regression.

Figure 2: Time series plot of reserves to GDP and LC ratio for inflation targeters



For each country i and time t, we define a variable $LCshare_{i,t} = \frac{\text{foreign held local currency sovereign debt}_{i,t}}{\text{foreign held total sovereign debt}_{i,t}}$. We follow most of the literature in focusing on external debt. Inspired by the studies cited above that focus on inflation commitment, we control for whether a country is an inflation targeter. We use the definition by Ogrokhina and Rodriguez (2018) that documents the explicit inflation targeting dates for these countries.[7] Whether a country is an inflation targeter is a time-varying dummy. The panel fixed effect regression takes the form:

$$LCshare_{i,t} = \alpha_i + \beta_1 IT_{i,t} + \beta_2 (IT_{i,t} = 0) \times ln(\frac{reserves}{GDP}) + \beta_3 (IT_{i,t} = 1) \times ln(\frac{reserves}{GDP}) + \gamma GC_t + \delta DC_{i,t} + \varepsilon_{i,t} \quad (1)$$

where $IT_{i,t}$ is a time-varying dummy for an inflation targeter, GC is a vector of global factor controls (VIX, US Treasury 5Y, US GDP growth) and DC is a vector of domestic

[7] They follow the definition from Mishkin (2004). There are a number of essential elements to an inflation targeting regime: (i) an explicit central bank mandate to pursue price stability as the primary objective of monetary policy; (ii) an explicit quantitative target for inflation; (iii) a high degree of transparency in monetary policy strategy and implementation; and (iv) monetary policy based on a wide set of information, including an inflation forecast. We updated India, Russia and Ukraine as inflation targeter since their paper. The detail list and year of inflation targeting is provided in Appendix Table 12.

variable controls (Domestic GDP growth, World Bank govt effectiveness index, World Bank policy stability index, the Chinn-Ito Index, domestic credit to GDP) from Engel and Park (2022). Since many of these variables are only available at annual frequency, we restrict this regression to an annual frequency (end of year data). Regressions in quarterly frequency give qualitatively the same results.

The regression estimates are displayed in Table 1. The first column shows estimates without controls. Column (2) to column (5) reports estimates with global controls alone, domestic controls alone, both domestic and global controls and domestic controls with a time fixed effect.[8] These estimates largely confirm the existing literature that finds a relationship between LC share and inflation targeting. The IT coefficient of the first four specifications are significantly positive but insignifcant for column (5). Inflation targeting countries tend to have a significantly higher LC ratio. For example, in the first column, the coefficient indicates inflation targeting countries tend to have 89.9% higher LC share than non-inflation targeter. In the sample, countries that adopt inflation targets never return to non inflation targeting. The positive relationship therefore captures a low frequency difference induced by changing from a non-inflation targeter to an inflation targeter within a country.

More interestingly, in addition to whether a country is an inflation targeter, we see that the coefficient estimates for $(IT_{i,t} = 1) \times ln(\frac{reserves}{GDP})$ are significantly positive in all specifications. This indicates that an inflation targeter with a higher reserves to GDP ratio tends to have a higher LC share of sovereign debt. This differs for non inflation targeters, who appear to have insignificant estimates across different specifications. The empirical results here suggests that in addition to being an inflation targeter, which is a long-term institution change for some EMs, the management of foreign reserves seems to have an interesting correlation with the sovereign currency portfolio at a higher frequency.[9]

[8] Global time trend is absorbed by the time fixed effect. The key variables $LCshare$ and $\frac{reserves}{GDP}$ are both stationary ratios.

[9] Note that the effect is not mechanically driven by exchange rate valuation. An depreciation of LC will lower the LC share but increase the reserves to GDP ratio, leading to a negative relationship.

Table 1: Local currency debt ratio and foreign reserves

$$LCshare_{i,t} = \alpha_i + \beta_1 IT_{i,t} + \beta_2 (IT=0) \times ln \frac{reserves}{GDP}_{i,t} + \beta_3 (IT=1) \times ln \frac{reserves}{GDP}_{i,t} + controls_{i,t} + \varepsilon_{i,t}$$

	no control	domestic controls	global controls	all controls	all controls +time fixed effect
	(1)	(2)	(3)	(4)	(5)
$IT\ dummy$	0.899***	0.604***	0.421**	0.211***	-0.001
	(0.233)	(0.096)	(0.149)	(0.057)	(0.080)
$(IT=0) \times ln\frac{reserves}{GDP}$	-0.051	-0.061**	0.011	-0.009	0.027
	(0.035)	(0.021)	(0.034)	(0.012)	(0.022)
$(IT=1) \times ln\frac{reserves}{GDP}$	0.326***	0.201***	0.176***	0.086***	0.049**
	(0.078)	(0.026)	(0.043)	(0.020)	(0.021)
Domestic GDP growth		0.425*		0.561**	0.278
		(0.230)		(0.249)	(0.351)
Chinn-Ito Index		-0.133**		-0.071	-0.047
		(0.050)		(0.067)	(0.066)
Govt Effectiveness		0.191***		0.158**	0.111
		(0.046)		(0.057)	(0.066)
Political Stability		-0.011		-0.001	0.001
		(0.021)		(0.028)	(0.035)
$\frac{Domestic\ credit}{GDP}$		0.007***		0.005***	0.004***
		(0.001)		(0.001)	(0.001)
VIX			-0.000	-0.004***	
			(0.002)	(0.001)	
US Treasury 5Y			-0.053***	-0.046***	
			(0.011)	(0.003)	
US GDP growth			1.979	-1.223	
			(1.785)	(1.096)	
N	311	311	270	270	270
Within R^2	0.15	0.38	0.28	0.47	0.52

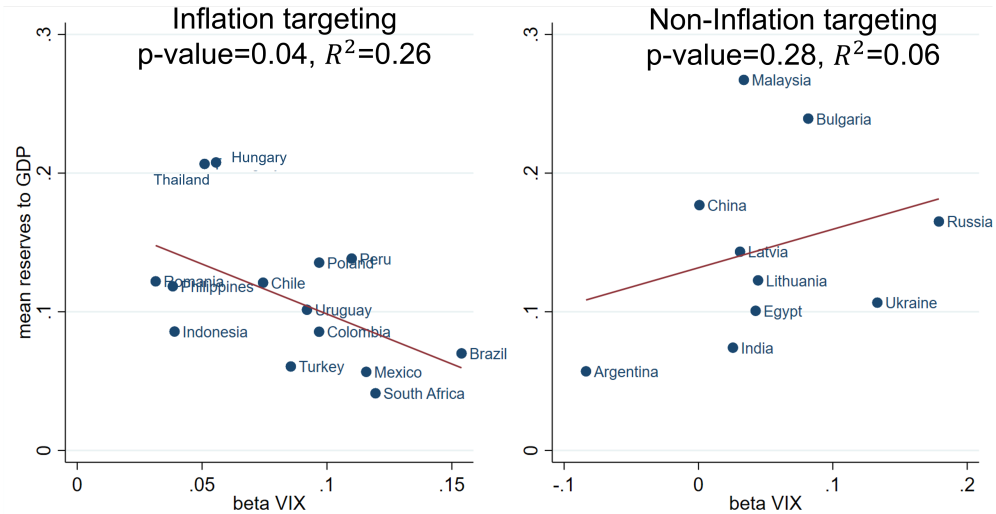
Discroll Kraay (1998) standard errors with 5 lags in parentheses. * p<0.1, ** p<0.05, *** p<0.01

2.2 Foreign reserves and exchange rate sensitivity

We next provide some empirical evidence on the relationship between foreign reserves and exchange rate movements in EMs that is attributable to global factors, as they would be non-diversifiable from the investor's point of view.

We first look at cross-sectional evidence. To capture exchange rate movements that

Figure 3: mean reserves to GDP and sensitivity of exchange rate to VIX index (β^{VIX})



are relevant for investors, we regress the log change of EM exchange rates on the log change of the VIX index country by country at quarterly frequency:

$$\Delta s_t = \alpha + \beta^{VIX} \Delta lnVIX_t + \varepsilon_t \tag{2}$$

where s_t is the log of the exchange rate of EM currency per USD.

In this quarterly regression, we obtain a β^{VIX} for each emerging country. This can be interpreted as a measure of how sensitive the EM currency is to changes of global shocks as measured by the VIX index, an often-used measure of foreign investor risk appetite/stochastic discount factor/state.

Figure 3 plots separately the time series mean of reserves to GDP v.s. the beta obtained from eq (2) for each country for inflation targeters and non inflation targeters.[10] On the left hand panel, we see a negative slope that indicates a country with a higher mean reserves to GDP tends to have a lower β^{VIX} for inflation targeting countries. The regression line of this scatter plot has a p-value of 0.04. In contrast, we see an insignificantly positive relationship on the right hand side for non-inflation targeters. Taken together, this empirical evidence indicates that inflation targeting countries with high reserves have their exchange rate less sensitive to global movements.

[10]Strictly speaking, inflation targeters in this cross-sectional picture are countries that become inflation targeter during first half of the sample period. India, Russia and Ukraine become inflation targeters at almost the end of the sample.

We further look at the exchange response to external shock and foreign reserves intervention (FXI) using daily data.[11] We investigate how exchange rates of EM responses to a high-frequency identified monetary shocks as in Nakamura and Steinsson (2018).

$$\Delta s_{i,t} = \alpha_i + \beta_1 \Delta i_t + \beta_2 \frac{FXI}{GDP}_{i,t} + \beta_3 \frac{FXI}{GDP}_{i,t} \times \Delta i_t + \varepsilon_{i,t} \tag{3}$$

where Δi_t is the first principal component of the changes in the next four maturing EuroDollar future (ED1-ED4) identified in a 30-minute window around FOMC announcements from Bauer and Swanson (2022). We look at the change of 5-day exchange rate and change of 5-day FX intervention in a fashion similar to Rodnyansky et al. (2022).

Table 2 reports the coefficient estimates. The standalone FOMC shock coefficient is 10.43, indicates the EM exchange rate depreciates by 10.43% in a 5-day horizon after a 1% tightening shock. Importantly, the interaction term has a negative coefficient of -244, meaning that if the EM central bank conducts a foreign reserves decumulation of 1% of its GDP, the exchange rate will appreciate by 2.44%. The regression exercise indicates an effective foreign reserves intervention can counteract the exchange rate movements driven by a US tightening. To completely neutralize a 1% tightening shock, it would require a typical country to conduct a foreign reserves decumulation of 4% of its GDP. Ahmed et al. (2023) evaluates the effect of reserves during the tightening monetary cycle in the US in 2021-2022 and conclude a similar effect of reserves management in fighting against depreciation of a local currency.

Table 2: 5-day exchange rate response to FOMC shocks and FXI

	Five-day exchange rate change
FOMC shock (Δi_t)	10.43***
	(1.920
$\frac{FXI}{GDP}_{i,t}$	-0.18*
	(0.09)
$\frac{FXI}{GDP}_{i,t} \times \Delta i_t$	-244.0**
	(123.2)
N	544
Within R^2	0.10

Robust standard errors clustered by time in parentheses. * p<0.1, ** p<0.05, *** p<0.01

[11] We appreciate the authors of Rodnyansky et al. (2022) sharing their daily FXI data. Limited by the availability of daily FXI data, the sample countries in this exercise are Argentina, Brazil, Chile, Colombia, Mexico, Peru and Turkey.

2.3 Lean against the global wind

Now we turn our focus to some panel regression evidence. We look at how changes of reserves to GDP are associated with changes of globally and locally driven exchange rates. To do so, we first conduct a panel regression at quarterly frequency:

$$\Delta s_{i,t} = \alpha_i + \beta_i^{VIX} \Delta lnVIX_t + \sum_{t=0}^{T} \delta_t T_t + \varepsilon_{i,t} \tag{4}$$

where T_t is a time dummy.

Note that we allow for country-dependent coefficients on the VIX as well as time fixed effects, so the regression can capture the most aggregate related exchange rate movements. We produce fitted values for the dependent variable $\Delta \hat{s}_{i,t}$ and fitted values for the residual $\hat{\varepsilon}_{i,t}$. $\Delta \hat{s}_{i,t}$ can be interpreted as the exchange rate movements that are related to global factors and the residual will be country specific exchange rate movements.

After decomposing the exchange rate, we investigate how a country's foreign reserves co-move with different components with the follow quarterly panel regression:

$$\Delta ln(\frac{reserves}{GDP})_{i,t} = \alpha_i + \beta_1 IT_{i,t} + \beta_2 (IT_{i,t} = 0)\Delta\hat{s}_{i,t} + \beta_3 (IT_{i,t} = 1)\Delta\hat{s}_{i,t} + \beta_4 (IT_{i,t} = 0)\hat{\varepsilon}_{i,t} + \beta_5 (IT_{i,t} = 1)\hat{\varepsilon}_{i,t} + \varepsilon_{i,t} \tag{5}$$

Table 3: Global factor exchange rates and change of reserves

	$\Delta ln(\frac{reserves}{GDP})_{i,t}$
IT dummy	-0.030***
	(0.009)
$(IT = 0) \times$ global exchange rate change	-0.129
	(0.190)
$(IT = 1) \times$ global exchange rate change	**-0.241***
	(0.105)
$(IT = 0) \times$ local exchange rate change	0.103
	(0.088)
$(IT = 1) \times$ local exchange rate change	-0.042
	(0.108)
N	2647
Within R^2	0.01

Discroll Kraay standard errors with 5 lags in parentheses. * p<0.1, ** p<0.05, *** p<0.01

In table 3, we report the regression estimates of eq (5). The coefficient on $IT_{i,t}$ is negative, meaning that inflation targeting countries decumulate reserves more than non-inflation targeter on average. The more interesting result is that only the coefficient on $(IT_{i,t}=1)\Delta\hat{s}_{i,t}$ is significant and the estimate is negative. This means that for inflation targeting countries, there is decumulation of reserves when the currency depreciates due to its global component. This is not the case for currency depreciation that is due to country specific factors, as the coefficient estimate of $(IT_{i,t}=1)\hat{\varepsilon}_{i,t}$ is small and insignificant. This foreign reserves management behaviour is consistent with a "lean against the wind" story for inflation targeters, in which Central Banks use their existing reserves to intervene in face of pressure on the currency coming from global shocks.[12,13]

2.4 Foreign reserves and local currency sovereign spreads

We now provide evidence directly from the pricing data. The sovereign spread is defined as the LC sovereign yield over the US treasury yield of the same maturity:

$$
\begin{aligned}
\text{LC sovereign spread} \equiv\ & \text{local currency sovereign yield} - \text{US Treasury yield} \\
& \underbrace{\text{local currency sovereign yield} - \text{US Treasury yield} - \text{CCS}}_{\text{local currency sovereign risk (LCCS, pure credit risk)}} + \underbrace{\text{CCS}}_{\text{exchange rate risk}}
\end{aligned} \tag{6}
$$

Since the LC sovereign yield is in a different currency denomination than the US Treasury, investing in local currency sovereign bonds involve two risks: pure credit risk and exchange rate risk. As in Du and Schreger (2017), one can use the cross currency swap (CCS) to eliminate the currency risk. This could be thought of as swapping the US Treasury to the EM currency and treat it a risk-free benchmark in EM currency. We add and subtract the CCS to the first line equation (6) and result in the decomposition in the 2nd line.

We directly obtain these sovereign spreads and the decomposition from the Du and Schreger database. Because of a non-nesting sample with Arslanalp and Tsuda (2014), we are left with 16 countries and one non-inflation targeter (see Table 10) so we do not split between inflation targeter and otherst. With a quarterly panel fixed effect setting, we

[12]The effect is not mechanical. A depreciation of the local currency (an increase Δs_t) will increase $\frac{reserves}{GDP}$ because reserves are in foreign currency and GDP is measured in local currency. Therefore, the mechanical relationship should result in a positive coefficient.

[13]To the extreme if a central bank completely stablizes exchange rate then β_i^{VIX} could be zero. However, it is never optimal in our model to do so because it is costly to accumulate reserves. The regression estimates here could be intepreted as moments under optimal policy, in which we compare them with the simulated regression in the quantitative section.

regress the local currency sovereign spread and each of its component with the reserves to GDP:

$$\begin{aligned} y_{i,t} &= \alpha_i + \beta_1 ln(\frac{reserves}{GDP})_{i,t} + \beta_2 ln(\frac{Govt\ debt}{GDP})_{i,t} + \beta_3 ln(\frac{Privdebt}{GDP})_{i,t} \\ &+ \beta_4 (E_t \Delta S_{t+1}) + \beta_5 GDPgrowth_{i,t} + \beta_6 ln \frac{Domestic\ credit}{GDP} + \beta_7 lnVIX_t + \varepsilon_{i,t} \end{aligned} \tag{7}$$

where $y_{i,t} = \{\text{local currency sovereign spread}, \text{exchange rate risk}, \text{pure credit risk}\}$ defined in eq (6) and we control for govt external debt to GDP, private external debt to GDP, domestic GDP growth, domestic credit to GDP and the VIX index, all of which are known to be important in explaining sovereign risk. $E_t \Delta S_{t+1}$ is a measure of expected exchange rate using survey forecasted exchange rates expected from Bloomberg.

The regression estimates are reported in Table 4 below. The first column reports the regression with the local currency sovereign spread as the dependent variable. The coefficient estimate of $ln(\frac{reserves}{GDP})$ is negative and significant, indicating an increase in reserves is associated with a reduction in the sovereign spread. When we look at the component that is associated with this reduction, we find that the reduction in the sovereign spread is primarily associated with a change in the exchange rate premium component in column (2), while there is no significant change attributable to the credit risk component in column (3).[14]

3 A model of currency composition and foreign reserves

In this section, we lay out a small open economy tradable-nontradable model to reconcile the empirical findings and study the interaction between the currency composition of sovereign debt and foreign reserves.The sovereign local currency debt is a good hedge for domestic shocks, since the local currency depreciates in domestic bad times. Inflation targeting makes local currency debt possible, because investors understand the sovereign will not completely inflate away the debt. However, risk averse international investors charge a premium on local currency bonds because of exchange rate risk, since global bad shocks are associated with depreciation of local currency and bad times for investors. The risk premium makes local currency debt less attractive for the sovereign. Reserves

[14]We control for survey forecast-based expected exchange rate appreciation as a proxy of controlling for first-moment exchange rate movement. It is the forecast value of the end of quarter exchange rate 1 year from the current quarter. We construct the value such that a positive value is an expected appreciation.

Table 4: Sovereign Spreads and Foreign Reserves

	LC spreads (%)	Exchange rate premium (%)	Local currency credit risk (%)
	(1)	(2)	(3)
$ln\frac{reserves}{GDP}$	-1.797**	-1.840***	0.043
	(0.709)	(0.445)	(0.520)
$ln\frac{govt\ debt}{GDP}$	0.843**	0.951***	-0.108
	(0.329)	(0.216)	(0.277)
$ln\frac{private\ debt}{GDP}$	2.117***	1.748***	0.369
	(0.464)	(0.239)	(0.325)
Survey expected	-0.549	-7.503***	6.955***
appreciation	(2.134)	(1.740)	(1.265)
GDP growth	-0.529	-0.181	-0.348
	(0.733)	(0.779)	(0.431)
$ln\frac{domesticcredit}{GDP}$	0.004	0.012	-0.008
	(0.013)	(0.008)	(0.008)
ln VIX	2.591***	1.773***	0.818***
	(0.257)	(0.154)	(0.211)
N	565	565	565
R^2	0.31	0.33	0.22

Notes: Discroll Kraay standard errors with 5 lags in parentheses. * p<0.1, ** p<0.05, *** p<0.01

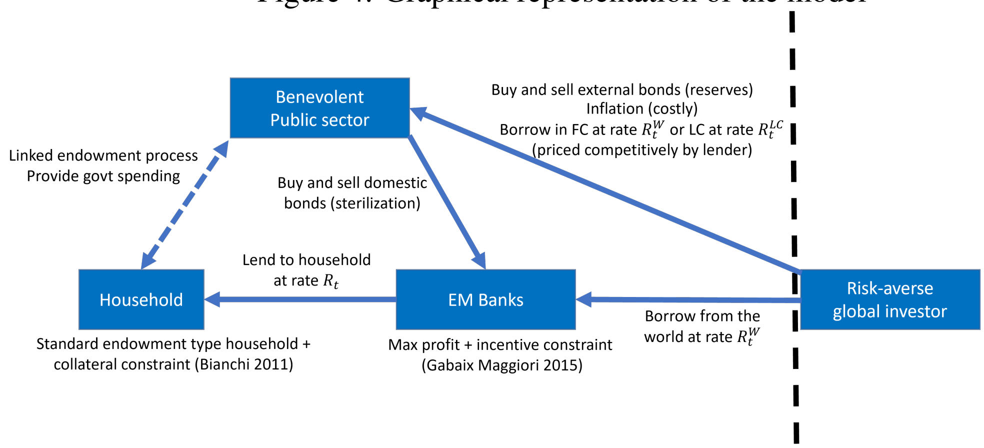
management dampens the sensitivity of the exchange rate to global shocks. This acts so as to reduce the local currency risk premium. The presence of financial frictions in the domestic economy, as described more fully below, allows the Central Bank to accumulate reserves in placid periods, and deploy reserves in response to downturns in the global economy. This acts so as to stabilize the domestic real exchange rate. Intuitively, by providing insurance to global investors via a more stable exchange rate during global bad times, the Sovereign enjoys a lower insurance premium (risk premium) in the issuance of instruments (local currency debt) that insure against domestic shocks.

Figure 4 displays a graphical illustration of the model. The small open economy has four types of agents: 1) Representative households who borrow subject to a collateral constraint as in Bianchi (2011); Davis et al. (2023). This constraint gives rise to a pecuniary externality and a role for reserves accumulation.[15] 2) Domestic financial intermediaries who purchase bonds from private households, financed by borrowing in international financial markets. This supports the effectiveness of foreign exchange rate intervention, as

[15]Foreign reserves accumulation as a tool to correct for pecuniary externality is studied in Arce et al. (2019) and Kim and Zhang (2020).

in Gabaix and Maggiori (2015). 3) A consolidated public sector that can be compartmentalized into a Central Bank and a Sovereign that chooses an optimal reserve management policy, inflation policy and optimal sovereign borrowing and its currency composition 4) Risk averse international investors who provide funding to the small open economy.

Figure 4: Graphical representation of the model



Exogenous processes and the law of one price. We assume there are two exogenous processes. The two shocks are assumed to be uncorrelated and represent different aspects of the world. The first one is tradable endowment y_t^T of the small open economy, which represents country-specific movements. The second one is the world interest rate (R_t^W) on foreign currency debt, which reflects global financial fluctuations.

The tradable endowment and world interest rate processes are

$$y_t^T = \rho^y y_{t-1}^T + \sigma^y \varepsilon_t^y \text{ , } R_t^W = \rho^R R_{t-1}^W + \sigma^R \varepsilon_t^R$$

Denote p_t^T as the tradable goods price at home, P_t^{T*} as the tradable goods price in the foreign country. S_t as the home currency price of a foreign currency (e.g. Peso per USD), and we assume that the law of one price is satisfied for traded goods, so that $p_t^T = S_t P_t^{T*}$. Without loss of generality, we normalize $P_t^{T*} \equiv 1$.

Household problem. We begin with a description of the private household problem. The household sector is same as Bianchi (2011), but using the calibration of Schmitt-Grohe and Uribe (2020) for the empirical relevance as shown in their paper.[16]

[16]This is critical, since as shown in Davis et al. (2023), it gives rise to a role for countercyclical foreign exchange rate intervention.

The households value consumption (c_t), government spending (G_t) and dislike inflation ($\frac{P_t}{P_{t-1}}$). ω_g is the Pareto weight on government spending less inflation costs. A continuum of identical households receive tradable (y_t^T) and non-tradable endowments (y^N, a constant). They choose the consumption of tradable goods (c_t^T) and non-tradable goods (c_t^N) within each period, together which are aggregated to a consumption composite c_t. Households can sell one-period maturity foreign currency denominated bonds (b_t^{FC}) to financial intermediaries at a gross rate R_t. By assumption, households and financial intermediaries do not trade in local currency bonds, perhaps due to "orginal sin" at the private level. This is empirically realistic, since it is widely recognized that most corporate EM borrowing is in FC (Du and Schreger (2017), Wu (2022)).

The household problem is described as:

$$V = \max_{c_t^T, c_t^{NT}, b_t^{FC}} E_0 \sum_{t=0}^{\infty} \beta^t \left[(1-\omega_g)u(c_t) + \omega_g[v(G_t) - l(\frac{P_t}{P_{t-1}})] \right] \tag{8}$$

subject to

$$c_t = [\alpha^{1/\xi}(c_t^T)^{(\xi-1)/\xi} + (1-\alpha)^{1/\xi}(c_t^N)^{(\xi-1)/\xi}]^{\xi/(\xi-1)}, \; u(c_t) = \frac{c_t^{1-\sigma}}{1-\sigma}$$

where c_t is aggregate consumption. The period budget constraint in domestic currency is:

$$p_t^T c_t^T + p_t^N c_t^N + S_t \frac{b_t^{FC}}{R_t} \leq p_t^T(1+\tau)y_t^T + p_t^N y^N + S_t b_{t-1}^{FC} - T_t + TR_t^{CB} + \Pi_t \tag{9}$$

where $b_t^{FC} < 0$ denotes new foreign currency borrowing at time t. p_t^T and p_t^N are the prices for tradable and non-tradable goods and S_t is the nominal exchange rate. $T_t = \tau y_t^T$ represents a tax paid to the government, and TR_t^{CB} is a transfer from the Central Bank. Finally, Π_t represents profits of financial intermediaries, which are described below.

Households face a borrowing constraint that depends on the value of GDP, given by:

$$\text{credit constraint:} \; -S_t \frac{b_t^{FC}}{R_t} \leq \kappa(p_t^T y_t^T + p_t^N y^N) \tag{10}$$

where κ is a parameter that determines the tightness of the borrowing constraint.

Financial sector. The financial sector operates similar to Gabaix and Maggiori (2015). Competitive two-period lived financial intermediaries borrow from international investors, in foreign currency, and lend to households. Financial intermediaries begin each period with zero net worth, and satisfy a balance sheet condition;

$$\left(b_t^{fs}+F_t^{fs}\right)=0$$

where b_t^{fs} represents bonds purchased from domestic households with a return R_t, and $F_t^{fs}<0$ represent bonds sold to international investors at return R_t^W. Intermediary profits are then maximizes the profit using the households SDF ($\Lambda_{t+1}\equiv\beta\frac{U'(c_{t+1})}{U(c_t)}$):

$$max\ V_t\equiv E_t\Lambda_{t+1}(R_t^W F_t^{fs}+R_t b_t^{fs})=E_t\beta\frac{U'(c_{t+1})}{U(c_t)})(R_t^W-R_t)F_t^{fs}$$

To prevent intermediaries from absconding with the assets, intermediaries are limited by the incentive constraint that discounted profits must be at least equal to $\Gamma\mid b_t^{fs}\mid$ times assets at the beginning of $t+1$, which are $b_t^{fs}=-F_t^{fs}$, which results in the follow incentive constraint:

$$E_t\beta\frac{U'(c_{t+1})}{U(c_t)})(R_t-R_t^W)F_t^{fs}\geq\Gamma\mid F_t^{fs}\mid\times F_t^{fs}$$

The constraint is always binding, this leads to a wedge between domestic and world returns given by

$$R_t=R_t^W-E_t\frac{\Gamma}{\beta\Lambda_{t+1}}\frac{F_t^{fs}}{R_t^W}\tag{11}$$

where $F_t^{fs}<0$ is the financial sector borrowing from abroad, which, in equilibrium will equal the household's borrowing from financial intermediaries. Higher household borrowing will increase the trading activity of financiers, increasing the wedge between domestic and world rates of return.

International investors. We assume that international investors are risk averse. They price the assets according to their asset pricing equation.

$$E_t[\Gamma_{t+1}^*R_t^W]=1\text{ and }E_t[\Gamma_{t+1}^*R_t^{LC}\frac{S_t}{S_{t+1}}]=1\tag{12}$$

where Γ_{t+1}^* is the stochastic discount used by the international investor. We assume Γ_{t+1}^* is a function of the world interest rate, $\Gamma_{t+1}^*=f(R_{t+1}^W)$ with $f'>0$ (parameterized in section 4). The idea is to capture the fact that when the international investors are in bad times, they charge a higher premium on assets. Therefore, a high interest rate state is associated with high marginal utility of the investors.

Collection of private optimal conditions

Before introducing the public sector decision, we collect the system of equations that describes the private sector behaviors and are relevant for the public section analysis.

Given prefences as described, the true price index for the household is

$$P_t \;=\; [\alpha(p_t^T)^{1-\xi} + (1-\alpha)(p_t^N)^{1-\xi}]^{\frac{1}{1-\xi}}$$

Rearranging the price index and use the fact that $p_t^T = S_t$ gives an expression for the nominal exchange rate:

$$S_t = \underbrace{P_t}_{\text{price index factor}} \times \underbrace{[\alpha + (1-\alpha)(\frac{p_t^N}{p_t^T})^{1-\xi}]^{-\frac{1}{1-\xi}}}_{\text{real exchange rate factor}} \tag{13}$$

The market clearing condition for tradable and non-tradable goods are:

$$y^N = c_t^N \,,\; p_t^T c_t^T + S_t \frac{b_t^{FC}}{R_t} \leq p_t^T(1+\tau)y_t^T + S_t b_{t-1}^{FC} - T_t + TR_t^{CB} + \Pi_t$$

Households first order conditions of tradable and non-tradable intratemporal tradeoff lead to the equilibrium relative price given by:

$$\frac{p_t^N}{p_t^T} = \frac{1-\alpha}{\alpha}(\frac{c_t^T}{y^N})^{\frac{1}{\xi}} \tag{14}$$

Combining the nominal exchange rate equation (13) and intratemporal tradeoff equation (14) results in the equilibrium condition of the nominal exchange rate

$$S_t = \underbrace{P_t}_{\text{price index factor}} \times \underbrace{[\alpha + (1-\alpha)(\frac{1-\alpha}{\alpha}(\frac{c_t^T}{y^N})^{\frac{1}{\xi}})^{1-\xi}]^{-\frac{1}{1-\xi}}}_{\text{real exchange rate factor}} \tag{15}$$

The nominal exchange rate is influenced by two factors, one is real exchange rate factor, when the real allocation of tradable and non-tradable goods change, it changes the relative price and therefore the real exchange rate. The other one is the price index factor, by choosing a higher price index (inflation), the nominal exchange rate depreciates, regardless of the relative price.

The household's Euler equation is:

$$u_{T,t} - \mu_t = \beta R_t [E_t u_{T,t+1}] \tag{16}$$

where $u_{T,t}$ is the derivative w.r.t to c_t^T and μ_t is the multiplier on the borrowing constraint.

The complementary slackness condition of the borrowing constraint is:

$$S_t \frac{b_t^{FC}}{R_t} + \kappa(p_t^T y_t^T + p_t^N y^N) \geq 0 \text{ and with equality if } \mu_t > 0 \tag{17}$$

The financial sector is described by profit maximizing condition:

$$R_t = R_t^W - E_t \frac{\Gamma}{\beta \Lambda_{t+1}} \frac{F_t^{fs}}{R_t^W} \tag{18}$$

Combining Euler equation equation (16) and financial sector maximization equation (18) results in a modified Euler equation:

$$u_{T,t} - \mu_t = \beta [R_t^W - E_t \frac{\Gamma}{\beta \Lambda_{t+1}} \frac{F_t^{fs}}{R_t^W}][E_t u_{T,t+1}] \tag{19}$$

The investor's no arbitrage condition following equation (12) then implies:

$$E_t[\Gamma_{t+1}^* R_t^W] \quad = \quad E_t[\Gamma_{t+1}^* R_t^{LC} \tfrac{S_t}{S_{t+1}}] = R_t^{LC}[E_t(\Gamma_{t+1}^*) E_t(\tfrac{S_t}{S_{t+1}}) + cov_t(\Gamma_{t+1}^*, \tfrac{S_t}{S_{t+1}})] \tag{20}$$

Equation (20) implies that investors will require a premium on local currency debt over foreign currency debt if $cov_t(\Gamma_{t+1}^*, \frac{S_t}{S_{t+1}}) < 0$, since the local currency will depreciate when investors have high marginal utility of funds - i.e. in bad times for investors.

Equations (15), (17), (19), (20) characterize the private sector equilibrium.

The Public Sector Decision

For the ease of exposition, the optimal policy of the public sector represents the solution to a single consolidated public sector optimization problem. In practice, one could understand it as two agents. A Central Bank engages in sterilized foreign exchange intervention and monetary/inflation policy. A Sovereign finances public sector spending by borrowing in both foreign currency and local currency.

The Central Bank. The Central Bank engages in sterilized intervention and monetary policy. We assume the Central Bank does not have commitment to both policies. Both actions could influence the equilibrium exchange rate and therefore potentially the debt currency choice.

Similar to Engel and Park (2022), Ottonello and Perez (2019) and Du et al. (2020), the monetary policy decision is simplified by assuming that the Central Bank can choose the price level (P_t) in every period (subject to utility inflation cost $l(\frac{P_t}{P_{t-1}})$). As state in equilibrium exchange rate equation (15) and because prices are fully flexible, changing P_t has no consequences for the relative price $\frac{p_t^N}{p_t^T}$. The inflation setter can therefore pick a P_t which determines S_t and changes the real value of local currency debt.

Sterilized intervention means that all its intervention by the Central Bank in the FX market (F_t^{cb}) is accompanied by exactly offsetting measures in the domestic bond market (b_t^{cb}). Besides taking in private households domestic bonds, as described above, the financial sector can also hold domestic bonds issued by the Central Bank.

Because it is a sterilized FX policy, the balance sheets of the Central Bank must be satisfied, as follows:

$$b_t^{cb} + F_t^{cb} = 0 \tag{21}$$

We also assume that the Central Bank is subject to a non-negative reserves constraint:

$$F_t^{cb} \geq 0 \tag{22}$$

The net profits and losses from FX intervention is rebated to the households through TR_t^{CB}in equation (9):

$$TR_t^{CB} = R_t^W F_t^{cb} + Rb_t^{cb} \tag{23}$$

Notation. By bond market clearing we have:

$$b_t^{FC} + b_t^{fs} + b_t^{cb} = 0 \text{ , } F_t = F_t^{fs} + F_t^{cb} \tag{24}$$

Here, the first equality is the domestic bond market clearing condition, which says that bonds traded within the domestic market must sum to zero, where b_t^{fs} and b_t^{cb} respectively represent bonds issues by the financial intermediaries and the Central Bank. **The second equation defines the net foreign assets (excluding the assets of the fiscal department), which are the sum of the claims of the financial intermediaries F^{fs} and the Central Bank F^{cb} on the rest of the world.**[17]

[17] **We note that the net foreign asset definition here represents the net claims that the private sector holds indirectly through the asset holdings of the Central Bank and the financial intermediaries. It excludes the debt of the Sovereign, as described below.**

The conditions (11) and (19) become:

$$R_t = R_t^W + E_t \frac{\Gamma}{\beta}(\frac{-F_t + F_t^{cb}}{\Lambda_{t+1} R_t^W}) \tag{25}$$

and

$$u_{T,t} - \mu_t = \beta [R_t^W - E_t \frac{\Gamma}{\beta \Lambda_{t+1}} \frac{-F_t + F_t^{cb}}{R_t^W}][E_t u_{T,t+1}] \tag{26}$$

Equation (26) implies that Central Bank sterilized intervention will have non-neutral effects on the economy. When the central bank increases its reserve holdings, it must have an equivalent sales of domestic bond to the private sector because of the sterilized intervention assumption. The private sector in turn will attempt to maintain its total borrowing $-F_t$ by selling more bonds to the intermediaries. But this will tend to push up domestic interest rates above foreign interest rates, and as a result, leads to a fall in total private sector borrowing and consumption. The change in consumption could affect the exchange rate through the real exchange rate channel in equation (15) and therefore asset pricing equation (20).

The Sovereign. We refer to the fiscal department of the public sector as the Sovereign, and denote sovereign borrowing as the debt issued by the fiscal department. The key function of the Sovereign is to provide government spending to private households and borrowing internationally with a debt currency choice. This give rises to the core economics we are interested in, how the sovereign currency composition is determined with various inflation policy and foreign reserves management.

As in Engel and Park (2022), Ottonello and Perez (2019) and Du et al. (2020), the Sovereign must provide public goods to citizens that are produced by government spending on the internationally traded goods. The Sovereign desires to front-load government spending and borrow from international investors.[18] The Sovereign can choose in which currency to issue debt.

The Sovereign receives tax revenue from the private households which is proportional to the country's tradable endowment. We assume the sovereign receives a constant share

[18] We assume that the Sovereign does not borrow directly from private households. This allows us to separate the problem of currency denomination of Sovereign debt cleanly from that of optimal FX intervention. Relaxing this assumption would not materially affect the benefits of intervention for the Sovereign portfolio choice, but would make the analysis more complicated.

τ of the tradable endowment.[19] If we let the total tradable endowment be $(1+\tau)y_t^T$, we assume that the Sovereign receives τy_t^T per period, and and the private economy gets y_t^T. To simplify notation, denote $\tau y_t^T \equiv y_t^G$. The Sovereign's budget constraint is:

$$\text{Budget constraint in LC: } p_t^T G_t + \frac{S_t}{R_t^{FC}} B_t^{FC} + \frac{1}{R_t^{LC}} B_t^{LC} \leq p_t^T y_t^G + S_t B_{t-1}^{FC} + B_{t-1}^{LC}$$

where G_t is government spending, R_t^{FC} and R_t^{LC} are respectively the rate of return on foreign and local currency debt. Using $p_t^T = S_t$ and rewrite this equation in real terms by dividing p_t^T gives:

$$\text{Real budget constraint: } G_t + \frac{1}{R_t^{FC}} B_t^{FC} + \frac{1}{R_t^{LC} S_t} B_t^{LC} \leq y_t^G + B_{t-1}^{FC} + \frac{B_{t-1}^{LC}}{S_t} \tag{27}$$

Noting that taxes paid to the Sovereign are given by $T_t = \tau y_t^T$ and profit of the Central Bank are rebated as in equation (23), it follows that the households balance of payments condition may be written as

$$p_t^T c_t^T - S_t \frac{F_t}{R_t^W} \leq p_t^T y_t^T - S_t F_{t-1} \tag{28}$$

Equation (28) indicates that despite the presence of financial intermediaries, households effectively borrow at the world interest rate, since both intermediary and Central Bank profits are rebated to households in a lump-sum transfer. But despite that, the household first order condition is influenced by the sterilized intervention indicated by equation (26).

Public Sector Optimization.

While the activities of the Central Bank and the Sovereign are compartmentalized, we can describe optimal policy as the solution to a single optimization problem where the public sector planner simultaneously chooses FX intervention, inflation and sovereign borrowing to maximize household welfare.

The Central Bank part of the optimal policy choice involved managing reserves through sterilized intervention, buying reserves with bonds issued to the private sector, while also

[19] We assume a constant tax rate for simplicity but it is in fact the optimal tax rate for this class of utility (See Fernandez et al. (2020)). Tax revenue could be proportional to GDP without changing any of the qualitative results, so long as real GDP and the tradable endowment are positively correlated. Assuming revenue as proportional to the tradable endowment simplifies the analysis.

depleting reserves by retiring bonds. As discussed above, sterilized intervention has real effects in this model due to the frictions in financial markets associated with financial intermediaries. Since the Central Bank can buy or sell reserves on the world market without going through the financial intermediaries, its intervention policy can affect the domestic rate of return faced by households and thereby affecting the total external position of households. In addition, in the absence of inflation commtment, the Central Bank chooses the ex-post inflation rate. In doing so, the Central Bank faces the trade off between costly inflation and the temptation to devalue the local currency sovereign debt through exchange rate depreciation.

The Sovereign borrowing side of optimal policy simultaneously chooses government spending and domestic and foreign currency sovereign borrowing.

The choice variables for the consolidated public sector are foreign exchange intervention (F_t^{cb}) , government spending (G_t), the aggregate price index (P_t), debt in foreign currency (B_t^{FC}) and local currency (B_t^{LC}).

$$W = \max_{G_t, B_t^{FC}, B_t^{LC}, P_t, F_t^{cb}} E_0 \sum_{t=0}^{\infty} \beta^t \left[(1-\omega_g) u(c_t) + \omega_g [v(G_t) - l(\frac{P_t}{P_{t-1}})] \right] \tag{29}$$

subject to

public sector constraints:

$$\text{Sovereign budget constraint: } G_t + \frac{1}{R_t^{FC}} B_t^{FC} + \frac{1}{R_t^{LC} S_t} B_t^{LC} \leq y_t^G + B_{t-1}^{FC} + \frac{B_{t-1}^{LC}}{S_t} \tag{30}$$

$$\text{None zero reserves:} F_t^{cb} \geq 0 \tag{31}$$

and private sector constraints, which are household balance of payments and the 4 equations (15), (17), (19), (20) that characterize the private sector equilibrium:

$$\text{Household BOP: } p_t^T c_t^T - S_t \frac{F_t}{R_t^W} \leq p_t^T y_t^T - S_t F_{t-1} \tag{32}$$

$$\text{Private borrowing constraint: } \quad -\frac{F_t}{R_t^W} \leq \kappa (y_t^T + \frac{1-\alpha}{\alpha} (\frac{c_t^T}{y^N})^{\frac{1}{\xi}} y^N) \tag{33}$$

$$\text{Household Euler equation: } u_{T,t} - \mu_t = \beta [E_t u_{T,t+1} (R_t^W - \frac{\Gamma}{\beta} (F_t - F_t^{cb})] \tag{34}$$

$$\text{Investor pricing: } E_t[\Gamma^*_{t+1} R^W_t] \quad = \quad R^{LC}_t [E_t(\Gamma^*_{t+1}) E_t(\frac{S_t}{S_{t+1}}) + cov_t(\Gamma^*_{t+1}, \frac{S_t}{S_{t+1}})] \tag{35}$$

$$\text{Eqm price index: } S_t = \underbrace{P_t}_{\text{price index factor}} \times \underbrace{[\alpha + (1-\alpha)(\frac{1-\alpha}{\alpha}(\frac{c^T_t}{y^N})^{\frac{1}{\xi}})^{1-\xi}]^{-\frac{1}{1-\xi}}}_{\text{real exchange rate factor}} \tag{36}$$

The detailed Ramsey problem, mechanics and intuition of foreign exchange intervention are described more fully in Appendix B. The critical feature of the model is that the collateral constraint depends on the real exchange rate, which in turn depends on the consumption of traded goods and total private sector borrowing. Since the presence of the real exchange rate in the collateral constraint represents a pecuniary externality, private agents do not explicitly take account of the effect of their borrowing on the real exchange rate and the probability of a sudden stop. The key difference between Central Bank optimization and the household optimization hinges on the Central Bank internalizing the pecuniary externality to respond via reserves management (F^{cb}_t). By changing the reserves level, the Central Bank alters the interest rate the households are facing (equation (34)). It therefore change the consumption path of households and the real exchange rate (via equation (32) and (36)). This in turn has an effect on the tightness of the household borrowing constraint (equation (33)). The financial friction associated with financial intermediaries guarantees that foreign exchange intervention can be effective, even when the borrowing constraint is slack.

Two forces drive the nominal exchange rate in this model, a nominal price index factor that depends on monetary policy and a real exchange rate factor that is related to the relative price of tradable and non-tradable goods. Without government intervention, a lower tradable endowment, or a higher R^W_t which results in endogenous deleveraging of the households, lowers c^T_t. Therefore, the real exchange rate endogenously depreciates in bad times for the local economy, which give rise to the hedging benefit of local currency debt. The real exchange rate also endogenously depreciates when R^W_t is high, which is the bad times for global investors. Foreign reserves management can stabilize the second factor, while inflation policy can influence the nominal price index. In both cases, a reduction in the sensitivity of the nominal exchange rate to global shocks can reduce the currency risk premium imposed by international investors, thereby fostering the issue of local currency debt on the part of the Sovereign.

To illustrate the mechanics of foreign exchange intervention, the appendix derives the following condition, which illustrates a case of an optimal FX policy where the current borrowing constraint is slack, but the expected future borrowing constraint may bind.[20]

$$\beta E_t u'(c^T_{t+1})(R_t - R^W_t) = \beta R^W_t E_t \left(\frac{\mu_{t+1}}{\xi} \kappa \left(\frac{1-\alpha}{\alpha} \right)^{\frac{1}{\xi}} \left(\frac{c^T_{t+1}}{y^N} \right)^{\left(\frac{1-\xi}{\xi}\right)} y^N \right)$$

In this expression, the term μ_t is the multiplier on the collateral constraint. The left hand side of this expression represents the cost of foreign exchange rate accumulation. When $R_t - R^W_t > 0$ households are borrowing at a higher rate than the world interest rate, which is costly given that $\beta R^w_t < 1$. In the absence of a borrowing constraint, and without any restriction on foreign reserves, FX intervention would ensure $R_t - R^W_t = 0$. But when the collateral constraint is expected to bind in the future, it is optimal to ensure that $R_t - R^W_t > 0$ so the cost FXI is balanced by the expected benefit of losening the collateral constraint. This involves the Central Bank accumulating reserves in times where the constraint doesn't bind, and deploying the reserves in states where the world interest rate spikes and the collateral constraint binds.

The first order condition for the price index (inflation) is:

$$\text{FOC } P_t : \underbrace{l'(\frac{P_t}{P_{t-1}})\frac{1}{P_{t-1}}}_{\text{MC of inflation}} = v'(G_t)\frac{\partial (p^T_t)^{-1}}{\partial P_t} B^{LC}_{t-1} = \underbrace{-v'(G_t)\frac{B^{LC}_{t-1}}{p^T_t}\frac{1}{P_t}}_{\text{MB of inflation}} \tag{37}$$

This is the inflating away the debt incentive. Since $B^{LC}_{t-1} < 0$ for borrowing, the marginal benefit of inflation is positive when there is some local currency borrowing. The marginal benefit of inflation is also higher when the marginal utility of G_t is high. In the extreme case where the planner has no disutility of inflation at all, the planner would always inflate away any existing debt with an arbitrarily high inflation rate. i.e. arbitrary high P_t, p^T_t and S_t.

As regards the optimal policy relating to the Sovereign, it is instructive to describe

[20] To simplify the discussion, we assume the non-negativity constraint on reserves in the future is non-binding here. The full expression including the non-negativity constraint is shown in the Appendix.

in more detail the optimality conditions with respect to government spending (G_t) and Sovereign borrowing (B_t^{FC}, B_t^{LC}). The first order conditions with respect to government spending and foreign currency debt are:

$$\text{FOC } G_t: \ \lambda_t = v'(G_t) = \frac{1}{G_t^{\sigma}} \text{ and FOC } B_t^{FC}\text{: } \underbrace{v'(G_t)}_{MBFC} = \underbrace{\beta R_t^W [E_t v'(G_{t+1})]}_{MCFC} \tag{38}$$

where λ_t represents the Lagrange multiplier on the Sovereign's budget constraint, and the last equality follows from a CRRA utility we assume in the quantitative section, *MBFC* and *MCFC* stand for marginal benefit and cost of FC debt for the Sovereign.

The choice of local currency Sovereign debt is determined by the first order condition:

$$\begin{aligned} \text{FOC } B_t^{LC}\text{: } \underbrace{v_{T,t}(1+\frac{\partial 1/R_t^{LC}}{\partial B_t^{LC}})}_{MBLC} &= \underbrace{\beta R_t^{LC}[E_t v'(G_{t+1})\frac{p_t^T}{p_{t+1}^T}]}_{MCLC} \\ &= \beta R_t^{LC}[E_t(v'(G_{t+1}))E_t(\tfrac{S_t}{S_{t+1}}) + cov_t(v'(G_{t+1}), \tfrac{S_t}{S_{t+1}})] \end{aligned} \tag{39}$$

where *MBLC* and *MCLC* represent the marginal benefit and cost of LC debt. This condition illustrates that the Sovereign has a hedging benefit of issuing debt in local currency due to the possibility that the nominal exchange rate depreciates ($\frac{S_t}{S_{t+1}} \downarrow$) in bad times for the Sovereign - i.e. when the marginal utility of government spending ($v'(G_{t+1})$) is high. The term $\frac{\partial 1/R_t^{LC}}{\partial B_t^{LC}}$ captures the impact of Sovereign local currency borrowing on the interest rate the sovereign must pay on local currency debt, and this depends on the behavior of international investors in equation (20).

Local currency premium. We plug in (20) into the Sovereign's first order condition for local currency debt (39), dividing the whole equation by $E_t(\Gamma_{t+1}^*)E_t(\frac{S_t}{S_{t+1}})$, to get:

$$\underbrace{u_{T,t}}_{MBFC}\underbrace{[1+\frac{cov_t(\Gamma_{t+1}^*, \frac{S_t}{S_{t+1}})}{E_t(\Gamma_{t+1}^*)E_t(\frac{S_t}{S_{t+1}})}](1+\frac{\partial 1/R_t^{LC}}{\partial B_t^{LC}})}_{\text{local currency risk premium}} = \underbrace{\beta R_t^W [E_t v'(G_{t+1})]}_{MCFC} + \underbrace{\beta R_t^W \frac{cov_t(v'(G_{t+1}), (\frac{S_t}{S_{t+1}}))}{E_t(\frac{S_t}{S_{t+1}})})}_{\text{hedging benefit}} \tag{40}$$

Compared to the first order condition of foreign currency debt, in additional to the hedging benefit, local currency debt is more expensive because of the local currency risk premium. The optimal portfolio choice between foreign currency debt and local currency debt boils down to how much the hedging benefit of local currency debt is worth relative to the higher interest cost compared to the foreign currency debt. But both the hedging benefit and the local currency premium are critically dependent on the stochastic process for the nominal exchange rate. These depend both the inflation policy and also the process for the real exchange rate. But as described above, the real exchange rate is affected by the foreign reserves management decision of the Central Bank. The means that the optimal FX decisions of the Central Bank impact on the costs and benefits of local currency borrowing by the Sovereign.

Competitive equilibrium definition. The competitive equilibrium consist of three blocks. The private block, the sovereign block and the central bank block.

Given the exogenous states (y^T and R^W), reserves policy $F^{cb}(B^{FC}, B^{LC}, b^{FC}, y^T, R^W)$ and the pricing function $\frac{p^N}{p^T}(B^{FC}, B^{LC}, b^{FC}, y^T, R^W)$, the financial intermediary operates according to equation (25) and gives $R(B^{FC}, B^{LC}, b^{FC}, y^T, R^W)$. The household optimization consists of consumption policy function $c^T(B^{FC}, B^{LC}, b^{FC}, y^T, R^W), c^N(B^{FC}, B^{LC}, b^{FC}, y^T, R^W)$ and debt policy $b^{FC'}(B^{FC}, B^{LC}, b^{FC}, y^T, R^W)$ such that it solves for (8) with a value function $V(B^{FC}, B^{LC}, b^{FC}, y^T, R^W)$, subjects to the household budget constraint (9) and credit constraint (10). Bond markets clear as in (24) and non-tradable goods market clear such that $c^N = y^N$.

Given the private and reserves policy F^{cb} and the price schedule $R^{LC}(B^{FC'}, B^{LC'}, b^{FC}, y^T, R^W)$ from investor's no arbitrage condition (20), the public sector chooses $F^{cb}(B^{FC}, B^{LC}, b^{FC}, y^T, R^W)$, $P(B^{FC}, B^{LC}, b^{FC}, y^T, R^W)$, $G(B^{FC}, B^{LC}, b^{FC}, y^T, R^W)$, $B^{FC'}(B^{FC}, B^{LC}, b^{FC}, y^T, R^W)$ and $B^{LC'}(B^{FC}, B^{LC}, b^{FC}, y^T, R^W)$ such that it maximizes (29) with a value function $W(B^{FC}, B^{LC}, b^{FC}, y^T, R^W)$. $P(B^{FC}, B^{LC}, b^{FC}, y^T, R^W)$ and $\frac{p^N}{p^T}(B^{FC}, B^{LC}, b^{FC}, y^T, R^W)$ deliver S (or p^T) according to (15).

An analytical example for the sovereign debt currency trade-off. Before going to the quantitative section with full global solution, we provide some intuition with the help of analytical approximation as in Devereux and Sutherland (2011) to understand the optimal local currency bond portfolio from the sovereign point of view, taking foreign reserves management as given. To keep things simple, we focus here only on the case with full inflation targeting and refer the readers to the Appendix for the derivation. The optimal local currency portfolio can be expressed as followed:

$$\frac{\bar{D}^L}{\beta\bar{G}} = \frac{1}{Var(\hat{S})}\left(\underbrace{\underbrace{-y_g Cov(\hat{Y}^g,\hat{S})}_{\text{hedging domestic shock}} + \underbrace{\beta(\frac{\bar{D}}{\beta}+\frac{1}{\sigma})Cov(\hat{R}^W,\hat{S})}_{\text{hedging global shock}}}_{\text{hedging benefit}} - \underbrace{\frac{\gamma}{1-\beta\vartheta}Cov(\hat{R}^W,\hat{S})}_{\text{local currency risk premium}}\right) \tag{41}$$

where $\bar{D}^L$, $\bar{D}$ and $\bar{G}$ are the steady-state LC debt, total debt and government spending, $\hat{S}$, $\hat{Y}^g$ and $\hat{R}^W$ are the log deviation from the steady state of the nominal exchange rate, government tax income and the world interest rate, y_g is the steady state tax income to government spending ratio.

Expression (41) captures the intuition discussed in the more general model above. The sovereign would like to have a positive local currency debt in order to hedge both income (tax revenue) risk and world interest rate risk, so long as the covariance between revenue and the nominal exchange rate is negative, and so long as the nominal exchange rate depreciates in response to positive world interest rate shocks, given that the sovereign is a debtor, and interest rate shocks tend to depress current fiscal spending. On the other hand, the final term in (41) indicates that a positive covariance between world interest rate shocks and the nominal exchange rate tends to increase the cost of local currency debt, given risk averse lenders, and ceteris paribus, would reduce the sovereign's optimal local currency portfolio.

This expression takes the process for the nominal exchange rate as given. But given condition (15) above, we know that nominal exchange rate process is driven partly by foreign exchange reserve management and the real exchange rate. Foreign reserve intervention can reduce the covariance between the world interest rate and the nominal exchange rate.[21] In principle, this could reduce or increases the sovereign's issue of local currency debt since the lower direct hedging benefit of a reduced covariance between world interest rates and the nominal exchange rate goes against the fall in costs of issuing local currency debt when this covariance falls. In the quantitative analysis below we find that the second channel clearly dominates, and foreign reserves management unambiguously leads to an

[21]The endogenous stochastic process for the nominal exchange rate implied by foreign reserve management cannot be described by first order approximation, since it is discontinuous at the point where the collateral constraint binds. This is described in Appendix A. Despite this, optimal foreign reserve management always reduces $Cov(\hat{R}_t^W,\hat{S}_t)$.

increase in local currency debt issue.[22]

4 Calibration and quantitative evaluation

Calibration strategy. We calibrate our model to Brazil, a typical and widely studied emerging economy. The model frequency is annual. We separate parameters into three blocks. The first block contains parameters that are standard and are directly taken from the literature. The second block is estimated from the data and the last block is calibrated to match data moments.

Symbols	Description	Values	Notes
Parameters from the literature			
σ	CRRA coefficient	5	Standard literature value
ξ	Elasticity of substitution between T and NT goods	0.45	Schmitt-Grohe and Uribe (2020),Akinci (2017)
α	weight on tradable goods	0.3	Standard literature value
w_g	weight on government spending	0.02	Bianchi et al. (2019)
Parameters from the dataρ			
ρ^{r^W}	Persistence of r^W	0.4	Estimated AR(1) of 1yr Brazilian CDS
σ^{r^W}	sd of innovation of r^W	0.038	Same
Er^W	Mean of r^W	0.02	Same
ρ^y	Persistence of y^T	0.65	Estimated AR(1) on Brazil GDP[23]
σ^y	sd of innovation of y^T	0.03	Same
π^{target}	Target inflation rate	4.5%	Central Bank of Brazil target
Parameters from moment matching			Jointly matching
β	Discount factor	0.9	External private debt to GDP (17.5%),
Γ	Financial friction parameter	0.056	regression β in table 3,
κ	Credit constraint parameter	0.203	Crisis probability (7%),
γ	Investor SDF $m = \exp^{-r+\gamma\varepsilon_\tau-0.5\gamma^2\sigma_\tau^2}$	$\gamma = 44.055$	Foreign exchange premium (6.9%),
k	Sovereign inflation disutility in of quadratic form cost $\delta(\pi-\pi^{target})^2$	δ=8170	Inflation rate of 6.3%,
τ	Share of endowment to the government	0.053	and Total government debt to GDP (9.8%)

[22]This example can be extended to allow for lack of inflation commitment on the part of the Sovereign. In that case, we must add (37) to (67) and (68) in the approximation. The approximation becomes more cumbersome however, due to the fact that the first order approximation of (37) involves a first order portfolio term $\hat{D}^L_{t-1}$, and requires a higher order (3rd order) approximation of the optimal portfolio equations, as in Devereux and Sutherland (2010).

[23]The GDP process is quadratic detrended.

For the first block, we take the CRRA coefficient (σ) of 5 for both households and the government. We take the elasticity of substitution between tradable and non-tradable goods (ξ) of 0.45, very close to estimate from Schmitt-Grohe and Uribe (2020), and Akinci (2017).[24] As discussed at length in their paper, this is the empirically relevant value.We take the weight on tradable goods (α) of 0.30, which is commonly used in the literature. The weight on government utility function is set at 0.02 from Bianchi et al. (2019).[25]

The second block is comprised of parameters directly estimated from the Brazilian data. We use the one year CDS rate to measure the r^W ($r^W \equiv R^W - 1$). To get the exogenous component of the country's funding cost, we first run a regression of $CDS_t^{1y} = \alpha_0 + \beta_0 VIX_t + \varepsilon_t$. We use the fitted value from this regression $C\hat{D}S_t^{1y}$ as r^W. We then estimate the AR(1) coefficient of r^W. The estimated persistent coefficient is 0.4 and the standard deviation of the innovation is 0.038. The mean value of r^W is 0.02. For the endowment process, we estimate the AR(1) process after a quadratic detrending. The estimated persistent coefficient is 0.65 and the standard deviation of innovation is 0.03.

For the last block, we use a set of parameters to match the data moments in the Brazilian data. Although the parameters are calibrated jointly, we can give a heuristic description of how the data moments inform specific parameters. The discount factor (β) and the parameter on the collateral constraint (κ) are useful to match the total private debt to GDP and implied crisis probability of 5-8% in the literature. The parameter on the financial friction (Γ) is useful for matching reserves sensitive to exchange rate movements (β_3 in table 3). The calibrated value is $\beta = 0.9$, $\kappa = 0.203$ and $\Gamma = 0.056$. We then parameterize the investor stochastic discount factor as $\Gamma_{t+1}^* = e^{(-r_t^W + \gamma \varepsilon_{t+1}^r - 0.5\gamma^2 \sigma_r^2)}$, where ε_{t+1}^r is the innovation of r^W and σ_r^2 is the unconditional variance of r_t^W. We calibrate the param-

[24]Calibrating the ξ to this value has two important implications. First, expressing c_t in collateral constraint equation(33) using household balance of payment equation(28) shows that both LHS and RHS of the collateral constraint depends on F_t and R_t^W. A low ξ ensures the LHS of the constraint (amount of debt) falls less than the RHS value (collateral value) when R_t^W rises. This results in an empirically realistic situation of a tighter constraint when world interest rates go up. Second, as shown by Schmitt-Grohe and Uribe (2020), if there is sufficient complementarity in consumption between traded and non-traded goods, a fall in private sector borrowing may give rise to multiple equilibrium and a self-fulfilling deleveraging driven sudden stop. The self-fulfilling sudden stop is not necessary for our analysis. Therefore, we follow the analysis of Davis et al. (2023) by assuming that the good equilibrium is selected when it exists. Allowing for multiple equilibrium will enhance the importance and the need of having FXI.

[25]Technically, we can calibrate this utility weight from the data. However, calibrating this value is computationally challenging given the rich structure of our model. We take this input from the literature and postpone a full calibration for future work.

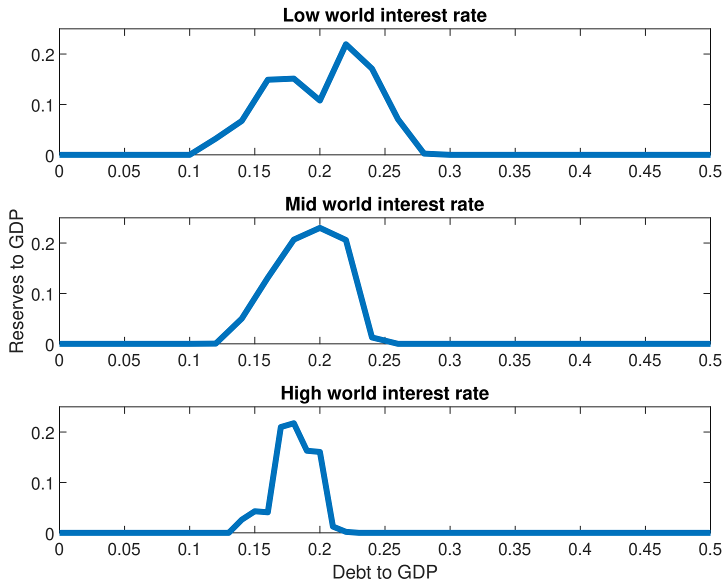
eter γ to 44.055 to match the average forward exchange premium of 6.9% in the data. We parameterize the inflation disutility cost using a quadratic cost function: $\delta(\pi_t - \pi^{target})^2$ where $\pi_t \equiv \frac{P_t}{P_{t-1}}$ and π^{target} is the inflation target by the Central Bank of Brazil (4.5%). The parameter δ is set to 8170 to match the realized inflation in Brazil (6.3%).

Policy functions. To understand the mechanism of foreign reserves management (FXI), Figure 5 shows the optimal reserves to GDP (y-axis) as a function of existing private debt to GDP (x-axis). We set the existing private debt at an arbitrary level from low to high and plot the optimal reserves to GDP accordingly. The next period debt is chosen optimally. The three panels correspond to the case when the world interest rate is at the second to the lowest, median and second to the highest interest rate level. Each of the figures demonstrates four phases of reserves management. First, when the private debt to GDP is low, there is no need to accumulate reserves. This is because the economy is very far from a sudden stop crisis. Second, as the private debt to GDP rises, there is an accumulation of reserves. This is because the economy enters a region where there is a positive probability that the collateral constraint is binding in the next period. The economy as a whole is overborrowing due to the pecuniary externality that households do not internalize. Therefore, the Central Bank accumulates reserves to save and to correct for the pecuniary externality. Third, when the debt to GDP is sufficiently high, the constraint binds and the households are underborrowing in the sense of Schmitt-Grohe and Uribe (2020). The central bank finds it optimal to decumulate reserves to support private consumption. In this sense, the severity of the crisis is dampened. Finally, as private debt to GDP goes even higher, the central bank hits the non-negative reserves constraint. The reserves to GDP goes to zero.

This figure illustrates the lean against the global wind property of the optimal reserves policy. As the interest rate goes up (from the upper to the lower panel), the central bank decumulates reserves earlier as the economy hits the binding region quicker. The peak of the reserves to GDP ratio is located at more towards to left-hand-side of the figure at higher realizations of the interest rate. The central bank also sells down its reserves more aggressively in this case, as shown by the fact that it reaches the zero reserves region earlier at the right-hand side of the figure.

Figure 6 shows the equilibrium price schedule offered by for the local currency debt price as a function of local currency debt derived from the simulated model. As local currency debt increases, risk averse investors increase the interest rate required on debt, so the price sinks. The red and blue lines illustrate the debt price in the absence of FXI

Figure 5: Policy function of optimal foreign reserves management



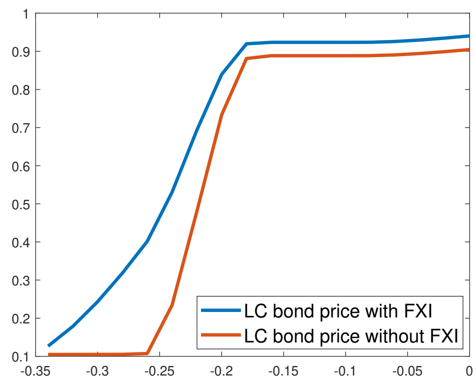
Notes: Y-axis refers to reserves to GDP ratio and x-axis is the existing private debt (b^{FC}_{t-1}) to GDP ratio. Existing private debt is set at an arbitrary level from low to high. The next period debt is chosen optimally. The three cases correspond to second to the lowest, median and second to the highest interest rate.

and in the case of optimal FXI respectively. FXI leads to an increase in the price at which the sovereign can issue local currency debt across the full range of debt issue.

Simulation of the dynamics of the exchange rate and foreign exchange reserves. Figure 7 below illustrates the workings of the part of the model that governs the dynamics of foreign reserves and the exchange rate. For a better illustration, this figure is generated using the parameterization discussed above, we allow the world interest rate to be simulated according to the stochastic process but we hold the ex-post realized path of endowment of traded goods always at one grid below the mean. So we can focus on the response to global shocks. The top panel of the figure shows the behavior of the nominal exchange rate with and without optimal foreign exchange intervention. The bottom panel of the figure shows an exogenous stochastic process for the world interest rate and the optimal foreign exchange reserve policy followed by the Central Bank.

It is clear from the figure that the optimal foreign exchange management policy dampens the covariance of the nominal exchange rate with global shocks. During normal times (in the middle of the time series), the exchange rate with FXI is less volatile (blue solid line) and is always further from the mean than the one without FXI (dashed blue line).

Figure 6: Price schedule of local currency debt



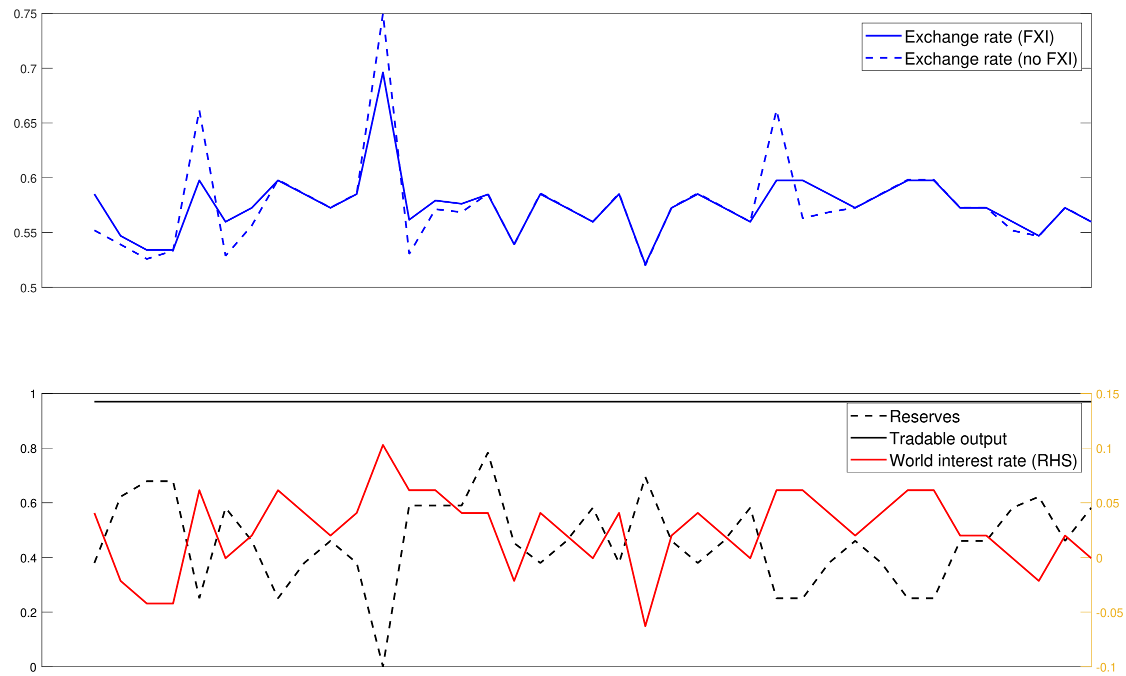
Notes: the X axis is the local currency sovereign debt (B^{LC}) and the Y axis is the corresponding local currency bond price. The endowment level is held at one grid point below the mean and all other choice variables are at the mean level. The blue line refers to the case with foreign exchange intervention (FXI) and the red line refers to the case without FXI.

There are three crises in the simulated path, which are all caused by a spike in the foreign interest rate (note that the endowment is assumed to be constant in this simulation). At the left and right of the time series, a sudden stop crisis is prevented by FXI, leading to a substantially less depreciated exchange rate. At the left one, with an FXI that sells down half of the reserves, the exchange rate depreciates from 0.54 by 7% to 0.58 rather than by 22% to 0.67 in the case without reserves management.[26] In the middle of the figure a sudden stop crisis still occurs regardless of the FXI. But even so, the case with FXI has a less depreciated exchange rate due to a decumulation of reserves (the bottom panel). In this manner, the FXI policy makes local currency debt safer from the perspective of international investors. Overall, the bottom panel of the figure shows that foreign exchange reserves are strongly negatively correlated with world interest rates - reserves increase in times of low interest rates and are then depleted in face of interest rate spikes. Ex-ante, investors understand that FXI leads to a more stable (less depreciated) exchange rate when global shocks hit (when the risk premium and marginal utility of the investor is high). The investor pricing therefore results in the higher price schedule as shown in Figure 6.

Quantitative results. Table 5 shows the results of the simulated model moments. The model targeted moments are all close to those of Brazilian data. The spread between

[26] Due to the non-linearity of the model, there is no simple formula for the reserves elasticity of exchange rate. In the Appendix, we provide a semi-closed form formula to hightlight the relevant forces.

Figure 7: Simulations for reserves and exchange rates under optimal FXI



Notes: For a better illustration, the model is solved under the assumed stochastic process but the realized simulation path of the tradable endowment is held at one grid below the mean. All the choice variables are chosen optimally according to the policy functions.

local and foreign currency rate for Brazil is measured at 6.9%. The mean spread of the simulations with optimal FXI (column (2)) matches this almost exactly at 7.0%. Without FXI but with endogenous inflation policy (column (3)), the risk premium is 10%. The inflation rate is calibrated at the historical inflation rate of 6.3 percent over the sample. Government debt is estimated at roughly 10 percent of GDP.

The untargeted moments in the simulated model concern the level of FX reserves and the ratio of debt in local currency. In the model with optimal FX policy (column (2)), the mean reserves to GDP ratio is 13.4 percent, very close to the level of 14 percent observed in the data sample. The baseline model matches well the mean share of government debt in local currency - 50 percent in the data, 53 percent in the model with FXI and endogenous monetary policy. We note that in the absence of endogenous FX intervention, but with endogenous monetary policy local currency debt represents a smaller fraction of the total at 39.8%. This accords well with the empirical evidence presented in section 2 above where we showed a robust association between FX reserves and local currency debt. It is noteworthy that despite a higher share of local currency debt, the risk premium on local currency sovereign debt is 300 basis points lower in the case of optimal FXI compared to the case of no FXI. This is because the exchange rate risk premium is much lower in the case of FXI than no FXI (0.7% vs 3.9%).[27] Intuitively, FXI leads to a more stable exchange rate response to global shocks, as described in the theoretical section and Figure 7. This is also reflected in the fact that the crisis probability with optimal FXI is much lower (3.3 percent, compared to 7.5 percent without FXI).

In column (4), we report the model moments for the case when we assume the inflation cost tends to infinity ($\delta \to \infty$), representing a full inflation targeting commitment, but assuming no FXI policy. Given the infinite inflation cost, the inflation rate is choosen at 4.5%, which is the central bank target. The infinite inflation cost case is associated with a LC share of 40.7%, slightly higher than the case in column (3). Comparing column (4) to FXI case in column (2), we can see that additional commitment cannot achieve the same LC share as FXI. This is because full inflation commitment cannot act countercyclically and therefore is not able to lower the risk premium on local currency debt. Further inflation commitment only lowers the exchange rate risk premium from 3.9% to 3.8%.

By contrast, column (5) reports the extreme case when there is no inflation cost at all ($\delta = 0$), but the CB follows an optimal FXI policy. The LC interest rate is infinite and the LC share is always zero. This accords with our empirical evidence that foreign reserves is only correlated with LC share when the country is an inflation targeting country.

Model simulated regressions. We conduct a simulation analysis with the model that can be compared to the results of Tables 1-4 in the empirical section. In Tables 6-8, using the calibration for Brazil, we simulate 14 separate economies, and then also simulate 10 economies with the same calibration except without a very low inflation cost (inflation disutility $\delta = 20$).[28] This mirrors the empirical results based on 14 inflation targeting countries and 10 countries without inflation targeting. We run each simulation for 520 periods, and drop the first 500 periods, giving us 20 years of data similar to the sample

[27] The simulated policy functions in Figure 6 confirm that the local currency spread is reduced by FXI for the same states of FC and LC holdings along each point of the interest rate grid.

[28] The average LC share for this low inflation cost calibration is 3%. We do not mimic the non-inflation targeter using $\delta = 0$ because that would imply a LC share of zero all the time and there will be no correlation with any variables by construction.

Table 5: Moments from the model simulation

Variable	Brazil Data	Model with FXI	Model with no FXI	Model with full IT and no FXI	Model with no IT cost but with FXI
	(1)	(2)	(3)	(4)	(5)
Targeted					
Private Debt to GDP	17.5%	17.0%	17.4%	17.4%	17.0%
Crisis Probability	-	3.3%	7.5%	7.5%	0.33%
regression β_3 in table 3 of $\Delta reserves$ on $\Delta\hat{s}_{i,t}$	-0.34 for Brazil , -0.24 for Panel	-0.30	NA	NA	NA
Government LC spread $r_{LC} - r_{FC}$	6.9%	7.0%	10%	8.3%	NA (LC rate = ∞)
Realized Inflation Rate	6.3%	6.3%	6.1%	4.5%	NA
Total Gov Debt to GDP	5+4.8%=9.8% (FC+LC)	9.9%	10.1%	10.3%	7.23%
Untargeted					
Reserves to GDP	14%	13.4%	0%	0%	13.4%
Expected exchange rate change		6.3%	6.1%	4.5%	NA
Exchange rate risk premium		0.7%	3.9%	3.8%	NA
FC gov. debt to GDP	5%	5.1%	5.8%	5.9%	7.23%
LC gov. debt to GDP	4.9%	5.7%	4.1%	4.2%	0%
LC gov debt share	50%	53%	39.8%	40.7%	0%

Notes: The model is simulated for 11,000 periods and dropping the first 1,000 periods. Mean values are reported. IT standards for inflation taregeting. Column (2) is the full model calibrated with foreign exchange intervention (FXI). Column (3) is the calibrated model but shutting down FXI channel. Column (4) is the calibrated model but setting inflation cost to infinity and no FXI channel. Column (5) is the calibrated model but setting inflation cost to zero.

size of our empirical results above.[29] In the model, there is no time variation of inflation targeting, so we run the regressions separately for inflation targeters and non-targeters to prevent collinearity of the IT dummy with country fixed effects.

Table 6 corresponds to Table 1 in the empirical section. We regress the model implied LC share on the reserves to GDP and other controls. For the simulated 'inflation targeting' sample, the coefficient on reserves to GDP is significant at the 1 percent level, and very close in size to that of Table 1. On the other hand, for the non-inflation targeters, the reserves to GDP has no significant effect on the LC ratio, similar to Table 1.

Table 7 reports the results of the simulated regression corresponding to Table 3 of the empirical section. We take the global shock in the model as a proxy for VIX in the data to construct the fitted values of the model-simulated equivalent to (4), and regress this on

[29]While many of our empirical regressions above are at quarterly frequency, the results are unchanged if we switch to annual frequency.

Table 6: Model simulated regression of local currency debt ratio and foreign reserves

$$LCratio_{i,t} = \alpha_i + ln\frac{reserves}{GDP}_{i,t} + controls_{i,t} + \varepsilon_{i,t}$$

	Inflation targeter	Non-inflation targeter
LHS:	$LCratio_{i,t}$	$LCratio_{i,t}$
	(1)	(2)
$ln\frac{reserves}{GDP}$	**0.120*** **	-0.005
	(0.020)	(0.016)
r^W	0.554	0.471*
	(0.566)	(0.243)
GDP growth	0.793***	0.009
	(0.129)	(0.032)
N	280	200

Notes: Discroll Kraay (1998) standard errors with 5 lags in parentheses. * p<0.1, ** p<0.05, *** p<0.01. Simulated sample of inflation targeter is based on simulations of the calibrated model 14 times. Simulated sample of non inflation target is based on simulations of the calibrated model 10 times. Each time we simulate 520 periods and dropping the first 500 periods.

the change in reserves to GDP from the simulated model. We see that the exchange rate response to the global interest rate shock in the model (the fitted value) is significant for the IT sample, but not for the non-IT sample. Just as in Table 3, the residual response of the exchange rate has no significant effect in either case. Finally Table 8 reports the model simulations for the local currency risk premium, and the relationship between the risk premium and FX reserves, along with other controls. For the IT sample, there is again a significant negative association between FX reserves and the risk premium, and the coefficient estimates are also very close to Table 4 at around 1.5%. The regression coefficient is insignificant for the non-IT sample.

Figure 8 shows a scatter plot of the level of FX reserves to GDP (on the y-axis) and the beta coefficient of the exchange rate regressed on the world interest rate (global factor in the model, on the x-axis) from the model simulations. For the IT countries, the scatter plot illustrates a negative relationship between the beta to global factor and the FX reserves to GDP. By contrast, the relationship is slightly positive for the non-IT simulations. Comparing this figure to Figure 3, the simulated model accurately represents the impact of FX reserves on the sensitivity of the exchange rate to global shocks.

In summary, these results from the model-based simulations provide strong quantita-

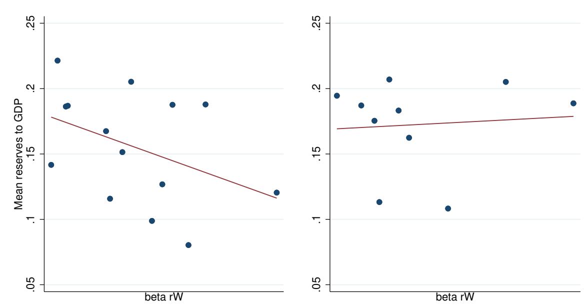
Table 7: Model simulated foreign reserves and global factor of exchange rate regressions

$$\Delta ln(\frac{reserves}{GDP})_{i,t} = \alpha_i + \beta_1 \text{global exchange rate change}_{i,t} + \beta_2 \text{local exchange rate change}_{i,t} + \varepsilon_{i,t}$$

	Inflation targeter	Non-inflation targeter
LHS	$\Delta ln(\frac{reserves}{GDP})_{i,t}$	$\Delta ln(\frac{reserves}{GDP})_{i,t}$
	(1)	(2)
global exchange rate change	**-0.077***	0.002
	(0.041)	(0.002)
local exchange rate change	0.057	0.002
	(0.091)	(0.001)
N	280	200

tive support for the dual importance of inflation targeting and the holding and deployment of foreign exchange rate reserves in fostering the growth of local currency sovereign debt for emerging market economies.

Figure 8: Scatter plot of model simulated reserves to GDP and exchange rate sensitivity to global shock



Notes: The Y axis shows mean reserves to GDP and the X-axis is the regression beta of a country's exchange rate on r^W. The simulated sample of inflation targeters is based on simulations of the calibrated model 14 times. The simulated sample of non inflation targeters is based on simulations of the calibrated model 10 times. For each time we simulate 520 periods and drop the first 500 periods.

Welfare analysis. How beneficial is FXI in avoiding welfare-reducing sudden stops and fostering local currency borrowing for the sovereign? In this final section, we report estimates of the welfare benefits of FXI both for households and for the sovereign. We compute the welfare estimates in the following way. Both for the FXI and no FXI case,

Table 8: Model simulated sovereign spreads and foreign reserves regressions

$$(r_{i,t}^{LC} - r_{i,t}^{W}) = \alpha_i + ln\frac{reserves}{GDP}_{i,t} + controls_{i,t} + \varepsilon_{i,t}$$

	Inflation targeter	Non-inflation targeter
LHS:	LC spreads (%)	LC spreads (%)
	(1)	(2)
$ln\frac{reserves}{GDP}$	**-1.5*** **	-50.7
	(0.3)	(150.1)
r^W	-28.8**	4126.8
	(11.0)	(2476.7)
$\frac{\text{public debt}}{GDP}$	38.4	3757.4
	(53.7)	(4203.4)
$\frac{\text{private debt}}{GDP}$	-22.7	-3278.0
	(21.8)	(4673.8)
GDP growth	-0.000	66.1
	(2.8)	(200.8)
N	280	200

Notes: Discroll Kraay (1998) standard errors with 5 lags in parentheses* p 0.1, ** p 0.05, *** p 0.01Simulated sample of inflation targeter is based on simulations of the calibratedmodel 14 times. Simulated sample of non inflation target is based on simulationsof the calibrated model 10 times. Each time we simulate 520 periods anddropping the first 500 periods.

we take the simulated data of column (2) and column (3) of Table 5. That is, we simulate the model for 11,000 periods, drop the first 1,000 periods, and record the values for consumption and government spending. We use the discounted sum of the utility of household consumption and sovereign government spending for the whole path, and then compute a constant c and constant G that gives the same utility as these sum of utilities.

Table 9 reports the welfare results in terms of consumption equivalent (for both household and government consumption). First, for households, the welfare gain of FXI is consumption is 0.25% of consumption equivalent. This is quite consistent with Bianchi (2011)'s estimated number for the benefits of prudential capital taxes, where he finds a small welfare gain (0.135%).

The welfare gain for the sovereign is much larger. It is 6.8% of government spending equivalent. In contrast to the welfare gains for households, the sovereign benefits in two ways from optimal FXI. First, the sovereign has smoother government spending, since it is issuing local currency debt which is a hedge against endowment and global shocks. But it benefits also from being able to borrow at a lower interest rate. It is the combination of these two effects which make the FXI policy so valuable for the sovereign. Table 9 provides further insight into the welfare benefits of FXI by showing two intermediate cases. Intermediate case FXI1 computes the outcome where agents decisions are guided by optimal FXI but the sovereign borrows at the LC interest rate that would pertain without FXI. This captures the benefits from using LC debt, but not the benefits of a lower borrowing cost (i.e. only hedging benefit). We see that the welfare gain in this case is 3 percent. Intermediate case FXI2 shows the polar opposite case, where there is no FXI, but governments face the lower interest cost of borrowing that would be implied by optimal FXI (i.e. only price effect). In this case, the welfare gain is 2.6 percent. In either case, Table 9 indicates that, given our calibrated model, the benefits of FXI and local currency borrowing for the sovereign can be substantial.

Table 9: Welfare analysis

	FXI	Intermediate FXI1 (only hedging but no price effect)	Intermediate FXI2 (only price but no hedging effect)	No FXI
Consumption equivalent	0.9872			0.9847
Welfare gain relative to no FXI	0.25%			
Government spending equivalent	0.04814	0.04640	0.04625	0.04506
Welfare gain relative to no FXI	6.8%	3.0%	2.6%	

Notes: The numbers are based on the model simulated data for 11,000 periods and dropping the first 1,000 periods. Intermediate FXI1 case allows the agents to make decisions based on optimal FXI, but government spending is computed based on the interest rate in the no FXI case.. Intermediate FXI2 case refers to a situation without FXI affecting agents behavior, but government spending is computed based on the interest rate in the FXI case.

5 Conclusion

The constraints faced by emerging market economies in the international financial system have been the subject of an enormous research effort over the last two decades. At the turn of the century, most emerging countries were constrained by original sin and unable to borrow in domestic currency. This limitation has been substantially relaxed for many emerging countries in the intervening decades. Our paper argue that the accumulation of foreign exchange reserves and active foreign exchange rate intervention to lean against the global wind represents an important reason that sovereigns have been able to issue own currency debt. We show substantial empirical evidence in support of our claim, and we build a rich open economy macroeconomic model that explains the transition towards a high sovereign local currency debt issue and potentially large welfare gain. Each element in the model is an important link explaining how sovereigns have managed to shed off the curse of original sin. However, some questions remain. One feature we do not fully explain is why advanced economies can easily issue local currency debt without holding or deploying large stocks of foreign reserves. We leave this question of the inherent difference between emerging economies and advanced economies for future research.

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Online Appendix (Not for publication)

A Sudden stops and the collateral constraint

Following Schmitt-Grohe and Uribe (2020), we assume that the collateral constraint on household borrowing depends on the current value of GDP, and that the elasticity of substitution in preferences between traded and non-traded goods is sufficiently low that there can arise an underborrowing equilibrium driven by self-fulfilling expectations. Figure A, taken from Schmitt-Grohe and Uribe (2020), illustrates the relationship between net private sector borrowing and the collateral constraint. The figure illustrates the left and right hand side of the borrowing constraint, written first as a steady state condition

$$-F = \kappa(y_t^T + \frac{(1-\alpha)}{\alpha}(y^T + F(1 - \frac{1}{R^W}))^{\frac{1}{\xi}}) \tag{42}$$

and secondly as a 'short-run' condition

$$-F_t = \kappa(y_t^T + \frac{(1-\alpha)}{\alpha}(y^T - \frac{F_t}{R_t^W} + F_{t-1})^{\frac{1}{\xi}}) \tag{43}$$

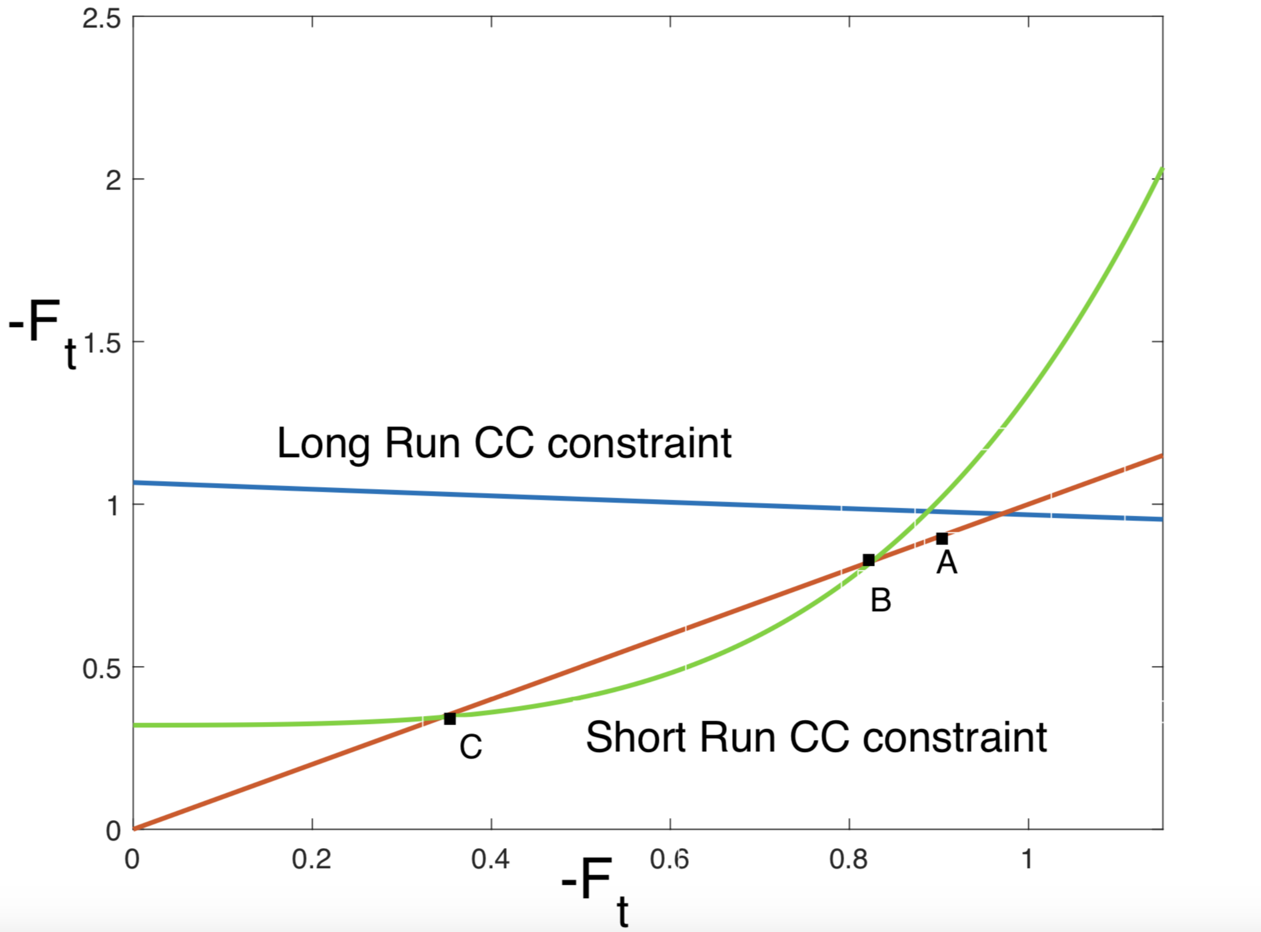
where for simplicity we have assumed that traded good output is constant at y^T, and non-traded good output is normalized to unity.

Equation (42) describes a downward sloping relationship on the left hand side, as a higher net foreign debt $-F$ tightens the borrowing constraint by reducing traded good consumption, depreciating the real exchange rate, and reducing the value of collateral. The intersection of this with the 45^0 line indicates the maximum possible long run level of net foreign debt. Point A in the graph could be a steady state debt level in which the collateral constraint is not binding.

However, note that, conditional on $-F_{t-1}$, the right hand side of (43) is increasing in $-F_t$. Then, in some cases, coinciding with point A, there may be other short run equilibria, where the collateral constraint binds if agents reduce their debt sufficiently, causing a fall in the right hand side of (43) more than the fall in the left hand side. This can occur if the elasticity of substitution between traded and non-traded goods is low, so a fall in current consumption of traded goods leads to a large fall in the relative price of non-traded goods, and the short run borrowing constraint intersects the 45^0 line with a slope greater than unity. Points B and C in the figure are both potential equilibria.

In the quantitative solution of the model, in the case of multiple equilibria, it is necessary to adopt an equilibrium selection mechanism. Schmitt-Grohe and Uribe (2020) discuss a number of alternative strategies for selecting an equilibrium in a quantitative evaluation of their model. We follow Davis et al. (2023) in assuming that if equilibrium A exists, agents coordinate on that equilibrium, but if not, then they coordinate on equilibrium C. The argument is that equilibrium B is unstable in a traditional sense. The implication of this equilibrium selection assumption is that small increases in the world interest rate can lead to precipitous declines in consumption of traded goods, depreciating real exchange rates, and reversals in the current account. This implies that a rising world interest rate can cause large 'sudden stops'. This link between world interest rates and sudden stops is present only due to the calibration of the model with low intra-temporal elasticity of demand and the potential for multiple equilibria as in figure A.

Figure 9: Long run and short run borrowing constraint



B Optimal Policy Problem

As stated in the text, the Central Bank manages reserves through sterilized intervention and conducts monetary policy by setting the price level. Intervention is implemented by buying reserves with bonds sold to private sector in some states, and selling reserves in order to retire debt to private sector in other states. By manipulating the domestic interest rate faced by households, FX intervention can affect the economy's net foreign assets. The presence of a pecuniary externality associated with the household borrowing constraint ensures a welfare role for the Central Bank in an optimal reserve management policy, subject to the non-negativity constraint on total reserve holdings. We keep the model as simple as possible by abstracting from the mechanics of monetary policy and simply assuming that the Central Bank can directly choose the price level. The Sovereign chooses government spending, total sovereign borrowing, and the currency composition of sovereign debt, subject to the implicit bond pricing function determined by investors. The full decision problem of the Central Bank and the Sovereign can be described by a single Ramsey maximization problem, assuming no commitment.

We may describe the Ramsey optimization problem as follows:

$$\max_{F_t^{cb}, c_t^T, F_t, G_t B_t^{FC}, B_t^{LC}, p_t^T} E_0 \sum_{t=0}^{\infty} \beta^t \left\{ (1-\omega_g) u(c_t^T) + \omega_g (v(G_t) - l(\frac{P_t}{P_{t-1}}) \right\} \tag{44}$$

subject to

$$y_t^T + F_{t-1} - \frac{F_t}{R_t^W} - c_t^T \geq 0 \tag{45}$$

$$u'(c_t^T) - \mu_t^p = \beta [E_t u'(c_{t+1}^T)(R_t^W - \frac{\Gamma}{\beta}(F_t - F_t^{cb})] \tag{46}$$

$$\frac{F_t}{R_t^W} + \kappa(y_t^T + \left(\frac{1-\alpha}{\alpha}\right)^{\frac{1}{\xi}} (\frac{c_t^T}{y^N})^{\frac{1}{\xi}} y^N) \geq 0 \tag{47}$$

$$F_t^{cb} \geq 0 \tag{48}$$

$$y_t^G + B_{t-1}^{FC} + \frac{B_{t-1}^{LC}}{p_t^T} - G_t - \frac{1}{R_t^{FC}} B_t^{FC} - \frac{1}{R_t^{LC} p_t^T} B_t^{LC} \geq 0 \tag{49}$$

$$P_t = p_t^T [\alpha + (1-\alpha) \left(\frac{1-\alpha}{\alpha}\right)^{\frac{1}{\xi}} (\frac{c_t^T}{y^N})^{\frac{1-\xi}{\xi}}]^{\frac{1}{1-\xi}} \tag{50}$$

where $u(c_t^T) = \frac{\{[\alpha^{1/\xi}(c_t^T)^{(\xi-1)/\xi} + (1-\alpha)^{1/\xi}(y^N)^{(\xi-1)/\xi}]^{\xi/(\xi-1)}\}^{1-\sigma}}{1-\sigma}$ incorporates market clearing in the non-traded good sector, and non-traded output is an exogenous constant. We denote $u_{T,t} \equiv \frac{\partial u}{\partial c_t^T}$. Constraint (64) represents the private sector budget constraint incorporating profits of financial intermediaries as well as Central Bank sterilized intervention. Constraint (46) represents the households first order condition where μ_t^prepresents the private sector's Lagrange multiplier on the collateral constraint. Then (47) represents the collateral constraint limiting net foreign borrowing by the private sector, while (48) is the non-negativity constraint on FX reserves. The sovereign budget constraint is given by(49) and the price index definition (50). [30]

Let the Lagrange multipliers on constraints (45)-(50) be denoted $\lambda_t, \eta_t, \mu_t, \phi_t, \psi_t$, and φ_t respectively. The first order conditions for the Ramsey problem can be written as:

$$(1-\omega_g)u'(c_t^T) - \lambda_t - \eta_t \frac{\partial u'(c_t^T)}{\partial c_t^T} + \frac{\mu_t}{\xi}\kappa\left(\frac{1-\alpha}{\alpha}\right)^{\frac{1}{\xi}}\left(\frac{c_t^T}{y^N}\right)^{\left(\frac{1-\xi}{\xi}\right)} y^N - \omega_g l'(\frac{P_t}{P_{t-1}})\left(\frac{\partial P_t}{\partial c_t^T}\right) = 0 \tag{51}$$

$$\eta_t E_t u_{T,t+1}\Gamma + \phi_t = 0 \tag{52}$$

$$\frac{\lambda_t}{R_t^W} = \frac{\mu_t}{R_t^W} - \eta_t E_t u_{T,t+1}\Gamma + \beta E_t \lambda_{t+1} \tag{53}$$

$$\omega_g v'(G_t) = \psi_t \tag{54}$$

$$\frac{\psi_t}{R_t^{FC}} = \beta E_t \psi_{t+1} \tag{55}$$

$$\frac{\psi_t}{R_t^{LC}}\left(1 + \frac{\partial \frac{1}{R_t^{LC}}}{\partial B_t^{LC}}\right) = \beta E_t \psi_{t+1}\frac{p_t^T}{p_{t+1}^T} \tag{56}$$

$$\psi_t \frac{B_{t-1}^{LC}}{p_t^T} = l'\left(\frac{P_t}{P_{t-1}}\right)\frac{P_t}{P_{t-1}} \tag{57}$$

It is useful to first look at the case where $\omega_g = 0$, so that the Ramsey planner places no weight on public spending or inflation. We can make use of (52) to substitute $\eta_t E_t u_{T,t+1}\Gamma$ in (53), then we can express divide (51) by R_t^R and express $\frac{\lambda_t}{R_t^W}$ using the combined equation of (52) and (53) to have the condition:

[30]The optimal policy choice variables do not include the private sector Lagrange multiplier μ_t^p. This is because constraint (46) will not bind when $\mu_t^p > 0$. See Davis et al. (2023).

$$\frac{1}{R_t^W}\left(u'(c_t^T)+\frac{\phi_t}{E_t u_{T,t+1}\Gamma}\frac{\partial u_{T,t}}{\partial c_t^T}+\frac{\mu_t}{\xi}\kappa\left(\frac{1-\alpha}{\alpha}\right)^{\frac{1}{\xi}}\left(\frac{c_t^T}{y^N}\right)^{\left(\frac{1-\xi}{\xi}\right)}y^N\right)=\frac{\mu_t}{R_t^W}+\phi_t$$

$$+\beta E_t\left(u'(c_{t+1}^T)+\frac{\phi_{t+1}}{E_t u_{T,t+2}\Gamma}\frac{\partial u_{T,t+1}}{\partial c_{t+1}^T}+\frac{\mu_{t+1}}{\xi}\kappa\left(\frac{1-\alpha}{\alpha}\right)^{\frac{1}{\xi}}\left(\frac{c_{t+1}^T}{y^N}\right)^{\left(\frac{1-\xi}{\xi}\right)}y^N\right) \quad (58)$$

This describes the optimal FX policy of the Central Bank, taking into account the pecuniary externality associated with the endogenous real exchange rate effect on the collateral constraint, the possibility of a binding collateral constraint, and the non-negativity constraint on reserves. To bring out the intuition, we can show the nature of the optimal policy in successive steps. First, we amend (58) by multiplying the equation by R_t^W and using the first order condition from the private sector, given by (46) to substitute away $u'(c_t^T)$. We then get:

$$\beta E_t u'(c_{t+1}^T)(R_t-R_t^W)+\frac{\mu_t}{\xi}\kappa\left(\frac{1-\alpha}{\alpha}\right)^{\frac{1}{\xi}}\left(\frac{c_t^T}{y^N}\right)^{\left(\frac{1-\xi}{\xi}\right)}y^N=R_t^W\phi_t\left(1-\frac{1}{E_t u_{T,t+1}\Gamma R_t^W}\frac{\partial u_{T,t}}{\partial c_t^T}\right)$$

$$+\mu_t-\mu_t^p+\beta R_t^W E_t\left(\frac{\phi_{t+1}}{E_t u_{T,t+2}\Gamma}\frac{\partial u_{T,t+1}}{\partial c_{t+1}^T}+\frac{\mu_{t+1}}{\xi}\kappa\left(\frac{1-\alpha}{\alpha}\right)^{\frac{1}{\xi}}\left(\frac{c_{t+1}^T}{y^N}\right)^{\left(\frac{1-\xi}{\xi}\right)}y^N\right) \quad (59)$$

In (59), consider an additional unit of reserves accumulation, the first term on the LHS is the marginal utility gain of future consumption. The second term is the effect of reserves accumulation on collateral constraint today. The first term on the RHS is the effect on the non-negative reserves constraint. The term $(\mu_t-\mu_t^p)$ captures the difference on the shadow value of collateral constraint of private and public due to pecuniary externality. The last term in the second line captures the effect of reserves accumulation today on the potential binding of collateral constraint and non-negative reserves constraint in the future. To see the intuition from (59), assume first that the collateral constraint doesn't bind now or in the future, and that there is no constraint on the sign of FX reserves so $\phi_t = \phi_{t+1} = \mu_t = \mu_{t+1} = \mu_t^p = \mu_{t+1}^p = 0$. In that case the optimal policy is to set $R_t = R_t^W$ continually, which amounts to the situation where the planner simply replaces the financial intermediaries and borrows for the private sector. On the other hand, if the collateral constraint never binds but the planner is constrained by non-negative reserves, we have the condition:

$$\beta E_t u'(c_{t+1}^T)(R_t - R_t^W) = R_t^W \phi_t \left(1 - \frac{\frac{\partial u_{T,t}}{\partial c_t^T}}{E_t u_{T,t+1} \Gamma R_t^W}\right) + \beta R_t^W E_t \frac{\phi_{t+1}}{E_t u_{T,t+2} \Gamma} \frac{\partial u_{T,t+1}}{\partial c_{t+1}^T} \quad (60)$$

This condition implies that if the planner is currently constrained by the non-negativity condition on FX reserves, it must be that it cannot reduce the domestic interest rate all the way to the world interest rate by sufficient borrowing on the part of the private sector.[31] If alternatively the non-negativity constraint is not currently binding, but is expected to be binding in the future, the Central Bank will borrow more and set $R_t < R_t^W$. This is a case where the social value of consumption in the future is less than in the current period due to the binding FX constraint in the future, so the planner will borrow more today, lending so much to the households that it drives the domestic interest rate below the world interest rate. Finally, if we impose a steady state condition on 60we get

$$\beta E_t u'(c^T)(R - R^W) = R^W \phi \left(1 - \frac{\frac{\partial u_T}{\partial c^T}(1 - \beta R^W)}{E_t u_T \Gamma R^W}\right) \quad (61)$$

The right hand side of this expression is positive when $\beta R^W < 1$. In a steady state where the non-negativity constraint on reserves is binding the domestic interest rate must be greater than the world rate.

To see the logic behind the FX policy highlighted in the paper, take the case where the collateral constraint is not currently binding $(\mu_t = \mu_{t+1} = 0)$ but might bind in the future, which gives a motive for the planner to accumulate reserves in advance. This can be described by the condition

[31] Note that $\frac{\partial u_{T,t}}{\partial c_t^T} < 0$.

$$\beta E_t u'(c^T_{t+1})(R_t - R^W_t) = \beta R^W_t E_t \left(\frac{\phi_{t+1}}{E_t u_{T,t+2}\Gamma} \frac{\partial u_{T,t+1}}{\partial c^T_{t+1}} + \frac{\mu_{t+1}}{\xi} \kappa \left(\frac{1-\alpha}{\alpha}\right)^{\frac{1}{\xi}} \left(\frac{c^T_{t+1}}{y^N}\right)^{\left(\frac{1-\xi}{\xi}\right)} y^N \right) \tag{62}$$

In this case, the planner has an incentive to accumulate reserves to set $R_t > R^W_t$ in anticipation of a future crisis, in which case it can deploy reserves, taking account of the pecuniary externality associated with the collateral constraint, captured by the second term on the right hand side. The first term, representing the possibility of the non-negativity constraint on reserves in period $t+1$, remains as in (53), but in practice is dominated by the pecuniary externality in the quantitative calibration.

Now let's introduce the presence of public spending and sovereign borrowing. The first order conditions 56 and 57are explored in detail in 38, 39, and 40 in the text. But there is an indirect effect of the FX policy on welfare through the impact on inflation, captured by the last term in 51. To illustrate this clearly, again take the extreme case where neither the collateral constraint nor the non-negatively constraint on reserves is binding. Then combining (51) and (53) we get the condition:

$$(1-\omega_g)\beta E_t u'(c^T_{t+1})(R_t - R^W_t) = \omega_g \left[l'\left(\frac{P_t}{P_{t-1}}\right)\left(\frac{\partial P_t}{\partial c^T_t}\right) - \beta R^W_t l'\left(\frac{P_{t+1}}{P_t}\right)\left(\frac{\partial P_{t+1}}{\partial c^T_{t+1}}\right)\right] \tag{63}$$

In the case $\omega_g = 0$, we are back to the case where the planner should set $R_t = R^W_t$ and act as the solve borrower for the economy. But when $\omega_g > 0$ the planner has to take account of the positive linkage between traded goods consumption and inflation through the appreciation of the real exchange rate. In a steady state, where $\beta R^W < 1$, this involves the Central Bank borrowing less than under the unconstrained policy, and setting $R > R^W$.

C Analytical approximation derivation

To begin, we rewrite the Sovereign's budget constraint as:

$$G_t + D_{t-1}R^W_{t-1} + D^L_{t-1}R^x_{t-1} = y^G_t + D_t \tag{64}$$

where $D_t \equiv -\left(\frac{B_t^{FC}}{R_t^W} + \frac{B_t^{LC}}{R_t^{LC} p_t^T}\right)$ represents the value of new debt issued by the Sovereign in time t, $D_t^L \equiv -\frac{B_t^{LC}}{R_t^{LC} p_t^T}$ is the value of local currency debt, and $R_{t-1}^x \equiv R_{t-1}^W - R_{t-1}^{LC} \frac{p_{t-1}^T}{p_t^T}$ is the ex-post (time t) excess return on foreign currency debt over local currency debt. For simplicity in what follows, we further define $\tilde{R}_{t-1}^{LC} = R_{t-1}^{LC} \frac{p_{t-1}^T}{p_t^T}$.

We may then rewrite the portfolio optimization conditions for the Sovereign and the international investors as

$$E_{t-1} R_{t-1}^x G_t^{-\sigma} = 0 \,, \; E_{t-1} R_{t-1}^x \Gamma_t^* = 0 \tag{65}$$

We then combine (65) with the Euler equation for optimal provision of government spending.

$$G_t^{-\sigma} = \tilde{\beta} E_t R_t^W G_{t+1}^{-\sigma} \tag{66}$$

Following Devereux and Sutherland (2011), we take a 2nd order approximation of (65) around a non-stochastic steady state, combined with a first order approximation of (64), and (66). Here we make a slight change in the model so as to ensure the existence of a non-stochastic steady state by re-defining the time discount factor to be endogenous to the size of government consumption, so that $\tilde{\beta} = \omega G_t^{-\eta}$. It is also assumed that in steady state, the Sovereign is a net debtor.[32]

For this example we make the additional assumptions about the shocks to Y_t^G and R_t^W;

$$\log(Y_t^G) = \bar{Y}_t^G + \varepsilon_y \,, \; \log(R_t^W) = \bar{R}_t^W + \varepsilon_R$$

where ε_y and ε_R are mean zero i.i.d. random variables.

For a variable z, we define $\hat{z}$ as the log deviation from steady state, except for $\hat{D}$ and $\hat{R}_t^x$, which is defined below. Then the 2nd order approximation of (65) can be written as

$$E_t \left(\hat{R}_t^x + \frac{1}{2} (\hat{R}_t^{2W} - \hat{\tilde{R}}_t^{2LC}) - \sigma \hat{G}_t \hat{R}_t^x \right) = \mathcal{O}(\varepsilon^3) \tag{67}$$

[32] In a steady state, we must have $\omega \bar{G}^{-\eta} R^W = 1$. We could alternatively introduce portfolio adjustment costs, which would serve the same purpose as an endogenous time discount factor, following the arguments of Schmitt-Grohé and Uribe (2003). Note to simplify the exposition, we assume that the Sovereign does not take into account the effect of spending on the time discount factor. This doesn't affect the qualitative results of this section. Moreover, the value of η can be very small while still ensuring the existence of a steady state.

$$E_t\left(\hat{R}_t^x + \frac{1}{2}(\hat{R}_t^{2W} - \hat{\hat{R}}_t^{2LC}) - \hat{\Gamma}_t^* \hat{R}_t^x\right) = \mathscr{O}(\varepsilon^3) \tag{68}$$

where $\hat{R}_t^x \equiv \hat{R}_t^W - \hat{\hat{R}}_t^{LC}$, and $\mathscr{O}(\varepsilon^3)$ denotes that the approximation is up to the second order.

We may approximate (64), and (66) up to the first order.[33] This gives:

$$\hat{G}_t + \frac{1}{\beta}\hat{D}_{t-1} + \frac{\bar{D}}{\beta\bar{G}}\hat{R}_{t-1}^W - \frac{\bar{D}}{\beta\bar{G}}^{L}\hat{R}_{t-1}^x = y_g\hat{y}_t^G + \hat{D}_t + \mathscr{O}(\varepsilon^2) \tag{69}$$

$$E_t\sigma\hat{G}_{t+1} = (\sigma - \eta)\hat{G}_t + \hat{R}_t^W + \mathscr{O}(\varepsilon^2) \tag{70}$$

where $\hat{D}_{t-1} = \frac{D_{t-1} - \bar{D}_t}{\bar{G}}$, $\bar{\beta} = \frac{1}{\bar{R}^W}$ is the reciprocal of the steady state world interest rate, and $y_g = \frac{\bar{Y}^G}{\bar{G}}$.

Equation (69) reflects the fact that up to a first order, the steady state value of R_t^x is zero, so the first order response of D_t^L does not enter (69). Moreover, from the definition of R_{t-1}^x, we may write

$$\hat{R}_{t-1}^x = \hat{R}_{t-1}^W - \hat{R}_{t-1}^{LC} + \hat{S}_t - \hat{S}_{t-1} \tag{71}$$

where we have used the fact that $p_t^T = S_t$ from above.

Finally, we make the assumption that the stochastic discount factor of international investors is a function of the global interest rate R_t^w, and moreover, up to a first order approximation, we have

$$\hat{\Gamma_t^*} = -\gamma\hat{R}_t^W + \mathscr{O}(\varepsilon^2) \tag{72}$$

We wish to obtain the optimal response of government spending $\hat{G}_t$ in order to obtain the equilibrium local currency portfolio from (67) and (68). Using (69) and iterating forward, we obtain the approximate inter-temporal budget constraint condition as:

$$E_t\sum_{i=0}^{\infty}\beta^i\left(y_gY_{t+i}^G - G_{t+i} - \frac{\bar{D}}{\beta\bar{G}}\hat{R}_{t+i-1}^W\right) + \frac{\bar{D}}{\beta\bar{G}}^{L}\hat{R}_{t-1}^x + \frac{1}{\beta}\hat{D}_{t-1} = \mathscr{O}(\varepsilon^2) \tag{73}$$

where we have used the fact that $E_tR_t^x = 0$ up to the first order.

Now substituting in (70) and summing, using the assumptions on $\hat{Y}_t^G$ and $\hat{R}_t^W$ we

[33] Note that because eq(67) and eq(68) are accurate only up to second order, in order to determine the optimal portfolio, the other equations can be approximated up to first order.

obtain (ignoring the order notation hereafter)

$$\hat{G}_t = (1-\beta\vartheta)\left(y_g Y_t^G - \frac{\bar{D}}{\beta\bar{G}}\hat{R}^W_{t-1} - \beta(\frac{\bar{D}}{\beta\bar{G}}+\frac{1}{\sigma})\hat{R}^W_t + \frac{\bar{D}}{\beta\bar{G}}^L \hat{R}^x_{t-1} + \frac{1}{\beta}\hat{D}_{t-1}\right) \tag{74}$$

where $\vartheta = \frac{\sigma-\eta}{\sigma} < 1$.

Now, equating (67) with (68), and using (72) and (74), dropping the time notation since this describes a constant portfolio, we may derive the optimal local currency portfolio

$$\frac{\bar{D}}{\beta\bar{G}}^L = \frac{1}{Var(\hat{S})}\left(\underbrace{\underbrace{-y_g Cov(\hat{Y}^g,\hat{S})}_{\text{hedging domestic shock}} \underbrace{+\beta(\frac{\bar{D}}{\beta}+\frac{1}{\sigma})Cov(\hat{R}^W,\hat{S})}_{\text{hedging global shock}}}_{\text{hedging benefit}} - \underbrace{\frac{\gamma}{1-\beta\vartheta}Cov(\hat{R}^W,\hat{S})}_{\text{local currency risk premium}}\right) \tag{75}$$

D Reserves elasticity of exchange rate

From (**??**), we have:

$$S_t = \underbrace{P_t}_{\text{price index factor}} \times \underbrace{[\alpha + (1-\alpha)(\frac{p_t^N}{p_t^T})^{1-\xi}]^{-\frac{1}{1-\xi}}}_{\text{real exchange rate factor}}$$

In order to know how a change in reserves could change the exchange rate, $\frac{\partial S_t}{\partial F_t^{cb}}$, we can break the partial derivative into:

$$\frac{\partial S_t}{\partial F_t^{cb}} = \frac{\partial S_t}{\partial \frac{p_t^N}{p_t^T}} \frac{\partial \frac{p_t^N}{p_t^T}}{\partial c_t^T} \frac{\partial c_t^T}{\partial C_t} \frac{\partial C_t}{\partial R_t} \frac{\partial R_t}{\partial F_t^{cb}}$$

which correspond to

$$\frac{\partial S_t}{\partial F_t^{cb}} = [P_t(-\frac{1}{1-\xi})(\alpha+(1-\alpha)(1-\xi)(\frac{1-\alpha}{\alpha}(c_t^T)^{\frac{1}{\xi}})^{-\xi})^{-\frac{\xi}{1-\xi}}][\frac{1}{\xi}(\frac{1-\alpha}{\alpha})(c_t^T)^{(1-\xi)/\xi}][(\frac{c_t^T}{\alpha C_t})^{1/\xi}][\frac{\partial C_t}{\partial R_t}][\frac{\Gamma}{\beta}]$$

E Data appendix

We use an inflation targeting definition from Ogrokhina and Rodriguez (2018), who follow the definition from Mishkin (2004). We also use the covered interest parity dataset from Du et al. (2018) to obtain a measure of LC spreads and the decomposition.

Table 10: Sample countries

Asia	Latin America	European Union	Europe, Middle East, Africa
China (DS) (IT=0)	Argentina (IT=0)	Bulgaria (IT=0)	Egypt (IT=0)
India (DS)	Brazil (DS)	Hungary (DS)	Russia (DS)
Indonesia (DS)	Chile (DS)	Latvia (IT=0)	South Africa (DS)
Malaysia (DS) (IT=0)	Colombia (DS)	Lithuania (IT=0)	Turkey (DS)
Philippines (DS)	Mexico (DS)	Poland (DS)	Ukraine
Thailand (DS)	Peru (DS)	Romania	
	Uruguay		

Notes: DS denotes countries that are available in the Du and Schreger dataset for pricing data. IT=0 denotes countries that are not inflation targeters in our entire sample period. Note that Latvia and Lithuania joined the Eurozone in the later part of the sample period. All the empirical results are robust to excluding them.

Table 11: Data source

Sample period: 2004Q1-2019Q1

Variable	Data source
Reserves	IMF IFS
Local currency share	Arslanalp and Tsuda (2014)
Local currency sovereign spreads	Du and Schreger (2016)
VIX	FRED
US Treasury 5Y	FRED
US GDP	World Bank WDI, IMF IFS
Domestic GDP	World Bank WDI, IMF IFS
Chinn-Ito Index	Chinn Ito (2006)
Govt Effectiveness	World Bank WDI
Political Stability	World Bank WDI
Domestic credit to GDP	World Bank WDI
External public debt to GDP	BIS International Debt Statistics
External private debt to GDP	BIS International Debt Statistics, BIS locational bank statistics
Nominal exchange rates	IMF IFS
Inflation targeting	Ogrokhina and Rodriguez (2018)

Table 12: Inflation targeting year

Inflation Targeter	Inflation target year	Non Inflation Targeter
Brazil	1999	Argentina
Chile	1999	Bulgaria
Colombia	1999	China
Hungary	2001	Egypt
India	2016	Lithuania
Indonesia	2005	Lativa
Korea	2001	
Mexico	2001	
Peru	2002	
Philippines	2002	
Poland	2002	
Romania	2005	
Russia	2013	
South Africa	2000	
Thailand	2000	
Turkey	2006	
Uruguay	2002	
Ukraine	2015	

F Computation algorithm

The computation of the model is based on Schmitt-Grohe and Uribe (2020), Davis et al. (2023) and Ottonello and Perez (2019). The model is solved by value function iteration. We iterate until the maximum distance of any point in the value function is smaller than 1^{-5} (convergence). There are three big blocks in the model. The households problem, the sovereign problem and the central bank problem. There are two useful observations. First, the central bank reserves decision is not a state variable, the endogenous states are private debt (b^{FC}) and public debt (B^{FC},B^{LC}). Second, the central bank internalizes its effect on the sovereign and private decisions subject to the equilibrium conditions.

Households problem.

Let the household value function be $V(B^{FC},B^{LC},b^{FC},y^T,R^W)$. However, sovereign decisions only matter in determining the reserves positions in a given state. Therefore, it is useful to rewrite the problem as $V(F^{cb}, b^{FC}, y^T, R^W)$. We solve the household problem given any reserves state and then the central bank picks the optimal reserves state when we solve the central bank problem. Specifically, given any F^{cb}, y^T and R^W, in iteration number n, the household chooses a policy $b^{FC'}_{temp,n}$ with an inital guess of $V_{temp,n}(F^{cb}, b^{FC}, y^T, R^W)$. We iterate until convergence of $V_{temp,n}$.

Sovereign problem.

Let the sovereign value function be $W(B^{FC}, B^{LC}, b^{FC}, y^T, R^W)$. Note that while the original in problem involves P_{t-1}, as in Ottonello and Perez (2019), we can detrend the problem and normalize the choice as choosing $P_t/P_{t-1} = \pi_t$ each period. Since the central bank decides the reserves policy given the private and sovereign decision, the reserves policy is unsolved in this stage. Therefore, we solve the sovereign problem given any reserves state and the central bank picks the optimal reserves state when we solve the central bank problem. Specifically, given any F^{cb}, y^T,R^W, b^{FC}, B^{FC} and B^{LC}, the sovereign chooses policies $B^{FC'}_{temp,n}$, $B^{LC'}_{temp,n}$ $\pi_{temp,n}$ with an initial guess of $W_{temp,n}(F^{cb}, B^{FC}, B^{LC}, b^{FC}, y^T, R^W)$. The foreign investors price the local currency sovereign bond $q^{LC}_{temp,n}(F^{cb}, B^{FC'}_{temp,n}, B^{LC'}_{temp,n}, b^{FC'}_{temp}, y^T, R^W)$ based on equation **??** given $B^{FC'}_{temp,n}$, $B^{LC'}_{temp,n}$ $\pi_{temp,n}$,$b^{FC'}_{temp}$.

Central bank problem.

Let the central bank value function be $U(B^{FC}, B^{LC}, b^{FC}, y^T, R^W)$. Given all temporary value function and policy function above, and state variables $(B^{FC}, B^{LC}, b^{FC}, y^T, R^W)$, the central bank choose the optimal reserves state $F^{cb}(B^{FC}, B^{LC}, b^{FC}, y^T, R^W)$. And after solving $F^{cb}(B^{FC}, B^{LC}, b^{FC}, y^T, R^W)$, we can plug this back to the temporary value function and policy function to obtain $V(B^{FC}, B^{LC}, b^{FC}, y^T, R^W)$, $W(B^{FC}, B^{LC}, b^{FC}, y^T, R^W)$, $b^{FC}(B^{FC}, B^{LC}, b^{FC}, y^T, R^W)$, $q^{LC}(B^{FC'}, B^{LC'}, b^{FC'}, y^T, R^W)$,
$B^{LC'}(B^{FC}, B^{LC}, b^{FC}, y^T, R^W)$, $B^{FC'}(B^{FC}, B^{LC}, b^{FC}, y^T, R^W)$, $\pi(B^{FC}, B^{LC}, b^{FC}, y^T, R^W)$.

Computation.

We discretize the exogenous process using Tauchen method. The y^T and r^W are discretized with 5 grids and 9 grids. Grid points for F^{cb}, b^{FC}, B^{FC}, B^{LC}, π are 100, 100, 15, 15, 15 respectively. As in Ottonello and Perez (2019), once the maximum was chosen on the grid, we use a numerical optimizer routine to find the maximum in a continuous neighborhood around the initially identified point. We use Gaussian quadrature methods to compute all expectations and piecewise linear interpolation to interpolate policies outside the grids.