Motivational and informational financial ratings

Pierre Chaigneau* Nicolas Sahuguet†
Queen's University HEC Montréal

March 17, 2025

Abstract

Financial information is produced and sold according to different business models. This paper analyzes the design of financial ratings that can be informative about the effort decision of a corporate insider and can also reduce uncertainty about firm value. Ratings can be paid either by issuers who want to maximize their firm's valuation or by investors who want to maximize their informational advantage. We show that aggregating information into one rating paid by the issuer but used by market participants maximizes effort and firm value. However, in sufficiently liquid financial markets or when the firm insider has a long-term horizon, the rating agency will rather sell ratings directly to investors. Even when competing rating agencies could in principle cater to different clienteles, a motivational rating will not be provided when an informational rating can be produced at a low cost and markets are sufficiently liquid. Then, bad information drives out good information.

Keywords: corporate governance, crowding-out effect, endogenous information acquisition, ESG ratings, ratings, short-termism.

*Address: Smith School of Business, Goodes Hall, 143 Union Street West, K7L 2P3, Kingston, ON, Canada. Email: pierre.chaigneau@queensu.ca. Tel: 613 533 2312.

†Address: Applied Economics Department, HEC Montréal, 3000 chemin de la côte Sainte Catherine, H3T 2A7 Montréal, QC, Canada. Email: nicolas.sahuguet@hec.ca. Tel: 514 340 6031.

Information about a firm can be valuable for two reasons: because it leads to a better assessment of its value, or because it changes decisions that matter for firm value. These two dimensions can be in conflict, so that a better information system does not simply generate more information on each dimension (Crémer (1995), Hörner and Lambert (2021)). In particular, the composition of a signal or "rating" matters for incentive provision and economic efficiency (Hörner and Lambert (2021), Saeedi and Shourideh (2024)).

Information about a firm does not fall from the sky; it must be financed and produced. In practice, the production and dissemination of financial ratings involves different business models. Credit ratings are purchased by firms that issue debt securities; equity research by large investment banks is often widely disseminated and either directly or indirectly paid for by issuers (e.g. via investment banking fees); smaller independent research firms and brokerages also produce equity research which is either directly or indirectly sold to their investor clientele (e.g. via trading fees); likewise, ESG ratings are purchased by investors.

These two types of economic agents, issuers and investors, have different objectives and therefore value information differently. Issuers will value information that improve the market value of their securities. This can result in rating inflation (Skreta and Veldkamp (2009), Bolton, Freixas, and Shapiro (2012), Sivan (2015), Goldstein and Huang (2020)). By contrast, investors will value information that gives them an informational advantage so that they can generate trading profits.

A rating agency will take into account the needs of its clientele: it will design the rating differently depending on who is purchasing it. In turn, the rating design matters because it affects the firm's market valuation, which in turns affects the incentives of firm insiders who might sell their shares. In sum, the financing of the rating ultimately influences the nature of the information produced and transmitted, which will have real effects. In our model, aggregating multidimensional information about a firm in one imperfectly informative rating paid by the issuer maximizes economic efficiency. However, it may not be the equilibrium outcome. In some cases, the equilibrium will involve highly informed investors and low output.

Instead of taking a business model as given, our paper endogenizes it, studies its determinants and its consequences. We consider a firm partly owned by an "insider". This insider can take value-increasing actions but might need to sell his stake at an interim date, as in Faure-Grimaud and Gromb (2004). This can be interpreted as the stochastic "exit" of founders, managers, or involved shareholders such as venture capitalists. Firm output is driven by an unknown component, the firm's "type", and by the insider's effort. The firm's shares are traded at an interim date on a financial market in the spirit of Kyle (1985), with a potentially informed investor who trades alongside liquidity traders, and a market maker. As in Faure-Grimaud and Gromb (2004), there are two sources of effort incentives. The insider, who might need to sell his shares, cares about the firm's output and about its market valuation. He may not internalize all the benefits of his effort if the market price at which he can sell does not fully reflect his actual effort.

In this context, we consider information about the firm in the form of a "rating", which is available at the time of establishing the firm's market value. Since firm output is driven by two components, the firm's type and the insider's effort, a rating may combine different sources of information. We follow Hörner and Lambert (2021) by letting the rating put some weight on a signal informative about effort, and some weight on a signal informative about the firm's type. In our paper, the weights are chosen by a rating agency to maximize its profits. The agency can sell the rating either to the investor or to the issuer.

The investor is willing to pay for the rating to the extent that it increases her informational advantage. In particular, she is not interested in purchasing information to better learn the manager's effort, since effort is correctly anticipated in equilibrium. Therefore, an agency that sells its rating directly to investors will design a rating that is only informative about the firm's type.[1]

By contrast, the firm insider is willing to pay for the rating agency to credibly produce information about firm value that is then observed by the investor or by all market participants. When it sells the rating to the issuer, the rating agency chooses the rating weights to maximize the insider's willingness to pay for it. This is achieved when the rating is informative about effort and the firm type, as in Hörner and Lambert (2021). Intuitively, when the rating is informative about effort, the market price at which the insider might need to sell his stake will (partly) reflect his actual effort. This allows him to implicitly commit to a higher effort. However, a rating that is not informative about the unknown firm type would be ignored by market participants, and would therefore fail to provide any effort incentives. As a result, the rating agency optimally provides a single rating that aggregates information about the insider's effort and information about the firm's type.

As long as the firm's type is not too uncertain, the optimal rating places a greater weight on information about effort as the prior uncertainty about the firm's type increases. Intuitively, the higher the uncertainty, the higher the price impact of the rating, which makes it possible to have the rating be more informative about effort and still matter a lot for firm valuation. Moreover, when there is enough uncertainty about firm value, the equilibrium rating weight is such that effort incentives are the same as if the insider could fully commit to effort. Intuitively, the rating agency caters to the issuer, and the level of effort under full commitment is his preferred outcome.[2] This is different from Faure-Grimaud and Gromb (2004) and Chemla and

[1]This is not because it would be more expensive for the rating to also be informative about effort: in our baseline model, the agency receives signals for free. Instead, this is because a rating that also puts some weight on the signal about effort would be more noisy and therefore less informative about the firm's type.

[2]The optimal rating design is not simply the one that maximizes the market value of the firm. Indeed, when uncertainty about firm value is sufficiently high, the firm value-maximizing rating design would involve effort

Hennessy (2014), where the possibility of exit or sale always decreases effort incentives, so that the equilibrium effort is always lower than under full commitment.

When there is a large volume of liquidity trades and a correspondingly very liquid stock market, the rating agency maximizes its profits by selling its rating to the investor. Indeed, the investor values private information about the firm type all the more that she can profitably trade on this information, which is easier in more liquid markets. This business model also arises when the insider is rarely hit by a liquidity shock, i.e. when he has a long-term horizon. In this case, the insider has a low willingness to pay for the rating since he already largely internalizes the benefits of his effort. Otherwise, the rating is purchased by the issuer and disseminated to market participants. Thus, the most profitable business model for the rating agency is not necessarily the one associated with the highest aggregate surplus. Selling the rating to the investor results in low effort incentives for the insider, and therefore low output.

This is not easily remedied. In an extension of the model with competition between rating agencies, we show that it is not in the interests of a rating agency to make public a potentially welfare-improving rating that is informative about both effort and firm value alongside a rating designed specifically for investors. Indeed, the latter already provides all the information available about firm value, so that the former would be ignored by market participants. In short, *bad information drives out good information.* Specifically, the potential availability of bad information may make it impossible to effectively provide good information that would improve economic efficiency. This will be the case if financial markets are sufficiently liquid and the cost of information production is sufficiently low. Paradoxically, "bad information" is more informative in the sense that it reduces uncertainty more than good information.

In this context, it would be welfare-improving if the availability of a "good rating" crowded out the production of other (bad) information. This is possible if the cost of information production is sufficiently high. This is in contrast to the well-known result that disclosure can crowd out (good) information acquisition by investors (e.g. Verrecchia (1982), Goldstein and Yang (2017)).[3]

The model has implications for short-termism. The propensity of corporate insiders to liquidate their stake early, which can be viewed as a proxy for their time horizon, may not affect economic efficiency. Indeed, when financial information is designed to cater to the issuer's preferences and there is enough uncertainty about firm value, the insider's time horizon does not change his effort incentives. This is a new result in a model of exit with moral hazard, which we refer to as the "time horizon irrelevance" result. Intuitively, in this case, the rating design can

incentives above the full commitment level, which would make the insider worse off. In this case, there are two rating weights that induce the socially optimal level of effort, which is higher than the level of effort under full insider commitment when the insider only has partial ownership of the firm.

[3]It is also related to the more recent result that public information can crowd in voluntary disclosure (Banerjee, Marinovic, and Smith (2024)).

be such that the insider has the same effort incentives as if he were never subject to a liquidity shock, i.e. as if he had a long-term horizon. When there is enough uncertainty about firm value, we show that this is approximately true if the rating is simply informative about firm output. This might be the most straightforward information design used in practice.

The time horizon irrelevance result can shed light on the business model of venture capitalists and its limitations. These shareholders with a short investment horizon are actively involved in the firms in their investment portfolio (Hellmann and Puri (2002), Bernstein, Giroud, and Townsend (2016)). They primarily target young firms in industries undergoing technological revolutions, which tend to be subject to a lot of uncertainty (Gompers and Lerner (2001), Janeway, Nanda, and Rhodes-Kropf (2021)). This narrow focus has been deplored by Lerner and Nanda (2020). However, our results suggest that a *partial* generalization of this model, such as venture capital investments in more mature industries, or short-term private equity investments in mature companies that need key shareholders to be actively involved, might lead to worse outcomes. Indeed, since there is not enough uncertainty about these firms, financial information cannot be designed to offset the effort-reducing effect of a short-term horizon. The time horizon irrelevance result can also contribute to explain the apparently puzzling fact noted by Lerner and Nanda (2020) that the time horizons of venture capital funds do not vary depending on the type of firms financed, even though "great diversity across industries exists in terms of the typical project length."[4]

The result that effort incentives with endogenous information design can be as high as when the insider can fully commit to effort has implications for security design and capital structures. When the outcome preferred by the insider is achievable, there is no benefit for the insider to issue anything other than equity, i.e. there is no role for security design. This can help explain the "zero leverage puzzle" (Strebulaev and Yang (2013)).[5]

Applications of the framework described in the paper are not limited to explicit "ratings". A "rating agency" can be an investment bank that produces equity research. It can also be an external auditor that certifies financial statements. Whereas audit reports of public companies must adhere to strict regulatory requirements, audit reports of private companies are more flexible and can be tailored to the specific needs of the firm.[6] These degrees of freedom may

[4]Their tentative explanations for this puzzling fact involve historical norms and conflicts of interest between limited partners and general partners.

[5]They note that firms with zero leverage tend to have higher R&D expenses and higher market-to-book ratios. Market-to-book ratios are usually viewed as a proxy for growth opportunities, which involve more uncertainty than established assets. This is in line with the implication from our model that issuing securities other than equity is not beneficial when information is designed for the issuer and there is enough uncertainty about firm type.

[6]For example, the information disclosed can include a combination of GAAP earnings and non-GAAP earnings (Leung and Veenman (2018)). The types of risks assessed and the procedures used for risk assessment must be determined (Knechel et al. (2013)). The audited information can also be presented with various levels of complexity and disaggregation (Chen, Miao, Shevlin (2015), Hoitash and Hoitash (2018), Noordermeerb and

allow to shift the extent to which the audit reflects insider effort rather than firm type, which is equivalent to shifting rating weights. These audit reports can be either kept private, or privately shared with some investors, or made publicly available.

The rating agency can also be an agency that produces ESG ratings. An ESG rating that is valuable for investors because it helps them know the firm's type, for example its exposure to climate risks, may not provide incentives for its managers to improve its social and environmental impact, for example by investing in mitigation.

The paper contributes to the debate on the funding of rating agencies via issuer fees or user fees. Issuer fees are paid by the entities whose financial products are being rated. Issuer fees may create conflicts of interest, leading to inflated ratings to cater to the issuers and secure future business (Hayward and Boeker (1998), Benmelech and Dlugosz (2010), Bolton, Freixas, and Shapiro (2012), Kraft (2015)). By contrast, having investors directly pay for the ratings that they use arguably aligns the interests of the rating agencies with those of the end-users of their ratings. It should accordingly mitigate conflicts of interest and related biases. Our model allows us to identify a new countervailing effect. We show that, when agencies design ratings to cater to investors rather than issuers, the outcome is inefficient and leads to welfare losses. Thus, we highlight a different inefficiency related to rating design rather than rating bias.

Related literature

Admati and Pfleiderer (1986, 1988) analyze how an informed agent should sell some exogenously given information. This started a large literature on the endogenous production of information in financial markets (Admati, Pfleiderer, and Zechner (1994), Fulghieri and Lukin (2001), Peress (2010), Ferreira, Manso, and Silva (2014), Dasgupta and Matthews (2024)). An important difference is that, in our paper, information is not only sold but also optimally designed by a third-party, the rating agency.

Moreover, the rating agency does not care about the actions taken by the firm or about its output, instead it is only maximizing the value of information sold. This differs from an information design problem, in which the information designer cares about actions or outcomes (Bergemann and Morris (2019)). In particular, in Hörner and Lambert (2021), the rating is designed to maximize the social surplus, which is equivalent to maximizing effort. In our model, the rating is designed to maximize the profitability of a rating agency that sells its rating. The resulting equilibrium rating weights differ from the rating weights that induce first-best effort.

In a model of certification, Stahl and Strausz (2017) point out that a buyer and a seller use information for different purposes, inspection and signaling. There is also a dual purpose

Vorst (2024)). Incidentally, the perspective in this accounting literature is that greater informativeness is better. On the contrary, we point out that economic efficiency, specifically insider effort, is maximized for an intermediate level of rating informativeness: the most informative rating, which is designed to cater to investors, induces the lowest level of effort.

in our model, where the investor and the insider are interested respectively in learning the firm's type and setting appropriate effort incentives. In their model, certification does not have any implications for effort. Bouvard and Levy (2018) also consider a model of certification in which the certifier chooses accuracy to maximize its profits. In their model, certification is informative about one dimension, "quality", whereas in our model the rating can be informative about effort and firm type. Faure-Grimaud, Peyrache, and Quesada (2009), Daley, Green, and Vanasco (2020), and Kashyap and Kovrijnykh (2016) consider contracting with ratings in models without effort from firm insiders or originators and uni-dimensional ratings that are informative about the asset type. In our model, ratings can also be informative about the insider's effort.

Pollrich and Strausz (2024) analyze a model with a seller and a buyer of information, and show that the fee structure is irrelevant. In their model, differences in transparency do not affect economic efficiency. In our model, by contrast, there are several potential buyers for the information, and differences in transparency affect economic efficiency.

Many papers, most notably Banker and Datar (1989), study the optimal weights to put on various signals to encourage effort in a principal-agent framework. In our model, the optimal rating is a combination of two signals, and only one of these two signals is informative about effort. The other signal is unrelated to effort, and it is not useful to reduce the risk borne by the insider since all agents are risk neutral.

Despite the fact that information about firms is multidimensional, ratings are often uni-dimensional. Our model can help explain this puzzle for financial ratings. As in Hörner and Lambert (2021), a rating that were just informative about effort would be ignored, and a rating that were just informative about firm value would not improve aggregate surplus. This contributes to a literature that studies properties of ratings.[7]

The finance literature has focused on credit ratings. In the models of Manso (2013) and Goldstein and Huang (2020), credit ratings have real effects because they affect the probability of default of the borrower, which results in a feedback loop. Bolton, Freixas, and Shapiro (2012) study conflicts of interest in credit rating agencies, including their reliance on fees from firms, and the ability of firms to selectively purchase ratings. In this context, they show that ratings can be inflated, and that competition between rating agencies can reduce efficiency by encouraging ratings shopping. Ratings shopping and selective disclosure is also studied in Sangiorgi and Spatt (2017), and competition between credit rating agencies is studied in Doherty, Kartasheva, and Phillips (2012). The notion that rating agencies cater to issuers is supported by empirical studies (Griffin, Nickerson, and Tang (2013), Kraft (2015)).[8]

[7]Even though they are less precise, discrete ratings can be justified because they preclude rating inflation (Goel and Thakor (2015)), or because they reduce the probability of misinterpretation (Martel, van Wesep and van Wesep (2022)).

[8]Mathis, McAndrews, and Rochet (2009) and Bar-Isaac and Shapiro (2013) study the reputational concerns of these agencies. Alessio and Shapiro (2022) study how the market for credit risk has a disciplinary effect on

The paper is related to the large literature on the real effects of financial markets, reviewed by Bond, Edmans, and Goldstein (2012) and Goldstein (2023). The outcome with highly informed investors and low economic efficiency (low effort by the firm insider) that we describe is reminiscent of the result by Dow and Gorton (1997) that the economy may be stuck in a bad equilibrium with informationally efficient markets and depressed corporate investment. However, the mechanism at play is very different. In Dow and Gorton (1997), this inefficiency is possible because managers learn from prices, as in the literature on feedback effects. In our model, the information in stock prices matters because it has implication for effort incentives.

The model is also related to the literature on exit as a governance mechanism (Admati and Pfleiderer (2009), Edmans (2009), Edmans and Manso (2011), and Cvijanović, Dasgupta, and Zachariadis (2022)). In these models, shareholders can sell their shares, which disciplines the manager. In our model, the "insider" can take actions to improve firm value but might need to sell his shares.

Thus, the paper is related to the literature on short-termism. This literature examines the possibility of short-termism (Narayanan (1985), Stein (1989), and Von Thadden (1995)), how to prevent short-termism (with contracting as in Marinovic and Varas (2019), with monitoring as in Von Thadden (1995)), and the endogenous determination of investment horizons (Gryglewicz, Mayer, and Morellec (2020), Hackbarth, Rivera, and Wong (2021), Thakor (2021)). By contrast, we take the insider's horizon as given and we study its consequences for value creation when ratings are optimally determined. Our time horizon irrelevance result shows that a short-term horizon is not necessarily detrimental when there is enough uncertainty about firm value. In Bolton, Scheinkman, and Xiong (2006), when stock prices deviate from their fundamental values, corporate short-termism can be optimal to take advantage of erroneous investor beliefs. In our model, all market participants have rational expectations and unbiased beliefs.

The adaptation of Gresham's law to the economics of information, the notion that bad information can drive out good information, was already proposed by Varian (1998) in another context. He argued that "Cheap, low quality information on the Internet can cause problems for providers of high-quality information." Instead, we are pointing out that the preferred information of investors and market participants focuses on reducing uncertainty rather than encouraging value-improving actions. Consequently, their demand for information may lead to the production of information that results in low economic output, in part because it makes irrelevant information that affects incentives and output.[9]

credit rating agencies. Opp, Opp and Harris (2013) study the biases that may arise when ratings are also used for regulatory purposes. Fulghieri, Strobl, and Xia (2014) show that credit rating agencies can benefit from the possibility of issuing unsolicited ratings at no fee, by using it as a threat to extract higher fees. Alissa et al. (2013) and Cohn, Rajan, and Strobl (2025) study credit ratings when the issuer can manipulate information.

[9]This is also related to the Hirshleifer effect that releasing information can prevent efficiency gains from optimal risk sharing (Hirshleifer (1971)). In our model, it is crucial that information can be obtained on two dimensions (effort and the firm type), whereas the Hirshleifer effect only relies on information about the asset

1 Model

1.1 The firm

A firm is owned and managed by an "insider" who exerts effort to improve its output. The insider initially owns a fraction $1-\delta$ of the shares. As in Faure-Grimaud and Gromb (2004), the insider can be viewed either as an entrepreneur or founder, or as an involved shareholders such as a venture capitalist. The output of the firm, which is realized at $t=2$, is given by:

$$\tilde{X} \;=\; A+\tilde{\theta}. \tag{1}$$

where A is the insider's effort, and θ is the firm productivity, also referred to as the firm's "type".[10] Effort A is unobservable and optimally chosen by the insider at $t=0$. The firm's type θ is drawn from a normal distribution $\mathcal{N}(\bar{\theta},\sigma_\theta^2)$. The realization θ of the random variable $\tilde{\theta}$ is unobservable. Effort is privately costly for the insider, with a monetary cost of $C(A)$, with $C'>0$, $C''>0$, $C(0)=0$, and $C'(0)=0$.

We assume universal risk neutrality and no discounting.

1.2 Financial market

As in Faure-Grimaud and Gromb (2004), with probability $\pi\in(0,1]$, the "insider" is hit by a publicly observable liquidity shock at $t=1$ and sells his shares in the firm.[11] As in Faure-Grimaud and Gromb (2004), the insider's trade is observable and cannot be anonymous.

Trading occurs at $t=1$. The financial market is in the spirit of Kyle (1985). In addition to the insider, it involves three economic agents.

- Noise traders submit a random market order $\tilde{u}$, which is normally distributed with mean 0 and variance σ_u^2 and is independent of the firm type $\tilde{\theta}$.
- A risk-neutral investor obtains information about the firm by acquiring a rating $\tilde{R}$, then he submits a market order $q(R)$.
- A competitive and risk-neutral market-maker observes whether the insider is hit by a liquidity shock as well as the total order flow. He sets the price of the asset to its expected value given his information.

type.

[10] We interpret output as firm profits, but other interpretations are also possible. For example, if economic agents also have nonpecuniary preferences, the "firm type" could be the position of the firm on the green-brown spectrum, and "effort" could be actions it takes to reduce its negative externalities, similar to the model of socially responsible investment of Edmans, Levit, and Schneemeier (2023).

[11] As in Faure-Grimaud and Gromb (2004), this assumption allows to abstract from adverse selection issues that could lead to a market breakdown when initial owners have private information about firm value.

The game is sequential: the insider and the informed investor submit market orders, then the market maker sets the market price p depending on the aggregate order flow.

1.3 Rating

A rating agency observes two signals at $t = 1$, one about effort and the other about the firm type:

$$\tilde{S}_1 = A + \tilde{\varepsilon}_1 \tag{2}$$

$$\tilde{S}_2 = \theta + \tilde{\varepsilon}_2 \tag{3}$$

where θ is the realization of $\tilde{\theta}$, and for $k \in \{1, 2\}$, $\tilde{\varepsilon}_k \sim \mathcal{N}(0, 1)$, and $\tilde{\varepsilon}_1$ and $\tilde{\varepsilon}_2$ are independent from other random variables. The unit variance assumption is to simplify expressions (it is not crucial and could be relaxed). The rating agency designs the rating R to maximize its expected profits. It chooses the rating design at $t = 0$, as specified in subsection 2.1 below, and it discloses the rating R at $t = 1$. Below we will consider different "business models" for the rating agency, depending on whether it sells its rating to the firm or to the investor.

The timeline of events is summarized in Figure 1.

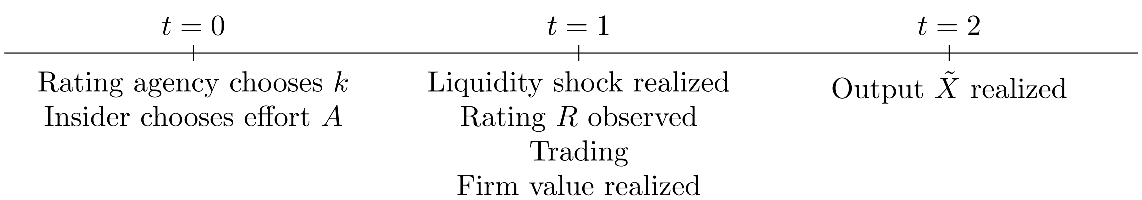

Figure 1: Timeline of the model.

1.4 Other interpretations

Throughout the paper, we will use the interpretation described above, in which output can be viewed as the firm's "profits", and the insider holds shares in the firm that he might have to sell before uncertainty is realized. However, the stylized model could also be interpreted in other ways. Below we sketch two possible reinterpretations.

First, in the context of ESG ratings, firm output could be viewed as its "social output" or its net externalities. As in equation (1), this has two components: one depends on the unknown firm type, the other depends on "effort", which can in this context be interpreted as mitigating actions. The firm's social output will be intrinsically valued and affect the stock price either when externalities expose the firm to future regulatory action, or when economic agents including insiders and investors have nonpecuniary preferences. In turn, this makes information about the firm's social output valuable.

Second, in the context of credit ratings, shares could be reinterpreted as debt, and firm output could be reinterpreted as a measure that captures not just the probability but also the severity of default. The higher this measure is, the lower the probability and the severity of default, and the higher the value of firm debt. In this context, the liquidity shock, which introduces a concern for the $t = 1$ valuation of claims on the firm's cash flows, could be more broadly interpreted as a possible liquidity need (e.g. a need for cash to continue operations). Moreover, this liquidity need can only be financed with debt because of prohibitive costs of equity issuance due either to equity market access or to a severe adverse selection problem on the equity market.

2 Analysis

2.1 Preliminaries

The objective of the rating agency is to maximize the revenues from its rating. We start by ruling out a simple rating design that maximizes information revelation. By contradiction, suppose that the rating agency sells separately two verifiable ratings on S_1 and S_2. The rating on S_1 is not valued by the investor in equilibrium, since the insider's effort is already known, i.e. this rating is not informative. The rating on S_1 is not valued by the insider either, since it is only a noisy measure of his effort, which is not informative and therefore not taken into account for the determination of firm value, even if it is disclosed publicly. Lemma 1 summarizes this reasoning.

Lemma 1 *Suppose that the rating agency sells two ratings which are linear functions of S_1 and S_2. The same information will be revealed by selling a single rating which is a linear function of S_1 and S_2.*

Selling two ratings which involve linearly independent combinations of the signals S_1 and S_2 is equivalent to selling only one rating on S_2. On the contrary, when a single rating depends both on firm type and effort, firm value is increasing in this rating if it is known by the investor, and the insider can increase the rating's value by increasing his effort. This increases his effort incentives, all the more that the rating has a large effect on firm value.

Lemma 1 implies that, considering linear ratings, there is no loss of generality in assuming that the rating agency designs a single rating which depends on signals S_1 and S_2. As in Hörner and Lambert (2021), we henceforth assume that the rating agency can commit to a rating design by choosing $k \in [0,1]$ such that its rating R is:

$$\tilde{R} = k\tilde{S}_1 + (1-k)\,\tilde{S}_2. \tag{4}$$

The rating design is parametrized by k, which captures the weight on each of the two signals. With $k > \frac{1}{2}$, the rating is more informative about effort; with $k < \frac{1}{2}$, the rating is more informative about the firm's type; in the special case $k = \frac{1}{2}$, the rating is simply a linear transformation of a noisy version of output, as is standard in career concerns model. For a given k, the variance of the rating is equal to:

$$\sigma_R^2 \equiv (1-k)^2\,\sigma_\theta^2 + k^2 + (1-k)^2\,. \tag{5}$$

The rating agency designs the rating by choosing parameter k.

2.2 Equilibrium when the rating is private information

Since the market maker observes the insider's market order and the total order flow, he can infer $q+u$, which is the part of the order flow emanating from the investor and the noise trader. Since the order from the insider is only driven by an exogenous shock whose probability is independent of firm value, it does not affect beliefs about firm value. Therefore, letting the event $l = 1-\delta$ be the insider selling his shares and $l = 0$ be the insider keeping his shares, belief updating is such that: $\mathbb{E}\left[\tilde{X}\middle|1+q+u, l = 1-\delta\right] = \mathbb{E}\left[\tilde{X}\middle|q+u, l = 0\right]$. As a result, regardless of whether the insider sells his shares, we define the equilibrium as follows:

Definition 1 *An equilibrium is a market order $q(R) = \alpha + \beta R$ and a price function $p(y) = \mu + \lambda y$ such that:*

$$q\,(R) = \arg\max_q \mathbb{E}\left[\left(\tilde{X} - p\,(q+u)\right)q\middle|R\right] \tag{6}$$

$$p(q+u) = \mathbb{E}\left[\tilde{X}\middle|q+u\right]. \tag{7}$$

As in the Kyle (1985) model, we consider linear equilibria.

Lemma 2 *When the investor privately observes the rating R, the stock market equilibrium is* *described by:*

$$\alpha = \frac{\sigma_u}{\sigma_R}\left(k\hat{A} + (1-k)\bar{\theta}\right) \quad (8)$$

$$\beta = \frac{\sigma_u}{\sigma_R} \quad (9)$$

$$\mu = \hat{A} + \bar{\theta} - \frac{(1-k)\,\sigma_\theta^2}{2\sigma_u\sigma_R} \quad (10)$$

$$\lambda = \frac{(1-k)\,\sigma_\theta^2}{2\sigma_u\sigma_R} \quad (11)$$

In particular, we show that the investor's optimal market order is:

$$q^*(R) = \frac{\mathbb{E}\left[\tilde{X}\middle|R\right] - (\mu+\lambda)}{2\lambda} \quad (12)$$

and the equilibrium price function is:

$$p(y) = \hat{A} + \bar{\theta} + \frac{\beta\,(1-k)\,\sigma_\theta^2}{\beta^2\sigma_R^2 + \sigma_u^2}\,(y - \mathbb{E}[\tilde{y}]) \quad (13)$$

Parameter λ, the sensitivity of the market price to the order flow, is a measure of market illiquidity. As in the Kyle (1985) model, it is increasing in σ_θ, which reflects the informational advantage of the investor, and decreasing in σ_u, which reflects the random part of the order flow. In addition, market illiquidity λ is decreasing in σ_R. Intuitively, holding σ_θ constant, a higher σ_R makes the investor's signal less precise, which makes her trade less informative, which reduces the price sensitivity to the order flow, i.e. the market is more liquid.

Parameter β represents the trading aggressiveness of the investor. As in Kyle (1985), it is increasing in the magnitude σ_u of noise trading. It is also decreasing is the standard deviation σ_R of the rating. This is for two reasons. First, σ_R is increasing in prior uncertainty about firm value, as measured by σ_θ. This is as in the Kyle model: the greater σ_θ, the more information is asymmetric, which reduces market liquidity, which reduces trading aggressiveness. Second, σ_R also measures the unreliability of the investor's information, which is driven by the noise in signals $\tilde{S}_1$ and $\tilde{S}_2$. This is different from the Kyle model, in which the insider is perfectly informed. Intuitively, prices move against the investor whenever she trades, so that she will refrain from trading based on unreliable information. Holding σ_θ constant, the greater σ_R, the more unreliable her information.

Let $\hat{A}$ be the expected effort of the insider, as opposed to his actual effort A. Even though $A = \hat{A}$ in equilibrium, implicit effort incentives emanating from the firm's market valuation depend on the relation between actual effort A and firm value. Accordingly, we start by expressing the expected market value of the firm as a function of A and $\hat{A}$.

Lemma 3 *The expected price of the firm when the investor purchases rating R is:*

$$\mathbb{E}\left[p \middle| A, \hat{A}\right] = \hat{A} + \bar{\theta} + \frac{1}{2}k(1-k)\frac{\sigma_\theta^2}{\sigma_R^2}\left(A - \hat{A}\right) \tag{14}$$

Informed trading on the market for firm shares at $t = 1$ generates implicit effort incentives for an insider who might need to sell his shares. Indeed, the market value of the firm will depend on the observed rating, which in turn depends on the insider's actual effort as long as $k > 0$. For $k = 0$, the rating is independent of effort and therefore does not provide effort incentives. For $k = 1$, the rating only depends on effort, not on firm type $\tilde{\theta}$, and it is consequently ignored in equilibrium, as already discussed. Therefore, it does not provide effort incentives either. As can be seen in equation (14), the weight k that maximizes the sensitivity of expected firm value to effort is in-between these two extremes. Moreover, the implicit effort incentives from market valuation are stronger when prior uncertainty about firm value, as measured by σ_θ, is higher ($\frac{\sigma_\theta^2}{\sigma_R^2}$ is strictly increasing in σ_θ, see equation (5)). Indeed, in this case, the rating is more informative about firm type $\tilde{\theta}$, and therefore it has a greater impact on firm valuation.

An investor who does not purchase the rating does not have an informational advantage, does not trade, and makes zero profits. By contrast, an investor who purchases the rating makes positive expected profit from informed trading, as described in Lemma 4.

Lemma 4 *The expected profit of the investor when she purchases rating R is $\frac{\sigma_u(1-k)\sigma_\theta^2}{2\sigma_R}$.*

This expected profit is important because it represents the investor's maximum willingness to pay for the rating.

We now analyse how the insider's effort depends on the rating. The first-best effort A^{FB} of the insider maximizes aggregate welfare, i.e. expected output minus the effort cost. It solves:

$$\max_A \{A - C(A)\} \tag{15}$$

which gives:

$$C'(A^{FB}) = 1 \tag{16}$$

The first-best outcome is trivially achieved at the second-best if the insider owns the whole firm and is never hit by a liquidity shock, i.e. $\delta = 0$ and $\pi = 0$. In this case, he knows ex-ante that he will own the whole firm ex-post, and therefore fully internalizes the effect of his effort on firm value.

Lemma 5 shows that, when the investor privately observes the rating and when uncertainty about firm value is sufficiently high, there are rating weights that induce the first-best effort.

Lemma 5 *When the investor privately observes the rating, the insider effort is first-best if* $\sigma_\theta \geq \sqrt{4\sqrt{\frac{\delta^2-2\delta+5}{(\delta-1)^4}}+\frac{8}{(\delta-1)^2}}$ *and* $k = k^{FB} \equiv \frac{(\delta-5)\sigma_\theta^2-4\pm\sqrt{(\delta-1)^2\sigma_\theta^4-16\sigma_\theta^2-16}}{2(\delta-3)\sigma_\theta^2-8}$.

The insider's effort is first-best when the implicit incentives from the resale value of the firm are sufficiently high to outweigh the insider's partial ownership of the firm (when $\delta > 0$). This is only possible for a subset of parameter values.

First, insider ownership $1-\delta$ must be high enough (see the right panel in Figure 2). Otherwise, there is no rating design such that market pricing based on the rating delivers first-best incentives. Intuitively, the insider's incentives can be decomposed into two components. First, the explicit incentives from his (stochastic) long-term ownership. These incentives are proportional to $(1-\delta)(1-\pi)$. Second, the implicit incentives from the firm's market valuation when he needs to liquidate his stake. These incentives are proportional to $(1-\delta)\pi$. By contrast, first-best effort incentives are independent of the manager's ownership $1-\delta$. As a result, when the insider does not own the whole firm, i.e. $1-\delta < 1$, a necessary condition to get first-best incentives is that the implicit incentives from market valuation be sufficiently stronger than the explicit incentives from the insider's long-term ownership.

Second, prior uncertainty about firm value, as captured by σ_θ, must be sufficiently high (see the left and right panels in Figure 2). Otherwise, the rating is not very informative, so that the market value of the firm is not very sensitive to the rating, and therefore the implicit incentives for effort emanating from the effect of the rating on the resale value of the firm cannot be high enough, for any rating weight k on the signal $\tilde{S}_1$ which is informative about effort.

If the first-best effort is achievable, there are two rating weights k^{FB} that induce it. Intuitively, there are two possibilities. The first is that the rating puts a high weight on effort, so that it is very informative about effort, but not very informative about firm value. Because of the latter, the price impact of the rating is small, so that the implicit incentives from the resale value of the firm are low. The second possibility is that the rating puts a relatively low weight on effort, so that it is not very informative about effort, but it is very informative about firm value. Because of the latter, the price impact of the rating is large, so that implicit incentives from the resale value of the firm are high.

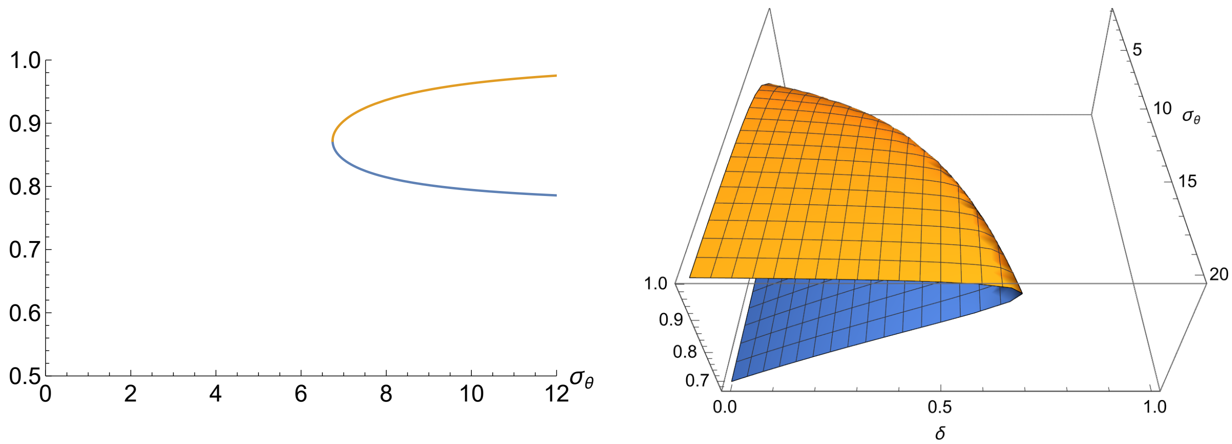

Figure 2: Left panel: rating weight k^{FB} that induces effort A^{FB}, as defined in Lemma 5, as a function of σ_θ, for $\delta = 0.25$ and $\pi = 0.5$. Right panel: rating weight k^{FB} as a function of σ_θ and δ. In each panel, the two different colors correspond to the two possible rating weights.

A rating that induces first-best effort with $k = k^{FB}$, if it exists, would be the rating used by a benevolent rating agency whose objective is to maximize social welfare. Of course, even if this rating exists, this is not necessarily the rating that arises at the second-best when the rating agency designs the rating to maximize its profits.

We now derive the level of effort that would be optimally chosen by the insider if he could commit to an observable effort level at $t = 0$. This level of effort, denoted by A^{LT}, solves:

$$\max_A \,(1-\delta)\left(\pi\mathbb{E}\left[p\middle|A,A\right] + (1-\pi)\left(A+\bar{\theta}\right)\right) - C(A) \quad \Leftrightarrow \quad \max_A \,\{(1-\delta)A - C(A)\} \tag{17}$$

where $\mathbb{E}\left[p\middle|A,A\right]$ is defined in Lemma 3 for $\hat{A} = A$. This gives:

$$C'(A^{LT}) = 1 - \delta \tag{18}$$

We call this level of effort A^{LT} because this is the level of effort that would be achieved at the second-best if the insider were never hit by a liquidity shock, i.e. if he had a long-term ("LT") horizon with $\pi = 0$. Instead, at the second-best when $\pi > 0$, since his effort is unobservable, the market value of the firm may fail to reflect his actual effort, depending on stock price informativeness. Consequently, the insider may not exert the level of effort that he would commit to if he could.

Lemma 6 shows that, if prior uncertainty about firm value is sufficiently high, there exists two rating weights such that the effort preferred by the insider under full commitment is achieved at the second-best when the rating is communicated to the investor only.

Lemma 6 *When the investor privately observes the rating, the insider exerts effort A^{LT} if $\sigma_\theta \geq 2\sqrt{2+\sqrt{5}}$ and $k = k^{LT} \equiv \frac{4+5\sigma_\theta^2 \pm \sqrt{\sigma_\theta^4 - 16 - 16\sigma_\theta^2}}{2(4+3\sigma_\theta^2)}$.*

The insider's optimal effort corresponds to his preferred effort under full commitment when the implicit incentives embedded in the rating are sufficiently high so that he exerts as much effort when he may have to sell the firm ($\pi > 0$) as he would if he never had to ($\pi = 0$). This is only possible if prior uncertainty about firm value, as captured by σ_θ, is sufficiently high – greater than $2\sqrt{2+\sqrt{5}} \approx 4.12$ (see Figure 3). The reason why there are two rating weights that induce this effort level is the same as for Lemma 5.

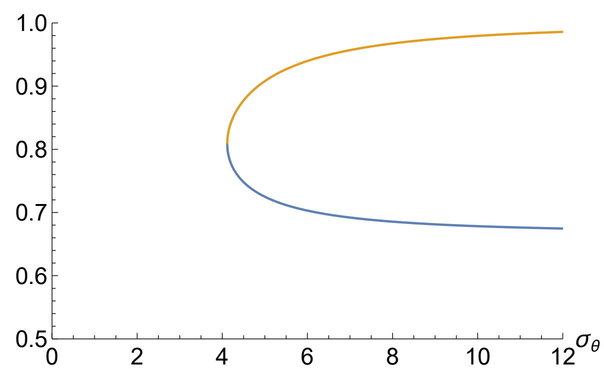

Figure 3: Rating weight k^{LT}, as defined in Lemma 6, as a function of σ_θ. The two lines with different colors correspond to the two possible rating weights.

We now consider the business model such that the rating agency sells the rating to the investor. Specifically, it sets the rating price and the rating weight, and makes a take-it-or-leave-it offer to the investor. This is the business model of ESG rating agencies, for example. It is also the business model for some equity research conducted by investment banks and brokerage houses, which is privately communicated to their investor clientele.

Proposition 1 *When the rating agency sells the rating to the investor, it optimally sets:*

$$k = k^{**} \equiv 0 \tag{19}$$

The investor who purchases the rating wants to maximize her informational advantage to generate trading profits. This is achieved by having a rating which is as informative as possible about firm type $\tilde{\theta}$. Indeed, information about effort is useless in that regard because it can be correctly anticipated in equilibrium. This implies that a rating that puts a positive weight k on the signal about effort would just be more noisy and less informative about $\tilde{\theta}$. As a result, the rating weight that maximizes the investor's willingness to pay is $k = k^{**} = 0$. It does not necessarily imply that the stock price will be most informative about the firm type under this business model, because the strategic trading of the informed investor reduces information transmission into the stock price.

We now consider the business model such that the firm insider pays the rating agency to communicate the rating to the investor. Specifically, the rating agency sets the rating price and the rating weight, and makes a take-it-or-leave-it offer to the insider. This case, which might be less practically relevant, is still useful as a benchmark against other business models. Define:

$$k^* \equiv \frac{1+\sigma_\theta^2 - \sqrt{1+\sigma_\theta^2}}{\sigma_\theta^2}, \qquad \sigma_\theta^* \equiv 2\sqrt{2+\sqrt{5}}. \tag{20}$$

Proposition 2 *When the insider pays the rating agency to communicate the rating to the investor, the rating agency optimally sets:*

$$k = \begin{cases} k^* & \text{if } \sigma_\theta < \sigma_\theta^* \\ k^{LT} & \text{if } \sigma_\theta \geq \sigma_\theta^* \end{cases} \tag{21}$$

The rating agency chooses the weight k of the rating to maximize the insider's willingness to pay. If the rating that induces the level of effort under full commitment is achievable, it is the most valuable rating for the insider. If it is not, then the most valuable rating is the one that maximizes the expected value of the firm when the investor trades based on the rating.

When prior uncertainty about firm value is sufficiently high, there are two equilibrium rating weights k^{LT}. To identify the equilibrium rating weight, we now let the rating agency have lexicographic preferences as follows.[12]

Definition 2 *An agency with lexicographic preferences will choose the rating weights to maximize the willingness to pay of the economic agent who purchases the rating, either the investor or the insider. When it is indifferent between several weights, it will choose the rating weights that maximize the willingness to pay of the other economic agent, either the insider or the investor.*

Corollary 1 *When the rating agency has lexicographic preferences and the insider pays for the rating to be communicated to the investor, the rating agency optimally sets:*

$$k = \begin{cases} k^* & \text{if } \sigma_\theta < \sigma_\theta^* \\ \frac{4+5\sigma_\theta^2 - \sqrt{\sigma_\theta^4 - 16 - 16\sigma_\theta^2}}{2(4+3\sigma_\theta^2)} & \text{if } \sigma_\theta \geq \sigma_\theta^* \end{cases} \tag{22}$$

Figure 4 depicts the rating weights corresponding to the business model in Proposition 2 and Corollary 1. It shows that, for any σ_θ, we have $k^* > k^{**}$, i.e. the weight on the signal informative about effort is always higher when the rating is designed to maximize the firm

[12]This could be motivated by the rating agency having to sell the rating to the other economic agent with a vanishingly small probability.

insider's willingness to pay. It also shows that, at $k = k^*$, the optimal weight on the signal S_1, which is informative about effort, is increasing in σ_θ. The greater the prior uncertainty about firm value, the more the rating is informative about the insider's actual effort. Intuitively, the greater the uncertainty about firm value, the more valuable is a rating informative about firm value, and the greater its price impact. As a result, as uncertainty about firm value increases, the rating agency can increase the weight on the signal S_1 about effort in the optimal rating. Indeed, this will allow the insider to commit to a higher level of effort while still maintaining a sufficiently high price impact of the rating.

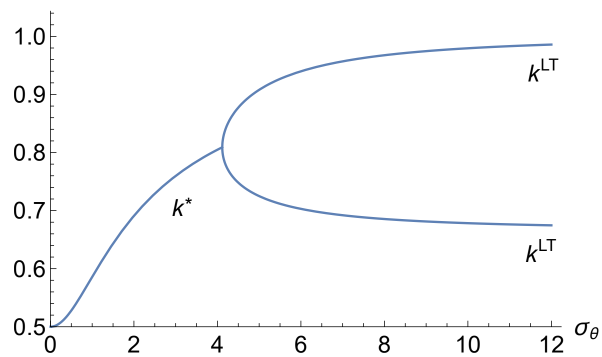

Figure 4: Optimal rating weight k from Proposition 2 (left panel) and Corollary 1 (right panel) as a function of σ_θ.

2.3 Equilibrium when the rating is public information

We now consider the business model such that the firm insider pays the rating agency, directly or indirectly, to give a public rating, observable by all market participants. This is the business model of credit rating agencies, for example. It is also the business model for some equity research conducted by investment banks, when it is made publicly available.

As in the previous subsection, the insider's effort incentives depend on the effect of his actual effort A on the expected market value of the firm.

Lemma 7 *The expected price of the firm when the rating R is public information is:*

$$\mathbb{E}\left[p \middle| A, \hat{A}\right] = \hat{A} + \bar{\theta} + k(1-k)\frac{\sigma_\theta^2}{\sigma_R^2}(A - \hat{A}) \tag{23}$$

The expression is similar to the one in Lemma 3. One difference is that the sensitivity of firm value to the rating, and therefore to actual effort, is twice as high in Lemma 7 relative to Lemma 3. In the case studied in Lemma 3, the informed investor trades strategically, which reduces price informativeness, which reduces the sensitivity of firm value to the actual effort of the insider. In Lemma 7, the rating is public information and therefore fully incorporated in the firm's market price. This emphasizes that, for any given positive rating weight k, implicit effort incentives are higher when the rating is public information (Lemma 7) than when it is privately observed by the investor (Lemma 3). Finally, as in Lemma 3, in Lemma 7 the implicit effort incentives from market valuation are stronger when prior uncertainty about firm value, as measured by σ_θ, is higher.

Lemma 8 shows that, when the rating is publicly disclosed and when uncertainty about firm value is sufficiently high, there are rating weights that induce the first-best effort defined in equation (16).

Lemma 8 *When the rating is publicly disclosed, the insider effort is first-best if* $\sigma_\theta \geq \Sigma(\delta, \pi)$ *and* $k = k_d^{FB}$*, where* $\Sigma(\delta, \pi)$ *and* k_d^{FB} *are defined in the proof of Lemma 8.*

Lemma 8 describes the rating weights that induce the first-best optimal effort. These rating weights are displayed in Figure 5. As when ratings are communicated privately to the investor (see Lemma 5 and Figure 2), insider ownership $1-\delta$ and prior uncertainty about firm value σ_θ, must be sufficiently high for these rating weights to exist. The difference is that these rating weights exist for a larger subset of parameter values when the rating is publicly disclosed. This is because a publicly disclosed rating will have a higher price impact. In particular, the market value of the firm will be more sensitive to the insider's actual effort, all else equal.

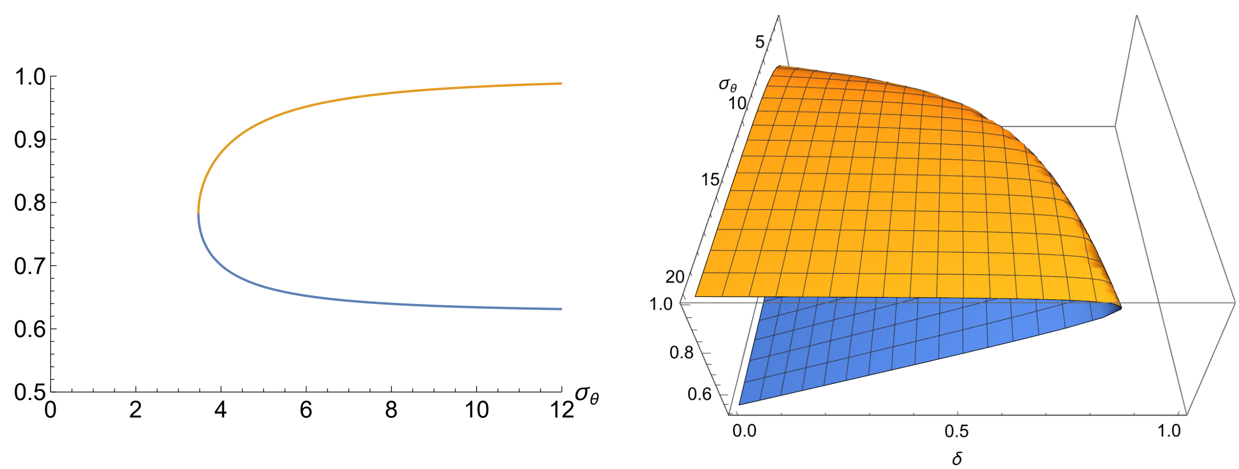

Figure 5: Left panel: rating weight k_d^{FB} that induces effort A^{FB}, as defined in Lemma 8, as a function of σ_θ, for $\delta = 0.25$ and $\pi = 0.5$. Right panel: rating weight k_d^{FB} as a function of σ_θ and δ. In each panel, the two different colors correspond to the two possible rating weights.

Lemma 9 *When the rating is publicly disclosed, the insider exerts effort* A^{LT} *if* $\sigma_\theta \geq \sigma_\theta^d \equiv \sqrt{2\left(1+\sqrt{2}\right)}$ *and* $k = k_d^{LT} \equiv \frac{3\sigma_\theta^2 \pm \sqrt{\sigma_\theta^4 - 4\sigma_\theta^2 - 4} + 2}{4\sigma_\theta^2 + 4}$.

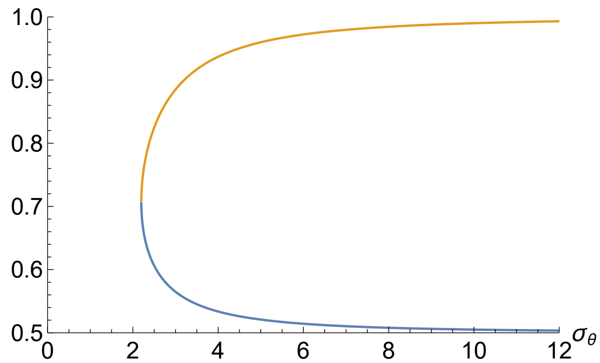

Figure 6: Rating weight k_d^{LT}, as defined in Lemma 9, as a function of σ_θ. The two lines with different colors correspond to the two possible rating weights.

The threshold above which there exists a rating weight that induces the level of effort under full commitment is lower when the rating is public information than when it is the investor's private information, i.e. $\sigma_\theta^d < \sigma_\theta^\star$. Indeed, in the latter case, the investor's strategic trading reduces the transmission of the information contained in the rating in the stock price.

Proposition 3 *When the insider pays the rating agency to publicly disclose the rating, the rating agency optimally sets:*

$$k = \begin{cases} k^* & \text{if } \sigma_\theta < \sigma_\theta^d \\ k_d^{LT} & \text{if } \sigma_\theta \geq \sigma_\theta^d \end{cases} \tag{24}$$

The rating weight is the same as in Proposition 2. Intuitively, the information mix preferred by the insider does not change when the rating is communicated to the investor as in Proposition 2, and when it is publicly revealed as in Proposition 3. These two cases are still different. As we make clear below, they have different implications for the transmission of information in market prices.

Corollary 2 *When the rating agency has lexicographic preferences and the insider pays for the rating to be publicly disclosed, the rating agency optimally sets:*

$$k = \begin{cases} k^* & \text{if } \sigma_\theta < \sigma_\theta^d \\ \frac{3\sigma_\theta^2 - \sqrt{\sigma_\theta^4 - 4\sigma_\theta^2 - 4} + 2}{4\sigma_\theta^2 + 4} & \text{if } \sigma_\theta \geq \sigma_\theta^d \end{cases} \tag{25}$$

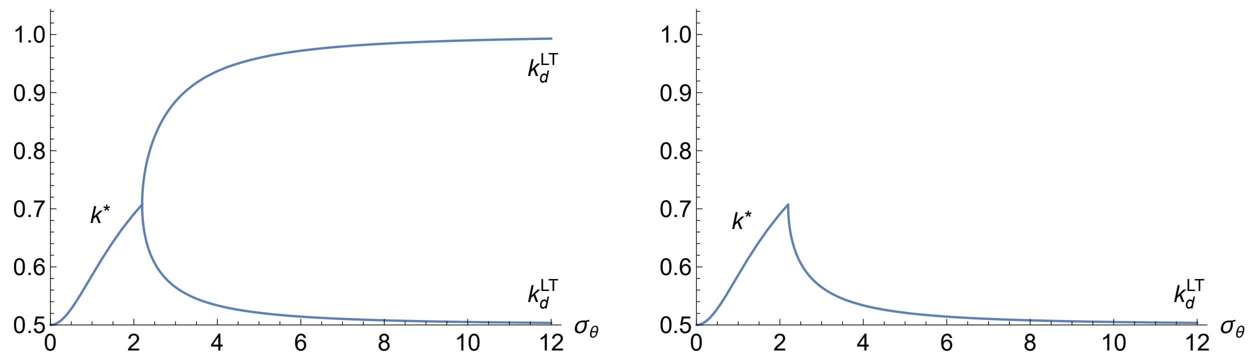

Figure 7: Optimal rating weight k from Proposition 3 (left panel) and Corollary 2 (right panel) as a function of σ_θ.

Corollary 3 *As σ_θ increases, one of the two optimal rating weights k_d^{LT} in Proposition 3 tends asymptotically to $\frac{1}{2}$:*

$$\lim_{\sigma_\theta \to \infty} \frac{3\sigma_\theta^2 - \sqrt{\sigma_\theta^4 - 4\sigma_\theta^2 - 4} + 2}{4\sigma_\theta^2 + 4} = \frac{1}{2} \tag{26}$$

The optimal rating corresponding to the lowest value of k_d^{LT} in Proposition 3 converges asymptotically to $k = \frac{1}{2}$ as uncertainty about firm value increases. For $k = \frac{1}{2}$, the rating is simply a (linear transformation of a) noisy version of output. Thus, as uncertainty about firm value is sufficiently high, it is approximately optimal to provide a rating which is simply informative about output. This rating will approximately induce the level of effort under full commitment.

2.4 Economic efficiency

The economic efficiency of various business models depends on their implications for the insider's equilibrium effort.

Figure 8 depicts the equilibrium insider effort corresponding to the three different business models and the associated optimal ratings derived in Propositions 1-3.

When the rating is designed to maximize the investor's willingness to pay, the equilibrium effort, A_1, is minimal, and proportional to the probability that the insider does not need to sell the firm. Indeed, the rating is uninformative about the insider's effort, so that the market valuation of the firm does not reflect his actual effort. Therefore an insider who always had to sell the firm ($\pi = 1$) would not have any effort incentives. More generally, effort incentives are decreasing in the probability π that the insider needs to exit.

When the rating is instead designed to maximize the insider's willingness to pay, the equilibrium effort, A_2 or A_3, is weakly increasing in the uncertainty about firm value σ_θ. When uncertainty is higher than a threshold, equilibrium effort is equal to the optimal effort under commitment, as already established. When uncertainty is lower than the threshold, the rating has a low price impact even when optimally designed, so that implicit incentives from the firm's resale value are low. In this case, the weight k on the signal informative about effort is increasing in the uncertainty σ_θ about firm value (see Figure 4). The greater the uncertainty, the more firm value is sensitive to the insider's actual effort, and the greater his effort incentives. In sum, a change in the probability π of the liquidity shock only matters for effort incentives and economic efficiency when σ_θ is low. When σ_θ is higher than a threshold, the effort under full commitment is feasible at the second-best when information is designed to cater to the insider, so that the probability π of the insider's exit does not matter. In simple words, the time horizon of corporate insiders does not matter for economic efficiency.

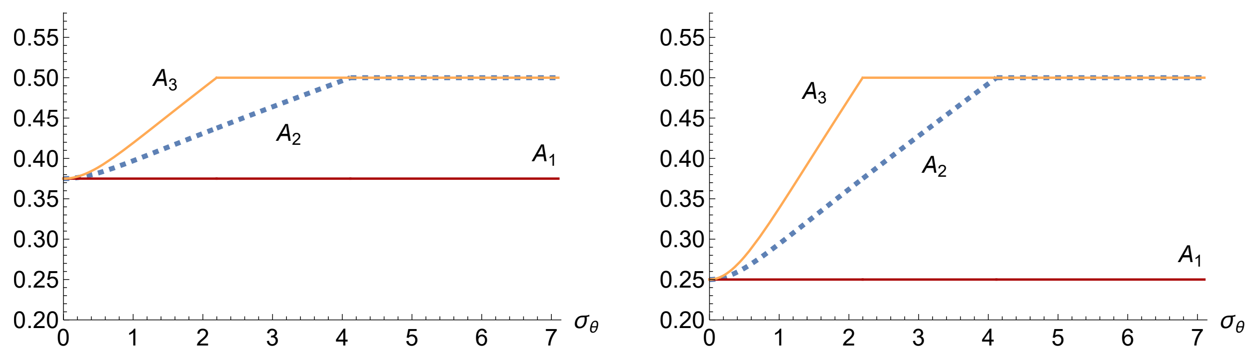

Figure 8: Equilibrium effort A_1, A_2, and A_3 corresponding to the equilibrium and rating weights as in Propositions 1, 2, and 3, respectively, as a function of σ_θ, with $C(A) = \frac{A^2}{2}$, $\delta = 0.5$, $\pi = 0.25$ (left panel), and $\pi = 0.5$ (right panel).

Interestingly, we don't have the result from career concerns models that the agent's short-term effort can be "excessive" when one of the purposes of effort is to improve beliefs about his ability. Our model is similar in that regard, with the slight difference that the insider is not concerned about his ability per se but about the type of his firm, which matters to the extent that he may need to sell it. There are two possibilities in our model. Either the rating is chosen to maximize the investor's willingness to pay, which generates low effort incentives, as already explained. Or the rating is chosen to maximize the insider's willingness to pay. In this latter case, the insider will never choose a rating such that his own effort incentives are higher than the preferred level under full commitment. Put differently, the implicit incentives generated via the market valuation of the firm will never be greater than the intrinsic effort incentives of a long-term insider, even though they could be for other (nonoptimal) rating weights.

2.5 Rating selection

We now determine the equilibrium outcome, by showing how the equilibrium business model depends on model parameters.

Proposition 4 *The rating agency optimally sells the rating to the investor rather than the firm insider if σ_u is sufficiently high or π is sufficiently low.*

In an illiquid stock market with a low σ_u, the expected profit of the informed investor is very low, and so is her willingness to pay for a rating. On the contrary, if the stock market is sufficiently liquid, the rating agency can sell its rating at a higher price to the investor than to the firm insider. Now consider the insider's liquidity shock, which proxies for his time horizon. If $\pi = 0$, the insider does not value the rating: he will never sell the firm and therefore does not care about its market value. As long as π is sufficiently low, the insider's willingness to pay is so low that the rating agency will rather sell a rating to the investor.

3 Competition between rating agencies

We now extend the model to address the possibility of competition between rating agencies and its possible side effects. We are considering a setting similar to the one described in section 1, except that a second rating agency can now decide to enter. After the first agency has designed and sold its rating $R_1(k_1)$, the second agency can acquire at cost C the same signals S_1 and S_2 and can design its rating $R_2(k_2)$ to sell to the firm's insider or to the investor.[13]

The first rating $R_1(k_1)$ provides valuable information to the investor. However, if a second agency provided a second rating $R_2(k_2)$, then the investor could use both ratings to recover the initial signals S_1 and S_2, as shown in the proof of Lemma 1. Thus, the second rating would be valuable for the investor. From Lemma 4, the expected profit of the investor when she purchases rating $R_1(k_1)$ is $\frac{\sigma_u(1-k)\sigma_\theta^2}{2\sigma_R}$. With a second rating, the investor's expected profit would be $\frac{\sigma_u\sigma_\theta^2}{2\sqrt{1+\sigma_\theta^2}}$. An investor who observed both ratings would then behave by only taking into account the information provided by S_2, i.e., as if the rating were designed with $k = 0$. In turn, this would minimize the insider's willingness to pay for the first rating. The first rating agency will therefore try to prevent entry by another rating agency.

We denote $\Delta(k_1)$ the additional value to the investor of a second rating. We have $\Delta(k_1) = 0$ if $k_1 = 0$. The second agency, after observing the first rating $R_1(k)$, acquires information only if a second rating is sufficiently valuable to the investor. The equilibrium depends on model parameters.

[13] A potential benefit of an additional rating agency is that it provides different information. We turn off this well-understood channel to focus on a new effect.

For high values of σ_u, the first agency designs a rating with $k_1 = 0$ even without the threat of entry of a second agency (see Proposition 4). Then, the second agency does not acquire information since the willingness to pay of any economic agent for an additional rating is zero.

For low values of σ_u, information has little value to investors. In that case, the first agency will sell a rating designed for the firm's insider as derived in Proposition 2 or 3. A second agency would not make enough profit by selling a second rating to investors to cover the cost of information acquisition. This happens when $\Delta(k_1) \leq C$. Using terminology from the industrial organization literature, these first two possibilities involve "blockaded entry".

For intermediate values of σ_u, the first agency will deter entry by altering its optimal rating k_1. This happens when the first agency sells a rating to the insider without the threat of entry and when $\Delta(k_1) \geq C$, i.e., when the second agency would enter profitably in that case. The first agency either chooses to sell a rating with $k = 0$ to the investor or will modify the design of the rating sold to the insider. The first agency then decreases the value of k so that $\Delta(k_1) = C$ thus making the entry of the second agency unprofitable.

We summarize this discussion in the following Proposition.

Proposition 5 *When a second rating agency can acquire information and design another rating, the first rating agency may adjust the design of its rating to prevent entry. The first agency blockades entry when σ_u is sufficiently high, by selling a rating to the investor with $k_1 = 0$, and when σ_u is sufficiently low, by selling to the firm insider the rating that maximizes his willingness to pay. Otherwise, the first agency deters entry by selling a rating with $k = 0$ to the investor.*

This result shows that bad information, with no efficiency implications, can drive out good information. The threat of entry by a second agency that provides information about the firm's type can distort the rating design away from economic efficiency. This is especially a concern in an era when the raw data necessary for information production are easily accessible (Gao and Huang (2020)).

4 Empirical predictions

The first two empirical implications relate the business model of rating agencies to the type of information that they will produce.

Empirical implication 1 *When financial information is designed for investors, it is only designed to reduce uncertainty about firm value.*

Empirical implication 2 *When financial information is designed for issuers, it is designed to reduce uncertainty about firm value and to reflect the actions taken by firm insiders.*

The third empirical implication emphasizes the nonmonotonic relation between uncertainty about firm value and the weight on the signal that is informative about effort in the equilibrium rating. When uncertainty is very low, the rating must be sufficiently informative about the firm type to have enough stock price impact. When uncertainty is very high, the rating is already very informative about the firm type and has a correspondingly high stock price impact, so that it should not be too informative about effort lest it induces excessive effort.[14]

Empirical implication 3 *When financial information is designed for issuers, it is most informative about effort for intermediary levels of uncertainty about firm value.*

The fourth empirical implication points out that strategic trading by investors does not necessarily reduce economic efficiency as long as there is enough uncertainty about firm value. Indeed, in this case, the rating weight can be such that the level of effort induced is the level that would be chosen by an insider who could fully commit to effort. Put differently, if there is enough uncertainty and issuers are paying for ratings, the design of ratings will be such that economic efficiency does not depend on which market participants observe the ratings.

Empirical implication 4 *When financial information is designed for issuers, whether it is privately communicated to investors or publicly disclosed only matters when there is not much uncertainty about firm value.*

The fifth empirical implications shows that a short-term horizon by corporate insiders (as reflected in a high π) does not necessarily reduce economic efficiency. The reason is as above: if there is enough uncertainty and issuers are paying for ratings, rating design will be such that the level of effort induced will be the level that would be chosen by an insider who could fully commit to effort. That is, the outcome does not depend on the insider's horizon – the time horizon irrelevance result.

Empirical implication 5 *The insider's effort does not depend on his time horizon if financial information is designed for issuers and if uncertainty about firm value is sufficiently high.*

This suggests that, when financial information is designed optimally, the short investment horizon of some key investors is not necessarily costly. For example, venture capitalists have limited horizons, yet they and the firms they support are often very successful. Our model emphasizes that two conditions are especially important: the optimal design of financial information, and the substantial uncertainty about the assets financed. This is consistent with the

[14] This refers to the equilibria studied in Corollaries 1 and 2. In the other equilibria in Propositions 2 and 3, the weight on the signal about effort can alternatively be very high when uncertainty is very high. Indeed, this reduces the stock price impact of the rating, so that effort incentives are not excessive.

business model of venture capitalists, who finance firms early in their life cycle, especially firms with a very uncertainty future, and generally liquidate their stake before the firm has reached its full potential. For example, venture capitalists tend to finance "technological revolutions" (Lerner and Nanda (2020)), in which the idiosyncratic uncertainty about each firm's potential is compounded by uncertainty about the space that the firm is trying to enter. Thus, our model can help explain the narrow focus of venture capitalists noted by Lerner and Nanda (2020),[15] and the finding that VC funding is especially beneficial for firms that face high uncertainty (Park and Steensma (2012)).

The sixth empirical implication studies the informativeness of ratings depending on the business model. It focuses on the informativeness of the rating, not the informativeness of the stock price at $t = 1$ (it is therefore unrelated to strategic trading by the informed investor).

Empirical implication 6 *When financial information is designed for issuers, it is more informative about firm type when it is publicly disclosed than when it is communicated to the investor.*

The seventh empirical implication considers the decision of potential entrepreneurs to found firms. In our model, the equilibrium level of effort might be lower than the level of effort that the insider would exert if he could commit to effort. In addition, the business model of rating agencies and associated design of financial information has implications for the equilibrium level of effort, which matters for the insider's utility. It matters all the more that the insider sells his stake with a high probability (as can be seen by comparing the two panels in Figure 9). In turn, although we do not model this decision, this has straightforward implications for the willingness of potential entrepreneurs to found firms – assuming that a higher insider utility helps in that regard. We summarize this reasoning in the empirical implication below.

Empirical implication 7 *Would-be entrepreneurs are more likely to found companies when financial information is designed for insiders and uncertainty is sufficiently high.*

[15] *The Economist* also notes that "historically, venture investors shied away from supporting hardware industries, especially those like defence that can gobble up lots of capital." Source: The warrior spirit, Feb 15 2025.

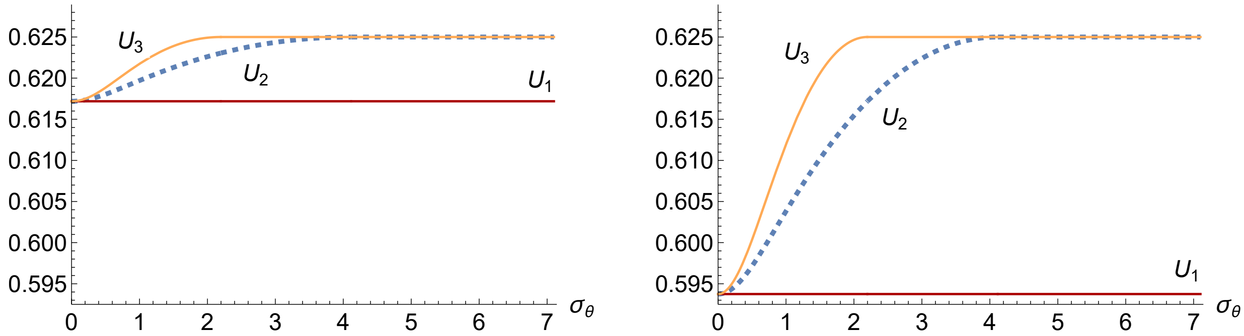

Figure 9: Insider's expected utility U_1, U_2, and U_3 corresponding to the equilibrium and rating weights as in Propositions 1, 2, and 3, respectively, as a function of σ_θ, with $C(A) = \frac{A^2}{2}$, $\delta = 0.5$, $\pi = 0.25$ (left panel), and $\pi = 0.5$ (right panel).

5 Appendix

Proof of Lemma 1:

The investor uses the ratings to update her beliefs about the value of the firm. As the signal S_1 does not provide additional information about θ, traders will disregard an additional rating based solely on S_1. Now, suppose that the investor has access to two ratings, R and R', which are independent linear combinations of signals S_1 and S_2 with weights k and k'.

$$R = kS_1 + (1-k)S_2. \tag{27}$$

$$R' = k'S_1 + (1-k')S_2. \tag{28}$$

From observing R and R', the investor can recover signals S_1 and S_2. We get:

$$S_1 = \frac{(1-k')R - (1-k)R'}{k-k'} \tag{29}$$

$$S_2 = \frac{kR' - k'R}{k-k'} \tag{30}$$

The investor will then use only S_2 to update her beliefs about θ. This is equivalent to providing a rating with $k = 0$. ■

Proof of Lemma 2:

The investor solves:

$$\max_q \mathbb{E}\left[(\tilde{X} - p(\tilde{y}))q \middle| R\right]$$

The second-order condition for a maximum is satisfied, so the optimum $q^*(R)$ is described by the first-order necessary condition:

$$q^*(R) = \frac{\mathbb{E}\left[\tilde{X}\middle|R\right] - (\mu+\lambda)}{2\lambda} \tag{31}$$

We have:

$$\mathbb{E}[\tilde{X}|R] = \hat{A} + \bar{\theta} + \frac{(1-k)\,\sigma_\theta^2}{\sigma_R^2}\left(R - k\hat{A} - (1-k)\,\bar{\theta}\right) \tag{32}$$

Denoting $b_I \equiv \frac{(1-k)\sigma_\theta^2}{\sigma_R^2}$, we have:

$$q^*(R) = \frac{\hat{A} + \bar{\theta} - (\mu+\lambda)}{2\lambda} + \frac{b_I\left(R - k\hat{A} - (1-k)\,\bar{\theta}\right)}{2\lambda} \tag{33}$$

So we get

$$\beta = \frac{b_I}{2\lambda} \tag{34}$$

$$\alpha = \frac{\hat{A} + \bar{\theta} - (\mu + \lambda)}{2\lambda} - \frac{b_I \left(k\hat{A} + (1-k)\,\bar{\theta}\right)}{2\lambda} \tag{35}$$

The price set by the market maker corresponds to the expected value of the firm given the total order flow observed.

$$\begin{aligned} p(y) &= \mathbb{E}\left[\tilde{X} \middle| y = 1 + q + u\right] \\ &= \hat{A} + \bar{\theta} + \frac{\beta\,(1-k)\,\sigma_\theta^2}{\beta^2(\sigma_R^2) + \sigma_u^2}\,(y - \mathbb{E}[\tilde{y}]) \end{aligned} \tag{36}$$

In particular:

$$p\,(\mathbb{E}[\tilde{y}]) = \mu + \lambda \left(1 + \alpha + \beta \left(k\hat{A} + (1-k)\,\bar{\theta}\right)\right) \tag{37}$$

We get:

$$\begin{aligned} \lambda &= \frac{\beta\,(1-k)\,\sigma_\theta^2}{\beta^2(\sigma_R^2) + \sigma_u^2} \\ \mu &= \hat{A} + \bar{\theta} - \lambda \left(1 + \alpha + \beta \left(k\hat{A} + (1-k)\,\bar{\theta}\right)\right. \end{aligned}$$

Solving for α, β, λ and μ yields:

$$\beta = \frac{\sigma_u}{\sigma_R} \tag{38}$$

$$\lambda = \frac{(1-k)\,\sigma_\theta^2}{2\sigma_u\sigma_R} \tag{39}$$

$$\alpha = \beta\mathbb{E}[\tilde{R}] = \beta\left(k\hat{A} + (1-k)\,\bar{\theta}\right) \tag{40}$$

$$\mu = \mathbb{E}[\tilde{X}] - \lambda = \hat{A} + \bar{\theta} - \lambda \tag{41}$$

■

Proof of Lemma 3:

The expected price of the firm as a function of the expected effort of the insider, $\hat{A}$, and his actual effort, A, is:

$$\begin{aligned}\mathbb{E}\left[p|A,\hat{A}\right] &= \mathbb{E}\left[\mu+\lambda y\right]\\ &= \mu+\lambda+\lambda\beta\mathbb{E}\left[R-\mathbb{E}[\tilde{R}]\right]\\ &= \hat{A}+\bar{\theta}+\frac{1}{2}\left(1-k\right)k\frac{\sigma_\theta^2}{\sigma_R^2}\left(A-\hat{A}\right)\end{aligned}$$

■

Proof of Lemma 4:

The expected (ex-ante) profit of the investor who is trading according to rating R is:

$$\begin{aligned}\mathbb{E}\left[\left(\tilde{X}-p(1+q+u)\right)q\right] &= \mathbb{E}\left[(\tilde{X}-\mu-\lambda-\lambda\beta(R-\mathbb{E}[\tilde{R}]))(\lambda\beta(R-\mathbb{E}[\tilde{R}])\right]\\ &= \frac{\sigma_u}{\sigma_R}\left(1-k\right)\sigma_\theta^2-\lambda\beta^2\sigma_R^2\\ &= \frac{\sigma_u\left(1-k\right)\sigma_\theta^2}{2\sigma_R}\end{aligned} \tag{42}$$

■

Proof of Lemma 5:

At the second-best, when the liquidity shock occurs with probability π, the insider maximizes:

$$\max_A\left(1-\delta\right)\left(\left(1-\pi\right)\left(A+\bar{\theta}\right)+\pi\left(\left(\hat{A}+\bar{\theta}\right)+\lambda\beta k\left(A-\hat{A}\right)\right)\right)-C(A) \tag{43}$$

The second-order condition for a maximization problem is satisfied, so that the optimal effort is given by the first-order necessary condition:

$$C'(A)=\left(1-\delta\right)\left(1-\pi+\pi\lambda\beta k\right) \tag{44}$$

From the solution to the insider's maximization problem in equation (44) and the definition of the first-best effort in equation (16), the rating induces the first-best effort if and only if:

$$\left(1-\delta\right)\left(1-\pi+\pi\lambda\beta k\right)=1 \quad\Leftrightarrow\quad \lambda\beta k=1+\frac{1}{\pi}\frac{\delta}{1-\delta} \tag{45}$$

Substituting for λ and β, the condition is:

$$\frac{k\left(1-k\right)\sigma_\theta^2}{2\sigma_R^2}=1+\frac{1}{\pi}\frac{\delta}{1-\delta} \quad\Leftrightarrow\quad k\left(1-k\right)\sigma_\theta^2=2\left(\left(1-k\right)^2\left(\sigma_\theta^2+1\right)+k^2\right)\left(1+\frac{1}{\pi}\frac{\delta}{1-\delta}\right)$$

Solving for k gives the optimal weights in Lemma 5. ■

Proof of Lemma 6:

At the second-best, when the liquidity shock occurs with probability π, the insider maximizes:

$$\max_{A} \, (1-\delta)\left((1-\pi)\left(A+\bar{\theta}\right)+\pi\left(\left(\hat{A}+\bar{\theta}\right)+\lambda\beta k\left(A-\hat{A}\right)\right)\right)-C(A) \tag{46}$$

The second-order condition for a maximization problem is satisfied, so that the optimal effort is given by the first-order necessary condition:

$$C'(A)=(1-\delta)\,(1-\pi+\pi\lambda\beta k) \tag{47}$$

From the solution to the insider's maximization problem in equation (47) and the definition of the effort preferred by the insider in equation (18), the rating induces the effort preferred by the insider if and only if:

$$(1-\delta)\,(1-\pi+\pi\lambda\beta k)=1-\delta \quad \Leftrightarrow \quad \lambda\beta k=1 \tag{48}$$

Substituting for λ and β, the condition is:

$$\frac{k\,(1-k)\,\sigma_\theta^2}{2\sigma_R^2}=1 \qquad \Leftrightarrow \qquad k\,(1-k)\,\sigma_\theta^2=2\left((1-k)^2\left(\sigma_\theta^2+1\right)+k^2\right) \tag{49}$$

Solving for k gives the optimal weights in Lemma 6. ■

Proof of Proposition 1:

$$\max_{k}\left(\frac{\sigma_u\,(1-k)\,\sigma_\theta^2}{2\sigma_R}\right) \tag{50}$$

Since σ_u and σ_θ are constants, and substituting for σ_R, this is equivalent to:

$$\max_{k}\left(\frac{1-k}{\sqrt{(1-k)^2\left(\sigma_\theta^2+1\right)+k^2}}\right) \tag{51}$$

The optimal weight k^{**} is given by:

$$k=k^{**}\equiv 0 \tag{52}$$

■

Proof of Proposition 2:

Given equilibrium effort $\hat{A}$ induced by the contract, the firm's willingness to pay is maximized by solving:

$$\max_k (1-\delta)\left(\pi\left(\hat{A}+\bar{\theta}\right)+(1-\pi)\left(\hat{A}+\bar{\theta}\right)\right)-C(\hat{A}) \quad \Leftrightarrow \quad \max_k (1-\delta)\hat{A}-C(\hat{A}) \tag{53}$$

The first-order condition is:

$$(1-\delta-C'(\hat{A}))\frac{d\hat{A}}{dk}=0 \tag{54}$$

There are two cases.

First, if parameters are such that there exists a rating weight k that induces $1-\delta=C'(\hat{A})$, then the optimal rating weight is implicitly determined to induce $1-\delta=C'(\hat{A})$, and it is derived in Lemma 6. This maximizes the insider's objective function in equation (53).

Second, if parameters are such that there does not exist a rating weight k that induces $1-\delta=C'(\hat{A})$, the first-order condition in equation (54) involves maximizing:

$$\max_k \lambda\beta k=\frac{k(1-k)}{\sigma_R^2}=\frac{k(1-k)}{(1-k)^2\left(\sigma_\theta^2+1\right)+k^2} \tag{55}$$

The solutions are thus the roots of:

$$k^2(\sigma_\theta^2)-2(\sigma_\theta^2+1)k+\left(\sigma_\theta^2+1\right) \tag{56}$$

We find:

$$k=k^*\equiv\frac{\sigma_\theta^2+1\pm\sqrt{1+\sigma_\theta}}{\sigma_\theta^2} \tag{57}$$

The maximum is reached for the smaller root. The larger root corresponds to a minimum. ■

Proof of Lemma 7:

With a public rating, the market maker sets the market value of the firm equal to its expected output given the rating. The order flows are irrelevant as they do not include any additional information about firm value. Given rating R, the market value of of the firm is:

$$\begin{aligned} p &= \hat{A}+\mathbb{E}\left[\tilde{\theta}\middle|R\right] \\ &= \hat{A}+\bar{\theta}+\frac{(1-k)\sigma_\theta^2}{\sigma_R^2}\left(R-\mathbb{E}\left[\tilde{R}\right]\right) \end{aligned}$$

The expected market value conditional on actual effort A and expected effort $\hat{A}$ is:

$$\mathbb{E}\left[p\middle|A,\hat{A}\right] \;=\; \hat{A}+\bar{\theta}+k(1-k)\frac{\sigma_\theta^2}{\sigma_R^2}(A-\hat{A}) \tag{58}$$

■

Proof of Lemma 8:

At the second-best, when the liquidity shock occurs with probability π, the insider maximizes:

$$\max_{A}\ (1-\delta)\left((1-\pi)\left(A+\bar{\theta}\right)+\pi\left(\hat{A}+\bar{\theta}+k(1-k)\frac{\sigma_\theta^2}{\sigma_R^2}(A-\hat{A})\right)\right)-C(A) \tag{59}$$

The second-order condition for a maximization problem is satisfied, so that the optimal effort is given by the first-order necessary condition:

$$C'(A)=(1-\delta)\left(1-\pi+\pi k(1-k)\frac{\sigma_\theta^2}{\sigma_R^2}\right) \tag{60}$$

From the solution to the insider's maximization problem in equation (60) and the definition of the first-best effort in equation (16), the rating induces the first-best effort if and only if:

$$(1-\delta)\left(1-\pi+\pi k(1-k)\frac{\sigma_\theta^2}{\sigma_R^2}\right)=1 \quad\Leftrightarrow\quad k(1-k)\frac{\sigma_\theta^2}{\sigma_R^2}=\frac{1}{\pi}\frac{\delta}{1-\delta}+1 \tag{61}$$

Solving for k gives the optimal weights $k=k_d^{FB}$, which exist for $\sigma_\theta\geq\Sigma(\delta,\pi)$, where k_d^{FB} and $\Sigma(\delta,\pi)$ are defined as:

$$\begin{aligned}
k_d^{FB} &\equiv \frac{\frac{2\delta\sigma_\theta^2}{(1-\delta)\pi}+\frac{2\delta}{(1-\delta)\pi}+3\sigma_\theta^2+2\pm\sqrt{-\frac{4\delta^2\sigma_\theta^2}{(1-\delta)^2\pi^2}-\frac{4\delta^2}{(1-\delta)^2\pi^2}-\frac{8\delta\sigma_\theta^2}{(1-\delta)\pi}-\frac{8\delta}{(1-\delta)\pi}+\sigma_\theta^4-4\sigma_\theta^2-4}}{2\left(\frac{\delta\sigma_\theta^2}{(1-\delta)\pi}+\frac{2\delta}{(1-\delta)\pi}+2\sigma_\theta^2+2\right)}\\
\Sigma(\delta,\pi) &\equiv \left(\frac{2\left(\delta^2\pi^2-2\delta^2\pi+\delta^2-2\delta\pi^2+2\delta\pi+\pi^2\right)}{(\delta-1)^2\pi^2}\right.\\
&\quad\left.+2\sqrt{\frac{2\delta^4\pi^4-6\delta^4\pi^3+7\delta^4\pi^2-4\delta^4\pi+\delta^4-8\delta^3\pi^4+18\delta^3\pi^3-14\delta^3\pi^2+4\delta^3\pi+12\delta^2\pi^4-18\delta^2\pi^3+7\delta^2\pi^2-8\delta\pi^4+6\delta\pi^3+2\pi^4}{(\delta-1)^4\pi^4}}\right)^{\frac{1}{2}}
\end{aligned}$$

■

Proof of Lemma 9:

At the second-best, when the liquidity shock occurs with probability π, the insider maximizes:

$$\max_{A}\ (1-\delta)\left((1-\pi)\left(A+\bar{\theta}\right)+\pi\left(\hat{A}+\bar{\theta}+k(1-k)\frac{\sigma_\theta^2}{\sigma_R^2}(A-\hat{A})\right)\right)-C(A) \tag{62}$$

The second-order condition for a maximization problem is satisfied, so that the optimal effort is given by the first-order necessary condition:

$$C'(A)=(1-\delta)\left(1-\pi+\pi k(1-k)\frac{\sigma_\theta^2}{\sigma_R^2}\right) \tag{63}$$

From the solution to the insider's maximization problem in equation (63) and the definition of the effort preferred by the insider in equation (18), the rating induces the effort preferred by the insider if and only if:

$$(1-\delta)\left(1-\pi+\pi k(1-k)\frac{\sigma_\theta^2}{\sigma_R^2}\right)=1-\delta \quad\Leftrightarrow\quad k\,(1-k)\,\sigma_\theta^2=\left((1-k)^2\left(\sigma_\theta^2+1\right)+k^2\right) \tag{64}$$

Solving for k gives the optimal weights in Lemma 9. ■

Proof of Proposition 3:

The insider chooses effort to maximize:

$$\max_A \,(1-\delta)\left((1-\pi)\left(A+\bar\theta\right)+\pi\left(\hat A+\bar\theta+k(1-k)\frac{\sigma_\theta^2}{\sigma_R^2}(A-\hat A)\right)\right)-C(A) \tag{65}$$

The second-order condition for a maximum is satisfied. At the time of exerting effort, the insider takes the rating k as given. The optimal effort, $\hat A$, is described by the first-order necessary condition:

$$C'(\hat A)=(1-\delta)\left(1-\pi+\pi k(1-k)\frac{\sigma_\theta^2}{\sigma_R^2}\right) \tag{66}$$

The agency chooses k to maximize the insider's objective function and thus his willingness to pay for the rating.

$$\max_k\,((1-\delta)\hat A-C(\hat A)) \tag{67}$$

The first-order condition is:

$$(1-\delta-C'(\hat A))\frac{d\hat A}{dk}=0 \tag{68}$$

There are two cases.

First, if parameters are such that there exists a rating weight k that induces $1-\delta=C'(\hat A)$, then the optimal rating weight is implicitly determined to induce $1-\delta=C'(\hat A)$, and it is derived in Lemma 6. This maximizes the insider's objective function in equation (65).

Second, if parameters are such that there does not exist a rating weight k that induces $1-\delta=C'(\hat A)$, the first-order condition in equation (68) involves maximizing:

$$\max_k \frac{k(1-k)}{\sigma_R^2}=\frac{k(1-k)}{(1-k)^2\left(\sigma_\theta^2+1\right)+k^2} \tag{69}$$

The first-order necessary conditions are

$$\frac{1+\sigma_\theta^2+k^2\sigma_\theta^2-2k(1+\sigma_\theta^2)}{(\sigma_R^2)^2}=0 \tag{70}$$

The solutions are thus the roots of

$$1 + \sigma_\theta^2 + k^2\sigma_\theta^2 - 2k(1 + \sigma_\theta^2) \tag{71}$$

We find:

$$k = k^* \equiv \frac{1 + \sigma_\theta^2 \pm \sqrt{1 + \sigma_\theta^2}}{\sigma_\theta^2} \tag{72}$$

The maximum is reached for the smaller root. The larger root corresponds to a minimum. ■

Proof of Proposition 4:

First, with $k = k^{**} = 0$, the expected profit of the investor derived in Lemma 4 becomes:

$$\frac{\sigma_u \sigma_\theta^2}{2\sigma_R} = \frac{\sigma_u}{2} \frac{\sigma_\theta^2}{\sqrt{\sigma_\theta^2 + 1}}, \tag{73}$$

which is linearly increasing in σ_u. Since the rating agency makes a take-it-or-leave-it offer, it captures the surplus from the investor's valuation.

Second, the insider's valuation of any given rating is linear in π. With $\pi = 0$, the insider does not value any rating. ■

References

Admati, A.R., and Pfleiderer, P., 1986. A monopolistic market for information. *Journal of Economic Theory*, 39(2), 400-438.

Admati, A.R., and Pfleiderer, P., 1988. Selling and trading on information in financial markets. *American Economic Review*, 78(2), 96-103.

Admati, A.R. and Pfleiderer, P., 2009. The "Wall Street Walk" and shareholder activism: Exit as a form of voice. *Review of Financial Studies*, 22(7), 2645-2685.

Admati, A.R., Pfleiderer, P. and Zechner, J., 1994. Large shareholder activism, risk sharing, and financial market equilibrium. *Journal of Political Economy*, 102(6), 1097-1130.

Alissa, W., Bonsall, S.B., Koharki, K. and Penn, M.W., 2013. Firms' use of accounting discretion to influence their credit ratings. *Journal of Accounting and Economics*, 55(2-3), 129-147.

Banerjee, S., Marinovic, I. and Smith, K., 2024. Disclosing to informed traders. *Journal of Finance*, 79(2), 1513-1578.

Banker, R.D. and Datar, S.M., 1989. Sensitivity, precision, and linear aggregation of signals for performance evaluation. *Journal of Accounting Research*, 27(1), 21-39.

Bar-Isaac, H. and Shapiro, J., 2013. Ratings quality over the business cycle. *Journal of Financial Economics*, 108(1), 62-78.

Bergemann, D. and Morris, S., 2019. Information design: A unified perspective. *Journal of Economic Literature*, 57(1), 44-95.

Bernstein, S., Giroud, X. and Townsend, R.R., 2016. The impact of venture capital monitoring. *Journal of Finance*, 71(4), 1591-1622.

Bolton, P., Freixas, X. and Shapiro, J., 2012. The credit ratings game. *Journal of Finance*, 67(1), 85-111.

Bolton P., Scheinkman J., Xiong W.. 2006. Executive compensation and short-termist behaviour in speculative markets. *Review of Economic Studies*, 73, 577-610.

Bond, P., Edmans, A. and Goldstein, I., 2012. The real effects of financial markets. *Annual Review of Financial Economics*, 4(1), 339-360.

Bouvard, M. and Levy, R., 2018. Two-sided reputation in certification markets. *Management Science*, 64(10), 4755-4774.

Chemla, G. and Hennessy, C.A., 2014. Skin in the game and moral hazard. *Journal of Finance*, 69(4), 1597-1641.

Chen, S., Miao, B. and Shevlin, T., 2015. A new measure of disclosure quality: The level of disaggregation of accounting data in annual reports. *Journal of Accounting Research*, 53(5), 1017-1054.

Cohn, J.B., Rajan, U. and Strobl, G., 2025. Credit ratings: strategic issuer disclosure and optimal screening. *Review of Finance*, 29(1), 169-199.

Crémer, J., 1995. Arm's length relationships. *Quarterly Journal of Economics*, 110(2), 275-295.

Cvijanović, D., Dasgupta, A. and Zachariadis, K.E., 2022. The Wall Street stampede: Exit as governance with interacting blockholders. *Journal of Financial Economics*, 144, 433-455.

Daley, B., Green, B. and Vanasco, V., 2020. Securitization, ratings, and credit supply. *Journal of Finance*, 75(2), 1037-1082.

Dasgupta, A. and Mathews, R.D., 2022. Delegated Blocks. Working paper, London School of Economics.

Doherty, N.A., Kartasheva, A.V. and Phillips, R.D., 2012. Information effect of entry into credit ratings market: The case of insurers' ratings. *Journal of Financial Economics*, 106(2), 308-330.

Dow, J. and Gorton, G., 1997. Stock market efficiency and economic efficiency: is there a connection? *Journal of Finance*, 52(3), 1087-1129.

Edmans, A., 2009. Blockholder trading, market efficiency, and managerial myopia. *Journal of Finance*, 64(6), 2481-2513.

Edmans, A., Levit, D. and Schneemeier, J., 2023. Socially responsible divestment. Working paper, London Business School.

Edmans, A. and Manso, G., 2011. Governance through trading and intervention: A theory of multiple blockholders. *Review of Financial Studies*, 24(7), 2395-2428.

Faure-Grimaud, A. and Gromb, D., 2004. Public trading and private incentives. *Review of Financial Studies*, 17(4), 985-1014.

Faure-Grimaud, A., Peyrache, E. and Quesada, L., 2009. The ownership of ratings. *RAND Journal of economics*, 40(2), 234-257.

Ferreira, D., Manso, G. and Silva, A.C., 2014. Incentives to innovate and the decision to go public or private. *Review of Financial Studies*, 27(1), 256-300.

Frenkel, S., 2015. Repeated Interaction and Rating Inflation: A Model of Double Reputation. *American Economic Journal: Microeconomics*, 7(1), 250-280.

Fulghieri, P. and Lukin, D., 2001. Information production, dilution costs, and optimal security design. *Journal of Financial Economics*, 61(1), 3-42.

Fulghieri, P., Strobl, G. and Xia, H., 2014. The economics of solicited and unsolicited credit ratings. *Review of Financial Studies*, 27(2), 484-518.

Gao, M. and Huang, J., 2020. Informing the market: The effect of modern information technologies on information production. *Review of Financial Studies*, 33(4), 1367-1411.

Goel, A.M. and Thakor, A.V., 2015. Information reliability and welfare: A theory of coarse credit ratings. *Journal of Financial Economics*, 115(3), 541-557.

Goldstein, I. and Yang, L., 2017. Information disclosure in financial markets. *Annual Review of Financial Economics*, 9(1), 101-125.

Goldstein, I. and Huang, C., 2020. Credit rating inflation and firms' investments. *Journal of Finance*, 75(6), 2929-2972.

Goldstein, I., 2023. Information in financial markets and its real effects. *Review of Finance*, 27(1), 1-32.

Gompers, P. and Lerner, J., 2001. The venture capital revolution. *Journal of Economic Perspectives*, 15(2), 145-168.

Grossman S.J., Stiglitz J.E. 1980. On the impossibility of informationally efficient markets. *American Economic Review*, 70(3), 393-408.

Gryglewicz, S., Mayer, S. and Morellec, E., 2020. Agency conflicts and short-versus long-termism in corporate policies. *Journal of Financial Economics*, 136(3), 718-742.

Hackbarth, D., Rivera, A. and Wong, T.Y., 2022. Optimal short-termism. *Management Science*, 68(9), 6477-6505.

Hayward, M.L. and Boeker, W., 1998. Power and conflicts of interest in professional firms: Evidence from investment banking. *Administrative Science Quarterly*, 43(1), 1-22.

Hellmann, T. and Puri, M., 2002. Venture capital and the professionalization of start-up firms: Empirical evidence. *Journal of Finance*, 57(1), 169-197.

Hirshleifer, J., 1971. The private and social value of information and the reward to inventive activity. *American Economic Review*, 61(4), 561-574.

Hoitash, R. and Hoitash, U., 2018. Measuring accounting reporting complexity with XBRL. *The Accounting Review*, 93(1), 259-287.

Holmström, B., 1999. Managerial incentive problems: a dynamic perspective. *Review of Economic Studies*, 66, 169-182.

Hörner, J. and Lambert, N.S., 2021. Motivational ratings. *Review of Economic Studies*, 88(4), 1892-1935.

Janeway, W.H., Nanda, R. and Rhodes-Kropf, M., 2021. Venture capital booms and start-up financing. *Annual Review of Financial Economics*, 13(1), 111-127.

Kaplan, S.N. and Strömberg, P., 2003. Financial contracting theory meets the real world: An empirical analysis of venture capital contracts. *Review of Economic Studies*, 70(2), 281-315.

Kashyap, A.K. and Kovrijnykh, N., 2016. Who should pay for credit ratings and how?. *Review of Financial Studies*, 29(2), 420-456.

Knechel, W.R., Krishnan, G.V., Pevzner, M., Shefchik, L.B. and Velury, U.K., 2013. Audit quality: Insights from the academic literature. *Auditing: A Journal of Practice & Theory*, 32(1), 385-421.

Kraft, P., 2015. Do rating agencies cater? Evidence from rating-based contracts. *Journal of Accounting and Economics*, 59(2-3), 264-283.

Kyle, A.S., 1985. Continuous auctions and insider trading. *Econometrica*, 53(6), 1315-1335.

Lerner, J. and Nanda, R., 2020. Venture capital's role in financing innovation: What we know and how much we still need to learn. *Journal of Economic Perspectives*, 34(3), 237-261.

Leung, E. and Veenman, D., 2018. Non-GAAP earnings disclosure in loss firms. *Journal of Accounting Research*, 56(4), 1083-1137.

Manso, G., 2013. Feedback effects of credit ratings. *Journal of Financial Economics*, 109, 535-548.

Marinovic, I., Varas, F., 2019. CEO Horizon, Optimal Pay Duration, and the Escalation of Short-Termism. *Journal of Finance*, 74, 2011-2053.

Martel, J., Van Wesep, E.D. and Van Wesep, R., 2022. Ratings and Cooperative Information Transmission. *Management Science*, 68(12), 9175-9197.

Mathis, J., McAndrews, J. and Rochet, J.C., 2009. Rating the raters: Are reputation concerns powerful enough to discipline rating agencies? *Journal of Monetary Economics*, 56(5), 657-674.

Narayanan, M., 1985. Managerial incentives for short-term results. *Journal of Finance*, 40(5), 1469-1484.

Noordermeer, B. and Vorst, P., 2024. The Informativeness of Balance Sheet Disaggregations: Evidence from Forecasting Operating Assets. *Management Science*, Forthcoming.

Opp, C.C., Opp, M.M. and Harris, M., 2013. Rating agencies in the face of regulation. *Journal of Financial Economics*, 108(1), 46-61.

Park, H.D. and Steensma, H.K., 2012. When does corporate venture capital add value for new ventures? *Strategic Management Journal*, 33(1), 1-22.

Parlour, C.A. and Rajan, U., 2020. Contracting on credit ratings: Adding value to public information. *Review of Financial Studies*, 33(4), 1412-1444.

Peress, J., 2010. The tradeoff between risk sharing and information production in financial markets. *Journal of Economic Theory*, 145(1), 124-155.

Piccolo, A. and Shapiro, J., 2022. Credit ratings and market information. *Review of Financial Studies*, 35(10), 4425-4473.

Pollrich, M. and Strausz, R., 2024. The irrelevance of fee structures for certification. *American Economic Review: Insights*, 6(1), 55-72.

Saeedi, M. and Shourideh, A., 2024. Optimal rating design under moral hazard. Working paper, Carnegie Mellon University.

Sangiorgi, F. and Spatt, C., 2017. Opacity, credit rating shopping, and bias. *Management Science*, 63(12), 4016-4036.

Skreta, V. and Veldkamp, L., 2009. Ratings shopping and asset complexity: A theory of ratings inflation. *Journal of Monetary Economics*, 56(5), 678-695.

Stahl, K. and Strausz, R., 2017. Certification and market transparency. *Review of Economic Studies*, 84(4), 1842-1868.

Stein J. 1989. Efficient capital markets, inefficient firms: A model of myopic corporate behavior. *Quarterly Journal of Economics*, 104, 655-669.

Strebulaev, I.A., and B. Yang., 2013. The Mystery of Zero-Leverage Firms. *Journal of Financial Economics*, 109, 1-23.

Thakor, R.T., 2021. Short-termism, Managerial Talent, and Firm Value. *Review of Corporate Finance Studies*, 10(3), 473-512

Varian H.R., 1998. *Markets for Information Goods*. University of California: Berkeley.

Verrecchia R.E. 1982. The use of mathematical models in financial accounting. *Journal of Accounting Research*, 20, 1-42.

Von Thadden E. 1995. Long-term contracts, short-term investment and monitoring. *Review of Economic Studies*, 62, 557-575.