Opportunism in Vertical Contracting: A Dynamic Perspective*

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Abstract

This paper proposes a dynamic approach to modeling opportunism in bilateral vertical contracting between an upstream monopolist and competing downstream firms. Unlike previous literature on opportunism which has focused on games in which the upstream firm makes simultaneous secret offers to the downstream firms, we model opportunism as a consequence of asynchronous recontracting in an infinite-horizon continuous-time model. We find that the degree of opportunism depends on the absolute and relative reaction speeds of the different bilateral upstream-downstream firm pairs and on the firms' discount rate. Patience, fast reaction speeds, and asymmetries in reaction speeds across upstream-downstream pairs are shown to alleviate opportunism. Our results are relevant for vertical merger policy and for competition policy on vertical restraints.

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1 Introduction

It is well-known that an upstream firm may suffer from an opportunism problem when it deals with multiple competing downstream firms through bilateral contracts. Although the upstream firm wants to restrict the total quantity of its input in order to maintain high prices and profits, it may be unable to commit to refrain from opportunistic moves that increase the bilateral surplus with one downstream firm at the expense of other firms. Therefore, the upstream firm may not be able to fully exert its market power, even if it enjoys a monopoly position in the upstream market. This opportunism problem, which was first analyzed by Hart and Tirole (1990), O'Brien and Shaffer (1992), and McAfee and Schwartz (1994), occupies a central place in the literature on vertical contracting and has been invoked as an explanation for vertical mergers and vertical restraints such as exclusive dealing that allow the supplier to restore its market power (see Rey and Tirole (2007) for an overview).[1]

The leading approach to modeling opportunism in vertical contracting is to assume that an upstream monopolist makes simultaneous *secret* offers to competing downstream firms. Under the commonly adopted equilibrium refinement that a downstream firm holds passive beliefs when it receives an out-of-equilibrium offer, that is, the downstream firm does not revise its beliefs about the offers received by its rivals upon reception of an 'unexpected' offer, the upstream firm is unable to fully exert its market power in equilibrium. If the upstream monopolist makes simultaneous public offers, on the other hand, then the monopoly outcome arises in equilibrium.

This paper proposes a different approach to modeling opportunism in vertical contracting. Our approach is dynamic in nature, capturing the notion that opportunism arises because an upstream "monopolist might gain by recontracting with another firm" once "the initial firm is somewhat locked in" (McAfee and Schwartz (1994, p. 210)). Although recontracting has long been recognized as an important source of opportunism, previous attempts to model the opportunism problem dynamically are scarce. McAfee and Schwartz (1994)'s seminal paper on opportunism in vertical contracting does consider a game in

[1]Important contributions on the opportunism problem and the various solutions to it include Rey and Vergé (2004), Marx and Shaffer (2004), White (2007), Montez (2015), Reisinger and Tarantino (2015), and Gaudin (2019).

which the upstream monopolist makes sequential offers (see Section II of their paper), but, as they explain, this game fails to capture that "*all* firms will be leery that the monopolist might recontract with their rivals" (McAfee and Schwartz (1994, p. 218), emphasis added). This weakness of the sequential-offers model prompts them to consider a game with simultaneous secret offers, which has since become the workhorse model of vertical contracting with opportunism in the literature. Our paper, in contrast, models recontracting explicitly, using an infinite-horizon model in which each downstream firm anticipates future recontracting between the upstream firm and its rival, as well as between the upstream firm and itself.

In addition to capturing the inherently dynamic recontracting explanation for opportunism, our dynamic modeling approach has a number of advantages over the standard secret-offers approach. First, the dynamic approach allows us to obtain new comparative statics results. It delivers predictions about the *degree* of opportunism and how it varies with the key parameters of our model—the discount rate and the absolute and relative recontracting reaction speeds of the various bilateral upstream-downstream pairs. This is useful for vertical merger policy and competition policy on vertical restraints, because when the supplier's opportunism problem is worse, the competitive damage from strategies such as vertical intergration that restore the supplier's monopoly power will be worse as well (and the supplier's incentive to use such strategies will be stronger).

Second and relatedly, our model yields less stark, and thus perhaps more realistic, predictions about the degree of opportunism. As a function of the different parameter values, our theory can explain steady-state outcomes that lie between the boundary cases of (i) the integrated monopoly outcome (no opportunism), which would arise in equilibrium under simultaneous public offers, and (ii) the pairwise-proof outcome (full opportunism), which would arise in equilibrium under simultaneous secret offers and passive beliefs.

Third, no assumptions about out-of-equilibrium beliefs are needed to characterize the degree of opportunism in our dynamic model. The equilibrium outcome of a game with simultaneous secret offers, on the other hand, is highly sensitive to which out-of-equilibrium belief refinement is adopted, with different assumptions often leading to radically different equilibrium outcomes.

Our setting embeds a simple Cournot-style model of bilateral contracting between one

upstream firm and two competing downstream firms, similar to the one set out by Rey and Tirole (2007), into an infinite-horizon continuous-time framework. The upstream firm (henceforth also called supplier) gets to make new contract offers to the downstream firms (henceforth also called retailers) according to independent Poisson processes, one for each retailer. The independence of the Poisson processes implies that recontracting occurs asynchronously, that is, the probability of simultaneous offers is zero. Moreover, because a higher Poisson rate implies that a retailer's contract reacts more quickly (in expectation) to changes in the other retailer's contract, the Poisson rate that governs the arrival of recontracting events between a supplier-retailer pair has a natural interpretation as the reaction speed of that retailer's contract. Contracts specify the quantity supplied by the supplier and a fixed fee to be paid by the downstream firm per unit of time, and we focus on Markov Perfect Equilibria (MPE).[2]

The supplier's Markov strategy involves dynamic quantity reaction functions that specify how the quantity that the supplier offers to a retailer depends on the quantity that the supplier currently supplies to the retailer's rival. We characterize the first-order conditions that differentiable dynamic quantity reaction functions must satisfy in an MPE, and use them to derive a series of results about the equilibrium steady-state quantities (assuming existence of a stable equilibrium with differentiable dynamic quantity reaction functions). For the case of linear demand functions, we establish the existence of a unique MPE with linear dynamic quantity reaction functions. This linear MPE is shown to be dynamically stable, with the retailers selling symmetric quantities in the equilibrium steady state when the reaction speeds are symmetric, and the retailer whose contract reacts faster selling a larger quantity than its rival when the reaction speeds are asymmetric.

The analysis yields three broad takeaways about the degree of opportunism, as measured by the aggregate quantity sold in the equilibrium steady state. First, patience reduces opportunism. Second, a faster average reaction speed reduces opportunism. And third, asymmetry in reaction speeds reduces opportunism.

The intuition for the first two insights can be understood as follows. When making an offer to retailer D_i, the supplier internalizes only the effect on D_i's own variable profit in the

[2]The restriction to Markov strategies reduces the large multiplicity of equilibria, and it allows us to focus on the key trade-off between short-term incentives to behave opportunistically and longer-term incentives to achieve collective surplus maximization.

time interval until the next recontracting with the retailer's rival D_{-i}, but it internalizes the future effects on *all* retailers' variable profits from the next recontracting with D_{-i} onwards (through its current and anticipated future fixed fees). Greater patience makes the supplier care relatively more about profits earned after the next recontracting with the rival retailer, the time for which it internalizes the effects of its current offer on total surplus, thus weakening the supplier's incentive to behave opportunistically. Similarly, fast reaction speed reduces the expected length of time until the next recontracting with the rival retailer, that is, the length of time during which effects on the rival retailer's variable profits are ignored, thereby weakening the supplier's incentive to behave opportunistically.[3]

Nonetheless, under symmetric reaction speeds, some degree of opportunism prevails in the equilibrium steady state even in the limit, when the discount rate goes to zero or the reaction speed goes to infinity. However, asymmetries in reaction speeds across supplier-retailer pairs—due to, for instance, asymmetries in the closeness of the relationships between the supplier and different retailers—can further reduce the extent of opportunism, and even eliminate it altogether in a limiting case when the retailers sell undifferentiated products. The intuition for why asymmetries in reaction speeds help to reduce opportunism can be understood as follow. Suppose retailer D_A's contract reacts faster to changes in retailer D_B's contract than D_B's contract reacts to changes in D_A's contract. The supplier can then exploit its relatively weaker incentive to behave opportunistically when making offers to D_B to also weaken its (otherwise stronger) incentive to behave opportunistically when making offers to D_A by offering a smaller quantity to D_B. Doing so reduces the conflict between bilateral profit maximization and industry profit maximization when the supplier makes its next offer to D_A, thereby leading to less opportunism overall and a lower aggegrate steady state equilibrium quantity (compared to a situation with the same average reaction speed but symmetry across supplier-retailer pairs).

Our results have implications for vertical merger policy and for competition policy on vertical restraints. Models of vertical contracting with opportunism have been used to understand anticompetitive effects of vertical mergers and vertical restraints by noting that

[3]This result may appear surprising in the sense that fast reaction speed can be thought of as a lack of commitment to refrain from recontracting. However, as our setting makes it clear, opportunism arises not due to a lack of commitment to long contracts, but due to a lack of commitment on the terms of future contracts.

if secret offers and passive beliefs are observed, then vertical intergration and contract provisions that eliminate opportunism reduce total output and harm downstream consumers. Our model shows that secret offers are not needed for opportunism to arise, and that vertical mergers and opportunism-avoiding contract provisions can have anticompetitive effects even when contracts are public. Moreover, our results offer new guidance on when vertical mergers and opportunism-avoiding contract provisions are likely to be more harmful, namely when firms are impatient, there are long time gaps between recontracting, and reaction speeds are symmetric.

The paper is organized as follows. In Section 2, we discuss the connections of our paper to the related literatures on dynamic vertical contracting and on dynamic oligopoly games. In Section 3, we describe the model setup. Section 4 offers a brief summary of the benchmark case of simultaneous offers in a static game. Section 5 contains our main analysis. In Section 5.1, we derive the equilibrium conditions, and we prove the existence of a unique MPE with linear dynamic quantity reaction functions in the linear-demand case. In Section 5.2, we derive limit results and comparative static results on the equilibrium steady-state quantities for the case of symmetric reaction speeds. In Section 5.3, we analyze the implications of asymmetries in reaction speeds across supplier-retailer pairs. Section 6 discusses several important extensions of the baseline model. In Section 6.1, we consider the effects of product differentiation, and in Section 6.2, we extend the analysis from quantity-forcing contracts to two-part tariffs. In Section 7, we discuss the policy implications of the results and some directions for further research. Proofs are relegated to the appendix.

2 Related literature

In addition to the aforementioned literature on opportunism in static models of vertical contracting, our paper contributes to the literatures on dynamic vertical contracting and on dynamic oligopoly games.

Dynamic vertical contracting Although the literature on vertical contracting is vast, previous attemps to model the opportunism problem dynamically are scarce. McAfee and Schwartz (1994), Marx and Shaffer (2004), and Bedre-Defolie (2012) consider models in which a supplier makes public sequential offers to competing retailers, so the supplier has

an incentive to behave opportunistically with the later retailer(s) in the sequence. However, sequential-offer models fail to capture that all retailers may be wary of future opportunistic behavior, and they impose a strong ex ante asymmetry between firms. Our infinite-horizon model is more general, encompassing sequential offers as a special limiting case when the reaction speeds are allowed to be asymmetric.

Dequiedt and Martimort (2015) consider a model in which offers are made simultaneously but that nonetheless has a dynamic flavor, because the supplier is allowed to move last, choosing each retailer's quantity from a menu contract accepted by the retailer. Offers are public but retailers have private information. The supplier's ability to choose quantities can thus be thought of as allowing the supplier to dynamically "respond to new information as it arrives." We focus on forcing contracts instead, and introduce dynamics explicitly by considering repeated asynchronous contracting in an infinite-horizon model. Opportunism arises due to the supplier's inability to commit to the terms of future contracts in our setting, whereas Dequiedt and Martimort (2015) focus on a new form of "informational" opportunism due to private information on the retailer side.[4]

Lee and Fong (2013) analyze MPE of an infinite-horizon dynamic seller-buyer network formation game with transfers. However, the focus of their work is on network formation (who trades with whom), whereas we are interested in the severity of the opportunism problem that a monopolistic supplier faces. Closer to our setting, Farrell (2019, Section 5) proposes to analyze a symmetric alternating-offers model of vertical contracting between an upstream monopolist and competing downstream firms. However, the formal analysis of the dynamic model in his paper is highly incomplete. It does not include a characterization of the MPE strategies and steady-state quantities, nor any comparative statics results. Moreover, Farrell (2019)'s main focus are partially exclusionary contracts rather than the degree of opportunism.[5]

[4]Segal and Whinston (2003) consider menu contracts, from which the principal chooses after the agents have made their acceptance decisions, in a model without private information. They show that when marginal production costs are nonincreasing (as we will assume), allowing menu contracts does not restrict the equilibrium set in a game in which a principal makes simultaneous secret offers to competing agents: Any profile of quantities and transfers such that total surplus is nonnegative can be sustained as a Weak Perfect Bayesian Equilibrium in this case (see Proposition 6 in their paper).

[5]The broader literature on dynamic vertical contracting also includes work on dynamic common agency games (Bergemann and Välimäki (2003), Pavan and Calzolari (2009)), in which multiple principals make offers to a common agent rather than the opposite scenario considered in this paper, and research on the implications of vertical market structure and contracting for firms' ability to sustain collusion in a repeated

Dynamic oligopoly and asynchronous moves This paper is also related to the literature on dynamic oligopoly, and more broadly the literature on dynamic games with asynchronous moves. In a series of seminal articles on dynamic oligopoly, Maskin and Tirole (1987, 1988a,b) analyze MPE of repeated games in which duopolists make alternating moves.[6] Our continuous-time model in which contract timing is governed by independent Poisson processes is inspired by the micro-foundation that Maskin and Tirole (1988a, Section 4) propose for such games.

A key difference to the models considered by Maskin and Tirole and other work on dynamic oligopoly is that we consider a vertical industry structure with a supplier that makes asynchronous offers to two competing firms (who then accept or reject and compete in the market), instead of two competing firms that make asynchronous strategic decisions. Unsurprisingly, this leads to qualitatively different results. For instance, under symmetry the equilibrium steady-state quantity lies below the static Cournot quantity in our setting, whereas Maskin and Tirole (1987) obtain the opposite result in a dynamic game of symmetric Cournot competition with linear demand functions.

An important feature of our dynamic model is that contract offers are asynchronous. Asynchronous moves have also been analyzed in repeated coordination games (Lagunoff and Matsui (1997)), and more recently in asynchronous revision games where players prepare some actions at the beginning and then obtain revision opportunities according to independent Poisson processes until some predetermined deadline (Kamada and Kandori (2012), Calcagno et al. (2014)). Ambrus and Lu (2015) analyze a continuous-time finite-horizon coalitional bargaining game in which opportunities to make offers arrive asynchronously according to independent Poisson processes, until an agreement is reached. While these papers share the asynchronous-moves assumption and some important modeling ingredients with our research, they differ substantially from our work in motivation, focus, and analysis.

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game (Nocke and White (2007), Piccolo and Miklós-Thal (2012), Gilo and Yehezkel (2020)).

[6]Other important contributions to this literature include Eaton and Engers (1990) who consider dynamic price competition with differentiated goods, De Fraja (1993) who analyzes the impact of staggered wage bargaining on wages in an oligopolistic industry, Davies (1991) who explores a dynamic entry deterrence model where two firms alternate in making price and entry choices, and Pastine and Pastine (2002) who consider dynamic competition when duopolists choose advertisement and price levels and there are consumption externalities among buyers. Jun and Vives (2004) analyze a dynamic duopoly model with adjustment costs.

3 Setting

We embed a simple Cournot-style model of vertical contracting, similar to the one set out by Rey and Tirole (2007), into a continuous-time dynamic setting. Consider a vertical structure with one upstream supplier, U, and two competing downstream firms D_i ($i = A, B$, also called retailers). The downstream firms purchase an input from the supplier, transform it into a final good using a one-to-one technology, and sell the final good to consumers. Upstream marginal costs are constant and equal to $c \geq 0$, downstream marginal costs are constant and normalized to zero.

Consumers have an inverse demand curve $P(Q) : \mathbb{R}_+ \to \mathbb{R}$ for the product, where $Q = q_A + q_B$ denotes the total quantity put on the market by the downstream firms. We make the following assumptions:

A1 $P(Q)$ is continuous and strictly decreasing for all $Q \geq 0$, and twice continuously differentiable for all $Q > 0$.

A2 $P'(Q) + P''(Q)\,Q < 0$ for all $Q > 0$.

A3 $P(0) > c$ and $\lim_{Q\to\infty} P(Q) < c$.

A1 and A3 implies that there exists a unique quantity $\overline{Q}_c > 0$ such that price is equal to total marginal cost: $P\left(\overline{Q}_c\right) = c$.[7] This quantity will be useful because it represents a natural upper bound to impose on quantities in order to obtain bounded action spaces.

The instantaneous variable profit of downstream firm D_i (gross of any payments to the supplier) is given by

$$\pi(q_i, q_{-i}) = q_i P(q_A + q_B),$$

where q_i denotes D_i's own quantity, and q_{-i} its competitor's quantity. We use subscripts to denote derivatives, e.g., $\pi_2(q_i, q_{-i}) = \frac{\partial \pi(q_i, q_{-i})}{\partial q_{-i}}$. Assumptions A1-A2 imply that $\pi_{11}(q_i, q_{-i}) = \frac{\partial^2 \pi(q_i, q_{-i})}{\partial q_i^2} < 0$ and $\pi_{12}(q_i, q_{-i}) = \frac{\partial^2 \pi(q_i, q_{-i})}{\partial q_i \partial q_{-i}} < 0$.

The instantaneous industry profit, that is, the sum of all three firms' profits, is given by

$$\Pi(q_A + q_B) = (q_A + q_B)\left(P(q_A + q_B) - c\right).$$

[7] Note that for $c = 0$, A3 implies that price must be negative for high quantities. This assumption is not crucial for our results. See Footnote 21 in Section 5.1, where we discuss the implications of restricting prices to be non-negative in the linear-demand case.

We denote by $R^C(q_{-i}) = \arg\max_q (q(P(q+q_{-i})-c)) = \arg\max_q (\Pi(q+q_{-i}) - \pi(q_{-i},q))$ the Cournot reaction function given the marginal production cost c. The per-firm Cournot quantity q^C is defined by $q^C = R^C(q^C)$. The quantity that maximizes industry profits is denoted by $Q^M = \arg\max_Q \Pi(Q)$, and we let $R^M(q_{-i}) = \arg\max_q \Pi(q+q_{-i}) = Q^M - q_{-i}$ denote the "monopoly reaction function" and $q^M = Q^M/2 = R^M(q^M)$ the quantity per downstream firm when they split the total monopoly quantity equally. Throughout the paper, we will frequently use the linear-demand specification $P(Q) = 1 - Q$.[8] In this case, $q^C = \frac{1-c}{3}$ and $q^M = \frac{1-c}{4}$, and A1 to A3 are satisfied for any $c < 1$.

Time is continuous and infinite, indexed by $t \in [0,\infty)$, and all firms discount future payoffs at a rate $r > 0$. A contract between U and D_i consists of a vector $(q_i, f_i) \in \left[0, \overline{Q}_c\right] \times \mathbb{R}$ that specifies a flow of input quantity q_i from the supplier to the retailer and a fixed payment f_i from the retailer to the supplier per unit of time.[9] For simplicity, we assume that the supply contracts are quantity-fixing, that is, they fix how much quantity the retailer transforms into the final output and sells to consumers per unit of time.[10] The absence of a contract between U and D_i is equivalent to $(q_i, f_i) = (0,0)$.

D_i's flow payoff given the current contracts is thus $\pi(q_i, q_{-i}) - f_i$, and U's flow payoff is

$$f_A + f_B - c(q_A + q_B).$$

The sum of the three firms' flow payoffs is equal to the industry profit $\Pi(q_A + q_B)$.

The timing of contracting is governed by two independent Poisson processes with rates $\lambda_A > 0$ and $\lambda_B > 0$, respectively. For a small time interval Δt, the probability that the current contract between U and D_i terminates and recontracting occurs is $\lambda_i \Delta t$. In this event, U instantaneously makes a new offer to D_i, and D_i immediately accepts or rejects

[8] Using $P(Q) = 1 - Q$ rather than $P(Q) = a - bQ$ is without loss of generality. Given that the marginal cost c is constant and a free parameter, setting $a = 1$ amounts to a choice of measurement units for output, and setting $b = 1$ is a normalization of the market size.

[9] Two-part tariffs will be considered in Section 6.2.

[10] An alternative would be that after purchasing their input quantities, the downstream firms play the Bertrand-Edgeworth game of downstream price competition with capacity constraints. In this alternative specification, both retailers find it optimal to transform all their input and set their price at $P(q_A, q_B)$ for any $(q_A, q_B) \in \left[0, \overline{Q}_c\right]^2$ if the following assumptions hold: the upstream marginal cost c is high enough, stockpiling is infeasible (i.e., a retailer whose contract specifies quantity q_i can sell at most $q_i \Delta t$ in a time interval Δt), and the retailers can adjust their prices instantaneously in response to a change in either retailer's contract. Our assumptions on contracts (quantity-transfer pairs, and quantity fixing) are in line with those in Segal (1999)'s general analysis of contracting with externalities between one principal and several agents.

the offer.[11] If D_i accepts (rejects) an offer (q_i, f_i), its quantity becomes q_i (zero) per unit of time until the contract terminates and the next recontracting between U and D_i occurs.

We will refer to λ_i as the "reaction speed" of D_i's contract. A higher λ_i (shorter commitment length) means that recontracting with D_i occurs more frequently, therefore the bilateral pair $U - D_i$ can react more quickly to changes in the contract between U and D_{-i}. Conversely, a lower λ_i (longer commitment length) means that recontracting with D_i occurs less frequently, therefore the bilateral pair $U - D_i$ reacts less quickly to changes in the contract between U and D_{-i}.

We focus on (stationary) pure Markov strategies. The state variable when U is about to make an offer to D_i is the quantity in U's current contract with D_{-i}.[12] Formally, a Markov strategy of U is given by a pair of mappings $(R_i, F_i)_{i=A,B}$ where $(R_i(q_{-i}), F_i(q_{-i}))$ is the contract offered to D_i when U currently sells quantity q_{-i} to D_{-i}. We will refer to (R_A, R_B) as the *dynamic quantity reaction functions*, because they capture how the quantity in U's contract with one retailer reacts to the quantity in the competing retailer's contract. The quantity action spaces are restricted to a bounded set; specifically, we assume that $R_i : \left[0, \overline{Q}_c\right] \to \left[0, \overline{Q}_c\right]$ for all i.[13] The fixed fee offers are allowed to take on any value in $\mathbb{R}$. For D_i, a pure Markov strategy is given by a function $M_i(q, f; q_{-i}) \in \{0, 1\}$, where $M_i(q, f; q_{-i}) = 1$ (resp. $M_i(q, f; q_{-i}) = 0$) means that D_i accepts (resp. rejects) the offer (q, f) when U currently sells quantity q_{-i} to D_{-i}. All actions are public.

A strategy profile is called a *Markov Perfect Equilibrium* (MPE) if it is a subgame

[11]The assumption that time lapses between offers, but not between offers and acceptance decisions, goes back to the classic bargaining models of Stahl (1972) and Rubinstein (1982). It has also been adopted more recently by Ambrus and Lu (2015) in a continuous-time bargaining model where proposal opportunities arrive according to a Poisson process over time. The assumption helps us to focus on the opportunism problem and how the supplier's contract with one retailer reacts to changes in its contract with the other retailer.

[12]One may wonder why the state when U makes an offer to D_i excludes the fixed fee f_{-i} in U's current contract with D_{-i}, although this fixed fee affects U's (and D_{-i}'s) payoff until the next recontracting with D_{-i}. The reason is that U's preferences over continuation strategies are independent of f_{-i}. The fixed fee f_{-i} does not affect D_i's payoffs when deciding whether to accept or reject an offer (q_i, f_i) from U, which implies that U's continuation payoff functions for the same q_{-i} but two different values of f_{-i} are positive affine transformations of one another. Applying the criterion set out in Maskin and Tirole (2001), Markov strategies therefore only depend on the quantity that U currently supplies to the rival retailer in our context.

[13]The assumption that the quantity action spaces are bounded ensures that the value functions are well defined and that dynamic programming techniques are applicable. Our approach in the analysis will be to ignore the upper and lower bounds on quantity when solving the supplier's optimization problem and to verify ex post that indeed $R_i(q_{-i}) \in \left[0, \overline{Q}_c\right]$ for all $q_{-i} \in \left[0, \overline{Q}_c\right]$ and i.

perfect equilibrium in Markov strategies. We restrict attention to equilibria in which all offers are accepted ($M_i\left(R_i\left(q_{-i}\right), F_i\left(q_{-i}\right); q_{-i}\right) = 1$ for all q_{-i} and all i), which is without loss of generality because an offer that would be rejected can be replaced by an accepted null contract $(q_i, f_i) = (0, 0)$ without any impact on expected present discounted payoffs. Henceforth, an MPE is also referred to simply as *equilibrium*, and we will say that an equilibrium is linear if the equilibrium dynamic quantity quantity reaction functions are linear. All firms are risk neutral and seek to maximize expected present discounted payoffs.

Remarks The results in our main model do not hinge on the assumption that contract offers are publicly observed. As the two Poisson processes according to which contracts terminate are independent, the probability that the supplier makes simultaneous offers to the two downstream firms is zero. Moreover, since contracts are quantity-forcing, a downstream firm can infer its rival's current quantity from its own variable profit. When receiving a contract offer, a downstream firm would thus be aware of the quantity in its rival's current contract even if the rival's offer had been privately observed. The issue of beliefs about the supplier's offer to another downstream firm, which is central to the analysis when the suplier makes secret simultaneous offers, is therefore moot in our setting.

It is also worth noting that the stochastic nature of the contract timing is not important for our results. Given that the firms are risk neutral, formally the model is equivalent to one in which all contracts have length $\Delta_A + \Delta_B$, where Δ_i $(i = A, B)$ denotes the time lag with which the bilateral pair $U - D_i$ reacts to a change in D_{-i}'s quantity.[14] By the Poisson property, the probability of contract termination is independent of a contract's age in our setting. Therefore, as in a model with alternating offers and deterministic contract lengths, only the quantity that the rival retailer is committed to under its current contract, and not contract age, is relevant. The critical feature of our model is that recontracting events are asynchronous, not that they are stochastic.

[14]Specifically, the setting with deterministic reactions lags Δ_A and Δ_B is analogous to our model when $e^{-r\Delta_i} = \frac{\lambda_i}{r+\lambda_i}$ for each i (or, in a discrete-time version of the model with deterministic reaction lags, when $\delta^{T_i} = \frac{\lambda_i}{r+\lambda_i}$, where T_i denotes the number of periods before i's contract reacts and δ the discount rate). See also Maskin and Tirole (1988a) and Lagunoff and Matsui (1997, Section 3) for discussions of various microfoundations of dynamic games with asynchronous moves.

4 Benchmark: Static Model

Before we analyze the dynamic game, let us set out the benchmark of simultaneous offers in a static game. Consider a game in which U makes simultaneous contract offers of the form (q_i, f_i) to the retailers, and the retailers simultaneously and independently decide whether to accept or reject these offers. The payoffs are the same as the flow payoffs in our dynamic setting, namely $\pi(q_i, q_{-i}) - f_i$ for D_i and $f_A + f_B - c(q_A + q_B)$ for U, with $(q_i, f_i) = (0, 0)$ if D_i rejects U's offer.

Public offers When the supplier's offers are publicly observed by the retailers, the supplier can fully exert its market power and obtains the entire monopoly profit in (a subgame perfect) equilibrium. For instance, U can achieve this by offering the contract $\left(q^M, \pi\left(q^M, q^M\right)\right)$ to each retailer. Both retailers will accept and together they will sell the monopoly quantity. The intuition for why the monopoly outcome arises in equilibrium is that the supplier internalizes the effects on *all* retailers' profits when making offers: Any change in the quantity offered to D_i affects the fixed fee that the supplier can obtain from retailer D_{-i} by an amount equal to the effect of the change on D_{-i}'s variable profit.

Secret offers When D_i cannot observe the contract offered to D_{-i}, the (perfect Bayesian) equilibrium of the game is sensitive to D_i's beliefs about the contract offered to D_{-i} when D_i receives an out-of-equilibrium offer. A sensible and widely-used assumption in Cournot settings like the one we consider is that retailers hold *passive beliefs*, whereby a retailer that receives an out-of-equilibrium offer believes that its rival was offered the equilibrium contract.[15]

Let $(\widehat{q}_A, \widehat{q}_B)$ denote the equilibrium quantities. With passive beliefs, retailer D_i is willing to accept an offer (q, f) if and only if $f \leq \pi(q, \widehat{q}_{-i})$. The equilibrium offer to D_i must therefore be pairwise-proof, i.e., maximize the bilateral surplus of the pair $U - D_i$ given $\widehat{q}_{-i}$, which in our setting means that D_i's equilibrium quantity must be the Cournot best

[15]The passive beliefs refinement is appealing in Cournot-like settings because U has no incentive to change the offer to D_{-i} when it changes the offer to D_i. See Hart and Tirole (1990), Rey and Vergé (2004), or Rey and Tirole (2007) for more detailed discussions.

response to $\widehat{q}_{-i}$:

$$\widehat{q}_i = R^C\left(\widehat{q}_{-i}\right) = \arg\max_{q_i}\left(\Pi\left(q_i + \widehat{q}_{-i}\right) - \pi\left(\widehat{q}_{-i}, q_i\right)\right).$$

In the unique equilibrium given passive beliefs, the retailers thus sell the Cournot quantities $\left(q^C, q^C\right)$ and the supplier earns $\Pi\left(2q^C\right) < \Pi\left(Q^M\right)$. Intuitively, the supplier is unable to fully exert its market power because when making an offer to D_i, it does *not* internalize the negative effect that a higher q_i has on D_{-i}'s variable profit.[16]

Different beliefs can lead to different equilibrium outcomes. In particular, the monopoly outcome arises in equilibrium under symmetric beliefs, whereby a retailer that receives an out-of-equilibrium offer believes that its rival was offered the same contract.[17,18]

The simultaneous-offer games analyzed in the literature thus all feature one of two polar outcomes in our setting. Either the monopoly outcome arises in equilibrium (when offers are public or offers are secret and beliefs symmetric) or opportunism leads to the Cournot competition outcome in equilibrium (when offers are secret and beliefs passive or wary). Other than through changes in parameters that affect the Cournot or the monopoly outcome, these models do not accommodate varying degrees of opportunism.

[16]Some papers in the vertical contracting literature (e.g., O'Brien and Shaffer, 1992) use the "contract equilibrium" concept pioneered by Cremer and Riordan (1987), which requires contracts to be pairwise proof (i.e., each contract must maximize bilateral surplus given the contracts of other retailers) but does not rule out multilateral deviations. In the model with Cournot competition and quantity-fixing contracts considered here, the quantities in a passive-beliefs perfect Bayesian equilibrium coincide with the quantities in such a contract equilibrium, hence this alternative solution concept would lead to the same conclusion.

[17]The third belief refinement in the literature is wary beliefs, first introduced by McAfee and Schwartz (1994). Wary beliefs coincide with passive beliefs in our setting with quantity-fixing contracts, hence the Cournot outcome would remain the equilibrium outcome under this alternative belief refinement. See In and Wright (2018) for a more general analysis of "endogenous signaling games" that offers a game-theoretic foundation for wary beliefs in vertical contracting games.

[18]In fact, in our setting, an equilibrium with any non-negative quantities $(\widehat{q}_A, \widehat{q}_B)$ such that $\widehat{q}_A + \widehat{q}_B < \overline{Q}_c$ can be sustained by appropriately defined out-of-equilibrium beliefs. To see this, suppose that if D_i receives an offer $q_i \neq \widehat{q}_i$, D_i believes that D_{-i} was offered the contract $\left(\overline{Q}_c - q_i, 0\right)$ and that D_{-i} holds similar beliefs (which makes accepting $\left(\overline{Q}_c - q_i, 0\right)$ optimal for D_{-i}). The highest fixed fee D_i is willing to pay is thus equal to $\pi\left(q_i, \overline{Q}_c - q_i\right) = q_i c$, which makes a deviation unprofitable for the supplier.

5 Dynamic Model

5.1 Equilibrium conditions and existence

To solve for an equilibrium, we define four value functions. Given the Markov strategies $((R_A, F_A, R_B, F_B), M_A, M_B)$, let $W_A(q_B, f_B)$ denote the expected present discounted value of U's profits when U is about to make an offer to D_A, the other retailer's current contract is (q_B, f_B), and all firms play according to their Markov strategies henceforth. $W_B(q_A, f_A)$ is defined symmetrically. Let $V_A(q, f; q_B)$ denote the expected present discounted value of D_A's profits when D_A accepts the contract offer (q, f) it just received, U currently supplies q_B to the other retailer, and all firms play according to their Markov strategies henceforth. $V_B(q, f; q_A)$ is defined symmetrically.

U's optimization problem when it is about to make an offer to D_i can then be written as[19]

$$W_i(q_{-i}, f_{-i}) = \max_{(q,f)} \left(\frac{f + f_{-i} - c(q + q_{-i})}{r + \lambda_i + \lambda_{-i}} + \frac{\lambda_i}{r + \lambda_i + \lambda_{-i}} W_i(q_{-i}, f_{-i}) + \frac{\lambda_{-i}}{r + \lambda_i + \lambda_{-i}} W_{-i}(q, f) \right) \tag{1}$$

$$\text{s.t. } V_i(q, f; q_{-i}) \geq V_i(0, 0; q_{-i}).$$

The first term in the objective function captures the discounted present value of U's profits in the time interval until the next recontracting. The second and third term capture the discounted present value of U's continuation profits from the next recontracting onwards, taking into account that either the contract with D_i or the contract with D_{-i} can terminate first. Morover, the supplier is constrained by the condition that D_i finds it optimal to accept U's offer (q, f). Note that it is without loss of generality to require D_i's acceptance in U's optimization problem, because any offer that D_i's strategy would reject can be replaced by an offer $(0, 0)$ such that D_i is indifferent between acceptance and rejection.

We first show that the retailer's acceptance condition boils down to an upper bound on the fixed fee:

Lemma 1 *It is optimal for D_i to accept offer (q, f) when the rival's current quantity is q_{-i}, i.e., $V_i(q, f; q_{-i}) \geq V_i(0, 0; q_{-i})$, if and only if*

$$f \leq f_i(q; q_{-i}) \equiv \frac{r + \lambda_i}{r + \lambda_i + \lambda_{-i}} \pi(q, q_{-i}) + \frac{\lambda_{-i}}{r + \lambda_i + \lambda_{-i}} \pi(q, R_{-i}(q)).$$

[19] Details on how to derive the objective function in (1) can be found in Online Appendix A.

The fixed payment $f_i(q; q_{-i})$ extracts the present discounted value of all variable profits that D_i earns (in expectation) during the contract. Capturing the possibility of a reaction in the rival's contract before D_i's own contract terminates, it is a weighted average of D_i's variable profit given its rivals' current quantity q_{-i} and D_i's variable profit given its rival's quantity $R_{-i}(q)$ after recontracting between U and D_{-i}. The weights depend on the relative reaction speeds of the two supplier-retailer pairs, with a smaller λ_i and a higher λ_{-i} increasing the relative weight of $\pi(q, R_{-i}(q))$, and on the discount rate, with a higher discount rate increasing the weight of the profit prior to the possible recontracting with D_{-i}. Variable profits beyond the current contract do not matter for the retailer's acceptance decision, because these profits will be fully extracted by the fixed fees in future contracts.

The objective function in the supplier's optimization problem in (1) is strictly increasing in f, as can be seen by noting that $\frac{\partial W_{-i}(q,f)}{\partial f} = \frac{1}{r+\lambda_i} > 0$. It follows that $f = f_i(q; q_{-i})$ at the solution of the supplier's problem. Substituting the binding constraint into the objective function, solving for $W_i(q_{-i}, f_{-i})$, and denoting by $\overline{W}_i(q_{-i}) = W_i(q_{-i}, f_{-i}) - \frac{f_{-i}}{r+\lambda_{-i}}$ the supplier's value function net of the fixed payments already committed to in the past, the supplier's problem can be rewritten as

$$\overline{W}_i(q_{-i}) = \frac{1}{r+\lambda_{-i}} \max_q \left(\pi(q, q_{-i}) - c(q + q_{-i}) + \frac{\lambda_{-i}}{r+\lambda_i} \pi(q, R_{-i}(q)) + \lambda_{-i} \overline{W}_{-i}(q) \right). \tag{2}$$

The strategies $((R_A, F_A, R_B, F_B), M_A, M_B)$ form an MPE if and only if there exist value functions $\left(\overline{W}_A, \overline{W}_B\right)$ such that, for every i and q_{-i}, (2) holds,

$$R_i(q_{-i}) \in \arg\max_q \left(\pi(q, q_{-i}) - c(q + q_{-i}) + \frac{\lambda_{-i}}{r+\lambda_i} \pi(q, R_{-i}(q)) + \lambda_{-i} \overline{W}_{-i}(q) \right),$$

$F_i(q_{-i}) = f_i(R_i(q_{-i}); q_{-i})$, and $M_i(q, f; q_{-i}) = 1$ if and only if $f \leq f_i(q; q_{-i})$.

For the remainder of the analysis, we will focus on characterizing the equilibrium dynamic quantity reaction functions (R_A, R_B), with the implicit understanding that $F_i(q_{-i}) = f_i(R_i(q_{-i}); q_{-i})$ and $M_i(q, f; q_{-i}) = 1$ if and only if $f \leq f_i(q; q_{-i})$.

Our first result on the equilibrium dynamic quantity reaction functions is that they must be downward sloping (if they exist). As in dynamic oligopoly models, this result follows from $\pi_{12} < 0$ (see Maskin and Tirole (1987, 1988a,b) or Vives (2005)).

Lemma 2 *When an equilibrium exists, the equilibrium dynamic quantity reaction functions*

are downward sloping: $R_i(q) \leq R_i(q')$ *if* $q > q'$*, for* $i \in \{A, B\}$.

Assuming that an equilibrium has differentiable dynamic quantity reaction functions, we can use the first-order conditions of the supplier's optimization problem to further characterize these functions. For each $i = A, B$, the first-order condition of the problem in (2) is that, at $q = R_i(q_{-i})$,

$$\pi_1(q, q_{-i}) - c + \frac{\lambda_{-i}}{r + \lambda_i}\left(\pi_1(q, R_{-i}(q)) + \pi_2(q, R_{-i}(q)) R'_{-i}(q)\right) + \lambda_{-i}\frac{d\overline{W}_{-i}(q)}{dq} = 0. \quad (3)$$

By the envelope theorem,

$$\frac{d\overline{W}_{-i}(q)}{dq} = \frac{1}{r + \lambda_i}\left(\pi_2(R_{-i}(q), q) - c\right),$$

which implies that the first-order condition for each $i = A, B$ can be written as follows: at $q = R_i(q_{-i})$,

$$\begin{aligned} 0 = &-c + \frac{r + \lambda_i}{r + \lambda_i + \lambda_{-i}}\pi_1(q, q_{-i}) \\ &+ \frac{\lambda_{-i}}{r + \lambda_i + \lambda_{-i}}\left(\pi_1(q, R_{-i}(q)) + \pi_2(q, R_{-i}(q)) R'_{-i}(q) + \pi_2(R_{-i}(q), q)\right). \end{aligned} \quad (4)$$

The first-order condition in (4) has the following interpretation. Consider U's contract offer to D_A given that D_B's current contract specifies a quantity q_B. At $q = R_A(q_B)$, a small change Δq in the quantity that U and D_A agree upon must have zero effect on the present discounted joint profit of the bilateral pair $U - D_A$. This effect can be decomposed as follows. First, there is a direct effect $-c\Delta q$ on upstream costs until U's next recontracting with D_A. Second, D_A's variable profit is affected, both at its rival's current quantity q_B and after a possible reaction in the rival's quantity during D_A's contract. At the rival's current quantity, the only effect on D_A's variable profit is the direct effect $\pi_1(q, q_B)\Delta q$, but after a reaction in D_B's contract there is both a direct effect $\pi_1(q, R_B(q))\Delta q$ and an indirect effect $\pi_2(q, R_B(q)) R'_B(q)\Delta q$ due to the marginal reaction in D_B's quantity. Third, there is the direct effect $\pi_2(R_B(q), q)\Delta q$ on D_B's variable profit after a reaction of D_B's contract during D_A's contract. This change in D_B's variable profit is part of the present discounted joint profit of $U - D_A$, because it will be fully extracted by U through the fixed fee in its next contract with D_B. From the envelope theorem, Δq has no additional first order effects. In particular, the indirect effect of Δq on D_B's variable profit due to the marginal reaction

in D_B's quantity in case of recontracting is not first order, because U's recontracting offer to D_B internalizes this change.[20]

The first-order conditions are necessary for an equilibrium given differentiable dynamic quantity reaction functions, but they are not sufficient. For the case of linear demand functions, however, we can show that there exists a unique set of linear dynamic quantity reaction functions such that the necessary and sufficient condition of U's dynamic optimization problem are satisfied:

Proposition 1 *Suppose* $P(Q) = 1 - Q$.[21] *For any parameter values* $r, \lambda_A, \lambda_B > 0$ *and* $c \in [0, 1)$*, there exists a unique linear MPE. This MPE is dynamically stable, i.e., for any history, quantities converge to a steady state.*

The next two subsections further analyze the properties of the steady state in a dynamically stable equilibrium. We first consider the case of symmetric reaction speeds and then turn to the implication of asymmetries in reaction speeds. In each case, we first derive results for the general model assuming equilibrium existence and differentiability of the dynamic quantity reaction functions, and then we analyze the comparative statics of the steady-state equilibrium quantities in the unique linear MPE under linear demand.

5.2 Symmetric reaction speeds

If $\lambda_A = \lambda_B = \lambda$, the equilibrium dynamic quantity reaction functions depend on $\frac{r}{\lambda}$, the discount rate scaled by the recontracting rate, but not on r and λ separately. This can be seen directly from the first-order conditions in (4) by rewriting the weights on the second and third term as $\frac{\frac{r}{\lambda}+1}{\frac{r}{\lambda}+2}$ and $\frac{1}{\frac{r}{\lambda}+2}$, respectively. Intuitively, what matters for the firms'

[20] In contrast, U's recontracting offer to D_B will *not* internalize the effect on D_A's variable profit in the time interval until the next recontracting with D_A, therefore the indirect effect $\pi_2(q, R_B(q)) R'_B(q)$ appears in the first-order condition for U's offer to D_A.

[21] Our specification allows for negative prices, but that is not essential for the results. The alternative specification $P(Q) = \max\{1 - Q, 0\}$ would yield the same equilibrium for high enough c, because the equilibrium dynamic reaction functions in our unrestricted specification satisfy $R_i(q_{-i}) + q_{-i} < 1$ for all $q_{-i} \in \left[0, \overline{Q}_c\right]$ and i if c is large enough. In the case of small c, where $R_i(q_{-i}) + q_{-i} > 1$ for q_{-i} close to $\overline{Q}_c = 1 - c$ (although on path aggregate quantity never exceeds 1, that is, $R_i(R_{-i}(q)) + R_{-i}(q) < 1$ for all $q \in \left[0, \overline{Q}_c\right]$), restricting the quantity action space to $\left[0, Q^M\right]$ re-establishes the equivalence between the two specifications, because the equilibrium dynamic reaction functions in our specification satisfy $R_i(q_{-i}) + q_{-i} < 1$ and $R_i(q_{-i}) \in \left(0, Q^M\right)$ for all $q_{-i} \in \left[0, Q^M\right]$ and i.

present discounted payoffs is not the absolute value of the discount rate but the discount rate relative to the expected time between recontracting events.

The next proposition uses the result that the equilibrium dynamic reaction functions must be downward-sloping together with the (local) stability condition for the steady-state quantities, $R'_A\left(q_B^e\right)R'_B\left(q_A^e\right)<1$,[22] to establish upper and lower bounds on the equilibrium steady-state quantity in a symmetric equilibrium:

Proposition 2 *Suppose $\lambda_A=\lambda_B=\lambda$. In a dynamically stable equilibrium with symmetric differentiable dynamic quantity reaction functions and a symmetric steady state,*

(i) *the steady-state quantity $q^e<q^C$.*

(ii) *the steady-state quantity $q^e\geq \underline{q}\left(\frac{r}{\lambda}\right)$, where the lower bound $\underline{q}\left(\frac{r}{\lambda}\right)\in\left(q^M,q^C\right)$ is uniquely defined by*

$$\Pi'\left(2\underline{q}\right)-\pi_2\left(\underline{q},\underline{q}\right)+\frac{1}{\frac{r}{\lambda}+1}\Pi'\left(2\underline{q}\right)=0,$$

strictly increasing in $\frac{r}{\lambda}$, and has limits

$$\lim_{\frac{r}{\lambda}\to 0}\underline{q}\left(\frac{r}{\lambda}\right)\in\left(q^M,q^C\right)\ \ and\ \ \lim_{\frac{r}{\lambda}\to\infty}\underline{q}\left(\frac{r}{\lambda}\right)=q^C.$$

Proof. Let $\lambda_A=\lambda_B=\lambda$, and suppose that a dynamically stable equilibrium with differentiable $R_A=R_B=R$ and a symmetric steady state (q^e,q^e) exists. The first-order condition (4) evaluated at (q^e,q^e) is

$$-c+\frac{\frac{r}{\lambda}+1}{\frac{r}{\lambda}+2}\pi_1\left(q^e,q^e\right)+\frac{1}{\frac{r}{\lambda}+2}\left(\pi_1\left(q^e,q^e\right)+\pi_2\left(q^e,q^e\right)R'\left(q^e\right)+\pi_2\left(q^e,q^e\right)\right)=0$$

for each $i=A,B$, which, using $\Pi'\left(2q^e\right)=\pi_1\left(q^e,q^e\right)+\pi_2\left(q^e,q^e\right)-c$, can be rewritten as

$$\Pi'\left(2q^e\right)-\pi_2\left(q^e,q^e\right)+\frac{1}{\frac{r}{\lambda}+1}\left(\Pi'\left(2q^e\right)+\pi_2\left(q^e,q^e\right)R'\left(q^e\right)\right)=0. \tag{5}$$

We first prove part (ii), and then part (i), of the proposition.

Part (ii): By Lemma 2, $R'\left(q^e\right)\leq 0$. Since $\pi_2\left(q,q\right)\leq 0$ for all $q\geq 0$, (5) and $R'\left(q^e\right)\leq 0$ imply that

$$0\geq\Pi'\left(2q^e\right)-\pi_2\left(q^e,q^e\right)+\frac{1}{\frac{r}{\lambda}+1}\Pi'\left(2q^e\right). \tag{6}$$

[22]The equilibrium reaction functions define a difference equation from $(q_{A,t},q_{B,t-1})$ to $(q_{A,t+2},q_{B,t+1})=\left(R_A\left(R_B(q_{A,t})\right),R_B\left(R_A(q_{B,t-1})\right)\right)$. Then, its Jacobian matrix evaluated at a steady state (q_A^e,q_B^e) has two equal real eigenvalues $R'_B(q_A^e)R'_A(q_B^e)$, which must be strictly less than one for (local) stability. See Vives (1999) for more details.

The function $G(q) \equiv \Pi'(2q) - \pi_2(q,q) + \frac{1}{\frac{r}{\lambda}+1}\Pi'(2q)$ is strictly decreasing in q, because both $\Pi'(2q) - \pi_2(q,q)$ and $\Pi'(2q)$ are strictly decreasing in q. Moreover, by the definitions of q^M and q^C, $G(q^M) = -\pi_2\left(q^M, q^M\right) > 0$ and $G(q^C) = \frac{1}{\frac{r}{\lambda}+1}\Pi'\left(2q^C\right) < 0$. Hence, for every $\frac{r}{\lambda} > 0$, there exists a unique $\underline{q}\left(\frac{r}{\lambda}\right) \in \left(q^M, q^C\right)$ such that

$$\Pi'\left(2\underline{q}\right) - \pi_2\left(\underline{q}, \underline{q}\right) + \frac{1}{\frac{r}{\lambda}+1}\Pi'\left(2\underline{q}\right) = 0, \tag{7}$$

and (6) implies that $q^e \geq \underline{q}\left(\frac{r}{\lambda}\right)$.

Applying the implicit function theorem to (7), we obtain that

$$\frac{d\underline{q}}{d\left(\frac{r}{\lambda}\right)} \overset{sign}{=} -\frac{1}{\left(\frac{r}{\lambda}+1\right)^2}\Pi'\left(2\underline{q}\right),$$

which is strictly positive because $\underline{q} > q^M$ and thus $\Pi'\left(2\underline{q}\right) < 0$. Moreover, it follows from (7) that $\lim_{\frac{r}{\lambda}\to\infty} \underline{q}\left(\frac{r}{\lambda}\right) = q^C$ and $\lim_{\frac{r}{\lambda}\to 0} \underline{q}\left(\frac{r}{\lambda}\right) \in \left(q^M, q^C\right)$.

Part (i): By the stability condition and Lemma 2, $R'(q^e) > -1$. Moreover, the result from part (ii) that $q^e > q^M$ implies that $\pi_2(q^e, q^e) < 0$. (5) and $R'(q^e) > -1$ thus imply that

$$0 < \left(1 + \frac{1}{\frac{r}{\lambda}+1}\right)\left(\Pi'(2q^e) - \pi_2(q^e, q^e)\right). \tag{8}$$

The function $\Pi'(2q) - \pi_2(q,q)$ is strictly decreasing in q and equal to zero for $q = q^C$. Hence, (8) implies that $q^e < q^C$. ■

Proposition 2 implies that any symmetric steady-state quantity lies strictly between q^M and q^C. The intution for this finding is as follows. Consider U's contract offer to D_A. When making an offer, the supplier internalizes only the effect on D_A's own variable profit in the time interval until the next recontracting with D_B, but it internalizes the effect on *all* retailers' variable profits from the next recontracting with D_B onwards (because the fixed fee offered to D_A takes into account expected changes in D_A's variable profit due to recontracting with D_B before D_A's own contract terminates, and similarly the fixed fees in future contracts extract all expected future changes in the retailers' variable profits). Hence, it is intuitive that the steady state in the dynamic model falls in between the benchmark cases of simultaneous secret offers with passive beliefs, in which the supplier only internalizes the effect on D_A's own variable profits when making an offer to D_A and each retailer sells q^C in equilibrium, and the case of simultaneous public offers, in which the

supplier internalizes the effects on all retailers variable profits when making an offer and each retailer sells q^M in the symmetric equilibrium.[23]

By this reasoning, one might expect that $\lim_{\frac{r}{\lambda}\to\infty} \underline{q}\left(\frac{r}{\lambda}\right) = q^C$ and $\lim_{\frac{r}{\lambda}\to 0} \underline{q}\left(\frac{r}{\lambda}\right) = q^M$, but the latter part of this intuition turns out to be wrong. While indeed $\lim_{\frac{r}{\lambda}\to\infty} \underline{q}\left(\frac{r}{\lambda}\right) = q^C$, we find that $\lim_{\frac{r}{\lambda}\to 0} \underline{q}\left(\frac{r}{\lambda}\right) > q^M$. Although the short-term gain from making an offer that raises bilateral profits at the expense of industry profit goes to zero when reactions become near instantaneous ($\lambda \to \infty$),[24] the equilibrium steady-state quantity must nonetheless lie strictly above the monopoly quantity. Intuitively, that the short-term gain from making an opportunistic offer goes to zero is not enough to guarantee that the supplier has no incentive to behave opportunistically, because the expected present discount value of the losses in future industry profits triggered by an opportunistic move also goes to zero in this limit case. In a dynamically stable equilibrium, quantities converge back to the steady-state level after a deviation from it (and do so "fast" for $\lambda \to \infty$), hence a deviation from a candidate steady-state $\left(q^M, q^M\right)$ does not have a persistent negative effect on industry profits.[25] For $\frac{r}{\lambda} \to \infty$, on the other hand, the short-term gain from an opportunistic move remains positive while the present discounted value of long-term effects goes to zero, hence the supplier makes offers that maximize bilateral profits in equilibrium.

Consistent with these intuitions, we obtain the following comparative statics results on the steady-state quantity in the linear-demand case:

Proposition 3 *Suppose* $P(Q) = 1 - Q$, $\lambda_A = \lambda_B = \lambda$, *and* $c \in [0, 1)$. *Then, the unique linear MPE is symmetric* ($R_A = R_B$) *with steady-state quantities* $q_A^e = q_B^e = q^e$. *The steady-state quantity* q^e *is strictly increasing in* $\frac{r}{\lambda}$, *with* $\lim_{\frac{r}{\lambda}\to 0} q^e = \frac{3(1-c)}{10} \in (q^M, q^C)$, *and*

[23]This insight may seem to suggest that $R^M(q_{-i}) < R(q_{-i}) < R^C(q_{-i})$ (which is equivalent to $q^M < q^e < q^C$ for $q_{-i} = q^e$) for all q_{-i}, but that intuition is incorrect. As will be illustrated later, $R^M(q_{-i}) > R(q_{-i})$ for small q_{-i} in the linear-demand case. Intuitively, maximizing current industry profits given q_{-i} is different from maximizing *future* industry profits, because the quantity offered to D_i today triggers reactions in future recontracting offers and thus dynamically affects the quantities of both retailers.

[24]Or, in the case of $r \to 0$, the weight put on the short-term gain goes to zero.

[25]The observation that both the short-term gain from an opportunistic offer and the net present value of the effects on future industry profits go to zero for $\frac{r}{\lambda} \to 0$ also holds for a deviation from a candidate equilibrium steady-state with a symmetric quantity $q^e > q^M$. However, intuitively the supplier's incentive to behave opportunistically is smaller when the hypothetical steady-state quantity is larger, because (i) the maximal gain in bilateral profits that $U - D_i$ can obtain is smaller for larger q_{-i}, and (ii) since industry profits are concave, the negative effect of a marginal increase in total quantity on industry profits is larger when the aggregate quantity is farther above q^M.

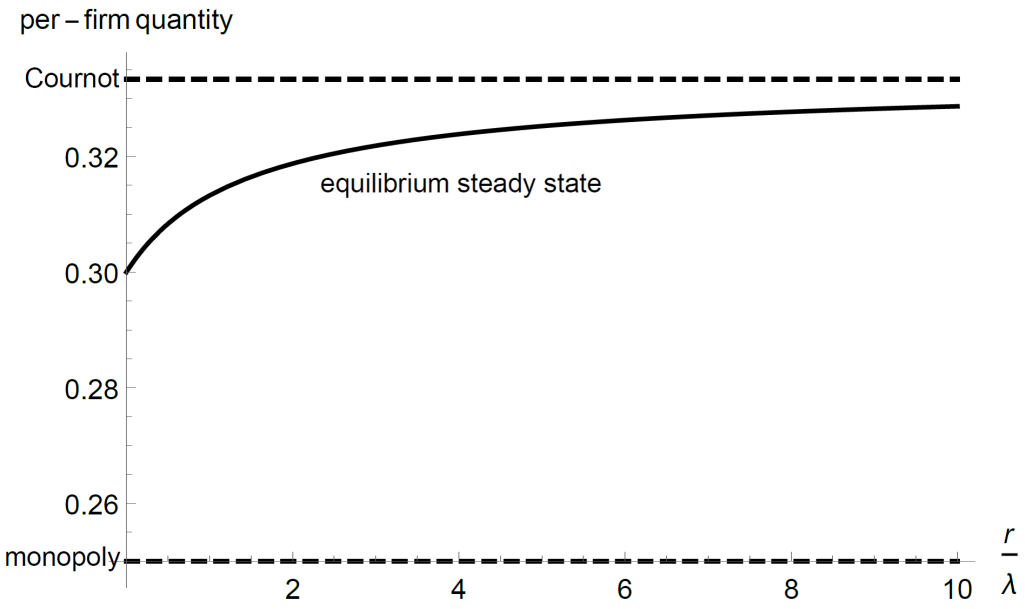

Figure 1: The solid line shows the equilibrium steady-state quantity q^e as a function of $\frac{r}{\lambda}$, the dashed lines the per-firm Cournot quantity (q^C) and the per-firm symmetric monopoly quantity (q^M). All quantities are computed for $P(Q) = 1 - Q$ and $c = 0$.

$\lim_{\frac{r}{\lambda} \to \infty} q^e = \frac{1-c}{3} = q^C$.

Figure 1 illustrates the comparative statics results of Proposition 3. The equilibrium steady-state quantity is smaller, that is, the opportunism problem is less severe, for greater patience (lower r) and faster reaction speed (higher λ). Intuitively, faster reaction speed alleviates the opportunism problem, for a given discount rate, because it decreases the length of time during which each supplier-retailer pair can gain from opportunistic moves at the expense of the rival retailer, and greater patience alleviates the opportunism problem, for a given reaction speed, because firms attach less weight to the short-term profit gains from opportunistic moves. As also illustrated in the figure, the equilibrium steady state approaches the Cournot quantity as $\frac{r}{\lambda}$ approaches infinity, but remains bounded strictly above the per-firm monopoly quantity as $\frac{r}{\lambda}$ approaches zero.

It is worth noting that although the aggregate quantity always lies between Q^M and $2q^C$ in the equilibrium steady state, the same need not be true out of steady state. This is illustrated in Figure 2, which shows how aggregate quantity changes with each reaction in the rival retailer's contract starting from the state $q_{-i} = 0$. Moreover, the pattern of

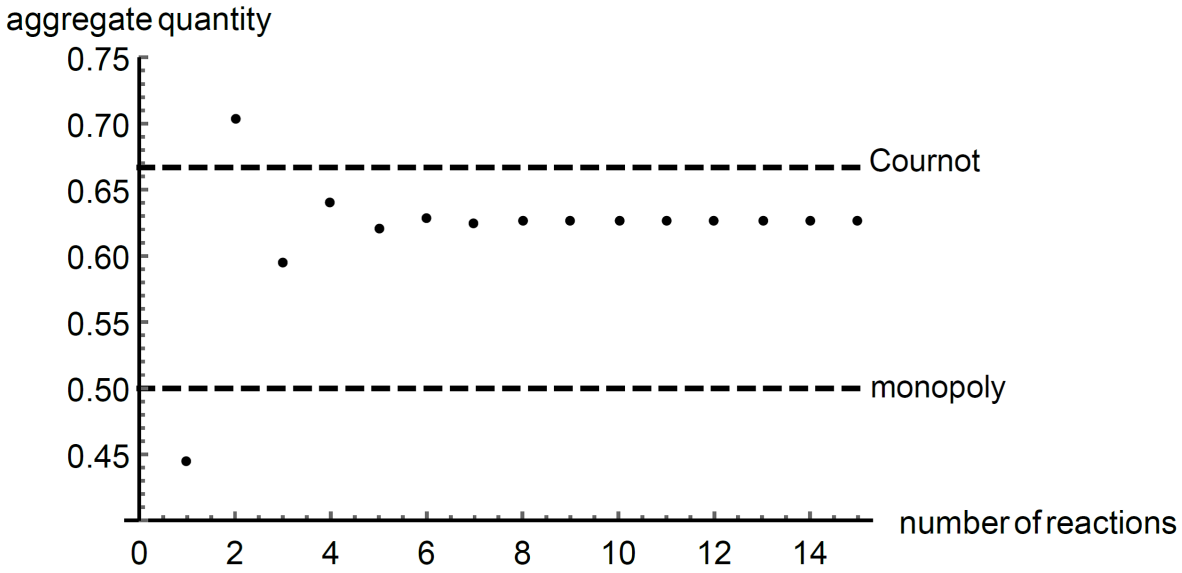

Figure 2: The dots show the sequence of aggregate quantities $0 + R(0)$, $R(R(0)) + R(0)$, $R(R(0)) + R(R(R(0)))$, etc. given the dynamic quantity reaction function R in the equilibrium from Proposition 3. The dashed lines show the aggregate Cournot quantity ($2q^C$) and the monopoly quantity (Q^M). All quantities are computed for $P(Q) = 1 - Q$ and $c = 0$.

convergence in Figure 2, whereby the aggregate quantity "zig-zags" around the steady state and the distance to the aggregate steady-state quantity decreases at each reaction, holds more generally, starting from any initial state:

Proposition 4 *Suppose $P(Q) = 1 - Q$, $\lambda_A = \lambda_B = \lambda$, and $c \in [0, 1)$. In the unique linear MPE,*

$$q + R(q) < 2q^e < R(q) + R(R(q)) \text{ if and only if } q < q^e,$$

with both inequalities reversed for $q > q^e$, and

$$|2q^e - (q + R(q))| > |2q^e - (R(q) + R(R(q)))| \text{ for any state } q \neq q^e.$$

5.3 Asymmetric reaction speeds

This section considers the implications of asymmetries in reaction speeds. Specifically, we assume that D_B's contract reacts faster than D_A's contract, i.e., that

$$\lambda_A \leq \lambda_B,$$

which is without loss of generality but simplifies the exposition.

In order to disentangle the effects of asymmetry from effects of changes in the aggregate recontracting rate, our comparative statics exercises will focus on the following two parameters. First, the mean reaction speed, denoted by

$$\lambda_M = \frac{\lambda_A + \lambda_B}{2},$$

and second, the degree of asymmetry in reaction speeds, defined as

$$\sigma = \frac{\lambda_M - \lambda_A}{\lambda_M} \in [0, 1).$$

The reaction speeds of the two supplier-retailer pairs can be expressed as functions of these two parameters as follows:

$$\begin{aligned} \lambda_A &= (1 - \sigma)\,\lambda_M, \\ \lambda_B &= (1 + \sigma)\,\lambda_M. \end{aligned}$$

For a small time interval Δt, the probability of a contract termination occuring is then given by $2\lambda_M \Delta t$, independently of the degree of asymmetry σ. Raising σ makes the reaction speeds more asymmetric without changing the mean recontracting rate.

Using the first-order conditions together with the result that the dynamic quantity reaction functions are non-increasing and the local stability condition, our next proposition establishes a series of insights about the steady-state quantities under asymmetric reaction speeds without imposing linear demand.

Proposition 5 *Suppose $\lambda_A \leq \lambda_B$. In any dynamically stable equilibrium with twice differentiable dynamic quantity reaction functions that have uniformly bounded first and second derivatives (when such an equilibrium exists),*

(i) *the aggregate steady-state quantity $q_A^e + q_B^e \geq \underline{Q}\left(\frac{r}{\lambda_M}, \sigma\right)$, where the lower bound $\underline{Q}\left(\frac{r}{\lambda_M}, \sigma\right) \in \left(Q^M, 2q^C\right)$ is uniquely defined by*

$$\Pi'\left(\underline{Q}\right) - \pi_2\left(\frac{1}{2}\underline{Q}, \frac{1}{2}\underline{Q}\right) + \frac{\frac{r}{\lambda_M} + 1 + \sigma^2}{\left(\frac{r}{\lambda_M} + 1 - \sigma\right)\left(\frac{r}{\lambda_M} + 1 + \sigma\right)}\Pi'\left(\underline{Q}\right) = 0,$$

strictly decreasing in σ, strictly increasing in $\frac{r}{\lambda_M}$, and has limits

$$\lim_{\sigma\to 0}\underline{Q}\left(\frac{r}{\lambda_M},\sigma\right) = 2\underline{q}\left(\frac{r}{\lambda_M}\right) > Q^M, \quad \lim_{\left(\frac{r}{\lambda_M},\sigma\right)\to(0,1)}\underline{Q}\left(\frac{r}{\lambda_M},\sigma\right) = Q^M, \text{ and}$$

$$\lim_{\frac{r}{\lambda_M}\to\infty}\underline{Q}\left(\frac{r}{\lambda_M},\sigma\right) = 2q^C;$$

(ii) *for any given $\epsilon > 0$, the steady-state quantities satisfy*

$$q_A^e < \epsilon \text{ and } \left|q_B^e - Q^M\right| < \epsilon$$

if $\frac{r}{\lambda_M}$ is close enough to 0 and σ is close enough to 1.

The key new insight in Proposition 5 is that asymmetry in reaction speeds can lead to a lower aggregate quantity in the equilibrium steady state (when an equilibrium exists). When the reaction speeds are symmetric, the aggregate steady-state quantity is bounded below by an amount strictly above the monopoly quantity. When the degree of reaction speed asymmetry is sufficiently large (and $\frac{r}{\lambda_M}$ is close enough to zero), however, the aggregate steady-state quantity is arbitrarily close to the monopoly quantity, as shown in part (ii) of the proposition.

Consistent with this, our next proposition shows that in the linear-demand case, the aggregate steady-state quantity is falling in the degree of reaction speed asymmetry in the unique equilibrium with linear dynamic quantity reaction functions.

Proposition 6 *Suppose $P(Q) = 1 - Q$, $c \in [0, 1)$, and $\lambda_A \leq \lambda_B$. The steady-state quantities (q_A^e, q_B^e) in the unique linear MPE vary with $\frac{r}{\lambda_M}$ and σ as follows:*

(i) *The aggregate quantity $q_A^e + q_B^e$ is strictly increasing in $\frac{r}{\lambda_M}$.*

(ii) *The aggregate quantity $q_A^e + q_B^e$ is strictly decreasing in σ, q_A^e is strictly decreasing in σ, and q_B^e is strictly increasing in σ.*

(iii) $\lim_{\left(\frac{r}{\lambda_M},\sigma\right)\to(0,1)}(q_A^e, q_B^e) = (0, Q^M)$.

Figures 3 and 4 provide graphical illustrations of these results. First, as illustrated in Figure 3, reaction speed asymmetry reduces the degree of opportunism: For a given mean reaction speed, the aggregative quantity in the steady state is lower for higher degrees of

asymmetry. Second, and consistent with the results in the symmetric case, for a given degree of asymmetry, the aggregate steady-state quantity is smaller when the firms are more patient (smaller r) or the mean reaction speed is higher (greater λ_M). Third, as illustrated in Figure 4, at the steady state the retailer whose contract reacts faster sells a larger quantity than the retailer whose contract reacts more slowly. Finally, as $\frac{r}{\lambda_M} \to 0$ and $\sigma \to 1$, the steady state approaches the "exclusive dealing outcome" in which one retailer sells the entire monopoly quantity and its rival sells zero.[26] For $q_{-i} = 0$, the conflict vanishes altogether, as $R^C(0) = R^M(0) = Q^M$. Hence, although reaction speed asymmetry strengthens the incentive for opportunism with D_B for a *given* current quantity q_A, when q_A is small the conflict between collective surplus maximization and bilateral surplus maximization between the supplier and D_B is weak to begin with. By selling a small quantity to D_A, exploiting the weakened incentive to behave opportunistically when making offers to D_A, the supplier thus also weakens its own incentive to behave opportunistically when making the next contract offer to D_B.

Overall, this intuition suggests that, consistent with our results, the aggregate steady-state quantity will be smaller under reaction speed asymmetry than under symmetry, and that it will be allocated asymmetrically across retailers, with the retailer whose contract reacts faster (D_B) selling a larger quantity than its rival. In the limit case where $\sigma \to 1$ and $\frac{r}{\lambda_M} \to 0$, D_A's quantity converges to zero, eliminating the conflict between collective surplus maximization and bilateral surplus maximization when U contracts with D_B. Anticipating the slow future reaction of D_A's contract (since $\lambda_A \to 0$ when $\sigma \to 1$), the supplier therefore optimally offers the quantity $R^C(0) = R^M(0) = Q^M$ to D_B. The supplier also has no profitable deviation to offering a larger quantity to D_A in this limit case, because doing so would harm future industry profits and these negative effects are persistent when $\lambda_A \approx 0$.

More generally, the steady state in the limit case where $\sigma \to 1$ and $\frac{r}{\lambda_M} \to 0$ corresponds to the equilibrium outcome of a sequential-move game in which U first offers a contract to D_A and then to D_B, and the second retailers (D_B) observes the first retailer's (D_A's) contract. The contract offered to the second retailer (D_B) then maximizes the bilateral surplus of the supplier and the second retailer given the contract accepted by the first

[26]Formally, $R^C(q_{-i}) - R^M(q_{-i})$ is strictly increasing for all $q_{-i} > 0$ because $\frac{\partial R^M}{\partial q_{-i}} = -1$ while $\frac{\partial R^C}{\partial q_{-i}} \in (-1, 0)$ by assumptions A1 and A2.

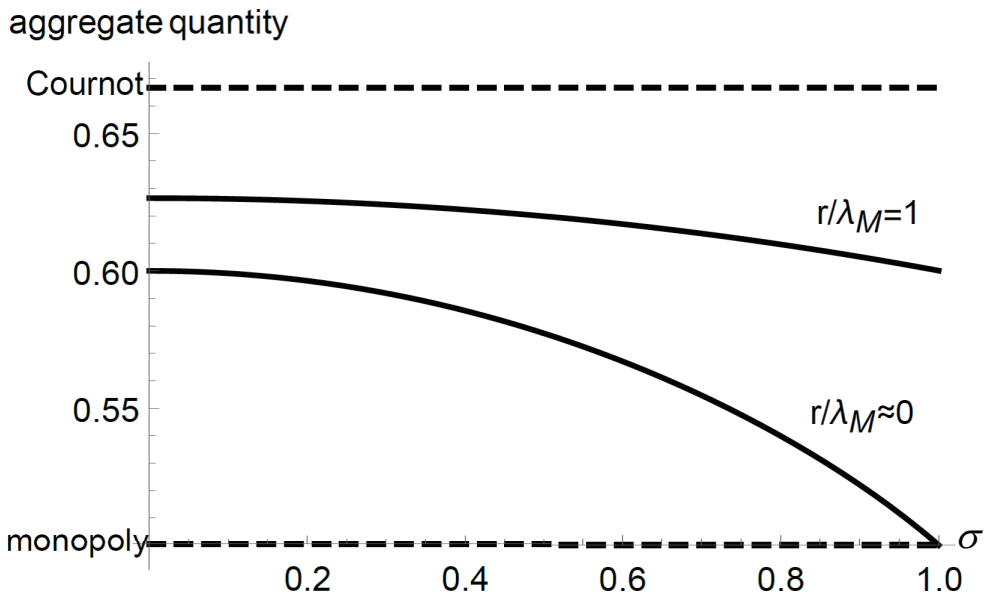

Figure 3: The solid lines show the aggregate equilibrium steady-state quantity $q_A^e + q_B^e$ as a function of the reaction speed asymmetry σ for two different values of $\frac{r}{\lambda_M}$, the dashed lines the aggregate Cournot quantity ($2q^C$) and the monopoly quantity (Q^M). All quantities are computed for $P(Q) = 1 - Q$ and $c = 0$.

retailer, while the contract offered to the first retailer (D_A) maximizes the bilateral surplus of the supplier and the first retailer anticipating that the later contract will maximize the bilateral surplus of the supplier and the second retailer (D_B). In our setting, this means that the supplier offers $R^C(q_A)$ to D_B, and, anticipating this, offers 0 to D_A, which leads to quantities $\left(0, Q^M\right)$ and a total profit of Π^M.

6 Extensions

6.1 Product differentiation

While the result that the opportunism problem is fully solved in the limit clearly hinges on the retailers selling undifferentiated products, the findings that reaction speed asymmetry and patience alleviate the opportunism problem hold more broadly. To see this, we now extend our baseline model to allow for differentiation between the final products sold by

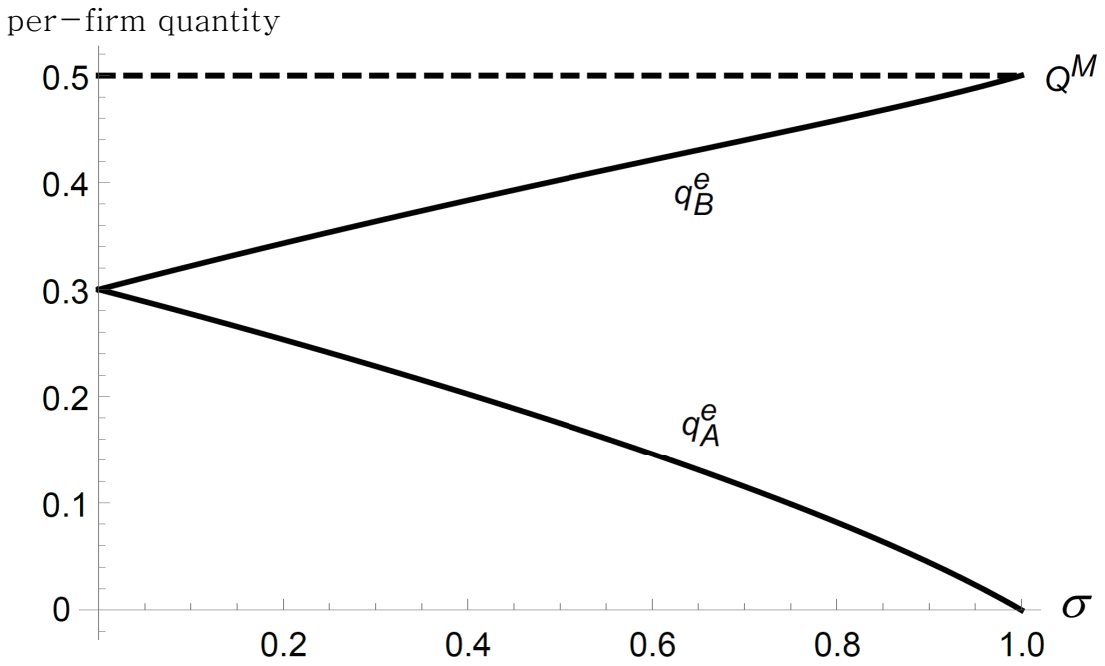

Figure 4: The solid lines show the equilibrium steady-state quantities q_A^e and q_B^e as a function of the reaction speed asymmetry σ, given $\frac{r}{\lambda_M} \approx 0$. The dashed line indicates the monopoly quantity Q^M. All quantities are computed for $P(Q) = 1 - Q$ and $c = 0$.

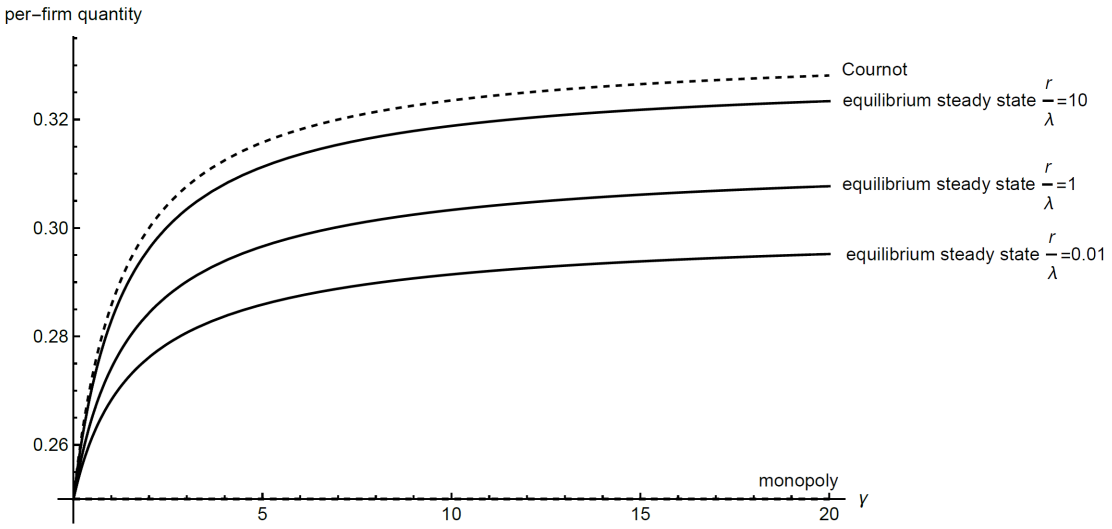

Figure 5: Illustration of the per-firm steady-state quantity under differentiated Cournot competition with symmetric reaction speeds when $c = 0$.

the retailers. Suppose the inverse demand curve for D_i's good is

$$P_i\left(q_i, q_{-i}\right) = 1 - \frac{2+\gamma}{1+\gamma}\left(q_i + \frac{\gamma}{2+\gamma} q_{-i}\right), \tag{9}$$

where $\gamma \geq 0$ measures the degree of product substitutability, with $\gamma = 0$ corresponding to independent goods and $\gamma \to \infty$ to perfect substitutes. Under this demand formulation, which is due to Levitan and Shubik (1971), the monopoly benchmark is independent of the degree of product substitutability: $q_{LS}^M = q^M = \frac{1-c}{4}$ and $P_i\left(q^M, q^M\right) = \frac{1+c}{2}$. The Cournot quantity given the marginal production cost c becomes $q_{LS}^C = \frac{(1+\gamma)(1-c)}{4+3\gamma}$, which is rising in the degree of substitutability γ. Competition is more intense when the retailers sell closer substitutes. The limit $\gamma \to \infty$ corresponds to our baseline model: $\lim_{\gamma\to\infty} q_{LS}^C = q^C$.

As in our baseline linear-demand setting, the game has a unique linear MPE that is dynamically stable.[27] Figure 5 illustrates the main comparative statics of the equilibrium steady-state quantity under reaction speed symmetry.[28] The per-firm steady-state quantity,

[27]The proof of equilibrium existence and uniqueness within the class of linear MPE proceeds in the same way as in our baseline model with perfect substitutes.

[28]For $\lambda_A = \lambda_B = \lambda$, the equilibrium steady-state quantity given the demand functions in (9) becomes

$$q_{LS}^e = \frac{3\left(1-c\right)\left(1+\gamma\right)\left(2+\widetilde{r}\right)}{10\left(2+\widetilde{r}\right)+\gamma\left(19+8\widetilde{r}\right)+\sqrt{4\left(1+\gamma\right)\left(2+\widetilde{r}\right)^2+\gamma^2\left(1+\widetilde{r}+\widetilde{r}^2\right)}},$$

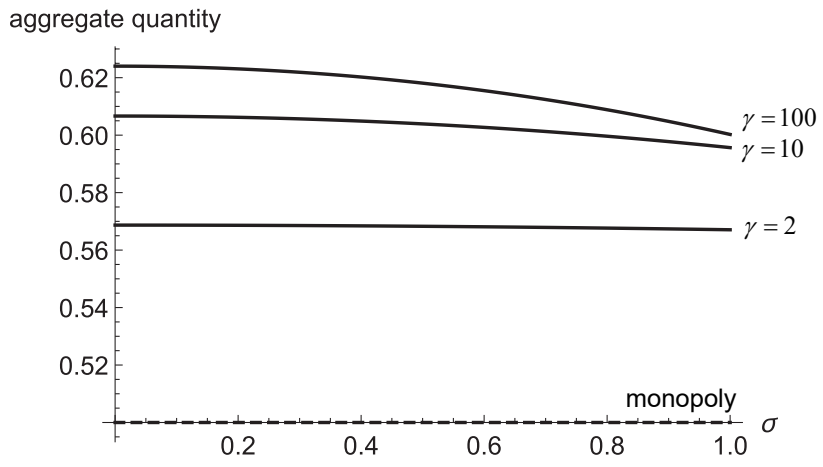

Figure 6: Illustration of the aggregate steady-state quantity under differentiated Cournot competition with asymmetric reaction speeds when $\lambda_A = (1-\sigma)\lambda_M, \lambda_B = (1+\sigma)\lambda_M, c = 0$ and $\frac{r}{\lambda_M} = 1$.

which lies between the monopoly quantity and the Cournot quantity, rises in the degree of product substitutability. Hence, the degree of opportunism, defined as the gap between the the steady-state equilibrium outcome and the monopoly outcome, is greater when the final products are closer substitutes. Intuitively, for given reaction speeds and discount rates, better substitutability leads to greater opportunism because each bilateral pair has greater incentives to free-ride on the rival downstream firm when the competitive externalities are stronger.[29] The comparative statics of the equilibrium steady-state quantity with respect to $\frac{r}{\lambda}$ are the same as before.

The insight that reaction speed asymmetry alleviates the opportunism problem also extends to the setting with differentiated final goods, as illustrated in Figure 6. Moreover, reaction speed asymmetry affects the aggregate steady-state equilibrium quantity more strongly when the downstream goods are closer substitutes (that is, for higher γ).

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where $\widetilde{r} = \frac{r}{\lambda}$. Straightforward calculations show that $\frac{\partial q^e_{LS}}{\partial \gamma} > 0$ for all $\gamma > 0$, and that $\frac{\partial q^e_{LS}}{\partial \widetilde{r}} > 0$.

[29] In Online Appendix D, we also analyze an extension of the model to three retailers, with a fixed order of recontracting. We find that (under reaction speed symmetry) the equilibrium degree of opportunism is greater with three retailers than with two retailers. Intuitively, all else equal, the incentive of each bilateral supplier-retailer to free-ride on the rival retailers is greater when there are more retailers and thus each individual supplier-retailer pair has a smaller market share in a symmetric equilibrium.

6.2 Two-part tariffs

We now consider two-part tariff contracts, which allow retailers to choose their quantities freely and adjust them in response to changes in the rival's contract. Suppose that a contract between U and D_i consists of a vector (w_i, f_i), where w_i is the wholesale price per unit of quantity ordered and $f_i \in \mathbb{R}$ is a flow of fixed payment per unit of time. Contract rejection is denoted by $(w, T) = (\infty, 0)$.

The retailers can adjust their quantities freely, and we assume that at any point in time, the retailers play the Nash equilibrium of the downstream competition game given the current wholesale price vector.[30] Retailer D_i's quantity given the wholesale price vector (w_i, w_{-i}) is

$$q(w_i, w_{-i}) = \arg\max_q \left(P\left(q + q\left(w_{-i}, w_i\right)\right) - w_i\right) q.$$

If D_{-i} does not have an accepted contract, then D_{-i}'s quantity is $q(\infty, w_i) = 0$ and D_i's quantity is $q(w_i, \infty) = \arg\max_q \left(P(q) - w_i\right) q$.

Retailer D_i's variable downstream profit can thus be written in reduced form as

$$\pi(w_i, w_{-i}) \equiv \pi\left(q\left(w_i, w_{-i}\right), q\left(w_{-i}, w_i\right)\right) = \left(P\left(q\left(w_i, w_{-i}\right) + q\left(w_{-i}, w_i\right)\right) - w_i\right) q\left(w_i, w_{-i}\right).$$

The supplier's profit is

$$f_A + f_B + (w_A - c)\, q(w_A, w_B) + (w_B - c)\, q(w_B, w_A),$$

and the industry profit becomes

$$\Pi(w_A, w_B) = \pi(w_A, w_B) + (w_A - c)\, q(w_A, w_B) + \pi(w_B, w_A) + (w_B - c)\, q(w_B, w_A).$$

Benchmarks The benchmarks against which we will compare the equilibrium steady state of the dynamic game are the monopoly wholesale prices and the pairwise-proof wholesale prices.

As in the case of quantity-forcing contracts, the equilibrium of the game with simultaneous public offers features monopoly profits. In equilibrium, the supplier offers the wholesale prices $\left(w^M, w^M\right)$ that induce the retailers to set the industry profit-maximizing quantities

[30] This requires that each retailer can adjust its own quantity without delay and observes changes in its rival's wholesale price, either because contract offers are publicly observed or because accepted wholesale prices are ex post observable.

(i.e., $q\left(w^M, w^M\right) = q^M$) and sets $f_i = \pi\left(w^M, w^M\right)$ for all i. The equilibrium wholesale price $w^M \in \left(c, P\left(2q^M\right)\right)$ and is equal to $w^M = \frac{1+3c}{4}$ in our baseline linear demand specification.

Our other benchmark are the pairwise-proof wholesale prices that satisfy the first-order equilibrium conditions of the simultaneous secret-offers game with passive beliefs and ex post observability, ensuring that the retailers play the Nash equilibrium of the downstream competition game given wholesale prices as in our dynamic setting.[31] Letting $(\widehat{w}_A, \widehat{w}_B)$ denote the equilibrium wholesale prices, under passive beliefs D_i is willing to accept (w, f) if and only if $f \leq \pi\left(w, \widehat{w}_{-i}\right)$. Thus, $\widehat{w}_i$ must maximize the bilateral profit of U and D_i given $\widehat{w}_{-i}$ (pairwise proofness):

$$\widehat{w}_i = \arg\max_{w_i} \pi\left(w_i, \widehat{w}_{-i}\right) + \left(w_i - c\right) q\left(w_i, \widehat{w}_{-i}\right) + \left(\widehat{w}_{-i} - c\right) q\left(\widehat{w}_{-i}, w_i\right).$$

As shown in McAfee and Schwartz (1995), the pairwise-proof wholesale price in a symmetric game lies below the supplier's marginal cost: $\widehat{w} < c$. In our linear-demand specification, $\widehat{w} = \frac{3c-1}{2}$, which is non-negative if and only if $c \geq \frac{1}{3}$, and the per-retailer quantity becomes $q\left(\widehat{w}, \widehat{w}\right) = \frac{1-c}{2}$.

It is important to note, however, that while the pairwise-proof wholesale prices satisfy the first-order equilibrium conditions in the simultaneous-offers game with passive beliefs and ex post observability, a perfect Bayesian equilibrium also requires immunity to multilateral deviations. And as shown by McAfee and Schwartz (1995) and Rey and Vergé (2004), multilateral deviations are profitable under linear demand and undifferentiated goods, leading to equilibrium inexistence. As we show next, however, our dynamic game can have an MPE, with a steady-state that approaches the pairwise-proof outcome in the limit, even in cases where the static game with secret offers, passive beliefs, and ex post observability does not have a perfect Bayesian equilibrium.

Dynamic model The state variable when U contracts with D_i is now the wholesale price in U's current contract with D_{-i}, and the dynamic reaction function $R_i\left(w_{-i}\right)$ determines the wholesale price offered to D_i given D_{-i}'s current wholesale price. We are interested in an MPE in which rejections do not occur on the equilibrium path.

[31] Under ex post (or interim) observability, the retailers observe each other's accepted contracts before competing in the downstream market. Ex post observability in simultaneous-offer games has been considered by McAfee and Schwartz (1994), McAfee and Schwartz (1995), and Rey and Vergé (2004), among others.

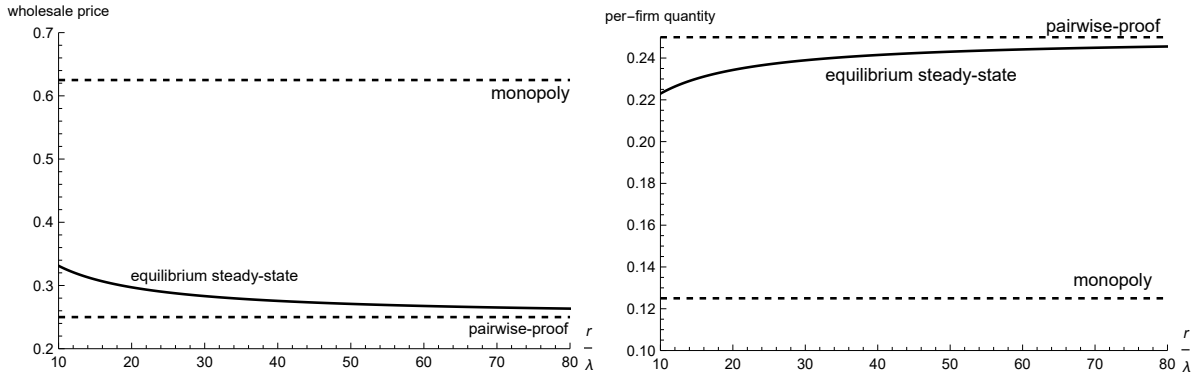

Figure 7: Illustration of the per-firm steady-state wholesale price and quantity under linear demand $P(Q) = 1 - Q$ with $\lambda_A = \lambda_B = \lambda$ and $c = 0.5$.

In Online Appendix C, we provide a detailed equilibrium analysis. Two new challenges arise relative to the case of quantity-forcing contracts. First, unlike with quantity-forcing contracts, there is no null contract whose acceptance is equivalent to contract rejection. "Large" deviations that induce contract rejection therefore need to be ruled out separately even when the second-order conditions for deviations that induce contract acceptance are satisfied. Second, solving for the equilibrium strategies in the linear-demand specification becomes more computationally difficult.

In spite of these challenges, we are able to establish existence of unique linear MPE and derive comparative statics results in our linear-demand setting under some restrictions on the parameter values.[32] Figure 7 illustrates the equilibrium steady-state under symmetric reaction speeds for parameter values such that deviations that induce rejection are unprofitable and wholesale prices, quantities, and prices are nonnegative on- and off the equilibrium path. The symmetric steady-state wholesale price decreases in $\frac{r}{\lambda}$ and converges to the pairwise-proof wholesale price level $\widehat{w}$ as $\frac{r}{\lambda}$ goes to infinity. The steady-state per-firm quantity increases in $\frac{r}{\lambda}$ and converges to the pairwise-proof quantity level $q(\widehat{w}, \widehat{w})$, consistent with the result in our baseline setting. Moreover, as shown formally in the Online Appendix, the opportunism problem cannot disappear, even in the limit $\frac{r}{\lambda} \to 0$.

Numerical analyses also suggest that asymmetry in reaction speeds continues to reduce

[32]For instance, we can show that if $c \in \left(\frac{1}{2}, \frac{5}{9}\right)$ and the upper bound on the supplier's wholesale price action space is chosen appropriately, there exists a unique linear MPE for sufficiently small λ. In this equilibrium, wholesale prices, quantities, and prices are nonnegative on and off the equilibrium path. A formal analysis can be found in Online Appendix C.

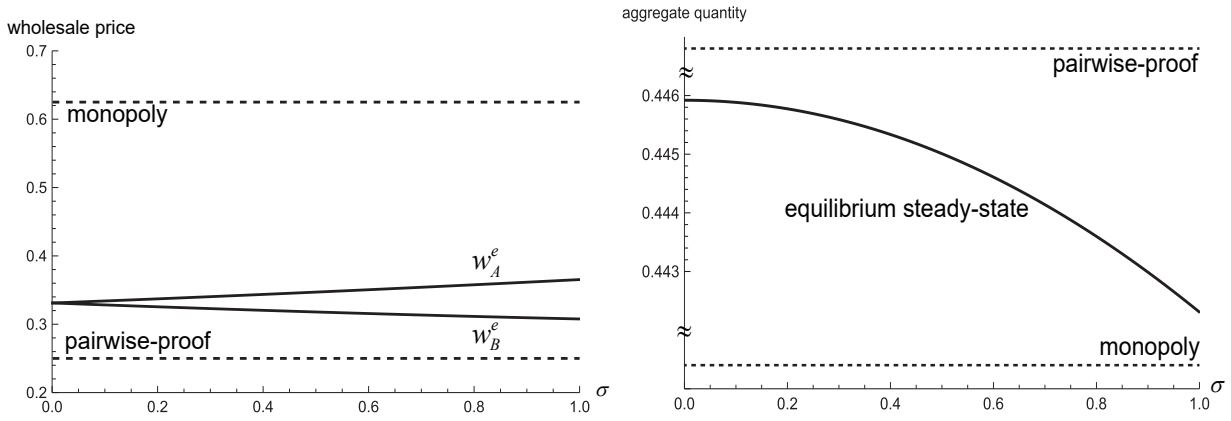

Figure 8: Illustration of the steady-state wholesale prices and aggregate quantities under linear demand $P(Q) = 1 - Q$ with $\lambda_A = (1-\sigma)\lambda_M, \lambda_B = (1+\sigma)\lambda_M, c = 0.5$ and $\frac{r}{\lambda_M} = 10$.

the degree of opportunism with two-part tariffs contracts. The left panel of Figure 8 depicts the steady-state equilibrium wholesale prices, showing that the wholesale price of D_B, whose contract reacts faster, is lower than the wholesale price of D_A, whose contract reacts more slowly. As a result, D_B sells a larger quantity than D_A in the steady state, which is qualitatively similar to the baseline model of quantity-forcing contracts. The overall impact of reaction asymmetry is illustrated in the right panel of Figure 8, which shows that the aggregate steady-state quantity is decreasing in the reaction speed asymmetry.

The analysis with two-part tariffs raises the exciting prospect that the dynamic modeling approach proposed in this paper could potentially lend a proper game-theoretic foundation for pairwise-proofness and the related "contract equilibrium" concept pioneered by Cremer and Riordan (1987). The contract equilibrium concept, which imposes pairwise-proofness but lacks an equilibrium foundation, is hard to defend in simultaneous-offer games because it ignores multilateral deviations by the supplier. And indeed, as shown by Rey and Vergé (2004), such multilateral deviations are profitable in many cases, leading to inexistence of a perfect Bayesian equilibrium with passive beliefs (in which pairwise proofness is implied by the first-order conditions). In our dynamic model, the issue of multilateral deviations does not arise because offers are made asynchronously, and we have shown that there can exist an MPE with a steady state that converges to the pairwise-proof outcome in a setting where the simultaneous secret-offers game with passive beliefs and ex post observability does not have an equilibrium. We leave a more complete analysis of the prospect that a dynamic

model could serve as a game-theoretic foundation for the contract equilibrium concept for future research.

7 Concluding Remarks

We have analyzed a dynamic model of bilateral contracting between one supplier and multiple competing downstream firms. In our setting, each downstream firm anticipates future recontracting between the supplier and its rival (as well as itself), and supplier suffers from opportunism even if it can make public contract offers. The proposed dynamic model offers an alternative to simultaneous secret-offers models of opportunism in vertical contracting. Although characterizing equilibria tends to be more difficult in the dynamic model, we have shown that a unique equilibrium in simple linear strategies, with closed-form solutions for the strategies and equilibrium steady-state quantities, exists under linear demand. Moreover, the dynamic model overcomes a key weakness of simultaneous secret-offers models, the sensitivity of the equilibrium outcome to out-of-equilibrium beliefs. The dynamic model also unifies existing literature, by offering a setting that generates the equilibrium outcomes of existing models, with either simultaneous or sequential offers, as special limit cases.

The focus of our analysis has been the extent of opportunism that prevails in the absence of any strategies used by the supplier to restore its monopoly power. In Online Appendix B, we discuss various strategies that would allow the supplier to overcome its opportunism problem, many of which mirror solutions in static games. The supplier can restore its monopoly power by vertically integrating with one of the retailers, by committing to deal exclusively with one retailer, or by offering the retailers opt-out contracts, which allow retailers to stop buying before contract termination and thereby protect them against future opportunistic moves.

Our results are relevant for vertical merger policy as well as for competition policy on vertical restraints. First, the results show that secret offers are not needed for opportunism to arise, and that vertical mergers and opportunism-avoiding contract provisions like exclusive dealing or opt-out clauses can have anticompetitive effects even when contracts are public. Second, the results offer guidance on when vertical mergers and opportunism-avoiding contract provisions are likely to be more harmful. We have found that the degree

of opportunism is greater when firms are impatient, there are long time gaps between recontracting, and reaction speeds are symmetric. This, in turn, suggests that vertical mergers and opportunism-avoiding contract provisions are likely to be more harmful for downstream consumers (and more attractive for suppliers) when firms are impatient, time gaps between recontracting are long, and reaction speeds are symmetric. Proxies for the model's key parameters that can potentially be observed and used by competition authorities to help assess the likely competitive harm include the average duration of downstream firm i's contracts (as a proxy for $\frac{1}{\lambda_i}$) and the interest rate (as a proxy for r). Moreover, useful information to assess the degree of asymmetries in reaction speeds would include observed asymmetries in the time gap between a change in downstream firm A's supply terms followed by a change in downstream firm B's supply terms versus the opposite order, and evidence of otherwise similar downstream firms being treated asymmetrically prior to a vertical merger.

There are several interesting issues that we leave for future research. First, an important yet difficult direction for future research is to endogenize the speed of contract reactions and the associated contract durations. Second, a more extensive analysis of non-linear vertical contracts could be done in more general environments. Finally, the model could potentially be extended to allow for upstream competition.

Appendix: Proofs

Proof of Lemma 1. Let $V_i(s) = V_i(R_i(s), F_i(s); s)$ and recall that $M_i(R_i(s), F_i(s); s) = 1$ for all i and s. We have that

$$\begin{aligned} V_i(q, f; s) &= \frac{\pi(q, s) - f}{r + \lambda_i + \lambda_{-i}} + \frac{\lambda_i}{r + \lambda_i + \lambda_{-i}} V_i(s) + \frac{\lambda_{-i}}{r + \lambda_i + \lambda_{-i}} \widetilde{V}_i(q, f), \\ \widetilde{V}_i(q, f) &= \frac{\pi(q, R_{-i}(q)) - f}{r + \lambda_i} + \frac{\lambda_i}{r + \lambda_i} V_i(R_{-i}(q)). \end{aligned}$$

The first term in $V_i(q, f; s)$ captures the flow profit that D_i earns until the next contract termination occurs. The second term captures the case in which D_i's own contract terminates before D_{-i}'s contract. The third term captures the case in which D_{-i}'s contract terminates first. In the latter case, D_i earns flow profit $(\pi(q, R_{-i}(q)) - f)$ in the time interval between the termination of D_{-i}'s contract and the termination of its own current contract,[33] and an expected discounted profit of $V_i(R_{-i}(q))$ thereafter.[34]

Substituting $\widetilde{V}_i(q, f)$ into $V_i(q, f; s)$ yields

$$\begin{aligned} V_i(q, f; s) &= -\frac{f}{r + \lambda_i} + \frac{\pi(q, s)}{r + \lambda_i + \lambda_{-i}} + \frac{\lambda_{-i}}{(r + \lambda_i + \lambda_{-i})(r + \lambda_i)} \pi(q, R_{-i}(q)) \\ &\quad + \frac{\lambda_i \lambda_{-i}}{(r + \lambda_i + \lambda_{-i})(r + \lambda_i)} V_i(R_{-i}(q)) + \frac{\lambda_i}{r + \lambda_i + \lambda_{-i}} V_i(s). \end{aligned}$$

Firm D_i prefers acceptance over rejection if $V_i(q, f; s) \geq V_i(0, 0; s)$, which holds if and only if

$$f \leq \frac{r + \lambda_i}{r + \lambda_i + \lambda_{-i}} \pi(q, s) + \frac{\lambda_{-i}}{r + \lambda_i + \lambda_{-i}} \pi(q, R_{-i}(q)) + \frac{\lambda_i \lambda_{-i}}{r + \lambda_i + \lambda_{-i}} (V_i(R_{-i}(q)) - V_i(R_{-i}(0))). \quad (10)$$

The supplier's objective function in (1) is strictly increasing in f, because $\frac{\partial W_{-i}(q,f)}{\partial f} = \frac{1}{r+\lambda_i} > 0$. Hence, the retailer's acceptance condition must be binding in equilibrium, otherwise U could increase its profit by offering a contract with the same quantity but a higher fixed payment. It follows that

$$V_i(s) \equiv V_i(R_i(s), F_i(s); s) = \frac{\lambda_i \lambda_{-i}}{(r + \lambda_i + \lambda_{-i})(r + \lambda_i)} V_i(R_{-i}(0)) + \frac{\lambda_i}{r + \lambda_i + \lambda_{-i}} V_i(s),$$

which can be rewritten as

$$V_i(s) = \frac{\lambda_i \lambda_{-i}}{(r + \lambda_i)(r + \lambda_{-i})} V_i(R_{-i}(0)).$$

Since this must hold for all s including $s = R_{-i}(0)$, we can conclude that

$$V_i(s) = 0 \text{ for all } s.$$

[33] Note that if D_{-i}'s contract terminates within this interval, D_{-i} will be offered and accept a contract with quantity $R_{-i}(q)$ again, hence D_i's flow profit remains constant until its own contract terminates.

[34] For details on how to derive the weights in $V_i(q, f; s)$ and $\widetilde{V}_i(q)$, see Online Appendix A, which presents the omitted details for the analysis of the supplier's objective function.

Hence, (10) simplifies to $f \leq f_i(q; q_{-i}) \equiv \frac{r+\lambda_i}{r+\lambda_i+\lambda_{-i}} \pi(q, q_{-i}) + \frac{\lambda_{-i}}{r+\lambda_i+\lambda_{-i}} \pi(q, R_{-i}(q))$. ■

Proof of Lemma 2. The proof strategy follows that of Lemma 1 in Maskin and Tirole (1988a). Suppose (in negation) that there exist $q_{-i} > q'_{-i}$ such that $R_i(q_{-i}) > R_i(q'_{-i})$. By the definition of R_i, we have that

$$\begin{aligned}
&\pi(R_i(q_{-i}), q_{-i}) - c(R_i(q_{-i}) + q_{-i}) + \frac{\lambda_{-i}}{r+\lambda_i}\pi(R_i(q_{-i}), R_{-i}(R_i(q_{-i}))) + \lambda_{-i}\overline{W}_{-i}(R_i(q_{-i})) \\
&\geq \pi(R_i(q'_{-i}), q_{-i}) - c(R_i(q'_{-i}) + q_{-i}) + \frac{\lambda_{-i}}{r+\lambda_i}\pi(R_i(q'_{-i}), R_{-i}(R_i(q'_{-i}))) + \lambda_{-i}\overline{W}_{-i}(R_i(q'_{-i})), \qquad (11)
\end{aligned}$$

and similarly,

$$\begin{aligned}
&\pi(R_i(q'_{-i}), q'_{-i}) - c(R_i(q'_{-i}) + q'_{-i}) + \frac{\lambda_{-i}}{r+\lambda_i}\pi(R_i(q'_{-i}), R_{-i}(R_i(q'_{-i}))) + \lambda_{-i}\overline{W}_{-i}(R_i(q'_{-i})) \\
&\geq \pi(R_i(q_{-i}), q'_{-i}) - c(R_i(q_{-i}) + q'_{-i}) + \frac{\lambda_{-i}}{r+\lambda_i}\pi(R_i(q_{-i}), R_{-i}(R_i(q_{-i}))) + \lambda_{-i}\overline{W}_{-i}(R_i(q_{-i})). \qquad (12)
\end{aligned}$$

Adding (11) to (12), we obtain that

$$\pi(R_i(q_{-i}), q_{-i}) - \pi(R_i(q'_{-i}), q_{-i}) \geq \pi(R_i(q_{-i}), q'_{-i}) - \pi(R_i(q'_{-i}), q'_{-i}),$$

which can be rewritten as

$$\int_{q'_{-i}}^{q_{-i}} \int_{R_i(q'_{-i})}^{R_i(q_{-i})} \pi_{12}(x, y) dx dy \geq 0.$$

This is a contradiction because $\pi_{12} < 0$. ■

Proof of Proposition 1. Suppose that $P(Q) = 1 - Q$ and $c \in [0, 1)$. We look for an equilbrium with linear dynamic quantity reactions functions of the form

$$R_i(q_{-i}) = \alpha_i - \beta_i q_{-i},$$

where $\beta_i \geq 0$ for each i. Given linearity of the dynamic reaction functions, the first-order conditions (4) simplify as follows: for $q = R_i(q_{-i})$,

$$1 - c - q_{-i} - 2q + \frac{\lambda_{-i}}{r+\lambda_i}(1 - 2\alpha_{-i} - c + (3\beta_{-i} - 2)q) = 0,$$

or, equivalently,

$$R_i(q_{-i}) = \frac{1}{2 - \frac{\lambda_{-i}}{r+\lambda_i}(3\beta_{-i} - 2)} \left(1 - c + \frac{\lambda_{-i}}{r+\lambda_i}(1 - 2\alpha_{-i} - c) - q_{-i}\right). \qquad (13)$$

Setting the right-hand side of (13) equal to $\alpha_i - \beta_i q_{-i}$ for each $i = A, B$, we obtain that either $(\beta_A, \beta_B) = (\beta_A^*, \beta_B^*)$ or $(\beta_A, \beta_B) = (\beta_A^{**}, \beta_B^{**})$, where

$$\beta_i^* = \frac{7\lambda_i + \lambda_{-i} + 4r - \sqrt{(\lambda_A + \lambda_B + 4r)^2 + 12\lambda_A\lambda_B}}{12\lambda_i} > 0, \qquad (14)$$

and

$$\beta_i^{**} = \frac{7\lambda_i + \lambda_{-i} + 4r + \sqrt{(\lambda_A + \lambda_B + 4r)^2 + 12\lambda_A\lambda_B}}{12\lambda_i} > 0. \qquad (15)$$

We first show that the solution (β_A^*, β_B^*) gives rise to a dynamically stable equilibrium, and then rule out an equilibrium in which $(\beta_A, \beta_B) = (\beta_A^{**}, \beta_B^{**})$. From the definition of β_i^*,

$$\beta_i^* < \frac{7\lambda_i + \lambda_{-i} + 4r - \lambda_A - \lambda_B - 4r}{12\lambda_i} = \frac{1}{2}.$$

To see the second-order conditions, note that the value functions of the supplier are quadratic under linear demand and dynamic quantity reaction functions. By taking derivatives of both sides in the first-order condition (3) with respect to q_{-i}, we have: for all q_{-i},

$$G\left(q_{-i}\right) R_i'\left(q_{-i}\right) + \pi_{12}\left(R_i\left(q_{-i}\right), q_{-i}\right) = 0,$$

where $G\left(q_{-i}\right)$ is the second derivative of the supplier's objective function,

$$\begin{aligned} G\left(q_{-i}\right) = \pi_{11}\left(q, q_{-i}\right) + \frac{\lambda_{-i}}{r+\lambda_i}\Big(&\pi_{11}\left(q, R_{-i}\left(q\right)\right) + \pi_{12}\left(q, R_{-i}\left(q\right)\right) R_{-i}'\left(q\right) + \pi_{12}\left(q, R_{-i}\left(q\right)\right) R_{-i}'\left(q\right) \\ &+ \pi_{22}\left(q, R_{-i}\left(q\right)\right) R_{-i}'\left(q\right) + \pi_2\left(q, R_{-i}\left(q\right)\right) R_{-i}''\left(q\right)\Big) + \lambda_{-i}\overline{W}_{-i}''\left(q\right), \end{aligned}$$

with $q = R_i\left(q_{-i}\right)$. Note that $G\left(q_{-i}\right)$ is constant and strictly negative since $\pi_{12} < 0$ and $R_i'\left(q_{-i}\right) = -\beta_i^* < 0$. Hence, the necessary conditions in (13) are also sufficient.

Given the slope parameters, we can characterize the intercepts of the dynamic quantity reaction functions. First, one can check from (13) that the following system of linear equations should be satisfied:

$$\alpha_i^* = \frac{(1-c)(r+\lambda_i) + \lambda_{-i}(1 - 2\alpha_{-i}^* - c)}{2(r+\lambda_i) - \lambda_{-i}(3\beta_{-i}^* - 2)}.$$

Solving for the intercepts yields

$$\alpha_i^* = \frac{\left(1-c\right)\left(\lambda_A + \lambda_B + r\right)\left(7\lambda_i + \lambda_{-i} + 4r - \sqrt{\left(\lambda_A + \lambda_B + 4r\right)^2 + 12\lambda_A\lambda_B}\right)}{2\lambda_i\left(7\lambda_A + 7\lambda_B + 10r - \sqrt{\left(\lambda_A + \lambda_B + 4r\right)^2 + 12\lambda_A\lambda_B}\right)}, \tag{16}$$

for each $i = A, B$.

Letting $R_i^*(q_{-i}) = \alpha_i^* - \beta_i^* q_{-i}$, we check that $R_i^*(q_{-i}) \in [0, \overline{Q}_c] = [0, 1-c]$ for all $q_{-i} \in [0, 1-c]$ and $i = A, B$, i.e., that it was innocuous to ignore the lower and upper bound on the action space in the supplier's optimization problem. To economize on notation, let us define

$$x = \left(\lambda_A + \lambda_B + 4r\right)^2 + 12\lambda_A\lambda_B.$$

We then obtain that

$$R_i^*(0) = \frac{\left(1-c\right)\left(\lambda_A + \lambda_B + r\right)\left(7\lambda_i + \lambda_{-i} + 4r - \sqrt{x}\right)}{2\lambda_i\left(7\lambda_A + 7\lambda_B + 10r - \sqrt{x}\right)},$$

and

$$R_i^*(\overline{Q}_c) = R_i^*(1-c) = \frac{\left(1-c\right)\left(\sqrt{x} - \lambda_A - \lambda_B - 4r\right)\left(7\lambda_i + \lambda_{-i} + 4r - \sqrt{x}\right)}{12\lambda_i\left(7\lambda_A + 7\lambda_B + 10r - \sqrt{x}\right)}.$$

It is immediate that $R_i^*(\overline{Q}_c) \geq 0$, and hence $R_i^*(q_{-i}) \geq 0$ for all $q_{-i} \in [0, \overline{Q}_c]$. Moreover, $R_i^*(0) \leq 1 - c$ is equivalent to

$$\begin{aligned} &\left(\lambda_A + \lambda_B + r\right)\left(7\lambda_i + \lambda_{-i} + 4r - \sqrt{x}\right) \leq 2\lambda_i\left(7\lambda_A + 7\lambda_B + 10r - \sqrt{x}\right) \\ \iff &\left(\lambda_{-i} - \lambda_i + r\right)\left(7\lambda_A + 7\lambda_B + 10r - \sqrt{x}\right) \leq 6\left(\lambda_{-i} + r\right)\left(\lambda_A + \lambda_B + r\right), \end{aligned}$$

which is true since $6\lambda_A + 6\lambda_B + 6r \geq 7\lambda_A + 7\lambda_B + 10r - \sqrt{x}$ and $\lambda_{-i} + r \geq \lambda_{-i} - \lambda_i + r$.

We can conclude that there exists an equilibrium with steady-state quantities (q_A^e, q_B^e) that satisfy $R_A^*(q_B^e) = q_A^e$ and $R_B^*(q_A^e) = q_B^e$ and are given by

$$(q_A^e, q_B^e) = \left(\frac{\alpha_A^* - \beta_A^* \alpha_B^*}{1 - \beta_A^* \beta_B^*}, \frac{\alpha_B^* - \beta_B^* \alpha_A^*}{1 - \beta_A^* \beta_B^*} \right). \tag{17}$$

The steady state is dynamically stable because the slopes of the reaction functions are less than one in absolute value.

It remains to rule out the existence of an equilibrium with slope parameters $(\beta_A, \beta_B) = (\beta_A^{**}, \beta_B^{**})$, which would give rise to an unstable dynamic path because $\beta_A^{**} \beta_B^{**} > 1$.[35] Computing the intercepts of the dynamic quantity reaction functions that correspond to $(\beta_A^{**}, \beta_B^{**})$, we obtain

$$\alpha_i^{**} = \frac{(1-c)(\lambda_A + \lambda_B + r)\left(7\lambda_i + \lambda_{-i} + 4r + \sqrt{(\lambda_A + \lambda_B + 4r)^2 + 12\lambda_A\lambda_B}\right)}{2\lambda_i\left(7\lambda_A + 7\lambda_B + 10r + \sqrt{(\lambda_A + \lambda_B + 4r)^2 + 12\lambda_A\lambda_B}\right)}. \tag{20}$$

Denoting $R_i^{**}(q_{-i}) = \alpha_i^{**} - \beta_i^{**} q_{-i}$, it is easy to check $R_i^{**}\left(\overline{Q}_c\right) < 0$. Thus, there is no MPE in which the dynamic quantity reaction functions are (R_A^{**}, R_B^{**}), because the restriction that $R_i : \left[0, \overline{Q}_c\right] \to \left[0, \overline{Q}_c\right]$ would be violated.

It is worth noting that this equilibrium inexistence result is not an artifact of the restriction to non-negative quantities. First, if quantities were allowed to take on negative values and the supplier were to

[35] Since each β_i^{**} is strictly increasing in r,

$$\begin{aligned} \beta_A^{**}\beta_B^{**} &> \frac{\left(7\lambda_A + \lambda_B + \sqrt{(\lambda_A+\lambda_B)^2 + 12\lambda_A\lambda_B}\right)\left(\lambda_A + 7\lambda_B + \sqrt{(\lambda_A+\lambda_B)^2 + 12\lambda_A\lambda_B}\right)}{144\lambda_A\lambda_B} \\ &= \frac{1}{3} + \frac{(\lambda_A+\lambda_B)^2 + (\lambda_A+\lambda_B)\sqrt{(\lambda_A+\lambda_B)^2 + 12\lambda_A\lambda_B}}{18\lambda_A\lambda_B}, \end{aligned} \tag{18}$$

where the last equality is obtained by straightforward algebra. From (18), $\beta_A^{**}\beta_B^{**} > 1$ if

$$\frac{(\lambda_A+\lambda_B)^2 + (\lambda_A+\lambda_B)\sqrt{(\lambda_A+\lambda_B)^2 + 12\lambda_A\lambda_B}}{18\lambda_A\lambda_B} \geq \frac{2}{3},$$

which can be rewritten as

$$(\lambda_A+\lambda_B)^2 + (\lambda_A+\lambda_B)\sqrt{(\lambda_A+\lambda_B)^2 + 12\lambda_A\lambda_B} - 12\lambda_A\lambda_B \geq 0. \tag{19}$$

For a given $\lambda_T = \lambda_A + \lambda_B$, the derivative of the left-hand-side of (19) with respect to $\lambda_A\lambda_B$ is

$$\frac{6\lambda_T}{\sqrt{\lambda_T^2 + 12\lambda_A\lambda_B}} - 12 < 0.$$

Hence, if (19) holds for $\lambda_A = \lambda_B = \frac{\lambda_T}{2}$, then it also holds for all other $\lambda_A, \lambda_B > 0$ such that $\lambda_A + \lambda_B = \lambda_T$. For $\lambda_A = \lambda_B = \frac{\lambda_T}{2}$, (19) becomes $\lambda_T^2 + \lambda_T\sqrt{\lambda_T^2 + 3\lambda_T^2} - 3\lambda_T^2 \geq 0$, which holds with equality. Thus, $\beta_A^{**}\beta_B^{**} > 1$.

follow (R_A^{**}, R_B^{**}), its present discounted payoff when contracting with one retailer would tend to $+\infty$ and its present discounted payoff when contracting with the other retailer would tend to $-\infty$. Hence, the value functions would not be well defined. Second, an equilibrium with piece-wise linear dynamic quantity reaction functions of the form $R_i(q_{-i}) = \max\{\alpha_i^{**} - \beta_i^{**} q_{-i}, 0\}$ does not exist either, as shown in the next lemma.[36] ■

Lemma A1 *Suppose $P(Q) = 1 - Q$ and $c \in [0,1)$. There does not exist a MPE in which the dynamic quantity reaction functions are*

$$R_i(q_{-i}) = \max\{\alpha_i^{**} - \beta_i^{**} q_{-i}, 0\} \text{ for each } i,$$

where α_i^{**} and β_i^{**} are as given in (20) and (15), respectively.

Proof of Lemma A1. Suppose (in negation) that there exists a MPE with the dynamic reaction functions

$$R_i(q_{-i}) = \max\{\alpha_i^{**} - \beta_i^{**} q_{-i}, 0\}$$

for each $i = A, B$. Moreover, without loss of generality, suppose that $\lambda_A \geq \lambda_B$.

For all $i = A, B$,

$$\alpha_i^{**} - \beta_i^{**} q_{-i} = 0 \iff q_{-i} = \overline{q} \equiv \frac{6(1-c)(\lambda_A + \lambda_B + r)}{7(\lambda_A + \lambda_B) + 10r + \sqrt{x}}.$$

It is easy to check that

$$\alpha_B^{**} > \overline{q},$$

and thus $R_B(0) > \overline{q}$, if and only if

$$24\lambda_B(\lambda_A - \lambda_B + 2r) > 0,$$

which is true for $\lambda_A \geq \lambda_B$.

Since $R_A(q_B) = 0$ for all $q_B \geq \overline{q}$, we have that for all $q_B \geq \overline{q}$,

$$\overline{W}_A(q_B) = -\frac{cq_B}{r + \lambda_B} + \frac{\lambda_B}{r + \lambda_B} \bar{W}_B(0).$$

Now consider U's optimal offer to D_B when D_A's current quantity is 0. According to the postulated equilibrium strategies, $R_B(0) = \alpha_B^{**} > \overline{q}$, hence

$$\begin{aligned} \overline{W}_B(0) &= \frac{\pi(\alpha_B^{**}, 0) - c\alpha_B^{**}}{r + \lambda_A} + \frac{\lambda_A}{(r + \lambda_A)(r + \lambda_B)} \pi(\alpha_B^{**}, 0) + \frac{\lambda_A}{r + \lambda_A} \overline{W}_A(\alpha_B^{**}) \\ &= \frac{\pi(\alpha_B^{**}, 0) - c\alpha_B^{**}}{r + \lambda_A} + \frac{\lambda_A}{(r + \lambda_A)(r + \lambda_B)} \pi(\alpha_B^{**}, 0) + \frac{\lambda_A}{r + \lambda_A} \left(-\frac{c\alpha_B^{**}}{r + \lambda_B} + \frac{\lambda_B}{r + \lambda_B} \overline{W}_B(0) \right). \end{aligned}$$

[36] In contrast, since $R_i^*(q_{-i}) > 0$ for all $q_{-i} \in [0, \overline{Q}_c]$, an equilibrium in which $R_i(q_{-i}) = \max\{R_i^*(q_{-i}), 0\}$ exists and exhibits the same dynamics and steady-state quantities as the equilibrium charaterized in this proof.

Now consider a one-shot deviation to offering $\alpha_B^{**}-\epsilon$, where $\alpha_B^{**}-\epsilon \geq \overline{q}$. U's deviation profit is

$$\begin{aligned}\widetilde{W}_B(\alpha_B^{**}-\epsilon,0) &\equiv \frac{\pi(\alpha_B^{**}-\epsilon,0)-c(\alpha_B^{**}-\epsilon)}{r+\lambda_A}+\frac{\lambda_A}{(r+\lambda_A)(r+\lambda_B)}\pi(\alpha_B^{**}-\epsilon,0)+\frac{\lambda_A}{r+\lambda_A}\overline{W}_A(\alpha_B^{**}-\epsilon)\\ &= \frac{\pi(\alpha_B^{**}-\epsilon,0)-c(\alpha_B^{**}-\epsilon)}{r+\lambda_A}+\frac{\lambda_A}{(r+\lambda_A)(r+\lambda_B)}\pi(\alpha_B^{**}-\epsilon,0)\\ &\quad +\frac{\lambda_A}{r+\lambda_A}\left(-\frac{c(\alpha_B^{**}-\epsilon)}{r+\lambda_B}+\frac{\lambda_B}{r+\lambda_B}\overline{W}_B(0)\right).\end{aligned}$$

Comparing U's deviation profit to its equilibrium profit, we obtain that

$$\widetilde{W}_B(\alpha_B^{**}-\epsilon,0)-\overline{W}_B(0)=\left(\frac{\lambda_A+\lambda_B+r}{(r+\lambda_A)(r+\lambda_B)}\right)\epsilon\left(c+2\alpha_B^{**}-(1+\epsilon)\right).$$

Thus, U has a profitable deviation if there exists an $\epsilon>0$ such that $\alpha_B^{**}-\epsilon\geq\overline{q}$ and

$$c>1+\epsilon-2\alpha_B^{**} \iff \alpha_B^{**}>\frac{1-c+\epsilon}{2}.$$

If $\alpha_B^{**}>\frac{1-c}{2}$, then both inequalities are satisfied for small enough $\epsilon>0$. Denoting $x=(\lambda_A+\lambda_B+4r)^2+12\lambda_A\lambda_B$, we obtain that $\alpha_B^{**}>\frac{1-c}{2}$ if and only if

$$(\lambda_A+\lambda_B+r)(7\lambda_B+\lambda_A+4r+\sqrt{x})>\lambda_B(7\lambda_A+7\lambda_B+10r+\sqrt{x}),$$

which can be simplified to

$$7\lambda_A+7\lambda_B+10r+\sqrt{x}>6\lambda_A+6\lambda_B+6r,$$

which is true. Hence, U has a strictly profitable deviation. ■

Proof of Proposition 3. Let $\lambda_A=\lambda_B=\lambda$, and suppose that $P(Q)=1-Q$ and $c\in[0,1)$. From the expressions for α_i^* and β_i^* in (16) and (14) in the proof of Proposition 1, it is immediate that $\alpha_A^*=\alpha_B^*=\alpha^*$ and $\beta_A^*=\beta_B^*=\beta^*$. Specifically, letting $\tilde{r}\equiv\frac{r}{\lambda}$, we obtain that

$$\beta^*=\frac{8\lambda+4r-\sqrt{(2\lambda+4r)^2+12\lambda^2}}{12\lambda}=\frac{2+\tilde{r}-\sqrt{1+\tilde{r}+\tilde{r}^2}}{3},$$

and

$$\begin{aligned}\alpha^* &= \frac{(1-c)(2\lambda+r)\left(8\lambda+4r-\sqrt{(2\lambda+4r)^2+12\lambda^2}\right)}{2\lambda\left(14\lambda+10r-\sqrt{(2\lambda+4r)^2+12\lambda^2}\right)}\\ &= (1-c)(2+\tilde{r})\left(\frac{2+\tilde{r}-\sqrt{1+\tilde{r}+\tilde{r}^2}}{7+5\tilde{r}-2\sqrt{1+\tilde{r}+\tilde{r}^2}}\right).\end{aligned}$$

Given that the dynamic quantity reaction functions are symmetric, the steady-state quantities are symmetric as well and given by

$$q^e=\frac{\alpha^*}{1+\beta^*}=(1-c)(2+\tilde{r})\,3\frac{2+\tilde{r}-\sqrt{1+\tilde{r}+\tilde{r}^2}}{\left(7+5\tilde{r}-2\sqrt{1+\tilde{r}+\tilde{r}^2}\right)\left(5+\tilde{r}-\sqrt{1+\tilde{r}+\tilde{r}^2}\right)},$$

which can be simplified to

$$q^e = \frac{(1-c)(2+\tilde{r})3}{19+8\tilde{r}+\sqrt{1+\tilde{r}+\tilde{r}^2}}. \tag{21}$$

The symmetric steady-state quantity q^e is strictly increasing in $\tilde{r}$ because

$$\begin{aligned}
\frac{\partial q^e}{\partial \tilde{r}} &\overset{sign}{=} 19+8\tilde{r}+\sqrt{1+\tilde{r}+\tilde{r}^2}-(2+\tilde{r})\left(8+\frac{1+2\tilde{r}}{2\sqrt{1+\tilde{r}+\tilde{r}^2}}\right) \\
&= 3+\sqrt{1+\tilde{r}+\tilde{r}^2}-\frac{2\left(1+\tilde{r}+\tilde{r}^2\right)}{2\sqrt{1+\tilde{r}+\tilde{r}^2}}-\frac{3\tilde{r}}{2\sqrt{1+\tilde{r}+\tilde{r}^2}} \\
&= 3\left(1-\frac{\tilde{r}}{2\sqrt{1+\tilde{r}+\tilde{r}^2}}\right)>0.
\end{aligned}$$

Next, observe that for $\tilde{r}=0$, (21) simplifies to $q^e=\frac{3(1-c)}{10}$. It remains to show that q^e converges to $q^C=\frac{1-c}{3}$ as $\tilde{r}$ tends to $+\infty$. From (21),

$$\begin{aligned}
\lim_{\tilde{r}\to\infty} q^e &= 3(1-c)\lim_{\tilde{r}\to\infty}\left(\frac{2+\tilde{r}}{19+8\tilde{r}+\sqrt{1+\tilde{r}+\tilde{r}^2}}\right) \\
&= 3(1-c)\lim_{\tilde{r}\to\infty}\left(\frac{\frac{2}{\tilde{r}}+1}{\frac{19}{\tilde{r}}+8+\sqrt{\frac{1+\tilde{r}}{\tilde{r}^2}+1}}\right) \\
&= 3(1-c)\frac{1}{9},
\end{aligned}$$

which is equal to the Cournot quantity q^C. ■

Proof of Proposition 4. As shown in Propositions 1 and 3, the unique linear MPE is symmetric with a slope $-\beta^* \in \left(-\frac{1}{2},0\right)$ of $R(q)$. It follows that $q'+R(q')>q+R(q)$ if and only if $q'>q$. Since $R(q^e)=q^e$, it follows that $q+R(q)<(>)2q^e$ if and only if $q<(>)q^e$. Moreover, $R'<0$ implies that $R(q)>(<)q^e$ if and only if $q<(>)q^e$. Together, these observations imply that if $q<q^e$, then

$$q+R(q)<2q^e<R(q)+R(R(q)),$$

with both inequalities reversed if $q>q^e$.

Next, note that $\frac{\partial(q+R(q)+R(q)+R(R(q)))}{\partial q}=(1-\beta^*)^2>0$. Hence, $q+R(q)+R(q)+R(R(q))<(>)4q^e$ if and only if $q<(>)q^e$. It follows that for any $q\neq q^e$,

$$|2q^e-(q+R(q))|>|2q^e-(R(q)+R(R(q)))|.$$

Proof of Proposition 5. Let $\tilde{r}=\frac{r}{\lambda_M}$ thoughout the proof. Note that the first-order equilibrium conditions in (4) can be rewritten as functions of $\tilde{r}$ and σ as follows: at $q=R_A(q_B)$,

$$0=-c+\frac{\tilde{r}+1-\sigma}{\tilde{r}+2}\pi_1(q,q_B)+\frac{1+\sigma}{\tilde{r}+2}\left(\pi_1(q,R_B(q))+\pi_2(q,R_B(q))R_B'(q)+\pi_2(R_B(q),q)\right), \tag{22}$$

and at $q=R_B(q_A)$,

$$0 = -c + \frac{\tilde{r}+1+\sigma}{\tilde{r}+2}\pi_1\left(q, q_A\right) + \frac{1-\sigma}{\tilde{r}+2}\left(\pi_1\left(q, R_A\left(q\right)\right) + \pi_2\left(q, R_A\left(q\right)\right) R'_A\left(q\right) + \pi_2\left(R_A\left(q\right), q\right)\right). \tag{23}$$

Part (i): The first-order conditions evaluated at a steady state (q_A^e, q_B^e) can be written as

$$\begin{aligned} R'_B(q_A^e) &= -\frac{\left(\tilde{r}+2\right)\Pi'\left(Q^e\right) - \left(\tilde{r}+1-\sigma\right)\pi_2(q_B^e, q_A^e)}{\left(1+\sigma\right)\pi_2(q_A^e, q_B^e)}, \\ R'_A(q_B^e) &= -\frac{\left(\tilde{r}+2\right)\Pi'\left(Q^e\right) - \left(\tilde{r}+1+\sigma\right)\pi_2(q_A^e, q_B^e)}{\left(1-\sigma\right)\pi_2(q_B^e, q_A^e)}, \end{aligned}$$

where $Q^e = q_A^e + q_B^e$ denotes the aggregate quantity. The stability condition $R'_A(q_B^e)R'_B(q_A^e) < 1$ therefore implies that

$$\frac{\left(\left(\tilde{r}+2\right)\Pi'\left(Q^e\right) - \left(\tilde{r}+1+\sigma\right)\pi_2(q_A^e, q_B^e)\right)\left(\left(\tilde{r}+2\right)\Pi'\left(Q^e\right) - \left(\tilde{r}+1-\sigma\right)\pi_2(q_B^e, q_A^e)\right)}{\left(1-\sigma\right)\pi_2(q_B^e, q_A^e)\left(1+\sigma\right)\pi_2(q_A^e, q_B^e)} < 1,$$

or, equivalently,

$$\begin{aligned} &\left(\left(\tilde{r}+1-\sigma\right)\left(P(Q^e) - c + q_A^e P'\left(Q^e\right)\right) + \left(1+\sigma\right)\left(P(Q^e) - c + Q^e P'\left(Q^e\right)\right)\right) \\ &\times\left(\left(\tilde{r}+1+\sigma\right)\left(P(Q^e) - c + q_B^e P'\left(Q^e\right)\right) + \left(1-\sigma\right)\left(P(Q^e) - c + Q^e P'\left(Q^e\right)\right)\right) \\ &< (1+\sigma)(1-\sigma)\left(P'\left(Q^e\right)\right)^2 q_A^e q_B^e. \end{aligned} \tag{24}$$

Now suppose (in negation) that $q_A^e \geq R^C(q_B^e)$ and $q_B^e \geq R^C(q_A^e)$. By the definition of the Cournot reaction function, we then have that $P(Q^e) - c + q_A^e P'\left(Q^e\right) \leq 0$ and $P(Q^e) - c + q_B^e P'\left(Q^e\right) \leq 0$. Moreover, adding up these two conditions yields $P(Q^e) - c + \frac{Q^e}{2}P'(Q^e) \leq 0$, and thus

$$P(Q^e) - c + Q^e P'(Q^e) \leq \frac{Q^e}{2}P'(Q^e) \leq 0,$$

It follows that the left-hand side of (24) is greater than or equal to

$$(1+\sigma)(1-\sigma)\frac{\left(Q^e\right)^2}{4}\left(P'(Q^e)\right)^2. \tag{25}$$

We now show that (25) is greater than or equal to the right-hand side of (24), thereby establishing a contradiction. Since $q_A^e + q_B^e = Q^e$ and $q_i^e \geq 0$, there exists an $\alpha \in [0,1]$ such that $q_A^e = \alpha Q^e$, $q_B^e = (1-\alpha)Q^e$, and thus $q_A^e q_B^e = \alpha\left(1-\alpha\right)\left(Q^e\right)^2$. As $\alpha\left(1-\alpha\right) \leq \frac{1}{4}$ for all $\alpha \in [0,1]$, it follows that the right-hand side of (24) is smaller than or equal to

$$(1+\sigma)(1-\sigma)\left(P'\left(Q^e\right)\right)^2\frac{\left(Q^e\right)^2}{4}. \tag{26}$$

Since (25) is equal to (26), condition (24) is violated. Hence, it cannot be that $q_A^e \geq R^C(q_B^e)$ and $q_B^e \geq R^C(q_A^e)$ in a stable equilibrium with differentiable dynamic quantity reaction functions.

Part (ii): From $R'_B \leq 0$, $R'_A \leq 0$, and the first-order conditions, we obtain that

$$\begin{aligned}\frac{\tilde{r}+1-\sigma}{\tilde{r}+2}\left(\Pi'\left(q_A^e+q_B^e\right)-\pi_2\left(q_B^e,q_A^e\right)\right)+\frac{1+\sigma}{\tilde{r}+2}\Pi'\left(q_A^e+q_B^e\right) &\leq 0,\\ \frac{\tilde{r}+1+\sigma}{\tilde{r}+2}\left(\Pi'\left(q_A^e+q_B^e\right)-\pi_2\left(q_A^e,q_B^e\right)\right)+\frac{1-\sigma}{\tilde{r}+2}\Pi'\left(q_A^e+q_B^e\right) &\leq 0,\end{aligned}$$

which can be rewritten as

$$\begin{aligned}q_B^e\frac{P'\left(Q^e\right)}{\Pi'\left(Q^e\right)} &\leq \frac{\tilde{r}+2}{\tilde{r}+1-\sigma},\\ q_A^e\frac{P'\left(Q^e\right)}{\Pi'\left(Q^e\right)} &\leq \frac{\tilde{r}+2}{\tilde{r}+1+\sigma}.\end{aligned}$$

Adding up these two conditions yields

$$\frac{Q^eP'\left(Q^e\right)}{\Pi'\left(Q^e\right)}\leq\frac{2\left(\tilde{r}+2\right)\left(\tilde{r}+1\right)}{\left(\tilde{r}+1+\sigma\right)\left(\tilde{r}+1-\sigma\right)}. \tag{27}$$

The left-hand side of (27) is strictly decreasing in Q^e, because

$$\frac{\partial}{\partial Q}\left(\frac{QP'\left(Q\right)}{\Pi'\left(Q\right)}\right)=\frac{\left(P'(Q)+QP''(Q)\right)\left(P(Q)-c\right)-Q\left(P'(Q)\right)^2}{\left(\Pi'(Q)\right)^2}<0, \tag{28}$$

where the inequality follows from assumption A2 and $P\left(Q^e\right)\geq c$. Moreover, $\lim_{Q^e\to Q^M}\frac{Q^eP'(Q^e)}{\Pi'(Q^e)}=\infty$ and $\frac{2q^CP'\left(2q^C\right)}{\Pi'(2q^C)}=2$, while the right-hand side of (27) lies strictly above 2 for all $\tilde{r}>0$ and $\sigma\in[0,1)$. It follows that, for each $\tilde{r}>0$ and $\sigma\in[0,1)$, there exists a unique $\underline{Q}\left(\tilde{r},\sigma\right)\in\left(Q^M,2q^C\right)$ such that

$$\frac{\Pi'\left(\underline{Q}\right)}{\underline{Q}P'\left(\underline{Q}\right)}=\frac{\left(\tilde{r}+1-\sigma\right)\left(\tilde{r}+1+\sigma\right)}{2\left(\tilde{r}+2\right)\left(\tilde{r}+1\right)}, \tag{29}$$

which can be rewritten as

$$\Pi'\left(\underline{Q}\right)-\pi_2\left(\frac{1}{2}\underline{Q},\frac{1}{2}\underline{Q}\right)+\frac{\tilde{r}+1+\sigma^2}{\left(\tilde{r}+1-\sigma\right)\left(\tilde{r}+1+\sigma\right)}\Pi'\left(\underline{Q}\right)=0. \tag{30}$$

The right-hand side of (29) is strictly decreasing in σ, and strictly increasing in $\tilde{r}$. Given (28), it follows that $\underline{Q}\left(\tilde{r},\sigma\right)$ is strictly decreasing in σ, and strictly increasing in $\tilde{r}$.

For $\sigma=0$, (30) coincides with the condition that defines the lower bound $\underline{q}\left(\tilde{r}\right)$ in the symmetric case (see Proposition 2), hence

$$\underline{Q}\left(\tilde{r},0\right)=2\underline{q}\left(\tilde{r}\right)>Q^M.$$

Also, note that the right-hand side of (29) converges to zero as $(\tilde{r},\sigma)\to(0,1)$. Since $\underline{Q}\in\left[Q^M,2q^C\right]$ and $P'\left(\cdot\right)$ is continuous, $|\underline{Q}P'\left(\underline{Q}\right)|$ is bounded above and

$$\lim_{(\tilde{r},\sigma)\to(0,1)}\underline{Q}\left(\tilde{r},\sigma\right)=Q^M.$$

Finally, as $\tilde{r}$ tends to ∞, the right-hand side of (29) converges to 2, which implies that

$$\lim_{\tilde{r}\to\infty}\underline{Q}\left(\tilde{r},\sigma\right)=2q^C.$$

This completes the proof of part (ii).

Part (iii): We define $\mathcal{E}$ as the set of equilibrium dynamic reaction functions (R_A, R_B) such that R_i's are twice continuously differentiable, and $\sup|R_i'|$ and $\sup|R_i''|$ are bounded by K. Although the set $\mathcal{E}$ can depend on the choice of bounds $K > 0$, as well as the parameters r, λ_M, and σ, we omit them to simplify the notations. In particular, it will be clear in the proof that, given equilibrium existence, the specific choice of $K > 0$ does not affect our limit result.[37]

Our first intermediate result is that R_B approaches the Cournot reaction function when the degree of asymmetry σ converges to 1.

Lemma A2 *Let $\epsilon > 0$ be given. There exists $\bar{\sigma} < 1$ such that*

$$\max_{q_A \in [0, \bar{Q}_c]} |R_B(q_A) - R^C(q_A)| < \epsilon,$$

for any $(R_A, R_B) \in \mathcal{E}$ when $\sigma \geq \bar{\sigma}$.

Proof of Lemma A2. Let $\epsilon > 0$ be given. Recall that for $q = R_B(q_A)$,

$$\Pi'(q + q_A) - \pi_2(q_A, q) + \frac{1-\sigma}{\tilde{r} + 1 + \sigma} \left(\Pi'(R_A(q) + q) + \pi_2(q, R_A(q))R_A'(q)\right) = 0,$$

while for $q = R^C(q_A)$,

$$\Pi'(q + q_A) - \pi_2(q_A, q) = 0.$$

Now, we choose $\bar{\sigma} < 1$ such that $\sigma \geq \bar{\sigma}$ implies

$$\left(\frac{1-\sigma}{\tilde{r} + 1 + \sigma}\right) \cdot \max|\Pi' + \pi_2 \cdot K| \leq \epsilon \cdot \min|\pi_{11}|.$$

Then, we have for any $q_A \in [0, \bar{Q}_c]$ and $(R_A, R_B) \in \mathcal{E}$ with $\sigma \geq \bar{\sigma}$,

$$\left|\pi_1(R_B(q_A), q_A) - \pi_1(R^C(q_A), q_A)\right| \leq \epsilon \cdot \min|\pi_{11}|.$$

By the mean-value theorem, there exists z between $\min\{R^C(q_A), R_B(q_A)\}$ and $\max\{R^C(q_A), R_B(q_A)\}$ such that

$$\left(R_B(q_A) - R^C(q_A)\right)\pi_{11}(z, q_A) = \pi_1\left(R_B(q_A), q_A\right) - \pi_1\left(R^C(q_A), q_A\right),$$

and hence

$$|R_B(q_A) - R^C(q_A)||\pi_{11}(z, q_A)| \leq \epsilon \cdot \min|\pi_{11}|.$$

We conclude that $\max_{q_A \in [0, \bar{Q}_c]} |R_B(q_A) - R^C(q_A)| < \epsilon$ for any $(R_A, R_B) \in \mathcal{E}$ with $\sigma \geq \bar{\sigma}$. ■

Our next intermediate result is that the slope of reaction function R_B also converges to that of the Cournot reaction function as σ approaches 1.

[37] The existence of equilibria with bounded first and second derivaties will be satisfied in the case of linear demand, where $|R_i^{*\prime}| \leq \frac{1}{2}$ for all i and all parameter values $r > 0$, $\lambda_M > 0$, and $\sigma \in [0, 1)$.

Lemma A3 *Let $\epsilon > 0$ be given. There exists $\bar{\sigma} < 1$ such that*

$$\max_{q_A \in [0, \bar{Q}_c]} |R_B'(q_A) - R^{C\prime}(q_A)| < \epsilon,$$

for any $(R_A, R_B) \in \mathcal{E}$ when $\sigma \geq \bar{\sigma}$.

Proof of Lemma A3. Let $\epsilon > 0$ be given. Recall that for $q = R_B(q_A)$,

$$\Pi'(q + q_A) - \pi_2(q_A, q) + \frac{1-\sigma}{\tilde{r} + 1 + \sigma} \left(\Pi'(R_A(q) + q) + \pi_2(q, R_A(q)) R_A'(q)\right) = 0,$$

while for $q = R^C(q_A)$,

$$\Pi'(q + q_A) - \pi_2(q_A, q) = 0.$$

By the implicit function theorem,

$$R_B'(q_A) = -\frac{\pi_{12}(R_B(q_A), q_A)}{H(R_B(q_A), q_A)}, \text{ and } R^{C\prime}(q_A) = -\frac{\pi_{12}(R^C(q_A), q_A)}{\pi_{11}(R^C(q_A), q_A)},$$

where

$$H(q, q_A) = \pi_{11}(q, q_A) + \frac{1-\sigma}{\tilde{r} + 1 + \sigma} \tilde{H}(q),$$

for some $\tilde{H}(q)$ such that there exists an upper bound $K' > 0$ with $\max |\tilde{H}| < K'$ for any $(R_A, R_B) \in \mathcal{E}$. Together with Lemma A2, this implies that for any given $\epsilon' > 0$, there exists $\bar{\sigma}' < 1$ such that for any $(R_A, R_B) \in \mathcal{E}$ with $\sigma \geq \bar{\sigma}'$,

$$\max_{q_A \in [0, \bar{Q}_c]} |H(R_B(q_A), q_A) - \pi_{11}(R^C(q_A), q_A)| \leq \epsilon'. \tag{31}$$

To see this, note first that

$$\begin{aligned}
&|H(R_B(q_A), q_A) - \pi_{11}(R^C(q_A), q_A)| \\
\leq &|\pi_{11}(R_B(q_A), q_A) - \pi_{11}(R^C(q_A), q_A)| + \frac{1-\sigma}{\tilde{r} + 1 + \sigma} K' \\
\leq &\max |P''| \cdot |R_B(q_A) - R^C(q_A)| + \max |R^C| \cdot |P''(R_B(q_A) + q_A) - P''(R^C(q_A) + q_A)| \\
&+ 2|P'(R_B(q_A) + q_A) - P'(R^C(q_A) + q_A)| + \frac{1-\sigma}{\tilde{r} + 1 + \sigma} K'.
\end{aligned}$$

Since P'' and P' are continuous functions on a compact interval, they are uniformly continuous. Thus, by Lemma A2, we can choose $\bar{\sigma}' < 1$ such that for any $(R_A, R_B) \in \mathcal{E}$ with $\sigma \geq \bar{\sigma}'$,

$$|H(R_B(q_A), q_A) - \pi_{11}(R^C(q_A), q_A)| < \epsilon'.$$

Now, notice that

$$\begin{aligned}
&|R_B'(q_A) - R^{C\prime}(q_A)| \\
= &\left| \frac{\pi_{12}(R_B(q_A), q_A)}{H(R_B(q_A), q_A)} - \frac{\pi_{12}(R^C(q_A), q_A)}{\pi_{11}(R^C(q_A), q_A)} \right| \\
= &\left| \frac{\pi_{12}(R_B(q_A), q_A) \pi_{11}(R^C(q_A), q_A) - H(R_B(q_A), q_A) \pi_{12}(R^C(q_A), q_A)}{H(R_B(q_A), q_A) \pi_{11}(R^C(q_A), q_A)} \right|.
\end{aligned}$$

Observe that the denominator is strictly bounded away from zero when σ beccomes arbitrarily close to 1. To see this, note that for each $q_A \in [0, \bar{Q}]$,

$$\begin{aligned}
&|H(R_B(q_A), q_A)\pi_{11}(R^C(q_A), q_A)| \\
=&\left|\left(H\left(R_B(q_A), q_A\right) - \pi_{11}\left(R^C(q_A), q_A\right)\right)\pi_{11}(R^C(q_A), q_A) + \pi_{11}\left(R^C(q_A), q_A\right)^2\right| \\
\geq&\pi_{11}(R^C(q_A), q_A)^2 - \left|H(R_B(q_A), q_A) - \pi_{11}(R^C(q_A), q_A)||\pi_{11}(R^C(q_A), q_A)\right|.
\end{aligned}$$

Then, from (31) and the fact that $\min|\pi_{11}^2| > 0$, we have $\bar{\sigma}'' > 0$ such that the minimum of the right-hand side is bounded below from some $K'' > 0$ for any $(R_A, R_B) \in \mathcal{E}$ with $\sigma \geq \bar{\sigma}''$. Therefore, we have

$$\begin{aligned}
&\left|R_B'(q_A) - R^{C\prime}(q_A)\right| \\
\leq&\frac{1}{K''}\left|\pi_{12}(R_B(q_A), q_A)\pi_{11}(R^C(q_A), q_A) - H(R_B(q_A), q_A)\pi_{12}(R^C(q_A), q_A)\right|.
\end{aligned}$$

Since $\pi_{12}(R_B(q_A), q_A)$ and $H(R_B(q_A), q_A)$ approach to $\pi_{12}(R^C(q_A), q_A)$ and $\pi_{11}(R^C(q_A), q_A)$ respectively as σ converges to 1, we can apply the same argument before and find $\bar{\sigma} > 0$ such that

$$\max_{q_A \in [0, \bar{Q}_c]}\left|R_B'(q_A) - R^{C\prime}(q_A)\right| < \epsilon,$$

for any $(R_A, R_B) \in \mathcal{E}$ with $\sigma \geq \bar{\sigma}$. ■

Given Lemmas A2 and A3, we now prove the statement in part (iii). Let $\epsilon > 0$ be given. Recall that the first-order conditions evaluated at steady-state quantities (q_A^e, q_B^e) are given by, for $R_A(q_B^e) = q_A^e$,

$$\left(\tilde{r} + 1 - \sigma\right)\left(\Pi'\left(Q^e\right) - \pi_2\left(q_B^e, q_A^e\right)\right) + \left(1 + \sigma\right)\left(\Pi'\left(Q^e\right) + \pi_2\left(q_A^e, q_B^e\right)R_B'\left(q_A^e\right)\right) = 0,$$

and for $R_B(q_A^e) = q_B^e$,

$$\left(\tilde{r} + 1 + \sigma\right)\left(\Pi'\left(Q^e\right) - \pi_2\left(q_A^e, q_B^e\right)\right) + \left(1 - \sigma\right)\left(\Pi'\left(Q^e\right) + \pi_2\left(q_B^e, q_A^e\right)R_A'\left(q_B^e\right)\right) = 0.$$

By Lemma A3, we have $\max|R_B'| \leq L$ for some $L \in (0, 1)$ for any $(R_A, R_B) \in \mathcal{E}$ with σ being close enough to 1, because the Cournot reaction function has a slope whose absolute value is strictly less than than 1. As a result, we obtain

$$\begin{aligned}
0 &= \left(\tilde{r} + 1 - \sigma\right)\left(\Pi'\left(Q^e\right) - \pi_2\left(q_B^e, q_A^e\right)\right) + \left(1 + \sigma\right)\left(\Pi'\left(Q^e\right) + \pi_2\left(q_A^e, q_B^e\right)R_B'\left(q_A^e\right)\right) \\
&\leq \left(\tilde{r} + 1 - \sigma\right)\left(\Pi'\left(Q^e\right) - \pi_2\left(q_B^e, q_A^e\right)\right) + \left(1 + \sigma\right)\left(\Pi'\left(Q^e\right) - L\pi_2\left(q_A^e, q_B^e\right)\right) \\
&= \left(\tilde{r} + 1 - \sigma\right)\left(\Pi'\left(Q^e\right) - \pi_2\left(q_B^e, q_A^e\right)\right) + \left(1 + \sigma\right)\left(\pi_1\left(q_B^e, q_A^e\right) - c\right) + \left(1 + \sigma\right)\left(1 - L\right)\pi_2\left(q_A^e, q_B^e\right),
\end{aligned}$$

which is equivalent to

$$0 \leq \left(\frac{\tilde{r} + 1 - \sigma}{1 + \sigma}\right)\left(\Pi'\left(Q^e\right) - \pi_2\left(q_B^e, q_A^e\right)\right) + \pi_1\left(q_B^e, q_A^e\right) - c + \left(1 - L\right)\pi_2\left(q_A^e, q_B^e\right).$$

Now, choose $\bar{\sigma} < 1$ and $\bar{r} > 0$ such that

$$\left(\frac{\tilde{r} + 1 - \sigma}{1 + \sigma}\right)\left(\Pi'\left(Q^e\right) - \pi_2\left(q_B^e, q_A^e\right)\right) \leq \frac{\epsilon\left(1 - L\right)\min|P'|}{2},$$

and

$$\pi_1(q_B^e, q_A^e) - c = \pi_1\left(R_B(q_A^e), q_A^e\right) - c \leq \frac{\epsilon(1-L)\min|P'|}{2},$$

for any $(R_A, R_B) \in \mathcal{E}$ with $\sigma \geq \bar{\sigma}$ and $\tilde{r} \leq \bar{r}$, where the latter inequality follows from Lemma A2. As a result, we have

$$\begin{aligned} 0 &\leq \left(\frac{\tilde{r}+1-\sigma}{1+\sigma}\right)\left(\Pi'(Q^e) - \pi_2(q_B^e, q_A^e)\right) + \pi_1(q_B^e, q_A^e) - c + (1-L)\pi_2(q_A^e, q_B^e) \\ &\leq (1-L)\left(\epsilon \min|P'| + P'(Q^e)q_A^e\right), \end{aligned}$$

implying

$$0 \leq \epsilon \min|P'| + P'(Q^e)q_A^e.$$

Therefore,

$$\left(-P'(Q^e)\right)q_A^e \leq \epsilon \min|P'| \iff q_A^e \leq \frac{\epsilon \min|P'|}{|P'(Q^e)|} < \epsilon.$$

By Lemma A2, we also obtain

$$|q_B^e - Q^M| = |R_B(q_A^e) - R^C(0)| < \epsilon,$$

provided that σ is sufficiently close to 1. This completes the proof of part (iii). ■

Proof of Proposition 6. Letting $\tilde{r} = \frac{r}{\lambda_M}$ and $\tilde{x} = (2+4\tilde{r})^2 + 12(1-\sigma^2)$, the intercepts and slopes of the equilibrium dynamic quantity reactions (R_A^*, R_B^*) from the proof of Proposition 1 become

$$\alpha_A^* = \frac{(1-c)(2+\tilde{r})\left(2+4\tilde{r}+6(1-\sigma)-\sqrt{\tilde{x}}\right)}{2(1-\sigma)\left(14+10\tilde{r}-\sqrt{\tilde{x}}\right)},$$

$$\alpha_B^* = \frac{(1-c)(2+\tilde{r})\left(2+4\tilde{r}+6(1+\sigma)-\sqrt{\tilde{x}}\right)}{2(1+\sigma)\left(14+10\tilde{r}-\sqrt{\tilde{x}}\right)},$$

$$\beta_A^* = \frac{1}{2} - \frac{\sqrt{\tilde{x}}-(2+4\tilde{r})}{12(1-\sigma)},$$

$$\beta_B^* = \frac{1}{2} - \frac{\sqrt{\tilde{x}}-(2+4\tilde{r})}{12(1+\sigma)}.$$

The steady-state quantities are given by

$$(q_A^e, q_B^e) = \left(\frac{\alpha_A^* - \alpha_B^*\beta_A^*}{1-\beta_A^*\beta_B^*}, \frac{\alpha_B^* - \alpha_A^*\beta_B^*}{1-\beta_A^*\beta_B^*}\right).$$

We first prove part (iii), and then parts (i) and (ii) of the proposition.

Part (iii): Note that

$$\lim_{\tilde{r}\to 0}\beta_B^* = \frac{1}{2} - \frac{\sqrt{4+12(1-\sigma^2)}-2}{12(1+\sigma)},$$

so, by continuity,

$$\lim_{(\tilde{r},\sigma)\to(0,1)}\beta_B^* = \frac{1}{2},$$

and

$$\lim_{(\tilde{r},\sigma)\to(0,1)} \alpha_B^* = \frac{1-c}{2} = Q^M.$$

Now, we show that β_A^* converges to 0 as $(\tilde{r},\sigma) \to (0,1)$. Note that

$$\frac{1}{2} \geq \frac{1}{2} - \beta_A^* = \frac{\sqrt{(2+4\tilde{r})^2 + 12\left(1-\sigma^2\right)} - (2+4\tilde{r})}{12\left(1-\sigma\right)}$$

The right-hand side is strictly decreasing in $\tilde{r}$ since

$$\frac{\partial}{\partial \tilde{r}} \left(\frac{\sqrt{(2+4\tilde{r})^2 + 12\left(1-\sigma^2\right)} - (2+4\tilde{r})}{12\left(1-\sigma\right)} \right) = \frac{1}{12\left(1-\sigma\right)} \left(\frac{8\left(2+4\tilde{r}\right)}{2\sqrt{(2+4\tilde{r})^2 + \left(1-\sigma^2\right)}} - 4 \right) < 0,$$

and strictly increasing in σ because

$$\begin{aligned}
&\frac{\partial}{\partial \sigma} \left(\frac{\sqrt{(2+4\tilde{r})^2 + 12\left(1-\sigma^2\right)} - (2+4\tilde{r})}{12\left(1-\sigma\right)} \right) \\
\overset{sign}{=} &\frac{-288\sigma\left(1-\sigma\right)}{2\sqrt{(2+4\tilde{r})^2 + 12\left(1-\sigma^2\right)}} + 12\left(\sqrt{(2+4\tilde{r})^2 + 12\left(1-\sigma^2\right)} - (2+4\tilde{r}) \right) \\
\overset{sign}{=} &-144\sigma\left(1-\sigma\right) + 12\left((2+4\tilde{r})^2 + 12\left(1-\sigma^2\right) - (2+4\tilde{r})\sqrt{(2+4\tilde{r})^2 + 12\left(1-\sigma^2\right)} \right) \\
\overset{sign}{=} &144\left(1-\sigma\right) + (2+4\tilde{r})\left(12\left(2+4\tilde{r}\right) - \sqrt{(2+4\tilde{r})^2 + 12\left(1-\sigma^2\right)} \right) \\
\geq &0,
\end{aligned}$$

where the last inequality follows from

$$12\left(2+4\tilde{r}\right) \geq \sqrt{(2+4\tilde{r})^2 + 12\left(1-\sigma^2\right)} \iff 143\left(2+4\tilde{r}\right)^2 \geq 12\left(1-\sigma^2\right).$$

Therefore, any point $(\tilde{r},\sigma)$ in the open ball around $(\tilde{r},\sigma) = (0,1)$ of radius $\epsilon > 0$ satisfies

$$\begin{aligned}
\frac{1}{2} \geq \frac{1}{2} - \beta_A^* &= \frac{\sqrt{(2+4\tilde{r})^2 + 12\left(1-\sigma^2\right)} - (2+4\tilde{r})}{12\left(1-\sigma\right)} \\
&\geq \frac{\sqrt{(2+4\epsilon)^2 + 12\left(1-(1-\epsilon)^2\right)} - (2+4\epsilon)}{12\epsilon} \\
&\to \frac{1}{2} \text{ as } \epsilon \to 0,
\end{aligned}$$

which shows that $\lim_{(\tilde{r},\sigma)\to(0,1)} \beta_A^* = 0$. This, in turn, implies that α_A^* converges to 0 as $(\tilde{r},\sigma) \to (0,1)$

because

$$\begin{aligned}\lim_{(\tilde{r},\sigma)\to(0,1)}\alpha_A^* &= \lim_{(\tilde{r},\sigma)\to(0,1)}\left(\frac{(1-c)(2+\tilde{r})}{2\left(14+10\tilde{r}-\sqrt{\tilde{x}}\right)}\right)\left(\frac{2+4\tilde{r}+6(1-\sigma)-\sqrt{\tilde{x}}}{1-\sigma}\right)\\ &= \lim_{(\tilde{r},\sigma)\to(0,1)}\left(\frac{(1-c)(2+\tilde{r})}{2\left(14+10\tilde{r}-\sqrt{\tilde{x}}\right)}\right)\left(6-\frac{\sqrt{\tilde{x}}-(2+4\tilde{r})}{1-\sigma}\right)\\ &= \lim_{(\tilde{r},\sigma)\to(0,1)}\left(\frac{(1-c)(2+\tilde{r})}{2\left(14+10\tilde{r}-\sqrt{\tilde{x}}\right)}\right)\left(6-12\left(\frac{1}{2}-\beta_A^*\right)\right)\\ &= 0\end{aligned}$$

By the above observations, we conclude that $(q_A^e, q_B^e) = \left(\frac{\alpha_A^*-\alpha_B^*\beta_A^*}{1-\beta_A^*\beta_B^*}, \frac{\alpha_B^*-\alpha_A^*\beta_B^*}{1-\beta_A^*\beta_B^*}\right) \to \left(0, Q^M\right)$ as $(\tilde{r},\sigma)\to(0,1)$.

Part (i): Note that the aggregate steady-state quantity is given by

$$Q^e = \left(\frac{12(1-c)(2+\tilde{r})}{14+10\tilde{r}-\sqrt{\tilde{x}}}\right)\left(\frac{36(1-\sigma^2)-(2+4\tilde{r}-\sqrt{\tilde{x}})^2}{108(1-\sigma^2)-12\left(2+4\tilde{r}-\sqrt{\tilde{x}}\right)-\left(2+4\tilde{r}-\sqrt{\tilde{x}}\right)^2}\right).$$

To simplify notations, let us write $X = 2+4\tilde{r}-\sqrt{\tilde{x}} < 0$. Then, we have

$$\tilde{x} = (2+4\tilde{r}-X)^2 \iff 12(1-\sigma^2) = X^2 - 2X(2+4\tilde{r}),$$

and so

$$\begin{aligned}Q^e &= \left(\frac{12(1-c)(2+\tilde{r})}{6(2+\tilde{r})+X}\right)\left(\frac{2X^2-6X(2+4\tilde{r})}{8X^2-18X(2+4\tilde{r})-12X}\right)\\ &= \left(\frac{12(1-c)(2+\tilde{r})}{12+6\tilde{r}+X}\right)\left(\frac{X-6-12\tilde{r}}{4X-24-36\tilde{r}}\right)\\ &= 12(1-c)\cdot\left(6+\frac{X}{2+\tilde{r}}\right)^{-1}\cdot\left(4+\frac{12\tilde{r}}{X-6-12\tilde{r}}\right)^{-1}.\end{aligned}$$

Since

$$\begin{aligned}&\left(6+\frac{X}{2+\tilde{r}}\right)\cdot\left(4+\frac{12\tilde{r}}{X-6-12\tilde{r}}\right)\\ =&24+4\left(\frac{18\tilde{r}}{X-6-12\tilde{r}}+\frac{X}{2+\tilde{r}}+\frac{3\tilde{r}X}{(2+\tilde{r})(X-6-12\tilde{r})}\right),\end{aligned}$$

it sufficies to show that

$$\frac{18\tilde{r}}{X-6-12\tilde{r}}+\frac{X}{2+\tilde{r}}+\frac{3\tilde{r}X}{(2+\tilde{r})(X-6-12\tilde{r})}$$

is strictly decreasing in $\tilde{r}$. One can check that the denominator of its first-order derivative is

$$(2+\tilde{r})^2\sqrt{4(1+\tilde{r}+\tilde{r}^2)-3\sigma^2}\left(2+4\tilde{r}+\sqrt{4(1+\tilde{r}+\tilde{r}^2)-3\sigma^2}\right)^2 > 0,$$

and the numerator is

$$-3\Big(8-6\sigma^4+2\tilde{r}\left(10+7\tilde{r}+3\tilde{r}^2\right)+\sigma^2\left(10+13\tilde{r}^2+16\tilde{r}\right)$$
$$+2\left(2+4\tilde{r}+3\tilde{r}^2+10\sigma^2+8\tilde{r}\sigma^2\right)\sqrt{4\left(1+\tilde{r}+\tilde{r}^2\right)-3\sigma^2}\Big)<0.$$

Therefore, we conclude that Q^e is strictly increasing in $\tilde{r}$.

Part (ii): Finally, we prove part (ii). Remark that if Q^e is strictly decreasing in σ, and q_B^e is strictly increasing in σ, then q_A^e must be strictly decreasing in σ. Therefore, it sufficies to show the monotonicity of Q^e and q_B^e with respect to σ.

Recall that with $X=2+4\tilde{r}-\sqrt{\tilde{x}}$,

$$Q^e=\left(\frac{12\left(1-c\right)\left(2+\tilde{r}\right)}{12+6\tilde{r}+X}\right)\left(\frac{X-6-12\tilde{r}}{4X-24-36\tilde{r}}\right).$$

Thus, Q^e is strictly decreasing in σ if and only if

$$\frac{\left(12+6\tilde{r}+X\right)\left(4X-24-36\tilde{r}\right)}{X-6-12\tilde{r}}$$

is strictly increasing in σ. Since $\frac{\partial X}{\partial\sigma}>0$, it sufficies to show that

$$\frac{\partial}{\partial X}\left(\frac{\left(12+6\tilde{r}+X\right)\left(4X-24-36\tilde{r}\right)}{X-6-12\tilde{r}}\right)>0,$$

which is equivalent to

$$\begin{aligned}
&\left(4X-24-36\tilde{r}+4\left(12+6\tilde{r}+X\right)\right)\left(X-6-12\tilde{r}\right)-\left(12+6\tilde{r}+X\right)\left(4X-24-36\tilde{r}\right)>0\\
\iff&\left(4X-24-36\tilde{r}\right)\left(X-6-12\tilde{r}-12-6\tilde{r}-X\right)+4\left(12+6\tilde{r}+X\right)\left(X-6-12\tilde{r}\right)>0\\
\iff&18\left(1+\tilde{r}\right)\left(24+36\tilde{r}-4X\right)>4\left(6+12\tilde{r}-X\right)\left(12+6\tilde{r}+X\right)\\
\iff&\left(\frac{3}{2}\left(24+36\tilde{r}-4X\right)\right)\left(12\left(1+\tilde{r}\right)\right)>\left(24+48\tilde{r}-4X\right)\left(12+6\tilde{r}+X\right).
\end{aligned}$$

Note that the first term in the left-hand side is greater than the first term in the right-hand side. To see this,

$$\frac{3}{2}\left(24+36\tilde{r}-4X\right)-24-48\tilde{r}+4X=12+6\tilde{r}-2X>0,$$

where the inequality follows from $X<0$. Similarly, the second term in the left-hand side is greater than the second term in the right-hand side, because

$$12\left(1+\tilde{r}\right)-12-6\tilde{r}-X=6\tilde{r}-X>0.$$

We conclude that the aggregate quantity Q^e is strictly decreasing in σ.

Now, we show that q_B^e is strictly increasing in σ, which completes the proof of part (ii). Note that

$$q_B^e=\left(\frac{6(1-c)(2+\tilde{r})}{14+10\tilde{r}-\sqrt{\tilde{x}}}\right)\left(\frac{36(1-\sigma^2)+12\sigma(\sqrt{\tilde{x}}-(2+4\tilde{r}))-(\sqrt{\tilde{x}}-(2+4\tilde{r}))^2}{108(1-\sigma^2)+12(\sqrt{x}-(2+4\tilde{r}))-(\sqrt{x}-(2+4\tilde{r}))^2}\right).$$

Using our previous notation $X = 2 + 4\tilde{r} - \sqrt{\tilde{x}} < 0$, it is rewritten as

$$\begin{aligned} q_B^e &= \left(\frac{6(1-c)(2+\tilde{r})}{12+6\tilde{r}+X}\right)\left(\frac{2X^2 - 6X(2+4\tilde{r}) - 12\sigma X}{8X^2 - 48X - 72\tilde{r}X}\right) \\ &= \left(\frac{6(1-c)(2+\tilde{r})}{12+6\tilde{r}+X}\right)\left(\frac{X - 3(2+4\tilde{r}) - 6\sigma}{4X - 24 - 36\tilde{r}}\right). \end{aligned}$$

Thus, it sufficies to show that

$$\frac{X - 3(2+4\tilde{r}) - 6\sigma}{(12+6\tilde{r}+X)(4X - 24 - 36\tilde{r})}$$

is strictly increasing in σ. Letting $X' = \frac{\partial X}{\partial \sigma}$, one can check that

$$\frac{\partial}{\partial \sigma}\left(\frac{X - 3(2+4\tilde{r}) - 6\sigma}{(12+6\tilde{r}+X)(4X - 24 - 36\tilde{r})}\right) > 0$$

is equivalent to

$$\begin{aligned} & X'(4X - 24 - 36\tilde{r})(12 + 6\tilde{r} + X - X + 3(2+4\tilde{r}) + 6\sigma) \\ & - X'(48 + 24\tilde{r} + 4X)(X - 6 - 12\tilde{r} - 6\sigma) - 6(12 + 6\tilde{r} + X)(4X - 24 - 36\tilde{r}) > 0 \\ \iff & X'((4X - 24 - 36\tilde{r})(18 + 18\tilde{r} + 6\sigma) - (48 + 24\tilde{r} + 4X)(X - 6 - 12\tilde{r} - 6\sigma)) \\ & > 6(12 + 6\tilde{r} + X)(4X - 24 - 36\tilde{r}) \\ \iff & X'\left(\frac{3 + 3\tilde{r} + \sigma}{12 + 6\tilde{r} + X} - \frac{X - 6 - 12\tilde{r} - 6\sigma}{6(X - 6 - 9\tilde{r})}\right) < 1, \end{aligned}$$

where the last equivalence follows from dividing each side by $6(4X - 24 - 36\tilde{r})(12 + 6\tilde{r} + X)$, which is negative. Since $\frac{\partial X}{\partial \sigma} = \frac{12\sigma}{\sqrt{\tilde{x}}} = \frac{12\sigma}{2+4\tilde{r}-X}$, it can be shown to be equivalent to

$$\begin{aligned} & (2\sigma(6 + 6\sigma + 12\tilde{r} - X) + (2 + 4\tilde{r} - X)(6 + 9\tilde{r} - X))(12 + 6\tilde{r} + X) \\ & > 12\sigma(6 + 9\tilde{r} - X)(3 + 3\tilde{r} + \sigma). \end{aligned}$$

Noting that $12\sigma = 2 \cdot 6\sigma$, we show that each term in the left-hand side is greater than each term in the right-hand side. More precisely, note first that

$$\begin{aligned} & 12 + 6\tilde{r} + X > 2(3 + 3\tilde{r} + \sigma) = (6 + 6\tilde{r} + 2\sigma) \\ \iff & 6 + X > 2\sigma \\ \iff & 2(2 + 4\tilde{r})(6 - 2\sigma) + (6 - 2\sigma)^2 > 12(1 - \sigma^2), \end{aligned}$$

where the last equivalence follows from $X = 2 + 4\tilde{r} - \sqrt{\tilde{x}}$. Certainly, the left-hand side is strictly greater than the right-hand side.

In addition, we have

$$\begin{aligned} & 2\sigma(6 + 6\sigma + 12\tilde{r} - X) + (2 + 4\tilde{r} - X)(6 + 9\tilde{r} - X) > 6\sigma(6 + 9\tilde{r} - X) \\ \iff & 2\sigma(6 + 6\sigma + 12\tilde{r} - X) > (6 + 9\tilde{r} - X)(6\sigma - 2 - 4\tilde{r} + X) \\ \iff & 2\sigma(6 + 9\tilde{r} - X + 6\sigma + 3\tilde{r}) > (6 + 9\tilde{r} - X)(6\sigma - 2 - 4\tilde{r} + X) \\ \iff & 2\sigma(6\sigma + 3\tilde{r}) > (6 + 9\tilde{r} - X)(4\sigma - 2 - 4\tilde{r} + X) \\ \iff & 6\sigma(2\sigma + \tilde{r}) > (6 + 9\tilde{r} - X)(4\sigma - 2 - 4\tilde{r} + X). \end{aligned}$$

Now, substituting $X = 2 + 4\tilde{r} - \sqrt{\tilde{x}}$, it reduces to

$$\begin{aligned} & 6\sigma(2\sigma + \tilde{r}) > (4 + 5\tilde{r} + \sqrt{\tilde{x}})(4\sigma - \sqrt{\tilde{x}}) \\ \iff & 6\sigma(2\sigma + \tilde{r}) - (4 + 5\tilde{r} + \sqrt{\tilde{x}})(4\sigma - \sqrt{\tilde{x}}) > 0. \end{aligned} \tag{32}$$

One can check that the left-hand side is strictly decreasing in σ:

$$\begin{aligned} & \frac{\partial}{\partial \sigma}\left(6\sigma(2\sigma + \tilde{r}) - (4 + 5\tilde{r} + \sqrt{\tilde{x}})(4\sigma - \sqrt{\tilde{x}})\right) \\ = & -\frac{2\left(16(1 + \tilde{r} + \tilde{r}^2) - 24\sigma^2 + 3\sigma(4 + 5\tilde{r}) + (8 + 7\tilde{r})\sqrt{4(1 + \tilde{r} + \tilde{r}^2) - 3\sigma^2}\right)}{\sqrt{4(1 + \tilde{r} + \tilde{r}^2) - 3\sigma^2}} \\ < & 0. \end{aligned}$$

Thus, it sufficies to show that the inequality (32) is satisfied at $\sigma = 1$:

$$\begin{aligned} & 6(2 + \tilde{r}) - (4 + 5\tilde{r} + 2 + 4\tilde{r})(4 - 2 - 4\tilde{r}) > 0 \\ \iff & 6(2 + \tilde{r}) - (6 + 9\tilde{r})(2 - 4\tilde{r}) = 36\tilde{r}^2 + 12\tilde{r} > 0. \end{aligned}$$

Thus, we conclude that q_B^e is strictly increasing in σ. ■

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