# Randomize at your own Risk: on the Observability of Ambiguity Aversion* 

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#### Abstract

Facing several decisions, people may consider each one in isolation, or integrate them into a single optimization problem. Isolation and integration may yield different choices, for instance, if uncertainty is involved and only one randomly-selected decision is implemented. We investigate whether the random incentive system in experiments that measure ambiguity aversion provide a hedge against ambiguity, making ambiguity-averse subjects who integrate behave as if they were ambiguity neutral. Our results suggest that about half of the ambiguity averse subjects integrated their choices in the experiment into a single problem, whereas the other half isolated. Our design further enable us to disentangle properties of the integrating subjects' preferences over compound objects induced by the random incentive system and the choice problems in the experiment.


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## 1 Introduction

For most decision makers, not all forms of uncertainty are equivalent. Following early contributions by Knight (1921) and Keynes (1921), Ellsberg's (1961) thought experiments demonstrated that many individuals do not reduce (subjective) uncertainty to (objective) risk, a behavior he termed ambiguity aversion and is inconsistent with Subjective Expected Utility (Savage, 1954). However, Raiffa (1961) questioned the normative appeal of ambiguity averse preferences, arguing that using an objective randomization device can reduce uncertainty to risk. How randomization interplays with ambiguity has been studied theoretically since then, with important recent contributions by Saito (2015) and Ke and Zhang (2020). The present paper investigates this interplay experimentally, while focusing on the challenge of observing ambiguity averse preferences.

To introduce the problem, consider a situation where an individual makes several risky decisions that will be simultaneously implemented. From a normative perspective, as only the final consequences matter, the decision maker should worry about the interactions between the choices. To do so, one can integrate all decisions into one overall maximization problem. In practice, however, people tend to focus too much on each decision independently, exhibiting isolation or 'narrow framing' (Tversky and Kahneman, 1981; Kahneman and Lovallo, 1993; Barberis et al., 2006; Rabin and Weizsäcker, 2009). Isolation is less cognitively demanding and gives the same solution as integration if the problem is linear (expected value maximization). Non-linearities in utility make isolation sub-optimal.

Now assume that not all decisions will be implemented, but only a single decision will be selected by a random device. The randomization creates a more complex decision environment, which people tend to simplify by isolating choices. Under expected utility, both isolation and integration yield the same optimal strategy because probabilities (of the risky decisions and of the random device) are all dealt with linearly. However, once non-expected utility is considered, there is no guarantee that isolation and integration lead to the same solution.

Finally, consider decisions involving uncertainty, and a random device determining which one is implemented. Here the decision maker does not need to re-invent the randomization device suggested by Raiffa (1961) to reduce uncertainty, as it has been already introduced into the description of the environment by the experimenter. The several decisions can be integrated into a single compound, multi-stage object involving (ambiguous) events and probabilities (from the randomization device). It is unknown empirically how this object will be dealt with. Does the order of resolution (between ambiguity and the randomization) matter? And if so, will the individual apply backward induction, conditioning on which uncertainty is resolved first? Facing this complexity, some individuals may simply isolate their decisions.

The interplay between risk and ambiguity poses a very practical challenge of mere
observation of ambiguity attitudes. In experimental economics, it is common to ask subjects to make several decisions and then to pay only one decision that is randomly selected. This mechanism is called the Random Incentive System (RIS). Theoretical arguments have been made that the usage of RIS makes it impossible to observe ambiguity aversion (Bade 2015, first version 2011; Kuzmics 2017, first version 2012; Baillon et al. 2019, first version 2013; Oechssler and Roomets, 2014; Azrieli et al. 2018, first version 2014). For instance, consider a subject who chooses to bet on an ambiguous event (to win a prize) in one decision and on its complementary event in a second decision. If a coin is tossed to determine which decision is implemented, the probability of winning the prize is one half, no matter which (ambiguous) event obtains. The random device used for RIS provides a complete hedge against the ambiguity. These arguments rely on integration and on uncertainty being resolved before the risky randomization. In spite of the above theoretical argument, most (but not all) of the experimental literature continues to use RIS to measure ambiguity attitude, with varying degrees of success (Trautmann and van de Kuilen, 2015), suggesting that isolation may take place, at least in some cases.

The present paper provides the first direct evidence on the impact of RIS on the observability of ambiguity aversion. By doing so, it provides a test of whether subjects isolate their decisions when responding to various decision problems in experiments, or behave as if they integrate them into a single decision. If they integrate, our experimental design allows us to further understand how people integrate risk with ambiguity, and especially whether the order matters, and if so - whether backward induction takes place.

We conducted a first experiment (the "main study") with more than 400 subjects divided into five between-subject treatments. In all treatments, subjects had to choose between known (risky) bets (on the color of a chip drawn from a bag containing one red and one blue chip) and unknown (ambiguous) bets (each chip could be red or blue, but the color composition of the chips in the bag was unknown). The ambiguous bets yielded slightly higher payments than the risky bets, allowing us to identify strict ambiguity aversion. In the control, called Single, no RIS was employed. Subjects chose the color they wanted to bet on (blue or red) and then chose a bag (known or unknown). The proportion of subjects choosing the known bag in the Single control served as a baseline for strict ambiguity aversion. All other treatments described next employed RIS and were compared to this baseline, enabling us to gauge the impact of RIS on the observability of ambiguity aversion. In one treatment, called Before, subjects had to make choices for both colors (which bag to draw from if the bets are on red and if the bets are on blue), and the choice they would be paid for was randomly determined (but was not disclosed to them) before their choices and before the ambiguity was resolved, i.e. before chips were drawn from the bags. Another treatment (After) reversed the order of random incentives and resolution of ambiguity, drawing chips from the bags before determining which choice would be paid. If people
were to condition on what is determined first and apply backward induction, which is consistent with the Statewise Monotonicity condition in Azrieli et al. (2018), we would expect to observe more ambiguity aversion in the Before treatment than in the After treatment. On the other hand, if they were indifferent between the timing of risk and ambiguity, satisfying Reversal of Order, the observed ambiguity aversion would not differ between Before and After. This way, these treatments enable us to focus on subjects' preferences over compound objects, specifically, how they integrate risk and ambiguity. Two additional treatments, Before-6 and After-6, included six choices to be able to identify various degrees of ambiguity aversion/seeking.

In the Single control, about $50 \%$ of the subjects exhibited strict ambiguity aversion. In all other treatments, the proportion of choices consistent with strict ambiguity aversion was between $25 \%$ and $29 \%$. First, this result implies that experimental work that relies on RIS may have underestimated the prevalence of ambiguity aversion in the population. Second, our results show that whereas half of the ambiguity averse subjects view each choice in isolation, the other half made choices as if they integrate all choices in the experiment into one meta-choice. Random incentives prevented the latter half (integrating) subjects from revealing their underlying ambiguity (averse) preferences. Specifically, the integrating subjects seem to satisfy Reversal of Order and not Statewise Monotonicity, which Azrieli et al. (2018) showed is equivalent to incentive compatibility of the RIS.

We conducted a follow-up study, to further investigate insights from recent models in the literature. We varied when the unknown bag was constructed in the Single and the Before treatment. Following the arguments in Saito (2015) and Ke and Zhang (2020), randomization is rendered ineffective if the ambiguous bag is constructed after the randomization takes place. In Single-Bag Later (Single-BL), the bag was prepared after the subjects made their decision, removing the possibility for ambiguity averse subjects to hedge by mentally randomizing during decision time (as proposed by Raiffa, 1961). We found no evidence that behavior in Single-BL differed from the behavior observed in the replication of the Single control in the follow-up study. Subjects did not seem to mentally randomize. We also varied when the bag was constructed in two variants of the Before treatment: after the random incentives, or after the random incentives and the decision. Following Saito (2015) and Ke and Zhang (2020), this should have eliminated the impact of the random incentives. It did not seem to be the case, as substantially fewer subjects exhibited ambiguity aversion in these two variants of the Before treatment than in Single and Single-BL.

Our results provide a valuable lesson on the challenge/impossibility of observing the complete spectrum ambiguity sensitive preferences. They suggest that one can design an experiment to observe ambiguity aversion (e.g. our baseline treatment with no random incentives). However, an experimenter who wishes to deliver richer and finer measurements of ambiguity attitudes runs the risk that ambiguity averse subjects will choose as if they integrate the decisions and hedge the ambiguity, making their
choices indistinguishable from those of subjective expected utility decision makers.

## 2 Main Study

### 2.1 The tasks

All subjects faced two bags, $\mathrm{K}\left(\right.$ nown ) and $\mathrm{U}($ nknown $) .{ }^{1}$ Each bag contained two poker chips that can be either red or blue. Bag K contained exactly one blue chip and one red chip, while the composition of bag $U$ was unknown. One chip would be drawn from each bag. The two bags, with bag K (opened) and bag U (closed), were presented to all subjects individually. ${ }^{2}$ After they finished reading the instructions, they received a choice sheet with two choice problems. In the red choice problem, subjects had to choose between winning $€ 10$ if a red chip was drawn from bag K and winning $€ 10.20$ if a red chip was drawn from bag U . In the blue choice problem, the choice was between winning $€ 10$ if a blue chip was drawn from bag K and winning $€ 10.20$ if a blue chip was drawn from bag U. Varying the payments slightly allows the subject to express strict preference, a technique introduced by Epstein and Halevy (2019).

We ran a control (Single) and four treatments: Before, After, Before-6 and After6.

- Single control: subjects first selected the choice problem - red or blue - that they wanted to determine their payment. Then, in the chosen problem, they made a choice between the two bets.
- Before and After treatments: subjects made a decision in both choice problems displayed on the choice sheet (see Figure 2.1). The experimental tasks are identical in these two treatments, but they differ in the timing of incentivizing, as explained in subsection 2.2.
- Before-6 and After- 6 treatments: subjects made a decision in six choice problems displayed on the choice sheet (see Figure 2.2). The design in these treatments is closer to choice lists (also called multiple price lists): the three choice problems for betting on red (blue) can be considered three sub-choice problems in a choice list. Nevertheless, choice problems Red 1 and Blue 4 are the same as the problems in the Before and After treatments. Hence, comparing proportions of subjects choosing bag K in these two problems in the Before-6 and After-6 treatments with those in the other treatments inform us whether introducing more choice problems further affect the measurement of ambiguity

[^1]aversion. The additional choice problems allowed for richer and more refined classification of ambiguity attitudes.

Figure 2.1: The two choice problems


## Red <br> (circle a or b)

a) $€ 10$ if a red chip is drawn from Bag K.
b) $€ 10.20$ if a red chip is drawn from Bag U.

Blue
(circle a or b)
a) $€ 10$ if a blue chip is drawn from Bag K.
b) $€ 10.20$ if a blue chip is drawn from Bag $U$.

Figure 2.2: The choice problems in additional treatments


## Red 1 (circle a or b)

a) $€ 10$ if a red chip is drawn from Bag K.
b) $€ 10.20$ if a red chip is drawn from Bag U.

## Red 2 (circle cor d)

c) $€ 10$ if a red chip is drawn from Bag K.
d) $€ 10$ if a red chip is drawn from Bag U .

## Red 3 (circle e or f)

e) $€ 10.20$ if a red chip is drawn from Bag K.
f) $€ 10$ if a red chip is drawn from Bag U.

## Blue 4 (circle a or b)

a) $€ 10$ if a blue chip is drawn from Bag K.
b) $€ 10.20$ if a blue chip is drawn from Bag U.

## Blue 5 (circle cor d)

c) $€ 10$ if a blue chip is drawn from Bag K.
d) $€ 10$ if a blue chip is drawn from Bag U.

## Blue 6 (circle e or f)

e) $€ 10.20$ if a blue chip is drawn from Bag K.
f) $€ 10$ if a blue chip is drawn from Bag $U$.

### 2.2 Incentives and ordering

As one of the main treatment manipulations, we varied the order of randomization, (subjects') choice making, and uncertainty resolution. To ensure fair and transparent implementation, we randomly selected one subject in each experimental session as the implementer, whose tasks differed across treatments.

- Single control: First all subjects made their choices. Then the implementer resolved the uncertainty by drawing one chip from each bag and announcing the color of each chip. Subjects were paid according to the outcome of their chosen bet in their chosen choice problem. The timeline in this treatment is hence: Choices - Chip draw.
- Before and Before-6 treatment: Before the subjects received the choice sheet, the implementer implemented random incentives by rolling a 6 -sided die. Three sides of the die were marked red and the others marked blue. The implementer threw the die for all subjects and put the corresponding choice problem into sealed envelopes (choice problems red or blue in the Before treatment, and choice problems 1 to 6 in the Before- 6 treatment). Each subject drew an envelope, knowing that it contained the choice problem that would determine their payment. Then, they received the choice sheet, and made their choices in all choice problems. Once the experimenters collected the choice sheet from all subjects, the subjects were asked to open their envelopes to find out which choice problem would be played for real. Finally, the implementer resolved the uncertainty. Subjects were paid according to their choice in the choice problem that was included in their envelope. The timeline in this treatment is hence: Randomization - Choices - Chip draw. With this procedure, it was maximally salient that the random incentives determining which choice problem matters for payment preceded the resolution of uncertainty concerning the draws from the bags. This implementation followed our proposal in Baillon et al. (2019), which was previously implemented in Epstein and Halevy (2019).
- After and After-6 treatment: Once all subjects were done with the experiment, the implementer drew a chip from each bag and announced the color of each chip. Afterwards, the implementer rolled the die for each subject. Subjects were paid according to their choice in the choice problem selected by die roll. The timeline in this treatment is hence: Choices - Chip draw - Randomization. The instructions, describing this process, made it salient that the random incentives followed the resolution of uncertainty.

Experimental instructions for all treatments are included in Appendix B. In total, 27 experimental sessions were conducted. Seventeen experimental sessions were randomly assigned to Single, Before, and After. Ten additional sessions were conducted at a different time and randomly assigned to Before- 6 and After-6. The sessions lasted on average 25 minutes. To ensure that all sessions would take about the same time, we asked the subjects in the Single control to answer an additional questionnaire after they completed the choice sheet. It was specified that this questionnaire was unrelated to payment. The subjects of the Before and After treatments answered a shorter questionnaire. The Implementers ( 27 subjects) received a fix payment of $€ 10$. All other subjects (84, 87, 87, 87, and 89 in the Single, Before, After, Before- 6 and After- 6 treatments, respectively) were guaranteed a show-up fee of $€ 5$ and a variable amount ( $€ 0, € 10$, or $€ 10.20$ ) depending on their choices.

### 2.3 Predictions

In this subsection, we provide intuitive explanations of what we can predict for each treatment. Section 4 derives these predictions formally. Our analysis focuses on strictly ambiguity averse (SAA) subjects (i.e., subjects who strictly prefer risk to ambiguity). We look at whether their choices are revealing strict ambiguity aversion ( $S A A^{r}$, i.e. are they willing to give up a positive amount to remove ambiguity). An incentive compatible experiment should guarantee that $S A A$ subjects are $S A A^{r}$. Preference conditions to define $S A A$ and $S A A^{r}$, as well as all other properties used in this subsection, are provided in Section 4.

Table 2.1 summarizes our predictions concerning $S A A^{r}$ choices. That is, selecting Bag K: in their chosen choice problem in the Single control; in both choice problems in the After and Before treatments; and in Red 1 and Blue 4 of the After-6 and Before-6 treatments.

In the Single control, there is no reason to expect that $S A A$ subjects will not reveal it. For the other treatments, $S A A$ and $S A A^{r}$ may differ because the other treatments involve several choice problems and a randomization device over these choice problems. The question is then whether subjects will isolate each choice problem in an experiment or integrate all choice problems in an experiment as a single compound object. We refer to the former view as isolation and the latter integration. ${ }^{3}$

The prediction of the isolation view is straightforward: all $S A A$ subjects should also be $S A A^{r}$ in all treatments. Under isolation, subjects treat each choice problem as if it were the only choice problem that determines their payoffs, therefore their reported preferences would coincide with their preferences in each choice problem. There is strong evidence in the economic and financial literature for isolation, also

[^2]known as narrow framing or narrow bracketing (Tversky and Kahneman, 1981; Kahneman and Lovallo, 1993; Barberis et al., 2006; Rabin and Weizsäcker, 2009). If subjects isolate choice problems, then all treatments are incentive compatible.

Predictions for the integration view is more nuanced and depend both on the incentives structure used by the experimenter and subjects' preferences over compound objects. In this case, their choices will not always reveal their 'true' preferences over acts. Next we develop our predictions for integrating subjects in each treatment. In the After treatment, consider the strategy of choosing Bag U in both choice problems. If a red chip is drawn from Bag $U$, then the chance of winning $€ 10.20$ is $50 \%$, which is the probability that the red choice problem is selected. If a blue chip is drawn, the chance of winning is also $50 \%$, the probability that the blue choice problem is selected. To sum up, no matter which chip is drawn from $U$, the probability of winning $€ 10.20$ is $50 \%$. By comparison, choosing Bag K twice induces a $50 \%$ chance of winning $€ 10$. Hence, choosing Bag U twice is not more ambiguous (in the integration view) than choosing Bag K twice, and it yields a higher payment ( $€ 10.20$ instead of $€ 10$ ). If follows that choosing Bag K in both choice problems is dominated by at least one other strategy (choosing Bag U twice) under integration (see Section 4). Consequently, integration predicts that $S A A$ subjects will not be $S A A^{r}$. This point is very closely related to Raiffa's (1961) argument and has recently been made by Bade (2015, first version 2011); Kuzmics (2017, first version 2012); Baillon et al. (2019, first version 2013); Azrieli et al. (2018, first version 2014) and Oechssler and Roomets (2014).

In the Before treatment, we can differentiate two cases. First, if subjects are not sensitive to the order in which risk and uncertainty are resolved, i.e., if they satisfy Reversal of Order, then the Before treatment is equivalent to the After treatment and the same prediction applies. That is, integration combined with Reversal of Order also predicts that $S A A$ subjects will not be $S A A^{r}$. But subjects may satisfy another property, that Azrieli et al. (2018) called Statewise Monotonicity. This property requires that preferring some bets to alternative bets (e.g., they prefer bet $a$ to bet $b$ and bet $c$ to bet $d$ ) implies preferring a randomization over the former to a randomization over the latter (they prefer a 50-50 chance of $a$ or $c$ to a 50-50 chance of $b$ or $d$ ). Statewise monotonicity directly implies incentive compatibility in the Before treatment (Baillon et al., 2019; Azrieli et al., 2018) and under this property, SAA subjects should be $S A A^{r}$. Note that $S A A$ subjects cannot satisfy both Reversal of Order and Statewise Monotonicity because it would lead to contradictions (violations of transitivity or stochastic dominance).

Our theoretical arguments carry through to the After-6 and Before-6 treatments. These treatments are closer to choice lists and we can study whether an increase in the number of tasks affects, for instance, subjects' tendency to isolate or integrate choice problems. Analyzing the four additional choice problems included in these treatments also allows us to infer whether $S A A$ subjects who integrate in the After treatment choose an ambiguity neutral strategy (as predicted by Bade (2015) for

Table 2.1: Summary of Predictions for $S A A$ subjects

|  | Treatments |  |  |
| :--- | :---: | :---: | :---: |
|  | Single | After \& After-6 | Before \& Before-6 |
| Isolation | $S A A^{r}$ | $S A A^{r}$ | $S A A^{r}$ |
| Integration + | $S A A^{r}$ | not $S A A^{r}$ | $S A A^{r}$ |
| Statewise Monotonicity | $S A A^{r}$ | not $S A A^{r}$ | not $S A A^{r}$ |
| Integration + Reversal <br> of Order |  |  |  |

instance) or if they even become ambiguity seeking. Experimental papers regularly find substantial ambiguity seeking. It is interesting to investigate whether it might be due to the incentive mechanism or if the mechanism only makes ambiguity averse subjects appear neutral. ${ }^{4}$

To sum up, the proportion of $S A A^{r}$ subjects in the Single control establishes a baseline for the rate of $S A A$ subjects in the population. In the After treatment, the integration view predicts that the rate of $S A A^{r}$ subjects drops with respect to the Single control, whereas isolation predicts it does not change. In the Before treatment, both isolation and integration with Statewise Monotonicity predict the same rate of $S A A^{r}$ subjects as in the Single control, while integration with Reversal of Order (and in the absence of Statewise Monotonicity) predicts a drop of $S A A^{r}$ subjects. Overall, we can therefore expect the highest rate of $S A A^{r}$ subjects in the Single control, followed by the Before treatment (where integration need not alter choices), and the lowest rate in the After treatment. The After-6 and Before-6 treatments are expected to produce similar results as the After and Before treatments respectively, unless the presence of additional choices affect subjects' propensity to isolate or integrate.

### 2.4 Results

Figure 2.3 presents the proportion of $S A A^{r}$ subjects in each treatment. The $S A A^{r}$ proportion in both Before ( $28.7 \%$ ) and After ( $25.3 \%$ ) treatments is significantly lower than in the Single control ( $50 \%$, p-values $<0.01$ in the proportion test). The $S A A^{r}$

[^3]Table 2.2: Choice pattern in Before and After treatments
Chosen bag in
Red and Blue problems

|  |  |  |  |  | KK |
| :---: | :---: | :---: | :---: | :---: | :---: |
|  | KU | UK | UU |  |  |
| Treatments | After | 22 | 24 | 6 | 35 |
|  | Before | 25 | 16 | 9 | 37 |

proportion is slightly higher in the Before treatment than in the After treatment, but the difference is not significant ( p -value $=0.73$ in the proportion test). Our results suggest that about half of the $S A A$ subjects are $S A A^{r}$ due to isolation. The proportion of $S A A$ subjects who satisfy integration and Statewise Monotonicity is negligible in our sample. Our results suggest that the introduction of RIS reduces the reported ambiguity aversion by almost a half, and the decrease mostly comes from subjects who integrate and use the RIS as a hedge against ambiguity.

At first sight, one may worry that our results are generated by random choice. If subjects randomly choose (with equal probability) between options in every choice problem, then we would mechanically have $50 \%$ of ambiguity averse subjects in the Single control and $25 \%$ in the Before and After treatments. But random choice would also imply that all possible choice patterns in Figure 2.1 (that is, KK, KU, UK, and UU ) are equally-likely. As can be seen in Table 2.2, this is not the case. For each treatment, the $\chi^{2}$ test rejects equal proportion of the four choice patterns ( p -values $<0.01$ ). For instance, only few subjects chose $U$ in the red problem and $K$ in the blue problem. As predicted, the most chosen patterns is UU, i.e. preferring Bag U in both choice problems. ${ }^{5}$ One could wonder whether the UU subjects really hedged or whether they actually became ambiguity seeking. Treatments Before-6 and After-6 provide further evidence.

The $S A A^{r}$ proportion is not different between Before-6 and After-6 ( p -value $=1$ in proportion test). Further, the $S A A^{r}$ proportions in the 6 -choice-problem treatments are not different than those in the 2-choice-problem treatments ( p -value $=0.86$ in the proportion test). Therefore, in our experimental setting, presenting more (than two) choice problems does not further decrease the rate of $S A A^{r}$. It did not seem to increase subjects' propensity to isolate or integrate.

[^4]Figure 2.3: Proportion of $S A A^{r}$ in each treatment


Notes: Each bar represents the number (and proportion) of $S A A^{r}$ participants in each treatment. Square brackets indicate proportion test results between corresponding treatments (or groups of treatments).
*** means significant at $1 \%$, and n.s stands for non-significant.

Figure 2.4: Ambiguity attitude categorization in After-6 and Before-6


We can also use the additional choices to further classify subjects. For instance, choosing Bag U in Red 3 and in Blue 6 reveals ambiguity seeking. Choosing Bag U in Red 1 and Blue 4 but Bag K in Red 2 and Blue 5 (the latter two choices corresponding to the Ellsberg Paradox as often implemented) reveals weakly disliking Bag U or being pretty much indifferent. Such subjects can be called weakly ambiguity averse. In Appendix A, we describe how each choice pattern indicates a type of behavior: ambiguity aversion $\left(A A^{r}\right)$, weak ambiguity aversion $\left(W A A^{r}\right),{ }^{6}$ ambiguity seeking $\left(A S^{r}\right)$, weakly ambiguity seeking $\left(W A S^{r}\right)$, ambiguity neutral $\left(A N^{r}\right)$, and non-monotonic ( $N M^{r}$ ).

Figure 2.4 presents the ambiguity attitude categorization in the After-6 and Before-6 treatments. The distribution across categories does not differ between the two treatments ( $p$-value $=0.85$ in the Fisher's exact test). Also, figure 2.4 clearly rejects random choice, knowing that random choice would result in $75 \%$ of $N M^{r}$ (see Table A. 1 in Appendix A). Finally, non- $S A A^{r}$ subjects were mostly not ambiguity seeking, but weakly ambiguity averse or ambiguity neutral. It suggests that the RIS does not make people ambiguity seeking, but, as predicted, makes them display less ambiguity aversion.

[^5]
## 3 Follow-up Study

### 3.1 Goals and predictions

So far, we investigated the interaction of the random incentive system with the timing of uncertainty, i.e., two stages, as in most traditional ambiguity models. Recently, Saito (2015) and Ke and Zhang (2020) (henceforth SKZ) have highlighted the importance of a third stage: when Nature 'chooses' a scenario. In our setting, this stage corresponds to the timing when Bag U is constructed. SKZ differentiate this stage from the stage of uncertainty resolution, which in our setting corresponds to the time when the color of the chip drawn from Bag U is revealed. Ambiguity aversion in these models reflect the decision maker's expectation that Nature will select the least favorable scenario (in our setting, an unfavorable Bag U composition). Following this line of reasoning, the time in which Nature 'chooses' is crucial. Randomization may be an effective hedge against ambiguity if Nature has played first, but it provides no hedge if Nature plays after the randomization. This follows because Nature could select the urn composition that is the least favorable given the randomization outcome.

In Study 1, Bag U was always constructed first, before subjects even entered the lab. This is the common practice in experimental economics as it avoids any form of suspicion towards the experimenter. Ambiguity aversion may arise from pessimism towards a malevolent Nature, but it should not be confounded by the intervention of the experimenter. Following SKZ's line of reasoning, participants could hedge against the ambiguity by mentally randomizing (randomly pick a color, as proposed by Raiffa, 1961) in the Single control and/or by using the RIS in the Before and After treatments. In this follow-up study, we further investigate the impact of the time in which Bag U was constructed. Note that these arguments assume that the participants do not satisfy reversal of orders, being sensitive to the timing of the three stages.

First, we replicated the Single control with one change: subjects were asked to seal their choice in an envelop as soon as they have made it, so that no one could know what they have chosen. It was opened by the experimenter at the conclusion of the experiment to determine the subject's payment. This change was not expected to have any impact on the Single control but ensured comparability with the treatments described next. The timeline of the Single control was thus: Bag construction - Choice - Chip draw.

We introduced a new treatment, called Single-Bag-Later (Single-BL), with the following timeline: Choice - Bag construction - Chip draw. For this treatment, subjects sealing choices in an envelop avoided suspicion concerns. It prevented the experimenters from knowing how to decrease the subjects' payoff when preparing Bag U. (Nature could still 'know' how to pick an unfavorable scenario). Comparing Single with Single-BL informed us if subjects mentally randomized. If some did in Single,

Table 3.1: Summary of Predictions for SAA subjects
Treatments

|  | Treatments |  |  |  |
| :--- | :---: | :---: | :---: | :---: |
| isolation | Single | Single-BL | Before-B | Before-BL |
| SKZ + mental <br> randomization | not $S A A^{r}$ | $S A A^{r}$ | $S A A^{r}$ | not $S A A^{r}$ |
| SKZ + no mental <br> randomization | $S A A^{r}$ | $S A A^{r}$ | $S A A^{r}$ |  |
| Reversal of Order + <br> mental randomization | not $S A A^{r}$ | not $S A A^{r}$ | not $S A A^{r}$ | not $S A A^{r}$ |
| Reversal of Order + no <br> mental randomization | $S A A^{r}$ | $S A A^{r}$ | not $S A A^{r}$ | not $S A A^{r}$ |

and therefore did not exhibit ambiguity aversion, we would expect more ambiguity aversion in Single-BL where mentally randomizing is futile. Recall that this line of reasoning assumes no reversal of orders.

We also introduced two new versions of the Before treatment.

- Before-Bag (Before-B), with the timeline: Randomization - Bag construction Choice - Chip draw;
- Before-Bag-Later (Before-BL), with the timeline: Randomization - Choice Bag construction - Chip draw.

The timelines of these two treatments only differ from those of Single and Single-BL by moving the construction of Bag U after the randomization. ${ }^{7}$ These two treatments hence offered no hedge against ambiguity and the SKZ interpretation therefore predicted Before-B and Before-BL to have the same proportion of $S A A^{r}$ as Single and Single-BL respectively. By contrast, reversal of orders would predict lower rates of $S A A^{r}$ in Before-B and Before-BL. For completeness, recall that isolation predicts no difference between the four treatments. ${ }^{8}$

[^6]
### 3.2 Procedure and incentives

The sessions for study 2 were run between July-2020 and Oct-2020, when the Dutch government relaxed the measures taken against COVID-19. We modified the experimental procedure in the additional treatments to effectively follow the hygiene protocol.

When entering the lab, all subjects had to sanitize their hands. Subjects saw the two bags. They were told that: bag K (bag A in the experiment) already contained one red chip and one blue chip, whereas bag $U$ (bag B in the experiment) was empty. The experimenter announced when he/she would put two chips into it, but subjects did not know the color composition.

- Single control: the experimenter filled Bag U with two chips after all subjects entered the lab but before they made their choices.
- Single-BL control: the experimenter filled Bag U with two chips after all subjects made their choices.

In these two treatments, subjects received a choice envelope together with the choice sheet. The experimenter asked each participant to put their choice sheet into the choice envelope once they made their choices, so that no one else could know what they chose. At the end of all sessions, one subject was invited to draw a chip from each bag (without looking). The choice envelopes were opened at the payment desk by the experimenter and subjects could not revise their choices after learning about the color draws. Subjects were paid the outcome of their chosen bet in their chosen choice problem.

In the following two treatments, each subject picked one sealed ticket envelope upon entering the lab (after hand sanitation). They read in the instructions that the envelope contained the choice problem that would determine their payment and were told not to open the ticket envelope until they were told to do so.

- Before- $B$ treatment: the experimenter filled Bag U with two chips after all subjects entered the lab but before they made their choices or opened the ticket envelope. The experimenter first checked that the choice envelope was properly sealed, and then asked the subject to open the ticket envelope.
- Before-BL treatment: subjects first sealed their choice sheets in the choice envelope and opened the ticket envelope. Then the experimenter filled Bag U with two chips.

Similarly, subjects were told to seal their choice sheet in the choice envelope, which was later opened by the experimenter at the payment desk. One subject was invited to draw a chip from each bag (without looking). Subjects were paid the outcome of their
chosen bet in the choice problem that was in their ticket envelopes. Experimental instructions for all treatments are included in Appendix B.

In total, 215 subjects participated in Study 2, in 42 small experimental sessions with no more than 9 subjects per session to ensure sufficient social distancing. Based on a power calculation using the effect size in the main experiment, our initial plan was to collect 80 observations per treatment. However, due to the second wave of COVID-19 in October 2020 and the more stringent social distancing measures in the Netherlands, we had to stop the data collection prematurely.

Subjects were randomly assigned to the four treatments. The sessions lasted on average 20 minutes. Subjects were guaranteed a show-up fee of $€ 5$ and a variable payment ( $€ 0, € 10$, or $€ 10.20$ ) depending on their choices and the outcome of the bets.

### 3.3 Results

Figure 3.1 presents the proportion of $S A A^{r}$ subjects in the follow-up experiment. Compared with the Main Study, we observe slightly less ambiguity aversion in the Single control. We conjecture that the uncertainty involved in participating in an inperson experiment during a pandemic led to a small under-representation of ambiguity averse subjects in the follow-up study.

Although none of the pairwise comparison was significant in the proportion tests, the proportion of $S A A^{r}$ subjects in Before-B and Before-BL (combined) is $11 \%$ lower than in the Single and Single-BL treatments combined (p-value $=0.093$ in the proportion test). In other words, roughly $25 \%$ ( $11 \%$ decrease divided by $42.5 \%$ ambiguity averse subjects in Single) of the ambiguity averse participants seemed to satisfy reversal of order. The others are compatible with isolation or with the SKZ interpretation, since in both the Before treatments the bag was filled after the randomization. ${ }^{9}$

[^7]Figure 3.1: Proportion of $S A A^{r}$ in the additional treatments


Notes: Each bar represents the number (and proportion) of $S A A^{r}$ participants in each treatment. Square brackets indicate proportion test results between corresponding treatments (or groups of treatments).

* means significant at $10 \%$.

To further analyze the strength of our empirical evidence, we followed the approach of Jamil et al. (2017) to obtain the Bayesian posterior of the difference in the $S A A^{r}$ proportion of subjects between every pair of treatments. ${ }^{10}$ Figure 3.2 presents these posteriors.

Comparing Single to Single-BL and Before-B to Before-BL (the two top posteriors) suggests that participants did not mentally randomize. Had they done so, it would have resulted in a positive differences in Figure 3.2, but the posteriors point to negative or null differences. Comparing the Single controls to the corresponding Before treatments we find that there is a $92.1 \%$ chance that Before-B led to fewer $S A A^{r}$ participants than Single and a $84.4 \%$ chance that Before-BL led to fewer $S A A^{r}$ participants than Single-BL. This is consistent with our finding above that there are many ambiguity averse subjects who satisfy reversal of order.

[^8]Figure 3.2: Posteriors of Bayesian Analysis


## 4 Theoretical Analysis

In Sections 2.3 and 3.1, we provided intuitive explanations for our predictions. In this section we elaborate and present formal definitions and theoretical analysis. The first subsections derives predictions for the Main Study and Subsection 4.5 for the Follow-up Study.

Consider a non-degenerate set $\mathcal{M}$ of monetary amounts. Let $S=$ $\{R R, R B, B R, B B\}$ be the state space, where the first letter of each state indicates the color of the chip drawn from bag K ( $R$ for red, $B$ for blue) and the second letter the color of the chip drawn from bag U . Denote by $\hat{x}=\left[x_{1}, x_{2}, x_{3}, x_{4}\right]$ the act assigning monetary amounts $x_{1}$ to $R R, x_{2}$ to $R B, x_{3}$ to $B R$, and $x_{4}$ to $B B$. Let us denote
$\tilde{x}=\left(x_{1}, \ldots, x_{n}\right)$ the lottery that yields $x_{1}, \ldots, x_{n} \in \mathcal{M}$ with equal chance. Lotteries are used to model the RIS, i.e. that each of the $n$ choices of the experiment will be paid with equal chance.

We will further consider two types of compound objects. The first type is $\left[\tilde{x}_{1}, \tilde{x}_{2}, \tilde{x}_{3}, \tilde{x}_{4}\right]$, which assigns lotteries to events (often called an Anscombe-Aumann act). The second type is ( $\hat{x}_{1}, \ldots, \hat{x}_{n}$ ), which yields $n$ possible acts with equal chance, with $n$ finite. The set of all possible acts is denoted $\mathcal{A}$, that of lotteries $\mathcal{L}$, and that of compound objects $\mathcal{C}$. With a slight abuse of notation, degenerate compound objects will be referred to by the corresponding act or lottery.

Definition 1. An act $\left[x_{1}, x_{2}, x_{3}, x_{4}\right]$ is ambiguous if $x_{1} \neq x_{2}$ or $x_{3} \neq x_{4}$; otherwise, it is unambiguous.

Unambiguous acts are constant in the color of the chip drawn from bag $U$.

### 4.1 Choices and types of preferences

Let $\succsim$ be the subject's weak preferences defined over $\mathcal{A}$, with $\sim$ and $\succ$ being the symmetric and asymmetric parts, as usual. These are the preferences that the experimenter would like to learn about. We assume $[x, x, y, y] \sim[y, y, x, x]$ for all $x, y$, which means that the subject has no intrinsic preferences for betting on red or blue in bag K, where both colors have an objective $50 \%$ chance to be drawn. In other words, he exhibits no color preference. Let $\succsim^{r}$ be the reported (revealed) preferences over acts in the experiment.

Definition 2. A treatment is incentive compatible if $\left[x_{1}, x_{2}, x_{3}, x_{4}\right] \succsim\left[y_{1}, y_{2}, y_{3}, y_{4}\right]$ implies not $\left[y_{1}, y_{2}, y_{3}, y_{4}\right] \succ^{r}\left[x_{1}, x_{2}, x_{3}, x_{4}\right]$.

Incentive compatibility means that subjects do not report preferences in the experiment that contradict their true preferences. It is not an equivalence because not all preferences $(\succsim)$ are elicited in the experiment. Indeed, it is the experimenter's discretion how much of $\succsim$ he wants to measure. One of the lessons of the current investigation (which we shall return to in the conclusion) is that there is a tradeoff between incentive compatibility and the domain on which $\succsim^{r}$ is revealed.

If subjects view each choice problem in an experiment in isolation, then $\succsim^{r}=\succsim$ in each choice of the experiment. Hence, if subjects satisfy isolation, all treatments are incentive compatible. This result is almost tautological but is necessary to understand the results of our experiments. By contrast, in the integration view, revealed preferences $\succsim^{r}$ concern compound objects $\mathcal{C}$. We therefore need to define what such preferences could be. Let $\succsim^{*}$ be the subject's preferences over compound objects.. We follow Azrieli et al. (2018) in defining admissible extensions of $\succsim$.
Definition 3. $\succsim^{*}$ is an admissible extension of $\succsim$ over $\mathcal{C}$ if $\hat{x}_{1} \succsim \hat{x}_{2} \Leftrightarrow \hat{x}_{1} \succsim^{*} \hat{x}_{2}$ for all acts in $\mathcal{A}$.

In the remainder of this section, we assume that $\succsim^{*}$ is an admissible extension of $\succsim$. Whether an experiment is incentive compatible will then depend on the properties of the incentive scheme and properties of $\succsim^{*}$. We assume $\succsim^{*}$ to be transitive and complete. For consistency, $[x, x, y, y] \sim^{*}(x, y)$ and all lotteries that are objectively equivalent lead to indifference. ${ }^{11}$ Preferences $\succsim^{*}$ are also assumed to satisfy firstorder stochastic dominance (FOSD), i.e. $\left[\tilde{x}_{1}, \tilde{x}_{2}, \tilde{x}_{3}, \tilde{x}_{4}\right] \succ^{*}\left[\tilde{y}_{1}, \tilde{y}_{2}, \tilde{y}_{3}, \tilde{y}_{4}\right]$ whenever $\tilde{x}_{i}$ strictly first-order stochastically dominates $\tilde{y}_{i}$ for all $i$. We consider two properties of $\succsim^{*}$.

Definition 4. $\succsim^{*}$ satisfies Statewise Monotonicity if $\left(\hat{x}_{1}, \ldots, \hat{x}_{n}\right) \succ^{*}\left(\hat{y}_{1}, \ldots, \hat{y}_{n}\right)$ whenever $\hat{x}_{i} \succ^{*} \hat{y}_{i}$ for all $i$.

Definition 5. $\succsim^{*}$ satisfies Reversal of Order if $\left[\left(x_{11}, \ldots, x_{1 n}\right), \ldots,\left(x_{41}, \ldots, x_{4 n}\right)\right] \sim^{*}$ $\left(\left[x_{11}, \ldots, x_{41}\right], \ldots,\left[x_{1 n}, \ldots, x_{4 n}\right]\right)$.

The Statewise Monotonicity property was proposed by Azrieli et al. (2018). It implies that a compound object is preferred when its second stage acts are all subjectively better. Unlike the commonly assumed FOSD property which says that an act yielding objectively better outcomes is always preferred, Statewise Monotonicity implicitly implies that the decision maker first considers what is subjectively better in the second stage, and therefore separates the evaluation of the two stages in a compound object. In this sense, Statewise Monotonicity is close to compound independence proposed by Segal (1990) (who studies compound objective lotteries), or recursivity in more general settings. ${ }^{12}$

Next, we analyze incentive compatibility of our treatments under different properties of $\succsim^{*}$. We are interested in strict ambiguity aversion, which is characterized by preferring bag K for both colors, even at the cost of a small payment. Note that this pattern of choice is sufficient for identification of ambiguity aversion, but is not necessary. It is possible that the premium some ambiguity averse subjects are willing to pay was smaller than .20 , so they chose bag U. Other subjects may believe that one of the two colors is much more likely than the other (but still be uncertain about the exact likelihood). In what follows, we focus on strict ambiguity aversion. As pointed out by Epstein and Halevy (2019), in experiments, we can at best observe weak preferences. So if a subject chooses to bet on a red chip from the Known bag with a prize of 10 to a bet on red chip from the Unknown bag, with a prize of 10.20 ,

[^9]and assuming he prefers more money to less and has transitive preference, we can conclude that he strictly prefers a bet on the Known bag when both prizes are equal.

Definition 6. If a subject has preferences $[10,10,0,0] \succsim[10,0,10,0]$ and $[0,0,10,10] \succsim[0,10,0,10]$, with at least one preference strict, then he shows Strict ambiguity aversion (SAA).

In the analysis below, we will focus on $S A A$ subjects. Let $S A A^{r}$ refer to subjects who are "reporting $[10,10,0,0] \succsim^{r}[10.20,0,10.20,0]$ and $[0,0,10,10] \succsim^{r}$ [ $0,10.20,0,10.20]$."

### 4.2 Single control

In the Single control, by selecting one color and then one bag for that color, subjects can effectively choose among four acts: $[10,10,0,0]$, $[10.20,0,10.20,0],[0,0,10,10]$, and $[0,10.20,0,10.20]$. There are no compound objects. Hence, the isolation and the integration views coincide. A subject who reports $[10,10,0,0] \succsim^{r}[10.20,0,10.20,0]$ or $[0,0,10,10] \succsim^{r}[0,10.20,0,10.20]$ (selecting one of the two colors and choosing bag K) is therefore $S A A$. As discussed above, the proportion of bag K choices in the Single control provides a lower bound and a benchmark for the rate of $S A A$ subjects in the population.

### 4.3 After treatment

In the After treatment, combinations of subjects' choices in the two choice problems generate four compound objects. With the RIS lottery following the resolution of uncertainty, each compound object is an Anscombe-Aumann act.

- Always choosing bag $\mathrm{K}\left([10,10,0,0] \succsim^{r}[10.20,0,10.20,0]\right.$ and $[0,0,10,10] \succsim^{r}$ $[0,10.20,0,10.20])$ reveals a preference for the following compound object $[(10,0),(10,0),(0,10),(0,10)]$, because irrespective of the color drawn from bag K, there is a $50 \%$ chance that the RIS selects it as the winning color. Since the order of outcomes is immaterial in 50-50 lotteries, the compound object can also be written as $[(10,0),(10,0),(10,0),(10,0)]$.
- Always choosing bag U ([10.20, 0, 10.20, 0] $\succsim^{r} \quad[10,10,0,0]$ and $\left.[0,10.20,0,10.20] \succsim^{r} \quad[0,0,10,10]\right)$ reveals a preference for $[(10.20,0),(10.20,0),(0,10.20),(0,10.20)]$. The reasoning is the same as in the first case: once the color of the chip drawn from bag U is determined, with $50 \%$ chance this color will match with the winning color determined by the RIS. But outcomes are higher and this choice pattern yields $[(10.20,0),(10.20,0),(0,10.20),(0,10.20)]$.
- Choosing bag K for red and bag U for blue reveals a preference for $[(10,0),(10,10.20),(0,0),(0,10.20)]$.
- Choosing bag U for red and bag K for blue reveals a preference for $[(0,10.20),(0,0),(10,10.20),(10,0)]$.

By FOSD, $\quad[(10.20,0),(10.20,0),(0,10.20),(0,10.20)] \quad \succ^{*}$ $[(10,0),(10,0),(0,10),(0,10)]$. Hence, subjects who employ an integration view and satisfy FOSD would never exhibit [10, 10, 0, 0] $\succsim^{r}$ [10.20, 0, 10.20, 0] and $[0,0,10,10] \succsim^{r}[0,10.20,0,10.20]$, even if they satisfy $S A A$. In other words, the incentive scheme in the After treatment is not incentive compatible for subjects with these preferences.

Proposition 7. RIS with the lottery stage after the resolution of uncertainty is not incentive compatible if subjects integrate the choice problems in the experiment and satisfy FOSD.

Although it is theoretically possible for subjects to integrate while violating FOSD, it is rather difficult to envision such a case in practice: for subjects who are able to perceive the whole experiment as a compound object, it is hard to imagine that they would fail to choose the dominant option. The next proposition provides predictions of subjects' preferences on their reported preferences in the After treatment.

Proposition 8. In the After treatment: (i) SAA subjects who satisfy isolation are $S A A^{r}$; (ii) SAA subjects who satisfy integration are not $S A A^{r}$.

### 4.4 Before treatment

In the Before treatment, the RIS lottery stage precedes the resolution of uncertainty. The integration view gives four possible compound objects that can be simply described as $\left(\hat{x}_{1}, \hat{x}_{2}\right)$ for some $\hat{x}_{1}$ and $\hat{x}_{2}$. For instance, the combination of choosing bag K in both choice problems ([10, 10, 0, 0] $\succsim^{r}$ [10.20, $\left.0,10.20,0\right]$ and $\left.[0,0,10,10] \succsim^{r}[0,10.20,0,10.20]\right)$ gives $([10,10,0,0],[0,0,10,10])$, whereas the combination of always choosing bag $U$ gives ( $[10.20,0,10.20,0],[0,10.20,0,10.20])$. For subjects who employ the integration view while satisfying Statewise Monotonicity, they will first evaluate the second stage acts according to their true preferences $\succsim$ over acts, and Statewise Monotonicity ensures that their reported preferences $\succsim^{r}$ reveals their true preferences. For instance, SAA subjects who satisfy Statewise Monotonicity will prefer (in the sense of $\succ^{*}$ ) ( $[10,10,0,0],[0,0,10,10]$ ) to all other possible compound objects. Hence, the incentive scheme in the Before treatment is incentive compatible for subjects who satisfy Statewise Monotonicity under the integration view.

Proposition 9. RIS with the lottery stage before the resolution of uncertainty is incentive compatible if subjects satisfy isolation, or integration with Statewise Monotonicity.

Interestingly, if subjects satisfy Reversal of Order, they exhibit $[(10,0),(10,0),(0,10),(0,10)] \quad \sim^{*} \quad([10,10,0,0],[0,0,10,10])$ and $[(10.20,0),(10.20,0),(0,10.20),(0,10.20)] \sim^{*}([10.20,0,10.20,0],[0,10.20,0,10.20])$. Consider an integrating $S A A$ subject who satisfies Statewise Monotonicity and Reversal of Order: $([10,10,0,0],[0,0,10,10]) \succsim^{*}([10.20,0,10.20,0],[0,10.20,0,10.20])$ together with Reversal of Order imply $[(10,0),(10,0),(0,10),(0,10)] \succsim^{*}$ $[(10.20,0),(10.20,0),(0,10.20),(0,10.20)]$, hence a violation of FOSD. Similarly, an integrating $S A A$ subject who satisfies FOSD and Reversal of Order (Statewise Monotonicity) must violate Statewise Monotonicity (Reversal of Order). This leads to the following proposition:

Proposition 10. For integrating SAA subjects: FOSD, Statewise Monotonicity and Reversal of Order cannot hold simultaneously.

As mentioned before, we assume that all integrating subjects satisfy FOSD, therefore integrating $S A A$ subjects violate either Statewise Monotonicity or Reversal of Order. If they satisfy Reversal of Order, then they treat the Before treatment similarly to the After treatment and cannot be $S A A^{r}$, as seen in Proposition 6.

Proposition 11. In the Before treatment: (i) SAA subjects who satisfy isolation are $S A A^{r}$; (ii) SAA subjects who satisfy integration with Statewise Monotonicity are $S A A^{r}$; (iii) $S A A$ subjects who satisfy integration with Reversal of Order are not $S A A^{r}$.

### 4.5 Scenario selection

In the Main study and the previous subsections we considered only two stages (subjective uncertainty and RIS), which allowed us to investigate the interaction between various properties of subjects' preferences and the incentive mechanism employed by the experimenter. As mentioned above, Saito (2015) and Ke and Zhang (2020) also included a stage of scenario selection (when 'Nature plays'). A Subject who is ambiguity averse and his preferences are represented by Maxmin expected utility (MEU) (Gilboa and Schmeidler, 1989) can be thought of as having a set of priors and evaluating each act by the lowest expected utility attained on this set. In other words, ambiguity aversion is modeled by Nature picking the least advantageous prior. In Double maxmin expected utility (DMEU) (Ke and Zhang, 2020), Nature can 'play' twice. First, it chooses a set of priors, called a scenario, from a collection. The subject makes a decision and then Nature chooses a prior from the scenario. Randomization at the decision time can render Nature's first move ineffective but not the second.

We follow the notation in Ke and Zhang (2020) and derive predictions for a subject represented by the DMEU for both studies.

Let us consider a compound object $\left(\hat{x}_{1}, \ldots, \hat{x}_{n}\right)$. DMEU assigns it the value

$$
\min _{M \in \mathcal{M}} \sum_{i=1}^{n} \frac{1}{n} \min _{\mu \in M} \sum_{s \in S} \mu(s) u\left(x_{i s}\right)
$$

where $\mathcal{M}$ is the scenario set and $x_{i s}$ is the monetary amount that act $\hat{x}_{i}$ assigns to state $s$. Each scenario is a set of priors $M$. Let $\mu_{t}$ refer to the probability measure over $S$ induced by having $t$ blue chips in Bag U. If Bag U is prepared first, then a plausible scenario set is $\mathcal{M}_{e x}=\left\{\left\{\mu_{0}\right\},\left\{\mu_{1}\right\},\left\{\mu_{2}\right\}\right\}$. The subject has in mind 3 scenarios, each representing a singleton prior over $S$. Ke and Zhang (2020) call this case ex-ante $M E U$. If Bag U is prepared after the randomization, then the plausible scenario set becomes the singleton $\mathcal{M}_{\text {post }}=\left\{\left\{\mu_{0}, \mu_{1}, \mu_{2}\right\}\right\}$. It corresponds to ex-post $M E U .{ }^{13}$

The number of blue chips in Bag U has no impact on the DMEU evaluation of $([10,10,0,0],[0,0,10,10])$, which is $0.5 u(10)$ for both ex-ante and ex-post MEU. However, with the scenario sets specified above, ex-ante MEU assigns $0.5 u(10.20)$ to $([10.20,0,10.20,0],[0,10.20,0,10.20])$, whereas ex-post MEU assigns it 0 . The DMEU model predicts ambiguity aversion only if Nature is perceived to play after any randomization. Subjects may have different scenario sets than assumed in this example, but the reasoning remains the same. In ex-ante MEU, the scenario set is made of singletons and randomization is effective against ambiguity. By contrast, in ex-post MEU, randomization is ineffective in hedging against ambiguity, and ambiguity aversion may be displayed.

## 5 Related Experimental Literature

Experiments with RIS differ in two ways from single-task experiments: (i) a random device creates compound objects and (ii) more choices are present (e.g. on the answer sheet). In this paper, we mainly considered the impact of (i). For risky decisions, Cox et al. (2014) and Cox et al. (2015) found that (some) subjects do not isolate, and Freeman et al. (2019) document that when one of the alternatives involves certainty, choice lists (with RIS) may underestimate risk aversion. Some studies controlled for (ii) by displaying all choice problems even in single-task treatments; see for instance Starmer and Sugden (1991) and Cubitt et al. (1998), who found (some) support for isolation (but Freeman and Mayraz (2019) show it fails in choice list when one alternative is certain). In our experiment, (ii) cannot explain the results because the

[^10]answer sheet of the Single control also displayed both Red and Blue problems (since the subjects had to select which choice problem would determine their payment).

Our results provide the first direct evidence that RIS affects the measurement of ambiguity aversion. The finding could be expected from the theoretical literature but less so from the experimental literature. The experimental literature to date (Dominiak and Schnedler, 2011; Oechssler et al., 2019) has relied on direct measurement of preference for randomization and its association to ambiguity attitude. That is, the subject was offered an objective tool (e.g. a coin toss) with which he can hedge the ambiguity, and the experimenter observed if he chose to employ it or how much he valued it. Both studies used a within-subject design, but employ different incentive schemes: Dominiak and Schnedler (2011) pay all decisions with trivial stakes (0.10 Euro) - creating perfect insurance for subjects who integrate decisions, while Oechssler et al. (2019) pay one decision randomly - which may interact (as they acknowledge) with their basic research question and confound any result. ${ }^{14}$ Both studies reported no relation between ambiguity aversion and preference for randomization. Dominiak and Schnedler (2011) found that only 6 out of 35 ambiguity averse subjects were also randomization loving. Oechssler et al. (2019) asked subjects to make two choices that together could potentially hedge ambiguity, but could not reveal strict preference for randomization. Approximately $50 \%$ of subjects made choices to hedge, a proportion that was independent of their ambiguity attitude. Crucially, since Oechssler et al. (2019) could not identify strict preference for randomization, this proportion is indistinguishable from indifference and choosing randomly. Our between-subject design, though considerably more expensive, bypasses these severe incentive problems. By focusing on a single measure - strict ambiguity aversion, we can investigate how it is affected by the potential for hedging. Our Main Study provides strong evidence that ambiguity averse subjects realized the hedging opportunity, by comparing Before and After to Single.

Oechssler et al. (2019) also varied the order of the objective hedge and the determination of the ambiguous state of the world. They found no difference between these treatments. In their "alternative specification", the authors also included a treatment in which, as in our Before treatment, the objective hedge preceded the decision. Again, they found no difference. Consistent with Oechssler et al. (2019), our main study supports reversal of order and finds no difference between Before and After. Moreover, our follow-up investigation further contributes to the field by varying when the bags were constructed, an aspect that was not considered previously. We find that the answer is more nuanced and some subjects are sensitive to this aspect, as proposed by Saito (2015) and Ke and Zhang (2020).

[^11]
## 6 Discussion

Our experiments test the impact of random incentives on the observability of ambiguity aversion. Further, our design allows us to draw conclusions about strictly ambiguity averse subjects, using the predictions displayed in Table 2.1. From the Single control, we expect that about $50 \%$ of the subjects are $S A A$. Observing about $25 \%$ of $S A A^{r}$ in the After and After-6 treatments in the main study suggests that half of the $S A A$ subjects isolate, while the other half integrate their decisions. Furthermore, the absence of difference between the After and Before treatments is compatible with Reversal of Order rather than Statewise Monotonicity.

The fact that half of the subjects integrate is worrisome, as ambiguity experiments may underestimate the prevalence of ambiguity aversion. It can be argued that our design made the possibility of integration especially salient, and that subjects of more complex experiments are less likely to integrate their choices. Adding a few more choices (in the After-6 and Before-6 treatments) did not seem to influence the rate of integrating subjects but even these treatments, with more choices, remained relatively simple relative to some experiments from the literature. Assessing the impact of more complex design is tricky if one wants to keep the same benchmark as we had (one single choice). More complex experiments may also introduce other confounds, such as order effects and fatigue.

While in the main study we could not exclude the possibility that some subjects mentally randomized, and therefore our measure of $S A A^{r}$ in the Single control was a lower bound for the level of strict ambiguity aversion in the population. The effects of the Before and After treatments are therefore also lower bounds of the effects of using RIS. In the follow-up study, however, we did not find that constructing the bag later increased the frequency of $S A A^{r}$, hence we found no experimental evidence that subjects mentally randomized in the SKZ framework. It can be that our manipulation was somewhat artificial. For many sources of ambiguity in real life, the timing of Nature's play is well-defined and cannot be manipulated. Coupled this observation with the fact that the effect of RIS was smaller in the follow-up study (we conjecture it was mainly due to sample selection during the pandemic but other sources are possible too), we believe that the SKZ interpretation when the source of ambiguity, and especially its timing, arises naturally, deserves further investigation.

Our aim was to test the theoretical claims that random incentives can provide a hedge against ambiguity. We used rather extreme conditions to give these predictions a chance and to identify how big the problem of using random incentives in ambiguity experiments might be. When implementing the various treatments, we made the order between the resolution of uncertainty and of random incentives as salient as possible. We also displayed all choices on single page, making hedging possibilities more salient. We tested whether the Before treatment and/or increasing the number of choices could reduce the prevalence of hedging. We leave for further research
whether other manipulations (for instance, not displaying all choices on the same page, using filler tasks) could reduce hedging when random incentives are used.

We were genuinely surprised by the absence of difference between the After and Before treatments. In a previously-circulating theoretical paper (Baillon et al., 2019), we conjectured that more subjects would exhibit ambiguity aversion in the Before treatment, especially if the difference between the two treatments were very salient. In other words, we expected (integrating) subjects to exhibit Statewise Monotonicity rather than Reversal of Order. Our instructions were written to enhance the perception of the order between risk and ambiguity. We also followed recommendations of Johnson et al. (2019) to use envelopes in the Before and Before-6 treatments. Making the prior selection of one-choice problem tangible (as the choice-problem is already in the envelope) can help subjects condition on choice problems, i.e. help them isolate or exhibit Statewise Monotonicity. Our implementation of the Before treatments did not seem to have any effect on this.

To conclude, one should recall that the main motivation to employ RIS in ambiguity experiments is to avoid relying on symmetric beliefs, i.e., not assuming that subjects consider both colors equally likely. Alternatively, letting subjects choose a color to bet on allows experimenters to unequivocally conclude that a subject preferring the unknown bag is ambiguity averse. However, experimenters must be willing to assume belief symmetry if they want to infer ambiguity seeking from the choice data. A general lesson from our empirical investigation is that if an experimenter wants to observe attitudes to ambiguity that are more general than ambiguity aversion, she must tradeoff identification assumptions that have only partial empirical support: either assuming that subjects isolate or that they hold symmetric beliefs. In this respect, there are behavioral constraints to observability, much like has been acknowledged long ago in other experimental sciences like Physics and Psychology.

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## Online appendix

## A Revealed ambiguity attitude in the six choice problems

The full choice patterns (beyond the choices in Red 1 and Blue 4) in the Before- 6 and After-6 treatments provide more information on subjects' ambiguity attitudes. All possible choice patterns can be assigned a (unique) six-letter code. For instance, we denote "acebdf" the choice pattern of a subject preferring "a" in Red 1, "c" in Red 2, and "e" in Red 3, but then " b ", " d " and " f " in the Blue problems of Figure 2.2. Based on observed choice patterns, we can classify subjects into one of the following six categories:

- Ambiguity Averse $\left(A A^{r}\right)$ : subjects who always chose bag K (KKKKKK), or chose bag K whenever it pays at least as much as bag U , and chose bag K at least once when it pays less than bag U (KKKUKK or UKKKKK). ${ }^{15}$
- Ambiguity Seeking $\left(A S^{r}\right)$ : subjects who exhibited exactly the opposite pattern than the ambiguity averse ones: UUUUUU, UUUUUK or UUKUUU.
- Weak Ambiguity Averse $\left(W A A^{r}\right)$ : subjects who chose bag K whenever it pays at least as much as bag U, and chose bag U when bag K pays strictly less than bag U (UKKUKK). Choosing K in Red 2 and in Blue 5 suggests disliking Bag U (and therefore ambiguity aversion) but it can also be that such subjects were indifferent (ambiguity neutral).
- Weak Ambiguity Seeking $\left(W A S^{r}\right)$ : subjects who chose bag U whenever it pays at least as much as bag K , and chose bag K when bag U pays strictly less than bag K (UUKUUK).
- Ambiguity Neutral $\left(A N^{r}\right)$ : subjects whose choices can be rationalized by subjective expected utility with arbitrary beliefs. For instance, a subject who chose "KKKUUK" can be ambiguity neutral believing that it is more likely that a blue chip will be drawn from Bag U. Similarly, a subject who chose "UKKUUK" might have been indifferent in problems Red 2 and Blue 5, but chose K in the former and U in the latter. Alternatively, the same subject may have held a belief that drawing blue from bag U is slightly more likely than drawing red.

[^12]- Non-monotonic or non-transitive preferences $\left(N M^{r}\right)$ : within all the red (blue) problems, the Bag K bet becomes better whereas the bag U bets becomes worse from left to right, subjects satisfying monotonicity, once choosing bag K in one option should not switch to bag U any more. For instance, a subject who chose "UKUUKU' violates monotonicity or transitivity.

Table A. 1 presents the full categorization of all possible choice patterns in the 6-choice-problem treatments.

| Category | Choice Patterns |
| :---: | :---: |
| $A A^{r}$ | KKKKKK, KKKUKK, UKKKKK |
| $A S^{r}$ | UUUUUU, UUUUUK or UUKUUU |
| $W A A^{r}$ | UKKUKK |
| $W A S^{r}$ | UUKUUK |
| $A N^{r}$ | UKKUUK, UUKUKK, KKKUUU, UUUKKK, KKKUUK, |
|  | UUKKKK, UKKUUU, UUUUKK |
| $N M^{r}$ | all the others |

Table A.1: Ambiguity attitude categorization by choice pattern

## B Instructions

The following pages include the Instructions for the five treatments as presented to the subjects.

## Instructions Single

There are two bags. Bag A has 2 chips, one of them is blue and the other red. Bag B also has 2 chips. Each chip in Bag B is either blue or red. However, the number of blue (and red) chips is unknown - it could be 0 blue ( 2 red) chips, 1 blue chip and 1 red chip, or 2 blue ( 0 red) chips.


Bag B


Below is an example of a choice problem that you will face during the experiment.

## Red choice problem

Which one do you prefer?
Option A: You win $€ 10.00$ if the implementer draws a red chip from Bag A, and $€ 0$ otherwise.
Option B: You win $€ 10.20$ if the implementer draws a red chip from Bag B, and $€ 0$ otherwise.
In the example, you need to choose between Option A and Option B. The two options have the same winning color but differ in the amount you can win and the bag from which the chip is drawn. A Blue choice problem is similar. The only difference is that the winning color is blue instead of red. You can select the color of your choice problem. The color that you select will be the color that you bet on.

You will receive a separate choice sheet.

- Firstly, select the color of your choice problem.
- Secondly, choose your preferred option in the choice problem that you selected.


## Payment:

At the end of the experiment, the implementer will draw a chip from Bag A and a chip from Bag B respectively without looking. He will announce the colors of both chips drawn and record them on a piece of paper.

You will be paid according to your choice in the choice problem that you selected. Below we give examples of how you will be paid.
Suppose you select the red problem.

- If you chose option A, you win if the implementer draws a red chip from Bag A (50\%).
- If you chose option B, you win if the implementer draws a red chip from Bag B.

Suppose you select the blue problem.

- If you chose option A, you win if the implementer draws a blue chip from Bag A (50\%).
- If you chose option B, you win if the implementer draws a blue chip from Bag B.


## Instructions Before

There are two bags. Bag A has 2 chips, one of them is blue and the other red. Bag B also has 2 chips. Each chip in Bag B is either blue or red. However, the number of blue (and red) chips is unknown - it could be 0 blue ( 2 red) chips, 1 blue chip and 1 red chip, or 2 blue ( 0 red) chips.


Bag B


Below is an example of a choice problem that you will face during the experiment.

## Red choice problem

Which one do you prefer?
Option A: You win $€ 10.00$ if the implementer draws a red chip from Bag A, and $€ 0$ otherwise.
Option B: You win $€ 10.20$ if the implementer draws a red chip from Bag B, and $€ 0$ otherwise.
In the example, you need to choose between Option A and Option B. The two options have the same winning color but differ in the amount you can win and the bag from which the chip is drawn.

During the experiment, you will also face a Blue choice problem. The only difference is that the winning color is blue instead of red.

You will receive a separate choice sheet.

## Payment:

You will be paid according to your choice in one of the two problems. To select the choice problem that will determine your payment, the implementer will throw a 6-sided die, 3 sides of which are marked red and the others marked blue. The implementer will throw the die for all participants and put the choice problems with the matching color into sealed envelopes. You will draw one envelope and write your subject ID on it. Please do not open the envelope until you are told to do so.
Remember that the choice problem that matters for your final payment is in your envelope, and it is chosen before you make any choices.

At the end of the experiment, the experimenters will ask you to open your envelope. Then, the implementer will draw a chip from Bag A and a chip from Bag B respectively without looking. He will announce the colors of both chips drawn and record them on a piece of paper.

You will be paid according to your choice in the problem in your envelope. Below we show how you will be paid.
Suppose the problem selected for you is red (50\%).

- If you chose option A, you win if the implementer draws a red chip from Bag A (50\%).
- If you chose option B, you win if the implementer draws a red chip from Bag B.

Suppose the problem selected for you is blue (50\%).

- If you chose option A, you win if the implementer draws a blue chip from Bag A (50\%). If you chose option B, you win if the implementer draws a blue chip from Bag B.


## Instructions After

There are two bags. Bag A has 2 chips, one of them is blue and the other red. Bag B also has 2 chips. Each chip in Bag B is either blue or red. However, the number of blue (and red) chips is unknown - it could be 0 blue ( 2 red) chips, 1 blue chip and 1 red chip, or 2 blue ( 0 red) chips.


Bag B


Below is an example of a choice problem that you will face during the experiment.

## Red choice problem

Which one do you prefer?
Option A: You win $€ 10.00$ if the implementer draws a red chip from Bag A, and $€ 0$ otherwise.
Option B: You win $€ 10.20$ if the implementer draws a red chip from Bag B, and $€ 0$ otherwise.
In the example, you need to choose between Option A and Option B. The two options have the same winning color but differ in the amount you can win and the bag from which the chip is drawn.

During the experiment, you will also face a Blue choice problem. The only difference is that the winning color is blue instead of red.

You will receive a separate choice sheet.

## Payment:

At the end of the experiment, the implementer will draw a chip from Bag A and a chip from Bag B respectively without looking. He will announce the colors of both chips drawn and record them on a piece of paper.

You will be paid according to your choice in one of the two problems. To select the choice problem that will determine your payment, the implementer will throw a 6-sided die for you. 3 sides of the die are marked red, and the others are marked blue. You will be paid according to the choice problem with the matching color. Below we show how you will be paid.

Suppose the implementer draws a red chip from Bag A and a red chip from Bag B.

- You win if the problem selected for you is red (50\%).

Suppose the implementer draws a red chip from Bag A and a blue chip from Bag B.

- If the problem selected for you is red ( $50 \%$ ), you win if you chose option A.
- If the problem selected is blue (50\%), you win if you chose option B.

Suppose the implementer draws a blue chip from Bag A and a blue chip from Bag B.

- You win if the problem selected for you is blue (50\%).

Suppose the implementer draws a blue chip from Bag A and a red chip from Bag B.

- If the problem selected for you is red (50\%), you win if you chose option B.
- If the problem selected is blue (50\%), you win if you chose option A.


## Instructions Before-6

In this session you will be asked to make 6 choices between bets. There are no correct choices. Your choices depend on your preferences and beliefs, so different participants will usually make different choices. You will be paid according to your choices, so read these instructions carefully and think before you decide.

In all the choice problems you will face during this experiment you will be asked to choose between two uncertain options. All choice problems will be organized in groups of three problems that share a simple structure, which is explained below.

Consider a choice between being paid:
(f) $€ 4.50$ for sure or (b) $€ 4.60$ for sure

Obviously, being paid $€ 4.60$ is better than being paid $€ 4.50$.
Similarly, consider a bet in which you can win some money with a chance of $50 \%$, and you are asked to choose between:
(a) $€ 10$ if you win or
(e) $€ 10.20$ if you win

Obviously, being paid $€ 10.20$ if you win is better than being paid $€ 10$ if you win.
Now, the following three choice problems ask you to choose between the bets and the sure payments above.

## Choice 1 (circle a or b) Choice 2 (circle $\mathbf{c}$ or d) Choice 3 (circle e or f)

a) $50 \%$ chance of $€ 10$.
b) $€ 4.60$ for sure.
c) $50 \%$ chance of $€ 10$.
d) $€ 4.50$ for sure.
e) $50 \%$ chance of $€ 10.20$.
f) $€ 4.50$ for sure.

Start with Choice 2: if you choose (c) in Choice 2, it makes sense to choose (e) in Choice 3 since the alternative ( $€ 4.50$ for sure) is the same while (e) is better than (c). Considering Choice 1 , you should consider whether (a) is better than $€ 4.60$ for sure (rather than $€ 4.50$ for sure as in (d)).
If you chose (d) in Choice 2, it makes sense to choose (b) in Choice 1 since the alternative ( $50 \%$ of winning $€ 10$ ) is the same while (b) is better than (d). Considering Choice 3, you should consider whether ( f ) is better than a $50 \%$ chance of winning $€ 10.20$ (rather than $€ 10$ as in (c)).
Therefore, choosing one or more of the combinations: (a) and ( f , (a) and (d), or (c) and (f) is not consistent with the reasoning above. If you find yourself choosing in such a way, please review the rationale presented above in order to better guide your choices.

## The experiment:

There are two bags. Bag A has 2 chips, one of them is blue and the other red. Bag B also has 2 chips. Each chip in Bag B is either blue or red. However, the number of blue (and red) chips is unknown - it could be 0 blue ( 2 red) chips, 1 blue chip and 1 red chip, or 2 blue ( 0 red) chips.


Below is an example of choice problem that you may face during the experiment.

## An example of a Red choice problem

Which one do you prefer?
Option A: You win $€ 10.00$ if the implementer draws a red chip from Bag A, and $€ 0$ otherwise.
Option B: You win $€ 10.20$ if the implementer draws a red chip from Bag B, and $€ 0$ otherwise.
In this example, you need to choose between Option A and Option B. The two options have the same winning color but differ in the amount you can win and the bag from which the chip is drawn.

During the experiment, you will also face Blue choice problems, in which the only difference is that the winning color is blue instead of red.

You will receive a separate choice sheet. On it, there are in total 6 problems: three red problems, numbered 1,2 , and 3 ; and three blue problems, numbered 4,5 , and 6 .

## Payment:

You will be paid according to your choice in one of the 6 problems. To select the choice problem that will determine your payment, the implementer will toss a 6 -sided die for all participants and put the choice problems with matching numbers into sealed envelopes. You will draw one envelope and write your subject ID on it. Please do not open the envelope until you are told to do so. Remember that the choice problem that matters for your final payment is in your envelope, and it is chosen before you make any choices.
At the end of the experiment, the experimenters will ask you to open your envelope. Then, the implementer will draw a chip from Bag A and a chip from Bag B respectively without looking. He will announce the colors of both chips drawn and record them on a piece of paper.
You will be paid according to your choice in the problem in your envelope. Below we give examples of how you will be paid.
Suppose the problem selected for you is red ( $50 \%$ ).

- If you chose option A, you win if the implementer draws a red chip from Bag A (50\%).
- If you chose option B, you win if the implementer draws a red chip from Bag B.

Suppose the problem selected for you is blue ( $50 \%$ ).

- If you chose option A, you win if the implementer draws a blue chip from Bag A (50\%).
- If you chose option B, you win if the implementer draws a blue chip from Bag B.


## Instructions After-6

In this session you will be asked to make 6 choices between bets. There are no correct choices. Your choices depend on your preferences and beliefs, so different participants will usually make different choices. You will be paid according to your choices, so read these instructions carefully and think before you decide.

In all the choice problems you will face during this experiment you will be asked to choose between two uncertain options. All choice problems will be organized in groups of three problems that share a simple structure, which is explained below.

Consider a choice between being paid:
(f) $€ 4.50$ for sure or (b) $€ 4.60$ for sure

Obviously, being paid $€ 4.60$ is better than being paid $€ 4.50$.
Similarly, consider a bet in which you can win some money with a chance of $50 \%$, and you are asked to choose between:
(a) $€ 10$ if you win or
(e) $€ 10.20$ if you win

Obviously, being paid $€ 10.20$ if you win is better than being paid $€ 10$ if you win.
Now, the following three choice problems ask you to choose between the bets and the sure payments above.

## Choice 1 (circle a or b) Choice 2 (circle $\mathbf{c}$ or d) Choice 3 (circle e or f)

a) $50 \%$ chance of $€ 10$.
b) $€ 4.60$ for sure.
c) $50 \%$ chance of $€ 10$.
d) $€ 4.50$ for sure.
e) $50 \%$ chance of $€ 10.20$.
f) $€ 4.50$ for sure.

Start with Choice 2: if you choose (c) in Choice 2, it makes sense to choose (e) in Choice 3 since the alternative ( $€ 4.50$ for sure) is the same while (e) is better than (c). Considering Choice 1 , you should consider whether $(\mathrm{a})$ is better than $€ 4.60$ for sure (rather than $€ 4.50$ for sure as in (d)).

If you chose (d) in Choice 2, it makes sense to choose (b) in Choice 1 since the alternative (50\% of winning $€ 10$ ) is the same while (b) is better than (d). Considering Choice 3, you should consider whether (f) is better than a $50 \%$ chance of winning $€ 10.20$ (rather than $€ 10$ as in (c)).

Therefore, choosing one or more of the combinations: (a) and (f), (a) and (d), or (c) and (f) is not consistent with the reasoning above. If you find yourself choosing in such a way, please review the rationale presented above in order to better guide your choices.

## The experiment:

There are two bags. Bag A has 2 chips, one of them is blue and the other red. Bag B also has 2 chips. Each chip in Bag B is either blue or red. However, the number of blue (and red) chips is unknown - it could be 0 blue ( 2 red) chips, 1 blue chip and 1 red chip, or 2 blue ( 0 red) chips.


Below is an example of choice problem that you may face during the experiment.

## An example of a Red choice problem

Which one do you prefer?
Option A: You win $€ 10.00$ if the implementer draws a red chip from Bag A, and $€ 0$ otherwise.
Option B: You win $€ 10.20$ if the implementer draws a red chip from Bag B, and $€ 0$ otherwise.
In the example, you need to choose between Option A and Option B. The two options have the same winning color but differ in the amount you can win and the bag from which the chip is drawn.

During the experiment, you will also face Blue choice problems, in which the only difference is that the winning color is blue instead of red.

You will receive a separate choice sheet. On it, there are in total 6 problems: three red problems, numbered 1,2 , and 3 ; and three blue problems, numbered 4,5 , and 6 .

## Payment:

At the end of the experiment, the implementer will draw a chip from Bag A and a chip from Bag B respectively without looking. He will announce the colors of both chips drawn and record them on a piece of paper.
You will be paid according to your choice in one of the 6 problems. To select the choice problem that will determine your payment, the implementer will toss a 6 -sided die for you. You will be paid according to the choice problem whose number matches the die throw. Below we give an example of how you will be paid.
Suppose the implementer draws a red chip from Bag A and a red chip from Bag B.

- You win if the problem selected for you is red ( $50 \%$ ).

Suppose the implementer draws a red chip from Bag A and a blue chip from Bag B.

- If the problem selected for you is red ( $50 \%$ ), you win if you chose option A.
- If the problem selected is blue ( $50 \%$ ), you win if you chose option B.

Suppose the implementer draws a blue chip from Bag A and a blue chip from Bag B.

- You win if the problem selected for you is blue (50\%).

Suppose the implementer draws a blue chip from Bag A and a red chip from Bag B.

- If the problem selected for you is red ( $50 \%$ ), you win if you chose option B.
- If the problem selected is blue ( $50 \%$ ), you win if you chose option A.


## Instructions Single

There are two bags. Bag A has 2 chips, one of them is blue and the other red. Bag B also has 2 chips. Each chip in Bag B is either blue or red. However, the number of blue (and red) chips is unknown - it could be 0 blue ( 2 red) chips, 1 blue chip and 1 red chip, or 2 blue ( 0 red) chips.


Bag B


Below is an example of a choice problem that you will face during the experiment.

## Red choice problem

Which one do you prefer?
Option A: You win $€ 10.00$ if the implementer draws a red chip from Bag A, and $€ 0$ otherwise.
Option B: You win $€ 10.20$ if the implementer draws a red chip from Bag B, and $€ 0$ otherwise.
In the example, you need to choose between Option A and Option B. The two options have the same winning color but differ in the amount you can win and the bag from which the chip is drawn. A Blue choice problem is similar. The only difference is that the winning color is blue instead of red. You can select the color of your choice problem. The color that you select will be the color that you bet on.

You will receive a separate choice sheet, and an empty envelope with "choice" written on it.

- Firstly, select the color of your choice problem.
- Secondly, choose your preferred option in the choice problem that you selected.

After you finish making your choices, please put your choice sheet into the choice envelope and seal it. Your choice envelope must then remain sealed until the end of the experiment.

## Payment:

After all participants have finished making their choices, the experimenter will first put two chips into bag B. The color composition of bag B remains unknown to all participants.
The experimenter will invite one volunteer participant to draw a chip from Bag A and a chip from Bag B respectively without looking. The volunteer will announce the colors of both chips drawn and the experimenter will record the colors on a piece of paper.
You will be paid according to your choice in the choice problem that you selected. Below we give examples of how you will be paid.
Suppose you select the red problem.

- If you chose option A, you win if the chip drawn from from Bag A is red (50\%).
- If you chose option B, you win if the chip drawn from from Bag B is red.

Suppose you select the blue problem.

- If you chose option A, you win if the chip drawn from from Bag A is blue (50\%).
- If you chose option B, you win if the chip drawn from from Bag B is blue.


## Instructions Single-BL

There are two bags. Bag A has 2 chips, one of them is blue and the other red. Bag B also has 2 chips. Each chip in Bag B is either blue or red. However, the number of blue (and red) chips is unknown - it could be 0 blue ( 2 red) chips, 1 blue chip and 1 red chip, or 2 blue ( 0 red) chips.


Before you start making your choices, the experimenter will put two chips into bag B. The color composition of bag B is unknown to all participants.

Below is an example of a choice problem that you will face during the experiment.

## Red choice problem

Which one do you prefer?
Option A: You win $€ 10.00$ if the implementer draws a red chip from Bag $\mathbf{A}$, and $€ 0$ otherwise.
Option B: You win $€ 10.20$ if the implementer draws a red chip from Bag B, and $€ 0$ otherwise.
In the example, you need to choose between Option A and Option B. The two options have the same winning color but differ in the amount you can win and the bag from which the chip is drawn. A Blue choice problem is similar. The only difference is that the winning color is blue instead of red. You can select the color of your choice problem. The color that you select will be the color that you bet on.

You will receive a separate choice sheet, and an empty envelope with "choice" written on it.

- Firstly, select the color of your choice problem.
- Secondly, choose your preferred option in the choice problem that you selected.

After you finish making your choices, please put your choice sheet into the choice envelope and seal it. Your choice envelope must then remain sealed until the end of the experiment.

## Payment:

After all participants have finished making their choices, the experimenter will invite one volunteer participant to draw a chip from Bag A and a chip from Bag B respectively without looking. The volunteer will announce the colors of both chips drawn and the experimenter will record the colors on a piece of paper.
You will be paid according to your choice in the choice problem that you selected. Below we give examples of how you will be paid.

Suppose you select the red problem.

- If you chose option A, you win if the implementer draws a red chip from Bag A (50\%).
- If you chose option B, you win if the implementer draws a red chip from Bag B.

Suppose you select the blue problem.

- If you chose option A, you win if the implementer draws a blue chip from Bag A (50\%).
- If you chose option B, you win if the implementer draws a blue chip from Bag B.


## Instructions Before B

There are two bags. Bag A has 2 chips, one of them is blue and the other red. Bag B also has 2 chips. Each chip in Bag B is either blue or red. However, the number of blue (and red) chips is unknown - it could be 0 blue ( 2 red) chips, 1 blue chip and 1 red chip, or 2 blue ( 0 red) chips.


Bag B


Before you start making your choices, the experimenter will put two chips into bag B. The color composition of bag B is unknown to all participants.

Below is an example of a choice problem that you will face during the experiment.

## Red choice problem

Which one do you prefer?
Option A: You win $€ 10.00$ if the implementer draws a red chip from Bag $\mathbf{A}$, and $€ 0$ otherwise.
Option B: You win $€ 10.20$ if the implementer draws a red chip from Bag B, and $€ 0$ otherwise.
In the example, you need to choose between Option A and Option B. The two options have the same winning color but differ in the amount you can win and the bag from which the chip is drawn.

During the experiment, you will also face a Blue choice problem. The only difference is that the winning color is blue instead of red.

You will receive a separate choice sheet, and an empty envelope with "choice" written on it.
After you finish making your choices, please put your choice sheet into the choice envelope and seal it. Your choice envelope must then remain sealed until the end of the experiment.

## Payment:

You will be paid according to your choice in one of the two problems. To select the choice problem that will determine your payment, you have randomly drawn one envelope from 10 ticket envelopes. Inside of the ticket envelopes, there is a ticket marked either red (5 out of 10) or blue ( 5 out of 10). Please do not open the ticket envelope until you are told to do so. Remember that the choice problem that matters for your final payment is in your envelope, and it is chosen before you make any choices.

After all participants have finished making their choices, the experimenter will first ask you to open your ticket envelope. The experimenter will invite one volunteer participant to draw a chip from Bag A and a chip from Bag B respectively without looking. The volunteer will announce the colors of both chips drawn and the experimenter will record the colors on a piece of paper.
You will be paid according to your choice in the problem of the same color as the ticket in your ticket envelope. Below we show how you will be paid.

Suppose the problem selected for you is red (50\%).

- If you chose option A, you win if the chip drawn from from Bag A is red (50\%).
- If you chose option B, you win if the chip drawn from from Bag B is red.

Suppose the problem selected for you is blue (50\%).

- If you chose option A, you win if the chip drawn from from Bag A is blue (50\%).
- If you chose option B, you win if the chip drawn from from Bag B is blue.


## Instructions Before-BL

There are two bags. Bag A has 2 chips, one of them is blue and the other red. Bag B also has 2 chips. Each chip in Bag B is either blue or red. However, the number of blue (and red) chips is unknown - it could be 0 blue ( 2 red) chips, 1 blue chip and 1 red chip, or 2 blue ( 0 red) chips.


Bag B


Below is an example of a choice problem that you will face during the experiment.

## Red choice problem

Which one do you prefer?
Option A: You win $€ 10.00$ if the implementer draws a red chip from Bag A, and $€ 0$ otherwise.
Option B: You win $€ 10.20$ if the implementer draws a red chip from Bag B, and $€ 0$ otherwise.
In the example, you need to choose between Option A and Option B. The two options have the same winning color but differ in the amount you can win and the bag from which the chip is drawn.

During the experiment, you will also face a Blue choice problem. The only difference is that the winning color is blue instead of red.

You will receive a separate choice sheet, and an empty envelope with "choice" written on it.
After you finish making your choices, please put your choice sheet into the choice envelope and seal it. Your choice envelope must then remain sealed until the end of the experiment.

## Payment:

You will be paid according to your choice in one of the two problems. To select the choice problem that will determine your payment, you have randomly drawn one envelope from 10 ticket envelopes. Inside of the ticket envelopes, there is a ticket marked either red (5 out of 10) or blue ( 5 out of 10). Please do not open the ticket envelope until you are told to do so. Remember that the choice problem that matters for your final payment is in your envelope, and it is chosen before you make any choices.

After all participants have finished making their choices, the experimenter will first ask you to open your ticket envelope. Then, the experimenter will put two chips into bag B. The color composition of bag B remains unknown to all participants.

The experimenter will invite one volunteer participant to draw a chip from Bag A and a chip from Bag B respectively without looking. The volunteer will announce the colors of both chips drawn and the experimenter will record the colors on a piece of paper.

You will be paid according to your choice in the problem of the same color as the ticket in your ticket envelope. Below we show how you will be paid.
Suppose the problem selected for you is red (50\%).

- If you chose option A, you win if the chip drawn from from Bag A is red (50\%).
- If you chose option B, you win if the chip drawn from from Bag B is red.

Suppose the problem selected for you is blue (50\%).

- If you chose option A, you win if the chip drawn from from Bag A is blue (50\%).
- If you chose option B, you win if the chip drawn from from Bag B is blue.


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[^1]:    ${ }^{1}$ In the experiment, they were referred to as bag A and B respectively.
    ${ }^{2}$ Subjects were not told who had prepared bag U.

[^2]:    ${ }^{3}$ In the literature, "integration" has been often used to refer to integration of outcomes in experiments with decision maker's wealth. We use "integration" to refer to integration of decision problems within the experiment.

[^3]:    ${ }^{4}$ Note that we concentrate on subjects whose choices revealed strict ambiguity aversion by choosing the Known bag in the Single control. Choosing to bet on the Unknown bag in the Single control is uninformative about the subject's ambiguity attitude: it is consistent with ambiguity neutrality and believing that the probability of drawing a red chip equals the probability of drawing a blue chip; or it could result from believing that the chip drawn is more likely to be of one color than another, while still being ambiguity neutral; or it is consistent with being ambiguity averse but preferring to bet on one color than another in the Unknown bag; or it could result from being ambiguity averse but having an ambiguity premium lower than 20 cents; or it could result from being ambiguity seeking. In other words, while choosing $K$ identifies strict ambiguity aversion, choosing U does not reveal the subject's ambiguity attitude.

[^4]:    ${ }^{5} 57$ out of the 174 subjects ( $32.75 \%$ ) in Before and After chose KU or UK. These choice patterns are consistent with various ambiguity attitudes (and with choosing U in the Single control, as noted in footnote 4), mostly suggesting that many subjects believed that one color was more likely to be drawn from the Unknown bag than another color. $70 \%$ of them chose KU , which is consistent with a belief that drawing a blue chip from the Unknown bag is more likely than drawing a red chip. Supportive evidence for asymmetric beliefs can also be found by examining the color choices in the Single control: among the 42 subjects who chose U, $32(76 \%)$ had selected the blue choice problem and only $10(24 \%)$ the red choice problem. Oechssler et al. (2019) also reported a large proportion of participants exhibiting such asymmetric beliefs or color preference.

[^5]:    ${ }^{6}$ Note that $W A A^{r}$ is also consistent with ambiguity neutrality. In the After- 6 treatment, $W A A^{r}$ FOSD $A A^{r}$ for integrating subjects. If the subject satisfies reversal-of-order then the same holds for Before-6.

[^6]:    ${ }^{7}$ In the follow-up study, we did not roll a die for the RIS like in the main study, but asked participants to pick one of eight envelopes, four containing to the Blue choice problem and the other four containing the red choice problem. See Appendix B for the other deviations from the main study due to health-safety measures.
    ${ }^{8}$ In table 3.1, isolation provides the same prediction as SKZ without mental randomization in the follow-up study. Nevertheless, when comparing Single and Before in the main study, they give different predictions. The $S A A^{r}$ subjects in Before are only compatible with isolation.

[^7]:    ${ }^{9}$ If we expect about $55 \%$ of ambiguity averse participants to isolate (based on the main Study, comparing Before to Single), then about 15 to $25 \%$ of the ambiguity averse participants seem to follow SKZ's interpretation.

[^8]:    ${ }^{10}$ The estimation was done using the function for contingency tables in the BayesFactor package in R. The posteriors were obtained from Markov chain Monte Carlo simulations.

[^9]:    ${ }^{11}$ For instance, $(x, y) \sim^{*}(y, x) \sim^{*}(y, x, x, y)$.
    ${ }^{12}$ Note that we define here Statewise Monotonicity for compound objects of the type ( $\hat{x}_{1}, \ldots, \hat{x}_{n}$ ) but it could also be defined for Anscombe-Aumann acts. The difference between Statewise Monotonicity and FOSD for Anscombe-Aumann acts is the same as the difference between Machina and Schmeidler's (1995) Axiom 4 (Substitution) and Axiom 5 (FOSD of Preferences). The former axiom states that the decision maker always prefers replacing one lottery of an act by a preferred lottery, while the latter requires that the decision maker always prefers replacing one lottery of an act by an objectively-better lottery.

[^10]:    ${ }^{13}$ Saito (2015) proposes a model that is a linear combination of ex-ante and ex-post MEU.

[^11]:    ${ }^{14}$ Their second experiment (called "alternative specification") did not have additional tasks so did not require the usage of RIS, but did not provide a benchmark for ambiguity aversion, making it impossible to identify non-hedging ambiguity averse subjects from ambiguity neutral ones.

[^12]:    ${ }^{15}$ Note that $A A^{r}$ may differ from $S A A^{r}$ (as displayed in Figure 2.3). For comparability with the Single, After, and Before treatments, we only considered problems Red 1 and Blue 4 to determine $S A A^{r}$, therefore including all patterns of the form K-- K --. The difference is negligible though. There were $25 S A A^{r}$ and also $25 A A^{r}$ subjects in the After- 6 treatment and there were $25 S A A^{r}$ and $24 A A^{r}$ subjects in the Before-6 treatment.

