Privacy, Transparency, and Policing

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Abstract

Citizens face heterogeneous returns to crime, or their types, and decide whether to commit a crime. Police acquire information about citizens' types and search them to deter and uncover crimes. If the police can covertly acquire information, the police fail to deter any crime in equilibrium because their ability to identify criminals erodes the deterrent effect of policing. If information acquisition is publicly observable, the police will acquire partial information that prevents the police from identifying those with high returns to crime. The result underscores the importance of transparency in data collection by law enforcement agencies.

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1 Introduction

Law enforcement agencies are increasingly relying on data and algorithms to predict and prevent crimes (Perry, 2013). They use a variety of data sources, such as criminal records, social media posts, financial records, and local environmental information. Private vendors, such as Palantir and PredPol, also offer predictive algorithms to police departments. This trend has raised a number of concerns, which include concerns about privacy and transparency: The use of data and algorithms by law enforcement can lead to data collection and prediction at individual-level, and it is often unclear what kinds of data, algorithms, and technologies are being used by law enforcement agencies (Hvistendahl, 2016; Robertson, Khoo, and Song, 2020).

This paper investigates the relationship between privacy, transparency, and the effectiveness of law enforcement. To do so, I study a model of endogenous information acquisition by police. In the model, there are police and a unit mass of citizens. At the outset, the police collect information about citizens' private types that capture their heterogeneous returns to crime. Formally, I model information acquisition as the choice of a signal, which reveals for each citizen a signal realization whose distribution depends on the citizen's type. The police can acquire any information at zero cost: For example, the police may collect full information or whether the type of each citizen exceeds some threshold. Given the information collected, the police allocate search resources across citizens, and simultaneously, each citizen decides whether to commit a crime. I assume that the police care about deterring and uncovering crimes.

To examine the impact of transparency in data collection, I study two versions of the game. One is the private regime, in which information acquired by the police is unobservable to citizens. In this regime, the police can deviate and collect any information without being observed by citizens. In equilibrium, each citizen anticipates only the distribution of signal realizations given his type. The other is the public regime, in which information acquired by the police is publicly observable. In this regime, every citizen knows what information (i.e., a posterior belief) the police have about him, both on and off the equilibrium paths.

First, I show that in any equilibrium of the private regime, the police fail to deter any crimes. Intuitively, when the police have limited search resources and can covertly acquire information, they will identify and search citizens who are most likely to commit crimes. Such a policing strategy turns out to maximize crime rate: Each citizen is not searched at all and thus commits crime, or the citizen is searched but still commits a crime because the returns to crime exceed the cost of being caught. As a result, the lack of transparency in data collection erodes the deterrence role of policing.

In contrast, in the public regime, the police commit to collecting partial information that minimizes a crime rate across all signals. This signal, which I call the crime-minimizing signal, has the following "truth or noise" structure: If a citizen's type (i.e., his returns to crime) is below some cutoff, the realized signal coincides with his true type; otherwise, the realized signal is a noise. For example, suppose that types are uniformly distributed on the unit interval $[0, 1]$, and consider the crime-minimizing signal with cutoff 0.5. For type $x \leq 0.5$, the signal draws realization $s = x$ with probability 1. For any type $x > 0.5$, the signal draws a realization uniformly randomly from $[0, 0.5]$. If the police observe realization s for a given citizen, they believe that the citizen's type is either equal to s or uniformly distributed on $[0.5, 1]$. However, the police cannot distinguish the two events.

The crime-minimizing signal captures the dual role of information for policing: The signal prevents the police from identifying individuals who face high returns to crime, so that the police will not target search resources only to them. At the same time, the realized signals differ in the lowest possible types, which enables the police to tailor search probabilities to those who face low returns to crime. In the above example, the signal keeps the police uninformed about whether types are above or below cutoff 0.5, i.e., the two cases are equally likely conditional on any realized signal. However, the signal enables the police to tailor search probabilities to types below 0.5.

The equilibrium in the public regime leads to the following comparative statics: If the environment is prone to crime—e.g., if the police have a lower search capacity or citizens face stochastically higher returns to crime—then the police collect less information. This result counters the intuition that data-driven policing is especially valuable in an environment that is prone to crime. Instead, in such a situation the police may need to restrict the use of information.

The equilibrium analysis of the private and public regimes lead to two main insights. First, the lack of transparency in data collection can reduce the effectiveness of law enforcement. This result holds even if the private regime does not involve excessive information collection. In fact, the lack of transparency may result in a high crime rate in equilibrium without affecting the police's behavior: There are equilibria in both regimes where the police behave the same way, but the crime rate is maximized in the private regime and minimized in the public regime.

Second, the equilibrium signal in the public regime—or the crime-minimizing signal—is an antithesis of predictive policing algorithms described in the public debate. This argument rests on two informational properties. First, the crime-minimizing signal prevents the police from identifying individuals whose returns to crime are above a cutoff. In contrast, predictive policing is typically described as a way to identify individuals or places that are prone to crime (Perry, 2013; Lau, 2020). Second, under the crime-minimizing signal, the realized signals are fully informative about the types of "innocents" but uninformative about "criminals." This property appears to be the opposite of information contained in past crime data, which are the primary inputs for predictive policing algorithms.

Finally, I extend the baseline model in two ways. First, I allow the police to increase search capacity at cost. In this case, citizens are on average exposed to more policing but also more likely to commit crimes in the private regime than the public regime. Second, I show that the crime-minimizing signal renders policing "unfair," i.e., if citizens in one group tend to face lower incomes (i.e., higher net returns to crime) than citizens in the other group, the lower-income group is exposed to higher average search intensities. The result implies that the tension between fairness and effectiveness of policing generally exists once we focus on the information structure that is most effective at deterring crimes.

Before proceeding, I clarify what this paper is not about: Even without the economic forces described in this paper, the use of data and algorithms may result in undesirable outcomes when the quality of data is bad, e.g., data suffer from econometric endogeneity problem (Cowgill and Tucker, 2020; Ludwig and Mullainathan, 2021). In this paper, I abstract away from this concern and focus on problems caused by the police's strategic incentives. In particular, the model abstracts away from an often criticized feature of data-driven policing—that if police use data that are generated by past policing decisions, they may exacerbate existing bias in policing (Ensign et al., 2018).

Related work. First, the paper relates to the economic literature on crime and policing, which starts from Becker (1968). My paper builds closely on Persico (2002), who studies whether requiring the police to behave more fairly reduces crime. Although Persico (2002) considers the impact of restricting the police's behavior to be fair, we can also view his paper as examining the impact of providing the police with a signal that has two signal realizations (i.e., each citizen's race). I highlight the transparency of data collection as a key variable that affects the impact of the police's ability to acquire information. The question of what information about citizens should (not) be used for policing is often discussed in the context of racial profiling (Knowles, Persico, and Todd, 2001; Persico and Todd, 2005; Bjerk, 2007; Persico, 2009). Law enforcement agencies will likely be collecting data that go beyond citizens' coarse characteristics, which renders it important to consider rich information structures. To focus on the role of information in policing, the model abstracts away from other important considerations, such as the design of judicial systems and richer responses by potential criminals and victims (e.g., Curry and Klumpp 2009; Cotton and Li 2015; Vasquez 2022).

Second, the paper relates to the literature on Bayesian persuasion and information design (see Kamenica (2019) and Bergemann and Morris (2019) for surveys). Papers such as Lazear (2006), Rabinovich, Jiang, Jain, and Xu (2015), and Hernández and Neeman (2022) use Bayesian persuasion and related tools to study what information to provide to agents who may take socially undesirable actions. In my paper the police disclose information to themselves, i.e, they choose what information to acquire about agents. The information design literature provides conditions under which an optimal signal takes a tractable form, such as monotone partitional signals, censorship policies, and nested intervals (e.g., Guo and Shmaya 2019; Dworczak and Martini 2019; Kolotilin, Mylovanov, and Zapechelnyuk 2022). The equilibrium signal in the public regime does not belong to these classes of signals, and to the best of my knowledge, the standard techniques to solve Bayesian persuasion do not apply to my model. The paper also relates to the computer science and economic literature on algorithmic fairness. Jung, Kannan, Lee, Pai, Roth, and Vohra (2020) relate algorithmic fairness to policing, and Liang, Lu, and Mu (2022) study the general trade-off between fairness and accuracy for the design of classification algorithms.

Finally, the paper contributes to the literature on the economic of privacy. The primary focus of the theoretical literature on privacy, reviewed by Acquisti, Brandimarte, and Loewenstein (2015), has been the ability of sellers and platforms to use consumer information for more profitable price discrimination, product matching, and service improvement. My paper contributes to the growing study of privacy in non-market contexts, such as mass surveillance (Tirole, 2021)

2 Model

This section sets up the model of policing with endogenous information acquisition. Citizens face heterogeneous returns to crime, which are their private types. The game has two stages. In the data collection stage, the police collect information about citizens' types. In the policing stage, the police allocate a finite search capacity, and simultaneously, citizens decide whether to commit a crime. We compare two versions of the game that differ in whether citizens observe what the police learn about their types.

2.1 Setup

There are police and a unit mass of citizens. Each citizen is privately informed of his returns to crime, or *type*, $x \in [0, 1]$. Types are distributed across the population according to a distribution function F_0, which has a positive density f_0 and is known to the police.

The game consists of the *data collection stage* and the *policing stage*. In the data collection stage, the police acquire information about citizens' types, which we model as the choice of a *signal*. A signal (S, π) is a pair of the set S of signal realizations and the collection $\pi = \{\pi(\cdot|x)\}_{x\in[0,1]}$ of conditional probability distributions on S for each possible type. For simplicity we write a signal as π. If the police choose signal π, then for each citizen who has type x, the police observe a signal realization $s \in S$ drawn according to distribution $\pi(\cdot|x) \in \Delta(S)$. Conditional on types, signal realizations are independent across citizens.

The police can choose any signal at no cost. For example, the police can choose the *fully informative signal*, which assigns signal realization $s = x$ to a citizen if only if his type is x. The police will then perfectly learn the type of every citizen from his signal realization. Alternatively, the police may acquire a signal that draws signal realization s_0 or s_1 if $x \leq x^*$ or $x > x^*$, respectively. The police will then learn whether the type of each citizen exceeds threshold x^*. The police could also choose to acquire no information by selecting an *uninformative signal* such that $\pi(\cdot|x)$ is independent of x.

At the end of the data collection stage, the types and signal realizations are jointly distributed across the population. Specifically, the mass of citizens whose types are in $X \subset [0, 1]$ and signal realizations are in S' is deterministic and equal to $\int_{X'} \pi(S'|x)F_0(dx)$.

We study two versions of the game that differ in what citizens observe in the data collection stage: In the *public regime*, every citizen observes the signal chosen by the police and the signal realization assigned to him. In the *private regime*, the citizens observe neither the signal nor signal realizations. The two regimes differ in whether the citizens observe what the police learn about citizens. We compare the equilibria of the two regimes to study the impact of transparency regarding data collection.

Both in the private and the public regimes, the data collection stage is followed by the policing stage. In the policing stage, citizens decide whether to commit a crime, and simultaneously, the police allocate search resources. Specifically, the police choose a *search strategy* $\sigma : S \to [0, 1]$, where the police search citizens with signal realization $s \in S$ with probability $\sigma(s)$. The police has a measure $\overline{P} \in (0, 1)$ of searches to allocate; thus the police can choose a search strategy σ if and only if

$$\int_0^1 \int_S \sigma(s)\pi(ds|x)F_0(dx) \leq \overline{P}. \tag{1}$$

Without observing the police's search strategy, each citizen decides whether to commit a crime.

A citizen's payoff of committing a crime is $x - \sigma L$, where x is the citizen's type, $\sigma \in [0, 1]$ is the probability with which the police search the citizen, and $L > 1$ is an exogenous cost of being caught. We normalize a citizen's payoff of not committing a crime to zero by interpreting type x as including the opportunity cost of crime such as legal earning opportunities. Because $x \in [0,1]$ and $L > 1$, any citizen will commit a crime for a sufficiently low search probability σ and will not commit a crime for a sufficiently high σ.

To define the police's payoffs, we introduce several notations. First, abusing notation, let $\pi(\cdot|s) \in \Delta([0,1])$ denote the posterior distribution of types conditional on signal realization s drawn according to signal π given prior F_0. Take any strategy profile, and let $c(x,s)$ denote the probability with which a citizen with type x and signal realization s commits a crime. The mass of citizens who commit crimes, which we call the *crime rate*, is defined as

$$r \triangleq \int_0^1 \left(\int_S c(x,s)\pi(ds|x)F_0(dx) \right).$$

We later use the *crime rate for signal realization* s, which is

$$r(s) \triangleq \int_0^1 c(x,s)\pi(dx|s).$$

The mass of citizens who commit a crime and are searched by the police, which we call the mass of *successful searches*, is defined as

$$a \triangleq \int_0^1 \int_S c(x,s)\sigma(s)\pi(ds|x)F_0(dx).$$

With these notations, the police's payoff is written as $\lambda r - a$ for some $\lambda \in (0,1)$. This specification implies that the police care about reducing and uncovering crimes. The assumption $\lambda < 1$ implies that the police prioritize reducing crimes, which excludes, for example, the preferences such that the police prefer a high crime rate because there are more crimes to uncover.

The timing of the game is summarized as follows. First, the police choose a signal, then nature draws the types of citizens and signal realizations. Citizens privately observe their types, and the police observe the signal realization of each citizen. In the public regime, each citizen also observes the chosen signal and his signal realization. Then the police choose a search strategy and citizens decide whether to commit a crime. Finally, the payoffs are realized. The solution concept is perfect Bayesian equilibrium, which we refer to as *equilibrium*. Finally, we focus on primitives that satisfy the following:

Assumption 1. The primitives satisfy $\overline{P} < \frac{\mathbb{E}_{F_0}[x]}{L}$.

The assumption is necessary and sufficient for the public and private regimes to have different equilibrium outcomes.[1] The assumption is more likely to hold if the search capacity $\overline{P}$ and the cost L of being caught is lower, and the average returns $\mathbb{E}_{F_0}[x]$ to crime is higher. Section 4.1 shows that the main insights continue to hold without Assumption 1 if the police can endogenously choose search capacity $\overline{P}$.

2.2 Interpretation as Location-Based Predictive Policing

According to Lau (2020), place-based predictive policing aims to identify places that have a high risk of crime. While I describe the model as a model of data collection on individuals, we can also interpret the model in the following way. There is a unit mass of locations. Each location $\ell \in [0,1]$ is associated with a location-specific return to crime, x_ℓ, which is distributed according to F_0. There are homogeneous citizens, and the crime opportunity at each location is discovered by exactly one citizen, who decides whether to commit a crime at that location. The police learn information about $(x_\ell)_{\ell \in [0,1]}$ and decides the probability to search each location. Given this interpretation, the model also captures place-based predictive policing.

3 Equilibrium

If the police do not have additional information beyond prior type distribution F_0, they will randomly search citizens with probability $\overline{P}$. A citizen will then commit a crime if his type x exceeds the expected cost of being caught, $\overline{P}L$. Thus the equilibrium crime rate is $1 - F_0(\overline{P}L)$. Compared to this no information benchmark, the police's data collection can increase or decrease crime depending on its transparency.

[1] If $\overline{P} \geq \frac{\mathbb{E}_{F_0}[x]}{L}$ then both regimes have an equilibrium in which the police search type x with probability $\frac{x}{L}$, and no citizen commits a crime.

3.1 The Private Regime

The following result shows that the police that can privately collect any information about citizens will fail to deter any crime.

Theorem 1. *In the private regime, in any equilibrium, almost all citizens commit a crime.*

Proof. Take any equilibrium. A citizen with type x is indifferent between committing a crime and not if he is searched with probability $\sigma(x) \triangleq \frac{x}{L}$. Thus to attain crime rate 0, the police need a search capacity of at least $\hat{P} \triangleq \frac{\mathbb{E}_{F_0}[x]}{L}$. However, Assumption 1 implies that the police's search capacity is less than $\hat{P}$. As a result, there is a positive mass of types, denoted by X^*, who commit a crime with probability 1. Also, let Y^* be the set of types who commit a crime with probability strictly less than 1. The police can profitably deviate by learning the type of every citizen, then shift search probabilities from types in Y^* to types in X^*. The deviation does not change the crime rate because the citizens do not observe it. At the same time, the deviation enables the police to catch more criminals and increases their payoffs. Therefore Y^* has measure zero, i.e., almost all citizens commit a crime. $\square$

Proposition 1 states that the police's ability to covertly gather information about citizens will eliminate the deterrent effect of policing: In order for policing to deter a citizen from committing a crime, the police must search the citizen with a positive probability even if the citizen does not commit a crime in equilibrium. However, the police have limited search resources and care about catching criminals. Moreover, in the private regime the police can secretly identify criminals without affecting the citizens' behavior. In this situation, instead of searching citizens who are unlikely to commit a crime, the police prefer to identify and search citizens who commit a crime. However, if citizens are not searched, they will also commit a crime. This leads to an equilibrium in which everyone commits a crime.

Proposition 1 states that the equilibrium crime rate is always 1. However, the equilibrium is not necessarily unique. In particular, the police may not acquire full information in equilibrium. To illustrate this point, I first present a necessary and sufficient condition for the police's choice to be part of an equilibrium. To state the result, let $\text{supp}\pi(\cdot|s) \subseteq [0,1]$ denote the support of posterior distribution $\pi(\cdot|s)$ conditional on signal realization s. Then

$\inf \operatorname{supp}\pi(\cdot|s) \in [0,1]$ denotes the infimum of the support, i.e., the lowest type that can have signal realization s.

Lemma 1. *A pair of signal π and search strategy σ is a part of some equilibrium if and only if the pair satisfies the police's resource constraint (1) with equality and $\sigma(s) \leq \frac{\inf \operatorname{supp}\pi(\cdot|s)}{L}$ for almost all $s \in S$.*

Proof. To show the "if" direction, take any pair (π, σ) that satisfies the two conditions. We have $\sigma(s) \leq \frac{\inf \operatorname{supp}\pi(\cdot|s)}{L}$. Thus for almost all $s \in S$, for any type $x \in \operatorname{supp}\pi(\cdot|s)$, we have $x - L\sigma(s) \geq 0$, i.e., it is optimal for almost all citizens to commit a crime. The police are then indifferent between any search strategies that satisfy the resource constraint (1) with equality. As a result, (π, σ) is part of an equilibrium. Conversely, take any (π, σ). If the police's resource constraint (1) holds with strict inequality in equilibrium, the police can profitably deviate by searching more citizens who commit a crime with probability 1. If we have $\sigma(s) > \frac{\inf \operatorname{supp}\pi(\cdot|s)}{L}$ for a positive mass of s, then a positive mass of citizens will prefer not to commit a crime, which contradicts Proposition 1. $\square$

Lemma 1 implies that any strategy of the police that exhausts the search capacity and leads all citizens to commit a crime can be a part of an equilibrium. This condition does not place many restrictions on the equilibrium signals: In some equilibria, the police may collect full information, while in others they may collect less information such as a signal with two signal realizations.

Corollary 1. *In the private regime, there is no equilibrium in which the police collect no information, and there is an equilibrium in which the police collect full information. If $\overline{P} < \frac{1}{L} \max_{x \in [0,1]} x[1 - F_0(x)]$, there is an equilibrium in which the police learn whether citizens' types exceed cutoff x^* that satisfies $\overline{P} = \frac{1}{L} x^*[1 - F_0(x^*)]$.*

Proof. To show the first part, suppose to the contrary that the police choose an uninformative signal. Then for any possible signal realization, we have $\frac{\inf \operatorname{supp}\pi(\cdot|s)}{L} = 0$. Lemma 1 then implies that $\sigma(s) = 0$ for all s, which contradicts the resource constraint holding with equality.

To show the second part, consider a fully informative signal that sends signal realization $s = x$ for each type $x \in [0,1]$. Let x_F satisfy $\overline{P} = \frac{\int_{x_F}^1 x F_0(dx)}{L}$, and consider search strategy such that $\sigma(x) = 1$ if $x \geq x_F$ and $\sigma(x) = 0$ if $x < x_F$. The police's strategy satisfies the condition in Lemma 1 and thus can be part of an equilibrium.

Finally, suppose that $\overline{P} < \frac{1}{L} \max_{x \in [0,1]} x[1 - F_0(x)]$. Define $R(x) = \frac{x}{L}[1 - F_0(x)]$. Because $R(0) = R(1) = 0 < \overline{P}$, there is some type x^* such that $\overline{P} = \frac{1}{L} x^*[1 - F_0(x^*)]$, i.e., the police's resource constraint holds with equality if they search every type $x \geq x^*$ with probability $\frac{x^*}{L}$. The police's search strategy and the signal that reveals whether citizens' types exceed x^* satisfy the condition in Lemma 1 and thus can be part of an equilibrium. □

Theorem 1 relies on the assumption that the police can potentially collect full information about citizens' types. Corollary 1 suggests, however, that the undesirable equilibrium outcome in the private regime does not necessarily manifest as intensive data collection on the equilibrium path. Indeed, we will later show that the private regime may not distort data collection at all, in that the private and public regimes have equilibria in which the police behave the same way on the equilibrium path.

3.2 The Public Regime

We have shown that the lack of transparency regarding data collection—or equivalently, the police's inability to commit to not collect certain information—leads to an extreme outcome where every citizen commits a crime. We now turn to the public regime, in which the police can publicly commit to an information policy upfront. We first define a class of signals that contains an equilibrium signal.

Definition 1. For any $z \in (0,1)$, define signal (S_z, π_z) as follows: The set of signal realizations is $S_z = [0, z]$. If a citizen's type is $x \leq z$, $\pi_z(\cdot|x)$ draws signal realization $s = x$ with probability 1. If $x > z$, $\pi_z(\cdot|x)$ draws signal realization $s \in [0, z]$ according to conditional distribution $F_0(\cdot|x \leq z)$.

Any signal π_z has a "truth or noise" structure: If the signal realization comes from types below z, it coincides with the true type. If the signal realization comes from types above

z, it is a noise. The police cannot differentiate between these two events, so signal π_z is neither fully informative nor uninformative. Signal π_z is similar to the "truth or noise" signals described in Lewis and Sappington (1994), but here the distinction between truth and noise depends on one's type. Signal π_z is also different from an upper-censorship signal, which pools the types above a cutoff and reveals the types below the cutoff (Kolotilin et al., 2022).

To gain some intuition, consider the distribution of types conditional on each signal realization drawn from signal π_z. Figure 1 depicts two such conditional distributions, G_s and $G_{s'}$, which correspond to signal realizations s and s'. The conditional distributions have two features: First, they contain the same mass $1 - F_0(z)$ of citizens whose types are distributed according to $F_0(\cdot|x > z)$. Second, the conditional distributions differ in the remaining mass $F_0(z)$ of citizens, who have the identical type, i.e., distribution G_s induced by signal realization s contains point mass $F_0(z)$ of type s. As a result, each conditional distribution has a non-convex support and has neither a density nor a probability mass function. Distributions $\{G_s\}_{s\in[0,z]}$ themselves are distributed according to $F_0(\cdot|x \leq z)$, which ensures that they average to the prior type distribution, F_0.

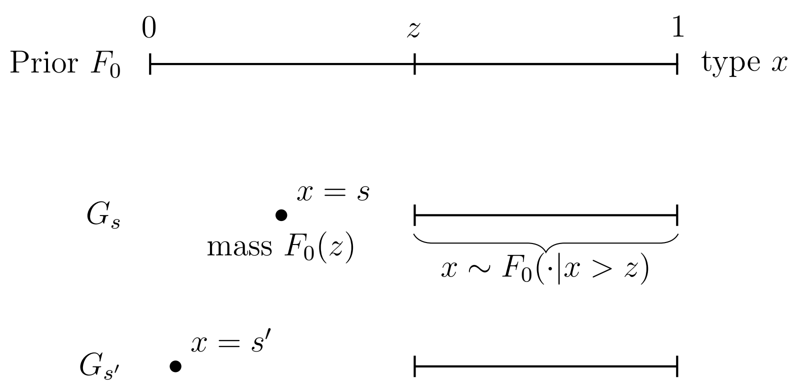

Figure 1: Distributions of types conditional on signal realizations s and s' under signal π_z.

The following result states that in the public regime, the police choose signal π_z with a suitably chosen cutoff z. This signal minimizes a crime rate across all signals, and moreover, the equilibrium outcome is unique (see Appendix A for the proof).

Theorem 2. *The public regime has an equilibrium in which the police choose signal* π_{z^*}, *where* $z^* \in (0,1)$ *solves*

$$\overline{P} = \frac{\mathbb{E}_{F_0}[x|x \leq z^*]}{L}. \tag{2}$$

In the equilibrium, a citizen commits a crime if and only if $x > z^*$*, and the crime rate is* $1 - F_0(z^*)$*. No signal attains a strictly lower crime rate than* $1 - F_0(z^*)$*. Moreover, the equilibrium outcome is unique: Any two equilibria have the same set of types committing a crime and the same search strategy, except with respect to a measure zero set of types.*

I describe how the equilibrium in the theorem looks like, then provide intuitions for why signal π_{z^*} minimizes a crime rate and why police choose such a signal in equilibrium. In an equilibrium of the public regime, the police choose signal π_{z^*}, which has the space of realizations $[0, z^*]$. Consider the set of citizens with signal realization $x \in [0, z^*]$. Recall that in this set, mass $F_0(z^*)$ of citizens have type x, and mass $1 - F_0(z^*)$ of citizens have types above x (see Figure 1). The police search any citizen with signal realization x with probability $\sigma^*(x) \triangleq \frac{x}{L}$, which makes it optimal for type x to not commit a crime and types above x to commit a crime. As a result, the crime rate for every signal realization is $1 - F_0(z^*)$. The police's search strategy $\{\sigma^*(x)\}_{x \in [0,1]}$ is optimal, because every signal realization has the same crime rate, and equation (2) ensures that the search strategy satisfies the capacity constraint with equality.

Theorem 2 states that signal π_{z^*} also minimizes the crime rate across all signals. Signal π_{z^*} effectively deters crime by preventing the police from identifying high types, who commit a crime in equilibrium, while still allowing the police to learn about low types, who choose not to commit a crime. Intuitively, to attain a low crime rate the police should deter crimes by low types, who would choose not to commit a crime at a low search probability. However, if the police knew that low types do not commit a crime whereas other higher types do, then the police would instead allocate search efforts to high types, which prevents the police from deterring a crime by low types. In order to incentivize the police to search low types, signal π_{z^*} pools low types, who respond to policing, with high types (i.e., types above z^*), who will commit a crime in equilibrium. At the same time, signal realizations drawn from π_{z^*} differ in the values of the lowest possible types. In equilibrium, the police search citizens with signal realization x with a probability that just deters type x from committing crime.

Overall, signal π_{z^*} renders policing more effective by garbling information to maintain its deterrence effect while providing information that guides the allocation of search resources.

In our model, the police care not only about deterring crimes but also uncovering them. Thus we cannot immediately conclude that the police choose the crime-minimizing signal π_{z^*} in equilibrium. For example, there might be a signal that leads to a slightly higher crime rate than signal π_{z^*} but enables the police to catch many criminals. However, the police indeed choose signal π_{z^*} in equilibrium. This result relies on the observation—which slightly extends a condition provided in Persico (2002)—that in any equilibrium of the policing subgame that follows any signal, the crime rate for every signal realization is equalized (see Lemma 2 in Appendix A).[2] Thus if some signal π leads to crime rate $r(\pi)$, then the crime rate for every signal realization is $r(\pi)$, which implies that the mass of successful searches is $r\overline{P}$. Consequently, the police's payoff becomes $\lambda r(\pi)\overline{P} - r(\pi) = -(1 - \lambda\overline{P})r(\pi)$, which implies that the police choose a signal that minimizes crime rate $r(\pi)$, e.g., signal π_{z^*}.

The above argument has a caveat: Given a signal, there may be multiple equilibria in the policing stage that differ in crime rates, and depending on which equilibrium is played, the police may choose a different signal. This concern is relevant to the crime-minimizing signal, because there are indeed multiple equilibria: For any $p \in [0, 1]$, the policing stage that follows signal π_{z^*} has an equilibrium in which all types below z^* commits a crime with probability p and all types above z^* commits a crime with probability 1.[3] To show the uniqueness of the equilibrium outcome, in the proof, I construct a signal such that the subsequent policing stage has a unique equilibrium and the crime rate is arbitrarily close to the minimum crime rate, $1 - F_0(z^*)$. Therefore the equilibrium outcomes, such as the set of citizens who commit a crime, will be unique.

Given the analysis so far, we may think that the private regime leads to a high crime rate by changing what information the police acquire and how they allocate search resources. This intuition is not precise, because the private regime has an equilibrium in which the

[2]The police care about catching criminals and will not search a citizen whose signal realization indicates a lower likelihood of crime. However, if the police do not search some citizens, they will commit a crime with probability 1. Thus in equilibrium all signal realizations have the same crime rate.

[3]In this equilibrium, the police still search citizens with signal realization s with probability $\frac{s}{L}$, so that any citizen with type $x \leq z^*$ randomizes between committing and not committing a crime because of indifference.

police behave the same way as in the public regime: Lemma 1 and Definition 2 imply that in the private regime, there is an equilibrium in which the police choose signal π_{z^*} and search any citizen who has realization $s \in [0, z^*]$ with probability $\frac{s}{L}$, which coincides with the police's behavior in the public regime. At the same time, by Theorem 1 and Theorem 2, all citizens commit crimes in the private regime, but only types above z^* commit crimes in the public regime. The following result summarizes these observations:

Corollary 2 (Crime-Minimizing Signal in the Private Regime). *The private regime has an equilibrium in which the police choose the same signal (i.e., signal π_{z^*}) and adopt the same search strategy as in the public regime.*

The result implies that the undesirable impact of the lack of transparency in data collection may not manifest in the police's strategy but only in the citizens' behavior.

3.3 Comparative Statics in the Public Regime

Theorem 2 enables us to examine how the equilibrium signal and crime rate respond to a change in parameters. First, consider a change in search capacity $\overline{P}$ or cost L of being caught. According to equation (2), if $\overline{P}L$ increases, then cutoff z^*, above which citizens commit a crime in the public regime, decreases. The decrease in z^* expands the set of types that commit a crime, leading to a higher crime rate. Moreover, the decrease in z^* leads to a less informative signal in equilibrium. Indeed, Definition 2 implies that if $\overline{z} > \underline{z}$, signal $\pi_{\overline{z}}$ is more informative than signal $\pi_{\underline{z}}$ in the sense of Blackwell (1951, 1953): We can replicate signal $\pi_{\underline{z}}$ by garbling signal $\pi_{\overline{z}}$ so that whenever the signal realization drawn from signal $\pi_{\overline{z}}$ belongs to $(\underline{z}, \overline{z}]$, we redraw signal $s \in [0, \underline{z}]$ randomly from distribution $F_0(\cdot|x \leq \underline{z})$. Thus the police will collect less information and more people commit a crime if the police's resource $\overline{P}$ is limited or the cost L of being caught is smaller, or more generally, if $\overline{P}L$ is smaller.

Corollary 3 (Comparative statics regarding $(\overline{P}, L)$). *Consider the equilibrium in Theorem 2. A higher $\overline{P}L$ leads to a higher crime rate and a less informative signal in the sense of Blackwell (1951, 1953).*

Corollary 3 contrasts with the intuition that the police benefit from information especially when they face limited search resources—e.g., a police chief quoted in Pearsall (2010) says that a predictive policing algorithm "is the perfect tool to help departments become more efficient as budgets continue to be reduced." On the contrary, the corollary states that if the police have limited resources, they may need to commit to using less information. Intuitively, if the police have a low search capacity, more people will commit crimes. The increase in the crime rate will exacerbate the distortion described in Proposition 1 by incentivizing the police to catch criminals rather than deterring crimes. To mitigate this distortion, the crime-minimizing signal reveals less information to the police and makes it harder for them to identify citizens who commit crimes.

We now turn to comparative statics regarding the prior distribution of returns to crime, F_0. In the benchmark of no information, the police can only search citizens randomly with probability $\overline{P}$, so the expected cost of committing a crime is $\overline{P}L$. The equilibrium crime rate is thus $1 - F_0(\overline{P}L)$. As a result, a greater F_0 in the sense of the first-order stochastic dominance leads to a higher crime rate in equilibrium.

A similar comparative statics no longer holds when the police can endogenously acquire information—i.e., for some parameters, the first-order stochastic increase in the distribution of returns to crime can *decrease* a crime rate. Figure 2 presents such an example. Type distribution G_0 first-order stochastically dominates F_0, so the equilibrium crime rate under no information is higher under G_0. At the same time, conditional distribution $F_0(\cdot|x \leq z)$ dominates $G_0(\cdot|x \leq z)$ for any $z \leq 0.5$, and thus we have $\mathbb{E}_{F_0}[x|x \leq z] > \mathbb{E}_{G_0}[x|x \leq z]$ for every $z \in (0.25, 0.5]$.[4] If $\overline{P}L$ is between $\mathbb{E}_{F_0}[x|x \leq 0.25]$ and $\mathbb{E}_{F_0}[x|x \leq 0.5]$, cutoff z^* that solves equation (2) is lower under F_0 than G_0. As a result, the equilibrium crime rate is lower under G_0 than F_0.

[4] For any $z \in (0.25, 0.5]$, $G_0(\cdot|x \leq z)$ is the uniform distribution on $[0, z]$. Distribution $F_0(\cdot|x \leq z)$ is uniform on each of the intervals $[0, 0.25]$ and $[0.25, z]$ and has a higher density on interval $[0.25, z]$. Thus conditional on $x \leq z$, F_0 dominates G_0 in the first-order stochastic sense.

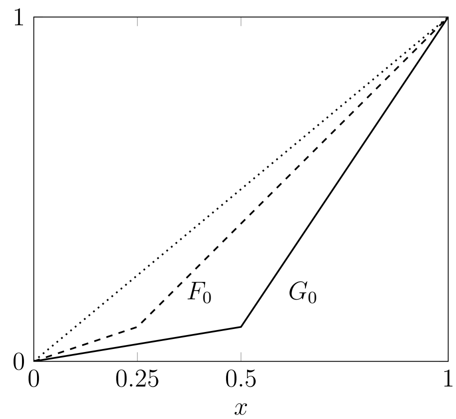

Figure 2: Reversal of the first-order stochastic dominance under truncation: Cumulative distribution function G_0 dominates F_0, but for any $z \leq 0.5$, $F_0(\cdot|x \leq z)$ dominates $G_0(\cdot|x \leq z)$.

Comparative statics based on the usual stochastic dominance fails because it can be reversed conditional on realized signals. A stronger condition, such as the likelihood ratio order, preserves the first-order stochastic dominance conditional on any event, so that we can establish a comparative statics with respect to the type distribution.[5] The following result summarizes the discussion:

Corollary 4 (Comparative statics regarding F_0). *Consider the equilibrium in Theorem 2. The following hold:*

1. *A greater F_0 in the sense of the likelihood ratio order results in a higher crime rate and a less informative signal.*

2. *There exist parameters, $\overline{P}, L, F_0$, and G_0, such that G_0 first-order stochastically dominates F_0, but the equilibrium crime rate in the public regime is lower under F_0.*

[5]If G_0 dominates F_0 in the likelihood ratio order—that is, if g_0/f_0 is increasing—then $G_0(\cdot|x \leq z)$ first-order stochastically dominates $F_0(\cdot|x \leq z)$ for any $z \in [0,1]$. Because we only consider truncations from G_0 and F_0 from above as opposed to general conditional distributions, a weaker ordering, such as the reverse hazard rate order, suffices for Corollary 4. See Shaked and Shanthikumar (2007) for the relation between these stochastic orderings.

3.4 The Crime-Minimizing Signal vs. Predictive Policing

In this section, I highlight two informational properties of the crime-minimizing signal and argue that it is an antithesis of predictive policing. First, the crime-minimizing signal prevents the police from identifying citizens who face high returns to crime: Any realized signal pools types above and below z^*, so that their relative fraction remains the same. This property seems to contrast with predictive policing, which is typically described as a way to identify individuals or places that are prone to crime (Perry, 2013; Lau, 2020). Indeed, some equilibria in the private regime capture this property. For example, the private regime has an equilibrium in which the police learn whether the types of citizens are above some cutoff or an equilibrium in which the police fully learn types above some cutoff and nothing below the cutoff.

Second, under the crime-minimizing signal, the signal realizations are fully informative about "innocents," who face low returns to crimes and do not commit crimes in equilibrium. In contrast, the realized signals are uninformative about "criminals," who face high returns to crime and commit crimes in equilibrium. To see this, suppose that the police learn that a citizen does not commit a crime in equilibrium. Conditional on this information, his realized signal reveals the exact type of the citizen, because each realized signal has a unique type that does not commit a crime. In contrast, suppose that the police learns that a given citizen committed a crime. Conditional on this information, his realized signal reveals no additional information, because the distribution of signal realizations are independent of types among criminals. This property is unlikely to arise from past crime data so long as the data contain more information about those with high returns to crimes.

Overall, even though the crime-minimizing signal might be "too complicated" to implement in practice, the result shows that the kind of predictive policing programs described in the public debate may lack desirable informational properties capture by the crime-minimizing signal.

4 Extensions

4.1 Endogenous Search Capacity

In this extension, the police can increase their search capacity at a cost. This extension results in two main differences. First, we no longer need exogenous restrictions on parameters, such as Assumption 1. Second, the extension highlights that the lack of transparency in data collection renders policing more intensive.

To accommodate endogenous search capacity, we modify the game as follows: The police choose a search capacity $\overline{P}$ at cost $C(\overline{P})$, which is strictly increasing, strictly convex, smooth, and satisfies $C(0) = 0$. In the policing stage, the police choose a search capacity and a search strategy at once, and simultaneously, citizens choose whether to commit a crime. The police's payoff is their original payoff minus cost, i.e., $\lambda r - a - C(\overline{P})$.

Proposition 1. *Suppose that the police can increase search capacity $\overline{P}$ at cost. Then the public and private regimes have equilibria that satisfy the following.*

1. *In the public regime, the police choose signal π_z for some $z \in (0, 1)$.*
2. *In the private regime, there is an equilibrium in which the police choose a higher search capacity and more citizens commit crimes than in the public regime.*

The result follows from the supply and demand interpretation of the model, where we interpret search capacity $\overline{P}$ as quantity and crime rate r as price (see Figure 3). First, we consider the "supply" side. The same argument as Lemma 5 implies that in any equilibrium, the crime rate is equalized across all realized signals. Thus if the equilibrium crime rate is r, the police must be choosing search capacity $\overline{P}$ to maximize $\lambda r\overline{P} - C(\overline{P})$, which gives the first-order condition $\lambda r = C'(\overline{P})$. Let $r^S(\overline{P}) \triangleq \frac{1}{\lambda}C'(\overline{P})$ denote the inverse supply curve, which is increasing because $C(\cdot)$ is convex.

For the "demand" side, if the equilibrium search capacity is $\overline{P}$, the crime rate $r^D(\overline{P})$ is determined as the equilibrium crime rate of the original model with exogenous search capacity $\overline{P}$.[6] Note that $r^D(\overline{P})$ implicitly depends on the chosen signal.

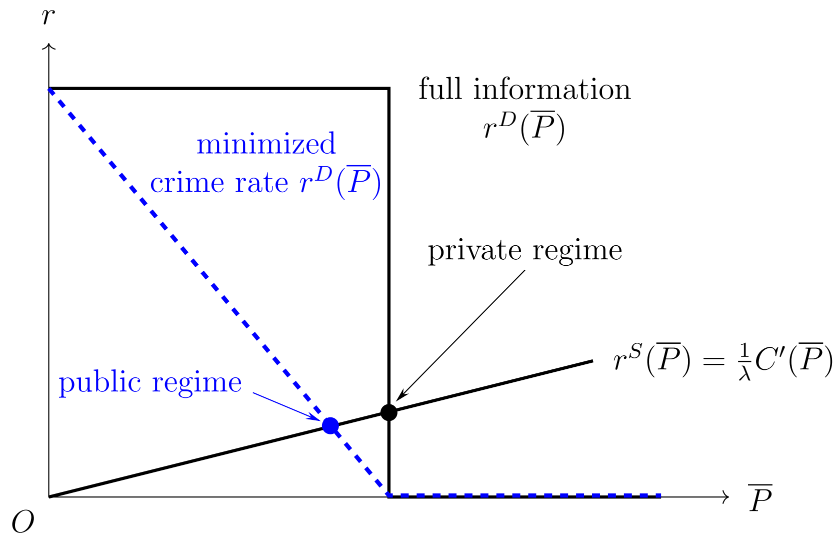

Figure 3: Equilibria with endogenous search capacity.

Any crossing point of the inverse supply curve $r^S(\overline{P})$ and the inverse demand curve $r^D(\overline{P})$ is an equilibrium of the game with endogenous $\overline{P}$, provided that the police has no incentive to deviate from the signal that generates $r^D(\overline{P})$ in the data collection stage. To find the equilibrium of the public regime in Proposition 1, we define $r^D(\overline{P})$ as the minimized crime rate of the original public regime with search capacity $\overline{P}$. For any $\overline{P}$, crime rate $r^D(\overline{P})$ is generated by signal π_z for some z in Definition 2. Because $r^D(\overline{P})$ is strictly decreasing and hits 0 at $\overline{P} = \frac{\mathbb{E}_{F_0}[x]}{L}$ whereas $r^S(\overline{P})$ is strictly increasing, the unique crossing point $(\overline{P}^*, r^*)$ satisfies $r^* > 0$ and $\overline{P}^* < \frac{\mathbb{E}_{F_0}[x]}{L}$. The corresponding signal is signal $\pi_{F_0^{-1}(1-r^*)}$. The police have no incentive to deviate from signal $\pi_{F_0^{-1}(1-r^*)}$ in the data collection stage, because any deviation shifts the inverse demand curve upwards, which leads to a higher crime rate and a lower payoff.

For the private regime, we define $r^D(\overline{P})$ as the crime rate(s) in equilibrium of the policing stage when the police have full information and search capacity $\overline{P}$. Specifically, $r^D(\overline{P})$ equals 1 if $\overline{P} < \frac{\mathbb{E}_{F_0}[x]}{L}$, can take any value in $[0, 1]$ if $\overline{P} = \frac{\mathbb{E}_{F_0}[x]}{L}$, and equals 0 if $\overline{P} > \frac{\mathbb{E}_{F_0}[x]}{L}$. The crossing point of the inverse supply and demand curve again gives an equilibrium. In this

[6] At search capacity $\overline{P} = \frac{\mathbb{E}_{F_0}[x]}{L}$, the private regime has a continuum of equilibria because for any given $p \in [0, 1]$, there is an equilibrium in which the police collect full information and search type x with probability $\frac{x}{L}$, and every citizen commits a crime with probability p. As a result, $r^D\left(\frac{\mathbb{E}_{F_0}[x]}{L}\right) = [0, 1]$.

case, the police have no incentive to deviate in the data collection stage because collecting less than full information only reduces the police's payoffs in the policing stage without affecting citizens' behavior.

Overall, as we move from the public regime to the private regime, the inverse demand curve shifts upwards whereas the inverse supply curve remain the same. Consequently, the equilibrium in the private regime has a higher crime rate and a higher search capacity than in the public regime.

4.2 Implication on the Fairness of Policing

Theorem 2 implies that the police that can commit to an information policy upfront can deter crime more effectively by collecting partial information. However, the resulting equilibrium is "unfair" in the sense that the equilibrium search strategy exposes different groups to different search intensities. To illustrate the idea, we introduce the group identity of each citizen into the model Each citizen is now endowed with his *group identity* that takes values r or b. For each group $g \in \{r, b\}$, let $p_g \in (0, 1)$ denote the fraction of the overall population that belongs to group g, and let F_g denote the type distribution across the citizens in group g, so that we have $p_b F_b + p_r F_r = F_0$. Distribution function F_g has a positive density, f_g. Throughout this section, we assume the following:

Assumption 2. Distribution F_r is greater than F_b in the likelihood ratio order.

If distribution F_r is greater than F_b in the likelihood ratio order, then F_r first-order stochastically dominates F_b. We need a condition stronger than the first-order stochastic dominance because we will compare the conditional distributions of F_r and F_b in terms of the first-order stochastic dominance. Interpreting type x as a fixed return to crime minus one's legal earning opportunity, the first part of the assumption implies that group r's income distribution is stochastically lower than group b, so that group r tends to face higher net returns to crime.

Because the group identity does not affect payoffs, the public regime continues to have an equilibrium described in Theorem 2. In this equilibrium, type $x \leq z^*$ will receive signal realization x and be searched with probability $\frac{x}{L}$. Any type $x > z^*$ will receive signal realization that is randomly drawn from $F_0(\cdot|x \leq z^*)$. Consequently, the expected search probability for type $x > z^*$ is $\overline{P}$. Overall, the expected search probability $\sigma^*(x)$ for type x is non-monotone in types: Compared to no information, where everyone is searched with probability $\overline{P}$, types below $\overline{P}L$ will be searched with a lower probability; types in $[\overline{P}L, z^*]$ will be searched with a higher probability; and types above z^* faces the same search probability. Even though the police do not learn group identity, it is correlated with types. As a result, the two groups will be searched with different probabilities on average:

Proposition 2. *In the equilibrium of the public regime described in Theorem 2, the following holds.*

1. *Group r is exposed to more policing than group b on average:*

$$\int_0^1 \sigma^*(x)F_r(dx) \geq \int_0^1 \sigma^*(x)F_b(dx). \tag{3}$$

2. *Across the citizens who do not commit crime, group r is exposed to more policing than group b on average:*

$$\int_0^1 \sigma^*(x)F_r(dx|x \leq z^*) \geq \int_0^1 \sigma^*(x)F_b(dx|x \leq z^*).$$

Part 1 uses the notion of fairness in Persico (2002), and Part 2 uses the notion of "fairness in the treatment of innocents" in Durlauf (2006).[7] Persico (2002) finds that an equilibrium crime rate may be lower when the police is required to search all citizens with equal probability than when the police bases search probabilities on group identity. In other words, there may be no trade-off between fairness and effectiveness of policing. Proposition 2 states that once we consider the set of all information structures, the trade-off generally exists, in the sense that the information structure that minimizes the equilibrium crime rate is unfair.

[7]The counterpart of Part 2 for "criminals" does not hold: The equilibrium is "fair among criminals" because any citizen who commits a crime faces search probability $\overline{P}$.

5 Conclusion

This paper studies a model in which police collect information about citizens in order to deter and uncover crimes. The paper highlights the importance of transparency in data collection: Opaque data collection leads to more crime and possibly more intensive policing. The police benefit from committing to not collect certain information. The resulting information policy, which prevents the police from identifying individuals and places that are prone to crime, starkly contrasts with predictive policing discussed in the public debate.

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Appendix

A Proof of Theorem 2: Equilibrium in the Public Regime

We begin with a series of lemmata. The following lemma generalizes an equilibrium condition in Persico (2002).

Lemma 2. *Take any signal. In any equilibrium of the subsequent policing stage, almost all signal realizations have the same crime rate.*

Proof. Suppose to the contrary that for some signal and some equilibrium in the policing stage, there are sets of signal realizations, $S_H, S_L \subset S$, such that they can arise with positive probabilities and $r(s) > r(s')$ for all $s \in S_H$ and $s' \in S_L$. If the police search a citizen with probability $\sigma \in [0, 1]$, the net benefit of committing a crime is $x - \sigma L$ with $L \geq 1$. Thus if $\sigma = 1$, no citizen commits a crime for any type x. If $\sigma = 0$, all citizens with $x < 1$ commit crime. Crime rates in S_H being positive implies that the average search probability across the realizations in S_H is strictly less than 1. Crime rates in S_L being less than 1 implies that the average search probability across the realizations in S_L is positive. Thus the police can profitably deviate by allocating search probabilities from signal realizations in S_L to realizations in S_H, which is a contradiction. As a result, almost all signal realizations must have the same crime rate. □

Lemma 2 implies that for any signal, if the equilibrium crime rate is r, every signal realization has crime rate r. The mass of successful searches will then be $r\overline{P}$ and the police's payoff will be $\lambda r\overline{P} - r = -(1 - \lambda\overline{P})r$.

Lemma 3. *Take any signal and any equilibrium of the corresponding policing stage. If the crime rate is r, the mass of successful searches is $r\overline{P}$ and the police's payoff is $-(1-\lambda\overline{P})r$.*

Because $1-\lambda\overline{P} > 0$, the police's payoff is decreasing in the crime rate. The lemma then implies that in the data collection stage, the police choose a signal that minimizes the crime rate. To formalize the idea, we define the crime-minimizing signal. Let $R(\pi)$ denote the set of all possible equilibrium crime rates in the policing stage that follows signal π.

Definition 2. Signal π is a *crime-minimizing signal* if $\min R(\pi)$ exists and for any signal π' and crime rate $r' \in R(\pi')$, we have $\min R(\pi) \leq r'$.

Lemma 4. *In some equilibrium of the public regime, the police choose a crime-minimizing signal whenever it exists.*

Proof. Construct a strategy profile as follows: If the police choose a crime-minimizing signal π, the equilibrium with crime rate $\min R(\pi)$ is played. If the police choose any other signal, an arbitrary equilibrium of the subgame is played. Lemma 3 implies that this is an equilibrium strategy profile. □

To prove that signal π_{z^*} is a crime-minimizing signal, we show the following:

Lemma 5. *If a signal attains crime rate $r \in [0,1]$, we have*

$$\overline{P} \geq \frac{\mathbb{E}_{F_0}[x|x \leq F_0^{-1}(1-r)]}{L}. \tag{4}$$

Proof. Throughout the proof, we fix a signal and an equilibrium in the policing stage. We define probability measure μ^{nc} as follows: For each measurable set $A \subset [0,1] \times S$, let $\mu^{nc}(A)$ be the fraction of citizens whose type x and signal realization s belong to A, among the mass $1-r$ of all citizens who do not commit a crime in the equilibrium. In particular, $\mu^{nc}(X \times S)$ is the fraction of citizens whose types are in $X \subset [0,1]$ among all citizens who do not commit a crime. Let G_s denote the posterior distribution on types conditional on realization s. Let $\sigma(x,s)$ denote the equilibrium search probability for type $x \in \text{supp}G_s$.

A citizen with type $x \in \operatorname{supp} G_s$ does not commit a crime only if $x \leq L\sigma(x,s)$. We then have

$$\begin{aligned} & x \leq L\sigma(x,s) \\ \Rightarrow & \frac{\int_{[0,1]\times S} x d\mu^{nc}}{L} \leq \int_{[0,1]\times S} \sigma(x,s) d\mu^{nc}. \end{aligned} \tag{5}$$

The search intensity allocated to citizens who do not commit a crime is

$$\overline{P}^{nc} \triangleq (1-r) \int_{[0,1]\times S} \sigma(x,s) d\mu^{nc}. \tag{6}$$

However, the crime rate is r for almost all signal realizations. Thus for every citizen with type $x \in \operatorname{supp} G_s$ who does not commit a crime, there are $\frac{r}{1-r}$ citizens who have realization s, commit a crime, and face search probability $\sigma(x,s)$. By equation (6), the total search intensity in equilibrium is

$$\overline{P}^{nc} + \frac{r}{1-r}\overline{P}^{nc} = \int_{[0,1]\times S} \sigma(x,s) d\mu^{nc}.$$

As a result, if the equilibrium crime rate is r, the total search capacity $\overline{P}$ must be at least the left-hand side of inequality (5).

Finally, because the mass of citizens who do not commit a crime is $1-r$, we have

$$\mathbb{E}[x|x \leq F_0^{-1}(1-r)] \leq \int_{[0,1]\times S} x d\mu^{nc}.$$

Combining this inequality with inequality (5), we conclude that

$$\overline{P} \geq \int_{[0,1]\times S} \sigma(x,s) d\mu^{nc} \geq \frac{\mathbb{E}[x|x \leq F_0^{-1}(1-r)]}{L}.$$

□

Lemma 6. *Signal π_{z^*} with cutoff $z^* \in (0,1)$ that solves equation (2) (i.e., $\overline{P} = \frac{\mathbb{E}_{F_0}[x|x\leq z^*]}{L}$) is a crime-minimizing signal.*

Proof. Lemma 5 implies that given search capacity $\overline{P}$, the crime rate is at least r^*, where

$$\overline{P} = \frac{1 - \mathbb{E}_{F_0}[x|x \leq F_0^{-1}(1-r^*)]}{L} \tag{7}$$

Indeed, if the equilibrium crime rate is $r < r^*$ then the total search capacity must be at least

$$\frac{\mathbb{E}_{F_0}[x|x \leq F_0^{-1}(1-r)]}{L} > \frac{\mathbb{E}_{F_0}[x|x \leq F_0^{-1}(r^*)]}{L} = \overline{P},$$

which is a contradiction. As crime rate r increases from 0 to 1, $\frac{\mathbb{E}_{F_0}[x|x\leq F_0^{-1}(1-r)]}{L}$ continuously and strictly decreases from $\frac{\mathbb{E}_{F_0}[x]}{L} > \overline{P}$ (by Assumption 1) to $0 < \overline{P}$. Thus equation (7) has a unique solution $r^* \in (0,1)$.

We show that in the public regime, signal π_{z^*} with $z^* \triangleq F_0^{-1}(1-r^*)$ attains the equilibrium crime rate r^*. Let $\sigma(x) \triangleq \frac{x}{L}$ be the search probability that makes type x indifferent between committing and not committing a crime. If the police allocate search probability $\sigma(x)$ to signal realization x, citizens with type x (weakly) prefer not to commit a crime, whereas any other citizens, whose types are above x, will commit a crime. As a result, the crime rate for any signal realization will be $1 - F_0(z^*)$. Under such a search strategy, the total search intensity is

$$\int_0^{z^*} \sigma(x) F_0(dx|x \leq z^*) = \frac{\mathbb{E}_{F_0}[x|x \leq z^*]}{L} = \overline{P}.$$

The first equality is by the definition of $\sigma(x)$ and that of signal π_{z^*} (see Definition 1), and the second equality follows from the definition of z^* (see equation (2)). According to the strategy profile constructed above, the police exhausts her search capacity $\overline{P}$ and the crime rates for all signal realizations are equalized. Thus the police is indifferent between any search strategy. By construction, each citizen maximizes his payoffs. As a result, we have an equilibrium with crime rate r^*. □

To show the uniqueness of the equilibrium outcome, we prove the following lemma:

Lemma 7. *Let $r^* \triangleq 1 - F_0(z^*)$ denote the minimized crime rate with z^* solving equation (2). For any $\epsilon > 0$, there is a signal π such that the corresponding policing stage has a unique equilibrium, which attains crime rate $r \in (r^* - \epsilon, r^*]$.*

Proof. Take any positive integer N. Define signal $(S^N, \pi^N_{z^*})$ as follows: The set of signal realizations is $S^N = \{s_1, ..., s_N\}$. If a type is $x \in [\frac{n-1}{N}z^*, \frac{n}{N}z^*)$ for some $n = 1, ..., N$, $\pi^N_{z^*}(\cdot|x)$ draws signal realization s_n with probability 1. If a type is $x \geq z^*$, distribution $\pi^N_{z^*}(\cdot|x)$ draws signal realization s_n with probability $\frac{F_0(\frac{n}{N}z^*)-F_0(\frac{n-1}{N}z^*)}{F_0(z^*)}$.

We show that signal $\pi^N_{z^*}$ leads to a unique equilibrium in the policing stage. Lemma 3 implies that in equilibrium, all the signal realizations have the same crime rate. Let r^N denote the crime rate. For ease of exposition we use the mass of innocents, $t^N = 1 - r^N$, instead of crime rate. Given t, the set of types who receive realization s_n and do not commit a crime is $[\frac{n-1}{N}z^*, x^N(t)]$ where $x^N(t)$ solves

$$t = \frac{F_0(x^N(t)) - F_0(\frac{n-1}{N}z^*)}{F_0(\frac{n}{N}z^*) - F_0(\frac{n-1}{N}z^*) + 1 - F_0(z^*)}.$$

To attain mass t of innocents—or equivalently, to deter types in $\cup_{n=1}^N [\frac{n-1}{N}z^*, x^N(t)]$ from committing a crime—the police need to search any citizens with realization s_n, who have mass $F_0(\frac{n}{N}z^*) - F_0(\frac{n-1}{N}z^*) + 1 - F_0(z^*)$, with probability $\frac{x^N(t)}{L}$. Thus the police need search capacity

$$P^N(t) \triangleq \sum_{n=1}^{N} \left[\frac{x^N(t)}{L} \cdot \frac{F_0(\frac{n}{N}z^*) - F_0(\frac{n-1}{N}z^*)}{F_0(z^*)} \right]. \tag{8}$$

Search capacity $P^N(t)$ satisfies $P^N(0) = 0$ and $P^N(F_0(z^*)) > \overline{P}$, and $P^N(t)$ is continuous and strictly increasing. Thus there is a unique t^N that satisfies equation (8) with equality. By construction, the unique equilibrium crime rate following signal $\pi^N_{z^*}$ is $r^N = 1 - t^N$.

We show that r^N can be arbitrarily close to $r^* = 1 - F_0(z^*)$ for a large N. To deter all types below z^* from crime, the police need search capacity

$$P^N(F_0(z^*)) \triangleq \sum_{n=1}^{N} \left[\frac{nz^*}{N} \cdot L \cdot \frac{F_0(\frac{n}{N}z^*) - F_0(\frac{n-1}{N}z^*)}{F_0(z^*)}, \right] \tag{9}$$

which converges to $\frac{\mathbb{E}_{F_0}[x|x\leq z^*]}{L} = \overline{P}$ as $N \to \infty$. Thus by taking N sufficiently large, the police can use signal $\pi^N_{z^*}$ to attain crime rate arbitrarily close to $1 - F_0(z^*)$ as a unique equilibrium outcome. □

Proof of Theorem 2. Lemma 4 and Lemma 6 imply that there is an equilibrium in which the police choose signal π_{z^*} and citizens commit a crime if and only if $x > z^*$. Moreover, there is no equilibrium in which the crime rate r is strictly greater than $r^* = 1 - F_0(z^*)$, because by Lemma 7 the police can then profitably deviate to a signal with crime rate $r' \in [r^*, r)$. As a result any equilibrium has crime rate r^*. Finally, take any equilibrium with crime rate r^*. The proof of Lemma 5 implies that if the probability measure of types across citizens who do not commit crime is given by $\mu \in \Delta([0,1])$ then we have $\frac{\int_0^1 x\mu(dx)}{L} \leq \overline{P}$. If measure μ is different from $F_0(\cdot|x \leq z^*)$ then we have $\frac{\mathbb{E}_{F_0}[x|x\leq z^*]}{L} < \frac{\int_0^1 x\mu(dx)}{L} \leq \overline{P}$, which is a contradiction. □

B Proof of Proposition 2

Proof. We have

$$\sigma^*(x) = \begin{cases} \frac{x}{L} & \text{if} \quad x \leq z^* \\ \overline{P} & \text{if} \quad x > z^*. \end{cases}$$

Because F_r dominates F_b in the hazard rate order, $F_r(\cdot|x \leq z^*)$ is greater than $F_b(\cdot|x \leq z^*)$ in the first-order stochastic dominance (Shaked and Shanthikumar, 2007). We then obtain Part 2 of the proposition, because $\sigma^*(x)$ is increasing in $x \leq z^*$. To show Part 1, note that $\overline{P} = \frac{\mathbb{E}_{F_0}(x|x\leq z^*)}{L}$ implies

$$\frac{\mathbb{E}_{F_r}(x|x \leq z^*)}{L} > \overline{P} > \frac{\mathbb{E}_{F_b}(x|x \leq z^*)}{L}. \tag{10}$$

The average search probability for group b is

$$[1 - F_b(z^*)]\overline{P} + F_b(z^*)\frac{\mathbb{E}_{F_b}(x|x \leq z^*)}{L} < \overline{P}. \tag{11}$$

The average search probability for group b is

$$[1 - F_r(z^*)]\overline{P} + F_r(z^*)\frac{\mathbb{E}_{F_r}(x|x \leq z^*)}{L} > \overline{P}. \tag{12}$$

Combining these inequalities, we conclude that group r faces a higher search probability than group b on average. $\square$