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A network-city model of spatial competition[☆]

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HIGHLIGHTS

- This paper investigates a spatial competition model of product differentiation.
- Each firm competes with all other firms in this model.
- City lengths and consumer densities need not be identical.
- The model has a unique and easily computable Nash equilibrium.
- The analysis provides a spatial microfoundation for a linear differentiated Bertrand oligopoly.

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ABSTRACT

In this paper, we analyze a spatial Bertrand oligopoly model called network-city model. Firms compete directly and simultaneously with all other firms. In the model, we allow for heterogeneous product differentiation, heterogeneous constant marginal costs of production, and heterogeneous consumer densities. We show that the equilibrium is unique and easily computable. Our model is more general than the existing spatial models, and more importantly, our model provides a spatial microfoundation for the traditional linear demand functions in a differentiated Bertrand oligopoly.

1. Introduction

Product differentiation is an important topic in microeconomics, especially in the theory of industrial organization. The traditional two-firm linear-city model (cf. the seminal paper of Hotelling, 1929) and multi-firm circular-city model (cf. Salop, 1979) assume that firms compete locally with their (at most two) neighboring firms only. von Ungern (1991) extends these localized competition models to a multidimensional pyramid model in which firms compete with all other firms directly along the edges of the pyramid. Chen and Riordan (2007) take a different approach, inventing the spokes model. In the model, firms are located at the endpoints of the spokes, along which the consumers are distributed. Recently, based on the pyramid model and the spokes model,[1] Somaini and Einav (2013) establish a dynamic spatial competition model to investigate the issue of customer retention in the presence of switching costs.

In the above models, the lengths of the edges are assumed to be identical and the consumer densities on the edges are assumed to be identical. We relax these restrictions in this paper, and we call our model a network-city model. In this network-city model, firms are located at the vertices of a network and consumers are located along the links connecting the vertices. Firms produce a physically identical product, but consumers need to pay transportation costs to buy from a firm. Our model allows for any arbitrary number of

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1 Note that Chen and Riordan's (2007) spoke model is isomorphic to the pyramid model when differentiations between products are symmetric and consumer densities are identical.

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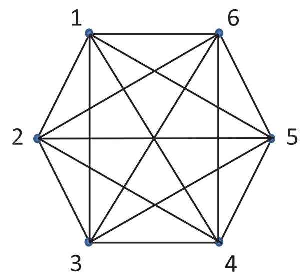

Fig. 1. A network city of six firms.

firms with differentiated constant marginal costs of production, and new entry of firms can be accommodated by adding new vertices to the network. Therefore, many issues in the theory of industrial organization, such as equilibrium entries of firms, product varieties and mergers can be analyzed using our model.

Our model is a spatial competition model. It extends the models in von Ungern (1991), Chen and Riordan (2007) and Somaini and Einav (2013) to accommodate unequal city lengths and consumer densities. With these extensions, we obtain standard demand functions for a linear differentiated Bertrand competition oligopoly. Therefore, our model provides a spatial microfoundation for the differentiated Bertrand competition demand functions.

Our model belongs to the class of global competition models. Firms can compete with all other firms directly and simultaneously, an advantage over the traditional localized models, such as Hotelling (1929) and Salop (1979). Meanwhile, it allows for asymmetric firms and an arbitrary degree of differentiation between any two firms, an advantage over the traditional non-localized models in the spirit of Chamberlin (1933), such as Hart (1985a, b).

2. The basic network-city model

We start with a basic model of identical lengths and densities. Suppose that there are n firms spatially located at the vertices of a n-vertex network, with one firm per vertex. Every vertex is connected to every other vertex by a link (called a linear city) of length 1. Therefore, there are a total of $n(n-1)/2$ different linear cities. (c.f. Fig. 1.) Consumers are uniformly distributed on these links, with density on each link given by f.

Firms are producing a physically identical product. They set prices simultaneously. Firm i has a constant marginal cost of production, c_i, and sets a price p_i, $i = 1, 2, \ldots, n$. Each consumer demands either 0 or 1 unit of the product, with willingness to pay denoted by v. Let t be a consumer's transportation cost per unit of distance.

Consider firm j's optimization problem. Demand for firm j's product comes from the $n-1$ cities connected to firm j. The demand from city ji (connecting firms j and i), for example, is determined by

$$x_{ji} = f\left(\frac{p_i - p_j}{2t} + \frac{1}{2}\right). \tag{1}$$

Therefore, the total demand for firm j is

$$D_j = f\sum_{i\neq j} x_{ji} = f\left(\frac{\sum_{i\neq j} p_i}{2t} - \frac{(n-1)p_j}{2t} + \frac{n-1}{2}\right).$$

Firm j chooses p_j to maximize its profit $D_j(p_j - c_j)$. Simplifying the FOC, we obtain

$$\sum_{i=1}^{n} p_i - (2n-1)p_j + (n-1)t + (n-1)c_j = 0. \tag{2}$$

Summing up these FOCs for all firms, we have

$$\sum_{i=1}^{n} p_i = nt + \sum_{i=1}^{n} c_i. \tag{3}$$

Using this, we can back out firm j's Nash equilibrium price p_j from (2):

$$p_j = t + \frac{1}{2n-1}\left(\sum_{i=1}^{n} c_i + (n-1)c_j\right), \tag{4}$$

and firm j's equilibrium profit becomes

$$\Pi_j = \frac{f}{nt}\left[t + \frac{1}{2n-1}\left(\sum_{i=1}^{n} c_i - nc_j\right)\right]^2, \quad j = 1, 2, \ldots, n. \tag{5}$$

In this equilibrium, we implicitly require that a consumer's w.t.p. v is large enough, so that every consumer will buy. We also require that the indifference consumer in city ji, $x_{ji} = \frac{p_i - p_j}{2t} + \frac{1}{2}$, is within the city; that is, $x_{ji} \in [0, 1]$. Equivalently, we require that

$$|c_i - c_j| \leq \frac{2n-1}{n-1}t, \tag{6}$$

that is, the marginal cost difference between any two firms cannot be too large. When the difference is large enough, the price competition between the two firms may attract consumers from other cities to cross over to buy.

We summarize the above results in the following proposition:

Proposition 1. *Suppose that v is large enough and that (6) is satisfied. The Nash equilibrium of the basic network-city model exists, and is uniquely characterized by (4) and (5).*

Example 1. When all firms are symmetric, i.e., $c_1 = c_2 = \cdots = c_n = c$, we have $p_1 = p_2 = \cdots = p_n = t + c$, and $\Pi_1 = \Pi_2 = \cdots = \Pi_n = \frac{ft}{n}$. Note that this is a special case of Chen and Riordan's (2007) model. Note also that the equilibrium price is independent of n in this example. In this basic model, we assume that the length of a city is always equal to 1. So the number of firms (n) does not affect the intensity of competition between firms, and the symmetric equilibrium price remains constant. This is true even in the case where the consumer density f changes with n to keep the total number of consumers constant. (If the lengths of the cities change with n, however, the equilibrium price would also change.)

Example 2. When k_1 firms have cost c, and $k_2 = n - k_1$ firms have cost $\tilde{c}$, a c-firm has a profit of $\Pi(c) = \frac{f}{nt}\left[t + \frac{k_2(\tilde{c}-c)}{2n-1}\right]^2$, while a $\tilde{c}$-firm has a profit of $\Pi(\tilde{c}) = \frac{f}{nt}\left[t + \frac{k_1(c-\tilde{c})}{2n-1}\right]^2$.

3. The general network-city model

In this section, we generalize the basic model to allow for cities of different lengths and different consumer densities. The city length characterizes the differentiation between the two firms at the two ends of the city, while the density indicates how many potential consumers are comparing the products of the two firms. A density of zero represents a missing link in the figure.

Let L_{ij} denote the length of the city connecting firms i and j and f_{ij} denote the corresponding density of consumers. Note that $L_{ij} = L_{ji}$ and $f_{ij} = f_{ji}$. The indifference consumer in city ji is given by $x_{ji} = \frac{p_i - p_j}{2t} + \frac{L_{ji}}{2}$, and the total demand for firm j is

$$D_j = \sum_{i\neq j} x_{ji} f_{ji}. = \sum_{i\neq j}\left(\frac{p_i - p_j}{2t} + \frac{L_{ji}}{2}\right) f_{ji}. \tag{7}$$

This is a standard linear demand function for a differentiated-good Bertrand competition oligopoly. Our model thus provides a spatial microfoundation for this differentiated-good demand function, a graphical alternative to the more general and less restrictive quadratic utility representative consumer microfoundation.

Firm j chooses p_j to maximize its profit $D_j(p_j - c_j)$. This is a concave function of p_j, with FOC

$$\sum_{i\neq j}\left(\frac{p_i - p_j}{2t} + \frac{L_{ji}}{2}\right) f_{ji} - \frac{p_j - c_1}{2t}\sum_{i\neq j} f_{ji} = 0. \tag{8}$$

Let $A_j = \sum_{i\neq j} f_{ji}(c_j + tL_{ji})$. We have

$$\begin{pmatrix} p_1 \\ p_2 \\ \vdots \\ p_n \end{pmatrix} = \begin{pmatrix} 2\sum\limits_{i\neq 1} f_{1i} & -f_{12} & \cdots & -f_{1n} \\ -f_{21} & 2\sum\limits_{i\neq 2} f_{2i} & \cdots & -f_{2n} \\ \cdots & \cdots & \cdots & \cdots \\ -f_{n1} & -f_{n2} & \cdots & 2\sum\limits_{i\neq n} f_{ni} \end{pmatrix}^{-1} \begin{pmatrix} A_1 \\ A_2 \\ \vdots \\ A_n \end{pmatrix}. \tag{9}$$

Proposition 2. *Suppose that v is large enough and that firms' marginal costs are close enough. The Nash equilibrium of the general network-city model exists, and is uniquely characterized by (9).*

Note that our identical-cost, identical-length and identical-density model is a special case of Chen and Riordan's (2007) spokes model. When the city lengths are not identical, however, our model and theirs are different. This is because we have $n(n-1)/2$ cities in our model, all of which can be of different lengths and densities. In Chen–Riordan's model, there are only n spoke lengths and densities.

4. Conclusion

In this paper, we analyze a spatial model of horizontal product differentiation. The model resembles a network, allowing for any number of firms competing against each other and any degree of differentiation between the products of any two firms. Given that consumers are uniformly distributed, an analytical solution to the model can be readily derived.

This paper offers a richer model to analyze product differentiation for a Bertrand oligopoly. Our model can accommodate any arbitrary degrees of differentiation between products, asymmetric production costs between firms, as well as non-identical consumer densities. Therefore, empirically, it should fit the data better than the linear- and circular-city models, the existing pyramid and spokes models and monopolistic competition models. Furthermore, our model provides a spatial microfoundation for the linear demand functions of a differentiated-good Bertrand oligopoly. Of course, the linearity in the demand functions generated by our model is unduly restrictive. In many applications, such as the mergers of firms where the pass-through rates depend on the curvature of the demand function (cf. Jaffe and Weyl, 2013), our model setting is insufficient in representing the richer market and competition environment.

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