ECON 811 Assignment #6 Fall 2022-23

Due in class on Thursday, November 24, 2022

#1. Consider the following **first-price, common-value** auction. There are N bidders bidding for an object. Each bidder receives a private signal, θ, which is a random and independent draw from a distribution with c.d.f. $F(\theta)$ and p.d.f. $f(\theta)$, where $\theta \in [\underline{\theta}, \bar{\theta}]$. Given θ_i, $i = 1, 2, ..., N$, each bidder values the object at

$$V = \theta_1 \theta_2 \cdots \theta_N.$$

In the auction, each bidder submits a sealed bid. The bidder with the highest bid wins the object and pays his bid to the seller. The losers (i.e., all other bidders) pay nothing.

(a) Set up bidder 1's maximization problem and derive the first-order condition assuming a symmetric strictly increasing Bayesian equilibrium bidding function $b_1 = B(\theta_1)$.

(b) Determine the initial value $B(\underline{\theta})$ and solve for the equilibrium bidding function.

#2. Consider the following **second-price, sealed-bid common-value** auction. There are N bidders bidding for an object. Each bidder receives a private signal, θ, which is a random and independent draw from a distribution with c.d.f. $F(\theta)$ and p.d.f. $f(\theta)$, where $\theta \in [\underline{\theta}, \bar{\theta}]$. Given θ_i, $i = 1, 2, ..., N$, each bidder values the object at

$$V = \theta_1 + \theta_2 + ... + \theta_N.$$

In this auction, each bidder submits a sealed bid, and the bidder with the highest bid wins the object and pays the second-highest bid. All other bidders pay nothing.

(a) Set up bidder 1'ss maximization problem and derive the first-order condition assuming a symmetric strictly increasing Bayesian equilibrium bidding function $b_1 = B(\theta_1)$.

(b) Solve for the equilibrium bidding function.

#3. Consider the following **1.5-price, sealed-bid common-value** auction. There are N bidders bidding for an object. Each bidder receives a private signal, θ, which is a random and independent draw from a distribution with c.d.f. $F(\theta)$ and p.d.f. $f(\theta)$, where $\theta \in [\underline{\theta}, \bar{\theta}]$. Given θ_i, $i = 1, 2, ..., N$, each bidder values the object at

$$V = \theta_1 + \theta_2 + ... + \theta_N.$$

In this auction, each bidder submits a sealed bid, and the bidder with the highest bid wins the object and pays the **average of the highest-bid and second-highest bid**. All other bidders pay nothing.

(a) Set up bidder 1'ss maximization problem and derive the first-order condition assuming a symmetric strictly increasing Bayesian equilibrium bidding function $b_1 = B(\theta_1)$.

(b) Solve for the equilibrium bidding function when the distribution $F(\cdot)$ is the uniform distribution $U[0, 1]$. (Hint: Guess $B(\theta) = \beta\theta$.)