# ECON 845 Midterm Practice Questions

## Answers

Fall 2022-23

#1. $R_1 = P_1 \cdot q_1 = (10-q_1)q_1$, $MR_1 = 10-2q_1$

$R_2 = P_2 q_2 = (20-q_2)q_2$, $MR_2 = 20-2q_2$

$R_3 = P_3 \cdot q_3 = (30-q_3)q_3$. $MR_3 = 30-2q_3$

$MR_1 = MR_2 = MR_3 = MC$

$10-2q_1 = 20-2q_2 = 30-2q_3$, $q_3 = 6q_1$

$10-2q_1 = 30-12q_1 \Rightarrow q_1 = 2$, $q_3 = 12$, $q_2 = 7$.

#2. $(T_1, Q_1) = (3, 1)$, $(T_2, Q_2) = (6, 3)$.

Type 1 do not buy $(T_2, Q_2)$, type 2 do not buy $(T_1, Q_1)$

All consumer surpluses are extracted. $\Rightarrow$ optimal.

#3. $\pi_1 = (a-bQ)q_1 - c_1 q_1 = [a-b(q_1+\cdots+q_n+q_{n+1}+\cdots+q_{2n})-c_1]q_1$

F.O.C. $\frac{\partial \pi_1}{\partial q_1} = -bq_1 + [a-bQ-c_1] = 0$

Let $q_1 = \cdots = q_n = q_1^*$, $q_{n+1} = \cdots + q_{2n} = q_2^*$ $\Rightarrow Q = nq_1^* + nq_2^*$

$$\Rightarrow \quad -bq_1^* + a - b(nq_1^* + nq_2^*) - c_1 = 0$$

$$-bq_2^* + a - b(nq_1^* + nq_2^*) - c_2 = 0$$

$+)$

$$-b(q_1^*+q_2^*) + 2a - 2nb(q_1^*+q_2^*) - c_1 - c_2 = 0$$

$$q_1^* + q_2^* = \frac{2a-c_1-c_2}{(2n+1)b}$$

$$\Rightarrow q_1 = \frac{1}{b}\left[a-c_1 - \frac{n}{2n+1}(2a-c_1-c_2)\right]$$

$$q_2 = \frac{1}{b}\left[a-c_2 - \frac{n}{2n+1}(2a-c_1-c_2)\right]$$

*Herfindah index*

$$I_H = \alpha_1^2 + \cdots + \alpha_n^2 + \alpha_{n+1}^2 + \cdots + \alpha_{2n}^2 = n\left(\frac{q_1^*}{n(q_1^*+q_2^*)}\right)^2 + n\left(\frac{q_2^*}{n(q_1^*+q_2^*)}\right)^2$$

#4. (a) $\pi_1 = (P-c)q_1 = [a-c-b(q_1+q_2)]q_1$

$$\frac{\partial \pi_1}{\partial q_1} = a-c-b(q_1+q_2)-bq_1 = 0.$$

Similarly, $\frac{\partial \pi_2}{\partial q_2} = a-c-b(q_1+q_2)-bq_2 = 0 \Rightarrow q_2 = \frac{a-c-bq_1}{2b} = R_2(q_1)$

$$\Rightarrow q_1 = q_2 = \frac{a-c}{3b}, \quad \alpha_1 = \frac{q_1}{q_1+q_2} = \frac{1}{2} = \alpha_2$$

$$I_H = \left(\tfrac{1}{2}\right)^2 + \left(\tfrac{1}{2}\right)^2 = \tfrac{1}{2}.$$

(b) $\pi_1(q_1, R_2(q_1)) = \left(a-c-bq_1-b\frac{a-c-bq_1}{2b}\right)q_1 = \frac{a-c-bq_1}{2}\cdot q_1$

F.o.c. $\frac{1}{2}[a-c-bq_1-bq_1] = 0 \Rightarrow q_1 = \frac{a-c}{2b}$

$$\Rightarrow q_2 = R_2(q_1) = \frac{a-c-b\cdot\frac{a-c}{2b}}{2b} = \frac{a-c}{4b}$$

$$\alpha_1 = \frac{q_1}{q_1+q_2} = \frac{2}{3}, \quad \alpha_2 = \frac{q_2}{q_1+q_2} = \frac{1}{3}$$

$$I_H = \left(\tfrac{2}{3}\right)^2 + \left(\tfrac{1}{3}\right)^2 = \frac{5}{9} > \frac{1}{2}$$

(c) Stackelberg has a higher concentration index. This is because the market becomes aymmetric, and compared to the Cournot equ, one firm becomes larger. That makes the market more concentrated.

#5. (a)

| | D | E | F |
|---|---|---|---|
| A | 2,3 | 0,0 | 2,1 |
| B | 0,0 | 3,2 | 0,1 |
| C | 1,0 | 0,0 | 1,5 |

① C is dominated by $\frac{2}{3}A + \frac{1}{3}B$

② F is dominated by $(\frac{1}{3}+\varepsilon)D + (\frac{2}{3}-\varepsilon)E$

Pure strategy Nash: (A,D), (B,E).

Mixed: $a_1 = (\frac{2}{5}, \frac{3}{5})$, $a_2 = (\frac{3}{5}, \frac{2}{5})$

#5 (b) Pure strategy equ: $(U,L)$, $(S,M)$, $(D,R)$

Mixed. 4 possibilities $(U,S)\times(L,M)$, $(U,D)\times(L,R)$
$(S,D)\times(M,R)$, $(U,S,D)\times(L,M,R)$.

$(U,S)\times(L,M)$: $a_1=(\frac{2}{3},\frac{1}{3},0)$, $a_2=(\frac{2}{3},\frac{1}{3},0)$

$(U,D)\times(L,R)$: $a_1=(\frac{3}{4},0,\frac{1}{4})$, $a_2=(\frac{3}{4},0,\frac{1}{4})$.

$(S,D)\times(M,R)$: $a_1=(0,\frac{3}{5},\frac{2}{5})$, $a_2=(0,\frac{3}{5},\frac{2}{5})$

$(U,S,D)\times(L,M,R)$: $a_1=(\frac{6}{11},\frac{3}{11},\frac{2}{11})$, $a_2=(\frac{6}{11},\frac{3}{11},\frac{2}{11})$

#6. SPE: $a_2=(c,f)$, $a_1=b$.

Nash but not SP: $a_2=(c,e)$, $a_1=a$

#7. $\pi_1(v_1,\tilde{v}_1)=\left(Pr\{v_j\le\tilde{v}_1\}\right)^{N-1}\cdot v_1-B(\tilde{v}_1)$

$$=F^{N-1}(\tilde{v}_1)\cdot v_1-B(\tilde{v}_1)$$

$$\frac{\partial\pi_1(v_1,\tilde{v}_1)}{\partial\tilde{v}_1}=(N-1)F^{N-2}(\tilde{v}_1)f(\tilde{v}_1)\cdot v_1-B'(\tilde{v}_1)=0 \quad \text{at } \tilde{v}_1=v_1$$

$$B'(v_1)=(N-1)F^{N-2}(v_1)f(v_1)v_1$$

$$B(v)-B(\underline{v})=(N-1)\int_{\underline{v}}^{v}\cancel{(N-1)}F^{N-2}(v_1)f(v_1)v_1\,dv_1$$

$B(\underline{v})=0$, since the lowest type wins with prob. 0.

$$\Rightarrow B(v)=(N-1)\int_{\underline{v}}^{v}F^{N-2}(v_1)f(v_1)v_1\,dv_1.$$

#8. At any price $p$, a firm can distinguish whether the demand is $H$ or $L$, or its price is undercut by its rival. So this becomes a regular infinitely repeated game. Given $P$, $p \in [0,6]$

$$E(Q) = \frac{1}{2}(12-2p) + \frac{1}{2}(6-p) = 9 - \frac{3}{2}p$$

$$\pi = (9-\frac{3}{2}p)p, \quad \frac{\partial \pi}{\partial p} = 9-3p = 0 \Rightarrow p = 3$$

$$\Rightarrow \pi^m = (9-\frac{3}{2}\times 3)3 = \frac{27}{2}$$

$$\frac{\pi^m}{2}(1+\delta+\delta^2+\cdots) \geq \pi^m + 0 + 0 \cdots\cdots$$

$$\frac{1}{2}\frac{1}{1-\delta} \geq 1 \quad \Rightarrow \delta \geq \frac{1}{2}$$

Firms use the following trigger strategy:

1. start with $p = p^m = 3$.
2. Continue to set $p=3$ if all previous demands were positive.
3. Switch to $p=0$ if any previous demand was zero, or it undercut the other firm's price in a previous period.