ECON 845 **Midterm Practice Questions** **Fall 2022-2023**

October 30, 2022

#1. Consider a third-degree price discrimination model. A profit-maximizing monopoly sells its product to three markets with demands $p_1 = 10 - q_1$, $p_2 = 20 - q_2$ and $p_3 = 30 - q_3$, respectively. There is no arbitrage between these markets and the monopoly can charge different prices in different markets. The monopoly produces this product using one factory, but its cost of production is unknown to us. What we know is that the monopoly sells 6 times as much in market 3 than in market 1; i.e., $q_3 = 6q_1$. Find q_1, q_2 and q_3 for this monopoly.

#2. Consider a second-degree price discrimination model. A monopoly produces a product with constant marginal cost $c = 1$ and sells it in a market with two types of consumers. Half of the consumers are type-1 consumers, with willingness to pay equal to 3 for 1 unit. They have no desire for more than 1 unit of the product. Half of the consumers are type-2 consumers, with willingness to pay equal to 2 per unit for up to 3 units. They have no desire for more than 3 units. Type is unobservable to the monopoly. Design a menu of two packages $\{(T_1, Q_1), (T_2, Q_2)\}$, so that type-1 consumers will choose (T_1, Q_1) and type-2 consumers will choose (T_2, Q_2), and the monopoly's profit is maximized. Here, T_i denotes the total cost and Q_i denotes the total quantity in the package.

#3. Suppose that $2n$ firms are competing in quantities. Among these firms, there are n firms with marginal cost c_1 and n firms with marginal cost c_2. The market demand is given by $P = a - bQ$, where Q is the total quantity from these firms. Derive the Cournot equilibrium in this game and calculate the Herfindah concentration index in this market.

#4. Suppose that 2 firms are competing in quantities. Both firms are producing an identical product with same marginal cost c. The market demand is given by $P = a - bQ$, where Q is the total quantity produced by these two firms.

(a) Suppose that these two firms set their quantities simultaneously. Derive the Cournot (Nash) equilibrium in this game and calculate the Herfindah concentration index in this market.

(b) Suppose that firm 1 sets its quantity first; observing firm 1's quantity, firm 2 sets its quantity. Derive the Stackelberg (subgame-perfect) equilibrium in this game and calculate the Herfindah concentration index in this market.

(c) Which market has a higher Herfindah concentration index? Briefly explain why.

#5. Find all pure and mixed strategy Nash equilibria in the following games. (Note: (b) has many mixed strategy equilibria.)

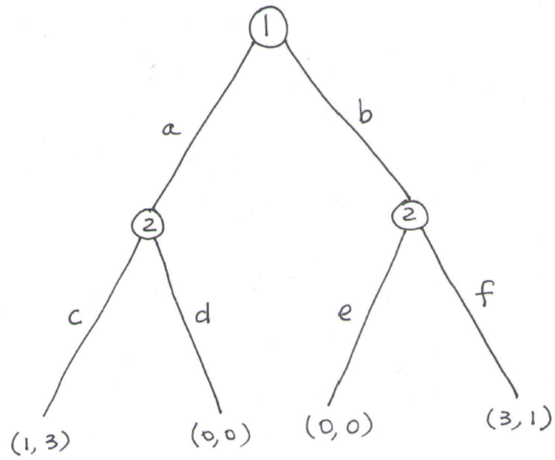
game (a)

	D	E	F
A	$2,3$	$0,0$	$2,1$
B	$0,0$	$3,2$	$0,1$
C	$1,0$	$0,0$	$1,5$

game (b)

	L	M	R
U	$1,1$	$0,0$	$0,0$
S	$0,0$	$2,2$	$0,0$
D	$0,0$	$0,0$	$3,3$

#6. Characterize the unique subgame-perfect equilibrium in the following extensive form game. Also, find one Nash equilibrium that is not subgame perfect in this game.



#7. Consider the following **All-Pay auction**. There are N bidders. Bidder i's valuation, denoted by v_i, is his private information, and is a random and independent draw from a distribution with c.d.f. $F(\cdot)$ and p.d.f. $f(\cdot)$, where $v_i \in [\underline{v}, \bar{v}]$. In the auction, each bidder submits a sealed bid. The bidder with the highest bid wins the object. Each bidder (including the losers) pays his own bid to the seller. Set up bidder 1's maximization problem and derive the first-order condition assuming a symmetric strictly increasing Bayesian equilibrium bidding function $b_i = B(v_i)$. Solve for the equilibrium bidding function.

#8. Suppose that two firms producing an identical product with zero marginal cost compete in prices for infinitely many periods. The demand in each period could be high ($Q = 12 - 2P$) with probability 0.5 and low ($Q = 6 - P$) with probability 0.5. Market demand is never observable by either firm, but a firm's sale in each period is observable to the firm itself. A firm can never observe the prices and sales of the other firm. Characterize the maximal profits for the firms that can be sustained in a subgame perfect equilibrium of this infinitely repeated game and find the condition under which the discount factor δ must satisfy.