QUEEN'S UNIVERSITY
Economics Department

ECON 811 Advanced Microeconomic Theory I Fall 2005

Instructor: Ruqu Wang

FINAL EXAMINATION

Thursday, 9:00am – 12:00noon, December 8, 2005

Instructions:

There are five questions in this exam.
You must answer ALL questions.
Do not forget to put your name and student I.D. number on each answer booklet.
Please read carefully and ASK if you have any questions!

Good Luck!

1. (15 points) Find all pure and mixed strategy Nash equilibria (if any) in the following normal-form game.

	E	F	G
A	$0,0$	$1,1$	$7,0$
B	$1,2$	$3,1$	$1,0$
C	$6,2$	$0,1$	$6,0$
D	$2,7$	$2,1$	$2,8$

2. (15 points) Consider the following entry game. There are N potential firms in a market. Each of these firms has equal marginal cost c and no fixed cost. However, firm i needs to pay a cost of entry F_i if it enters the market. The timing of the game is that, in stage 1, each firm decides simultaneously whether or not to enter the market; in stage 2, all entered firms compete in prices.

Assume that $0 < F_1 < F_2 < ... < F_N < \pi^m$, where π^m is the monopoly profit (not considering any cost of entry) for this market. Find all pure-strategy Nash equilibria in this game.

3. (15 points) Let $u_i(x)$ be the Bernoulli utility function of person i, and $r_A(x, u_i)$ be his absolute measure of risk aversion, where $i = 1, 2$ and x is his wealth. Show that if $r_A(x, u_2) < r_A(x, u_1)$ for all x, then there exists an increasing convex function $\phi(\cdot)$ such that $u_2(x) = \phi(u_1(x))$ for all x.

4. (25 points) Consider the usual T-period Rubinstein alternating-offer bargaining game. Two risk-neutral players share a pie of size 1. In the first period, player 1 makes an offer, and player 2 can accept or reject it. If player 2 accepts the offer, the game is over. If player 2 rejects the offer, player 2 makes a counter-offer in the second period. If player 1 accepts, the game is over. If player 1 rejects, player 1 makes the counter-offer in the third period. ... If no agreement is reached by the end of the T periods, each player gets 0.

Let δ be the players' common discount factor. The only difference from the usual Rubinstein bargaining game is that player 1 pays c each time he makes an offer. Player 2 has no cost of making offers.

Characterize the subgame perfect equilibrium in this game when

(a) $T = 3$, and

(b) $T = \infty$.

5. (30 points) Consider the following averaged-highest-two-price, sealed-bid, common-value auction. There are N bidders. Each bidder receives a private signal, θ, which is random and independent draw from a distribution with c.d.f. $F(\theta)$ and p.d.f. $f(\theta)$, where $\theta \in [\underline{\theta}, \bar{\theta}]$. Given θ_i, $1 = 1, 2, ..., N$, each bidder values the object at

$$V = \theta_1 + \theta_2 + ... + \theta_N.$$

In the auction, each bidder submits a sealed bid, and the bidder with the highest bid wins the object and pays the average of the highest and second highest bids. All other bidders pay nothing.

(a) Set up bidder i's maximization problem and derive the first-order condition assuming a symmetric Bayesian equilibrium bidding function $b_i = B(\theta_i)$.

(b) Solve for bidding function when the distribution $F(\cdot)$ is the uniform distribution $U[0, 1]$. *(Hint. Guess $B(\theta) = \beta\theta$.)*

QUEEN'S UNIVERSITY

Economics Department

ECON 811 Advanced Microeconomic Theory I Fall 2006

Instructor: Ruqu Wang

FINAL EXAMINATION

Thursday, 9:00am – 12:00noon, December 7, 2006

Instructions:

There are five questions in this exam.

You must answer ALL questions.

Do not forget to put your name and student I.D. number on each answer booklet.

Please read carefully and ASK if you have any questions!

Good Luck!

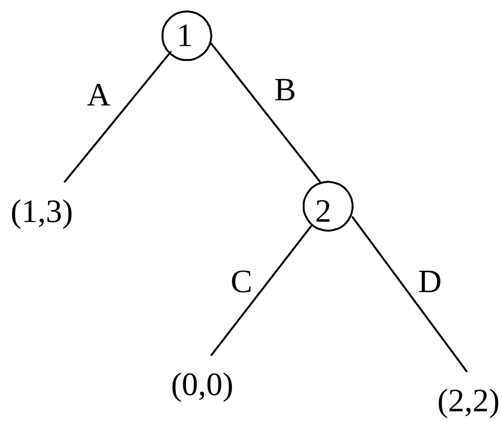
1. (15 points) Consider the following normal-form game:

	E	F	G
A	$1,0$	$1,0$	$1,8$
B	$3,0$	$0,3$	$0,1$
C	$0,0$	$3,3$	$0,1$
D	$0,3$	$0,0$	$6,1$

(a) (10 points) Find all pure and mixed strategy Nash equilibria (if any).

(b) (5 points) Find the pure-strategy minmax for each player (i.e., mixed strategies need not be considered).

2. (15 points) Consider the following dynamic game.



(a) (5 points) Find all subgame-perfect equilibria in this game.

(b) (10 points) Find all pure and mixed strategy Nash equilibria in this game. Identify which ones are subgame-perfect and which ones are not.

3. (25 points) Consider the following coordination game with switching costs. Suppose that the following stage game is played 3 times. In any period, actions in previous periods are known to the players. It costs a player $\varepsilon = 2.5$ each time he switches his action. (There is no cost for any action to any player in the first period.) Show that (A, A) in every period is the unique subgame-perfect equilibrium outcome.

	A	B
A	$2, 2$	$0, 0$
B	$0, 0$	$1, 1$

4. (20 points) Consider the usual T-period Rubinstein alternating-offer bargaining game, but with risk averse players. Two risk-averse players share \$1. In the first period, player 1 makes an offer, and player 2 can accept or reject it. If player 2 accepts the offer, the game is over. If player 2 rejects the offer, player 2 makes a counter-offer in the second period. If player 1 accepts, the game is over. If player 1 rejects, player 1 makes the counter-offer in the third period. ... If no agreement is reached by the end of the T periods, each player gets 0.

Let $U(x) = \sqrt{x}$ be the utility function for both players, where x is the amount a player gets in the negotiation. Let δ be the players' common discount factor.

(a) (10 points) Characterize the subgame perfect equilibrium when $T = 3$, and

(b) (10 points) Characterize the stationary subgame perfect equilibrium when $T = \infty$.

5. (30 points) Consider the following second-price, sealed-bid, common-value auction. There are N bidders. Each bidder receives a private signal, θ, which is a random and independent draw from a distribution with c.d.f. $F(\theta)$ and p.d.f. $f(\theta)$, where $\theta \in [\underline{\theta}, \bar{\theta}]$. Given θ_i, $1 = 1, 2, ..., N$, each bidder values the object at

$$V = \theta_1 + \theta_2 + ... + \theta_N.$$

In the auction, each bidder submits a sealed bid, and the bidder with the highest bid wins the object and pays the second highest bid. All other bidders pay nothing.

(a) (20 points) Set up bidder i's maximization problem and derive the first-order condition assuming a symmetric Bayesian equilibrium bidding function $b_i = B(\theta_i)$.

(b) (10 points) Solve for the equilibrium bidding function for any $F(\theta)$.

QUEEN'S UNIVERSITY

Economics Department

ECON 811 Advanced Microeconomic Theory I Fall 2007

Instructor: Ruqu Wang

FINAL EXAMINATION

Wednesday, 9:00am – 12:00noon, December 12, 2007

Instructions:

There are five questions in this exam.

You must answer ALL questions.

Do not forget to put your name and student I.D. number on each answer booklet.

Please read carefully and ASK if you have any questions!

Good Luck!

1. (20 points) Consider the following normal-form game with incomplete information:

	C	D
A	$2,2$	$0,0$
B	$x,0$	$1,1$

where x is equal to 0 with probability 0.5 and equal to 3 with probability 0.5. Player 1 knows the value of x but player 2 knows its distribution only. By introducing Nature, convert the game into a complete information game in extensive form. (You can choose to have either player 1 or player 2 move first.) Find all **pure-strategy** weak Perfect Bayesian equilibria in this game.

2. (20 points) Consider the usual Rubinstein alternating-offer bargaining game but with an opt-out option. Two risk-neutral players want to share a pie of size 1. The players make offers alternatively. Player 1 makes the first offer; player 2 has three options. Option 1 is to accept the offer and end the game. Option 2 is to reject the offer and make a counter-offer in the next period. Option 3 is to opt out of the negotiation and end the game. If player 2 decides to make a counter-offer in period 2, then in period 2 after seeing the counter-offer, player 1 has the same three options above. ... The players need to reach an agreement or opt out by the end of period T; if no agreement is reached or no one opts out by then, they both get zero. If any player opts out in a period, then in that period, player 1 gets u_1^*, player 2 gets u_2^*, with $u_1^*, u_2^* \in (0,1)$, and $u_1^* + u_2^* < 1$. Let δ be the players' common discount factor.

(a) **(10 points)** Find the unique subgame perfect equilibrium when $T = 2$;

(b) **(10 points)** Find the stationary subgame perfect equilibrium when $T = \infty$.

3. (20 points) Consider the following mechanism design problem. There are one risk neutral seller and one risk neutral buyer. The seller has two identical but indivisible objects to sell. These objects have no value to the seller. With probability μ, the buyer (named type 1 buyer) wants at most one unit, with willingness to pay v_1. With probability $1-\mu$, the buyer (named type 2 buyer) wants at most two units, with willingness to pay for each unit being v_2, where $v_2 > v_1$. Design an optimal direct mechanism that maximizes the seller's expected revenue. (*Hint. Define p_i as the expected number of units a type i buyer will receive, which then can be greater than 1.*)

4. (20 points) Consider a linear city of length 1 on [0,1]. There are a total of one unit of consumers. These consumers live uniformly in the first half of the city, from 0 to $\frac{1}{2}$. Nobody lives in $[\frac{1}{2}, 1]$. (Imagine that $[0, \frac{1}{2}]$ is city center and $[\frac{1}{2}, 1]$ is suburb.) There are two firms producing an identical good, with firm 1 located at 0 and firm 2 located at 1. Firm i has a marginal cost of production c_i, $i = 1, 2$. Consumers have linear transportation cost; it costs t per unit distance travelled. A consumer wants either 0 or 1 unit of the good. Assume that each consumer's willingness to pay for a unit of the good, w, is high enough that each consumer will buy from some firm and each firm will have some demand in equilibrium. Firms set prices simultaneously; seeing the prices, consumers decide where to buy. Find the equilibrium prices in this game.

5. (20 points) Consider the following private-value auction with a special payment rule. There are N risk neutral bidders bidding for an object. Bidder i's valuation, v_i, is a random and independent draw from a distribution with c.d.f. $F(v_i)$ and p.d.f. $f(v_i)$., $v_i \in [\underline{v}, \overline{v}]$, $i = 1, 2, ..., N$. This v_i is the private information of bidder i. In the auction, each bidder submits a sealed bid, and the bidder with the highest bid wins the object. This winner pays his bid to the seller. However, other bidders (losers) pay their own bids to the winner. (For example, suppose that $N = 3$ and $b_1 = 4$, $b_2 = 5$, $b_3 = 2$. Then bidder 2 wins the object and pay \$5 to the seller; however, bidder 2 also receives \$4 from bidder 1 and \$2 from bidder 3. Bidders 1 and 3 get nothing. The seller receives \$5.)

(a) (**10 points**) Set up bidder i's maximization problem.

(b) (**10 points**) Derive the first-order condition in the Bayesian equilibrium assuming a symmetric and strictly increasing bidding function $b_i = B(v_i)$. Do NOT solve for the bidding function.

QUEEN'S UNIVERSITY

Economics Department

ECON 811 Advanced Microeconomic Theory I Fall 2008

Instructor: Ruqu Wang

FINAL EXAMINATION

Wednesday, 9:00am – 12:00noon, December 17, 2008

Instructions:

There are five questions in this exam.

You must answer ALL questions.

Put your name and student I.D. number on each answer booklet.

Please read carefully and ASK if you have any questions!

Good Luck!

1. (10 points) Consider the following cooperative game. There are four risk neutral players. Any one player can earn 0 by himself. Any two players can earn 1 together. Any three players can earn 1 together. All four players can earn 2 together. (That is, each pair of players can earn 1. One extra player can earn nothing.) First, construct the characteristic functions; that is, $v(\{i\}) =?$ $v(\{i,j\}) =?$ ... Then, find the core for this game.

2. (20 points) Consider the following dynamic game of a lion and N fat guys, where $N > 2$. The lion is attacking these N fat guys, and each fat guy has to decide **sequentially** whether or not to fight the lion. (The lion is not a player of this game.) If at least two fat guys decide to fight the lion, the lion will be defeated, and each fighter earns a payoff of -5 (due to the cost of fighting); every non-fighter free rides and earns 0 . If there is only one or none fat guy deciding to fight, then the lion wins and eats everyone; every player earns a payoff of -100. So in this game, player 1 (fat guy #1) first chooses between "F" (fight) and "NF" (not fight). Then player 2 observes the choices of the previous player and decides "F" or "NF". ... ... The fight happens after the N^{th} player makes his decision.

(a) **(10 points)** Find a subgame perfect equilibrium in this game.

(b) **(10 points)** Find a Nash equilibrium which is not subgame perfect in this game.

3. (25 points) Consider the following private-value auction with a special payment rule. There are N risk neutral bidders bidding for an object. Bidder i's valuation, v_i, is a random and independent draw from a distribution with c.d.f. $F(v_i)$ and p.d.f. $f(v_i)$., $v_i \in [\underline{v}, \overline{v}]$, $i = 1, 2, ..., N$. This v_i is the private information of bidder i. In the auction, each bidder submits a sealed bid, and the bidder with the highest bid wins the object. This winner pays his bid to the seller **and to all other bidders**. That is, the winner pays out N times his bid and wins the object.

(a) (**15 points**) Set up bidder i's maximization problem.

(b) (**5 points**) Derive the first-order condition in the Bayesian equilibrium assuming a symmetric and strictly increasing bidding function $b_i = B(v_i)$.

(c) (**5 points**) Solve for the equilibrium bidding function when F is the uniform distribution on [0,1]. (Hint: Guess that the bidding function is linear.)

4. (20 points) Suppose that there is a lottery which costs \$1 per ticket but pays out a random x, where $x \in (-\infty, \infty)$ is distributed according to c.d.f. F. Assume that person B is strictly more risk averse than person A and that both persons have the same wealth W to begin with. Suppose that person A optimally buys $a > 0$ tickets and that person 2 optimally buys b tickets. Show that $a > b$. (You will get 10 points for formulating the problem correctly and 10 points for the proof. You can use any of the definitions for more risk aversion we discussed in class.)

5. (25 points) Consider the following Rubinstein alternating-offer bargaining game with a random end time. Two risk-neutral players want to share a pie of size 1. Player 1 makes the first proposal. If the proposal is accepted, the game ends and each player receives what is in the proposal. If the proposal is rejected, however, then with probability $1 - p$ the game ends and each player ends up with 0; with probability p, the game continues to the next period and player 2 makes the final offer which player 1 can accept or reject. As usual, both players end up with 0 if the proposal is rejected. (Therefore, this game lasts for at most two periods.) Let δ be the players' common discount factor.

(a) **(5 points)** Find the subgame perfect equilibrium of this game when in period 1 both players do not know for sure whether the game will end in one period.

(b) **(10 points)** Now suppose that, in period 1, player 2 knows whether or not the game will end in one period BEFORE he makes his "accept" or "reject" decision while player 1 knows only the probability. Find a perfect Bayesian equilibrium for this game.

(c) **(10 points)** Finally, suppose that in period 1 the player 1 knows whether the game will end in that period BEFORE he proposes, while player 2 knows only the probability when he makes his "accept" or "reject" decision. Find a perfect Bayesian equilibrium for this game.

QUEEN'S UNIVERSITY

Economics Department

ECON 811 Advanced Microeconomic Theory I Fall 2009

Instructor: Ruqu Wang

FINAL EXAMINATION

Wednesday, 9:00am – 12:00noon, December 16, 2009

Instructions:

There are five questions in this exam.

You must answer ALL questions.

Put your name and student I.D. number on each answer booklet.

Please read carefully and ASK if you have any questions!

Good Luck!

1. (10 points) Consider the following cooperative game. There are N risk neutral players. Any 1 player can earn 1 by himself. Any 2 players can earn 2 together. ... Any n players can earn n together. ... Finally, N players can earn N together. First, construct the characteristic functions; that is, $v(\{i_1\}) =?$ $v(\{i_1, i_2\}) =?$ ... Then, find the core for this cooperative game.

2. (20 points) For the following 2×2 game,

(a) **(10 points)** find all pure-strategy and mixed-strategy Nash equilibria;

(b) **(10 points)** find the minmax for each player.

	C	D
A	$2, 2$	$0, 0$
B	$3, 0$	$1, 1$

3. (20 points) Consider a boy trying to signal his love to a girl by taking some painful task which does not have any direct economic benefits. Both persons are risk neutral.

∘ The boy can be of two types: Y (Yes, he loves the girl) or N (No, he does not love the girl). Let μ be the probability that the boy is of type Y.

∘ There is one task, A, that the boy can take. Let C denote the disutility of taking this task, with $C > 0$. The disutility of taking 0 task is 0.

∘ The boy's utility of staying with the girl is B_Y and B_N, respectively, for types Y and N, and $B_Y > B_N > 0$. The boy's utility of not staying with the girl is 0.

∘ The girl's utility of staying with a type Y boy is G, where $G > 0$. Her utility staying with a type N boy is $-G$. Her utility of not staying with the boy is 0.

The timing of this signaling game is as follows. First, nature chooses the type of the boy and it becomes this boy's private information. Second, the boy chooses to take the task or not. Third, observing the task chosen by the boy, the girl decides whether or not to stay with the boy.

Characterize the strategies and beliefs in each separating and pooling equilibrium in this game and analyze the conditions under which it is an equilibrium.

4. (25 points) Consider the following special **All-Pay Common-Value** auction. There are N bidders. Each bidder receives a private signal, θ, which is a random and independent draw from a distribution with c.d.f. $F(\theta)$ and p.d.f. $f(\theta)$, where $\theta \in [\underline{\theta}, \bar{\theta}]$. Given θ_i, $1 = 1, 2, ..., N$, each bidder values the object at

$$V = \theta_1 + \theta_2 + ... + \theta_N.$$

In the auction, each bidder submits a sealed bid. The bidder with the highest bid wins the object and pays his bid to the seller. The losers (i.e., all other bidders) pay half of their respective bids to the seller. (For example, suppose that $N = 3$, and the bids are $b_1 = 2$, $b_2 = 6$, $b_3 = 9$. Then bidder 3 wins the object and pays 9. Bidder 1 pays 1 and gets nothing. Bidder 2 pays 3 and gets nothing.)

(a) **(15 points)** Set up bidder i's maximization problem and derive the first-order condition assuming a symmetric strictly increasing Bayesian equilibrium bidding function $b_i = B(\theta_i)$.

(b) **(10 points)** Explain why $B(\underline{\theta}) = 0$ and solve for the equilibrium bidding function.

5. (25 points) Consider the following two-period bargaining game. Two risk-neutral players want to share a pie of size 1.

In period 1, both players make demands simultaneously. Let a_1 and a_2 be the demands made by players 1 and 2, respectively. (a_i denotes the share player i demands.) If $a_1 + a_2 \leq 1$, then the game is over and player i obtains $a_i + \frac{1-a_1-a_2}{2}$, $i = 1, 2$. (That is, when the total demand does not exceed 1, each player gets his demand plus half of the surplus.) If $a_1 + a_2 > 1$, then the game goes to the second period.

In period 2, the player who made the smaller first-period demand gets the chance to make an offer, and the other player can choose to accept or reject. For example, if player i makes offer b_i and it is accepted by j, then the pie is divided accordingly: player i gets b_i and player j gets $1 - b_i$. If the offer is rejected, then both players get 0. (If the two players make identical offers in period 1 and they add up greater than 1, then each of them gets to make the second period offer with probability 50%.)

Assume that the discounting factor is $\delta \in (0, 1)$. Characterize all pure-strategy subgame-perfect Nash equilibria in this game.

QUEEN'S UNIVERSITY

Economics Department

ECON 811 Advanced Microeconomic Theory I Fall 2018

Instructor: Ruqu Wang

FINAL EXAMINATION

Monday, 9:00am – 12:00noon, December 10, 2018

Instructions:

There are five questions in this exam.

You must answer ALL questions.

Put your name and student I.D. number on each answer booklet.

Please read carefully and ASK if you have any questions!

Good Luck!

1. (20 points) For the following 3×3 game, find all pure-strategy and mixed-strategy Nash equilibria.

	D	E	F
A	$1,2$	$3,5$	$5,3$
B	$5,5$	$3,2$	$1,3$
C	$2,1$	$2,2$	$2,8$

2. (15 points) Consider an infinitely repeated game of the following stage game:

	C	D
A	$4,3$	$0,4$
B	$6,0$	$1,1$

Let δ be the discount factor for both players. Determine the condition on δ under which (4,3) can be sustained as the per-period payoffs for the players in a subgame-perfect equilibrium.

3. (20 points) (a) **(5 points)** Consider the following multi-period, alternating-offer bargaining game. Two risk-neutral players want to share a pie. There is no discounting, but the size of the pie is reduced by 0.2 every period. That is, the size of the pie is equal to 1 in period 1, 0.8 in period 2, ..., 0 in period 6. Therefore, if these two players cannot reach an agreement by the end of period 5, both players will get 0.

The timing of this alternating-offer bargaining game is as follows. In period 1, player 1 makes an offer, and player 2 accepts or rejects. If the offer is rejected, in period 2, player 2 makes an offer and player 1 accepts or rejects. etc. Characterize all pure-strategy subgame-perfect Nash equilibria in this game.

(b) **(15 points)** Now suppose that the size of the pie is reduced by 10% every period, and there is still no discounting. Characterize the stationary pure-strategy subgame-perfect Nash equilibria in this game.

4. (15 points) Consider a public project that needs \$100 to complete. The project is a common good; Once completed, it has a value of \$200 to every player in the game. There are three players who make contributions to this project sequentially. The timing of this dynamic game is as follows. Player 1 contributes first (amount denoted by x_1). Observing x_1, player 2 contributes next (amount denoted by x_2). Observing x_1 and x_2, player 3 contributes last (amount denoted by x_3). If $x_1 + x_2 + x_3 \geq 100$, then the project will be completed and any extra money will be thrown away. If $x_1 + x_2 + x_3 < 100$, then the project will not be completed, and all contributions will be thrown away. Find the subgame perfect equilibrium in this game. Are there any Nash equilibria which are not subgame perfect?

5. (30 points)

Consider the following **First-Price, Common-Value** auction. There are N bidders. Each bidder receives a private signal, θ, which is a random and independent draw from a distribution with c.d.f. $F(\theta)$ and p.d.f. $f(\theta)$, where $\theta \in [\underline{\theta}, \bar{\theta}]$. Given θ_i, $1 = 1, 2, ..., N$, each bidder values the object at

$$V = \theta_1 \theta_2 \cdots \theta_N.$$

In the auction, each bidder submits a sealed bid. The bidder with the highest bid wins the object and pays his bid to the seller. The losers (i.e., all other bidders) pay nothing.

(a) **(15 points)** Set up bidder i's maximization problem and derive the first-order condition assuming a symmetric strictly increasing Bayesian equilibrium bidding function $b_i = B(\theta_i)$.

(b) **(15 points)** Determine the initial value $B(\underline{\theta})$ and solve for the equilibrium bidding function.

QUEEN'S UNIVERSITY

Economics Department

ECON 811 Advanced Microeconomic Theory I Fall 2020

Instructor: Ruqu Wang

FINAL EXAMINATION

Friday, 9:00am – 1:00pm, December 11, 2020

Instructions:

There are four questions in this exam.

You must answer ALL questions.

Please upload your answers to OnQ by 1:15pm.

Please read carefully and ask in MS Teams if you are unclear about the questions.

Good Luck!

1. (20 points) For the following 3×3 game, find all pure-strategy and mixed-strategy Nash equilibria.

	D	E	F
A	$1,2$	$2,5$	$3,6$
B	$3,3$	$1,2$	$2,1$
C	$2,0$	$3,3$	$4,2$

2. (20 points) Consider the following multi-period, alternating-offer bargaining game. **Three** risk-neutral players want to share a pie. They have T periods to reach an agreement. If no agreement is reached by the end of the T^{th} period, then everyone will get zero. In each period, one player proposes a division of the pie, and the other two players say YES or NO sequentially. If both of these players say YES, then the agreement is reached. If one of them says NO, then the agreement is not reached and the bargaining proceeds to the next period.

Players take turns making offers in the following order: 1, 2, 3, 1, 2, 3, ... Player 1 makes the offer in periods 1, 4, 7, ..., and in these periods player 2 says YES or NO first, and then player 3 says YES or NO. Player 2 makes the offer in periods 2, 5, 8, ..., and in these periods player 3 says YES or NO first, and then player 1 says YES or NO. Player 3 makes the offer in periods 3, 6, 9, ..., and in these periods player 1 says YES or NO first, and then player 2 says YES or NO. *(For example, in period 1, player 1 proposes the division* (x_{11}, x_{12}, x_{13})*, where* x_{ti} *denotes how much player i gets in period t. Then player 2 says YES or NO, and then player 3 says YES or NO. If both players 2 and 3 say YES, the agreement is reached. Otherwise, the bargaining goes to period 2.)*

Let the discount factor be $\delta \in (0,1)$ for the players.

(a) **(10 points)** Characterize the subgame perfect equilibrium in this game when $T = 3$.

(b) **(10 points)** Characterize the stationary subgame perfect equilibrium in this game when $T = \infty$.

3. (20 points) Calculate the mixed strategy Minmax for players 1 and 2 in the following normal-form game:

	C	D
A	$4,6$	$0,4$
B	$1,0$	$3,2$

4. (40 points) Consider the following **Correlated-Value** auction. There are 2 bidders. Each bidder receives a private signal, θ, which is a random and independent draw from a distribution with c.d.f. $F(\theta)$ and p.d.f. $f(\theta)$, where $\theta \in [\underline{\theta}, \bar{\theta}]$. Given θ_i, $1 = 1, 2$, bidder 1 values the object at

$$V_1 = 2\theta_1 + \theta_2,$$

and bidder 2 values the object at

$$V_2 = 2\theta_2 + \theta_1,$$

(a) **(20 points)** Suppose that a **first-price, sealed-bid auction** with no reserve price is held. Set up bidder 1's maximization problem and derive the first-order condition assuming a symmetric strictly increasing Bayesian equilibrium bidding function $b_i = B(\theta_i)$, $i = 1, 2$. Determine the initial value $B(\underline{\theta})$ and solve for the equilibrium bidding function.

(b) **(20 points)** Now suppose that a **second-price, sealed-bid auction** with no reserve price is held. Assume that bidders use a symmetric strictly increasing Bayesian equilibrium bidding function $b_i = \beta(\theta_i)$, $i = 1, 2$. Determine this bidding function by setting the surplus of a bidder with signal θ to zero if he wins and pays his bid $\beta(\theta)$, i.e., if the other bidder happens to bid exactly the same amount $\beta(\theta)$ in the equilibrium. Prove that the bidding function you obtained is indeed an equilibrium bidding function.