ECONOMICS 811 Midterm Examination Fall 2005

11:30am – 1:00pm, October 27, 2005

1. **(20 points)** Consider the following choice structure.

$$\mathbf{X} = \{a, b, c, d\}, \quad \mathcal{B} = \{\{a, b\}, \{b, c\}, \{c, d\}, \{d, a\}, \{a, b, c, d\}\},$$

and $C(\{a, b\}) = \{a\}$, $C(\{b, c\}) = \{b\}$, $C(\{c, d\}) = \{c\}$, and $C(\{d, a\}) = \{d\}$. What can $C(\{a, b, c, d\})$ be to satisfiy the Weak Axiom of Revealed Preference (WARP)? If WARP cannot be satisfied given the information above, explain why.

2. **(20 points)** Let $u(x)$ be a continuous utility function representing a locally nonsatiated and convex preference, where x is the consmption vector. For any price vector $p \gg 0$, denote $e(p, u)$ as the expenditure function. Assume that $u(x)$ is homogeneous of degree $\frac{1}{2}$ in x. Is $e(p, u)$ homogeneous in u? If so, of what degree? Explain.

3. **(20 points)** Prove that the cost function $C(w_1, w_2, q)$ is concave in (w_1, w_2).

4. **(20 points)** Let the profit function of a firm be $\pi(p, w_1, w_2)$. Suppose that

$$\pi(p, 1, 1) = p^3.$$

That is, we know the firm's profit function only at $w_1 = w_2 = 1$. Derive the sum of optimal inputs $z_1 + z_2$ at $w_1 = w_2 = w$.

5. **(20 points)** Assume that a person's preference over lotteries is rational, satisfies the Independence Axiom, but is <u>NOT</u> continuous. Let L_1, L_2, and L_3 be three simple lotteries. Furthermore,

$$L_1 \succ L_2.$$

Suppose that there exists $\alpha \in (0, 1)$, such that

$$\alpha L_1 + (1 - \alpha) L_2 \sim L_3.$$

Show that

$$L_1 \succ L_3 \quad \text{and} \quad L_3 \succ L_2.$$

(Note: You can use any form of the Independence Axiom that we discussed in class.)

ECONOMICS 811 Midterm Examination Fall 2006

2:30pm – 4:00pm, October 25, 2006

1. **(20 points)** Consider the following choice structure:

$$\mathbf{X} = \{a, b, c\}, \quad \mathcal{B} = \{\{a, b\}, \{b, c\}, \{c, a\}\},$$

$C(\{a, b\}) = \{a\}$, and $C(\{b, c\}) = \{c\}$. Write down all possible values of $C(\{c, a\})$ that can satisfiy the Weak Axiom of Revealed Preference (WARP).

2. **(20 points)** Let $u(x)$ be a continuous utility function representing a locally nonsatiated and strictly convex preference, where x is the consmption vector. For any price vector $p \gg 0$, denote $e(p, u)$ as the expenditure function. Assume that $u(x)$ is concave in x. Show that $e(p, u)$ is convex in u.

3. **(20 points)** Prove that the cost function $C(w_1, w_2, q)$ is concave in (w_1, w_2).

4. **(20 points)** Let the profit function of a firm be $\pi(p, w_1, w_2)$. Suppose that

$$\pi(p, 1, 1) = p^3.$$

That is, we know the firm's profit function only at $w_1 = w_2 = 1$. Derive the sum of optimal inputs $z_1 + z_2$ at $w_1 = w_2 = w$.

5. **(20 points)** Assume that a person's preference ($\succsim$) over all lotteries is rational, continuous, and satisfies the Independence Axiom. Let L_1, L_2, L_3 and L_4 be four simple lotteries satisfying

$$L_1 \succ L_2 \succ L_3 \succ L_4.$$

Show that there must exist an $\alpha \in (0, 1)$ such that

$$\alpha L_1 + (1 - \alpha)L_4 \sim \alpha L_2 + (1 - \alpha)L_3.$$

ECONOMICS 811 Midterm Examination Fall 2007

1:00pm – 2:20pm, October 24, 2007

1. **(20 points)** Suppose that an individual's utility function is given by $U(x) = \max\{x_1, x_2\}$, where $x = (x_1, x_2)$. Is the demand correspondence $x(p_1, p_2, w)$ always convex? Explain.

2. **(30 points)** Let $u(x)$ be a continuous utility function representing a locally nonsatiated and strictly convex preference, where x is the consmption vector. For any price vector $p \gg 0$, and budget $w > 0$, denote $v(p, w)$ as the indirect utility function. Assume that $u(x)$ is concave in x. Show that $v(p, w)$ is concave in w.

3. **(20 points)** Let $\pi(p, w_1, w_2)$ be the profit function of a firm. Suppose that $\pi(p, w_1, w_2)$ is homogeneous of degree -1 in (w_1, w_2) and that

$$\pi(1, 1, w_2) = \frac{1}{\sqrt{w_2}}.$$

Find $\pi(p, w_1, w_2)$.

4. **(30 points)** Consider two risk averse persons. Person i has a Bernoulli utility function $u_i(x)$, $i = 1, 2$. Suppose that person 1 is more risk averse than person 2 (according to any of the equivalent definitions we discussed in class), and that $\underline{x} < \overline{x}$. Show that, if $u_1(\underline{x}) = u_2(\underline{x})$ and $u_1(\overline{x}) = u_2(\overline{x})$, then $\forall x \in (\underline{x}, \overline{x})$, $u_1(x) \geq u_2(x)$.

ECONOMICS 811 Midterm Examination Fall 2008

2:30pm – 3:50pm, October 16, 2008

1. **(20 points)** Suppose that a consumer's preference satisfies the Weak Axiom of Revealed Preference and his demand correspondence is always a singleton. Show that if he does not buy any good 1 at the current price, he will not buy any good 1 when good 1's price has increased, keeping other parameters (such as the prices of other goods and his income) constant.

2. **(20 points)** Let $u(x)$ be a continuous utility function representing a locally nonsatiated and strictly monotone preference, where x is the consmption vector. Assume that $u(x)$ is strictly convex in x. For any price vector $p \gg 0$, and wealth $w > 0$, show that this consumer will spend all of his wealth on one of the commodities only; that is, it is never optimal for him to spend money on two or more commodities at the same time.

3. **(20 points)** Let $u(x_1, x_2)$ be a continuous utility function representing a locally nonsatiated and strictly convex preference, where (x_1, x_2) is the consmption vector. Assume that the expenditure function $e(p_1, p_2, u)$ is linear in (p_1, p_2). Show that this preference is of Leontief type. That is, there exists a function $f(\cdot)$, such that the optimal consumption bundle satisfies $x_1 = f(x_2)$ regardless of the prices.

4. **(20 points)** Let $C(w_1, w_2, q) = w_1^{\frac{1}{3}} w_2^{\frac{2}{3}} q^2$ be the cost function of a firm, where w_1, w_2 are the input prices, q is the output level. Find this firm's production function $q = f(z_1, z_2)$, where z_1, z_2 are the inputs.

5. **(20 points)** Assume that a person's preference ($\succsim$) over the set of all lotteries is rational, satisfies the Independence Axiom, but **may not be continuous**. Let L_1, L_2, L_3 and L_4 be four simple lotteries satisfying

$$L_1 \succ L_2 \succ L_3 \succ L_4.$$

Show that if

$$\alpha L_1 + (1 - \alpha) L_3 \sim \beta L_2 + (1 - \beta) L_4,$$

then $\alpha < \beta$.

ECONOMICS 811 Midterm Examination Fall 2009

1:00pm – 2:20pm, October 16, 2009

1. **(20 points)** Consider the following choice structure:

$$\mathbf{X} = \{a, b, c, d\}, \quad \mathcal{B} = \{\{a, b\}, \{b, c\}, \{c, d\}, \{d, a\}, \{a, b, c\}\},$$

$C(\{a, b\}) = \{a\}$, $C(\{b, c\}) = \{b\}$, $C(\{c, d\}) = \{c\}$, and $C(\{d, a\}) = \{d\}$. Write down all possible values of $C(\{a, b, c\})$ that can satisfiy the Weak Axiom of Revealed Preference (WARP).

2. **(20 points)** Let $v(p_1, p_2, w)$ be the indirect utility function for a consumer's utility maximization over two goods, where p_1 and p_2 are the prices for the two goods and w is the consumer's wealth. Show that at any point where $x_1(p_1, p_2, w) = x_2(p_1, p_2, w)$, it must be true that

$$\frac{\partial v(p_1, p_2, w)}{\partial p_1} = \frac{\partial v(p_1, p_2, w)}{\partial p_2}.$$

3. **(20 points)** Prove that the cost function $C(w_1, w_2, q)$ is concave in (w_1, w_2).

4. **(20 points)** Assume that a person's preference ($\succsim$) over the set of all lotteries is rational, satisfies the Independence Axiom, but **may not be continuous**. Let L_1, L_2, and L_3 be three simple lotteries satisfying

$$L_1 \succ L_2 \succ L_3.$$

Show that if

$$\frac{1}{2}L_1 + \frac{1}{2}L_3 \succsim L_2,$$

then

$$\frac{3}{4}L_1 + \frac{1}{4}L_3 \succ L_2.$$

5. **(20 points)** Prove that any risk averse person prefers lottery $(-1, 1; \frac{1}{2}, \frac{1}{2})$ to $(-2, 2; \frac{1}{2}, \frac{1}{2})$. That is, prove that for any concave utility function $u(\cdot)$, the following holds:

$$\frac{1}{2}u(W-1) + \frac{1}{2}u(W+1) \geq \frac{1}{2}u(W-2) + \frac{1}{2}u(W+2),$$

where W is the person's initial wealth. (Note: you cannot just draw a figure to show this. You need to prove it analytically.)

ECONOMICS 811 **Midterm Examination** **Fall 2018**

11:30am – 1:00pm, October 18, 2018

1. **(20 points)** Consider the following choice structure:

$$\mathbf{X} = \{a, b, c, d\}, \quad \mathcal{B} = \{\{a, b\}, \{c, d\}, \{a, c\}, \{a, b, c, d\}\},$$

$C(\{a, b\}) = \{a\}$, $C(\{c, d\}) = \{c\}$, and $C(\{a, c\}) = \{a\}$. Suppose that the choice rule satisfies the Weak Axiom of Revealed Preference (WARP). Prove that $C(\{a, b, c, d\}) = \{a\}$.

2. **(20 points)** Suppose that the production function for a single output $y = f(z)$ is homogeneous of degree $\frac{1}{2}$ in z, where z is the input vector. Prove that the profit function $\pi(p, w)$ is homogeneous of degree 2 in p. (Note that profit is equal to revenue minus cost.)

3. **(20 points)** Let $y = f(z)$ be the production function of a firm, where y is the single output and z is the single input. Show that if $f(z)$ is convex in z, then the cost function $C(w, q)$ is concave in q, where w denotes the input price and q denotes the firm's output.

4. **(20 points)** Let $h_i(p, u)$ be a person's Hicksian demand for good i, where $p = (p_1, ..., p_L)$ is the price vector and u is the utility level. Show that

$$p_1 \frac{\partial h_i}{\partial p_1} + p_2 \frac{\partial h_i}{\partial p_2} + ... + p_L \frac{\partial h_i}{\partial p_L} = 0.$$

5. **(20 points)** Assume that a person's preference ($\succsim$) over the set of all lotteries is rational, satisfies the Independence Axiom, but **may not be continuous**. Let L_1, L_2, and L_3 be three simple lotteries satisfying

$$L_1 \sim 0.5L_2 + 0.5L_3$$

and

$$L_2 \sim 0.5L_1 + 0.5L_3$$

Show that $L_1 \sim L_2 \sim L_3$.

ECONOMICS 811 Midterm Examination Fall 2021

10:00am – 11:30am, November 28, 2021

Answer all questions. Please write on one side of the printing paper. Write your name, student I.D. and page number on each page. Hand in your answers and scrap work.

1. **(20 points)** Consider the following choice structure:

$$\mathbf{X} = \{a, b, c\}, \quad \mathcal{B} = \{\{a, b\}, \{b, c\}, \{a, c\}, \{a, b, c\}\},$$

$C(\{a, b\}) = \{a\}$, $C(\{b, c\}) = \{b\}$. List all possible answers for $C(\{a, c\})$, such that the choice rule satisfies the Weak Axiom of Revealed Preference (WARP). Explain your answers.

2. **(20 points)** Let x be the commodity vector, p be the price vector and w be the consumer's wealth. Suppose that utility function $u(x)$ is homogeneous of degree 2 in x. Then the indirect utility function $v(p, w)$ is homogeneous of what degree in w? Show your analysis.

3. **(20 points)** Consider a firm's profit maximization problem. Let y be the output, p be the output price, z be the input vector, w be the input price vector and $y = f(z)$ be the production function. Let $y = y(p, w)$ be the profit maximizing output and $z = z(p, w)$ be the profit maximizing input vector. Suppose that the profit function $\pi(p, w)$ is homogeneous of degree 2 in p. Show that $\pi(p, w) = \frac{1}{2} p y(p, w)$.

4. **(20 points)** Let y be the output, z be the input vector and w be the input price vector. Suppose that the production function $y = f(z)$ is convex and homogeneous in z. Show that the cost function $C(q, w)$ must be concave and homogeneous in q, where q is the output.

5. **(20 points)** Assume that a person's preference ($\succsim$) over the set of all lotteries is rational, satisfies the Independence Axiom, but **may not be continuous**. Let L_1, L_2, and L_3 be three simple lotteries satisfying

$$L_1 \sim \frac{1}{3} L_2 + \frac{2}{3} L_3$$

and

$$L_1 \sim \frac{2}{3} L_2 + \frac{1}{3} L_3$$

Show that $L_1 \sim L_2 \sim L_3$.