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Schumpeterian Restructuring

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Abstract

We develop a Shumpeterian theory of business cycles that relates job creation, job destruction and wages over the cycle to the processes of firm restructuring, innovation and implementation that drive long–run growth. Due to incentive problems, production workers are employed via relational contracts and experience involuntary unemployment. Job destruction and firm turnover are counter–cyclical, but labour productivity growth and job creation are procyclical. Endogenous fluctuations in job creation on the intensive margin are the dominant source of changes in employment growth. Our framework also highlights the countercyclical forces on wages due to restructuring, and illustrates the relationship between the cyclicality of wages and long–run productivity growth.

Key Words:
JEL: E0, E3, O3, O4

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"[Depressions] are the means to reconstruct each time the economic system on a more efficient plan. But they inflict losses while they last ... before the ground is clear and the way paved for new achievement ..."
"The proper role of a healthily functioning economy is to destroy jobs and put labor to use elsewhere. Despite this truth, layoffs and firings will always sting, as if the invisible hand of enterprise has slapped workers in the face."
Joseph A. Schumpeter (1934).

1 Introduction

Schumpeter's ideas have had a lasting impact on macroeconomic thought. For example, the process of "growth through creative destruction" has become central to theories of endogenous long–run growth.[1] However, as others have emphasized before (e.g. Caballero and Hammour, 1994), the Schumpeterian paradigm is as much about the consequences of the growth process for employment over the business cycle as it is about the determinants of long–run growth. Following the development of alternative measures of job creation and job destruction by Blanchard and Diamond (1990) and Davis and Haltiwanger (1992), the Schumpeterian paradigm has influenced the development of the macro labor literature. Although Schumpeter's writing emphasizes the feed–back between changes in the labor market and fluctuations at the aggregate level, formal work has tended to treat these separately. Recent theoretical work has gone a long way in understanding the effects of productivity changes on the labor market, but treats the process driving these movements as entirely exogenous. Conversely, a recent strand of the quantitative macroeconomics literature and much of the policy literature comparing the recent performance of OECD countries, emphasizes the role of labor market frictions in affecting economic growth.[2]

In this article we emphasize the two–way linkages between job creation, job destruction and wages over the business cycle, and the process of innovation, implementation and firm turnover that drive long run growth. We study the equilibrium of an endogenous growth model exhibiting two key features: (1) business cycle fluctuations are an intrinsic part of the growth process — expansions reflect the endogenous, clustered implementation of productivity improvements, and recessions are the negative side–product of the restructuring that anticipates them — and (2) the wages of production workers are endogenously determined by relational contracts designed to resolve incentive problems. We argue that the interactions between these two features plays a crucial role in determining the joint cyclical movements in firm entry and exit, worker flows and wages, and have important implications for the relationship between institutional features of the labor market and long–run productivity growth.

Our analytical framework is based on the business cycle paradigm we have developed elsewhere

[1] See Aghion and Howitt (1992), Grossman and Helpman (1991) and Segerstrom, Anant and Dinopoulos (1990).
[2] See, for example, the contributions to the January 2002 issue of the *Review of Economic Dynamics*.

(Francois and Lloyd-Ellis 2003).[3] This is modified to allow for relational contracts in labor markets, yielding endogenous fluctuations in involuntary unemployment and a novel framework in which to consider cyclical labor flows.[4] Output in each sector of the economy is produced using the most productive methods available combined with the services of managers and production workers. Managers' time may also be allocated away from supervising production workers in order to search for new ideas or ways of raising productivity. This ongoing innovation implies that the profits associated with productivity improvements are temporary. In order to protect the knowledge embedded in them, firms optimally delay implementation until macroeconomic conditions are most favorable. In the presence of imperfect competition, implementation by some firms increases the demand for others' products by raising aggregate demand, thereby creating such favorable conditions. This leads to mutually reinforcing endogenous clustering of implementation, causing booms in productivity.

Since firms optimally allocate managers to innovative activities just before the boom, this creates downturns in aggregate output which are also self–reinforcing.[5] During these recessions, the aggregate demand for complementary production workers falls. However, the implications for underlying firm turnover, worker flows and wages depends on the employment contract between firms and workers. Since the output of production workers is dependent on their imperfectly observable effort, firms must offer forward–looking implicit contracts to induce effort. However, such contracts can only work if firms are expected to last indefinitely with positive probability. If, during a recession, it becomes clear that a firm is about to be made obsolete, even when it has yet to be made so, its ability to motivate its workers is severely compromised. When this happens, firms are forced to shut down immediately causing the rate of job destruction during recessions to rise endogenously.

Our theory delivers joint predictions regarding key features of the labor market over the business cycle. Although previous work has tended to analyze each of these features in isolation, treating other factors as exogenous, here we argue that allowing for the endogenous interaction between them provides a useful perspective on some of the central puzzles in macro labor:

- *The relative contributions of job destruction and job creation to changes in employment growth.*

[3] This paradigm captures the idea that the cycle is an intrinsic part of the growth process. In this sense, our framework is related to the work of Matsuyama (1999, 2001) and Waelde (2003, 2004). The basic idea of demand externalties in this context is related to that of Shleifer (1986).

[4] A pioneering formalization of the relational contract paradigm is that of Macleod and Malcomson (1989).An earlier, though less formally complete contribution is the efficiency wage model of Shapiro and Stiglitz (1986).

[5] As should be clear, a key feature of our theory is the counter–cyclical allocation of managerial effort away from production (supervision) towards longer term productivity enhancing activities (entrepreneurship). We discuss some of the evidence relating to this in Francois and Lloyd-Ellis (2003).

This is an issue that has recently received considerable attention. The traditional view is that most of the cyclical movements in employment growth are the result of counter–cyclical fluctuations in job destruction. This view was supported by both aggregate worker–level data (see Blanchard and Diamond, 1990, and Bleakley, Ferris and Fuhrer 1999) and disaggregate plant–level data (see Davis and Haltiwanger 1992, 1996). More recently, however, Shimer (2005) has shown that the CPS worker flows data used by Blanchard and Diamond and others, overstates the variation in the true rate of job destruction, by effectively including declines in job creation between monthly observations.[6] When he corrects for this, he finds that job creation is, in fact, significantly more variable than job destruction.[7] But how can this be consistent with the observations of Davis and Haltiwanger ? Hall (2005) suggests one possibility: since they define job destruction as the sum of all declines in employment at plants where employment declines, Davis and Haltiwanger may be overstating movements in job destruction by including significant declines in job creation on the intensive margin.[8]

In our analysis, job creation on the intensive and extensive margins move in very different ways precisely because of the intrinsic nature of our cycle. The initial decline in aggregate labor demand is reflected in a sharp drop in the rate of job creation, as firms adjust immediately on the *intensive margin*. Job destruction rises only gradually as the recession proceeds, reflecting the gradual allocation of managerial effort away from production. The first part of a recession is therefore characterized by *low* "reallocation" of workers,[9] with reduced job creation contributing to almost all of the increasing unemployment. Towards the end of the recession however, as the next expansion approaches, adjustment on the extensive margin becomes more important: both job destruction and job creation pick up, as firms shut down and new firms take their place. Although worker reallocation then becomes relatively high, it actually peaks during the boom when job creation spikes upward as demand picks up and firms adjust employment upwards to its normal level.

Consequently, as we demonstrate, most of the variation in employment growth is caused by movements in job creation. Since most of this variation occurs on the intensive margin, the measurement approach of Davis and Haltiwanger would attribute some of this variation to changes

[6] A number of authors had also previously questioned the generality of the earlier empirical findings. Boeri (1996), who emphasized the importance of including establishments with less than 5 employees, and Burgess, Lane and Stevens (2001) who analysed flows at the individual level, and found dramatic differences between contracting and expanding firms.

[7] Hall (2005) comes to an even stronger conclusion by extrapolating backwards using recent JOLTS data. He finds that job destruction exhibits almost no variability over the business cycle. However, Davis et al. (2005) argue that that his backcasting methodology is problematic.

[8] Cabellero and Hammour (1994) also note this possibility but, in their model, all employment changes are on the extensive margin

[9] Here we are defining re–allocation as the sum of creation and destruction, following Davis and Haltiwanger (1992).

in job destruction. Indeed, we show that when creation and destruction are measured according to their methodology, the job destruction becomes much more variable and job creation less so. Moreover, when incorrectly measured in this way, job destruction (and re–allocation) appears to rise rather than fall at the beginning of recessions.

- *Creative destruction and the cyclical behavior of average labor productivity.*

Caballero and Hammour (1994) develop a partial equilibrium industry model in which production units with different efficiency levels co–exist due to re–allocation frictions. Exogenous declines in industry demand, drive down profitability, causing the least efficient units to shut down, thereby freeing up resources for more productive uses. Although their model has a similar Schumpeterian flavor to ours, it has a starkly counter–factual implication: average labour productivity is countercyclical (see Yi, 2004). Moreover, their story relies heavily on partial equilibrium reasoning in that unit wage costs are held fixed exogenously over the cycle. If labour markets were perfectly flexible, wages would fall allowing inefficient firms to remain in business. While the evidence suggests that wages are not perfectly flexible, understanding the endogenous interactions between employment flows and wages seems important.[10]

In our model, expansions are associated with increases in total factor productivity, but downturns are the result of restructuring in anticipation of future movements in productivity.[11] If labor productivity were correctly measured, it would remain constant during recessions.[12] As we demonstrate, although firms implement their improvements during booms, it is optimal for them to enter production during recessions, displacing and re–hiring workers as they do so. Unlike the previous incumbents, new entrants can credibly guarantee to honour contracts with workers and so can profitably produce. In equilibrium, new entrants optimally take over production using existing methods of production, but then delay implementation of their own improvements until the subsequent expansion.

- *The cyclical behavior of real wages.*

Real wages are commonly characterized as being mildly pro–cyclical on average (see Abraham and Haltiwanger 1995, Stock and Watson, 1998). Within reasonable parameters ranges, the canonical Real Business Cycle (RBC) model predicts excessively pro–cyclical real wages, even when compared with post–1970 data and controlling for composition bias (see Chang, 2000).

[10] This point has also been emphasized by Shimer (2004) in the context of labor search models.

[11] Because of this restructuring process, the anticipated growth in TFP starts to be reflected in stock–prices during the recession, prior to implementation. Beaudry and Portier (2004) find compelling evidence that aggregate US stock–price movements partially anticipate movements in total factor productivity, and that the economy reacts to this anticipation well before TFP rises.

[12] However, because the reallocation of skilled labor effort to innovative activities is likely to be unmeasured, it would look like labor hoarding, and measured labour productivity would fall.

Efficiency wage models have been suggested by some as a way of offsetting this pro–cyclical wage behavior (e.g. Romer, 2001). However, Gomme (1999) finds that, while the variability of the average wage is reduced, it remains just as pro-cyclical. His analysis does not, however, allow for crucial counter–cyclical forces implicit in the efficiency wage structure – in particular, the fact that increases in the rate of job destruction require a compensating increase in current wages to maintain incentive compatibility.[13] These effects can offset, and even outweigh, the downward pressure on wages due to falling demand and the declining opportunity cost of leisure. The converse holds in expansions when relationships are relatively stable, but employment is tight.

Once again, it is the fact that this restructuring is concentrated during recessions which drives this counter–cyclical effect on wages. This timing is associated with the early entry, but delayed implementation of productivity improvements, by firms. This behavior, in turn, is central to our theory of endogenous cyclical growth in which downturns are the result of anticipated booms. The wage behavior implied here would not arise in a model where productivity movements were treated exogenously. Note that one factor contributing to the mild pro–cyclicality of average wages is the so-called "composition bias", which results from the fact that low wage workers are more likely to become employed during recessions.[14] This effect is also present in our model because only production workers face employment fluctuations (managers are re–allocated between tasks).

• *Long–run productivity growth and the cyclicality of wages.*

The characterization of average post–war wages as being mildly pro–cyclical masks the fact that until 1970, average real wages were acyclical, or even counter cyclical, whereas between the early 1970s and the early 1990s wages became more pro-cyclical. As previously noted by Abraham and Haltiwanger (1995), the correlation rose significantly between the pre–1970 period and the 1970–1993 period. It fell again after 1994. This apparent increase in the cyclical behavior of average wages coincided with the 'productivity slowdown' which extended from the early 1970's until the mid 1990's, and the subsequent correlation decline in the mid 1990's corresponds with the relatively high productivity growth subsequently.[15] Was this timing a mere coincidence, or were both a function of changes in the same underlying factors?

Our model offers a parsimonious explanation for why these two phenomena may occur simul-

[13] In his model, firms do not shut down to be replaced by new ones, but rather adjust their rate of hiring downwards as demand falls. A more extensive discussion of offsetting effects in an efficiency wage framework was provided in earlier work – Macleod, Malcomson and Gomme (1994) – but again Schumpeterian firm shut-downs played no role. Several authors have shown that some alternative efficiency wage model can dampen the procyclicality of wages (see Alexopolous, 2004 and Danthine and Kurman, 2004).

[14] Bils (1985) and Solon, Barsky, and Parker (1994) document that the entry and exit of low-wage workers creates a counter–cyclical composition bias in aggregate wages in post–1970 data. Assessing the importance of the composition bias prior to 1970 is difficult because PSID date was only available after that date.

[15] See Table 3 below.

taneously. In particular, any factor that induces greater innovation on average, thereby fueling faster long–run productivity growth, also induces more obsolescence and more restructuring to occur during recessions. This implies that the counter–cyclical force on wages due to turnover tends to be greatest when average productivity growth is high, and less during productivity slowdowns.

Our framework is related to that of Ramey and Watson (1997), who explore the effects of exogenous transitory shocks in inducing permanent increases in separations, and to den Haan, Ramey and Watson (1999), who also explore relational contracting in a similar framework. They utilize a much richer framework of contracting possibilities between workers and firm than we do here, and pay more careful attention to the parameters of contract design imposed by imperfect verifiability and limited liquidity. In their framework, the macroeconomy impinges on worker–firm relationships by reducing the surplus to maintaining matches and rendering incentive compatibility infeasible. This can lead to break-ups which propagate shocks. Our framework, in contrast, emphasizes the Schumpeterian necessity of break-up as part of the economy's rejuvenation process. Worker reallocations which accompany contractual break up are also an integral part of the economy's endogenous cycle, and aggregate productivity changes here.

Our paper also bears some relation to the search/matching literature on labor flows (for a recent survey see Rogerson, Shimer, and Wright, 2004). These models typically have a richer view of the frictions in the labor market, but a less sophisticated view of cyclical firm behavior, and wage determination than here. These models can accommodate numerous labor market responses to aggregate shocks depending on the model specification (see Mortensen and Pissarides 1994). A more profound difference, however, is that like all previous work they treat aggregate demand and its fluctuations as exogenous. As will be demonstrated, the endogeneity of the economy's cyclical process plays a key role here.

The remainder of the paper is laid out as follows. Section 2 sets up the building blocks of the model. Section 3 posits and describes behavior in the cyclical equilibrium and elaborates the dynamics over the phases of the cycle, with particular emphasis on labor flows. Section 4 discusses and specifies sufficient conditions for existence to be met. Section 5 demonstrates existence and explores the equilibrium's qualitative characteristics. The model's comparative statics are also examined, and applied to the productivity slowdown. Section 6 concludes.

2 The Model and Optimal Behavior

2.1 Final Goods Production

Final output is produced by competitive firms according to a Cobb–Douglas production function utilizing intermediates, x, indexed by i, over the unit interval:

$$Y(t) = \exp\left(\phi t + \int_0^1 \ln x_i(t)di\right). \tag{1}$$

Final goods production is subject to exogenous productivity growth at a constant rate ϕ. Final output is costlessly storable, but cannot be converted back into an input for use in production.[16] We let p_i denote the price of intermediate i. Final goods producers choose intermediates to minimize costs. The implied demand for intermediate i is then

$$x_i^d(t) = \frac{Y(t)}{p_i(t)} \tag{2}$$

2.2 Intermediate Goods Production

The output of intermediate i depends upon a productivity level, $a_i(t)$, and on the labor allocated to production. There are two types of labor — managers who earn a salary $s(t)$, and production workers who are paid a wage $w_i(t)$ in sector i. There are two distinct modes of production which intermediate firms can use:

• Large scale production — an incumbent firm operates a constant returns to scale technology that requires both managers and production workers. Provided there is no shirking, the firm combines $m_i(t)$ managers and $l_i(t)$ production workers to produce output according to the Leontief production function:[17]

$$x_i^s(t) = a_i(t) \min\left[m_i(t), \frac{l_i(t)}{\theta L}\right]. \tag{3}$$

In equilibrium, firms optimally hire workers so that

$$l_i(t) = \theta L m_i(t) \tag{4}$$

Thus, we effectively assume that managers have a fixed "span of control" with one manager required to supervise $\theta L > 1$ production workers, where $\theta < 1$.[18]

[16] That is, there is no tangible capital in the model. We explore a version of the model with physical capital in Francois and Lloyd–Ellis (2004). This adds considerably to the technical complexity of the model so that capital is not included here in order to focus on the labor market.

[17] As we describe below, shirking would imply that $l_i(t) = 0$, so that output would be zero.

[18] This production function simplifies the exposition considerably, but could be generalized (allowing for some degree of substitution) without changing our main results. It is important, however, that managers and skilled labor remain net complements.

• Small scale production — a manager can set up production on her own and produce a "small" amount of output. Since this individual works alone, there is no need to supervise other workers, and the unit labor cost is simply the salary $s(t)$. Any individual holding the state of the art technology could produce using this method, but the profit would be negligible.

2.3 Innovation

Ongoing marginal improvements in productivity arise in each sector through the diversion of managerial effort away from the supervision of production and towards innovative activities.[19] These activities are financed by selling equity shares to households. The probability of an innovative success over an instant dt is $\delta h_i(t)dt$, where δ is a parameter, and h_i is the managerial effort allocated to innovation in sector i. The aggregate managerial effort allocated to innovation is given by

$$H(t) = \int_0^1 h_i(t)di. \tag{5}$$

The productivity of new innovations is assumed to dominate that of old ones by a factor e^{γ}.

Entrepreneurs with innovations face two choices: (1) they must choose the timing of their entry into production, and (2) they must choose whether or not to implement their innovation immediately upon entering production, or delay until a later date. Once they implement, the knowledge associated with the innovation becomes publicly available (even though they can protect a use right) and can be built upon by rival entrepreneurs.[20] However, prior to implementation, the knowledge is privately held by the entrepreneur. By delaying implementation the entrepreneur loses the productive advantage of the new innovation, but gains from ensuring that the content of the innovation is secret, and will thus not be built upon.

An innovator must have control of the productive resources of the firm prior to implementation. This is intended to capture the idea that some degree of reorganization is required to take advantage of new approaches or innovations. Once an innovation has been implemented, the entrepreneur with the knowledge of how to implement can costlessly enter or exit production at any time. The information embedded in a new productive technology will be used by the next generation of innovators to design an improvement which will make the current incumbent's technology redundant.

[19]This process can equivalently be thought of as a search for product improvements, process improvements, organizational advances or anything else which creates a productive advance over the existing state of the art.

[20]It is not necessary to assume that implementation reveals all of the information embedded in an improvement. Provided there is some information generated through implementation which is useful to entrepreneurs searching for productivity improvements that will supercede the incumbent, then implementation will be 'costly' in terms of hastening redundancy and qualitatively identical results will hold.

We let the indicator function $Z_i(t)$ take on the value 1 if there exists a successful innovation in sector i which has not yet been implemented, and 0 otherwise. The set of instances at which innovations are implemented in sector i is denoted by Ψ_i. We let $V_i^I(t)$ denote the expected present value of profits from implementing an innovation at time t, and $V_i^D(t)$ denote that of delaying implementation from time t until the most profitable time in future.

2.4 The Labor Market

In aggregate there is a unit measure of managers and a measure L of potential production workers. We assume that the owners of a firm can perfectly contract with managers so that these individuals are hired in a fully competitive labor market.[21] In contrast, production workers are working in teams or are otherwise more difficult to monitor. Consequently, "shirking" is a problem that firms must deal with. Specifically, production workers can choose to shirk by providing zero effort, while potentially retaining their jobs. If they do not shirk, workers in sector i are subject to a rate of "job destruction" $\mu_i(t)$. Jobs may be destroyed for two reasons. First, there is a constant, exogenous "normal" rate of job turnover, $\bar{\mu}$, which is independent of the business cycle and which is not associated with firm shut–downs. Second, there may be job destruction due to firms' exit decisions. If a worker does shirk, the rate of job loss increases to $\mu_i(t) + q$, where q depends on the ability of the firm to detect shirking. Since detection is imperfect, firms must pay workers a sufficiently high wage to ensures incentive compatibility, otherwise they will produce no output.

As we shall see, the rate of job destruction may vary across sectors. We denote the equilibrium average rate of job destruction across all sectors by $\mu^A(t)$. If workers do lose their job, they enter a pool of unemployed workers who are viewed as homogenous by firms and, hence, face an equal probability of being re–hired. We denote the fraction exiting the unemployment pool each instant by $d\Lambda(t)$.[22] In aggregate the change in the level of employment, $n(t)$, must equal the number of jobs created less the number destroyed:

$$dn(t) = (L - n(t))\, d\Lambda(t) - n(t)\mu^A(t)dt \tag{6}$$

[21] An alternative assumption is that the owner of a firm can observe the work effort, or that managers' reputations are long–lived, so that if they "shirk" they are forever marked and never again re-hired. Our aim is parsimony here, not realism. Introducing a moral hazard problem at this level as well would not change the qualitative behavior of our model.

[22] We assume throughout that incoming firms do not hire workers directly from the incumbents they replace. This assumption implies that all plant shut-downs are accompanied by job destruction and unemployment without allowing firm to firm transitions. A more complicated version of our model would parameterize this and allow for patial transitions directly to new employers. This has the slight advantage of allowing plant level destruction to be de-coupled from job destruction but changes nothing qualitatively significant.

In the absence of discontinuous jumps in employment levels, the hiring rate is given by the derivative $\lambda(t) = d\Lambda(t)/dt$.

2.5 Goods Market Competition

In order to extract rent from the leading edge technology, intermediate producers must utilize the large scale mode of production and hire workers.[23] Given the unit elasticity of demand, the producer holding the state of the art technology optimally limit prices at the marginal cost of his next best competitors. If $\eta(t)$ denotes the unit labor cost, the limit price in sector i is then given by

$$p_i(t) = \frac{\eta(t)}{e^{-\gamma} a_i(t)}. \tag{7}$$

We assume that intermediates are completely used up in production, but can be produced and stored for use at a later date. Incumbent intermediate producers must therefore decide whether to sell now, or store and sell later.

2.6 Households

The economy is populated by a unit measure of infinitely–lived "large" households. Each household consists of a unit measure of managers and a measure $L > 1$ of potential production workers. Managers supply labor inelastically, but the supply of production worker effort is determined by the household in response to the employment opportunities and wages offered. Households are assumed to have preferences given by

$$U(t) = \int_t^\infty e^{-\rho(\tau - t)} u(c(t), n(t)) d\tau, \tag{8}$$

where ρ denotes the rate of time preference, $c(t)$ denotes household consumption and $n(t)$ denotes the measure of production workers that are exerting effort. That is

$$n(t) = \int_0^L \varepsilon_j(t) dj, \tag{9}$$

where $\varepsilon_j(t) \in \{0, 1\}$ denotes the effort level exerted by worker j in period t. We assume that the instantaneous utility function takes the Cobb–Douglas form

$$u(c(t), n(t)) = c(t)^{1-\sigma} (L - n(t))^{\sigma}. \tag{10}$$

[23] In order to sustain meaningful innovation, there must exist some way for successful innovators to extract, at least temporarily, rents from their knowledge. At the most formal end, this may be through the use of patents and the threat of a law suit, more informally, rent dissipation may simply be a matter of time, or may be preserved by the threat that, if copied, the innovator will drive copiers out of the market. We leave unspecified the precise means by which this occurs and simply assume that the leading technology is the exclusive province of the innovator. Once the innovation has been superceded, others may enter and use it at will, since, as it can no longer generate rents, the innovator controlling it no longer has incentive to limit (by whatever means) its use elsewhere.

We focus on preferences of this form for expositional simplicity, but they can be generalized to allow for homogeneity of degree other than one without changing our qualitative conclusions.[24] Note that we assume throughout that $\rho > (1-\sigma)\phi$.

Workers do not have independent preferences. The household chooses the supply of labor effort for each worker so as to maximize this/her marginal contribution to household utility. For any production worker who is offered employment in sector i at the beginning of time t, the household chooses whether he/she should accept the offer or remain unemployed. If the offer is accepted, the household chooses whether or not the worker should exert effort. If the production wage in sector i is $w_i(t)$, and the worker supplies one unit of labor effort, the marginal contribution to household utility in period t is

$$w_i(t)u_c(t) + u_n(t), \tag{11}$$

where

$$\begin{aligned} u_c(t) &= (1-\sigma)c\,(t)^{-\sigma}\,(L-n(t))^{\sigma} > 0 & (12)\\ u_n(t) &= -\sigma c\,(t)^{1-\sigma}\,(L-n(t))^{\sigma-1} < 0. & (13)\end{aligned}$$

Note that the household's valuation of the wage earned by each worker takes as given aggregate household consumption and employment.

The marginal utility contributed to the household by worker j if he/she is offered employment in sector i can be written as

$$\psi_{ij}^*(t) = \max\left[\psi_{ij}^E(t), \psi_{ij}^S(t), \psi_j^U(t)\right] \tag{14}$$

where

$$\psi_{ij}^E(t) = [w_i(t)u_c(t) + u_n(t)]\,dt + e^{-\rho dt}\left[e_i^{-\mu_i(t)dt}\psi_{ij}^*(t+dt) + (1-e^{-\mu_i(t)dt})\psi_j^U(t+dt)\right] \tag{15}$$

represents the value of being employed in sector i and supplying effort $\varepsilon_j(t) = 1$,

$$\psi_{ij}^S(t) = w_i(t)u_c(t)dt + e^{-\rho dt}\left[e_i^{-(\mu_i(t)+q)dt}\psi_{ij}^*(t+dt) + (1-e^{-(\mu_i(t)+q)dt})\psi_j^U(t+dt)\right] \tag{16}$$

represents the value of being employed in sector i and shirking, $\varepsilon_j(t) = 0$, and

$$\psi^U(t) = e^{-\rho dt}\left[e^{-d\Lambda(t)}\psi_j^U(t+dt) + (1-e^{-d\Lambda(t)})\psi_{ij}^*(t+dt)\right], \tag{17}$$

represents the value of being unemployed. For workers who are not offered employment at time t the household receives $\psi^U(t)$.

[24] Note that the implied intertemporal elasticity of substitution exceeds unity. This feature can be relaxed in a model with physical capital (see Francois and Lloyd–Ellis, 2004), but is essential here.

The "large" household assumption effectively implies that total household wage income, $\omega(t) = \int_0^L w_j(t)\varepsilon_j(t)dj$, is certain and identical across households. Each household chooses consumption over time to maximize (8) subject to the intertemporal budget constraint

$$\int_t^{\infty} e^{-[R(\tau)-R(t)]}c(\tau)d\tau \leq S(t) + \int_t^{\infty} e^{-[R(\tau)-R(t)]}\left[s(\tau)+\omega(\tau)\right]d\tau \tag{18}$$

where $S(t)$ denotes the household's stock of assets at time t and $R(t)$ denotes the market discount factor from time zero to t. The stock of assets could potentially include claims to the profits of intermediate firms and stored output. The first–order conditions of the household's dynamic optimization require that

$$dR(t) = \rho dt + \sigma\left[\frac{dc(t)}{c(t)} + \frac{dn(t)}{L-n(t)}\right] \qquad \forall\ t \tag{19}$$

and that (18) holds with equality. The Euler equation is expressed in the form above to allow for the possibility of discontinuous jumps. Over intervals during which neither the discount factor nor employment jumps, household consumption satisfies

$$\frac{\dot{c}(t)}{c(t)} + \frac{\dot{n}(t)}{L-n(t)} = \frac{r(t)-\rho}{\sigma}, \tag{20}$$

where $r(t) = \dot{R}(t)$.

2.7 General Equilibrium

Given an initial stock of implemented innovations represented by a cross–sectoral distribution of productivities $\{a_i(0)\}_{i=0}^1$ and an initial distribution of unimplemented innovations, $\{Z_i(0)\}_{i=0}^1$, an equilibrium for this economy satisfies the following conditions:

• Households allocate consumption optimally over time, (19).

• Households only accept employment offers for worker j if the contribution to household utility for that worker is no less than that of remaining unemployed:

$$\psi_{ij}^E(t) \geq \psi_j^U(t). \tag{21}$$

• Final goods producers choose intermediates to minimize costs, (2).

• Intermediate producers set prices so as to maximize profits, given demand, (7).

• Intermediate producers choose the mode of production which maximizes their profits.

• The skilled–labor market clears:

$$\int_0^1 m_i(t)di + H(t) = 1. \tag{22}$$

• In the face of unemployment, intermediate producers offer a path of production wages so as to maximize profits subject to the participation constraint and the incentive compatibility condition:

$$\psi_{ij}^{E}(t) \geq \psi_{ij}^{S}(t). \tag{23}$$

• Free entry into arbitrage. For all assets that are held in strictly positive amounts by households, the rate of return between time t and time s must equal $\frac{R(s)-R(t)}{s-t}$.
• There is free entry into innovation. Managerial innovative effort is allocated to the sector which maximizes the expected present value of the innovation. Also

$$\delta \max[V_i^D(t), V_i^I(t)] \leq s(t), \quad h_i(t) \geq 0 \qquad \text{with at least one equality} \tag{24}$$

• Entrepreneurs with innovations choose whether to enter production using the previous technology.
• In periods where there is implementation, entrepreneurs with innovations must prefer to implement rather that delay until a later date

$$V_i^I(t) \geq V_i^D(t) \quad \forall\, t \in \Psi_i \tag{25}$$

• In periods where there is no implementation, either there must be no innovations available to implement, or entrepreneurs with innovations must prefer to delay rather than implement:

$$\begin{aligned} \text{Either } Z_i(t) &= 0, \\ \text{or if } Z_i(t) &= 1,\ V_i^I(t) \leq V_i^D(t) \quad \forall\, t \notin \Psi_i. \end{aligned} \tag{26}$$

3 The Cyclical Equilibrium

Although there exists an acyclical equilibrium growth path that satisfies the conditions stated above, our focus here is on a cyclical equilibrium growth path. In this section, we start by positing a temporal pattern of entrepreneurial behavior in innovation, entry into production, and implementation of productivity improvements. We then derive the implications of this posited pattern for relative returns between entrepreneurship and management, innovation levels, and firms' choice of production mode and evolution of aggregate variables. Section 4 then derives a set of sufficient conditions under which the implied evolution of aggregate variables, and market clearing, yield optimal entrepreneurial behavior corresponding with the originally posited behavior.

3.1 Posited Entrepreneurial Behavior

Suppose implementation occurs at discrete dates denoted by T_ν where $v \in \{1, 2, ..., \infty\}$. We adopt the convention that the vth cycle starts at time T_{v-1} and ends at time T_ν. The posited behavior of entrepreneurs over this cycle is illustrated in Figure 1. After implementation at date T_{v-1} there is an interval during which there is no innovation and consequently all managers are used to supervise production. At some time T_v^E, innovation commences again. Innovative activity is allocated symmetrically across sectors which have not yet had a success in the current cycle. Once a success occurs, all innovative activity ceases in the sector and the successful entrepreneur enters production displacing the existing incumbent. Implementation of the productivity improvement is, however, withheld until time T_v.

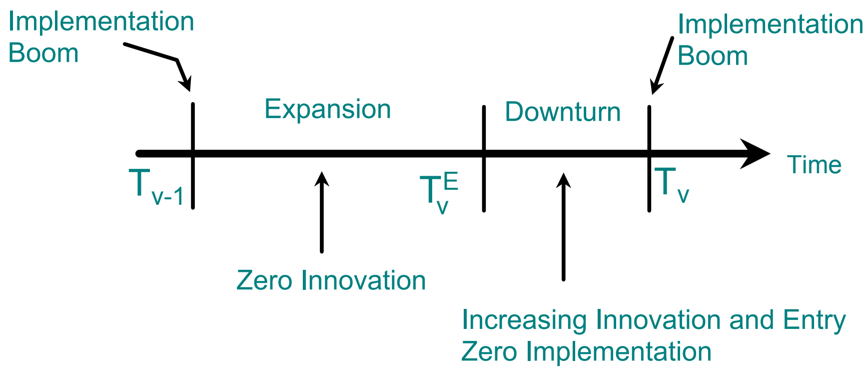

Figure 1: Innovation, Implementation and Entry in the vth cycle

3.2 Within–Cycle Implications

Lemma 1 *An employer who holds a redundant technology cannot profitably produce using the large scale mode of production.*

A firm with an obsolete technology cannot maintain a relational contract with production workers since it cannot promise employment into the future. Its production is thus taken over by the newly successful entrepreneur whose termination date, though not yet known with certainty, will at least not occur before the next cyclical downturn. Its lowest cost competitors are the competitive fringe who, though producing at small scale, can in aggregate, steal the new incumbent's whole market if too high a price is charged. These competitors avoid the costs of setting up large scale production and hiring multiple workers, but can only produce using the previous state of the art technology. Since the unit labor cost of this competitive fringe is $\eta(t) = s(t)$, the limit price charged by intermediate producer i is

$$p_i(t) = \frac{s(t)}{e^{-\gamma} a_i(t)}. \tag{27}$$

It follows from (2), (4) and (27), that the skilled work force receives a constant share of output:

$$s(t)(1 - H(t)) = e^{-\gamma} Y(t). \tag{28}$$

It also follows that the profits of an intermediate producer can be expressed as

$$\pi\left(w_i(t), t\right) = \left(1 - e^{-\gamma} - e^{-\gamma}\theta L \frac{w_i(t)}{s(t)}\right) Y(t). \tag{29}$$

Note that, since all sectors face the same skilled salary $s(t)$ and revenue shares are symmetric, profits vary across sectors only due to differences in the production wage, $w_i(t)$. A final implication is that, in equilibrium, the managerial salary is tied to the level of technology. Since within a cycle, the state of intermediate technology in use is unchanging, it follows that

Lemma 2 *Within the cycle, the managerial salary grows at the rate of technological change in the final output sector:*

$$\frac{\dot{s}(t)}{s(t)} = \phi. \tag{30}$$

Utilizing the incentive compatibility condition (23), net present value of each of the three states implied by the (15), (16) and (17) yields the following binding incentive compatible wage for production workers.

Lemma 3 *: The production wage in sector i is given by*

$$w_i(t) = -\frac{u_n(t)}{u_c(t)}\left[1 + \frac{1}{q}\left(\rho + \mu_i(t) + \lambda(t) - \frac{\dot{u}_n(t)}{u_n(t)}\right)\right]. \tag{31}$$

This expression summarizes the key forces acting on the incentive compatible production wage. If $q \to \infty$, then detection of shirking becomes perfect, so that the incentive problem disappears, and the expression simplifies to the standard first–order condition for household labor supply: $w_i(t) = -u_n(t)/u_c(t)$. In this case, the wage is determined by two forces, both of which are pro–cyclical. As $n(t)$ rises during an expansion the marginal utility cost to the household of supplying additional workers falls, so that a lower wage is needed induce it. Also, as consumption $c(t)$ rises the marginal benefit of supplying additional labor falls, so that firms must raise wages to induce labour effort.

However, imperfect detection ($q \to 0$) of shirking introduces three other forces. One of these is pro–cyclical — as the hiring rate, $\lambda(t)$, rises, being fired is a less costly threat, so that firms must raise the wage to provide greater incentives not to shirk. However, the other two forces are countercyclical. First, a higher rate of job–destruction, $\mu_i(t)$, implies that workers must be compensated for the increased likelihood of job loss. Second, with negative employment growth, the marginal cost of supplying effort is lower tomorrow than today $\dot{u}_n(t)/u_n(t) > 0$. Other things equal, this makes workers more willing to risk unemployment by shirking, so firms must raise wages to compensate.

3.3 The Expansion

Since all managers are used in production, it must be the case that production worker employment is at its maximum:

$$n(t) = \theta L. \tag{32}$$

Let the level of output immediately following the boom be given by $Y_0(T_{v-1})$.[25] With a constant level of employment it follows that output grows during the expansion at the rate ϕ, so that $Y(t) = e^{\phi(t-T_{v-1})}Y_0(T_{v-1})$. Since the economy is closed, and there is no incentive to store either intermediate or final output across periods (provided $r(t) \geq 0$), it must be the case that consumption grows at this rate too:

$$\frac{\dot{c}(t)}{c(t)} = \frac{\dot{Y}(t)}{Y(t)} = \phi. \tag{33}$$

Substituting these facts into (20) yields the implied interest rate during the expansion,

$$r(t) = \rho + \sigma\phi. \tag{34}$$

Since there is no firm turnover, flows out of employment are given by exogenous separation rate $\bar{\mu}$. Using (6) and (32), it follows that the rate at which workers are hired from the unemployment pool is then given by

$$\bar{\lambda} = \frac{\bar{\mu}\theta}{1-\theta} \tag{35}$$

Substituting these facts into (31) implies that the production wage also grows at the rate ϕ during the expansion:

Proposition 1 *: During the expansion unemployment is constant at* $(1-\theta)L$ *and the production wage is given by*

$$w^A(t) = \frac{A}{L}e^{\phi(t-T_{v-1})}Y_0(T_{v-1}) \tag{36}$$

where $A = \sigma\left(\rho + q + \frac{\bar{\mu}}{1-\theta}\right) \Big/ [(1-\sigma)(1-\theta)q]$.

[25] Throughout, we use the subscript 0 to denote the value of a variable immediately after the boom. Formally, $X_0(T) = \lim_{t\to T^+} X(t)$.

During this phase, the expected value of innovating and then delaying implementation until the subsequent boom, $\delta V^D(t)$, is necessarily growing at the rate of interest, $r(t) = \rho + \sigma\phi$, as the date of implementation draws closer. Since there is no innovation, it must be the case that $\delta V^D(t) < s(t)$. However, since $\rho > (1-\sigma)\phi$, $\delta V^D(t)$, must eventually equal $s(t)$. At this point, entrepreneurship commences. The following Lemma demonstrates that it does so smoothly:

Lemma 4 *At time T_v^E, when entrepreneurship first commences in a cycle, $s(T_v^E) = \delta V^D(t)$ and $H\left(T_v^E\right) = 0$.*

3.4 The Contraction

For the posited positive entrepreneurship phase to occur under free entry, it must be that $s_v = \delta V^D(t)$ while it does so. Since the skilled wage grows at rate ϕ throughout the cycle, $\delta V^D(t)$ must also grow at the same rate during this phase. Since the time until implementation for a successful entrepreneur is falling and there is no stream of profits, because implementation is delayed, an instantaneous interest rate equal to ϕ over this phase is required.

$$r(t) = \frac{\dot{V}^D(t)}{V^D(t)} = \frac{\dot{s}(t)}{s(t)} = \phi. \tag{37}$$

The household's Euler equation can be expressed as

$$\frac{\dot{c}(t)}{c(t)} + \frac{\dot{n}(t)}{L - n(t)} = -\left(\frac{\rho - \phi}{\sigma}\right) \tag{38}$$

Since the economy is closed, it follows once again that, because there is no incentive to store output, (33) holds. Hence, consumption growth must equal the sum of the growth in final goods productivity, ϕ, and the growth (negative) in employment. Solving the implied first order differential equation in $n(t)$ we have:

Lemma 5 *: Employment during the contraction evolves according to*

$$n(t) = \frac{\theta L e^{-\frac{(\rho-(1-\sigma)\phi)}{\sigma}(t-T_v^E)}}{1-\theta+\theta e^{-\frac{(\rho-(1-\sigma)\phi)}{\sigma}(t-T_v^E)}}. \tag{39}$$

Note that $n(T_v^E) = \theta L$. This expression implies that $n(t)$ must be declining during the downturn because, as high–skilled labor flows out of production and into entrepreneurship, the demand for low-skilled workers falls in proportion. The implied skilled labor that flows into innovation is therefore

$$H(t) = 1 - \frac{n(t)}{\theta L} = \frac{(1-\theta)\left(1 - e^{-\frac{(\rho-(1-\sigma)\phi)}{\sigma}(t-T_v^E)}\right)}{1-\theta+\theta e^{-\frac{(\rho-(1-\sigma)\phi)}{\sigma}(t-T_v^E)}}. \tag{40}$$

The proportion of sectors that have not yet experienced an entrepreneurial success by time $t \in (T_v^E, T_v)$ is given by

$$P(t) = \exp\left(-\int_{T_v^E}^{t} \delta h(\tau) d\tau\right). \tag{41}$$

Recalling that labor is only devoted to entrepreneurship in sectors which have not innovated since the start of the cycle, the labor allocated to entrepreneurship in each sector where no innovation has yet occurred is then

$$h(t) = \frac{H(t)}{P(t)}. \tag{42}$$

In the measure $(1 - P(t))$ of sectors where restructuring has occurred, the only source of job destruction is normal turnover, $\bar{\mu}$. However, in sectors where no restructuring has occurred yet, the rate of job destruction also includes the probability of a restructuring occurring, $\delta h(t)$, which increases as the downturn proceeds. It follows that the aggregate rate of job destruction is given by

$$\mu^A(t) = (1 - P(t))\bar{\mu} + P(t)\left[\bar{\mu} + \delta \frac{H(t)}{P(t)}\right] = \bar{\mu} + \delta H(t) \tag{43}$$

In sectors where innovation has occurred, wages are lower, denoted by $w^L(t)$, as there is no chance of further restructuring in the contraction, and so

$$w^L(t) = \frac{\sigma c(t)}{(1-\sigma)(L - n(t))}\left[1 + \frac{1}{q}\left(\rho + \bar{\mu} + \lambda(t) - \frac{\dot{u}_n(t)}{u_n(t)}\right)\right]. \tag{44}$$

When a restructuring has not yet occurred, there is higher probability of job destruction and a higher wage is required to ensure incentive compatibility:

$$w^H(t) = \frac{\sigma c(t)}{(1-\sigma)(L - n(t))}\left[1 + \frac{1}{q}\left(\rho + \bar{\mu} + \delta H(t) + \lambda(t) - \frac{\dot{u}_n(t)}{u_n(t)}\right)\right]. \tag{45}$$

Intuitively, this is because newly started firms have a longer expected duration of incumbency than existing firms, and can thus promise non-shirking workers a longer expected span of employment.

Since the unskilled wage in each sector is linearly related to the rate of job destruction, it follows that the average efficiency wage for the unskilled can be expressed as

$$w^A(t) = \frac{\sigma c(t)}{(1-\sigma)(L - n(t))}\left[1 + \frac{1}{q}\left(\rho + \mu^A(t) + \lambda(t) - \frac{\dot{u}_n(t)}{u_n(t)}\right)\right] \tag{46}$$

Substituting in for the endogenous variables yields the following result:

Proposition 2 *: During the contraction, the average production wage is given by*

$$w^A(t) = \left[Be^{-\frac{\rho-(1-\sigma)\phi}{\sigma}(t-T_v^E)} - Ce^{-\frac{2(\rho-(1-\sigma)\phi)}{\sigma}(t-T_v^E)} + \frac{De^{-\frac{\rho-(1-\sigma)\phi}{\sigma}(t-T_v^E)}}{1-\theta+\theta e^{-\frac{\rho-(1-\sigma)\phi}{\sigma}(t-T_v^E)}}\right]\frac{e^{\phi(t-T_{v-1})}Y_0(T_{v-1})}{L} \tag{47}$$

where $B = \frac{\sigma(q+\delta+\bar{\mu})}{(1-\sigma)(1-\theta)q}$, $C = \frac{\sigma\left(\delta - \frac{\bar{\mu}\theta}{1-\theta}\right)}{(1-\sigma)(1-\theta)q}$ *and* $D = \frac{\rho-(1-\sigma)\phi}{(1-\sigma)q}$.

The evolution of the production wage during downturns is ambiguous due to the interaction of the various pro– and counter– cyclical forces mentioned earlier. However, as we document below, in general the wage traces out a hump–shaped pattern. If countercyclical forces dominate, the wage rises to begin with largely reflecting the rising rate of job destruction. However, as the downturn proceeds, more and more sectors are taken over by new entrants who then pay the relatively low wage, $w_L(t)$, and fewer sectors are left facing imminent exit an paying the wage $w_H(t)$. If the downturn continues for long enough, this change in sectoral composition, eventually drives down the average wage.

3.5 The Boom

We denote the improvement in aggregate productivity during implementation period T_v by e^{Γ_v}, where

$$\Gamma_v = \int_0^1 [\ln a_i(T_v) - \ln a_i(T_{v-1})]\, di \tag{48}$$

Productivity growth at the boom is given by $\Gamma_v = (1 - P(T_v))\gamma$, where $P(T_v)$ is defined by (41). Substituting in the allocation of labor to entrepreneurship through the downturn given by (40) and integrating over the interval

$$\Delta_v^E = T_v - T_v^E. \tag{49}$$

yields the following implication:

Proposition 3 *The growth in productivity during the boom is given by*

$$\Gamma_v = \delta\gamma\Delta_v^E + \frac{\sigma\delta\gamma}{(\rho - (1-\sigma)\phi)\theta} \ln\left(1 - \theta\left(1 - e^{-\frac{(\rho-(1-\sigma)\phi)}{\sigma}\Delta_v^E}\right)\right). \tag{50}$$

Equation (50) tells us how the size of the productivity boom depends positively on the amount of time the economy is in the entrepreneurship phase, Δ_v^E. The amount of innovation in that phase is determined by the movements in the interest rate, so once the length of the entrepreneurship phase is known, the growth rate over the cycle is pinned down. The size of the boom is convex in Δ_v^E, reflecting the fact that as the boom approaches, the labor allocated towards innovation is increasing. This also implies that the boom size is increasing in the depth of the downturn, since the longer the downturn the greater the allocation of innovative effort and hence the larger the decline in output.

During the boom, productivity and production employment both jump up. Since consumption is the product of these two, consumption must also increase discontinuously. For this to be consistent with optimal household behavior, it follows that the discount factor must also rise discontinuously. The long run discount factor during the boom is given by the household's Euler equation

$$R_0(T_v) - R(T_v) = \sigma \ln \frac{c_0(T_v)}{c(T_v)} - \sigma \ln \frac{L - n_0(T_v)}{L - n(T_v)}. \tag{51}$$

Since consumption growth at the boom results both from implementation and the reallocation of labour into production it follows that

$$R_0(T_v) - R(T_v) = \sigma\Gamma_v + \sigma \ln \frac{n_0(T_v)}{n(T_v)} - \sigma \ln \frac{L - n_0(T_v)}{L - n(T_v)}. \tag{52}$$

Now, $n_0(T_v) = \theta L$ and using (39) to determine $n(T_v)$, we get

$$R_0(T_v) - R(T_v) = \sigma\Gamma_v + (\rho - (1-\sigma)\phi)\,\Delta_v^E. \tag{53}$$

Over the boom, the asset market must simultaneously ensure that entrepreneurs holding innovations are willing to implement immediately (and no earlier) and that, for households, holding equity in firms dominates holding claims to alternative assets (particularly stored intermediates). The following Proposition demonstrates that these conditions imply that during the boom, the discount factor must equal productivity growth:

Proposition 4 *: Asset market clearing at the boom requires that*

$$R_0(T_v) - R(T_v) = \Gamma_v \tag{54}$$

Combining (53) with (54) yields

$$\Gamma_v = \frac{(\rho - (1-\sigma)\phi)\Delta_v^E}{1-\sigma}. \tag{55}$$

Combining (50) and (55) yields a unique (non–zero) equilibrium pair (Γ, Δ^E) that is consistent with the within–cycle dynamics and asset market clearing. Equating them implies that Δ^E must satisfy

$$\left(1 - \frac{(\rho - (1-\sigma)\phi)}{\delta\gamma(1-\sigma)}\right)\Delta^E = -\frac{\sigma}{(\rho - (1-\sigma)\phi)\theta} \ln\left(1 - \theta\left(1 - e^{-\frac{(\rho-(1-\sigma)\phi)}{\sigma}\Delta^E}\right)\right) \tag{56}$$

Note that although we did not impose any stationarity on the cycles, the equilibrium conditions imply stationarity of the size of the boom and the length of the downturn. For a unique positive value of Δ^E that satisfies this condition to exist it is sufficient that $0 < \rho - (1-\sigma)\phi < \delta\gamma(1-\sigma)$.

4 Optimal Entrepreneurial Behavior

This section derives a set of sufficient conditions under which the behavior posited in Figure 1 is optimal given the implied behavior derived above.

4.1 Optimal Restructuring

The present value of profits earned in a sector where no future restructuring is anticipated up to the end of the current cycle is

$$V^* (t) = \int_t^{T_v} e^{-\int_t^\tau r(s)ds} \pi(w^L(\tau), \tau) d\tau$$

In the cyclical equilibrium considered here, secrecy (i.e. protecting the knowledge embodied in a new innovation by delaying implementation) can be a valuable option.[26] Since innovations are withheld until a common implementation time, simultaneous implementation is feasible. However, as the following Lemma demonstrates, such duplications do not arise in the cyclical equilibrium because successful innovators enter production to displace previous incumbents, sending a credible signal that stops subsequent entrepreneurial efforts in their sector.

Proposition 5 *: Given that innovations are implemented at the subsequent boom, a successful entrepreneur transfers $V^* (t) + \varepsilon$ to use the incumbent's technology until T_{v+1}, and takes over production in that sector. All innovative activity in their sector then stops until the next round of implementation.*

The payment $V^* (t) + \varepsilon$ by a successful entrepreneur to the previous incumbent acts as a credible signal that this entrepreneur has had an innovation, but does not reveal the content of that success.[27] If an entrepreneur's announcement is credible, other entrepreneurs will exert their efforts in sectors where they have a chance of becoming the sole dominant entrepreneur. One might imagine that unsuccessful entrepreneurs would have an incentive to mimic successful ones by falsely announcing success to deter others from entering the sector. But doing this yields a flow of profits for the interval $t \to T_{v+1}$ which is ε less than paid for it, and is thus not worthwhile.

[26] As Cohen, Nelson and Walsh (2000) document, delaying implementation to protect knowledge is a widely followed practice in reality.

[27] The payment does not have to be a literal transfer for technology but would instead more realistically take the form of the entrant renting the incumbent's machines, plant and production methods for the remainder of the recession. For example, this could occur at a fire-sale over the incumbent firm's assets. After that, the entrant will implement his own methods so that the rental rate on such equipment is zero anyway. The important point is that the purchase of the assets at a price that would only be worthwhile to an entrant with a valuable innovation sends a credible signal to other innovators to avoid the sector.

It follows that the value of an incumbent firm in a sector where no innovation has occurred by time t during the vth cycle can be expressed as

$$V_i^I(t) = \pi_i(w_H(t), t)dt + e^{-r(t)dt}\left[e^{-\delta h_i dt}V^I(t+dt) + (1-e^{-\delta h_i dt})V^*(t+dt)\right] \tag{57}$$

Integrating over time yields

$$V_i^I(t) = \int_t^{T_v} e^{-\int_t^\tau [r(s)+\delta h_i(s)]ds}\left[\pi(w^H(\tau),\tau) + \delta h_i(\tau)V_i^*(\tau)\right]d\tau + \frac{P_i(T_v)}{P_i(t)}e^{-[R_0(T_v)-R(t)]}V_{0,i}^I(T_v). \tag{58}$$

The first term here represents the expected discounted profit stream that accrues to the entrepreneur during the current cycle, and the second term is the expected discounted value of being an incumbent thereafter. The amount of entrepreneurship varies over the cycle, but at the beginning of each cycle all industries are symmetric with respect to this probability: $P_i(T_v) = P(T_v)\ \forall i$.

4.2 Optimal Innovation and Implementation

The willingness of entrepreneurs to delay implementation until the boom and to just start engaging in innovative activities at exactly T_v^E depends crucially on the expected value of monopoly rents resulting from innovation, relative to the current skilled labor returns. This is a forward looking condition: given Γ and Δ^E, the present value of these rents depend on the length of the subsequent cycle, $T_{v+1} - T_v$, which we denote by the term Δ_{v+1}.

The expected value of an entrepreneurial success occurring at some time $t \in \left(T_v^E, T_v\right)$ but whose implementation is delayed until time T_v is thus:

$$V^D(t) = e^{-[R_0(T_v)-R(t)]}V_0^I(T_v). \tag{59}$$

Since Lemma 4 implies that entrepreneurship starts at T_v^E, free entry into entrepreneurship, requires that

$$\delta V^D(T_v^E) = \delta e^{-[R_0(T_v)-R(T_v^E)]}V_0^I(T_v) = s(T_v^E) \tag{60}$$

The increase in the wage across cycles reflects the improvement in overall productivity: $s(T_{v+1}^E) = e^{\Gamma+\Delta_v\phi}s(T_v^E)$, and from the asset market clearing conditions, we know that $R_0(T_v) - R(T_v^E) = \Gamma + \phi\Delta^E$ is a constant across cycles. It immediately follows that the increase in the present value of monopoly profits from the beginning of one cycle to the next must satisfy:

$$V_0^I(T_v) = e^{\Gamma+\Delta_v\phi}V_0^I(T_{v-1}). \tag{61}$$

As the following proposition demonstrates, given stationary values of Γ and Δ^E, equation (61) is solved by a unique value of Δ_v:

Proposition 6 *Given the boom size, Γ, and the length of the entrepreneurial innovation phase, Δ^E, there exists a unique cycle length, Δ, such that entrepreneurs are just willing to commence innovation, Δ^E periods prior to the boom.*

Notice, once again that this stationarity is not imposed, but is an implication of the equilibrium conditions.

The equilibrium conditions (24), (25) and (26) on posited entrepreneurial behavior also impose the following requirements on our hypothesized cycle:

• Successful entrepreneurs at time $t = T_v$, must prefer to implement immediately, rather than delay implementation until later in the cycle or the beginning of the next cycle:

$$V_0^I(T_v) > V_0^D(T_v). \tag{E1}$$

This condition is also sufficient to ensure that household utility is bounded in equilibrium, since it implies that[28]

$$\frac{1}{\Delta}\ln\left(\frac{c_0(T_{v+1})}{c_0(T_v)}\right) = \frac{\Gamma}{\Delta} + \phi < \frac{\rho}{1-\sigma}. \tag{62}$$

$>$From (55) this condition must hold if $\Delta > \Delta^E$.

• Entrepreneurs who successfully innovate during the downturn must prefer to wait until the beginning of the next cycle rather than implement earlier:

$$V^I(t) < V^D(t) \qquad \forall\, t \in (T_v^E, T_v) \tag{E2}$$

• No entrepreneur wants to innovate during the expansion of the cycle. Since in this phase of the cycle $\delta V^D(t) < s(t)$, this condition requires that

$$\delta V^I(t) < s(t) \qquad \forall\, t \in (0, T_v^E) \tag{E3}$$

Note finally that in constructing the equilibrium above we have implicitly imposed two additional requirements:

• The downturn is not long enough that all sectors innovate:

$$P(T_v) > 0. \tag{E4}$$

• Firm operating profits are always positive:

$$\pi(t) > 0 \qquad \forall\, t \tag{E5}$$

[28] To see this observe that (E1) can be expressed as $V_0^I(T_v) > e^{-[R_0(T_{v+1})-R_0(T_v)]}e^{\Gamma+\Delta\phi}V_0^I(T_v)$, which holds only if $R_0(T_{v+1}) - R_0(T_v) = \sigma(\Gamma + \Delta\phi) + \rho\Delta > \Gamma + \Delta\phi$. Re-arranging yields (62).

5 Baseline Example

In this section we demonstrate that there is a non–empty parameter space such that the triple $(\Delta^E, \Delta, \Gamma) > 0$ solving (55), (56) and (118) exists, and the conditions (E1) through (E5) are satisfied. For these values, posited entrepreneurial behavior is optimal given the implied cyclical behavior of aggregates that are generated by this behavior. Hence the cycling steady state exists.

We do this by first solving the model for a baseline set of parameters, given in Table 1. This exercise is not an attempt to assess the quantitative significance of the model. Rather, our aim here is instead to establish that the existence conditions can be simultaneously satisfied for values that are within reasonable bounds. The second aim is to gain some understanding of the model's comparative static properties by varying parameters around this baseline case.

Table 1: Baseline Parameters

Parameter	Value
ρ	0.03
ϕ	0.01
σ	0.30
θ	0.40
L	10.00
γ	0.60
$\overline{\mu}$	0.10
δ	0.85
q	0.40

We choose annual parameters using a rate of time preference $\rho = .03$, and intertemporal substitution parameter $\sigma = 0.3$. As already stated, this implies a high value for the intertemporal elasticity of substitution, which is necessary in the absence of physical capital to obtain existence.[29] We consider a baseline detection rate of q =0.4, implying that a shirker will be detected with probability $1 - e^{-0.4} = \frac{1}{3}$ within a year of shirking. With 168 hours in a week we assume ten hours per day are needed for basic self maintenance (eating, cleaning and sleeping). This leaves 98 hours in total available for work/leisure so that the average work week of 37.5 hours implies a value of θ around 0.4. We set the ratio of managers to workers, $\theta L = 4$.[30] The parameter $\bar{\mu}$ is the minimum rate of job destruction over the cycle, which we set equal to 10% per annum

[29] In Francois and Lloyd-Ellis (2005) we show that, with capital, pro-cyclical variation in the rate of investment allows for aggregate output fluctuations that exceed those in consumption.

[30] Estimates of this ratio vary considerably depending on industry and precise definition of managerial labor. Our calibration is close to the lower bound of these estimates, but the nature of the cycles we study is not affected by this variable. All that changes is the degree of wage pro-cyclicality.

The value of the direct technological advance embedded in new innovations, γ, is set equal to 0.6 which yields a mark–up rate of approximately 5% at the end of the expansion and contraction. We choose the unobservable productivity of innovation parameter, $\delta = 0.85$ to yield an annual growth rate close to two percent for most parameter variations.

For these values as the model's baseline we obtain an average long–run growth rate of $\frac{\Gamma}{\Delta} =$ 1.93% and a correlation of detrended log average wages and log output equal to 0.28 (both of which are close to the corresponding post–war averages in the US). The length of an expansion, Δ^X, is 7.25 years and of a contraction, Δ^E, is 2.85 years in this baseline. Figure 2 illustrates the evolution of key aggregates over a cycle implied by simulating the baseline example.

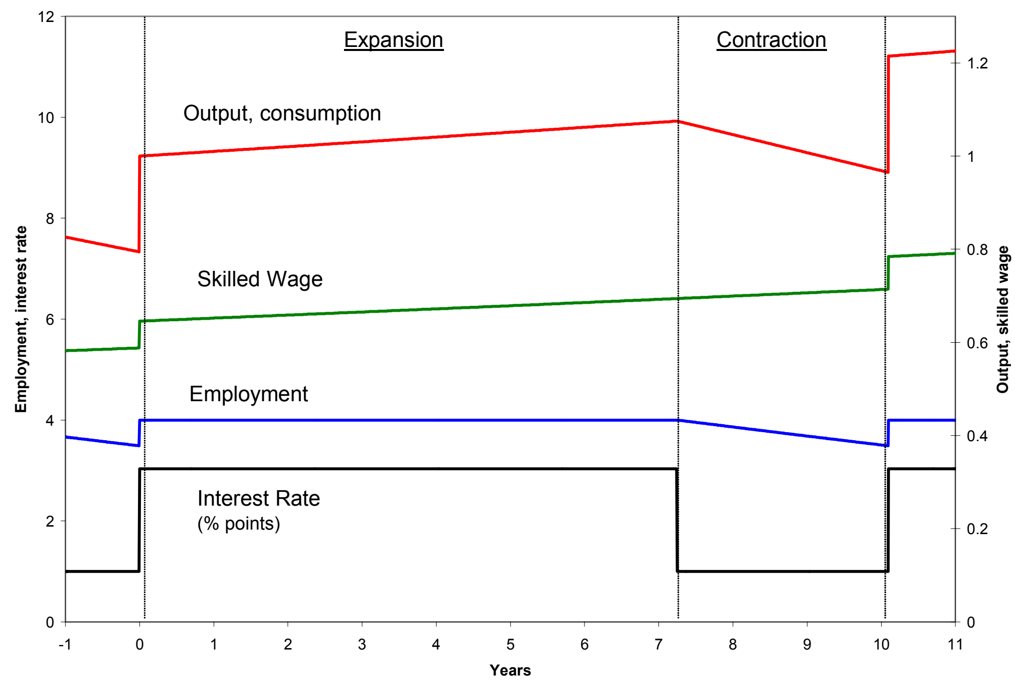

Figure 2: Evolution of Key Variable in Baseline Example

Figure 3 illustrates the evolution of the relevant value functions and the productivity adjusted wage $s(t)/\delta$ in the baseline example. At the beginning of the cycle $s(t) = \delta V^I(T_v) > \delta V^D(T_v)$. The value $\delta V^D(t)$ grows while $\delta V^I(t)$ declines during the first phase of the cycle, this condition implies that $\delta V^D(t)$ and $\delta V^I(t)$ must intersect before $\delta V^D(t)$ reaches $s(t)$. It follows that when entrepreneurship starts, it is optimal to delay implementation, $V^D(T_v^E) > V^I(T_v^E)$. Over time, during the entrepreneurship phase, the probability of not being displaced at the boom if implementing early declines so that $V^I(t)$ rises. Eventually, an instant prior to the boom, $V^I(T_{v+1}) = V^D(T_{v+1})$, but until that point it continues to be optimal to delay. At the boom, the

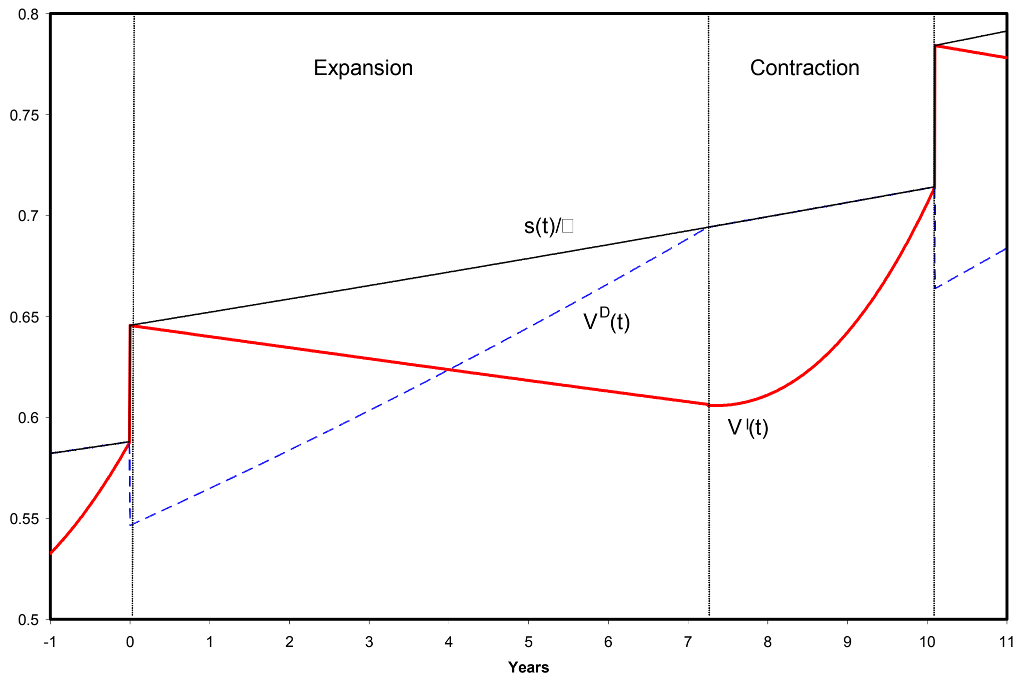

Figure 3: Evolution of Value Functions in Baseline Example

value of immediate implementation rises, while the value of delayed implementation falls, so that all existing innovations are implemented. However, since the skilled wage increases by as much as $V^I(t)$, entrepreneurship ceases and the cycle begins again.

5.1 Restructuring: Intensive and Extensive Margin Adjustment

The pattern of flows between unemployment and employment implied by this baseline case through the contraction are plotted in Figure 4. Flows into and out of employment are equivalent and stable over the expansion and only occur because of turnover on the intensive margin, $\bar{\mu}$. Upon entering the contractionary phase, job creation falls as firms cut back on the intensive margin in response to falling aggregate demand: $-\dot{n}(T_v^E)\Omega_v/\theta L$. This is generated by the endogenous recession occurring through increased restructuring. Simultaneous with this, the steady increase in innovation leads to gradually increasing destruction through this phase on the extensive margin; plants are being driven out of production by newly successful entrepreneurs. This occurs due to the increased entrepreneurship, $\delta H\left(t\right)$. This entrepreneurship is also reflected in the gradual but

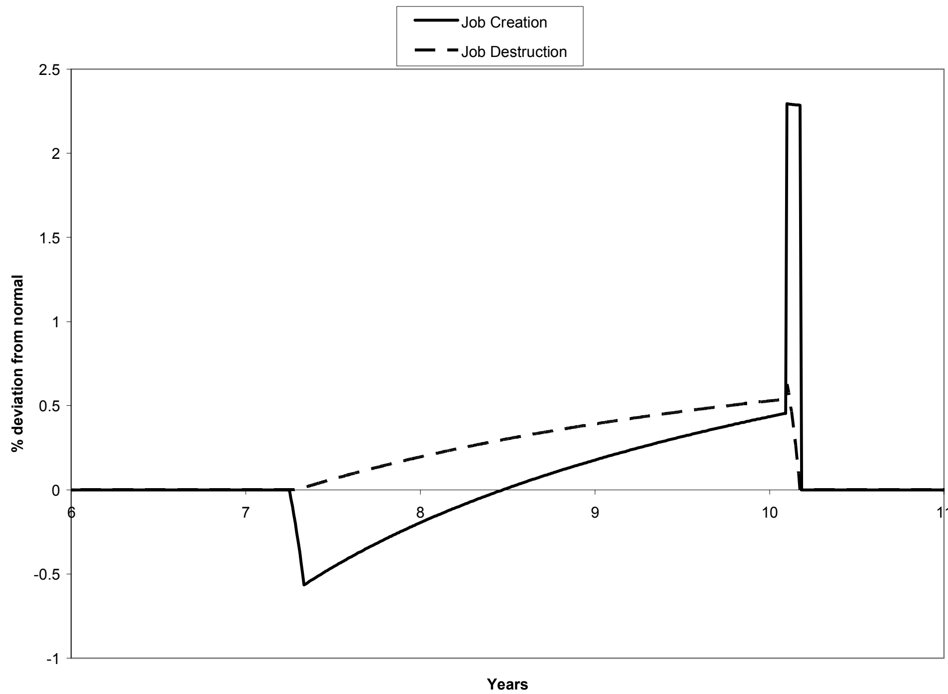

Figure 4: Employment Flows in Baseline Example

steady pick up in hiring that occurs in anticipation of the forthcoming boom. At the boom, existing firms then adjust employment on the intensive margin, increasing employment to meet the newly increased aggregate demand. This leads to a surge in job creation and an increase in employment amounting to $\theta L - n(T_v)$, which starts off the next cycle.[31]

Though successful entrepreneurs entering in recessions find it profitable to delay implementation of their own innovation until the boom, they start up production and thus hire workers out of the employment pool. Since the economy's output is contracting, and firms are reducing output to meet the lower aggregate demand, the rate of flow into employment is less than the rate of flow into unemployment, so that unemployment monotonically increases through the recession. Although the forces underlying the timing of the aggregate downturn here are related to the 'opportunity cost' view of recessions (e.g. Caballero and Hammour (1994), Aghion and Saint-Paul (1991) and Hall 1991), the productivity sequence is quite different. Both flows into unemployment and flows into employment increase when aggregate demand is falling, in the recession, and are de-coupled from increases in productivity which do not arise until the boom.

[31] This pattern of broadly felt changes on the intensive margin, coupled with large plant-specific changes (shutdowns) matches well with the pattern of employment adjustment reported by Schuh and Triest (1998). They report that dramatic employment growth rate changes (including start-ups and shut-downs) are a large proportion of flows – two thirds of jobs created and destroyed occur in plants that experience a more than 25% change in employment.

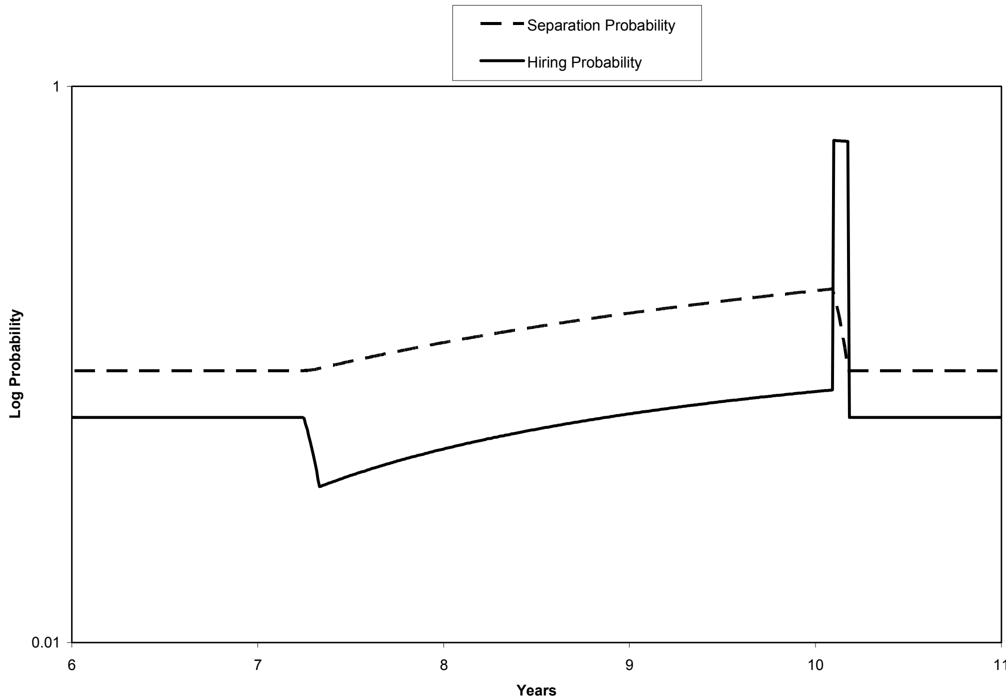

Figure 5: Transition Probabilities in Baseline Example

As Figure 4 suggests, the volatility of creation is greater than that of destruction — in the baseline example, the variance of job creation (.0045) is 5 times that of job destruction (.0009). Even though Schumpeterian creative destruction drives fluctuations here, firms adjust to reduced demand by cutting back on their hiring of labor, not by displacing workers. This adjustment is thus reflected in a lower probability of workers being re–employed once losing a job, illustrated in 5. As Shimer (2005) has argued, using the duration of unemployment spell information in the CPS, increased unemployment in recessions is due largely to increases in worker numbers with relatively long unemployment spells.

It is interesting to compare the relative variances of creation and destruction implied using measures of firm level flows corresponding to those used by Davis, Haltiwanger and Schuh (1996). Specifically, they define job destruction as the sum of declines in employment across plants where employment declines. In our framework, this would imply that the reduction in job creation on the intensive margin at the beginning of a recession would be measured as a sharp increase in job destruction.[32] Applying this measure to the data generated by the model here corresponds precisely with what Hall (2005) has argued occurs when these measures are used for actual data.

[32]During the recession, job destruction according the the DHS measure would be $n(t)\,\delta H(t) - \dot{n}(t)$, and job creation would be $n(t)\,\delta H(t)$.

He emphasizes the bias towards finding measured increases in destruction as recessions commence, as opposed to the actual decline in creation that occurs then. Recall that the decline in creation is what occurs in our model when the job-level measure is used — as in our Figure 4. Consequently, measuring employment flows using firm level flows overstates the volatility of job destruction and understates that of job creation. In our baseline example, the variances of job creation and job destruction measured in this way are .0025 and .0023 respectively.

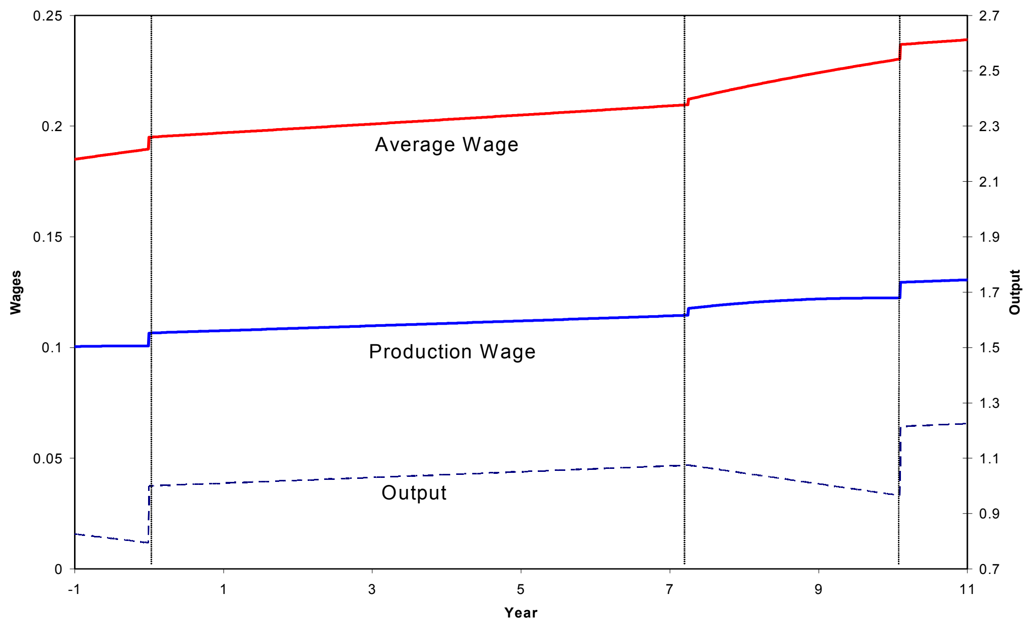

Figure 6: Wages in Baseline Example

5.2 Low Wage Pro-Cyclicality

The production wage grows at the rate ϕ through the expansion and thus tracks output through this phase, it does, however, move very differently through the recession. In the baseline case, the production wage rises at the start of the recession and then follows an inverted U shape through the downturn.[33] However, the increase in unemployment of production workers leads the model to replicate the counter–cyclical composition bias which emerges in the data for the same reason; see Bils (1984) and Abraham and Haltiwanger (1995). This is reflected in the continuously increasing average wage (i.e. the average of production workers and managers) through the recession, depicted in Figure 6. Thus, although through the expansion wages and

[33]This is admittedly hard to see in the Figure for the baseline case, but is generally the case.

output perfectly correlate, this is large offset by the negative correlation during the downturn. The baseline correlation of output with average wages (both detrended) is 0.28.

Note that average wages increase slightly upon entering the recession. This is because household employment growth, which was previously zero, becomes negative. Consequently, the marginal cost of providing effort is expected to be lower tomorrow than today which, on the margin, increases the willingness of workers to risk unemployment by shirking today. To compensate for this effect, firms must raise the wage in order to maintain incentives.

5.3 Comparative Statics

The following table shows the variation in output/wage correlation, growth, and cycle length for changes in each of the underlying parameters. The table lists the single parameter varied, and its value in the first column with the endogenous results in the columns to the immediate right.

Table 2: Growth, Wage Cyclicality, and Cycle Lengths

Parameter Values	$g(\%)$	Corr$(\hat{w}, \hat{y})$	Δ^X	Δ^E
Baseline Case	1.93	0.28	7.25	2.85
Variation: $\delta = 0.900$	1.96	0.15	6.53	2.69
$\delta = 0.800$	1.89	0.39	8.11	3.03
$\phi = 0.015$	2.65	0.28	4.02	2.84
$\phi = 0.005$	1.25	0.34	11.6	2.86
$\sigma = 0.301$	1.77	0.33	9.44	2.87
$\sigma = 0.299$	2.10	0.24	5.63	2.84
$\bar{\mu} = 0.105$	1.44	0.46	18.0	2.85
$\bar{\mu} = 0.095$	2.38	0.18	3.93	2.85
$q = 0.410$	2.23	0.27	4.11	2.85
$q = 0.390$	1.62	0.35	8.10	2.85
$\gamma = 0.610$	2.40	0.23	3.78	2.80
$\gamma = 0.590$	1.46	0.44	17.9	2.90
$\rho = 0.031$	1.82	0.32	9.00	2.85
$\rho = 0.029$	2.03	0.24	5.84	2.85

The intuition for most of the comparative static results in the table is relatively straightforward. Changes in parameters that reduce incentives to engage in entrepreneurship: lowering δ or γ (which have a direct effect on returns to entrepreneurship); lowering q and raising $\overline{\mu}$ (which raises the efficiency wage); raising σ and ρ (which makes consumers less willing to delay consumption) all lower the growth rate, as one would expect in a model of endogenous growth. Most of these changes also leave the length of the economy's contraction relatively unchanged, but imply (sometimes large) changes in the length of expansions. This is because with weaker underlying incentives to invest in entrepreneurship, longer expansionary phases, and therefore a longer reign of incumbency and profit, are required to provide sufficient incentives for entrepreneurship.

5.4 The Productivity Slowdown and Wage Pro-Cyclicality

As mentioned in the introduction the correlation between real wages and US industrial production has varied throughout the post-war period. Table 3, column 1, shows the correlation between the cyclical components of U.S. real manufacturing wages and industrial production over various periods (extracted using a HP filter). The last column reports annualized U.S. non-farm productivity growth (from the Bureau of Labor Statistics) over the sub-periods.

Table 3: U.S. Wage Cyclicality and Productivity Growth

Period	Correlation	Productivity Growth
1948:1–2003:4	0.27	1.8%
1948:1–1969:4	0.12	2.2%
1970:1–1993:4	0.43	1.1%
1994:1–2003:4	0.01	2.5%

From the second column in Table 2, compared with the baseline wage/output correlation of 0.28, changes in any or all of the following variables lead to an increase in the correlation between wages and production: lowering $\delta, \phi, \sigma, q, \gamma$ or increasing $\bar{\mu}, \rho$. Interestingly, for each one of these changes the growth rate, reported in column 1 of Table 2, also falls from the baseline case of 1.93%. Consequently, the pattern of co–movement between productivity and wage cyclicality generated by the model is consistent with that observed in post-war US data for *every* one of the models' comparative statics. Each one of these changes in parameters lowers the relative returns to innovation and lowers innovation on average. This lowers long-run productivity growth but also reduces firm obsolescence and implies less restructuring in recessions. This implies that the counter–cyclical force on wages due to turnover tends to be low when average productivity growth is also low, and high during phases of rapid productivity growth, explaining a pattern like that observed in Table 3.

6 Concluding Remarks

A Schumpeterian process of creative destruction implies a cyclical pattern of firm turnover, employment flows, wage movements and aggregate demand, that is qualitatively consistent with several key features of US data. Specifically, it can generate counter–cyclical restructuring, pro–cyclical productivity, and wages fluctuations which, depending on parameters, may exhibit any form of cyclicality. Moreover, in contrast to Caballero and Hammour (1994), the model can generate employment growth movements which are dominated by changes in job creation, while still preserving the Schumpeterian story. These patterns are derived in a model where the underlying source of productivity growth is partially endogenous, as is the clustering of activities across disparate sectors.

The Schumpeterian specification implies that the increased turnover integral to the recessionary phase developed here leads to increased volatility in employment. By increasing the fragility of the employment relationship in recessions, the incentive providing effect of the long–term relational contract is reduced, causing upward pressure on wages. This counter–cyclical mechanism, which offsets the standard pro–cyclical mechanisms that are also present here, leads to the possibility of low wage cyclicality in general. As the model is a unified framework for cycles, growth, and labor flows, it is a natural one to examine the interaction between changes in cyclical components and changes in the secular. Correlated changes in secular and cyclical aggregates, consistent with those observed between the productivity slowdown and increased wage pro–cyclicality of the early 70's, are generated by the current model.

Some features of our model's prediction are clearly at odds with the facts. However, we believe it is possible to extend the model in various ways to address some of these issues. In particular, the productivity boom and the associated jump in job creation are rather abrupt. As we show in a companion paper, Francois and Lloyd–Ellis (2005), adding capital to the model can help to smooth out the boom to some extent. If capital and production labor are strong complements in the short run, job creation may then be relatively slow following the boom. Alternative ways to do this may be to allow for some learning period or a stochastic implementation process. Another unrealistic feature of the cyclical process that we generate is that every cycle is the same and all fluctuations are deterministic. Extending the model to allow for some stochastic elements would relax some of these strong predictions. One approach that we are exploring is to allow the exogenous component of productivity growth to be subject to temporary i.i.d. shocks. This can change the length and amplitude of each cycle without changing the basic story.

A final comment we should make is that the model developed here captures an essential feature of the Schumpeterian paradigm. Although recessions allow skilled labor to be re–directed towards growth–promoting activities, this does not mean they are unambiguously a good thing. Recessions here are costly because they leave some resources (production labor) under–utilized. In Francois and Lloyd–Ellis (2003), we show formally that the cyclical equilibrium generates lower welfare than the corresponding acyclical one. A similar implication is likely to carry over to the framework developed here.

7 Appendix

Proof of Lemma 1:. Consider a firm holding an obsolete technology at time t. For there to exist a technology better than the firm's, an innovator must have allocated h to innovation in the firm's sector. But only firms planning to implement an innovation will find it worthwhile to undertake innovative effort, consequently, for the new technology, there exists some planned optimal date at which the technology will be implemented. Denote this date by t^*. Using the large scale of production requires hiring workers who do not shirk. This is only possible if the wage is incentive compatible. Clearly, at t^* the firm using the obsolete technology is not able to compete in production, so the large scale of production cannot be used. However it is also the case that incentive compatibility cannot hold, and the large scale of production not be used, if employment terminates with probability one in the next instant. Consequently at time $t^* - dt$ it is also not possible to use the large scale technology. However, the same argument applies at time $t^* - 2dt$ since the worker then knows that at $t^* - dt$ employment will be terminated. The same reasoning applies for each instant until time t.

Proof of Lemma 3: Profit maximization implies that $\psi_i^E(t) = \psi_i^S(t) \geq \psi^U(t)$ and so $\psi_i^*(t) = \psi_i^E(t)$. It follows that (15) can be expressed as

$$\psi_i^E(t) = (w_i(t)u_c(t) + u_n(t))\, dt + e^{-\rho dt}\left[e_i^{-\mu_i(t)dt}\psi_i^E(t+dt) + (1 - e^{-\mu_i(t)dt})\psi^U(t+dt)\right]. \quad (63)$$

Subtracting $e_i^{-(r(t)+\mu_i(t))dt}\psi_i^E(t)$ from both sides we get

$$\begin{aligned}(1 - e^{-(\rho+\mu_i(t))dt})\psi_i^E(t) &= (w_i(t)u_c(t) + u_n(t))\, dt + e^{-(\rho+\mu_i(t))dt}\left[\psi_i^E(t+dt) - \psi_i^E(t)\right] \quad (64)\\ &\quad +(1 - e^{-\mu_i(t)dt})e^{-\rho dt}\psi^U(t+dt)\end{aligned}$$

Dividing by dt we get

$$\begin{aligned}\frac{(1 - e^{-(\rho+\mu_i(t))dt})}{dt}\psi_i^E(t) &= w_i(t)u_c(t) + u_n(t) + e^{-(\rho+\mu_i(t))dt}\left[\frac{\psi_i^E(t+dt) - \psi_i^E(t)}{dt}\right] \quad (65)\\ &\quad +(\frac{1 - e^{-\mu_i(t)dt}}{dt})e^{-r(t)dt}\psi^U(t+dt)\end{aligned}$$

Letting $dt \to 0$ and noting that $\lim_{dt\to 0}\left(1 - e^{-xdt}\right) = x$ we get

$$[\rho + \mu_i(t)]\,\psi_i^E(t) = w_i(t)u_c(t) + u_n(t) + \dot{\psi}_i^E(t) + \mu_i(t)\psi^U(t) \quad (66)$$

Re–arranging yields

$$\rho\psi_i^E(t) = (w_i(t)u_c(t) + u_n(t)) - \mu_i(t)\left[\psi_i^E(t) - \psi^U(t)\right] + \dot{\psi}_i^E(t) \quad (67)$$

Similar reasoning can be applied to (16) and (17) to show that

$$\rho\psi_i^S(t) = w_i(t)u_c(t) - [\mu_i(t) + q]\left[\psi_i^S(t) - \psi^U(t)\right] + \dot{\psi}_i^S(t) \tag{68}$$

$$\rho\psi^U(t) = \lambda(t)\left[\psi_i^E(t) - \psi^U(t)\right] + \dot{\psi}^U(t) \tag{69}$$

Profit maximization subject to the incentive compatibility condition implies that $\psi_i^E(t) = \psi_i^S(t)$ and so subtracting (67) from (68) we get

$$\psi_i^E(t) - \psi^U(t) = -\frac{u_n(t)}{q} \tag{70}$$

It follows that $\psi_i^E(t) = \psi^E(t)\ \forall i$ and that

$$\dot{\psi}^E(t) - \dot{\psi}^U(t) = -\frac{\dot{u}_n(t)}{q} \tag{71}$$

Subtracting (69) from (67) we get

$$\rho\left[\psi^E(t) - \psi^U(t)\right] = w_i(t)u_c(t) + u_n(t) - [\mu_i(t) + \lambda(t)]\left[\psi^E(t) - \psi^U(t)\right] - \frac{\dot{u}_n(t)}{q} \tag{72}$$

Substituting in (70) and re-writing yields

$$-\rho\frac{u_n(t)}{q} = w_i(t)u_c(t) + u_n(t) + [\mu_i(t) + \lambda(t)]\frac{u_n(t)}{q} - \frac{\dot{u}_n(t)}{q} \tag{73}$$

$$-[\rho + \mu_i(t) + \lambda(t)]\frac{u_n(t)}{q} = w_i(t)u_c(t) + u_n(t) - \frac{\dot{u}_n(t)}{q}. \tag{74}$$

Re–arranging yields (31).■

Proof of Lemma 2: From the production function we have $\ln y(t) = \phi t + \int_0^1 \ln\frac{y(t)}{p_i(t)}di$. Substituting for $p_i(t)$ using (7) re-arranges to

$$s(t) = e^{-\gamma}\exp\left(\phi t + \int_0^1 \ln a_i(T_{v-1})di\right) = e^{\phi(t-T_{v-1})}s\left(T_{v-1}\right). \tag{75}$$

Proof of Proposition 1: Substituting for $c\,(t)$ from (33), setting $\dot{u}_n = 0$, and noting that when $dn = 0$ and $\mu^A = \overline{\mu}$, $\lambda = \frac{\overline{\mu}}{1-\theta}$, equation (31) rearranges to (36).

Proof of Lemma 4: Note that in any preceding no-entrepreneurship phase, $r\,(t) = \rho + \sigma\phi$. Thus, since, in a cycling equilibrium, the date of the next implementation is fixed at T_v, the expected value of entrepreneurship, δV^D, also grows at the rate $\rho + \sigma\phi > 0$. Thus, if under $H(T_v^E) = 0$, $\delta V^D(T_v^E) > s(T_v^E)$, then the same inequality is also true the instant before, i.e. at $t \to T_v^E$, since

$s(t)$ grows at the slower rate, $\phi < \rho + \sigma\phi$, within the cycle. But this violates the assertion that entrepreneurship commences at T_v^E. Thus necessarily, $\delta V^D(T_v^E) = s(T_v^E)$ at $H\left(T_v^E\right) = 0$.

Proof of Lemma 5: The Euler equation can be expressed as

$$\left(\frac{\dot{n}(t)}{n(t)} + \phi\right) + \frac{\dot{n}(t)}{L - n(t)} = -(\rho - \phi). \tag{76}$$

Re–arranging yields

$$\frac{\dot{n}(t)}{n(t)} + \frac{\dot{n}(t)}{L - n(t)} = -\left(\frac{\rho - (1-\sigma)\phi}{\sigma}\right). \tag{77}$$

Integrating, we get

$$\ln\frac{n(t)}{\theta L} - \ln\left(\frac{L - n(t)}{L - \theta L}\right) = -\frac{\rho - (1-\sigma)\phi}{\sigma}(t - T_v^E) \tag{78}$$

Solving for $n(t)$ yields (39).

Proof of Proposition 2: From (6) we can express the rate of job creation as

$$\lambda(t) = \frac{\mu^A(t)n(t) + \dot{n}(t)}{L - n(t)}. \tag{79}$$

Substituting into the expression for the efficiency wage we get

$$w^A(t) = \frac{\sigma c(t)}{(1-\sigma)(L - n(t))q}\left[\rho + q + \frac{\mu^A(t)L + \dot{n}(t)}{L - n(t)} - \frac{\dot{u}_n(t)}{u_n(t)}\right] \tag{80}$$

$$w^A(t) = \frac{\sigma c(t)}{(1-\sigma)(L - n(t))q}\cdot \left[\rho + q + \frac{\bar{\mu}L + \delta(L - n(t)/\theta)}{L - n(t)} + \frac{\dot{n}(t)}{L - n(t)} - \frac{\dot{u}_n(t)}{u_n(t)}\right] \tag{81}$$

$$w^A(t) = \frac{\sigma c(t)}{(1-\sigma)(L - n(t))q}\left[\rho + q + \frac{\delta}{\theta} - \frac{\frac{\delta(1-\theta)}{\theta}L - \bar{\mu}L}{L - n(t)} + \frac{\dot{n}(t)}{L - n(t)} - \frac{\dot{u}_n(t)}{u_n(t)}\right] \tag{82}$$

Differentiating (39) yields

$$\dot{n}(t) = \frac{-(1-\theta)\frac{\rho-(1-\sigma)\phi}{\sigma}\theta L e^{-\frac{\rho-(1-\sigma)\phi}{\sigma}(t - T_v^E)}}{\left[1 - \theta + \theta e^{-\frac{\rho-(1-\sigma)\phi}{\sigma}(t - T_v^E)}\right]^2}. \tag{83}$$

From (78) we have

$$\frac{n(t)}{L - n(t)} = \left(\frac{\theta}{1-\theta}\right) e^{-\frac{\rho-(1-\sigma)\phi}{\sigma}(t - T_v^E)} \tag{84}$$

Using (39) it follows that

$$L - n(t) = \frac{(1-\theta)L}{1 - \theta + \theta e^{-\frac{\rho-(1-\sigma)\phi}{\sigma}(t - T_v^E)}} \tag{85}$$

Diving (83) by (85) implies

$$\frac{\dot{n}(t)}{L-n(t)} = -\frac{\left(\frac{\rho-(1-\sigma)\phi}{\sigma}\right)\theta e^{-\frac{\rho-(1-\sigma)\phi}{\sigma}(t-T_v^E)}}{1-\theta+\theta e^{-\frac{\rho-(1-\sigma)\phi}{\sigma}(t-T_v^E)}} \tag{86}$$

Differentiating (13) w.r.t. time yields

$$\frac{\dot{u}_n(t)}{u_n(t)} = (1-\sigma)\frac{\dot{c}(t)}{c(t)} + (1-\sigma)\frac{\dot{n}(t)}{L-n(t)} \tag{87}$$

Using (38) to substitute we get

$$\frac{\dot{u}_n(t)}{u_n(t)} = \rho - \frac{\rho-(1-\sigma)\phi}{\sigma} \tag{88}$$

It follows that

$$w^A(t) = \frac{\sigma Y(t)}{(1-\sigma)(L-n(t))q}\left[\frac{\rho-(1-\sigma)\phi}{\sigma} + q + \frac{\delta}{\theta} - \frac{\frac{\delta(1-\theta)}{\theta}L - \bar{\mu}L}{L-n(t)} - \frac{\frac{\rho-(1-\sigma)\phi}{\sigma}\theta e^{-\frac{\rho-(1-\sigma)\phi}{\sigma}(t-T_v^E)}}{1-\theta+\theta e^{-\frac{\rho-(1-\sigma)\phi}{\sigma}(t-T_v^E)}}\right]. \tag{89}$$

Note that $Y(t) = Y_0(T_{v-1})e^{\phi(t-T_{v-1})}n(t)/\theta L$ and using (85) we get

$$w^A(t) = \frac{\sigma Y_0(T_{v-1})e^{\phi(t-T_{v-1})}e^{-\frac{\rho-(1-\sigma)\phi}{\sigma}(t-T_v^E)}}{(1-\sigma)(1-\theta)qL}\left[\begin{array}{c} q + \frac{\delta}{\theta} - \frac{\frac{\delta}{\theta}(1-\theta)-\bar{\mu}}{(1-\theta)}\left(1-\theta+\theta e^{-\frac{\rho-(1-\sigma)\phi}{\sigma}(t-T_v^E)}\right) \\ + \frac{\frac{\rho-(1-\sigma)\phi}{\sigma}(1-\theta)}{1-\theta+\theta e^{-\frac{\rho-(1-\sigma)\phi}{\sigma}(t-T_v^E)}} \end{array}\right] \tag{90}$$

$$w^A(t) = \frac{\sigma Y_0(T_{v-1})e^{\phi(t-T_{v-1})}}{(1-\sigma)(1-\theta)qL}\left[\begin{array}{c} (q+\delta+\bar{\mu})e^{-\frac{\rho-(1-\sigma)\phi}{\sigma}(t-T_v^E)} - \left(\delta - \frac{\bar{\mu}\theta}{1-\theta}\right)e^{-\frac{2(\rho-(1-\sigma)\phi)}{\sigma}(t-T_v^E)} \\ + \frac{\frac{\rho-(1-\sigma)\phi}{\sigma}(1-\theta)}{1-\theta+\theta e^{-\frac{\rho-(1-\sigma)\phi}{\sigma}(t-T_v^E)}} \end{array}\right] \tag{91}$$

Re–arranging yields (47).

Proof of Proposition 3: Long–run endogenous productivity growth in intermediates is given by

$$\Gamma_v = \gamma(1-P(T_v)) = \delta\gamma\int_{T_v^E}^{T_v} H(\tau)d\tau \tag{92}$$

Substituting using (40) implies

$$\Gamma_v = \delta\gamma\int_{T_v^E}^{T_v}\left(\frac{(1-\theta)\left(1-e^{-\frac{\rho-(1-\sigma)\phi}{\sigma}(\tau-T_v^E)}\right)}{1-\theta+\theta e^{-\frac{\rho-(1-\sigma)\phi}{\sigma}(\tau-T_v^E)}}\right)d\tau. \tag{93}$$

Integrating yields (50).[34]

[34] Details of integration are available from the authors.

Proof of Proposition 4: Just prior to the boom, when the probability of displacement is negligible, the value of implementing immediately must equal that of delaying until the boom:

$$\delta V^I(T_v) = \delta V^D(T_v) = s(T_v). \tag{94}$$

During the boom $V_0^I(T_v) > V_0^D(T_v)$. Thus the return to innovation at the boom is the value of immediate incumbency. It follows that free entry into entrepreneurship at the boom requires that

$$\delta V_0^I(T_v) \leq s_0(T_v) \tag{95}$$

The opportunity cost to financing entrepreneurship is the rate of return on shares in incumbent firms in sectors where no innovation has occurred. Just prior to the boom, this is given by the capital gains in sectors where no innovations have occurred. Combined with (94) and (95) it follows that asset market clearing at the boom requires

$$R_0(T_v) - R(T_v) = \log\left(\frac{V_0^I(T_v)}{V^I(T_v)}\right) \leq \log\left(\frac{s_0(T_v)}{s(T_v)}\right) = \Gamma_v. \tag{96}$$

Provided that $R_0(T_v) - R(t) > 0$, households will never choose to store final output from within a cycle to the beginning of the next. However, the return on stored intermediate output in sectors with no innovations, is strictly positive because its price increases at the boom. If innovative activities are to be financed at time t, households cannot be strictly better off buying claims to stored intermediate goods. Consider a sector where no innovation has occurred just prior to the boom. Since the cost of production is the same whether the good is stored or not, the rate of return on claims to stored intermediates in sector i is $\log p_{i,v+1}/p_{i,v} = \Gamma_v$. It follows that the long run rate of return on claims to firm profits an instant prior to the boom must satisfy

$$R_0(T_v) - R(T_v) \geq \log p_{i,v+1}/p_{i,v} = \Gamma_v. \tag{97}$$

Combining (96) and (97) yields the result.

In sectors with unimplemented innovations, entrepreneurs who hold innovations and are currently producing with the previous technology have the option of implementing the new technology before the boom, storing and then selling at the boom. Any such path of implementation will, however, affect the incentive compatible wage stream offered to production workers. Intuitively, producing and storing today for sale tomorrow implies a higher rate of layoffs tomorrow and hence a higher efficiency wage today. This upward effect on incentive compatible wages is what rules out such storage as a profitable option. Since the stream of revenue is unaffected by such storage, in order for firms' expected discounted profits to rise it must be that the discounted wage bill for production workers falls. To see this clearly, consider the expected value of the firm with an innovation at the beginning of the cycle, $V^I(T_v)$. Since $Y(\cdot), s, P(\cdot)$ and all discount rates are taken as given by the firm, V^I can only rise if the value of being employed at the firm $\psi^E(T_v)$ falls. But at all instants, the incentive compatible wage, $w^L(\tau)$ is given by the solution to $\psi^E(t) - \psi^U(t) = -\frac{u_n(t)}{q}$. Thus any such storage which raises profits for the firm will necessarily violate incentive compatibility for unskilled workers and will lead to shirking. The same argument rules out altering the production stream to benefit from discrete jumps in the wage anywhere along the cycle.

Proof of Proposition 5: Given that implementation of a new technology is delayed until the subsequent boom, T_v, we show: (1) An entrepreneur with a successful innovation offering amount $V^*(t) + \varepsilon$ to an incumbent to stop producing until T_{v+1} takes over production in that sector, and (2) other entrepreneurs stop innovation in that sector;.
Part (1): If a potential entrepreneur offers amount $V^*(t) + \varepsilon$ to an incumbent to stop producing until T_{v+1} and undertakes production itself using the existing technology up to time T_{v+1} then the incumbent correctly conjectures that this entrepreneur has discovered an innovation. This is because the value of the profit stream to the new entrant (whose incentive compatible wage is $w^L(t)$) is only $V^*(t)$. Since, before implementation, a successful entrepreneur must be producing with the technology in order to implement, and since buying out the previous incumbent stops all research, for $\varepsilon \to 0$, the entrepreneur is willing to pay this transfer to the previous incumbent. The previous incumbent is also willing to accept the transfer and shut-down production as $V^*(t)$ strictly exceeds the value of profits to the incumbent (whose incentive compatible wages is the higher $w^h(t)$) over the remaining interval to T_{v+1}. Part (2): Given (1) then all other entrepreneurs believe that the entrant in i has a productivity advantage which is e^γ times better than that currently available. If continuing to innovate in that sector, another entrepreneur will, with positive probability, also develop a productive advantage of e^γ over the current technology. Such an innovation yields expected profit of 0, since, in developing their improvement, they do not observe the non-implemented improvements of the new entrant, so that both firms Bertrand compete with the same technology. Returns to attempting innovation in another sector where there has been no signal of success, or from simply working in production, $s(t) > 0$, are thus strictly higher. An entrepreneur without an innovation would not find it worthwhile to take over production paying $V^*(t) + \varepsilon$ for the interval $t \to T_{v+1}$. At time T_{v+1} the previous incumbent is free to re-enter production using the same technology so that the total profit stream to an entrepreneur taking over production for the interval would be $V^*(t)$ which is ε less than the amount transferred.

Proof of Proposition 6: The value of an incumbent firm over a small interval dt during the cycle is

$$V_i^I(t) = \pi_i(w_H(t),t)dt + e^{-r(t)dt}\left[e^{-\delta h_i dt}V^I(t+dt) + (1-e^{-\delta h_i dt})V^*(t+dt)\right] \quad (98)$$

$$(1-e^{-(r(t)+\delta h_i)dt})V_i^I(t) = \pi_i(w_H(t),t)dt + e^{-r(t)dt}e^{-\delta h_i dt}\left[V^I(t+dt) - V_i^I(t)\right] \quad (99)$$
$$+(1-e^{-\delta h_i dt})e^{-r(t)dt}V^*(t+dt)$$

Dividing by dt and letting $dt \to 0$ we get

$$(r(t)+\delta h_i(t))V_i^I(t) = \pi_i(w_H(t),t) + \delta h_i(t)V^*(t) + \dot{V}_i^I(t) \quad (100)$$

Given some initial time period t and the final period T_v, the solution to this first–order differential equation is

$$V_i^I(t) = \int_t^{T_v} e^{-\int_t^s (r(\tau)+\delta h_i(\tau))d\tau}\left[\pi_i(w_H(s),s) + \delta h_i(s)V^*(s)\right]ds + e^{-\int_t^{T_v}(r(s)+\delta h_i(s))ds}V_0^I(T_v) \quad (101)$$

$$V_i^I(t) = \int_t^{T_v} e^{-\int_t^s r(\tau)d\tau}\frac{P(s)}{P(t)}\left[\pi_i(w_H(s),s) + \delta h_i(s)V^*(s)\right]ds + e^{-[R_0(T_v)-R(t)]}\frac{P(T_v)}{P(t)}V_0^I(T_v) \quad (102)$$

If $t \geq T_v^E$ we know that $r(\tau) = \phi$ to the end of the cycle. Hence

$$V_i^I(t) = \int_t^{T_v} e^{-\phi(s-t)}\frac{P(s)}{P(t)}\left[\pi_i(w_H(s),s) + \delta h_i(s)V^*(s)\right]ds + e^{-[R_0(T_v)-R(t)]}\frac{P(T_v)}{P(t)}V_0^I(T_v)$$
$$= \frac{1}{P(t)}\int_t^{T_v} e^{-\phi(s-t)}\left[P(s)\pi_i(w_H(s),s) + \delta H(s)V^*(s)\right]ds + e^{-[R_0(T_v)-R(t)]}\frac{P(T_v)}{P(t)}V_0^I(T_v) \quad (103)$$

Note that by partial integration

$$\int_t^{T_v} e^{-\phi(s-t)}\delta H(s)V^*(s)ds = \int_t^{T_v} e^{-\phi(s-t)}V^*(s)d(1-P(s)) \quad (104)$$

$$= \left[e^{-\phi(s-t)}V^*(s)(1-P(s))\right]_t^{T_v} - \int_t^{T_v}(1-P(s))de^{-\phi(s-t)}V^*(s) \quad (105)$$

But $V^*(T_v) = 0$ and $de^{-\phi(s-t)}V^*(s) = -e^{-\phi(s-t)}\pi_i(w_L(s),s)ds$, so that

$$\int_t^{T_v} e^{-\phi(s-t)}\delta H(s)V^*(s)ds = \int_t^{T_v}(1-P(s))e^{-\phi(s-t)}\pi(w_L(s),s)ds - V^*(t)(1-P(t)) \quad (106)$$

Since profits are a linear function of the production wage we get $\forall\ t \geq T_v^E$:

$$V_i^I(t) = \frac{1}{P(t)}\int_t^{T_v} e^{-\phi(s-t)}\pi(w^A(s),s)ds + e^{-[R_0(T_v)-R(t)]}\frac{P(T_v)}{P(t)}V_0^I(T_v) - V^*(t)\left(\frac{1-P(t)}{P(t)}\right) \quad (107)$$

Now if $t = T_v^E$, $P(T_v^E) = 1$ and this can be expressed as

$$V_i^I(T_v^E) = \int_{T_v^E}^{T_v} e^{-\phi(s-T_v^E)}\pi(w^A(s),s)ds + e^{-\left[R_0(T_v)-R(T_v^E)\right]}P(T_v)V_0^I(T_v). \quad (108)$$

It follows that the discounted monopoly profits from owning an innovation at time T_{v-1} is given by

$$\begin{aligned}
& V_0^I(T_{v-1}) && (109)\\
= & \int_{T_{v-1}}^{T_v} e^{-\int_{T_{v-1}}^{\tau} r(s)ds}\left(1-e^{-\gamma}\right)Y(\tau)d\tau \\
& -\int_{T_{v-1}}^{T_v} e^{-\int_{T_{v-1}}^{\tau} r(s)ds} e^{-\gamma}\theta L\frac{w^A(\tau)}{s(\tau)}Y(\tau)d\tau + P(T_v)e^{-[R_0(T_v)-R(T_{v-1})]}V_0^I(T_v) && (110)\\
= & \left(1-e^{-\gamma}\right)Y_0(T_{v-1})\left(\int_{T_{v-1}}^{T_v^E} e^{-(\rho-(1-\sigma)\phi)(\tau-T_{v-1})}d\tau + e^{-(\rho-(1-\sigma)\phi)(T_v^E-T_{v-1})}\int_{T_v^E}^{T_v}\frac{n(\tau)}{\theta L}d\tau\right) && (111)\\
& -\left(\int_{T_{v-1}}^{T_v^E} e^{-(\rho+\sigma\phi)(\tau-T_{v-1})}w^A(\tau)\theta L d\tau + e^{-(\rho+\sigma\phi)(T_v^E-T_{v-1})}\int_{T_v^E}^{T_v} e^{-\phi(t-T_v^E)}w^A(\tau)n(\tau)d\tau\right)\\
& +P(T_v)e^{-[R_0(T_v)-R(T_{v-1})]}V_0^I(T_v)\\
= & \left(1-e^{-\gamma}\right)Y_0(T_{v-1})\left(\int_{T_{v-1}}^{T_v^E} e^{-(\rho-(1-\sigma)\phi)(\tau-T_{v-1})}d\tau + e^{-(\rho-(1-\sigma)\phi)(T_v^E-T_{v-1})}\int_{T_v^E}^{T_v}\frac{n(\tau)}{\theta L}d\tau\right)\\
& -\left(\int_{T_{v-1}}^{T_v^E} e^{-(\rho-(1-\sigma)\phi)(\tau-T_{v-1})}A\theta Y_0(T_{v-1})d\tau + e^{-(\rho+\sigma\phi)(T_v^E-T_{v-1})}\int_{T_v^E}^{T_v} e^{-\phi(t-T_v^E)}w^A(\tau)n(\tau)d\tau\right)\\
& +P(T_v)e^{-[R_0(T_v)-R(T_{v-1})]}V_0^I(T_v)
\end{aligned}$$

Using (61) and re–arranging we get

$$\begin{aligned}
& \left(1-P(T_v)e^{\Gamma-[R_0(T_v)-R(T_{v-1})]}\right)V_0^I(T_{v-1})\\
= & \left(1-e^{-\gamma}\right)Y_0(T_{v-1})\left(\frac{1-e^{-b(T_v^E-T_{v-1})}}{b} + e^{-\rho(T_v^E-T_{v-1})}\int_{T_v^E}^{T_v}\frac{e^{-\frac{b}{\sigma}(t-T_v^E)}}{1-\theta+\theta e^{-\frac{b}{\sigma}(t-T_v^E)}}dt\right)\\
& -\left(\frac{1-e^{-b(T_v^E-T_{v-1})}}{b}\right)A\theta Y_0(T_{v-1}) && (112)\\
& -e^{-b(T_v^E-T_{v-1})}Y_0(T_{v-1})\theta\int_{T_v^E}^{T_v}\left[\frac{Be^{-2\frac{b}{\sigma}(t-T_v^E)}-Ce^{-3\frac{b}{\sigma}(t-T_v^E)}}{1-\theta+\theta e^{-\frac{b}{\sigma}(t-T_v^E)}}+\frac{De^{-2\frac{b}{\sigma}(t-T_v^E)}}{\left[1-\theta+\theta e^{-\frac{b}{\sigma}(t-T_v^E)}\right]^2}\right]dt,
\end{aligned}$$

where it is analytically convenient to let

$$b=\rho-(1-\sigma)\phi.$$

Integrating and dividing through by $Y_0(T_{v-1})$ yields

$$\begin{aligned}
& \left(1-P(T_v)e^{\Gamma-[R_0(T_v)-R_0(T_{v-1})]}\right)\frac{V_0^I(T_{v-1})}{Y_0(T_{v-1})}\\
= & \left(1-e^{-\gamma}\right)\left[\frac{1-e^{-b(T_v^E-T_{v-1})}}{b}-e^{-\rho(T_v^E-T_{v-1})}\left(\frac{\sigma}{b\theta}\ln\left(1-\theta\left(1-e^{-\frac{b}{\sigma}\Delta^E}\right)\right)\right)\right]
\end{aligned}$$

$$
-\left(\frac{1-e^{-\rho(T_v^E-T_{v-1})}}{\rho}\right)A\theta \tag{113}
$$

$$
-e^{-b(T_v^E-T_{v-1})}\frac{\sigma}{b}\left[\begin{array}{c}\left(B+\frac{1-\theta}{\theta}C\right)\left(1-e^{-\frac{b}{\sigma}\Delta^E}+\frac{1-\theta}{\theta}\ln\left(1-\theta\left(1-e^{-\frac{b}{\sigma}\Delta^E}\right)\right)\right)\\ -\frac{1}{2}C\left(1-e^{-\frac{2b}{\sigma}\Delta^E}\right)-D\left(\frac{1}{\theta}\ln\left(1-\theta\left(1-e^{-\frac{\rho}{\sigma}\Delta^E}\right)\right)+\frac{(1-\theta)\left(1-e^{-\frac{b}{\sigma}\Delta^E}\right)}{1-\theta\left(1-e^{-\frac{b}{\sigma}\Delta^E}\right)}\right)\end{array}\right]
$$

Collecting terms

$$
\begin{aligned}
&\left(1-P(T_v)e^{\Gamma-[R_0(T_v)-R_0(T_{v-1})]}\right)\frac{V_0^I(T_{v-1})}{Y_0(T_{v-1})}\\
=\ &\left(1-e^{-\gamma}\right)\frac{1-e^{-b(T_v^E-T_{v-1})}}{b}\\
&-e^{-(\rho-(1-\sigma)\phi)(T_v^E-T_{v-1})}\left(\frac{\sigma}{b\theta}\ln\left(1-\theta\left(1-e^{-\frac{b}{\sigma}\Delta^E}\right)\right)\right)\left[\left(1-e^{-\gamma}\right)+\left(B+\frac{1-\theta}{\theta}C\right)(1-\theta)-D\right]\\
&-\left(\frac{1-e^{-b(T_v^E-T_{v-1})}}{b}\right)A\theta \qquad (114)\\
&-e^{-\rho(T_v^E-T_{v-1})}\frac{\sigma}{b}\left[\left(B+\frac{1-\theta}{\theta}C\right)\left(1-e^{-\frac{b}{\sigma}\Delta^E}\right)-\frac{1}{2}C\left(1-e^{-\frac{2b}{\sigma}\Delta^E}\right)-D\frac{(1-\theta)\left(1-e^{-\frac{b}{\sigma}\Delta^E}\right)}{1-\theta\left(1-e^{-\frac{b}{\sigma}\Delta^E}\right)}\right]
\end{aligned}
$$

Substituting using (55) and (56),

$$
\begin{aligned}
&\left(1-\left(1-\frac{b\Delta^E}{\gamma(1-\sigma)}\right)e^{-b(\Delta-\Delta^E)}\right)\frac{e^{-\gamma}}{\delta}\\
=\ &\left(1-e^{-\gamma}\right)\frac{1-e^{-b(\Delta-\Delta^E)}}{b}\\
&+e^{-\rho(\Delta-\Delta^E)}\left[\left(1-e^{-\gamma}\right)+\left(B+\frac{1-\theta}{\theta}C\right)(1-\theta)-D\right]\left(1-\frac{\rho}{\delta\gamma(1-\sigma)}\right)\Delta^E\\
&-\left(\frac{1-e^{-b(\Delta-\Delta^E)}}{b}\right)A \qquad (115)\\
&-e^{-b(\Delta-\Delta^E)}\frac{\sigma}{b}\left[\left(B+\frac{1-\theta}{\theta}C\right)\left(1-e^{-\frac{b}{\sigma}\Delta^E}\right)-\frac{1}{2}C\left(1-e^{-\frac{2b}{\sigma}\Delta^E}\right)-D\frac{(1-\theta)\left(1-e^{-\frac{b}{\sigma}\Delta^E}\right)}{1-\theta\left(1-e^{-\frac{b}{\sigma}\Delta^E}\right)}\right]
\end{aligned}
$$

$$
\begin{aligned}
&\left(e^{b(\Delta-\Delta^E)}-1+\frac{b\Delta^E}{\gamma(1-\sigma)}\right)\frac{e^{-\gamma}}{\delta}\\
=\ &\left(1-e^{-\gamma}\right)\frac{e^{b(\Delta-\Delta^E)}-1}{b}+\left[\left(1-e^{-\gamma}\right)+\left(B+\frac{1-\theta}{\theta}C\right)(1-\theta)-D\right]\left(1-\frac{b}{\delta\gamma(1-\sigma)}\right)\Delta^E\\
&-\left(\frac{e^{b(\Delta-\Delta^E)}-1}{b}\right)A\theta \qquad (116)
\end{aligned}
$$

$$-\frac{\sigma}{b}\left[\left(B+\frac{1-\theta}{\theta}C\right)\left(1-e^{-\frac{b}{\sigma}\Delta^E}\right)-\frac{1}{2}C\left(1-e^{-\frac{2b}{\sigma}\Delta^E}\right)-D\frac{(1-\theta)\left(1-e^{-\frac{b}{\sigma}\Delta^E}\right)}{1-\theta\left(1-e^{-\frac{b}{\sigma}\Delta^E}\right)}\right]$$

Collecting terms we get

$$\begin{aligned}
&\left(e^{b(\Delta-\Delta^E)}-1\right)\left[\frac{e^{-\gamma}}{\delta}-\left(\frac{1-e^{-\gamma}}{\rho}\right)+\frac{A\theta}{\rho}\right] \qquad (117)\\
=\ &\left[(1-e^{-\gamma})-\frac{b}{\delta\gamma(1-\sigma)}+\left[\left(B+\frac{1-\theta}{\theta}C\right)(1-\theta)-D\right]\left(1-\frac{b}{\delta\gamma(1-\sigma)}\right)\right]\Delta^E\\
&-\frac{\sigma}{b}\left[\left(B+\frac{1-\theta}{\theta}C\right)\left(1-e^{-\frac{b}{\sigma}\Delta^E}\right)-\frac{1}{2}C\left(1-e^{-\frac{2b}{\sigma}\Delta^E}\right)-D\frac{(1-\theta)\left(1-e^{-\frac{b}{\sigma}\Delta^E}\right)}{1-\theta\left(1-e^{-\frac{b}{\sigma}\Delta^E}\right)}\right]
\end{aligned}$$

Dividing through by $\left[\frac{e^{-\gamma}}{\delta}-\left(\frac{1-e^{-\gamma}}{b}\right)+\frac{A\theta}{b}\right]$ and solving for Δ yields

$$\Delta=\Delta^E+\frac{1}{b}\ln\left[1+\alpha\Delta^E+\zeta_1\left(1-e^{-\frac{b}{\sigma}\Delta^E}\right)-\zeta_2\left(1-e^{-\frac{2b}{\sigma}\Delta^E}\right)-\frac{\zeta_3\left(1-e^{-\frac{b}{\sigma}\Delta^E}\right)}{1-\theta\left(1-e^{-\frac{b}{\sigma}\Delta^E}\right)}\right], \qquad (118)$$

where

$$\alpha=\frac{\frac{b}{\delta\gamma(1-\sigma)}-(1-e^{-\gamma})-\left[\left(B+\frac{1-\theta}{\theta}C\right)(1-\theta)-D\right]\left(1-\frac{b}{\delta\gamma(1-\sigma)}\right)}{\left(\frac{1-e^{-\gamma}}{b}\right)-\frac{e^{-\gamma}}{\delta}-\frac{A\theta}{b}}, \qquad (119)$$

$$\begin{aligned}
\zeta_1 &= \frac{\frac{\sigma}{b}\left(B+\frac{1-\theta}{\theta}C\right)}{\left(\frac{1-e^{-\gamma}}{b}\right)-\frac{e^{-\gamma}}{\delta}-\frac{A\theta}{b}},\ \zeta_2=\frac{\frac{\sigma}{b}\frac{1}{2}C}{\left(\frac{1-e^{-\gamma}}{b}\right)-\frac{e^{-\gamma}}{\delta}-\frac{A\theta}{b}} \qquad (120)\\
\text{and } \zeta_3 &= \frac{\frac{\sigma}{(\rho-(1-\sigma)\phi)}D(1-\theta)}{\left(\frac{1-e^{-\gamma}}{b}\right)-\frac{e^{-\gamma}}{\delta}-\frac{A\theta}{b}}.
\end{aligned}$$

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