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Intrinsic Business Cycles with Pro-Cyclical R&D

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Abstract

Recent empirical work finds that R&D expenditures are quite procyclical, even for firms that are not credit-constrained during downturns. This has been taken as strong evidence against Schumpeterian-style theories of business cycles that emphasize the idea that downturns in production may be good times to allocate labor towards innovative activities. Here we argue that the procyclicality of R&D investment is, in fact, quite consistent with at least one of these theories. In our analysis, we emphasize three key features of R&D investment relative to other types of innovative activity: (1) it uses knowledge intensively, (2) it is a long-term investment with uncertain applications and (3) it suffers from diminishing returns over time.

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1 Introduction

Empirical evidence suggests that R&D expenditures in the US and other OECD economies are pro-cyclical. This evidence appears to run counter to recent theories which have revived the Schumpeterian notion that economic downturns play an important role in promoting long run productivity growth. In this article we demonstrate that, in fact, the pro-cyclical behavior of R&D is quite consistent with at least one of these theories. Our analysis emphasizes the fact that R&D is just one part of a multi-stage innovative process by which basic discoveries are eventually translated into productivity improvements. We show that the inherent uncertainty regarding the timing and eventual application of new ideas implies that R&D investment naturally exhibits very different business cycle properties to other forms of innovative activity.

The notion that downturns may induce greater R&D spending, is often associated with the impact of negative shocks in Schumpeterian endogenous growth models. By lowering wages, negative shocks reduce the opportunity costs of innovative effort and induce higher long term productivity growth (see Aghion and Saint Paul (1998) for a survey). A number of recent theories of “endogenous growth cycles” imply the economy alternates between phases of high productivity growth and high fixed capital formation and phases of low productivity growth, but intensive R&D (e.g. Bental and Peled (1996), Matsuyama (1999, 2001) and Wälde (1999, 2001). However, because they feature no absolute downturns in economic activity and/or are single-sector models, it is difficult to relate these theories to the actual business cycle. In contrast, Francois and Lloyd-Ellis (2003) develop a multi-sector Schumpeterian paradigm in which expansions and absolute downturns are an intrinsic part of the long-term growth process. Expansions reflect the endogenous, clustered implementation of productivity improvements (as in Shleifer, 1986), and recessions are the negative side-product of the restructuring that anticipates them.¹

Most empirical evidence on R&D spending appears to run counter to the predictions of these theories. Looking at aggregate NSF data on R&D spending in the post-war US, Saint-Paul (1993) finds “little evidence of pro- or countercyclical behavior”. However, Fatas (2000) documents that growth in real R&D expenditures in the US is positively correlated with real GDP growth. Moreover, Barlevy (2005a), focussing only on the growth rate of real R&D expenditures financed by private industry (NSF data), finds that it tends to be significantly higher during periods of rapid economic growth. Wälde and Woitek (2004) study the cyclical behavior of R&D expenditures by business enterprises in G7 countries over the period 1973–2000. On balance, they also find stronger evidence in favour of pro-cyclical rather than countercyclical R&D spending.

¹Francois and Lloyd-Ellis (2006) extend the model to allow for capital accumulation and show how fluctuations in the investment rate support the incentives needed to generate the multi-sector cycle.

Recently, a number of theories have been advanced to explain why R&D spending may be pro-cyclical. Aghion et al. (2005), for example, show how R&D may fall during recessions because of tighter credit constraints. However, using Compustat data on the R&D expenditures of publicly traded companies in the US, Barlevy (2005a) finds that the tendency for R&D growth to fall during recessions is actually more pronounced in firms that are *less* likely to be credit constrained.² Barlevy (2005b) develops a stochastic Schumpeterian growth model in which, although it is socially optimal for R&D to be concentrated during downturns, short-term behavior by innovators results in an inefficiently counter-cyclical allocation of resources. In a business cycle model with endogenous R&D spending, Comin and Gertler (2006) find that exogenous mark-up shocks can also induce pro-cyclical movements in R&D.³

Here, we demonstrate that explicitly introducing R&D into the intrinsic business cycle model of Francois and Lloyd-Ellis (2003), as the first step in a multi-stage innovative process, implies that R&D investment inherently evolves in a pro-cyclical manner.⁴ Our explanation does not depend on the existence of tightening credit constraints during downturns nor on short-term behavior by innovators. Moreover, it arises in a model in which both the cyclical process and growth are endogenously determined. Here the pro-cyclical behavior of R&D is the result of three assumed characteristics of R&D: (1) its productivity is enhanced by implemented technology, (2) it is a long-term investment with uncertain applications and (3) there are diminishing returns to existing knowledge.

Although it is common to assume that ideas discovered through R&D are immediately translated into productivity gains, in reality R&D (as typically defined) is just the first step in the overall innovative process. According to the official OECD definition, R&D is distinguished from other innovation costs in that it must have “an appreciable element of novelty and the resolution of scientific and/or technological uncertainty.” However, as others have argued, this is a rather narrow definition of innovation. For example, Kamin, Bijaoui and Horesh (1982), Evangelista et al. (1997) and Baldwin et al. (2004) identify many activities (e.g. product development and design, product specification, prototype construction, manufacturing startup and organizational adjustments) that are crucial for adapting and implementing newly developed technology into production, but which are not generally classified as R&D. In all these studies, R&D spending accounts for less than 50% of the overall costs of innovation.

In the model developed here, we decompose the innovation process into three distinct stages: R&D, commercialization and implementation (see Figure 1). R&D is modelled as a costly process

²Aghion et al. (2005) do find cross-country empirical evidence in support of their theory.

³Tarashima (2005) argues that to resolve the pro-cyclical R&D puzzle one must abandon the conjecture that cycles are driven by technology shocks altogether.

⁴In our earlier work, the implications for the cyclical behaviour of R&D was left unspecified.

which generates potentially productive “ideas” whose exact application and timing thereof (if ever) is uncertain. We assume that these ideas are patented immediately, even though their exact application is unknown, so that some share of the return can be reaped by investors. We use the term commercialization to refer to the process of matching these ideas with particular applications and adapting them for use. Commercialization is modelled as a form of costly search by entrepreneurs and/or managers who are motivated by a share of the expected profits.⁵ In particular, the rate at which existing ideas are commercialized depends on this entrepreneurial search effort. Once commercially-viable uses have been identified, they can then be implemented in production at an optimally chosen date by licensing to intermediate goods producers. The resulting profits are divided between investors in R&D and the entrepreneur/managers according to a simple Nash bargain.

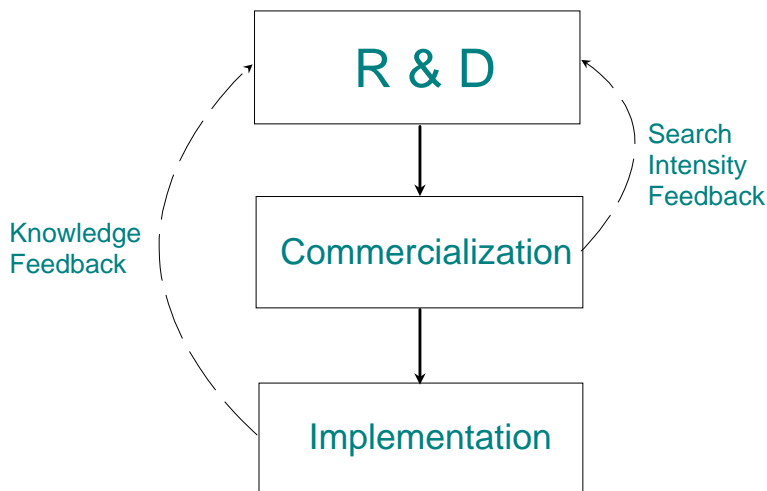


Figure 1: Multi-stage innovative process

As in Francois and Lloyd-Ellis (2003), commercialization is concentrated towards the end of downturns, peaking just prior to the subsequent boom. The very fact that this search activity intensifies during recessions, implies that the value of ideas whose applications have yet to be determined is maximized at the cyclical peak.⁶ After this, the value of these “unmatched ideas” declines temporarily as the likelihood of identifying a commercially viable application before the next expansion declines. Since the expected cost of obtaining each idea does not also fall, R&D actually ceases altogether during recessions.⁷ Following an implementation boom, the interest

⁵One could interpret these agents more narrowly as venture capitalists.

⁶Nickell, Nicolitsas and Patterson (2001) find that many “managerial innovations” (e.g. changes in structure, more decentralization, changes in human resources management practices, the implementation of just in time technologies) are concentrated during in downturns.

⁷In a model with fixed capital formation, R&D would decline but not necessarily fall to zero. See Francois and

rate rises and the value of unmatched ideas grows as the next phase of intensified commercialization approaches. This induces increased investment in R&D, causing the stock of potentially productive knowledge to rise. Due to diminishing returns to existing knowledge, the equilibrium unit cost of R&D consequently rises with the value of new ideas through the expansion. Thus, the incentives to undertake R&D move in exactly the opposite way over the cycle to those faced by entrepreneur/managers engaged in commercialization.

The remainder of the paper is laid out as follows. Section 2 develops the building blocks of the model. Section 3 posits and describes behavior in the cyclical equilibrium and elaborates the dynamics over the phases of the cycle. Section 4 derives sufficient conditions for existence to be met. Section 5 demonstrates existence of the equilibrium for various sets of parameter values and explores the equilibrium's qualitative characteristics. Section 6 concludes.

2 The Model

2.1 Assumptions

There is no aggregate uncertainty. Time is continuous and indexed by $t \geq 0$. The economy is closed and there is no government sector. There are L infinitely-lived households with identical iso-elastic preferences:

$$U(t) = \int_t^\infty e^{-\rho(\tau-t)} \frac{C(\tau)^{1-\sigma} - 1}{1-\sigma} d\tau \quad (1)$$

where ρ denotes the rate of time preference and σ is the inverse of the elasticity of intertemporal substitution. Each household maximizes (1) subject to the intertemporal budget constraint

$$\int_t^\infty e^{-[R(\tau)-R(t)]} C(\tau) d\tau \leq B(t) + \int_t^\infty e^{-[R(\tau)-R(t)]} w(\tau) d\tau \quad (2)$$

where $w(t)$ denotes wage income, $B(t)$ denotes the household's stock of assets at time t and $R(t)$ denotes the discount factor from time zero to t .

Final output is produced by competitive firms according to a Cobb-Douglas production function utilizing intermediates, x , indexed by i , over the unit interval:

$$Y(t) = \exp \left[\int_0^1 \ln x_i(t) di \right] \quad (3)$$

We let p_i denote the price of intermediate i . Final output can be used for consumption, $C(t)$, investment in R&D, $I_R(t)$, or (potentially) stored:

$$C(t) + I_R(t) \leq Y(t). \quad (4)$$

Lloyd-Ellis (2006).

Output of intermediate i depends upon the state of technology in sector i , $A_i(t)$, and the labor hours, l_i , according to a simple linear technology:

$$x_i^s(t) = A_i(t)l_i(t). \quad (5)$$

Labor receives the equilibrium wage $w(t)$. There is no imitation, so the dominant entrepreneur in each sector undertakes all production and earns monopoly profits by limit pricing until displaced by a higher productivity rival. We assume that intermediates are completely used up in production, but can be produced and stored for use at a later date. Incumbent intermediate producers must therefore decide whether to sell now, or store and sell later.

For simplicity we assume that, wherever they are ultimately used, new ideas always dominate old ones by a productivity factor e^γ . This implies that the total potential productivity of the stock of existing knowledge can be expressed as $Z(t) = e^{\gamma N(t)}$, where $N(t)$ is the measure of existing ideas. We assume that this potential productivity evolves according to a Cobb-Douglas function

$$\dot{Z}(t) = \mu [I_R(t)]^\theta [Z(t)]^{1-\theta}, \quad (6)$$

where $\mu > 0$ and $\theta \in (0, 1]$. It follows that the measure of new ideas emanating from R&D per period is

$$\dot{N}(t) = \frac{1}{\gamma} \frac{\dot{Z}(t)}{Z(t)} = \frac{\mu}{\gamma} \frac{I_R(t)}{Z(t)}^{1-\theta}.$$

R&D investment is financed by selling claims to households. This specification implies that one more new idea requires $\frac{\gamma}{\mu} \frac{1}{\theta} Z(t)$ units of investment — effectively there are diminishing returns to existing ideas. The expected value of the share of claims to such an idea, accruing to investors in R&D, is denoted by $\Omega(t)$.

2.2 The Market for Ideas

Although R&D adds to the stock of potentially productive ideas, these ideas are not immediately commercially viable. We model the market for these ideas as a one-sided matching process in which entrepreneur/managers allocate labor effort to searching amongst the stock of potential ideas for those that will be commercially viable in a particular application.⁸ The rate of success of this search activity is given by $\delta h_i(t)$, where δ is a parameter, and h_i is the labor effort allocated to search in sector i . At any point in time, entrepreneur/managers decide whether or not to allocate labor effort to searching for commercially viable ideas, and if they do so, how much.

⁸Comin and Gertler (2006) and Paterson (2006) use a related two-stage framework to capture the delay between R&D and the adoption of ideas into production. A key difference here is that, once the commercial viability of an idea is identified, there may be a further (strategic) delay before implementation.

The aggregate labor effort allocated to search is given by $H(t) = \int_0^1 h_i(t) dt$. As with R&D, entrepreneurial search is financed by selling claims to households.

Entrepreneurs with commercially-viable innovations must choose whether or not to implement immediately or delay until a later date. Once they implement, the knowledge of how the idea can be made commercially viable becomes publicly available, and can be built upon by rival entrepreneurs. However, prior to implementation, this knowledge is privately held by the entrepreneur. We let $V_i^I(t)$ denote the expected present value of profits from implementing at time t , and $V_i^D(t)$ denote that of delaying implementation from time t until the most profitable time in the future. It follows that the value of a commercially viable idea is

$$V_i^*(t) = \max[V_i^D(t), V_i^I(t)] \quad (7)$$

Once an idea is implemented in production, the households who financed R&D receive $(1-\eta)V_i^*(t)$ and those that financed entrepreneurial search receive $\eta V_i^*(t)$. As is common in models of search and matching, the share parameter η is treated as an exogenous outcome of a bilateral bargaining process.

It follows that the expected value of a claim to an unmatched idea, $\Omega(t)$, depends on the eventual payoff, the rate at which the idea is matched with an application and the delay before it is implemented. The stock $S(t)$ of “untapped ideas” — ideas emanating from R&D which have not been matched with a particular application — evolves according to

$$\dot{S}(t) = \dot{N}(t) - \delta H(t). \quad (8)$$

We assume that the ideas which constitute this untapped stock are equally likely to be commercially viable. It follows that the rate at which a given idea is matched is given by $\delta H(t)/S(t)$.

Note that we have implicitly imposed the assumption that each idea emanating from R&D has a unique application, so that once it has been matched with a sector, no subsequent matches can occur. This greatly simplifies the exposition, with little loss of generality. The model could be extended to allow for multiple applications without changing the main results.

2.3 Definition of Equilibrium

Given an initial stock of implemented innovations represented by a cross-sectoral distribution of productivities $\{A_i(0)\}_{i=0}^1$ an equilibrium for this economy satisfies the following conditions:

- Households allocate consumption over time to maximize (1) subject (2). The first-order conditions of the household’s optimization require that

$$C(t)^\sigma = C(s)^\sigma e^{R(t)-R(s)-\rho(t-s)} \quad \forall t, s, \quad (9)$$

and that the transversality condition holds: $\lim_{s \rightarrow \infty} e^{-R(s)}B(s) = 0$

- Final goods producers choose intermediates to maximize profits. The derived demand for intermediate i is then

$$x_i^d(t) = \frac{Y(t)}{p_i(t)} \quad (10)$$

- Intermediate producers set prices. It follows that the price of intermediate i is given by

$$p_i(t) = \frac{w(t)}{e^{-\gamma}A_i(t)}, \quad (11)$$

and the instantaneous profit earned is

$$\pi_i(t) = (1 - e^{-\gamma})Y(t). \quad (12)$$

Note crucially that firm profits are proportional to aggregate demand.

- Labor market clearing:

$$\int_0^1 l_i(t)di + H(t) = L \quad (13)$$

Labor market equilibrium also implies

$$w(t)(L - H(t)) = e^{-\gamma}Y(t) \quad (14)$$

- Free entry into arbitrage. For all assets that are held in strictly positive amounts by households, the rate of return between time t and time s must equal $\frac{R(s)-R(t)}{s-t}$.
- There is free entry into search

$$\delta\eta V_i^*(t) \leq w(t), \quad h_i(t) \geq 0 \quad \text{with at least one equality, } \forall i \quad (15)$$

- In periods where there is implementation, entrepreneurs with commercially viable ideas must prefer to implement rather than delay until a later date.
- In periods where there is no implementation, either there must be no commercially viable available to implement, or entrepreneurs must prefer to delay rather than implement.
- Free entry into R&D:

$$\Omega(t) \leq \frac{\mu}{\mu} \frac{\gamma}{\theta} Z(t), \quad I_R(t) \geq 0 \quad \text{w.a.l.o.e.} \quad (16)$$

- The aggregate resource constraint must be satisfied:

$$C(t) + I_R(t) = Y(t). \quad (17)$$

3 The Acyclical Equilibrium Growth path

Although we are mainly concerned with the cyclical equilibrium growth path, it is useful for comparison purposes to briefly consider the stationary growth path of the acyclical equilibrium. Along this path all commercially-viable ideas are implemented immediately and aggregates grow at the same constant rate

$$g^a = \delta\gamma H. \quad (18)$$

Consequently, the Euler equation yields a constant interest rate given by

$$r^a = \rho + \sigma g^a. \quad (19)$$

Along the balanced growth path, the search no-arbitrage equation must also hold:

$$r^a + \delta H = \frac{\dot{V}^I}{V^I} + \frac{\pi}{V^I}. \quad (20)$$

Along this path, free entry into commercialization requires that

$$\eta\delta V^I(t) = w(t) \quad (21)$$

and profits are given by

$$\pi(t) = (e^\gamma - 1)(L - H)w(t). \quad (22)$$

Substituting into (20) using (18), (21) and (22) yields

$$r^a = \eta(e^\gamma - 1)\delta L - [\eta(e^\gamma - 1) + 1 - \gamma]g^a/\gamma \quad (23)$$

Assuming $\gamma < 1 + \eta(e^\gamma - 1)$, this equation yields a negative relationship between r and g . The main reason is that a high steady-state growth rate, g , implies more labor allocated to search which deflates profits and raises the risk of obsolescence. These tend to offset the positive impact of higher profit growth. Equating (19) and (23) yields the steady state growth rate

$$g^a = \gamma \frac{\eta(e^\gamma - 1)\delta L - \rho}{\eta(e^\gamma - 1) + 1 - \gamma(1 - \sigma)} \quad (24)$$

A no-arbitrage equation must also hold for R&D. This is given by

$$r^a = \frac{\delta H}{S} \frac{(1 - \eta)V^I(t) - \Omega(t)}{\Omega(t)} + \frac{\dot{\Omega}(t)}{\Omega(t)}. \quad (25)$$

Free entry into R&D requires that

$$\Omega(t) = \frac{\mu}{8} \frac{\gamma}{\mu} \frac{\mu}{8} Z(t), \quad (26)$$

and the productivity of implemented knowledge can be expressed as

$$\bar{A}(t) = Z(t)e^{-\gamma S(t)}. \quad (27)$$

Substituting into (25), using (18), (21), (26) and (27) yields

$$\frac{\mu}{g^a} - 1 - \gamma S^* = \frac{(1 - \eta)e^{-\gamma}e^{-\gamma S^*}}{\delta\eta(\gamma/\mu)^{\frac{1}{\theta}}} - 1 \quad (28)$$

Given r^a and g^a , this equation pins down the steady-state stock of unmatched ideas, S^* .

Notice that, in this acyclical equilibrium, the R&D sector plays an essential, but largely supportive role: it produces and maintains a sufficiently large stock of knowledge to ensure that the economy grows at the required rate. The parameters of the R&D technology, μ and θ , have no impact on long-run growth. In effect the role is very similar to that of capital accumulation in a standard endogenous growth model.

4 The Cyclical Equilibrium Growth Path

Suppose that implementation occurs at discrete dates denoted by T_v where $v \in \{1, 2, \dots, \infty\}$. We adopt the convention that the v th cycle starts at time T_{v-1} and ends at time T_v . We denote values of variables the instant after implementation by a 0 subscript. After implementation at date T_{v-1} , an **expansion** is triggered by a productivity boom and continues through subsequent consumption growth. During this phase, commercialization ceases and consequently all labor effort is used in production. R&D spending is highest during this phase so that the stock of knowledge grows. At some time T_v^* , search commences and labor starts to be withdrawn from production. Commercially viable ideas are not implemented immediately but are withheld until time T_v . During this **contraction** phase, investment in R&D slows and search continues to accelerate in anticipation of the subsequent boom. As aggregate demand falls, labor continues to be released from production into the increased search.

Let $P_i(s)$ denote the probability that, since time T_{v-1} , no entrepreneurial success has been made in sector i by time s . It follows that the probability of there being no innovation by time T_v conditional on there having been none by time t , is given by $P_i(T_v)/P_i(t)$. Hence, the value of an incumbent firm in a sector where no innovation has occurred by time t during the v th cycle can be expressed as

$$V_i^I(t) = \int_t^{T_v} e^{-\int_t^\tau r(s)ds} \pi_i(\tau) d\tau + \frac{P_i(T_v)}{P_i(t)} e^{-\beta(t)} V_{0,i}^I(T_v), \quad (29)$$

where

$$\beta(t) = R_0(T_v) - R(t) \quad (30)$$

denotes the discount factor used to discount from time t during the cycle to the beginning of the next cycle. The first term in (29) represents the discounted profit stream that accrues with certainty during the current cycle, and the second term is the expected discounted value of being an incumbent at the beginning of the next cycle.

Lemma 1 *In a cyclical equilibrium, the identification of commercially viable ideas can be credibly signalled immediately and all search in that sector stops until the next round of implementation.*

If an entrepreneur's announcement is credible, other entrepreneurs will exert their search efforts in sectors where they have a better chance of becoming the dominant entrepreneur. One might imagine that unsuccessful entrepreneurs would have an incentive to mimic successful ones by falsely announcing success to deter others from entering the sector. But there is no advantage to this strategy relative to the alternative of allocating effort to the sector until, with some probability, another entrepreneur is successful, and then switching to another sector.

In the cyclical equilibrium, entrepreneurs' conjectures ensure no more entrepreneurship in a sector once a signal of success has been received, until after the next implementation. The expected value of an entrepreneurial success occurring at some time $t \in (T_v^*, T_v)$ but whose implementation is delayed until time T_v is thus:

$$V_i^D(t) = e^{-\beta(t)} V_{0,i}^I(T_v), \quad (31)$$

Since no implementation occurs during the cycle, the entrepreneur implementing at T_v is assured of incumbency until at least T_{v+1} . Incumbency beyond that time depends on the probability that no commercially viable improvement has been identified in that sector up until then.

The symmetry of sectors implies that innovative effort is allocated evenly over all sectors that have not yet experienced an innovation within the cycle. Thus the probability of not being displaced at the next implementation is

$$P_i(T_v) = \exp \left[- \int_{T_v^*}^{T_v} \delta h_i(\tau) d\tau \right]. \quad (32)$$

Given the simplifying assumption that all ideas have equal likelihood of being commercialized, it follows that $\Omega(t)$, the value of a claim to a new idea that has yet to be matched with a particular application must satisfy the Bellman equation, which is identical to that in the acyclical equilibrium, equation (25):

$$r(t)\Omega(t) = \frac{\delta H(t)}{S(t)} \mathbb{E} \left[(1 - \eta)V^D(t) - \Omega(t) \right] + \dot{\Omega}(t) \quad (33)$$

Note that since the probability of being matched is no greater than 1, it must be the case that $(1 - \eta)V^D(t) \geq \Omega(t)$.

5 Within-cycle dynamics

Within a cycle, $t \in [T_{v-1}, T_v]$, the state of technology in use is unchanging. The appendix demonstrates that the producers' limit pricing behavior ensures, the wage is also constant during the cycle and is pinned down by the level of technology:

$$w(t) = e^{-\gamma} \bar{A}_{v-1} \quad (34)$$

where

$$\bar{A}_{v-1} = \exp \left[\int_0^1 \ln A_i(T_{v-1}) di \right]. \quad (35)$$

Note that the wage is less than its marginal product by a constant factor $e^{-\gamma}$, reflecting the fact that a fraction $1 - e^{-\gamma}$ goes in the form of profits to intermediate producers. Consequently, standard aggregation results hold and aggregate output can be expressed as

$$Y(t) = \bar{A}_{v-1} [L - H(t)], \quad (36)$$

In order to afford a stationary representation of the economy, it is convenient to normalize aggregates by dividing by total factor productivity using lower-case letters to denote these deflated variables:

$$c(t) = \frac{C(t)}{\bar{A}_{v-1}}, \quad y(t) = \frac{Y(t)}{\bar{A}_{v-1}}, \quad z(t) = \frac{Z(t)}{\bar{A}_{v-1}}. \quad (37)$$

Consequently, the intensive form production function is given by

$$y(t) = L - H(t). \quad (38)$$

The household's Euler equation during the cycle can be expressed as

$$\frac{\dot{c}(t)}{c(t)} = \frac{r(t) - \rho}{\sigma} \quad (39)$$

where $r(t) = \dot{R}(t)$. The normalized potential productivity evolves according to

$$\frac{\dot{z}(t)}{z(t)} = \mu \frac{L - H(t) - c(t)}{z(t)} \theta, \quad (40)$$

since $\dot{z} > 0$ implies $\Omega(t) = \frac{\sigma}{\mu} \frac{1}{\theta} Z(t)$.

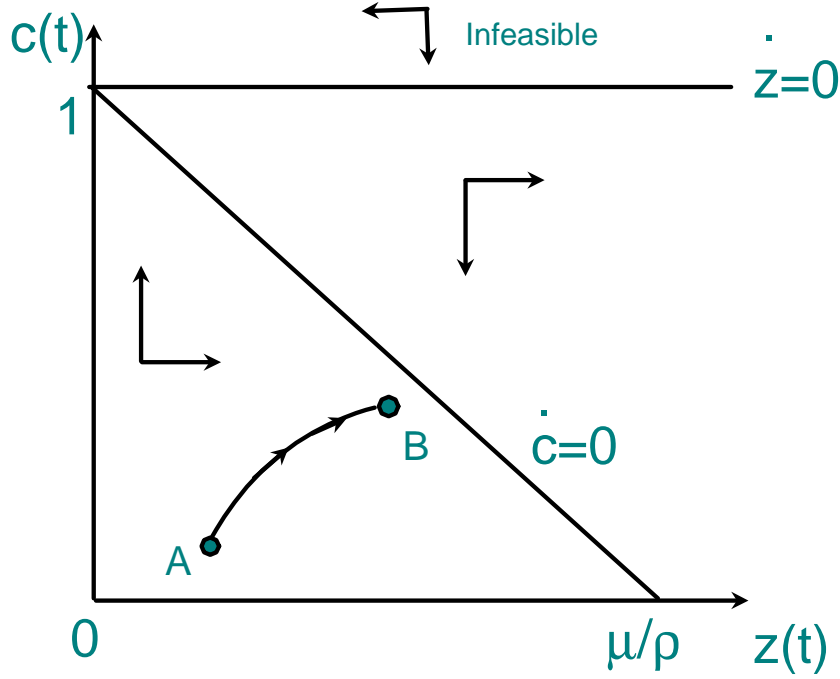


Figure 2: Phase Diagram

5.1 The Expansion ($T_{v-1} \rightarrow T_v^*$)

During the expansion all labour is used in the production of consumption goods and R&D, $H(t) = 0$. From (16) and (33) it follows that

$$\frac{\dot{z}(t)}{z(t)} = \frac{\dot{\Omega}(t)}{\Omega(t)} = r(t). \quad (41)$$

Combing these conditions with (36), (39) and (40) yields the dynamical system:

$$\frac{\dot{c}(t)}{c(t)} = \frac{\mu}{\sigma} \frac{\mu L - c(t)}{z(t)} \mathbb{1}_\theta - \frac{\rho}{\sigma} \quad (42)$$

$$\frac{\dot{z}(t)}{z(t)} = \mu \frac{\mu L - c(t)}{z(t)} \mathbb{1}_\theta. \quad (43)$$

These dynamics are illustrated using the phase diagram in Figure 2.

In the equilibrium that we study both consumption and the stock of knowledge grow during the expansion, so we restrict attention to the left of the $\dot{c} = 0$ locus. The economy evolves along a transition path like AB in Figure 2. Given initial values for consumption and the stock of knowledge, the dynamical system above therefore yields a unique path for $c(t)$ and $z(t)$ at each date t during the expansion. In particular, we can describe the path of consumption as

$$c(t) = F(t; c_0(T_{v-1}), z_0(T_{v-1})) \quad \forall t \in [T_{v-1}, T_v^*], \quad (44)$$

where the function $F(t; \cdot)$ is implicitly defined.

As a result of the boom, wages rise rapidly. Since the next implementation boom is some time away, the present value of allocating a unit of labour effort to search falls below the wage, $\delta\eta V^D(t) < w(t)$. During the expansion, the expected value of search, $\delta\eta V^D(t)$, grows at the rate of interest, but continues to be dominated by the wage. As a result of ongoing R&D, the stock of ideas expands. However none of these ideas is matched with a sector, so that

$$\dot{S}(t) = \dot{N}(t). \quad (45)$$

At date T_v^* , $\delta\eta V^D(T_v^*) = w(T_v^*)$ for the first time. If all labor were to remain in production, the returns to search effort would strictly dominate those in production. As a result, labor effort is reallocated from production into search, triggering the next phase of the cycle. The following Lemma demonstrates that during this transition, labor effort shifts rapidly from one activity to the other:

Lemma 2 : At T_v^ , investment in R&D falls discretely to zero and entrepreneurial search effort jumps discretely to $H_v = H_0(T_v^*) > 0$.*

5.2 The Contraction ($T_v^* \rightarrow T_v$)

During this phase, there is search, so that $H(t) > 0$. Since there is free entry into search, $w(t) = \delta\eta V^D(t)$, and so the value of entrepreneurship, $\delta\eta V^D(t)$, must be constant. Since the time until implementation for a successful entrepreneur is falling and there is no stream of profits (because implementation is delayed), the instantaneous interest rate necessarily equals zero:

$$r(t) = \frac{\dot{V}^D(t)}{V^D(t)} = \frac{\dot{w}(t)}{w(t)} = 0. \quad (46)$$

Since $H(t) > 0$, $r(t) = 0$ and $(1 - \eta)V^D(t) > \Omega(t)$ it follows that the value of claims to unmatched ideas starts to decline:

$$\frac{\dot{\Omega}(t)}{\Omega(t)} = -\frac{\delta H(t)}{S(t)} \frac{\mu}{\Omega(t)} \frac{(1 - \eta)V^D(t) - \Omega(t)}{\Omega(t)} < 0 \quad (47)$$

Since the accumulation of ideas is inherently irreversible ($\dot{z}(t) \geq 0$), it follows that during this phase, R&D optimally ceases and the expected value of an idea falls below the cost of producing it:

$$\frac{\Omega(t)}{A_{v-1}} < \frac{\mu}{\mu} \frac{\mu}{\mu} z(t) = \frac{\mu}{\mu} \frac{\mu}{\mu} z(T_v^*). \quad (48)$$

Intuitively, as the downturn proceeds, the likelihood that a given idea will be matched with a sector before the subsequent boom declines. Since the interest rate is zero during this phase, the fact that the boom is getting closer yields no offsetting growth in the value.

Although investment in R&D falls discretely at $t = T_v^*$, consumption is constant across the transition between phases because the discount factor does not change discretely. It follows that the decline in output due to the fall in R&D investment demand must be proportional to the fraction of labor hours withdrawn from production:

$$H_v = I_R(T_v^*) = L - c(T_v^*). \quad (49)$$

Lemma 3 : *During the downturn the value of an unmatched idea at time t is given by*

$$\Omega(t) = (1 - \eta)V^D(t) \left[1 - \frac{S(T_v)}{S(t)} \right] + \frac{S(T_v)}{S(t)} e^{-\Gamma_v} \Omega_0(T_v) \quad (50)$$

Since no R&D takes place during the downturn, the stock of potential knowledge remains the same until the beginning of the subsequent cycle, $Z_0(T_v) = Z(T_v^*)$. But since R&D is positive at the beginning of the next cycle, it must also be true that $\Omega_0(T_v) = \frac{\gamma}{\mu}^{\frac{1}{\theta}} Z_0(T_v)$. Taken together these facts imply that while the value of claims to R&D falls during the downturn, their increase at the boom must exactly offset this: $\Omega(T_v^*) = \Omega_0(T_v)$. Combining this with (50) implies the following:

Proposition 1 : *The stock of unmatched ideas at the cyclical peak, $S(T_v^*)$, must satisfy*

$$\frac{(1 - \eta)e^{-\gamma}}{\eta\delta} \left[\frac{\mu}{\gamma} \right]^{\frac{1}{\theta}} \frac{\Gamma_v}{\gamma S(T_v^*)(1 - e^{-\beta(T_v^*)}) + \Gamma_v e^{-\beta(T_v^*)}} = e^{\gamma S(T_v^*)}. \quad (51)$$

Note that it must be the case that $\gamma S(T_v^*) > \Gamma_v$. Since the term in brackets must be less than unity, an additional necessary (but not sufficient) condition on parameters that must hold is $(1 - \eta)e^{-\gamma} > \eta\delta \left[\frac{\gamma}{\mu} \right]^{\frac{1}{\theta}}$. This expression is analogous to (28) for the acyclical growth path — given growth in productivity at the boom, Γ_v , and the discount factor to the beginning of the next cycle, $\beta(T_v^*)$, it pins down the stock of unmatched ideas available at the previous cyclical peak.

Since the economy is closed, it follows once again that, because there is no incentive to store output,⁹ consumption must decline in this phase:

$$\frac{\dot{c}(t)}{c(t)} = -\frac{\rho}{\sigma} < 0. \quad (52)$$

⁹Although $r = 0$, strict preference for zero storage results from arbitrarily small storage costs.

This occurs during the downturn because labor gradually flows out of production and into search. Using (49) and (52), yields the following expression for aggregate effort allocated to search at time t :

$$H(t) = L - e^{-\frac{\rho}{\sigma}[t-T_v^*]}(L - H_v). \quad (53)$$

This, in turn, determines the measure of sectors in which commercially viable ideas are identified at each date:

$$-\dot{P}(t) = \delta H(t), \quad (54)$$

where $P(T_v^*) = 1$.¹⁰ At the end of the cycle, the fraction of sectors that have identified commercially viable ideas is therefore

$$1 - P(T_v) = \int_{T_v^*}^{T_v} \delta H(\tau) d\tau. \quad (55)$$

5.3 The Implementation Boom

We denote the growth in aggregate productivity during the implementation period T_v by $\Gamma_v = \ln(\bar{A}_v/\bar{A}_{v-1})$. Since $\Gamma_v = \gamma(1 - P(T_v))$, (55) and (53) determine the size of the boom as a function of the length of the downturn, $\Delta_v^C = T_v - T_v^*$:

$$\Gamma_v = \delta\gamma L\Delta_v^C - \delta\gamma(L - H_v) \frac{\bar{A} \left(1 - e^{-\frac{\rho}{\sigma}\Delta_v^C}\right)}{\rho/\sigma}. \quad (56)$$

At the beginning of each cycle all labor is used in production. Since output is only augmented by the increase in aggregate productivity $C_0(T_v) = e^{\Gamma_v}C_0(T_{v-1})$. The Euler equation therefore implies a long run discount factor given by

$$R_0(T_v) - R_0(T_{v-1}) = \rho\Delta_v + \sigma\Gamma_v \quad (57)$$

During the expansion, (41) implies that the discount factor must grow by $\ln \frac{z(T_v)}{z(T_{v-1})}$, recalling that there is no R&D in the recession. During the downturn the interest rate is zero. Combining these facts with (57), it follows that across the boom the discount factor must satisfy

$$\beta(T_v) = \rho\Delta_v + \sigma\Gamma_v - \ln \frac{z(T_v)}{z(T_{v-1})} \quad (58)$$

Over the boom, the asset market must simultaneously ensure that entrepreneurs holding viable ideas are willing to implement immediately (and no earlier) and that, for households, holding equity in firms dominates holding claims to alternative assets (particularly stored intermediates). The following Proposition demonstrates that these conditions imply that during the boom, the discount factor must equal productivity growth:

¹⁰The rate of change in P is given by $\frac{\dot{P}}{P} = -\delta h_i$. But since labor is allocated symmetrically to innovation only in the measure P of sectors where no innovation has occurred, $h_i = \frac{H}{P}$, so that $\dot{P} = -\delta H$.

Proposition 2 : *Asset market clearing at the boom requires that*

$$\beta(T_v) = \Gamma_v \quad (59)$$

Since the interest rate is zero through the downturn it is also the case that $\beta(T_v^*) = \Gamma_v$ and that, using the household Euler equation:

$$\rho \Delta^X + \sigma \ln \frac{c(T_v^*)}{c_0(T_v)} = \Gamma. \quad (60)$$

5.4 Optimal Entrepreneurial Behavior

The willingness of entrepreneurs to delay implementation until the boom and to just start engaging in search activities at exactly T_v^* depends crucially on the expected value of monopoly rents, relative to the current labor costs. This is a forward looking condition: given Γ and Δ^C , the present value of these rents depend crucially on the length of the subsequent cycle, $T_{v+1} - T_v$.

Since Lemma 2 implies that entrepreneurship starts at T_v^* , free entry into entrepreneurship, requires that

$$\delta \eta V^D(T_v^*) = \delta \eta e^{-\beta(T_v^*)} V_0^I(T_v) = w_v \quad (61)$$

Since the increase in the wage across cycles reflects only the improvement in productivity: $w_{v+1} = e^\Gamma w_v$, and since from the asset market clearing conditions, we know that $\beta(T_v^*) = \Gamma$, it immediately follows that the increase in the present value of monopoly profits from the beginning of one cycle to the next must, in equilibrium, reflect only the improvements in aggregate productivity:

$$V_0^I(T_{v+1}) = e^\Gamma V_0^I(T_v). \quad (62)$$

Equation (62) implies that, given some initial implementation period, and stationary values of Γ and Δ^C , the next implementation period is determined. Letting the total cycle length be denoted $\Delta_v = T_v - T_{v-1}$, and the expansion length be denoted $\Delta_v^X \equiv \Delta_v - \Delta_v^C$ we therefore have the following result:

Proposition 3 *Given the boom size, Γ_v , the contraction length, Δ_v^C , and the dynamic path followed by $z(t)$, there exists a unique expansion length, Δ_v^X , such that entrepreneurs are just willing to commence search, Δ_v^C periods prior to the boom. This is given by Δ_v^X solving:*

$$\frac{e^{-\gamma}}{\delta \eta} = \frac{(1 - e^{-\gamma}) \int_{T_{v-1}}^{T_{v-1} + \Delta_v^X} e^{-\rho(\tau - T_{v-1})} \frac{c(\tau)}{c_0(T_{v-1})} d\tau + e^{-\Gamma_v} (L - H_v) \frac{1 - e^{-\frac{\rho}{\sigma} \Delta_v^C}}{\rho/\sigma}}{1 - (1 - \Gamma_v/\gamma) e^{(1-\sigma)\Gamma_v - \rho(\Delta_v^C + \Delta_v^X)}} \quad (63)$$

The equilibrium conditions on entrepreneurial behavior also impose the following requirements on our hypothesized cycle:

- Successful entrepreneurs at time $t = T_v$, must prefer to implement immediately, rather than delay implementation until later in the cycle or the beginning of the next cycle:

$$V_0^I(T_v) > V_0^D(T_v). \quad (\text{E1})$$

- Entrepreneurs who find commercially viable ideas during the downturn must prefer to wait until the beginning of the next cycle rather than implement earlier:

$$V^I(t) < V^D(t) \quad \forall t \in (T_v^*, T_v) \quad (\text{E2})$$

- No entrepreneur wants to search for commercially viable ideas during the expansionary phase of the cycle. Since in this phase of the cycle $\delta V^D(t) < w(t)$, this condition requires that

$$\delta \eta V^I(t) < w(t) \quad \forall t \in (0, T_v^*) \quad (\text{E3})$$

- Finally, in constructing the equilibrium above, we have implicitly imposed the requirement that the downturn is not so long that commercially viable applications are identified in every sector:

$$P(T_v) > 0. \quad (\text{E4})$$

6 Stationary Cyclical Equilibrium Growth Path

We focus on a stationary cyclical equilibrium in which the boom in productivity, Γ , and the length of each phase of the cycle (Δ^X, Δ^C) are constant. Along such a growth path, the potential productivity of unmatched ideas increases by

$$\ln \frac{z(T_v)}{z_0(T_{v-1})} = \Gamma = \gamma [N(T_v) - N(T_{v-1})] \quad (\text{64})$$

during the expansion, and an equal measure of ideas is matched with a profitable application during downturns. Combining (58), (59) and (64) yields the following implication:

Proposition 4 : *Along the stationary cyclical growth path, average growth is given by*

$$\bar{g} = \frac{\Gamma}{\Delta} = \frac{\rho}{2 - \sigma}. \quad (\text{65})$$

Thus, long-run growth along this path is increasing in the rate of time preference and decreasing in the elasticity of intertemporal substitution. This is in sharp contrast to the acyclical growth path discussed in Section 3. To understand this, consider two key relationships. Firstly, the consumer's Euler equation implies, as usual, that a higher rate of return over the cycle yields a more rapidly growing consumption path. In equilibrium it follows that

$$\bar{g} = \frac{\bar{r} - \rho}{\sigma}, \quad (66)$$

where $\bar{r} = [R(T_v) - R(T_{v-1})] / \Delta$ denotes the average interest rate over the cycle.

In the cyclical steady state, the rate of return must be sufficient to induce investment in R&D during the expansion and to induce the financing of commercialization during the downturn. Both of these require a rate of return equal to Γ — the former because this is how much the unit cost of R&D grows during the expansion, and the latter because this ensures that intermediates are not stored. Since the interest rate during the downturn is zero, it follows that the average rate of return over the entire cycle required to induce the investment that supports a growth rate \bar{g} must be given by

$$\bar{r} = 2\bar{g}. \quad (67)$$

Note that, in contrast to the acyclical steady-state, the required average rate of return is unambiguously increasing in the average growth rate.

A strong implication of (65) is that the average growth rate is pinned down entirely by the preference parameters and is independent of the technological parameters, and the economy's size L . Once again, this contrasts starkly with the growth rate in the acyclical equilibrium, (24), and also contrasts with a recent literature which has endeavoured to construct technology based endogenous growth models that do not exhibit scale effects. Here, there are no direct scale effects on long run growth. Note however that changes in the size of the work force and other technological parameters do effect the length and nature of each phase of the cycle (see below), and whether or not a cyclical equilibrium exists.

Using (44), (56), (51), (60), (63) and (65), the stationary cyclical equilibrium is fully described by the vector $(\Gamma, \Delta^X, \Delta^C, \hat{z}, \hat{S})$ and a recurring expansion path for consumption $\{\hat{c}(t)\}_{\tau=0}^{\Delta^X}$ which satisfy the following system:

$$\hat{c}(\tau) = F(\tau, e^{-\Gamma/\sigma + \frac{\rho}{\sigma}\Delta^X} \hat{c}(\Delta^X), e^{-\Gamma} \hat{z}) \quad \forall \tau \in [0, \Delta^X] \quad (68)$$

$$\Gamma = \delta\gamma L\Delta^C - \delta\gamma \hat{c}(\Delta^X) \frac{\tilde{\Lambda}}{\rho/\sigma} \quad (69)$$

$$\frac{(1-\eta)e^{-\gamma}}{\eta\delta} \frac{\mu}{\gamma} \frac{\mu}{\theta} \frac{\tilde{A}}{\gamma\hat{S}(1-e^{-\Gamma}) + \Gamma e^{-\Gamma}} = e^{\gamma\hat{S}} = \hat{z} \quad (70)$$

$$\frac{e^{-\gamma}}{\delta\eta\gamma} = \frac{\mu}{\gamma(1-e^{-\Gamma}) + \Gamma e^{-\Gamma}} \frac{1}{L} \int_0^{\Delta^X} \frac{e^{-\rho\tau}}{e^{-\rho\Delta^X}} \frac{c(\tau)}{\hat{c}(\Delta^X)} \frac{\mu}{\theta} d\tau + \hat{c}(\Delta^X) \frac{\tilde{A}}{\rho/\sigma} \quad (71)$$

$$(2-\sigma)\Gamma = \rho(\Delta^X + \Delta^C) \quad (72)$$

Recall that the function $F(\cdot)$ represents the transitional dynamics during the expansion, given by (42) and (43). Although average growth depends only on preference parameters, technological parameters influence short-run growth and the nature of cycles. In order to characterize these effects, however, we turn to numerical methods.

6.1 Baseline Example

We numerically solve the (68) through (72) for various combinations of parameters and check the existence conditions (E1)–(E4). The parameters for our baseline example are given in Table 1.¹¹ The parameter γ implies a labor share of about 0.7. We chose ρ and σ to yield a long run growth rate of 2% and an average risk-free real interest rate of 4% (these values roughly correspond to average data for the post-war US.). The remaining parameters are chosen fairly arbitrarily, but imply a cycle length of about 10 years, with an expansion of almost 7 years and a contraction of just over 3 years.

Table 1: Baseline Parameters

Parameter	Value
δ	1.25
γ	0.30
σ	1.00
ρ	0.02
μ	0.20
η	0.20
θ	0.95
L	1.00

¹¹The Gauss program used to generate the numerical simulations and the diagrams contained here is downloadable from the following URL: <http://qed.econ.queensu.ca/pub/faculty/lloyd-ellis/research.html>

Figure 3 depicts the evolution of some key aggregates over the cycle. Clearly, R&D investment moves very pro-cyclically and is strongly correlated with movements in consumption. Consequently, the stock of potentially productive knowledge grows steadily during expansions and comes to a halt during contractions. Note that even if we include the wage costs associated with search in an aggregate measure of “all innovation” costs, this aggregate remains quite pro-cyclical.¹²

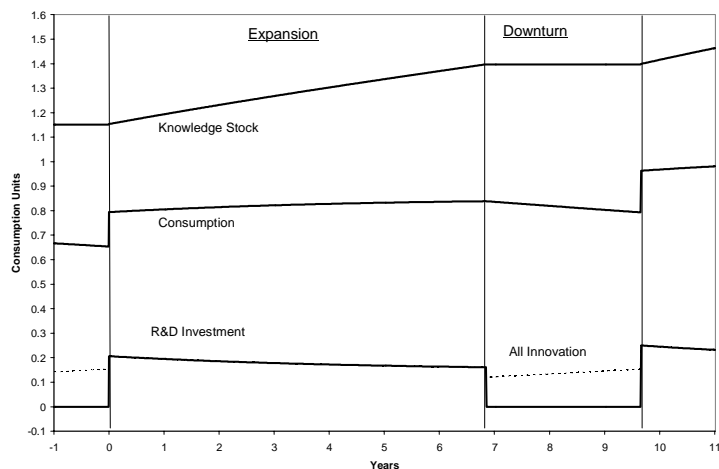


Figure 3: Key Aggregates in Baseline Cycle

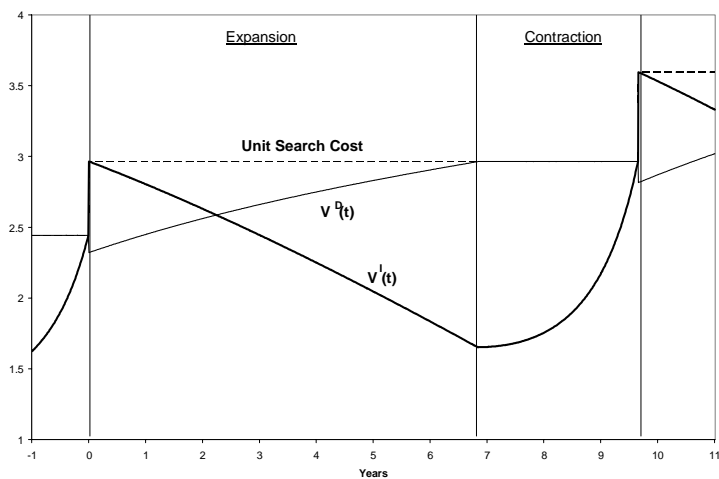


Figure 4: Incentives to search for commercially viable ideas

¹²In general, neither R&D investments nor other organizational investments are capitalized correctly in the national accounts.

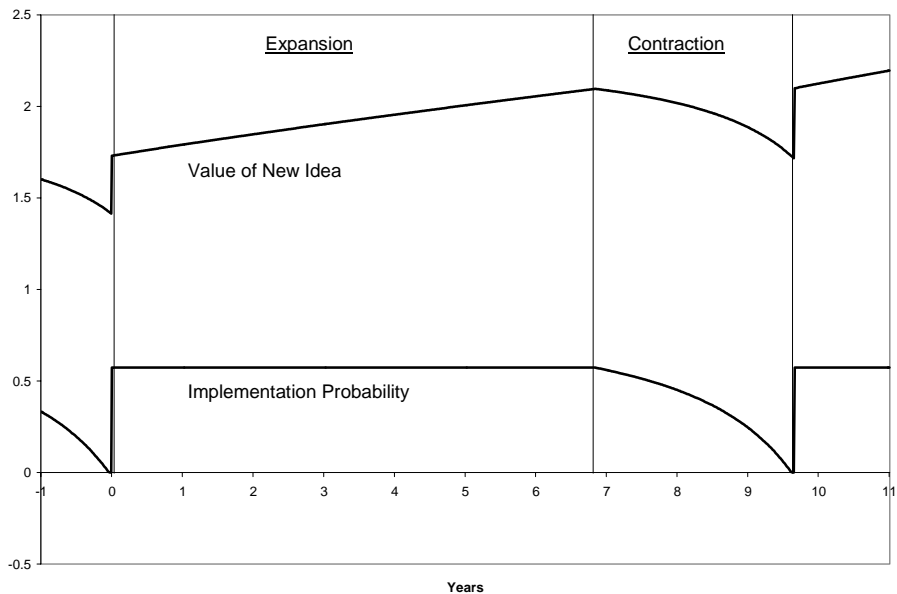


Figure 5: Incentives to undertake R&D

Figure 4 illustrates the factors affecting the incentives to search for commercially viable ideas and to implement them at each stage of the cycle. During expansions, wages are relatively high and the subsequent boom far is away. Consequently, the values newly commercialized ideas lie below the units cost of search effort, whether or not implementation is immediate or delayed. Eventually, as the next boom approaches, the value of delayed implementation becomes high enough to warrant the cost of search effort and commercialization starts to pick up. The value of immediate implementation remains below that of delay because of the risk to profits of implementing too early.

Figure 5 illustrates the factors affecting the incentives to undertake R&D at each stage of the cycle. The “value of a new idea” corresponds to $\Omega(t)$ in the model and “implementation probability” refers to the probability that a commercially viable application will be found for an existing idea prior the subsequent boom. Since commercialization is concentrated during the contraction, the implementation probability is constant during the expansion. However, as the contraction proceeds, the likelihood that any new idea created by the R&D sector will find a commercially viable application before the next boom falls gradually to 0. Thus, the value of new ideas grows with the unit cost of generating them (proportional to the knowledge stock) during the expansion, but then falls with the declining implementation probability during the contraction.

Table 2: Comparative Steady States

Parameters	Γ	Δ^X	Δ^C	\hat{z}	\bar{g} (%)	\hat{S}	$P(T)$	
Baseline	0.20	6.79	3.19	1.39	2.00	1.10	0.33	
$\delta =$	1.23	0.29	10.1	4.46	1.46	2.00	1.27	0.03
$\frac{1}{2}$	1.27	0.13	4.40	2.02	1.33	2.00	0.95	0.57
$\gamma =$	0.299	0.24	8.12	3.70	1.41	2.00	1.17	0.21
$\frac{1}{2}$	0.301	0.17	5.76	2.78	1.37	2.00	1.04	0.43
$\sigma =$	0.99	0.15	5.18	2.52	1.36	1.98	1.02	0.49
$\frac{1}{2}$	1.01	0.26	8.80	3.98	1.43	2.02	1.19	0.26
$\rho =$	0.0199	0.18	6.08	2.90	1.37	1.99	1.06	0.40
$\frac{1}{2}$	0.0201	0.23	7.84	3.60	1.41	2.01	1.14	0.23
$\mu =$	0.15	0.19	6.64	2.74	1.18	2.00	0.54	0.37
$\frac{1}{2}$	0.25	0.22	7.34	3.64	1.60	2.00	1.57	0.27
$\eta =$	0.199	0.23	7.78	3.56	1.41	2.00	1.15	0.24
$\frac{1}{2}$	0.201	0.18	5.94	2.86	1.37	2.00	1.05	0.41
$\theta =$	0.9	0.22	7.02	3.78	1.38	2.00	1.08	0.28
$\frac{1}{2}$	1.0	0.19	6.92	2.84	1.40	2.00	1.13	0.35
$L =$	0.95	0.25	8.82	3.74	1.44	2.00	1.22	0.16
	1.05	0.12	4.34	1.88	1.35	2.00	1.01	0.59

Table 2 details the consequences of varying each of the parameters of the model around the baseline example. In general, the nature of the cycle is quite sensitive to small parameter changes and the size of the changes considered is partly dictated by the conditions required for existence. As noted earlier, changes in technological parameters have no impact on long run growth, but do affect growth in the short run and the lengths of each phase of the cycle.

Increases in the commercialization success rate, δ , the size of the work force, L , or the size of productivity increments, γ , all shorten the length of both phases of the cycle. The length of the contraction declines because a higher rate of success induces entrepreneurs to want to implement earlier. Consequently, the size of the productivity boom declines, inducing less R&D and a shorter expansion. Overall, these adjustments are such that the steady state average growth rate remains unchanged. In contrast, an increase in the productivity of R&D, μ , lengthens both phases of the cycle and increases the size of the boom.

7 Concluding Remarks

Several recent theories of endogenous business cycles involve investments in growth-promoting activities that are countercyclical. In contrast, empirical observations suggest that one important category of these investments, those in R&D, are quite pro-cyclical. Here we show that explicitly introducing endogenous R&D investment into the “intrinsic business cycle” model of

Francois and Lloyd–Ellis (2003) yields an inherently pro–cyclical process. This conclusion is based on several key assumptions about the characteristics of R&D investment, as the first step in a multi–stage innovative process. In particular, we assume that investments in R&D (1) use implemented knowledge intensively, (2) are long-term investment with uncertain applications and (3) are characterized by diminishing returns to existing knowledge.

An interesting feature of the cyclical equilibrium growth path is that the steady state long–run growth rate depends only on preferences and not on any technological parameters. This is in stark contrast to the acyclical equilibrium growth path associated with the same underlying model. In particular, the long run growth path exhibits no scale effects in the sense that small changes in the size of the population do not affect the long–run growth rate. However, the size of the population does affect the short–run evolution of the economy and can be important for existence of the cyclical equilibrium. Moreover, long–run population growth is not consistent with a steady state cyclical growth path.

Some features of our model’s prediction are clearly at odds with the facts. However, it is possible to extend the model in various ways to address some of these issues. In particular, the productivity boom and the associated jump in output are rather abrupt. As we show in a recent paper, Francois and Lloyd–Ellis (2006), adding capital can help to smooth out the boom to some extent. Another unrealistic feature of the cyclical process that we generate is that every cycle is the same and all fluctuations are deterministic. Extending the model to allow for some stochastic elements relaxes some of these strong predictions. In particular, temporary i.i.d. shocks can change the length and amplitude of each cycle without changing the basic story.

Appendix

Proof of Lemma 1 We show: (1) that if a signal of success from a potential entrepreneur is credible, other entrepreneurs stop innovation in that sector; (2) given (1) entrepreneurs have no incentive to falsely claim success.

Part (1): If entrepreneur i 's signal of success is credible then all other entrepreneurs believe that i has a productivity advantage which is e^γ times better than the existing incumbent. If continuing to innovate in that sector, another entrepreneur will, with positive probability, also develop a productive advantage of e^γ . Such an innovation yields expected profit of 0, since, in developing their improvement, they do not observe the non-implemented improvements of others, so that both firms Bertrand compete with the same technology. Returns to attempting innovation in another sector where there has been no signal of success, or from simply working in production, $w(t) > 0$, are thus strictly higher, .

Part (2): If success signals are credible, entrepreneurs know that upon success, further innovation in their sector will cease from Part (1) by their sending of a costless signal. They are thus indifferent between falsely signalling success when it has not arrived, and sending no signal. Thus, there exists a signalling equilibrium in which only successful entrepreneurs send a signal of success.

Proof that w_i is pinned down by the level of technology: From (10) $x_i^d(t) = \frac{Y(t)}{p_i(t)}$, so that from (11) $Y(t) = x_i^d(t) \frac{w(t)}{e^{-\gamma} A_i(t)}$, but since the intermediate technology is linear (from (5) this is $x_i^s(t) = A_i(t) l_i(t)$ and $x_i^s(t) = x_i^d(t)$ in equilibrium) we thus have $Y(t) = l_i(t) \frac{w(t)}{e^{-\gamma}}$. Substituting from the final output production function (3) and substituting again for x_i yields: $\exp \int_0^1 \ln [A_i(t) l_i(t)] di = l_i(t) \frac{w(t)}{e^{-\gamma}}$. But the symmetry of sectors implies, again in equilibrium, that $l_i(t) = l(t) \forall i$, so that we have $\exp \int_0^1 \ln [A_i(t)] di = \frac{w(t)}{e^{-\gamma}}$. Rearranging yields: $w(t) = \exp \int_0^1 \ln [A_i(t)] di e^{-\gamma} \equiv \bar{A}_{v-1} e^{-\gamma}$. \pounds

Proof of Lemma 2: There are two possible alternatives which can be ruled out by contradiction.

(1) Suppose instead that at T_v^* , $\dot{Z} = 0$ and $H = 0$. From 33 it follows that $\dot{\Omega}/\Omega = r(T_v^*) > 0$. But then $\Omega(T_v^*) > \frac{\gamma}{\mu} \frac{1}{\theta} Z(T_v^*)$, so there would be entry into R&D, contradicting the supposition.

(2) Suppose instead that at T_v^* , $\dot{Z} > 0$ and $H > 0$. Since the wage is constant, free entry into search implies that $r(t) = \dot{V}^D/V^D = 0$. It follows from 33 that $\dot{\Omega}/\Omega < 0$. But then $\Omega(T_v^*) < \frac{\gamma}{\mu} \frac{1}{\theta} Z(T_v^*)$, so there would be no entry into R&D, contradicting the second supposition. \pounds

Proof of Lemma 3: During the downturn the value of untapped ideas can be expressed as

$$\begin{aligned}
\Omega(t) &= (1-\eta) \int_t^{T_v} e^{-\int_t^\tau \frac{\delta H(s)}{S(s)} ds} \frac{\delta H(\tau)}{S(\tau)} V^D(\tau) d\tau + e^{-\beta(t) - \int_t^{T_v} \frac{\delta H(\tau)}{S(\tau)} d\tau} \Omega(T_v) \\
\Omega(t) &= -(1-\eta) V^D(t) \int_t^{T_v} e^{\int_t^\tau \frac{\dot{S}(s)}{S(s)} ds} \frac{\dot{S}(\tau)}{S(\tau)} d\tau + e^{-\beta(t) + \int_t^{T_v} \frac{\dot{S}(\tau)}{S(\tau)} d\tau} \Omega(T_v) \\
\Omega(t) &= -(1-\eta) V^D(t) \int_t^{T_v} e^{\ln S(\tau) - \ln S(t)} \frac{\dot{S}(\tau)}{S(\tau)} d\tau + e^{-\beta(t) + \ln S(T_v) - \ln S(t)} \Omega(T_v) \\
\Omega(t) &= -(1-\eta) \frac{V^D(t)}{S(t)} \int_t^{T_v} \dot{S}(\tau) d\tau + e^{-\beta(t) + \ln S(T_v) - \ln S(t)} \Omega(T_v) \\
\Omega(t) &= (1-\eta) V^D(t) \left[1 - \frac{S(T_v)}{S(t)} \right] + \frac{S(T_v)}{S(t)} e^{-\beta(t)} \Omega(T_v)
\end{aligned}$$

Proof of Proposition 1: Since no new ideas emanate from the R&D sector between dates T_v^* and T_v , it follows that $\Omega(T_v^*) = \Omega_0(T_v)$. Using (50) we can write

$$\begin{aligned}
\Omega(T_v^*) \left[1 - \frac{S(T_v)}{S(T_v^*)} e^{-\beta(T_v^*)} \right] &= (1-\eta) V^D(T_v^*) \frac{S(T_v^*) - S(T_v)}{S(T_v^*)} \\
\Omega(T_v^*) &= (1-\eta) V^D(T_v^*) \frac{S(T_v^*) - S(T_v)}{S(T_v^*) - S(T_v) e^{-\beta(T_v^*)}} \\
\Omega(T_v^*) &= \frac{(1-\eta) e^{-\gamma \bar{A}_{v-1}}}{\eta \delta} \frac{S(T_v^*) - S(T_v)}{S(T_v^*) - S(T_v) e^{-\beta(T_v^*)}}
\end{aligned}$$

Now since $i_R(T_v^*) > 0$, we know that $\Omega(T_v^*) = \frac{\gamma}{\mu} \frac{1}{\theta} Z(T_v^*) = \frac{\gamma}{\mu} \frac{1}{\theta} Z(T_v)$. Also by definition, $\bar{A}_v = Z(T_v) e^{-\gamma S(T_v)}$ and so

$$\begin{aligned}
\frac{\gamma}{\mu} \frac{1}{\theta} Z(T_v) &= \frac{(1-\eta) e^{-\gamma \bar{A}_{v-1}}}{\eta \delta} \frac{S(T_v^*) - S(T_v)}{S(T_v^*) - S(T_v) e^{-\beta(T_v^*)}} \\
\frac{\gamma}{\mu} \frac{1}{\theta} \bar{A}_v e^{\gamma S(T_v)} &= \frac{(1-\eta) e^{-\gamma \bar{A}_{v-1}}}{\eta \delta} \frac{S(T_v^*) - S(T_v)}{S(T_v^*) - S(T_v) e^{-\beta(T_v^*)}} \\
\frac{\gamma}{\mu} \frac{1}{\theta} e^{\Gamma_v + \gamma S(T_v)} &= \frac{(1-\eta) e^{-\gamma}}{\eta \delta} \frac{S(T_v^*) - S(T_v)}{S(T_v^*) - S(T_v) e^{-\beta(T_v^*)}}
\end{aligned}$$

But $S(T_v) = S(T_v^*) - \Gamma_v/\gamma$, so that

$$\begin{aligned}
\frac{\gamma}{\mu} \frac{1}{\theta} e^{\gamma S(T_v^*)} &= \frac{(1-\eta) e^{-\gamma}}{\eta \delta} \frac{\Gamma_v}{\gamma S(T_v^*) - (\gamma S(T_v^*) - \Gamma_v) e^{-\beta(T_v^*)}} \\
\frac{(1-\eta) e^{-\gamma}}{\eta \delta} \frac{\gamma}{\mu} \frac{1}{\theta} &= \frac{\Gamma_v}{\gamma S(T_v^*) - (\gamma S(T_v^*) - \Gamma_v) e^{-\beta(T_v^*)}} = e^{\gamma S(T_v^*)}.
\end{aligned}$$

Re-arranging yields (51).

Proof of Proposition 2: For an entrepreneur who is holding a commercial viable idea, $\eta V^I(t)$ is the value of implementing immediately. Just prior to the boom, when the probability of displacement is negligible, the value of implementing immediately must equal that of delaying until the boom:

$$\delta\eta V^I(T_v) = \delta\eta V^D(T_v) = w(T_v). \quad (73)$$

During the boom, since entrepreneurs prefer to implement immediately, it must be the case that $V_0^I(T_v) > V_0^D(T_v)$. Thus the return to innovation at the boom is the value of immediate (rather than delayed) incumbency. It follows that free entry into entrepreneurship at the boom requires that

$$\delta\eta V_0^I(T_v) \leq w_0(T_v) \quad (74)$$

The opportunity cost to financing entrepreneurship is the rate of return on shares in incumbent firms in sectors where no innovation has occurred. Just prior to the boom, this is given by the capital gains in sectors where no innovations have occurred

$$\beta(T_v) = \log \frac{V_0^I(T_v)}{V^I(T_v)}. \quad (75)$$

Note that since the short-term interest rate is zero over this phase, $\beta(t) = \beta(T_v), \forall t \in (T_v^*, T_v)$. Combined with (73) and (74) it follows that asset market clearing at the boom requires

$$\beta(T_v) \leq \log \frac{w_0(T_v)}{w(T_v)} = \Gamma_v. \quad (76)$$

Proof of Proposition 3: The discounted monopoly profits from owning an innovation at time T_v is given by

$$\begin{aligned} V_0^I(T_{v-1}) &= (1 - e^{-\gamma}) \int_{T_{v-1}}^{T_v} e^{-\int_{T_{v-1}}^{\tau} r(s) ds} Y(\tau) d\tau + P(T_v) e^{-\beta(T_{v-1})} V_0^I(T_v) \\ &= (1 - e^{-\gamma}) \bar{A}_{v-1} \int_{T_{v-1}}^{T_v^*} e^{-\int_{T_{v-1}}^{\tau} r(s) ds} d\tau + e^{-\int_{T_{v-1}}^{T_v^*} r(\tau) d\tau} (L - H_v) \int_{T_v^*}^{T_v} e^{-\frac{\rho}{\sigma}(\tau - T_v^*)} d\tau \\ &\quad + P(T_v) e^{-\beta(T_{v-1})} V_0^I(T_v) \\ &= (1 - e^{-\gamma}) \bar{A}_{v-1} \int_{T_{v-1}}^{T_v^*} e^{-\rho(\tau - T_{v-1})} \frac{c(\tau)}{c_0(T_{v-1})} d\tau + e^{-\Gamma_v} (L - H_v) \frac{\tilde{A}}{\rho/\sigma} \\ &\quad + P(T_v) e^{-\beta(T_{v-1})} V_0^I(T_v) \end{aligned}$$

Substituting for $V_0^I(T_{v+1})$ using (62), and integrating yields

$$V_0^I(T_{v-1}) = \frac{(1 - e^{-\gamma}) \bar{A}_{v-1}}{1 - P(T_v) e^{\Gamma_v - \beta(T_{v-1})}} \int_{T_{v-1}}^{T_v^*} e^{-\rho(\tau - T_{v-1})} \frac{c(\tau)}{c_0(T_{v-1})} d\tau + e^{-\Gamma} (L - H_v) \frac{\tilde{A}}{\rho/\sigma}. \quad (77)$$

Noting that $\delta\eta V_0^I(T_{v-1}) = w_0(T_{v-1}) = e^{-\gamma}\bar{A}_{v-1}$.

$$\frac{e^{-\gamma}}{\delta\eta} = \frac{(1 - e^{-\gamma})}{1 - (1 - \Gamma_v/\gamma)e^{\Gamma_v - \beta(T_{v-1})}} \int_{T_{v-1}}^{T_v^*} e^{-\rho(\tau - T_{v-1})} \frac{c(\tau)}{c_0(T_{v-1})} d\tau + e^{-\Gamma}(L - H_v) \frac{1 - e^{-\frac{\rho}{\sigma}\Delta_v^C}}{\rho/\sigma}. \quad (78)$$

But $\beta(T_{v-1}) = \sigma\Gamma_v + \rho\Delta_v$, so that

$$\frac{e^{-\gamma}}{\delta\eta} = \frac{(1 - e^{-\gamma})}{1 - (1 - \Gamma_v/\gamma)e^{(1-\sigma)\Gamma_v - \rho\Delta_v}} \int_{T_{v-1}}^{T_v^*} e^{-\rho(\tau - T_{v-1})} \frac{c(\tau)}{c_0(T_{v-1})} d\tau + e^{-\Gamma_v}(L - H_v) \frac{1 - e^{-\frac{\rho}{\sigma}\Delta_v^C}}{\rho/\sigma}.$$

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