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Counterparty Risk in Insurance Contracts: Should the Insured Worry about the Insurer?

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Abstract

We analyze the effect of counterparty risk on insurance contracts using the case of credit risk transfer in banking. In addition to the familiar moral hazard problem caused by the insuree's ability to influence the probability of a claim, this paper uncovers a new moral hazard problem on the other side of the market. We show that the insurer's investment strategy may not be in the best interests of the insuree. The reason for this is that if the insurer believes it is unlikely that a claim will be made, it is advantageous for them to invest in assets which earn higher returns, but may not be readily available if needed. This paper models both of these moral hazard problems in a unified framework. We find that instability in the insurer can create an incentive for the insuree to reveal superior information about the risk of their "investment". In particular, a unique separating equilibrium may exist even in the absence of any signalling device. We extend the model and show that increasing the number of insurers with which the insuree contracts can exacerbate the moral hazard problem and may not decrease counterparty risk. Our research suggests that regulators should be wary of risk being offloaded to other, possibly unstable parties, especially in newer financial markets such as that of credit derivatives.

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1 Introduction

In this paper, we develop an agency model to analyze an insurer's optimal investment decision when failure is a possibility. We demonstrate that an insurer's investment choice may be suboptimal compared to the first best by showing how a moral hazard problem exists on this side of the market. This insurer moral hazard problem does have an upside however, as we show that it can alleviate the adverse selection problem on the part of the insuree.

The market for risk protection is one of the most important markets available today. Although we model a general insurance problem, a particularly relevant example in today's financial markets is that of credit risk transfer.[1] Banks, who were once confined to a simple borrow short and lend long strategy, can now disperse credit risk through credit derivatives markets to better implement risk management policies.[2] The rapid growth in these financial markets requires us to think of insurance in a different way than is standard in the literature. The reason is that banks are ceding potentially large credit risks to parties such as Hedge Funds which may or may not be in a better position to handle them.[3] Furthermore, Allen and Gale (2006) wonder whether credit risk transfer is done simply as a form of regulatory arbitrage. It would seem prudent then to ask the question of how stable is, and what are the incentives of the risk taker (insurer)? This entails a study of counterparty risk. In what is to follow, we define counterparty risk as the risk that when a claim is made, the insurer is insolvent and not able to fulfil its obligations.

This paper arrives at two novel results. The first is that there can exist a moral hazard on the part of the insurer. This moral hazard arises because the insurer may choose an excessively risky portfolio. The intuition behind this result is as follows. There are two key states of the world that enter into the insurer's decision problem: the first in which a claim is not made, and the second in which it is. We assume that the insurer can default in both of these states if they get an unlucky draw. However, they can invest to help minimize the chances that they become insolvent. This investment choice comes with a tradeoff: what reduces the probability of insolvency the most in the state in which a claim is not made, makes it more likely that the insurer will become insolvent in the state in which it is. If we consider a situation in which the insurers beliefs are such that the contract is relatively safe, it may be optimal to put capital into less liquid assets to reap higher returns, and lower the chance of failure in the state in which a claim is not made. However, assets which yield these higher returns can also be more costly to liquidate, and therefore, make it more difficult to free up capital if a claim is made. The moral hazard arises because the insurance premium must be

[1] Another example of a financial insurance market is the market for reinsurance.

[2] A credit derivative, and specifically a *credit default swap* is a contract whereby an insurer agrees to cover the losses of an insuree that take place if pre-defined events happen to an underlying borrower (In many cases, this event is the default of the underlying bond. However, some contracts include things like re-structuring and ratings downgrades as triggering events). In exchange for this protection, the insuree agrees to pay an ongoing premium at fixed intervals for the life of the contract.

[3] Fitch (2006) reports that banks are the largest insuree in this market. On the insurer side, banks and hedge funds are the largest, followed by insurance companies and other financial guarantors. It should be noted that the author's suspect that banks are the largest insurers, followed by hedge funds, however, they admit that the data is poor and that other research reports do not support this.

made up front, which introduces a contracting imperfection. This imperfection means the insuree cannot condition its premium on an observed outcome. Consequently, there is no way to provide incentives to the insurer to influence its investment decision.

The second result deals with the adverse selection problem that may be present because of the superior information that the insuree has about the underlying claim. Akerloff (1970) represents a fundamental paper on the dangers of informational asymmetries in insurance markets. In this seminal paper, it is shown how the market for good risks may break down, and one is left with insurance only being issued on the most risky of assets, or in Akerloff's terminology, lemons. The incentive that underlies this result is that the insuree is only interested in obtaining the lowest insurance premium possible.

In contrast, in this paper we show that the safer the underlying claim is perceived to be, the more severe the moral hazard problem is. Consequently, conditional on a claim being made, counterparty risk is higher for safer assets. Therefore, it can optimal for an insuree with a poor quality asset to reveal its type truthfully. In other words, if it is revealed as poor quality, the insurer will have incentives more in line with the insuree, and consequently it is subjected to less counterparty risk. We show that this new effect, which we call the *counterparty risk effect* allows a unique separating equilibrium to be possible. This result is new in that separation occurs without the existence of a signalling device. After Akerloff (1970) showed that no separating equilibrium can exist, the literature developed the concept of signalling devices with such famous examples as education in Spence's job market signalling paper. These papers allowed the high (safe) type agents to separate themselves by performing a task which is "cheaper" for them than for the low (risky) type agents. Our paper can achieve separation by the balance between the insuree's desire for the lowest insurance premium, and the desire to be exposed to the least counterparty risk. One can think of this result as adding to the *cheap talk* literature by showing an insurance problem in which costless communication can bring about separation of types.[4]

We enrich the model to the case of multiple insurers. The result is that as the size of the contract that each insurer takes on decreases, the moral hazard problem will increase. In a setup in which all the insurers are ex-ante identical (but not necessarily ex-post, i.e. they receive IID portfolio draws), we obtain the result that counterparty risk may remain unchanged. Next, we consider the case of multiple insurees. We show that even when each individual insuree is insignificant to the insurer (resembling that of a traditional insurance market such as health or automobile), our results carry through when there is aggregate risk that is private information to the insurees. Furthermore, our moral hazard result still remains if this assumption on risk is not met.

Finally, we enrich the model to include a possible moral hazard problem on the part of the insuree by their ability to affect the probability that a claim is made. If we use the example of a bank being insured on one of their loans, the banking literature typically assumes that a bank has superior information about that loan (due to their relationship with the borrower). On account of this, it is straightforward to see that if the bank is fully insured, they may not have the incentive

[4]For a nice review of the cheap talk literature, see Farrell and Rabin (1996).

to monitor the loan, and consequently, the probability of default could rise. This represents the classical moral hazard problem in the insurance literature. Our extension to the model shows that the new moral hazard of this paper may increase the desire of the insuree to monitor. This happens because counterparty risk forces the bank to internalize some of the default risk, which they otherwise would not if there is no counterparty risk. In this section we show that with a redefinition of the insuree's payoff distribution function, the addition of this insuree moral hazard problem does not effect the results of the paper.

1.1 Related Literature

This paper contributes to two streams of literature: that of insurance economics and that of credit risk transfer and credit derivatives. We contribute to the literature on insurance economics by raising the issue of counterparty risk which has not received much attention. The reason for this is that the traditional insurance economics literature imagines the insurer as large, and the insuree, small. This may be acceptable for some traditional markets, but modern financial markets require a new framework to asses insurance contracts. Henriet and Michel-Kerjan (2006) recognize that insurance contracts need not fit the traditional setup in which the insurer is the principal and the insuree, the agent. The authors relax this assumption and allow the roles to change. Their paper however does not consider the possibility of counterparty risk as ours does, as they assume that neither party can fail. Plantin and Rochet (2007) raise the issue of prudential regulation of insurance companies. They give practical recommendations for countries to better design their regulation of insurance companies. This work does not consider the insurance contract itself under counterparty risk as is done in this paper. Consequently, the author's do not analyze the effects of counterparty risk on the informational problems of insurance. Instead, they conjecture an agency problem arising from a corporate governance standpoint. We analyze a more abstract agency problem driven entirely by the investment incentives of the insurer.

The literature on credit risk transfer (CRT) is relatively small but is growing. Allen and Gale (2006) motivate a role for CRT in the banking environment. Using the same framework, Allen and Carletti (2006) show how a default by an insurance company can cascade into the banking sector causing a contagion effect when these two parties are linked through the transfer of credit risk. Wagner and Marsh (2006) argue that setting regulatory standards that reflect the different social costs of instability in the bank and insurance sector will be welfare improving. Our paper differs from these because they do not consider the agency problems of insurance contracts. As a result, they do not discuss the consequences that instability can have on the contracting environment, and how this affects the behavior of the parties involved. Duffee and Zhou (2001) and Thompson (2007) both analyze informational problems in insurance contracts, however, they focus on the factors that affect the choice between sales and insurance of credit risk. In contrast, we do not focus on the choice of an optimal risk transfer technique, but rather, we look deeper into one of them: insurance.

The paper proceeds as follows: section 2 outlines the model and solves the insurer's problem. Section 3 determines the equilibria that can be sustained when adverse selection is present. Furthermore, this section determines the first best investment choice and proves the existence of a moral hazard problem on the part of the insurer. Section 4 analyzes the following extensions: 1) multiple insurers, 2) multiple insurees, and 3) classical moral hazard on the side of the insuree. In section 5 we conclude. Many of the longer proofs are relegated to the appendix in section 6.

2 The Model Setup

The model is in three dates indexed $t = 0, 1, 2$. There are three agents, an insuree, whom we will call a bank, and multiple risk insurers, whom we will call Insuring Financial Institutions (IFIs). As well, there is an underlying borrower who has a loan with the bank. We will model this party simply as a return structure. The size of the loan will be normalized to 1 for simplicity. We motivate the need for insurance through an exogenous parameter (to be explained below) which makes the bank display risk aversion.[5] We assume there is no discounting, however, adding this feature will not effect our results.

2.1 The Bank

The bank is characterized by the need to shed credit (loan) risk. We use the example of a bank that faces capital regulation and who must reduce their risk, or else could face a cost (which we will call Z). It is this cost that makes the bank averse to holding the risk and so finds it advantageous to shed it through insurance. We can think of this situation as arising from an endogenous reaction to a shock to the banks portfolio, however for simplicity, we will not model this here. There are two types of loans that a bank can insure, a safe type (S) and a risky type (R). A bank is endowed with one or the other (for simplicity we assume with equal probability, however, it is not required for our results). The loan type is private knowledge for the bank and reflects the unique relationship between them and the underlying borrower. We assume that the loan can be costlessly monitored, so that there is no moral hazard problem in the bank-borrower relationship. In section 4.3 we will relax this assumption and show that introducing costly monitoring does not change the results of the paper. The expected return on the two loans described are given as:

$$E(S) = \int_0^{\overline{S}} \psi h_S(\psi) \partial\psi \tag{1}$$

$$E(R) = \int_0^{\overline{R}} \psi h_R(\psi) \partial\psi \tag{2}$$

Where ψ is the total return from the loan and h_S and h_R are return density functions, with corresponding distributions: H_S and H_R. We define the upper bound of these two functions as $\overline{S}$ and $\overline{R}$ respectively. Note that there is nothing in the analysis to follow that requires this to be a

[5] A smooth concave utility function will only distract from our analysis and will not yield new insights. As will soon become apparent, the issue of how much risk to shed will not play a role in the analysis. Instead, we show that we can obtain a separating equilibrium without the amount of insurance being used a signal.

single loan. When we interpret this as a single loan, the insurance contracts to be introduced in section 2.3 will resemble that of a *credit default swap*. In the case that this is a return on many loans, the insurance contract will closely resemble that of a *portfolio default swap* or *basket default swap*.[6]

Both loan types are assumed to default if the realized value is $\tilde{\psi} \in [0,1]$. To distinguish the safe from the risky loan, we assume that $H_S(1) < H_R(1)$ so that the probability of default of the safe loan is less than that of the risky loan.[7] We use a continuous set of states as opposed to a discrete set because it is useful for section 4.3 where we exploit this enriched view of the bank loan.

The regulator requires the bank to transfer a set amount of default risk. For simplicity, the bank must transfer a proportion γ of their loan, regardless of its type.[8] We impose the exogenous cost Z on the bank if the loan defaults and they are not insured for the appropriate amount, or if they are insured for the appropriate amount, but their counterparty is not able to fulfil the insurance contract (we will discuss the reasons this may occur below).

In what is to follow, we only model the payoff to this loan for the bank, however, it can be viewed as only a portion of its total portfolio. For simplicity, we assume that the bank cannot fail in the model. Having the bank able to fail will not affect our qualitative results since it will not affect the insurance contract to be introduced in section 2.3. We now turn to the modelling of the IFI.

2.2 The Insuring Financial Institution

Without the sale of the insurance contract, we assume that the IFI has a payoff function of the form (denoting NI as 'No Insurance'):

$$\Pi_{IFI}^{NI} = \int_0^{\overline{R}_f} \theta f(\theta) d\theta - \int_{\underline{R}_f}^0 (G - \theta) f(\theta) d\theta, \tag{3}$$

where f(θ) is a probability density function with corresponding distribution F(θ) representing the random return or valuation of the IFI's portfolio, and G is the cost of bankruptcy. One inter-

[6] A portfolio or basket default swap is a contract written on more than one loan. There are many different configurations of these types of contracts. For example, a first-to-default contract says that a claim can be made as soon as the first loan in the basket defaults.

[7] We need not assume anything about the shape of either distribution function. For example, it could be the case that $E(R) > E(S)$ and $var(R) > var(S)$, but it is not required.

[8] The fact that the bank must insure the same amount, regardless of the type of loan is not crucial. We can think of γ being solved for by the bank's own internal risk management. Therefore, we could have a differing γ depending on loan quality. What is important in this case is that the IFI is not able to perfectly infer the probability of default from γ. This assumption is justified when the counterparty does not know the exact reason the bank is insuring. To know so would require them to go in depth into the bank's book, which should be excluded as a possibility. In this enriched case, we could make γ stochastic for each loan type reflecting different (private) financial situations for the bank. In this case, the IFI may not be able to infer the loan quality from the amount that the bank wishes to have insured. This topic has been addressed in the new Basel II accord which allows the bank to use their own internal risk management system in some cases to calculate needed capital holdings. One reason for this change is because of the superior information banks are thought to have on their own assets; regulators have acknowledged that the bank itself may be in the best position to evaluate its own risk.

pretation of G is lost goodwill, but any reason for which the IFI would not like to go bankrupt will suffice. Note that bankruptcy occurs when the portfolio draw is in the set $[\underline{R}_f, 0]$, where it is assumed $\underline{R}_f < 0$. In what is to follow, we simplify the analysis by defining our portfolio distribution as uniform.[9] It is assumed that the IFI receives this payoff at time $t = 2$, so that at time $t = 1$, the random variable θ represents the portfolio value if it could be costlessly liquidated at that time. However, the IFI's portfolio is assumed to be composed of both liquid and illiquid assets. In practice, we observe financial institutions holding both liquid (e.g t-bills, money market deposits) and illiquid (e.g loans, some exotic options, some newer structured finance products) investments on its books. Because of this, if the IFI wishes to liquidate some of their portfolio at time $t = 1$, they will be subject to a liquidity cost which we discuss below in section 2.3.

2.3 The Insurance Contract

We now introduce the means by which the bank is insured by the IFI. Because of the possible cost Z, at time $t = 0$ the bank requests an insurance contract in the amount of γ for one period of protection. Therefore, the insurance coverage is from $t = 0$ to $t = 1$. To begin, we assume that the bank contracts with one IFI who is in bertrand competition. This assumption will be relaxed in section 4.1 when we allow the bank to spread the contract among multiple IFIs. The IFI forms a belief b about the probability that the bank loan will default. In section 3 we will show how b is formed endogenously as an equilibrium condition of the model. In exchange for this protection, the IFI receives an insurance premium $P\gamma$, where P is the per unit price of coverage. The IFI chooses a proportion β of this premium to put in a liquid asset that, for simplicity, has a rate of return normalized to one in both $t = 1$ and $t = 2$, but can be accessed at either time period. The remaining proportion $1 - \beta$ is put in an illiquid asset with an exogenously given rate of return of R_I which pays out at time $t = 2$.[10] This asset can be thought of as a two period project that cannot be terminated early. It is this property that makes it illiquid. As we shall soon see, the payoff to the IFI is linear in the state in which a claim is not made and therefore a redefinition of the return would allow us to capture uncertainty in the illiquid asset to make it risky as well as illiquid. Therefore there is no loss of generality assuming this return is certain. We assume R_I is paid at $t = 2$ when the portfolio pays off.[11] The key difference between these two assets is that the liquid asset is accessible at $t = 1$ when the underlying loan may default, whereas the illiquid asset is only available at $t = 2$.[12]

[9] This assumption can be relaxed to a general distribution, provided that it satisfies some conditions. For example, there must be mass in a region above and below zero. We explore this extension in a previous version of the paper and is available from the author upon request.

[10] We can think of these as two assets that are in the IFI's portfolio, however, we assume that this amount is small so that the illiquid asset and the original portfolio are uncorrelated. Note that adding correlation would only complicate the analysis and would not change the qualitative results.

[11] The choice between the liquid and illiquid assets is not crucial. The choice can be between a risky and riskless asset (both liquid) and the qualitative results of the paper will still hold. We explore this in a previous version of the paper that is available from the author upon request.

[12] This can be relaxed to allow the recovery of the illiquid asset at fire sale prices, but the qualitative results of the model would remain the same.

For the remaining capital needed (net of the premium put in the liquid asset) if a claim is made, we assume that the IFI can liquidate its portfolio. Recall that the IFI's initial portfolio contains both liquid and illiquid assets of possibly varying degrees with return governed by F. We assume that the IFI has a liquidation cost represented by the invertible function $C(\cdot)$ with $C'() > 0$, $C''() \geq 0$, and $C(0) = 0$. The weak convexity of $C(\cdot)$ represents the various assets of differing liquidity in the IFI's portfolio.[13] The IFI will choose to liquidate the least costly assets first, but as more capital is required, the cost of liquidating increases as they are forced to liquidate illiquid assets at potentially fire sale prices.[14] $C(\cdot)$ takes as its argument the amount of capital needed from the portfolio, and returns a number that represents the actual amount that must be liquidated to achieve that amount of capital. This implies that $C(x) \geq x \; \forall x \geq 0$ so that $C'(x) \geq 1$. For example, if there is no cost of liquidation and if $\$x$ is required to be accessed from the portfolio, the IFI can liquidate $\$x$ to satisfy its capital needs. However, because liquidation may be costly in this model, the IFI must now liquidate $\$y \geq \$x$ so that by the time the liquidation function $C(\cdot)$ shrinks the value of the capital, the IFI is left with $\$x$. If $C(\cdot)$ is linear, our problem becomes a linear program, and as will soon become apparent, this yields an extreme case of moral hazard.

At time $t = 1$, the IFI learns what the return on its portfolio will be (or leans a valuation of its portfolio), however, the return is not realized until $t = 2$. This could be relaxed so that the IFI receives a fuzzy signal about the return, however, this would yield no further insight into the problem. Also at $t = 1$, a claim is made if the underlying borrower defaults. If a claim is made, the IFI can liquidate its portfolio to fulfil its obligation of γ.[15] If the contract cannot be fulfilled, the IFI defaults. At time $t = 2$, conditional on the IFI's survival at time $t = 1$, the payoff to the IFI from its portfolio is realized. Also at time $t = 2$, the payoff to the bank's loan is realized. Figure 1 summarizes the timing of the model.

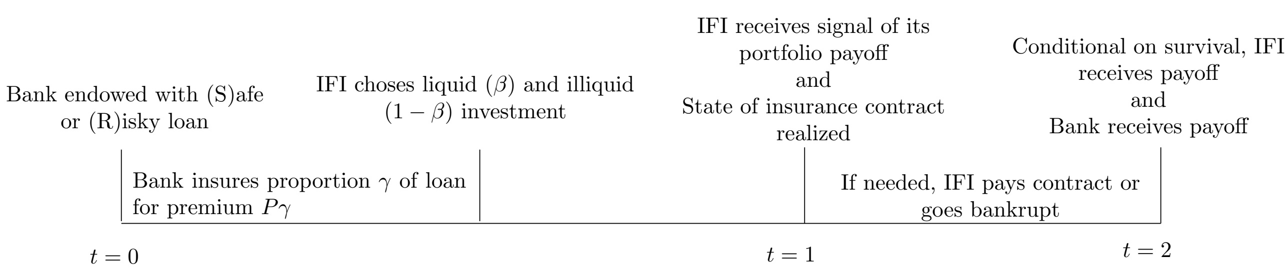

Figure 1: Timing of the model

[13] If we think of a bank as the IFI, it is obvious that they have many illiquid assets on their books. However, This is also the very nature most insurance companies and hedge funds businesses. In the case of insurance companies as the IFI, substantial portions of their portfolios may be in assets which cannot be liquidated easily (see Plantin and Rochet (2007)). In the case of Hedge Funds as the IFI, many of them specialize in trading in illiquid markets (see Brunnermeier and Pederson (2005) for example).

[14] There is a growing literature on trading in illiquid markets and fire sales. See for example Subramanian and Jarrow (2001), and Brunnermeier and Pedersen (2005).

[15] Note that this contract structure is assumed for simplicity. In reality, the insuring institution would typically pay the full protection value, but would receive the bond of the underlying borrower in return, which may still have a recovery value. Inserting this recovery value into the model only complicates the mathematics without changing the qualitative results.

The payoff function for the IFI can be written as (we suppress the superscript I denoting insurance as the remainder of the paper will analyze with this setting):

$$
\begin{aligned}
\Pi_{IFI} \quad = \quad & (1-b)\left[\int_{-P\gamma(\beta+(1-\beta)R_I)}^{\overline{R}_f} \theta f(\theta)d\theta - \int_{\underline{R}_f}^{-P\gamma(\beta+(1-\beta)R_I)} (G-\theta)f(\theta)d\theta\right] \\
& + (b)\left[\int_{C(\gamma-\beta P\gamma)}^{\overline{R}_f} (\theta - C(\gamma-\beta P\gamma) - \beta P\gamma)\, f(\theta)d\theta - \int_{\underline{R}_f}^{C(\gamma-\beta P\gamma)} (G-\theta)\, f(\theta)d\theta\right] \\
& + P\gamma(\beta+(1-\beta)R_I) \qquad (4)
\end{aligned}
$$

The first term is the expected payoff when a claim is not made, which happens with probability $1-b$ given by to the IFI's beliefs.[16] The $-P\gamma(\beta+(1-\beta)R_I)$ term in the integrand represents the positive diversification benefit that engaging in these contracts can have: it reduces the probability of portfolio default when a claim is not made. We assume that $\underline{R}_f$ is sufficiently negative so that $P\gamma\,(\beta+(1-\beta)R_I) < |\underline{R}_f|$. Since P and β are both bounded from above (by 1)[17], it follows that this inequality is satisfied for a finite $\underline{R}_f$. This assumption ensures that the IFI cannot completely eliminate its probability of default in this state. Recall that before the IFI engaged in the insurance contract, they would be forced into insolvency when their portfolio draw was less than zero. However, we see that in the state in which a claim is not made, they can receive a portfolio draw that is less than zero and still remain solvent (so long as their draw is greater than $-P\gamma(\beta+(1-\beta)R_I)$) . Since the IFI is able to lower its chances of defaulting in this state by investing some of the proceeds of the contract, we refer to this as the IFI *diversifying* its portfolio.

The second term is the expected payoff when a claim is made, which happens with probability B given by the IFI's beliefs. The term $C(\gamma-\beta P\gamma)$ represents the cost to the IFI of accessing the needed capital to pay a claim. Notice that the loans placed in the illiquid asset are not available if a claim is made. Furthermore, the probability of default for the IFI increases in this case. To see this, notice that before engaging int eh insurance contract, the IFI would default is its portfolio draw $\tilde{\theta} \in [\underline{R}_f, 0]$. After the insurance contract, we see that they default if the draw $\tilde{\theta} \in [\underline{R}_f, C(\gamma-\beta P\gamma) > 0]$. To ensure that the IFI prefers to pay the insurance contract when they are solvent, we assume that $G \geq C(\gamma-\beta P\gamma)+\beta P\gamma$. Intuitively, if this condition does not to hold, the IFI would rather declare bankruptcy than fulfil the insurance contract, no matter what their portfolio draw is. The final term in (4) ($P\gamma(\beta+(1-\beta)R_I)$) is the payoff of the insurance premium given how it was invested.

As stated previously, we define counterparty risk as the probability that the IFI defaults, conditional on a claim being made. Therefore, it is now clear that in the context of the model, counterparty risk is given by $\int_{\underline{R}_f}^{C(\gamma-\beta P\gamma)} f(\theta)d\theta$.

[16]Note that this does not necessarily have to be the true probability of default, as we shall discover in section 3.
[17]This is true for β by construction and will be proven for P in Lemma 1.

2.4 IFI Behavior

We now characterize the optimal investment choice of the IFI and the resulting market clearing price. We begin by looking at the IFI's optimal investment decision. In section 3.1 we will show that the optimal choice of β^* for the IFI is less than the first best choice, β^{fb}. In other words, the IFI acts riskier than they would if there were no contracting imperfections. The following proposition characterizes the optimal behavior conditional on an equilibrium belief (b) and an equilibrium price (P). The IFI is shown to invest more in the liquid asset if it believes a claim is more likely to be made.

Proposition 1 *The optimal investment in the liquid asset (β^*) is increasing in b.*

Proof. *See appendix.*

From the implicit solution for β^* derived in the proof to Proposition 1, we see that the result is conditional on a price P. We define P^* as the equilibrium market clearing price. To find it, we require the IFI to earn zero profit from engaging in the insurance contract.[18] We do not implement the zero profit condition on total profit. The reason for this is that if we take zero total profit, the additional profit earned from engaging in the insurance contract may be negative. Therefore, the payoff from the initial portfolio must be excluded. We re-write (4) in terms of the payoff to the IFI from only the insurance contract. We obtain this equation by subtracting (3) from (4). Call this payoff V_{IFI}.

$$\begin{aligned} V_{IFI} &= (1-b)\left[\int_{-P\gamma(\beta+(1-\beta)R_I)}^{0} Gf(\theta)d\theta\right] - b\left[\int_{C(\gamma-\beta P\gamma)}^{\overline{R}_f} (C(\gamma-\beta P\gamma)+\beta P\gamma)\right] \\ &\quad -b\left[\int_{0}^{C(\gamma-\beta P\gamma)} Gf(\theta)d\theta\right] + P\gamma(\beta+(1-\beta)R_I). \end{aligned} \tag{5}$$

It is straightforward to verify that the optimal β^* as derived in Proposition 1 from the total profit is the same as would be derived from optimizing V_{IFI}. Furthermore, the following lemma yields both existence and uniqueness of the market clearing price P^*.

Lemma 1 *There exists a unique market clearing price in the open set $(0, 1)$.*

Proof. *See appendix.*

We now analyze the properties of the equilibrium price P^*. The following lemma shows that as the IFI's beliefs about the probability a claim increases, so too must the premium increase to compensate them for the additional risk.

[18] Lemma 3 shows that this assumption can be relaxed to allow more market power to the IFI without affecting our results.

Lemma 2 *The market clearing price P^* is increasing in the belief of the probability of a claim* (b).

Proof. *See appendix.*

The lemma yields the intuitive result that our pricing function $P(b)$ is increasing in b. The price itself is not the focus of this paper; it can be solved for from the IFI's zero profit condition, $V_{IFI} = 0$ where V_{IFI} is given by (5).

We now turn to the issue of bargaining power. In the preceding analysis, we assumed that there was bertrand competition amongst the IFIs. We then invoked a zero profit condition to pin down the equilibrium price P^*. This turns out not to be a crucial assumption. The following lemma shows that if we allow the IFI to make positive profit, this will have no effect on counterparty risk, unless the underlying loan is "very" risky.

Lemma 3 *If the IFI can make positive profit so that P^* increases, counterparty risk remains unchanged unless $\beta^* = 1$, where it decreases.*

Proof. *See appendix.*

The intuition behind this result is that if we increase the amount of money given to the IFI without changing their beliefs, this will have no effect on the marginal benefit of choosing the liquid asset. The IFI makes its optimal choice by putting money into the liquid asset until the marginal benefit of doing so falls to the level of that of putting it into the illiquid asset. Since increasing just the premium will not change their beliefs (b), this will not change the absolute amount of the premium they put in the liquid asset. Instead, they will put all additional capital into the illiquid asset (which will have a higher marginal return at that point). The lemma shows that the only time counterparty risk will decrease is when $\beta^* = 1$, or in other words, when the loan is 'very' risky (recall that Proposition 1 showed that β^* is increasing in b). This case can only be obtained when both before and after the price increase, the underlying loan is so risky that it is never optimal to put any capital in the illiquid asset, so that all additional capital goes into the liquid asset.

3 Equilibrium Beliefs

Akerlof (1970) showed how insurance contracts can be plagued by the 'lemons' problem. One underlying incentive in his model that generates this result is that the insuree wishes only to minimize the premium they pay. It is for this reason that high risk agents would wish to conceal their type. In this environment, only pooling equilibria can be supported. Subsequent literature to the Akerlof (1970) result showed how the presence of a signalling device can allow a separating equilibrium to exist. What is new in our paper is that no signalling device is needed to justify the existence of a separating equilibrium. We will call the act of concealing one's type for the benefit of a lower insurance premium the *premium effect*. For the situation that we have been analyzing, we show in this section that this effect may be subdued in the presence of counterparty risk. We show that there is another effect that works against the premium effect that we will call the *counterparty risk effect*. The intuition of this new effect is that if high risk agents attempt to be revealed as low risk, they may obtain a better insurance premium, but the following lemma shows that their counterparty risk will increase.

Lemma 4 *If b decreases, but the actual probability of a claim does not, counterparty risk rises whenever $\beta \in (0, 1]$.*

Proof. *See appendix.*

There are two factors that contribute to this result. First, Lemma 2 showed that as the perceived probability of default decreases, the premium also decreases and therefore leaves less capital available to be invested in either asset. Second, since the IFI believes that the loan has a lower probability of failing, it puts more weight on the state in which a claim is not made. Consequently, it is optimal for the IFI to decrease β so as to capitalize on the increased return from the illiquid asset. When we combine these two facts, the counterparty risk unambiguously increases. The only case when the counterparty risk will not rise is when the bank is already investing everything in the illiquid asset, and the decrease in perceived probability of default does not change this.

To analyze the resulting equilibria, we employ the equilibrium concept of a perfect Baysian Nash Equilibrium (PBE). We now define an equilibrium in our model.

Definition 1 *An equilibrium is defined as a β, a price P, and a belief b such that:*

1. *b is consistent with Bayes' rule where possible.*

2. *The bank optimally chooses to reveal or not reveal truthfully its type.*

3. *Choosing P, The IFI earns zero profit with β derived according to the IFI's problem.*

We look first at the equilibrium that is unique to the insurance market under counterparty risk. We ask: is there a separating equilibrium in which the safe loans are revealed as such? The answer without counterparty risk is *no*. The reason is that in this setting, there is no signalling device present, making it costless for the bank with a risky loan to imitate a bank with a safe loan. However, with counterparty risk, it is possible that both types credibly reveal themselves so that separation occurs. Define the type space $i \in \{S, R\}$ to represent the two possible bank types, and define the message space $\mathcal{M} \in \{S, R\}$ to represent the report that bank type i sends to the IFI. Let the payoff be $\Pi(i, \mathcal{M})$ which represents the profit that a type i bank receives from sending the message $\mathcal{M}$. To begin, assume that the IFI's beliefs are such that we are in a separating equilibrium. Therefore, if $\mathcal{M} = S$ ($\mathcal{M} = R$) then $b = \int_0^1 dH_S$ ($\int_0^1 dH_R$). Denote the resulting price of the safe (risky) contract as P_S (P_R). Finally, let the investment in the liquid asset for the bank with safe (risky) loan be derived according to Proposition 1 and be denoted by β_S (β_R). We now write the profit for the bank with a safe loan, given that they are report they are safe (and are revealed as such).

$$\begin{aligned}\Pi(S,S) &= \int_1^{\overline{S}} \psi dH_S(\psi) + (1-\gamma)\int_0^1 \psi dH_S(\psi) + \gamma \int_{C(\gamma-\beta_S P_S \gamma)}^{\overline{R}_f} \int_0^1 (1+\psi) dF(\theta) dH_S(\psi) \\ &\quad -\gamma \int_{\underline{R}_f}^{C(\gamma-\beta_S P_S\gamma)} \int_0^1 (Z-\psi)\, dF(\theta) dH_S(\psi) - \gamma P_S \qquad (6)\end{aligned}$$

The first term represents the expected payoff to the bank conditional on the loan not defaulting. The second term represents the expected payoff (conditional on loan default) to the bank of the uninsured portion of the loan. The third term represents the expected payoff (conditional on loan default) in the case in which the IFI remains solvent and is able to pay γ to the bank as per the insurance contract. The fourth term represents the expected payoff (conditional on loan default) in the case in which the IFI is insolvent and cannot fulfil the terms of the insurance contract. The final term is the insurance premium that the bank must pay to the IFI for protection. We now look at the profit of a risky bank who reports that they are risky.

$$\begin{aligned}\Pi(R,R) &= \int_1^{\overline{R}} \psi dH_R(\psi) + (1-\gamma)\int_0^1 \psi dH_R(\psi) + \gamma \int_{C(\gamma-\beta_R P_R \gamma)}^{\overline{R}_f} \int_0^1 (1+\psi) dF(\theta) dH_R(\psi) \\ &\quad -\gamma \int_{\underline{R}_f}^{C(\gamma-\beta_R P_R\gamma)} \int_0^1 (Z-\psi)\, dF(\theta) dH_R(\psi) - \gamma P_R \qquad (7)\end{aligned}$$

We check the conditions under which neither bank type would like to deviate and report the wrong type. In this case, the IFI chooses (wrongly) β_R (β_S) when the bank type is safe (risky). Therefore, we are checking the conditions that could sustain a separating equilibrium.

$$\Pi(S,S) \geq \Pi(S,R) \Rightarrow$$

$$\underbrace{\int_{C(\gamma-\beta_R P_R\gamma)}^{C(\gamma-\beta_S P_S\gamma)} dF(\theta) \left(\int_0^1 (1+Z)\, dH_S(\psi)\right)}_{\text{expected cost of the additional counterparty risk}} \leq \underbrace{P_R - P_S}_{\text{amount to be saved in insurance premia}} \qquad (8)$$

From Lemma 4 we know that $C(\gamma - \beta_R P_R \gamma) < C(\gamma - \beta_S P_S \gamma)$ so that the left hand side represents the amount of counterparty risk the bank will save if it conceals its type. This is what we call the *counterparty risk effect*. The right hand side represents the amount of insurance premium that the bank will save if it is able to credibly reveal itself as being a safe loan. This is the *premium effect*. We now turn to a bank with a risky loan and repeat the same exercise.

$\Pi(R,R) \geq \Pi(R,S) \Rightarrow$

$$\underbrace{\int_{C(\gamma-\beta_R P_R\gamma)}^{C(\gamma-\beta_S P_S\gamma)} dF(\theta)\left(\int_0^1 (1+Z)\, dH_R(\psi)\right)}_{\text{expected saving in counterparty risk}} \geq \underbrace{P_R - P_S}_{\text{amount extra to be paid in insurance premia}} \tag{9}$$

Again, from Lemma 4 we know that $C(\gamma-\beta_R P_R\gamma) < C(\gamma-\beta_S P_S\gamma)$ and therefore the left hand side represents the additional counterparty risk that a bank with a risky loan will have to take on if they do not reveal their type. The right hand side represents the savings in insurance premium that the bank would receive if they did not reveal their type truthfully.

Therefore, when (8) and (9) simultaneously hold, this equilibrium exists. To give an example of when this can hold, we look at the case in which the safe loan is "very" safe. In particular, we let $\lim \int_0^1 dH_G(\theta) \to 0$ and we obtain:

$$\underbrace{0}_{\text{expected cost of the additional counterparty risk}} \leq \underbrace{P_R - P_S}_{\text{amount to be saved in insurance premia}} \tag{10}$$

$$\underbrace{\int_{C(\gamma-\beta_R P_R\gamma)}^{C(\gamma-\beta_S P_S\gamma)} dF(\theta)\left(\int_0^1 (1+Z)\, dH_R(\psi)\right)}_{\text{expected saving in counterparty risk}} \geq \underbrace{P_R - P_S}_{\text{amount extra to be paid in insurance premia}} \tag{11}$$

Note here that $P_S = 0$ since the probability of default is tending to zero. Inequality (10) is satisfied trivially, while (11) is satisfied when Z is sufficiently large. Recall that Z is the cost of counterparty failure when a claim is made. Therefore this outcome can be achieved by having a high enough penalty on the bank for taking on counterparty risk. This is intuitive since a larger penalty makes them internalize the counterparty risk more, and as a result, greater transparency is achieved in the market. This is a sense in which counterparty risk may be beneficial to the market, since it can help alleviate the adverse selection problem caused by one party having superior information. In this case, the IFI's beliefs are fully defined by Bayes' rule. We can now state the first main result of the paper.

Proposition 2 *In the absence of counterparty risk, no separating equilibrium can exist. When there is counterparty risk, the moral hazard problem allows a unique separating equilibrium to exist in which each type of bank truthfully announces its loan risk. A sufficient condition for this is either that the safe loan is relatively safe, the bankruptcy cost Z is large, or both.*

Proof. *See appendix.*

This proposition shows that a moral hazard problem on the part of the insurer can alleviate the adverse selection problem on the part of the insuree. The separating equilibrium is the case in which the *premium effect* dominates for the bank with the safe loan, while the *counterparty risk* *effect* dominates for the bank with the risky loan. Note that the case in which the safe type wishes to be revealed as risky, and the risky type wishes to be revealed as safe is ruled out in the proof to Proposition 2.

There are also two other pooling equilibria that may exist. The first pooling equilibrium occurs when both the safe and risky bank report that they are safe. In this case, the *premium effect* dominates for both types. Therefore, no information is gleaned by the IFI by the message sent from the banks. Any off-the-equilibrium path belief with $b > \frac{1}{2}\int_0^1 dH_S + \frac{1}{2}\int_0^1 dH_R$ if risky is reported is consistent for the IFI with the Cho-Kreps intuitive criterion.

The second pooling equilibrium occurs when both the safe and risky bank report that they are risky. In this case, the *counterparty risk effect* dominates for both types. Any off-the-equilibrium path belief with $b < \frac{1}{2}\int_0^1 dH_S + \frac{1}{2}\int_0^1 dH_R$ if safe is reported is consistent with the Cho-Kreps intuitive criterion. We formalize both of these pooling equilibrium in the proof to Proposition 2.

We now compare the above analysis with the first best outcome to highlight the inefficiencies and to formally prove the existence of a moral hazard problem.

3.1 The First Best Contract

To find the first best contract between the bank and the IFI, we look at the case in which there are no contracting imperfections. In particular, we do away with the ability of the IFI to invest the premium however it wishes, while maintaining the zero profit assumption. It is appropriate to look at a first best choice of β for both a separating and pooling equilibrium. In this way, we can focus on the effects of contracting imperfections and isolate them from the well understood effects of the standard adverse selection problem. Therefore, the first best can be found by maximizing the bank's profit, subject to the IFI's zero profit condition. We denote the first best solution for β in the separating case as β_{sp}^{fb}, and the first best price in the separating case as P_{sp}^{fb}. As well, let the equilibrium β and the resulting price from Proposition 1 be denoted by β_{sp}^* and P_{sp}^* respectively. The following Lemma shows that the equilibrium price P_{sp}^* must be weakly less than the first best price P_{sp}^{fb}.

Lemma 5 *There is no price $\tilde{P} < P_{sp}^*$ such that the IFI can earn zero profit. This implies that $P_{sp}^* \leq P_{sp}^{fb}$.*

Proof. It is straight-forward to see that $V_{IFI}(\beta_{sp}^*, P_{sp}^*) = 0$ (where V_{IFI} is defined by (5)) implies that $V_{IFI}(\tilde{\beta}, \tilde{P}) \neq 0 \; \forall \; \tilde{\beta} \in [0,1]$ and for $\tilde{P} < P_{sp}^*$.

Since Proposition 1 and Lemma 1 show that with (β_{sp}^*, P_{sp}^*) zero profit is attained, it must be the case that with $\tilde{\beta} \in [0,1] \neq \beta_{sp}^*$ and P_{sp}^*, the IFI earns negative profits. It follows that if $\tilde{P} < P_{sp}^*$, with $\tilde{\beta}$, the IFI must earn negative profits. Since the IFI must earn zero profits, $\tilde{P} \geq P_{sp}^*$.
■

This lemma is valid for the pooling equilibrium case by redefining β^* and P^* from Proposition 1 and Lemma 1.

We are now ready to state the second main result of the paper. The following proposition shows that the IFI chooses a β^* that is too small as compared to the first best choice β^{fb}. From this and Lemma 5, it follows that the insurer moral hazard problem causes the level of counterparty risk in equilibrium to be strictly higher than that of the first best case whenever $\beta^* \in [0, 1)$.

Proposition 3 *There exists a moral hazard problem when $\beta^* \in [0, 1)$ in which the IFI chooses to invest strictly too little in the liquid asset. This insurer moral hazard causes the counterparty risk to be too high in equilibrium.*

Proof. *See appendix.*

The intuition behind this result comes from two sources. First, since the first best case corresponds to optimizing the banks payoff while keeping the IFI at zero profit, the bank strictly prefers to have the IFI invest more in the liquid asset. Second, the IFI must be compensated for this *individually* sub-optimal choice of β by an increase in the premium. Since both β and P rise, counterparty risk falls (i.e. $\int_{\underline{R}_f}^{C(\gamma - \beta P \gamma)} f(\theta) d\theta$ falls). In other words, the moral hazard problem on the part of the IFI creates an inefficiency in the choice of the investment of capital. The key restriction on the contracting space that yields this result is that the insurance premium is paid upfront. Because of this, the bank cannot condition its payment on an observed outcome. In the competitive equilibrium case, the bank knows that the IFI will invest too little into the liquid asset, and therefore lowers its payment accordingly (as from Lemma 3, any additional payment beyond what would yield zero profit to the IFI would be put into the illiquid asset and have no effect on counterparty risk).
We now develop some extensions of the model.

4 Extensions

4.1 Multiple IFIs

Let us assume now that the bank is no longer restricted to insuring with only one IFI. We assume that the bank insures with a finite (and exogenous) number, N, IFIs in the market. For tractability, we assume that the N IFIs all have a portfolio that takes an IID draw from the distribution F with corresponding density f. Therefore, this gives the bank a chance to reduce how much each counterparty holds (as compared to the case in which there was only one counterparty). To contrast with the case in which $N = 1$, we assume that the penalty incurred (Z) is now linearly proportional to the number of IFIs that fail (e.g. if 1 out of 2 IFIs fail, the bank faces a cost of $\frac{Z}{2}$). The following lemma shows that in the environment described, it is equivalent to view the bank interacting with only one (modified) representative IFI. More specifically, if we imagine the bank insuring with all N of the IFIs, the expected profit is derived given that there can be up to N failures. The result says that there is an equivalent problem in which there is one representative IFI, however, that IFI solves its investment problem as though it was only insuring $\frac{\gamma}{N}$ of the loan.

The result comes from two key features of the setup: first, we are analyzing an extreme case in which each IFI takes a IID draw from the same distribution. Second, the bank has a linear payoff function.

Lemma 6 *Aggregation - Denote p as the probability that 1 of the N counterparties fail. Also, denote $\Pi_{bk}(i,k)$ as the (expected) profit of the bank when i counterparties fail and k counterparties succeed. Given this setup, the expected profit of the bank can be written in a simple form as follows:*

$$\sum_{n=0}^{N} \frac{N!}{n!(N-n)!} p^n (1-p)^{N-n} \Pi_{bk}(n, N-n) \; = \; p\Pi_{bk}(1,0) + (1-p)\Pi_{bk}(0,1). \tag{12}$$

Proof. *See appendix.*

The proof proceeds by rearranging the expected profit of the bank to apply the binomial theorem to show that it collapses down to that as if only one IFI was providing the insurance. However, each IFI is now responsible for $\frac{\gamma}{N}$; a reduction in their liability.

Casual intuition should tell us that when the number of IFIs increases, the counterparty risk should decrease. In Lemma 7, we show that when the optimal choice of each IFI is a corner solution, this intuition holds true. However, when an interior solution is achieved, we get the startling result that counterparty risk remains unchanged. The reason for this counterintuitive result is that when $N > 1$, each IFI behaves differently than when $N = 1$. For what is to follow, we denote the optimal β when $N = 1$ ($N > 1$) as β_1^* (β_N^*). Similarly, we denote the optimal price per unit of protection that the bank must pay to each IFI when $N > 1$ as P_N^*. The following proposition shows that the smaller the size of the contract that an IFI engages in, the riskier they will behave, and consequently counterparty risk will remain unchanged (provided an interior solution for β_N^* is attained). What is happening is that the IFI has less obligation so that the state of the world in which a claim is made will see them liquidating less of their portfolio. Therefore, the IFI will have an incentive to put more into the illiquid asset to take advantage of its higher return.

Proposition 4 *Given an interior solution for β_N^*:*

1. *The optimal proportion of the illiquid asset bought is decreasing in the amount of insurance contracts per IFI.*

2. *Counterparty risk that the bank is subjected to remains unchanged.*

Proof. *See appendix.*

This proposition shows that as the amount of the insurance contract that each IFI takes on decreases, the IFI reduces the percentage of the premium they put in the liquid asset. This reduction is by the exact amount so that the counterparty risk remains unchanged. This yields the counterintuitive result that even though each IFI is insuring less of the loan, the counterparty risk that the bank must endure does not decrease.

We now look at the consequences of a corner solution in the IFI's problem. If the IFI is being as risky as it can be ($\beta_N^* = 0$), counterparty risk decreases from the case in which $N = 1$ whenever the boundary constraint is strictly binding.[19] What is happening is that as N becomes large, the savings in the counterparty risk comes entirely from the reduction in liability of each IFI.

The final case in which counterparty risk decreases when $N > 1$ compared to when $N = 1$ is when $\beta_N^* = 1$. What is happening is that the IFI is so cautious that even after their liability decreases, they still invest as safe as possible. One case this may apply is when the insurance contract is being written on a "very" risky loan (b close to one). The following lemma formalizes these two cases.

Lemma 7 *If $\beta_N^* = 0$ or $\beta_N^* = 1$, counterparty risk decreases when $N > 1$ from the case where $N = 1$.*

Proof. Plug $\beta_N^* = 0$ and $\beta_N^* = 1$ into the counterparty risk term $C(\gamma - \gamma P\beta)$ and notice that this is an increasing function of γ. This in turns implies that counterparty risk decreases with respect to a decrease in γ.

■

We now relax the assumption that there is only one bank by allowing many banks to simultaneously purchase insurance contracts with a single IFI.

4.2 Multiple Banks

In this section, we analyze the case of multiple banks and one insurer. We assume there are a measure $M < 1$ of banks. This assumption is meant to approximate the case where there are many banks, and the size of each individual bank's insurance contract is insignificant for the IFI's investment decision.[20] Using an uncountably large number instead of a countably finite but large number of banks helps simplify the analysis greatly. Each bank requests an insurance contract of size γ. At time $t = 0$, each bank receives both an aggregate and idiosyncratic shock which assigns them a probability of default of their loan. For simplicity, this loan is assumed to have a rate of return of R_B if it succeeds and 0 if it does not. The idiosyncratic shock assigns the banks a probability of default which we define as ξ and let it be uniformly distributed. The CDF can then be written as follows.

$$\Phi(\xi) = \begin{cases} 0 & \text{if } \xi \leq 0 \\ \frac{\xi}{M} & \text{if } \xi \in (0, M) \\ 1 & \text{if } \xi \geq M \end{cases}$$

[19] This implies there is one technical case in which the counterparty risk would remain constant. It arises when the $\beta_N^* = 0$ but with the lower bound constraint not binding.

[20] This is the setup we would expect in a traditional insurance market such as health or automobile. However, we will continue to use the example of credit risk transfer in banking.

Next, denote the aggregate shock as p_A and let it take the following form:

$$p_A = \begin{cases} r & \text{with probability } \frac{1}{2} \\ s & \text{with probability } \frac{1}{2}, \end{cases}$$

where $0 < s < r < 1 - M$. It follows that the probability of default of a bank is $p = p_A + \xi$. The example we use is that the aggregate shock puts the banks in one of two industries, a (s)afe or a (r)isky one. Furthermore, the idiosyncratic shock assigns a level of risk to each bank within an industry. It follows that the conditional distribution determining the measure of banks that default in the safe industry, $p(\xi|p_A = s)$ first order stochastically dominates that of the risky industry $p(\xi|p_A = r)$, since $p(\xi|p_A = s) \geq p(\xi|p_A = r)\ \forall\ \xi$. Note that this is in contrast to the usual definition of first order stochastic dominance which entails higher draws providing a 'better' outcome. In the case of this model, the opposite is true, since lower draws refer to a lower probability of default; a 'better' outcome.

4.2.1 The IFI's Problem

Because of the asymmetric information problem, the IFI does not know ex-ante whether the loans are in the safe or risky industry (i.e. whether the aggregate shock was $p_A = s$ or $p_A = r$). It seems reasonable to imagine that banks have superior knowledge about the state of the industry in which they are extending loans. The IFI, even though they do not know the quality of the industry, is assumed to know what banks belong to the same industry. That is, they know that all banks belong to the same industry, but they do not know the quality of that industry. Therefore, if only a subset of the banks can successfully reveal their types, this reveals it for the rest of them.

The IFI is assumed contractually obligated to pay γ to each bank who's loan defaults, conditional on them being solvent. In Lemma 9 we will show that there can be no separation of types within the idiosyncratic shock. Because of this, it follows that given a fixed realization of the aggregate shock, each bank pays the same premium P.[21] We assume that the IFI has the same choice as in section 2.3, so that it invests a proportion β of the premium in the liquid storage asset and $(1 - \beta)$ in the illiquid asset with return R_I. We let the IFI's beliefs distribution over the measure of banks that will default be given by $b(\xi)$ defined over the interval $[0, M]$. Since each bank insures γ, the total size of contracts insured by the IFI is: $\int_0^M \gamma d\Phi(\xi) = M\gamma$. The IFI's payoff can now be written

[21] We are not concerned with pinning down the price in this section. However, we are interested in whether the IFI offers a single aggregate pooling price, or two separating prices. We can imagine the IFI having market power in this section, however, it is not crucial.

as follows (denoting 'MB' as 'Multiple Banks').

$$
\begin{aligned}
\Pi_{IFI}^{MB} &= \underbrace{\int_0^{\beta PM}\left[\int_{-PM\gamma(\beta+(1-\beta)R_I)+\xi\gamma}^{\overline{R}_f}\theta dF(\theta)+\int_{\underline{R}_f}^{-PM\gamma(\beta+(1-\beta)R_I)+\xi\gamma}(\theta-G)\,dF(\theta)\right]db(\xi)}_{\text{Term 1}} \\
&\quad \underbrace{+\int_{\beta PM}^{M}\left[\int_{C(\xi\gamma-\beta PM\gamma)}^{\overline{R}_f}(\theta-C\left(\xi\gamma-\beta PM\gamma\right)-\beta PM\gamma)\,dF(\theta)+\int_{\underline{R}_f}^{C(\xi\gamma-\beta PM\gamma)}(\theta-G)\,dF(\theta)\right]db(\xi)}_{\text{Term 2}} \\
&\quad +\underbrace{(\beta+(1-\beta)\,R_I)\,PM\gamma}_{\text{Term 3}} \qquad (13)
\end{aligned}
$$

The first term represents the case when the IFI has put sufficient capital into the liquid asset so that there is no need to liquidate its portfolio to pay claims. This happens if a sufficiently small measure of banks make claims. Since the IFI receives $PM\gamma$ in insurance premia, it puts $\beta PM\gamma$ into the liquid asset. It follows that if less than $\beta PM\gamma$ is needed to pay claims (i.e. less than βPM banks failed), portfolio liquidation is not necessary. The second term represents the case in which the IFI must liquidate its portfolio if a claim is made. This happens if the amount they need to pay in claims is greater than $\beta PM\gamma$. $C\left(\xi\gamma-\beta PM\gamma\right)-\beta PM\gamma$ represents the total cost of their claims payment, where $\xi M\gamma-\beta P\gamma$ is the total amount of capital the IFI needs to liquidate from its portfolio. The final term represents the direct proceeds from the insurance premium. We need to make the usual assumption that $G\geq C\left(\xi\gamma-\beta PM\gamma\right)-\beta PM\gamma$ so that the IFI wishes to fulfil the contract when they are solvent.

The following lemma both derives the optimal β^* and proves that counterparty risk is less when a set of beliefs first order stochastically dominates another.

Lemma 8 *For a given aggregate shock, there is less counterparty risk when the IFI's beliefs put more weight on the industry as being risky ($p_A = r$) as opposed to it being safe ($p_A = s$).*

Proof. *See appendix.*

The intuition for this result is similar to that of Lemma 4. If the IFI believes that the pool of loans is risky, it is optimal for them to invest more in the liquid asset. This happens because the expected number of claims is higher in the risky case so that the IFI wishes to prevent costly liquidation by investing more in assets that will be easily available if a claim is made.

We now give the conditions under which the IFI's beliefs ($b(\xi)$) are formed.

4.2.2 Equilibrium Beliefs

No Aggregate Shock

To analyze how the beliefs of the IFI are formed, we first consider the case where there is no aggregate shock. Since there is no uncertainty in what industry the IFI is insuring, its optimal investment choice remains the same regardless of whether it offers a pooling price or individual separating prices.[22] It follows that since an individual's choice will have no effect on counterparty risk, only the *premium effect* is active. It is for this reason that a separating equilibrium cannot exist. To see this, assume that each bank reveals its type truthfully. Now consider the bank with the highest probability of default, call it bank M. Since it is paying the highest insurance premium, it can lie about its type without any effect on counterparty risk, and obtain a better premium, and consequently, a better payoff. The following lemma formalizes.

Lemma 9 *There can be no separating equilibrium in which the idiosyncratic shock is revealed.*

We now introduce the aggregate shock and show that separation of industries can occur.

Aggregate and Idiosyncratic Shock

Each individual bank now receives both an aggregate and an idiosyncratic shock. We can think of this procedure as putting the banks in one of two intervals (either $[s, s+M]$ or $[r, r+M]$). We know that if one bank is able to successfully reveal its industry (aggregate shock), then the industry is revealed for all other banks. The following proposition shows that a unique separating equilibrium can exist in this setting.

Proposition 5 *There exists a parameter range in which a unique separating equilibrium in the aggregate shock can be supported.*

Proof. *See appendix.*

This insight follows from the structure of the industry. If a single bank could reveal only its own industry, and not the industry of the other banks at the same time, their premium would be insignificant to the IFI's investment decision. However, since by successfully revealing itself a bank also reveals every other bank's type, their individual problem has a significant effect on IFI's investment choice. The parameter range that can support this equilibrium is similar to the case in which there was only one bank. Some conditions that can support this equilibrium as unique are: Z sufficiently high, and the safe aggregate shock sufficiently low.

In a typical problem with a continuum of agents, no single agent can affect the equilibrium outcome through his or her choices. However, with informational problems like the one we have analyzed here, an individual who has a measure zero can affect the equilibrium outcome.

We now revert to the case of one bank and turn our attention to the bank-borrower relationship and show the consequences that the traditional moral hazard problem has in the model.

[22] To see this, note that with no aggregate risk, the IFI knows the average quality of banks and will use that to make its investment decision based. Any bank claiming that they received the lowest idiosyncratic shock will not change the IFIs beliefs about the average quality.

4.3 Moral Hazard in the Bank-Borrower Relationship

We now relax the assumption that monitoring of the borrower is costless for the bank thereby introducing the traditional moral hazard problem into our framework. For simplicity, we do away with the adverse selection problem, or alternatively, assume that the needed parameterization underlying Proposition 2 is satisfied.[23] Define M as the amount monitored that takes a value in the compact interval $[0, \overline{M}]$. We introduce a cost of monitoring function for a loan: $c(M)$ with $c'() > 0$, $c''() > 0$ and $c(0) = 0$. For simplicity, we rule out corner solutions by assuming $c(\cdot)$ satisfies the Inada conditions: $c'(0) = 0$ and $c'(\overline{M}) = +\infty$. As well, we redefine the return cdf (pdf) of the loan to be $H(\psi; M)$ $(h(\psi; M))$ that satisfies the usual Monotone Likelihood Ratio Property (MLRP) so that $\frac{\partial}{\partial \psi}\left(\frac{h_M(\psi;M)}{h(\psi;M)}\right) > 0$. Finally, we make the standard assumption that the distribution satisfies the convexity-of-distribution function (CDFC) assumption (as in Hart and Holmstrom, 1987).[24] This assumption implies that for any $\lambda \in [0, 1]$, and for any M, M':

$$h(\psi; \lambda M + (1-\lambda)M') \leq \lambda h(\psi; M) + (1-\lambda)h(\psi; M') \tag{14}$$

MLRP and CDIC assumptions together intuitively say that increasing the monitoring, increases, at a decreasing rate, the probability that the return will be above some level ψ.[25] We begin by analyzing the case in which the bank cannot insure itself to avoid the penalty (Z) if the loan fails.

4.3.1 No Insurance

When the bank does not use insurance, the optimal amount of monitoring is the incentive feasible level as follows:

$$\begin{aligned} &\int_1^{\overline{S},\overline{R}} \psi dH(\psi; M) + (1-\gamma)\int_0^1 dH(\psi; M)\psi - \gamma \int_0^1 (Z-\psi) dH(\psi; M) - c(M) \\ \geq &\int_1^{\overline{S},\overline{R}} \psi dH(\psi; M') + (1-\gamma)\int_0^1 dH(\psi; M') - \gamma \int_0^1 (Z-\psi) dH(\psi; M') - c(M') \ \ \forall\, M' \neq M. \end{aligned} \tag{15}$$

Where we use $(\overline{S}, \overline{R})$ as an the upper bound of the return distribution because it could be either loan type we are analyzing. Hart and Holmstrom (1987) showed that given MLRP and CDIC,

[23] All the analysis of this section carries through if we allow for adverse selection; there will be expressions for each loan type in both the pooling and separating equilibria yielding the same qualitative results.

[24] This assumption is used in the first order approach to principal agent problems when the monitoring space is continuous. It allows us to write the infinite number of incentive constraints in one equation. We do not wish to weigh in on the debate that began in the late 1970's as to the validity of the first order approach. For those who find the CDIC assumption unpalatable, Jewitt (1988) Theorem 1 shows how it can be relaxed with additional assumptions on the utility function. The alternative approach to the continuous case is to discretize the monitoring space so that there is a finite number of incentive constraints. The qualitative results of this section follow through with such a procedure, provided there are greater than 2 levels of monitoring (for reasons which will soon become apparent). We use the continuous setup for convenience, as it maps back easily to our previous results.

[25] See Laffont and Martimort (2002) for a nice review of the first order approach to principal agent problems.

these constraints can be re-written as:

$$c'(M) = \int_0^{\overline{S},\overline{R}} \psi dH_M(\psi; M) + \gamma \int_0^1 Z dH_M(\psi; M) \tag{16}$$

Where H_M is the partial with respect to M. Note that we have used the fact that the MLRP assumption implies the weaker condition of First Order Stochastic Dominance (FOSD) which implies that $\int_0^{\overline{S},\overline{R}} \psi dH_M(\psi; M) > 0$. The left hand side represents the marginal cost of increasing monitoring and is given by the marginal increase in the cost of monitoring itself. The right hand side represents its marginal benefit and is comprised of both the increase in the expected value of the loan, and the reduced probability of being subjected to the cost of Z that monitoring brings.

We now look at the case in which the bank can perfectly insure (i.e. no counterparty risk) themselves to avoid the possible cost of Z.

4.3.2 Insurance, No Counterparty Risk

When the bank uses insurance with no counterparty risk, the optimal amount of monitoring is as follows:

$$\int_1^{\overline{S},\overline{R}} \psi dH_M(\psi; M) + (1-\gamma) \int_0^1 \psi dH_M(\psi; M) + \gamma \int_0^1 (1+\psi) dH_M(\psi; M) + \gamma P_M^{NCR} - c'(M) = 0. \tag{17}$$

Where P_M^{NCR} represents the marginal price with no counterparty risk. Note that FOSD implies $\int_0^1 dH_M(\psi; M) < 0$, and Lemma 2 implies $\frac{\partial P}{\partial b} > 0$. Therefore, $P_M = \frac{\partial P}{\partial M} = \frac{\partial b}{\partial M}\frac{\partial P}{\partial b} = \int_0^1 dH_M(\psi; M)\frac{\partial P}{\partial b} < 0$. Finally, since FOSD implies both $\int_0^1 dH_M(\psi; M) < 0$ and $\int_1^{\overline{S},\overline{R}} dH_M(\psi; M) > 0$, we can rewrite (17) in a more intuitive form:

$$\underbrace{c'(M) + \gamma \int_0^1 dH_M(\psi; M)}_{\text{Marginal Cost of Monitoring}} = \underbrace{\int_0^{\overline{S},\overline{R}} \psi dH_M(\psi; M) + \gamma P_M^{NCR}}_{\text{Marginal Benefit of Monitoring}} \tag{18}$$

Again, the left hand side represents the marginal cost of monitoring. For an increase in monitoring, the bank incurs the monitoring cost itself, plus a decrease in expected payout from the claim (because claims are made less with more monitoring). The benefits to monitoring are the increase in the expected return of the loan, plus the reduced insurance premium the bank will enjoy by reducing the probability that a claim will be made.

Comparing (16) and (18) we see that insurance reduces the incentive to monitor when the following holds:

$$Z > \frac{P_M^{NCR} - \int_0^1 dH_M(\psi; M)}{\int_0^1 dH_M(\psi; M)} \tag{19}$$

In other words, when default of the loan without protection is sufficiently costly, the firm will choose to monitor more when it is not insured. Note that the sign of $P_M - \int_0^1 dH_M(\psi; M)$ is ambiguous and depends on the underlying parameters of the model. When $P_M \leq \int_0^1 dH_M(\psi; M)$, the bank will always monitor more when they are not insured (for any $Z > 0$).

We continue by adding counterparty risk to the insurance contract and show that the moral hazard problem may be less severe than in the current case.

4.3.3 Insurance with Counterparty Risk - Double Moral Hazard

When the bank uses insurance and is subject to counterparty risk, a double moral hazard problem is present: Both the hidden actions of monitoring by the bank, and investing by the IFI occur simultaneously in our model. Therefore, we must respect the incentive constraints of both the bank and the IFI. We now write the first order condition for the bank taking β^* as given.

$$\int_0^{\overline{S},\overline{R}} \psi dH_M(\psi; M) + \gamma \int_0^1 dH_M(\psi; M) \int_{C(\gamma-\beta^* P^* \gamma)}^{\overline{R}_f} dF(\theta)$$
$$-Z\gamma \int_0^1 dH_M(\psi; M) \int_{\underline{R}_f}^{C(\gamma-\beta^* P^* \gamma)} dF(\theta) - c'(M) - \gamma P_M^{CR} = 0 \tag{20}$$

Where P_M^{CR} is the marginal price with counterparty risk. Because $P_M^{CR} < 0$, $\int_0^1 dH_M(\psi; M) < 0$ and $\int_0^{\overline{S},\overline{R}} dH_M(\psi; M) > 0$, we can rewrite (20).

$$\underbrace{c'(M) + \gamma \int_0^1 dH_M(\psi; M) \int_{C(\gamma-\beta^* P^* \gamma)}^{\overline{R}_f} dF(\theta)}_{\text{Marginal Cost of Monitoring}} =$$
$$\underbrace{\int_0^{\overline{S},\overline{R}} \psi dH_M(\psi; M) + Z\gamma \int_0^1 dH_M(\psi; M) \int_{\underline{R}_f}^{C(\gamma-\beta^* P^* \gamma)} dF(\theta) + \gamma P_M^{CR}}_{\text{Marginal Benefit of Monitoring}} \tag{21}$$

Altering Proposition 1 to include an optimal choice of monitoring by the bank, we obtain β^* for a given M^*.

$$\begin{cases} \beta^* = 0 & \text{if } b(M^*) \leq b^* \\ -(1-b(M^*))(R_I - 1)G + b(M^*)[C'(\gamma - \beta^* P\gamma)\left(G - C(\gamma - \beta^* P\gamma) - \beta^* P\gamma\right) & \\ + \left(\overline{R}_f - C(\gamma - \beta^* P\gamma)\right)\left(C'(\gamma - \beta^* P\gamma) - 1\right)] = (R_I - 1)(\overline{R}_f - \underline{R}_f) & \text{if } b(M^*) \in (b^*, b^{**}) \\ \beta^* = 1 & \text{if } b(M^*) \geq b^{**} \end{cases}$$

where $b^* = \frac{(R_I-1)(G+\overline{R}_f-\underline{R}_f)}{G(R_I-1)+C'(\gamma)(G-C(\gamma))-(\overline{R}_f-C(\gamma))(C'(\gamma)-1)}$,
and $b^{**} = \frac{(R_I-1)(G+\overline{R}_f-\underline{R}_f)}{G(R_I-1)+C'(\gamma-P\gamma)(G-C(\gamma-P\gamma)-P\gamma)-(\overline{R}_f-C(\gamma-P\gamma)))(C'(\gamma-P\gamma))-1)}$.

Comparing (21) and (18) we derive a condition under which the bank strictly monitors more with counterparty risk.

$$Z > \frac{-\int_0^1 dH_M(\psi;M)\left(1-\int_{C(\gamma-\beta^* P^*\gamma)}^{\overline{R}_f} dF(\theta)\right) + P_M^{NCR} - P_M^{CR}}{\int_{\underline{R}_f}^{C(\gamma-\beta^* P^*\gamma)} dF(\theta)} \tag{22}$$

We can conclude that if Z is sufficiently high, the traditional moral hazard problem will be less severe. The parameter Z works to tie the bank to the loan. If the loan defaults, and so to does the bank's counterparty, the bank is not protected and is subject to the cost Z. Therefore, the higher is Z, the more intensely the bank will monitor the loan.

One of the key elements that emerges from this section is that when we introduce the classical moral hazard into the model, we need only modify the distribution function to include an optimal monitoring amount. In other words, the IFI readjusts its belief of the probability of a claim given the amount of monitoring that the bank will engage in.

5 Conclusion

In a setting in which insurers can fail, we construct a model to show a new moral hazard problem that can arise in insurance contracts. We model a situation in which the insurer itself may default in some states of the world. Taking account of this, when the insurer sells an insurance contract, it uses its evaluation of the risk in the contract to optimally invest its capital. If it suspects that the contract is safe, it puts capital into less liquid assets, which minimizes the probability they fail in the state when a claim is not made. However, the downside of this is that when a claim is made, they are more likely to be illiquid or insolvent and not able to fulfil the contract. We show that the insurers investment choice is too risky when compared to the first best. The existence of this moral hazard is shown to allow a unique separating equilibrium to exist wherein the insuree freely and credibly relays its superior information. In other words, the new moral hazard problem can alleviate the adverse selection problem. We extend the model by increasing the number of insurers that an insuree contracts with. We show that in this situation the counterparty risk may not decrease as one would expect. Next we allow for multiple insurees to investigate the case where each insurance contract is insignificant to the insurers investment choice. We show that our moral hazard problem still exists, and we can obtain the separating equilibrium result when there is private aggregate risk. In a final extension, we show how the classical moral hazard problem on the part of the insuree can have a positive effect on counterparty risk.

6 Appendix

Proof Proposition 1. Using the assumption that $f(\theta)$ is distributed uniform over the interval $[\underline{R}_f, \overline{R}_f]$, we solve for the optimal choice of β for the IFI, given b and P.

$$\max_{\beta\in[0,1]} \Pi_{IFI}$$

Using Leibniz rule to differentiate the choice variable in the integrands, we obtain the following first order equation:

$$\begin{aligned} 0 \;=\; & \frac{bP\gamma}{\overline{R}_f - \underline{R}_f}\left[C'(\gamma-\beta P\gamma)\left(G - C(\gamma-\beta P\gamma) - \beta P\gamma\right) + \left(\overline{R}_f - C(\gamma-\beta P\gamma)\right)\left(C'(\gamma-\beta P\gamma)-1\right)\right] \\ & +(1-b)\frac{G}{\overline{R}_f - \underline{R}_f}\left[-R_I\gamma P + \gamma P\right] + P\gamma(1-R_I) \end{aligned} \tag{23}$$

Where $G - C(\gamma-\beta P\gamma) - \beta P\gamma \geq 0$ by assumption, and $C'(\gamma-\beta P\gamma) - 1 \geq 0$ since $C(x) \geq x \; \forall \; x \geq 0$. To ensure a maximum, we take the second order condition and show the inequality that must hold.

$$\begin{aligned} C''(\gamma-\beta P\gamma)\left(G - C(\gamma-\beta P\gamma) - \beta P\gamma\right) + \left(\overline{R}_f - C(\gamma-\beta P\gamma)\right)C''(\gamma-\beta P\gamma) \\ \geq 2C'(\gamma-\beta P\gamma)\left(C'(\gamma-\beta P\gamma)-1\right) \end{aligned} \tag{24}$$

Plugging in the boundary conditions for β into the FOC, we now derive the optimal proportion of capital put in the liquid asset as an implicit function.

$$\begin{cases} \beta^* = 0 & \text{if } \; b \leq b^* \\ -(1-b)(R_I-1)G + b[C'(\gamma-\beta^*P\gamma)\left(G - C(\gamma-\beta^*P\gamma) - \beta^*P\gamma\right) & \\ +\left(\overline{R}_f - C(\gamma-\beta^*P\gamma)\right)\left(C'(\gamma-\beta^*P\gamma)-1\right)] = (R_I-1)(\overline{R}_f - \underline{R}_f) & \text{if } \; b \in (b^*, b^{**}) \\ \beta^* = 1 & \text{if } \; b \geq b^{**} \end{cases}$$

where $b^* = \frac{(R_I-1)(G+\overline{R}_f-\underline{R}_f)}{G(R_I-1)+C'(\gamma)(G-C(\gamma))-(\overline{R}_f-C(\gamma))(C'(\gamma)-1)}$,
and $b^{**} = \frac{(R_I-1)(G+\overline{R}_f-\underline{R}_f)}{G(R_I-1)+C'(\gamma-P\gamma)(G-C(\gamma-P\gamma)-P\gamma)-(\overline{R}_f-C(\gamma-P\gamma)))(C'(\gamma-P\gamma))-1)}$.

We now show that the optimal proportion of capital put in the liquid asset is increasing in b by finding $\frac{\partial\beta}{\partial b}$ from the FOC.

$$
\begin{aligned}
0 &= A+b[(-C'(\gamma-\beta P\gamma)(-\frac{\partial\beta}{\partial b}P\gamma)(C'(\gamma-\beta P\gamma)-1) \\
&+(\overline{R}_f-C(\gamma-\beta P\gamma))(C''(\gamma-\beta P\gamma)(-\frac{\partial\beta}{\partial b}P\gamma) \\
&+C''(\gamma-\beta P\gamma)(-\frac{\partial\beta}{\partial b}P\gamma)(G-C(\gamma-\beta P\gamma)-\beta P\gamma)+C'(\gamma-\beta P\gamma)(-C'(\gamma-\beta P\gamma)(-\frac{\partial\beta}{\partial b}P\gamma) \\
&-(-\frac{\partial\beta}{\partial b}P\gamma))]+G(R_I-1)P\gamma \qquad (25)
\end{aligned}
$$

Where we define:

$$
A=C'(\gamma-\beta P\gamma)P\gamma\left(G-C(\gamma-\beta P\gamma)-\beta P\gamma\right)+\left(\overline{R}_f-C(\gamma-\beta P\gamma)\right)P\gamma\left(C'(\gamma-\beta P\gamma)-1\right)\geq 0.(26)
$$

Rearranging for $\frac{\partial\beta}{\partial b}$ yields to following.

$$
\begin{aligned}
\frac{\partial\beta}{\partial b} &= \frac{-C'(\gamma-\beta P\gamma)\left(G-C(\gamma-\beta P\gamma)-\beta P\gamma\right)-\left(\overline{R}_f-C(\gamma-\beta P\gamma)\right)\left(C'(\gamma-\beta P\gamma)-1\right)-G(R_I-1)}{-C''(\gamma-\beta P\gamma)\left(G-C(\gamma-\beta P\gamma)-\beta P\gamma\right)-\left(\overline{R}_f-C(\gamma-\beta P\gamma)\right)C''(\left(\overline{R}_f-C(\gamma-\beta P\gamma)\right))+2C'(\gamma-\beta P\gamma)\left(C'(\gamma-\beta P\gamma)-1\right)} \\
&\geq 0 \qquad (27)
\end{aligned}
$$

Where the numerator is trivially negative while the denominator is negative because of condition (24) imposed by the SOC to achieve a maximum.

■

Proof of Lemma 1.
Step 1: Existence

We prove that there exists a P^* that satisfies the following:

$$
\begin{aligned}
0 &= (1-b)\left[\int_{-P^*\gamma(\beta+(1-\beta)R_I)}^{0} Gf(\theta)d\theta\right]-b\left[\int_{C(\gamma-\beta P^*\gamma)}^{\overline{R}_f}(C(\gamma-\beta P^*\gamma)+\beta P^*\gamma)\right] \\
&-b\left[\int_{0}^{C(\gamma-\beta P^*\gamma)} Gf(\theta)d\theta\right]+P^*\gamma(\beta+(1-\beta)R_I). \qquad (28)
\end{aligned}
$$

Consider $P^*\leq 0$. In this case, the IFI earns negative profits. To see this, notice all terms on the right hand side of (28) are weakly negative, with the second and third terms strict (since $C(\gamma-\beta P^*\gamma)>\beta P^*\gamma$ when $P^*\leq 0$). Therefore, it must be that $V_{IFI}(\beta^*,P^*\leq 0)<0$. This contradicts the fact that $V_{IFI}(\beta^*,P^*)=0$ in equilibrium.
Next, consider $P^*\geq 1$, and $\beta=1$ (not necessarily the optimal value). In this case, the first term on the right hand side of (28) is strictly positive, the second and third terms are zero, while the fourth is strictly positive. Since β^* can yield no less profit than $\beta=1$ by definition of it being an optimum, it must be that $V_{IFI}(\beta^*,P^*\geq 0)>0$. This contradicts the fact that $V_{IFI}(\beta^*,P^*)=0$ in equilibrium. Therefore, if it exists, $P^*\in(0,1)$.
To show that P^* exists in the interval $(0,1)$, we differentiate the right hand side of (28) to show

that profit is strictly increasing in P.

$$\begin{aligned}
\frac{\partial V_{IFI}}{\partial P} &= b\beta P\left[C'(\gamma-\beta P\gamma)\left(G-C(\gamma-\beta P\gamma)-\beta P\gamma\right)+\left(\overline{R}_f-C(\gamma-\beta P\gamma)\right)\beta\gamma\left(C'(\gamma-\beta P\gamma)-1\right)\right] \\
&\quad +(1-b)\left[G\gamma\left(\beta+(1-\beta)R_I\right)\right]+\gamma\left(\beta+(1-\beta)R_I\right) \qquad (29)\\
&\geq 0 \qquad (30)
\end{aligned}$$

Where the inequality follows from the assumption that $G \geq C(\gamma-\beta P\gamma)-\beta P\gamma$ and the assumption that $C(x) \geq x \; \forall x \geq 0$ (which implies $C'(x) \geq 1$). Therefore, since profit is negative when $P^* \leq 0$ and positive when $P^* \geq 1$, and since profit is a (monotonically) increasing function of P^*, profit must equate to zero within $P^* \in (0,1)$.

Step 2: Uniqueness

Assume the following holds: $V_{IFI}(\beta^*, P_1^*) = 0$. Since we have already shown that profit is a strictly increasing function of P^*, then if $P_2^* > P_1^*$ ($P_2^* < P_1^*$) this implies $V_{IFI}(\beta^*, P_2^*) > 0$ ($V_{IFI}(\beta^*, P_2^*) < 0$). Therefore, since given P_2^* and P_1^* and $V_{IFI}(\beta^*, P_1^*) = 0$ implies that $P_1^* = P_2^*$ must hold, our price is unique.

■

Proof of Lemma 2.
From the envelop theorem, we can ignore the effect that changes in b have on β when we evaluate the payoff at β^*. Plugging $\beta = \beta^*$ into (4) and taking the partial derivative with respect to b yields:

$$\begin{aligned}
\left.\frac{\partial V_{IFI}}{\partial b}\right|_{\beta=\beta^*} &= -\frac{\left(\overline{R}_f-C(\gamma-\beta^*P\gamma)\right)\left(C(\gamma-\beta^*P\gamma)+\beta^*P\gamma\right)+C(\gamma-\beta^*P\gamma)G+P\gamma G(\beta^*+(1-\beta^*)R_I)}{\overline{R}_f-\underline{R}_f} \\
&< 0 \qquad (31)
\end{aligned}$$

The inequality follows because $C(\cdot) > 0$ by assumption. Since the envelop theorem is a local condition and does not hold for large changes in b, it serves as an upper bound on the decrease in profits. It follows that an increase in b must be met with an increase in P otherwise the IFI would make negative profit and would not participate in the market.

■

Proof of Lemma 3. Since counterparty risk is defined as $\int_{\underline{R}_f}^{C(\gamma-\beta P\gamma)} f(\theta)d\theta$, we find the effect that a change in P has on $C(\gamma-\beta^*P\gamma)$. Since $C(\cdot)$ is monotonic, we focus on $(\gamma-\beta^*P\gamma)$. It should be immediately apparent that when $\beta^* = 0$, changes in P have no effect. Intuitively, if the IFI is already putting everything into the illiquid asset, any additional capital will be put into the illiquid asset.
We now take the following partial derivative and show that it equates to zero.

$$\frac{\partial\left(\gamma-\beta^* P\gamma\right)}{\partial P} = -\gamma\left(\frac{\partial \beta^*}{\partial P}P+\beta^*\right) \tag{32}$$

We find $\frac{\partial\beta^*}{\partial P}\equiv\left.\frac{\partial\beta}{\partial P}\right|_{\beta=\beta^*}$ (where β^* is defined implicitly in the FOC).

$$\begin{aligned} 0 \;=\; & \left[-C'(\gamma-\beta^*P\gamma)\left(-\frac{\partial\beta^*}{\partial P}P\gamma-\beta^*\gamma\right)\right]\left[C'(\gamma-\beta^*P\gamma)-1\right] \\ & +\left[\overline{R}_f-C(\gamma-\beta^*P\gamma)\right]\left[C''(\gamma-\beta^*P\gamma)\left(-\frac{\partial\beta^*}{\partial P}P\gamma-\beta^*\gamma\right)\right] \\ & +\left[C''(\gamma-\beta^*P\gamma)\left(-\frac{\partial\beta^*}{\partial P}P\gamma-\beta^*\gamma\right)\right]\left[G-C(\gamma-\beta^*P\gamma)-\beta^*P\gamma\right] \\ & +C'(\gamma-\beta^*P\gamma)\left[-C'(\gamma-\beta^*P\gamma)\left(-\frac{\partial\beta^*}{\partial P}P\gamma-\beta^*\gamma\right)-\beta^*\gamma-\frac{\partial\beta^*}{\partial P}P\gamma\right] \end{aligned} \tag{33}$$

Rearranging for $\frac{\partial\beta^*}{\partial P}$ yields the following.

$$\begin{aligned} \frac{\partial\beta^*}{\partial P}P\gamma A &= -\beta^*\gamma A \\ \Rightarrow \frac{\partial\beta^*}{\partial P} &= -\frac{\beta^*}{P} \end{aligned} \tag{34}$$

Where we define:

$$\begin{aligned} A \;=\; & C''(\gamma-\beta P\gamma)\left(G-C(\gamma-\beta P\gamma)-\beta P\gamma\right)+\left(\overline{R}_f-C(\gamma-\beta P\gamma)\right)C''(\gamma-\beta P\gamma) \\ & -2C'(\gamma-\beta P\gamma)\left(C'(\gamma-\beta P\gamma)-1\right). \end{aligned} \tag{35}$$

Note that $A<0$ from the assumption on the SOC (24) to ensure a maximum. Substituting (34) into (32) yields the desired result:

$$\frac{\partial\left(\gamma-\beta^* P\gamma\right)}{\partial P} = 0. \tag{36}$$

Therefore changes in P have no effect on counterparty risk when β attains an interior solution. The final situation is where $\beta^*=1$. We obtain:

$$\frac{\partial(\gamma-\gamma P)}{\partial P} = -\gamma < 0. \tag{37}$$

In this case, the IFI puts all additional premia in the liquid asset and thus reduces the counterparty risk.

■

Proof of Lemma 4. Since counterparty risk is defined as $\int_{\underline{R}_f}^{C(\gamma-\beta P\gamma)} f(\theta)d\theta$, we are interested in what happens to $C(\gamma-\beta^*P^*\gamma)$ as b changes.

We first focus on the case in which $\beta^* \in (0,1)$. We now take following partial derivative where we define $\frac{\partial \beta^*}{\partial b} \equiv \frac{\partial \beta}{\partial b}\Big|_{\beta=\beta^*}$ and $\frac{\partial P^*}{\partial b} \equiv \frac{\partial P}{\partial b}\big|_{P=P^*}$.

$$\frac{\partial\left(\gamma-\beta^* P^* \gamma\right)}{\partial b}=-\gamma\left(\frac{\partial \beta^*}{\partial b} P^*+\beta^* \frac{\partial P^*}{\partial b}\right) \tag{38}$$

From Proposition 1 we know $\frac{\partial \beta^*}{\partial b} \geq 0$. As well, from Lemma 2 we know $\frac{\partial P^*}{\partial b} > 0$. Since $\beta^* \in (0,1)$ and $P^* > 0$ (from Lemma 1), it follows that:

$$\frac{\partial\left(\gamma-\beta^* P^* \gamma\right)}{\partial b}<0 \tag{39}$$

Therefore, as b increases, counterparty risk decreases when $\beta \in (0,1)$. Next, consider the case of $\beta^* = 1$. Again, from Lemma 2 we know $\frac{\partial P^*}{\partial b} > 0$. Therefore, $\frac{\partial(\gamma-\beta^* P^* \gamma)}{\partial b} < 0$ regardless of whether $\frac{\partial \beta^*}{\partial b} = 0$ or $\frac{\partial \beta^*}{\partial b} > 0$. Thus, counterparty risk decreases when b decreases if $\beta^* = 1$.
It is obvious that if $\beta^* = 0$ there will be no change in counterparty risk by noting that $\beta^* P \gamma$ will be independent of b.

■

Proof of Proposition 2. We begin by ruling out a separating equilibrium when there is no counterparty risk. Without counterparty risk: $\int_{\underline{R}_f}^{0} dF(\theta) = 0$. It follows that the left hand side of (8) and (9) are both zero. Since $P_R - P_S > 0$, (8) and (9) cannot be simultaneously satisfied so that the separating equilibrium cannot exist.
We now introduce counterparty risk and show that the separating equilibrium in which the safe (risky) type reports they are risky (safe) cannot exist. The condition under which the safe type would prefer to be revealed as risky can be written as follows:

$$\Pi(S,R) \geq \Pi(S,S) \Rightarrow$$
$$\underbrace{\int_{C(\gamma-\beta_R P_R \gamma)}^{C(\gamma-\beta_S P_S \gamma)} dF(\theta)\left(\int_0^1 (1+Z)\, dH_S(\psi)\right)}_{\text{expected saving in counterparty risk}} \geq \underbrace{P_R - P_S}_{\text{amount extra to be paid in insurance premia}} \tag{40}$$

Next, we write the condition under which the risky type would prefer to be revealed as safe as:

$$\Pi(R,S) \geq \Pi(R,R) \Rightarrow$$
$$\underbrace{\int_{C(\gamma-\beta_R P_R \gamma)}^{C(\gamma-\beta_S P_S \gamma)} dF(\theta)\left(\int_0^1 (1+Z)\, dH_R(\psi)\right)}_{\text{expected cost of the additional counterparty risk}} \leq \underbrace{P_R - P_S}_{\text{amount to be saved in insurance premia}} \tag{41}$$

Since $\int_0^1 dH_S(\psi) < \int_0^1 dH_R(\psi)$, it follows that the left hand side of (40) is unambiguously smaller than the left hand side of (41). It follows that (40) and (41) cannot simultaneously hold, and thus this separating equilibrium cannot exist.

We proceed by showing the conditions for which the two pooling equilibria can exist. We begin with the case in which both types wish to be revealed as safe. We define $\beta_{1/2}$ and $P_{1/2}$ as the equilibrium result from the IFI's problem when the belief of the probability of a claim cannot be further updated: $b = \frac{1}{2}\int_0^1 dH_S(\psi) + \frac{1}{2}\int_0^1 dH_R(\psi)$. Finally, we let β_{OE} and P_{OE} be the result from the IFI's problem when a bank gives an off the equilibrium path report of R. The following two conditions formalize this case:

$$\Pi(S,S) \geq \Pi(S,R) \Rightarrow$$

$$\underbrace{\int_{C(\gamma-\beta_{OE}P_{OE}\gamma)}^{C(\gamma-\beta_{1/2}P_{1/2}\gamma)} dF(\theta)\left(\int_0^1 (1+Z)\,dH_S(\psi)\right)}_{\text{expected cost of the additional counterparty risk}} \leq \underbrace{P_{OE} - P_{1/2}}_{\text{amount to be saved in insurance premia}} \tag{42}$$

$$\Pi(S,R) \geq \Pi(R,R) \Rightarrow$$

$$\underbrace{\int_{C(\gamma-\beta_{OE}P_{OE}\gamma)}^{C(\gamma-\beta_{1/2}P_{1/2}\gamma)} dF(\theta)\left(\int_0^1 (1+Z)\,dH_R(\psi)\right)}_{\text{expected cost of the additional counterparty risk}} \leq \underbrace{P_{OE} - P_{1/2}}_{\text{amount to be saved in insurance premia}} \tag{43}$$

The binding condition (43) is satisfied for Z sufficiently small. The intuition is that if counterparty risk is not too costly, the bank would wish to obtain lowest insurance premium. In other words, the *premium effect* dominates. To sustain this equilibrium, we see that $b > \frac{1}{2}\int_0^1 dH_S(\psi) + \frac{1}{2}\int_0^1 dH_R(\psi)$.

We continue by analyzing the case in which both types report that they are risky. In this case, we will use the notation β_{OE2} and P_{OE2} to indicate the off the equilibrium path beliefs if a bank reports that they are safe. The conditions can be characterized as follows:

$$\Pi(S,S) \geq \Pi(S,R) \Rightarrow$$

$$\underbrace{\int_{C(\gamma-\beta_{1/2}P_{1/2}\gamma)}^{C(\gamma-\beta_{OE2}P_{OE2}\gamma)} dF(\theta)\left(\int_0^1 (1+Z)\,dH_S(\psi)\right)}_{\text{expected saving in counterparty risk}} \geq \underbrace{P_{1/2} - P_{OE2}}_{\text{amount extra to be paid in insurance premia}} \tag{44}$$

$$\Pi(S,R) \geq \Pi(R,R) \Rightarrow$$

$$\underbrace{\int_{C(\gamma-\beta_{1/2}P_{1/2})}^{C(\gamma-\beta_{OE2}P_{OE2}\gamma)} dF(\theta)\left(\int_0^1 (1+Z)\,dH_R(\psi)\right)}_{\text{expected saving in counterparty risk}} \geq \underbrace{P_{1/2} - P_{OE2}}_{\text{amount extra to be paid in insurance premia}} \tag{45}$$

The binding condition (44) is satisfied for Z sufficiently high. Intuitively, the bank is so averse to counterparty risk, that the *counterparty risk effect* dominates. It follows that for this equilibrium to exist, $b < \frac{1}{2}\int_0^1 dH_S(\psi) + \frac{1}{2}\int_0^1 dH_R(\psi)$.

We now show that the separating equilibrium defined by (8) and (9) can be unique. Combining (8) and (9) we get the following condition for when the separating equilibrium exists:

$$\frac{P_R - P_S}{\int_{C(\gamma-\beta_R P_R\gamma)}^{C(\gamma-\beta_S P_S\gamma)} dF(\theta)\int_0^1 dH_R(\psi)} \leq Z+1 \leq \frac{P_R - P_S}{\int_{C(\gamma-\beta_R P_R\gamma)}^{C(\gamma-\beta_S P_S\gamma)} dF(\theta)\int_0^1 dH_S(\psi)} \tag{46}$$

Turning to the pooling equilibria, we use extreme off the equilibrium path beliefs for simplicity (the following is valid for the general belief as well). Let $OE = R$ and $OE2 = S$. The condition under which the pooling equilibrium cannot exist can be written as:

$$\frac{P_R - P_{1/2}}{\int_{C(\gamma-\beta_R P_R\gamma)}^{C(\gamma-\beta_{1/2} P_{1/2}\gamma)} dF(\theta)\int_0^1 dH_R(\psi)} \leq Z+1 \leq \frac{P_{1/2} - P_S}{\int_{C(\gamma-\beta_{1/2} P_{1/2}\gamma)}^{C(\gamma-\beta_S P_S\gamma)} dF(\theta)\int_0^1 dH_S(\psi)} \tag{47}$$

It follows that if (46) and (47) are satisfied, the separating equilibrium exists and is unique. To see that these conditions can be simultaneously satisfied, let $\lim \int_0^1 dH_S(\psi) \to 0$ so that the right hand side of both (46) and (47) are satisfied. It follows that if Z is sufficiently large, the left hand side of these two inequalities can be satisfied yielding a unique separating equilibrium.

■

Proof of Proposition 3. The Proof proceeds in 3 steps. Step 1 derives the first order condition for the first best problem. Step 2 assumes the equilibrium solution and derives an expression for $\frac{\partial P}{\partial \beta}$ from the IFI's zero profit condition. Step 3 shows that β^{fb} and P^{fb} must be greater than in the equilibrium case when $\beta^* < 1$. Since the results apply for either the separating or pooling equilibrium, we show that $\beta^* < \beta^{fb}$, where these can be either equilibrium case.

Step 1

The profit for the bank (bk) is written as follows (note here we leave the banks return distribution function as H).

$$\begin{aligned}\Pi_{bk} &= \int_1^{(\overline{S},\overline{R})} \psi dH(\psi) + (1-\gamma)\int_0^1 \psi dH(\psi) + \gamma\int_{C(\gamma-\beta P\gamma)}^{\overline{R}_f}\int_0^1 (1+\psi)dF(\theta)dH(\psi) \\ &\quad -\gamma\int_{\underline{R}_f}^{C(\gamma-\beta P\gamma)}\int_0^1 (Z-\psi)\,dF(\theta)dH(\psi) - \gamma P\end{aligned} \tag{48}$$

For the first best case, P^{fb} is now endogenous and determined by $V_{IFI}(\beta^{fb}, P^{fb}) = 0$ (where V_{IFI} is defined by (5)). Using the uniform assumption on F yields the following first order condition.

$$\frac{\partial P}{\partial \beta} = \gamma C'(\gamma - \beta P\gamma)\left(P + \frac{\partial P}{\partial \beta}\beta\right)(1+Z)\int_0^1 dH(\psi) \tag{49}$$

The left hand side represents the marginal cost of increasing β, while the right hand side represents the marginal benefit of doing so.

Step 2
We show that if $\beta^{fb} = \beta^*$, then (49) cannot hold. We know that from the IFI's problem, the following must hold (see Proposition 1 for its derivation):

$$\begin{aligned} 0 &= \frac{b}{\overline{R}_f - \underline{R}_f}\left[C'(\gamma - \beta^* P^*\gamma)\left(G - C(\gamma - \beta^* P^*\gamma) - \beta^* P^*\gamma\right) + \left(\overline{R}_f - C(\gamma - \beta^* P^*\gamma)\right)\left(C'(\gamma - \beta^* P^*\gamma) - 1\right)\right] \\ &\quad + (1-b)\frac{G}{\overline{R}_f - \underline{R}_f}\left[-R_I\gamma P^* + \gamma P^*\right] + P^*\gamma(1 - R_I) \end{aligned} \tag{50}$$

We now find an expression for $\left.\frac{\partial P}{\partial \beta}\right|_{\beta=\beta^*, P=P^*}$ by implicitly differentiating the equation $V_{IFI}(\beta^*, P^*) = 0$.

$$\begin{aligned} 0 &= (1-b)\left[\int_{-P\gamma(\beta+(1-\beta)R_I)}^{0} Gf(\theta)d\theta\right] - b\left[\int_{C(\gamma-\beta P\gamma)}^{\overline{R}_f}\left(C(\gamma - \beta P\gamma) + \beta P\gamma\right)\right] \\ &\quad - b\left[\int_0^{C(\gamma-\beta P\gamma)} Gf(\theta)d\theta\right] + P\gamma(\beta + (1-\beta)R_I) \end{aligned} \tag{51}$$

Implicitly differentiating this equation to find $\frac{\partial P}{\partial \beta}$ yields the following.

$$\begin{aligned} A\left.\frac{\partial P}{\partial \beta}\right|_{\beta=\beta^*, P=P^*} &= (1-b)\frac{G}{\overline{R}_f - \underline{R}_f}\left[-R_I\gamma P^* + \gamma P^*\right] + P^*(\gamma(1 - R_I) \\ &\quad + \frac{bP^*\gamma}{\overline{R}_f - \underline{R}_f}[C'(\gamma - \beta^* P^*\gamma)\left(G - C(\gamma - \beta^* P^*\gamma) - \beta^* P^*\gamma\right) \\ &\quad + \left(\overline{R}_f - C(\gamma - \beta^* P^*\gamma)\right)\left(C'(\gamma - \beta^* P^*\gamma) - 1\right)] \end{aligned} \tag{52}$$

Where we define:

$$\begin{aligned} A &= b\beta^*\gamma[C'(\gamma - \beta^* P^*\gamma)(C(\gamma - \beta^* P^*\gamma) + \beta^* P^*\gamma) - \left(\overline{R}_f - C(\gamma - \beta^* P^*\gamma)\right)\left(C'(\gamma - \beta^* P^*\gamma) - 1\right) \\ &\quad + C'(\gamma - \beta^* P^*\gamma)G]. \end{aligned} \tag{53}$$

It follows that $\left.\frac{\partial P}{\partial \beta}\right|_{\beta=\beta^*, P=P^*} = 0$ since the right hand side of (52) is the FOC derived in Proposition 1 and must equate to 0 at the optimum, β^*.

Step 3
Substituting $\left.\frac{\partial P}{\partial \beta}\right|_{\beta=\beta^*, P=P^*} = 0$ into (49) yields:

$$0 = \gamma C'(\gamma - \beta^* P^*\gamma)\left(P^*\right)(1 + Z)\int_0^1 dH(\psi), \tag{54}$$

which cannot hold since $\gamma > 0$, $\int_0^1 dH(\psi) > 0$ and $Z > 0$. Therefore, $\beta^{fb} \neq \beta^*$ and $P^{fb} \neq P^*$. To satisfy (49), it must be the case that $\beta^{fb} > \beta^*$. From Lemma 5 we know that $P^{fb} \geq P^*$. However, if $\beta^{fb} > \beta^*$, then $P^{fb} > P^*$. It follows that $\int_0^{C(\gamma-\beta^{fb}P^{fb}\gamma)} f(\theta)d\theta < \int_0^{C(\gamma-\beta^* P^*\gamma)} f(\theta)d\theta$. Therefore,

counterparty risk is strictly smaller in the first best case as compared to the equilibrium case.
It is obvious that if $\beta^* = 1$, it is not possible for the first best to be any less risky. This is the case in which the IFI is already investing everything in the liquid asset.

■

Proof of Lemma 6. Define the payoff for a bank (bk) who contracts with N IFIs. For simplicity, we suppress the loan type as the proof does not depend on it.

$$\begin{aligned}\Pi_{bk} &= \int_0^{\overline{S},\overline{R}} \psi dH(\psi) + (1-\gamma)\int_0^1 \psi dH(\psi) + \gamma \sum_{n=1}^{N}\left(prob(\text{n IFIs do not fail})\frac{n}{N}\right)\int_0^1 dH(\psi) \\ &\quad -\gamma\sum_{n=1}^{N}\left(prob(\text{n IFIs fail})\frac{Zn}{N}\right)\int_0^1 dH(\psi) - \gamma N P_N \end{aligned} \tag{55}$$

Where $prob(\text{n IFIs fail})$ represents the probability that n IFIs fail, $prob(\text{n IFIs do not fail})$ represents the probability that n IFIs do not fail, and $\gamma N P_N$ represents the total premium paid by the bank. For simplicity (and since the IFIs are ex-ante identical) we assume that they each individually receive P_N in exchange for their coverage of $\frac{\gamma}{N}$. We now find $\sum_{n=1}^{N}\left(prob(\text{n IFIs do not fail})\frac{n}{N}\right)$ which represents the expected payment from the IFIs when a claim is made. Expanding this term yields the following.

$$= \sum_{n=0}^{N}\left(\frac{n}{N}\right)\left(\frac{N!}{n!(N-n)!}\right)\left[\left(\int_{C(\frac{\gamma}{N}-\beta P_N\frac{\gamma}{N})}^{\overline{R}_f} dF(\theta)\right)^n\left(1-\int_{C(\frac{\gamma}{N}-\beta P_N\frac{\gamma}{N})}^{\overline{R}_f} dF(\theta)\right)^{N-n}\right] \tag{56}$$

Where β is solved for from the IFIs problem according to Proposition 1. We let $a = \int_{C(\frac{\gamma}{N}-\beta P_N\frac{\gamma}{N})}^{\overline{R}_f} dF(\theta)$ to obtain:

$$\begin{aligned} &= \sum_{n=0}^{N}\left(\frac{n}{N}\right)\left(\frac{N!}{n!(N-n)!}\right)\left[a^n(1-a)^{N-n}\right] && (57)\\ &= \left[a^N + \left(\frac{N-1}{N}\right)\left(\frac{N!}{1!(N-1)!}\right)a^{N-1}(1-a) + ... + a(1-a)^{N-1}\right] && (58)\end{aligned}$$

Where the second equality follows by expanding the summation and reversing the order. We now factor out a and simplify:

$$= a\left[a^{N-1} + \frac{(N-1)!}{1!(N-2)!}a^{N-2}(1-a) + \frac{(N-2)!}{2!(N-3)!}a^{N-3}(1-a)^2 + ... + (1-a)^{N-1}\right] \tag{59}$$

Using a change of variable $M = N - 1$:

$$= \quad a\left[a^M + \frac{M!}{1!(M-1)!}a^{M-1}(1-a) + \frac{(M-1)!}{2!(M-2)!}a^{M-2}(1-a)^2 + ... + (1-a)^M\right] \tag{60}$$

$$= \quad \sum_{m=0}^{M} \frac{M!}{m!(M-m)!}\left(a^{M-m}(1-a)^m\right) \tag{61}$$

$$= \quad a\left[a + (1-a)\right]^M \tag{62}$$

Where the final equality follows from the binomial theorem. Since N is finite, this implies M is finite, and therefore $1^{N-1} = 1$. We obtain:

$$\sum_{n=1}^{N}\left(prob(\text{n IFIs do not fail})\frac{n}{N}\right) \quad = \quad \int_{C(\frac{\gamma}{N}-\beta P_N\frac{\gamma}{N})}^{\overline{R}_f} dF(\theta). \tag{63}$$

By letting $a = \int_{\underline{R}_f}^{C(\frac{\gamma}{N}-\beta P_N\frac{\gamma}{N})} dF(\theta)$, we can repeat the above analysis to show:

$$\sum_{n=1}^{N}\left(prob(\text{n IFIs fail})\frac{n}{N}\right) \quad = \quad \int_{\underline{R}_f}^{C(\frac{\gamma}{N}-\beta P_N\frac{\gamma}{N})} dF(\theta). \tag{64}$$

Substituting (63) and (64) into (55) yields an expression for the expected profit of the bank.

$$\begin{aligned}\Pi_{bk} \quad = \quad & \int_0^{\overline{S},\overline{R}} \psi dH(\psi) + (1-\gamma)\int_0^1 \psi dH(\psi) + \gamma\int_{C(\frac{\gamma}{N}-\beta P_N\frac{\gamma}{N})}^{\overline{R}_f} dF(\theta)\int_0^1 dH(\psi) \\ & -\gamma\int_{\underline{R}_f}^{C(\frac{\gamma}{N}-\beta P_N\frac{\gamma}{N})} ZdF(\theta)\int_0^1 dH(\psi) - \gamma N P_N \end{aligned} \tag{65}$$

We can see that the expected payoff of the bank is given by the expected payoff as if the bank were dealing with only one IFI, but the IFI was making its investment decision with a contract size of $\frac{\gamma}{N}$.

■

Proof of Proposition 4. The proof proceeds in two steps. In step one we show that as γ decreases, the IFI decreases β_N^* (compared to the case of β_1^*). Step two shows that as a result of the decrease of β_N^*, counterparty risk remains unchanged. This proof will follow closely the proof of Lemma 3.

Since counterparty risk is defined as $\int_{\underline{R}_f}^{C(\gamma-\beta P\gamma)} f(\theta)d\theta$, we find the effect that changes in γ have on $C(\gamma - \beta^* P\gamma)$. Since $C(\cdot)$ is monotonic, we focus on $\gamma - \beta^* P\gamma$. This proposition focuses only on the case in which β^* achieves an interior solution.

Step 1

In this step we take the following partial derivative and show that it equates to zero.

$$\frac{\partial\left(\gamma-\beta^{*} P \gamma\right)}{\partial \gamma}=1-\left(\frac{\partial \beta^{*}}{\partial \gamma} P \gamma+\beta^{*} P\right) \tag{66}$$

We now find $\frac{\partial \beta^*}{\partial \gamma} \equiv \left.\frac{\partial \beta}{\partial \gamma}\right|_{\beta=\beta^*}$ (where β^* is defined implicitly in the FOC found in the proof to Proposition 1).

$$\begin{aligned}
0 \;=\; & \left[C'(\gamma-\beta^* P\gamma)\left(1-\beta^* P-\frac{\partial \beta^*}{\partial \gamma}P\gamma\right)\right]\left[C'(\gamma-\beta^* P\gamma)-1\right] \\
& +\left[\overline{R}_f - C(\gamma-\beta^* P\gamma)\right]\left[C''(\gamma-\beta^* P\gamma)\left(1-\beta^* P-\frac{\partial \beta^*}{\partial \gamma}P\gamma\right)\right] \\
& +\left[C''(\gamma-\beta^* P\gamma)\left(1-\beta^* P-\frac{\partial \beta^*}{\partial \gamma}P\gamma\right)\right]\left[G-C(\gamma-\beta^* P\gamma)-\beta^* P\gamma\right] \\
& +C'(\gamma-\beta^* P\gamma)\left[-C'(\gamma-\beta^* P\gamma)\left(1-\beta^* P-\frac{\partial \beta^*}{\partial \gamma}P\gamma\right)-\beta^* P-\frac{\partial \beta^*}{\partial \gamma}P\gamma\right]
\end{aligned} \tag{67}$$

Rearranging for $\frac{\partial \beta^*}{\partial P}$ yields the following.

$$\frac{\partial \beta^*}{\partial P} P\gamma A=(1-\beta^* P) A \tag{68}$$

Where we define:

$$\begin{aligned}
A \;=\; & C''(\gamma-\beta^* P\gamma)\left(G-C(\gamma-\beta^* P\gamma)-\beta^* P\gamma\right)+\left(\overline{R}_f-C(\gamma-\beta^* P\gamma)\right) C''(\gamma-\beta^* P\gamma) \\
& -2C'(\gamma-\beta^* P\gamma)\left(C'(\gamma-\beta^* P\gamma)-1\right).
\end{aligned} \tag{69}$$

Note that $A<0$ from the assumption on the SOC to ensure a maximum. Therefore, $\frac{\partial \beta^*}{\partial \gamma}=\frac{1-\beta^* P}{P\gamma} \geq 0$. This implies that as γ decreases (N increases), β_N^* decreases as desired.

Step 2

Substituting (68) into (66) yields the following.

$$\begin{aligned}
\frac{\partial\left(\gamma-\beta^* P\gamma\right)}{\partial P} &= 1-P\gamma\frac{(1-\beta^* P)}{P\gamma}-\beta^* P \\
&= 0
\end{aligned} \tag{70}$$

By Lemma 6, we can view the counterparty risk as the probability that one IFI is insolvent when a claim is made (given its investment choice is solved for with a contract size of $\frac{\gamma}{N}$). Since this probability does not change in the case when $N>1$ from $N=1$, it follows that counterparty risk remains unchanged.

■

Proof of Lemma 8. Optimizing Π_{IFI}^{MB} choosing β yields the following first order condition (recall

F is assumed to be uniformly distributed):

$$
\begin{aligned}
0 \quad = \quad & \frac{1}{\overline{R}_f - \underline{R}_f} \int_0^{\beta^* PM} \left(-PM\gamma(1-R_I)\right) G\partial b(\xi) + \left(-PM\gamma(\beta^* + (1-\beta^*)R_I) + \beta^* PM\gamma - \underline{R}_f\right) GPM \\
& + \frac{1}{\overline{R}_f - \underline{R}_f} \int_{\beta^* PM}^{M} [-C'(\xi\gamma - \beta^* PM\gamma)(-PM\gamma)\left(-C(\xi\gamma - \beta^* PM\gamma) - \beta^* PM\gamma\right) \\
& + \left(\overline{R}_f - C(\xi\gamma - \beta^* PM\gamma)\right)\left(-C'(\xi\gamma - \beta^* PM\gamma)(-PM\gamma) - PM\gamma\right) + C'(\xi\gamma - \beta^* PM\gamma)(-PM\gamma)(-G)] \\
& - \frac{1}{\overline{R}_f - \underline{R}_f} \left[\left(\overline{R}_f - C(0)\right)\left(-C(0) - \beta^* PM\gamma\right) + \left(C(0) - \underline{R}_f\right)(-G)\right] PM + (1-R_I)PM\gamma \qquad (71)
\end{aligned}
$$

Recalling $C(0) = 0$ we simplify the above.

$$
\begin{aligned}
0 \quad = \quad & -\int_0^{\beta^* PM} \gamma(R_I - 1)Gdb(\xi) - PM\gamma(1-\beta^*)R_I G \\
& + \gamma \int_{\beta^* PM}^{M} [C'(\xi\gamma - \beta^* PM\gamma)\left(G - C(\xi\gamma - \beta^* PM\gamma) - \beta^* PM\gamma\right) \\
& + \left(\overline{R}_f - C(\xi\gamma - \beta^* PM\gamma)\right)\left(C'(\xi\gamma - \beta^* PM\gamma) - 1\right)] \\
& + \overline{R}_f \beta^* \gamma - \underline{R}_f G - \gamma(R_I - 1) \qquad (72)
\end{aligned}
$$

The SOC implies that the right hand side of (72) is decreasing in β^* so that our problem achieves a maximum. Define two belief distributions $b_1(\xi)$ and $b_2(\xi)$ such that $b_1(\xi) \geq b_2(\xi)\ \forall \xi$. As well, let $(\beta_1^*, b_1(\xi))$ solve the first order condition (71). Intuitively, moving from $b_1(\xi)$ to $b_2(\xi)$, mass shifts from the interval $[0, \beta^* PM]$ to $[\beta^* PM, M]$. Formally:

$$
\int_0^{\beta^* PM} db_1(\xi) \quad > \quad \int_0^{\beta^* PM} db_2(\xi) \qquad (73)
$$

$$
\int_{\beta^* PM}^{M} db_1(\xi) \quad < \quad \int_{\beta^* PM}^{M} db_2(\xi). \qquad (74)
$$

Since it is assumed that the FOC holds with $(\beta_1^*, b_1(\xi))$, given (73) and (74) that with $(\beta_1^*, b_2(\xi))$, it follows that β_1^* must increase for (72) to hold. In other words, the riskier the distribution of loans that the IFI insures, the more that it invests in the liquid asset.

To proceed we use a similar result to that of Lemma 2. It is straight forward to see that when the beliefs of default are higher (as in the risky case), so must the price of the contracts be higher (this can be shown in the same way that Lemma 2 was proved by showing that a net profit function is decreasing in the amount of risk in the loans). Next we find what happens to counterparty risk. What is different about the case of multiple banks is that the counterparty risk is defined relative to the number of banks that default: $\int_{\beta PM}^{M} \int_{\underline{R}_f}^{C(\xi\gamma - \beta PM\gamma)} dF(\theta)db(\xi)$.

In the case where the IFI puts more weight on the loans being risky ($p_A = r$), β^* and P^* increase, so that $C(\gamma - \beta P\gamma)$ decreases. Furthermore, since from the point of view of a bank the probability of a claim does not change, counterparty risk decreases as compared to when the IFI puts more weight on the loans being safe ($p_A = s$).

■

Proof of Proposition 5. The proof proceeds in 3 steps. Steps 1 and 2 determine when the pooling equilibria cannot exist. In particular, we use beliefs of the IFI for which banks have the greatest incentive to pool. In step 1 we assume that all banks report that they are in the safe industry ($p_A = s$) and find a condition wherein at least one bank who received the aggregate shock $p_A = r$ wishes to reveal it truthfully. In the second step we assume that all banks are reporting that they are in the risky industry ($p_A = r$) and find a condition wherein at least one bank who received the aggregate shock $p_A = s$ wishes to reveal it truthfully.[26] Step 3 determines when a unique separating equilibrium can exist. We use beliefs such that the banks have the least incentive to separate. In this step we assume banks are separating and find the condition wherein both bank types do not wish to deviate and be revealed as the other.

Step 1

Consider all banks reporting that they are safe ($p_A = s$), regardless of the aggregate shock. Now consider the incentive of banks who have received the aggregate shock $p_A = r$. Given that all banks are reporting that they are safe, we need to find one bank who wishes to send the message that they are risky ($p_A = r$). If every bank reports that it is in the safe industry, the IFI does not update its beliefs. However, if at least one bank deviates and says that it in the risky industry, then all banks are believed risky, with the deviating bank(s) believed to have received highest idiosyncratic shock ($\xi = M$) (or, if there is a measure of deviating banks, the highest measure of the idiosyncratic shock). We know that the bank with the greatest incentive to be revealed as risky is the one with the highest idiosyncratic shock, which we denote as bank M. Denote the probability of default of the loan of this bank as p_M^r and the individual (total) pooling price as $P_M^{\frac{1}{2}}$ ($P^{\frac{1}{2}}$).[27] Let the individual separating price for a risky bank with the highest idiosyncratic shock be P_M^r, and let the total price be P^r. Next, we denote the optimal investment choice of the IFI in the pooling (separating) case by $\beta^{1/2}$ (β^r). Finally, we will let $D^{1/2}$ (D^r) represent the probability that upon a claim being made in the pooling (separating) case, the IFI is insolvent and cannot pay. It follows that $D^{1/2} = \int_{\beta^{1/2}P^{1/2}M}^{M} \int_{\underline{R}_f}^{C(\xi\gamma - \beta^{1/2}P^{1/2}M\gamma)} dF(\theta)db(\xi)$ and $D^R = \int_{\beta^r P^r M}^{M} \int_{\underline{R}_f}^{C(\xi\gamma - \beta^r P^r M\gamma)} dF(\theta)db(\xi)$. The condition under which this bank has the incentive to reveal its type truthfully can be written as follows.

$$(1 - p_M^r)R_B + p_M^r\gamma(1 - D^r) - p_M^r\gamma D^r Z - \gamma P_M^r \geq (1 - p_M^r)R_B + p_M^r\gamma(1 - D^{1/2}) - p_M^r\gamma D^{1/2}Z - \gamma P_M^{1/2}$$
$$\Rightarrow p_M^r\left(D^{1/2} - D^r\right)(1 + Z) \geq P_M^r - P_M^{1/2} \quad (75)$$

Step 2

Consider all banks reporting that they are risky ($p_A = r$), regardless of the aggregate shock. Now consider the incentive of banks who receive the aggregate shock $p_A = s$. We find the condition under which a bank would like to reveal that it is safe ($p_A = s$). Let the beliefs of the IFI be

[26] Note that there are other pooling equilibria in which some banks report differently than others. These can arise when the IFI's beliefs are such that no new information is gleaned from the reports. Since these equilibria yield the same outcome, we will focus only on the cases described.

[27] Note that the total price is the per unit price that the IFI receives, while the individual price is the per unit price that a bank pays.

that if all banks report that they are risky, with at least one reporting that it is safe, then the banks are safe, with the deviating banks believed to have received the lowest idiosyncratic shock ($\xi = 0$) (again, if there is a measure of deviating banks, than they receive the lowest measure of the idiosyncratic shocks). We know that the bank with the greatest incentive to be revealed as safe is the one with the lowest idiosyncratic shock, call it bank 0. Denote the probability of default for the loan of this bank as p_0^s and the individual price if they reveal themselves as safe as P_M^s.[28] We let the total price be P^s. Finally, let β^s represent the optimal investment choice of the IFI if bank 0 reports that it is safe. It follows that counterparty risk in this case can be defined by: $D^s = \int_{\beta^s P^s M}^{M} \int_{\underline{R}_f}^{C(\xi\gamma - \beta^s P^s M\gamma)} dF(\theta)db(\xi)$. The condition under which this bank has the incentive to reveal its type truthfully can be written as follows.

$$
\begin{aligned}
(1-p_0^s)R_B + p_0^s\gamma(1-D^s) - p_0^s\gamma D^s Z - \gamma P_M^s &\geq (1-p_0^s)R_B + p_0^s\gamma(1-D^{1/2}) - p_0^s\gamma D^{1/2}Z - \gamma P_0^{1/2} \\
&\Rightarrow p_0^s\left(D^s - D^{1/2}\right)(1+Z) \leq P_0^{1/2} - P_M^s \qquad (76)
\end{aligned}
$$

From Lemma 9, we know that in the (aggregate) pooling case, there can be no separating equilibrium in the idiosyncratic shock so that $P_0^{1/2} = P_M^{1/2} = P^{1/2}$. Therefore (75) and (76) are simultaneously satisfied when:

$$
\frac{P^{1/2} - P_M^s}{p_0^s(D^s - D^{1/2})} \geq 1 + Z \geq \frac{P_M^r - P^{1/2}}{p_M^r(D^{1/2} - D^r)}. \tag{77}
$$

To see that (77) can hold, take the limit as p_0^s approaches zero and set Z sufficiently high.

Step 3

We now find the conditions under which the separating equilibrium exists. Consider the case where each bank is revealing its aggregate shock.[29] Let the beliefs of the IFI be as follows: if there is at least one bank reporting that they are in the safe industry, with the rest reporting risky, then everyone is believed to be in the safe industry. Because we are trying to show that a separating equilibrium can exist, we take the simple case in which the resulting price for the deviating bank(s) is P^s. Since there can be no separating equilibrium in the idiosyncratic shock, we let $P_0^r = P^r$. Next, if at least one bank reports that it is in the risky industry while the rest report that they are safe, then everyone is believed risky, with the deviating bank(s) receiving the price P^r. Again, since there can be no separation in the idiosyncratic shock, let $P_M^s = p^s$. The two cases are analyzed below.

[28] Note that the price corresponds to the highest idiosyncratic shock because we find the condition under which a bank is least likely to default.

[29] Note that there are other separating equilibria where some report their type truthfully, while others do not. For simplicity we will not consider these here.

$$(1-p_0^r)R_B + p_0^r\gamma(1-D^r) - p_0^r\gamma D^r Z - \gamma P^r \geq (1-p_0^r)R_B + p_0^r\gamma(1-D^s) - p_0^r\gamma D^s Z - \gamma P^s$$
$$\Rightarrow p_0^r\left(D^s - D^r\right)(1+Z) \geq P^r - P^s \quad (78)$$

Turning to the second case, we derive the following condition.

$$(1-p_M^s)R_B + p_M^s\gamma(1-D^s) - p_M^s\gamma D^s Z - \gamma P^s \geq (1-p_M^s)R_B + p_M^s\gamma(1-D^r) - p_M^s\gamma D^r Z - \gamma P^r$$
$$\Rightarrow P^r - P^s \geq p_M^s\left(D^s - D^r\right)(1+Z) \quad (79)$$

It follows that (78) and (79) are simultaneously satisfied when:

$$\frac{P_0^r - P_M^s}{p_M^s\left(D^s - D^r\right)} \geq (1+Z) \geq \frac{P_0^r - P_0^s}{p_0^r\left(D^s - D^r\right)} \quad (80)$$

Since $p_0^r > p_M^s$, these inequalities can be satisfied by choosing Z appropriately. It follows that the separating equilibrium exists and is unique when both (77) and (80) are satisfied. To see that this is possible, consider p_M^s and p_0^s sufficiently small so that for Z sufficiently large both the right hand side of (77) and (80) are satisfied.

■

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