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Declining Exhaustible Resource Rent with Small, Distinct Extractive Firms

John Hartwick
Queen's University

Andrei Bazhanov
Far Eastern National University, Vladivostok

Zhen Song
Central University of Finance and Economics, Beijing

Department of Economics
Queen's University
94 University Avenue
Kingston, Ontario, Canada
K7L 3N6

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Declining Exhaustible Resource Rent with Small, Distinct Extractive Firms

Andrei Bazhanov, John Hartwick and Zhen Song*

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Abstract

We consider a competitive extraction industry comprising many small firms, each with a slightly different quality of mineral holdings. With "rapidly" declining quality of holding per firm we observe rent declining over and interval. We do not work with the planning solution, commonly invoked in the study of firms with distinct qualities of stock.

- Keywords: exhaustible resources; resource rent; competitive extraction
- Journal classification: Q31, D41

1 Introduction

An issue of long-standing in oil-extraction economics is extending the Hotelling [1931] case of the competitive industry with identical small firms to one with small firms with distinct qualities of respective small holdings of stock. The classic contribution is Levhari and Leviatan [1977] and a recent investigation is Livernois and Martin [2001]. Here we draw back from the matter of formulating the problem as one in dynamic optimization at the industry level and focus instead on the local-in-time arbitrage condition for a single competitive extractive firm, a firm in a competitive industry. We then move up from zero profit arbitrage for a competitive firm to the long run equilibrium for the extractive industry, the collectivity comprising many firms each with its very small holding of stock with a distinct quality (and in consequence with a distinct extraction cost). As time passes, we observe market rent rising at a rate less than the rate

*Far Eastern National University, Vladivostok, Queen's University, Kingston, Ontario and Central University of Finance and Economics, Beijing

of interest. A special case of our illustrative example is the classic Hotelling outcome, market rent rising at $r\%$. This corresponds to each firm having the same quality of stock.

With quality declining quite rapidly, we observe that market rent declines over time, an outcome ruled out in the analysis of Livernois and Martin [2001]. Livernois and Martin follow tradition and work with an INDUSTRY extraction cost function and a PLANNER maximizing discounted surpluses. They sidestep the issue of defining firms in their analysis. We break with tradition and focus first on an extractive firm with zero profit from intertemporal arbitrage and then "work up" to the industry "equilibrium" at each moment. The marginal firm at each date in our analysis is indifferent between extracting its small holding of the stock currently or a moment later. In our view to extend Hotelling [1931] to the case of heterogeneous stock, one must not only focus on the competitive firm, first off, but one must also pay close attention to the implicit model of the "geology" of the stock that one is working with. Our formulation makes sense when total stock is located in a very long and thin pipe extending down into the earth's crust. The stock may be of uniform intrinsic quality but each unit is more costly to extract as one moves deeper into the stock. We associate a competitive firm with the holding of one unit of the stock. As time passes, firms located further down in the pipe extract their units and place them on the market.

We are dealing with a concept of industry equilibrium, one built up directly from extractive frims, different from that in Livernois and Martin and many other contributors. Hence when our result contradicts the central result of Livernois and Martin we are not indicating that the details of their analysis are faulty, rather we suggest that the concept of industry equilibrium which they work with, one "embedded in" a planning framework, may not be good at capturing the problem of extraction with many competitive firms, each with a distinct small holding of the total stock. We return to the matter of the appropriate model below.

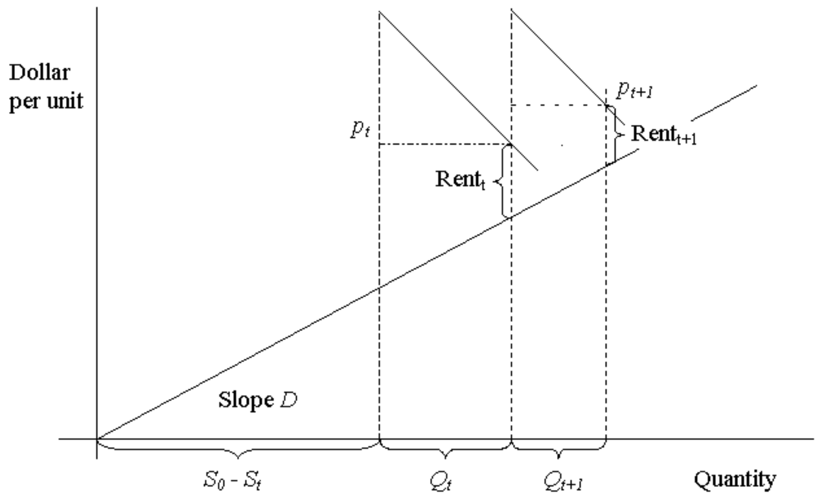

Figure 1: $Q(t)$ in period t and $Q(t+1)$ in period t+1, with the marginal firm in period t being the intra marginal firm in period t+1. For the marginal firm to be indifferent between extracting between periods, its profit on its ton must differ between periods by $r\%$. Hence $p(t+1) - C(Q(t), S(t)) = (1+r)[p(t) - C(Q(t), S(t))]$.

2 The Analysis

We focus first on the condition for zero profit arbitrage for the marginal firm in period t. Our set up involves each firm with a distinct extraction cost and each firm owning a very small amount of oil, say 1 ton for concreteness. In our industry equilibrium there will be Q(t) firms extracting at date t and the marginal firm, will have extraction costs for its ton as $C(Q(t), S(t))$. Each other firm extracting at date t will have extraction costs lower than $C(Q(t), S(t))$. Hence the marginal firm will earn a "Hotelling" rent on its unit extracted whereas each intramarginal firm which is active at the same instant will earn the same "Hotelling" rent plus some "quality" (Ricardian) rent. In Figure 1 we illustrate extraction of $Q(t)$ firms at date t and $Q(t+1)$ firms t+1 in a discrete time "approximation".

In Figure 1, we have the arbitrage condition for the marginal firm in period t, in

$$p(t+1) - C(Q(t), S(t)) = (1+r)[p(t) - C(Q(t), S(t))].$$

In continuous time this condition is taken as

$$\dot{p}(t) = [p(t) - C(Q(t), S(t))]r.$$

We turn to an example in order to illustrate industry equilibrium and rent declining over time. We take $C(Q(t), S(t))$ as $[S_0 - S(t) + Q(t)]D$ for D a positive constant and S_0 the initial stock in the deposit being "shared by" the S_0 firms. We assume that the market (industry) demand schedule is linear as in $p(t) = A - BQ(t)$ for A and B positive constants. (For the special case of $D = 0$ we have Hotelling's original competitive industry with his linear demand schedule.) Then industry equilibrium involves solving

$$\begin{aligned} -B\dot{Q}(t) &= [A - BQ(t) - [S_0 - S(t) + Q(t)]D]r \\ \text{and } \dot{S}(t) &= -Q(t). \end{aligned}$$

Note that the first equation above indicates that $-\dot{Q}(t)$ is proportional to current rent, the factor of proportionality being r/B. Thus if rent is declining, $-\dot{Q}(t)$ will be declining. In textbook renderings of Hotelling's competitive industry model, $-\dot{Q}(t)$ is usually indicated to be increasing with time and we observe this behavior for the case of D near zero. The "geology" of this example was sketched in the introduction. One thinks of S_0 firms each owning one unit of homogeneous stock, with each unit stacked on others as with a straight pipe moving down toward the center of the earth. The pipe would have depth S_0. "Early" firms face lower extraction costs because their holding is less deep than "later" firms. The cost of extraction per firm increases in a linear fashion as "the process" moves deeper as time passes.

The above pair of equations becomes the second order linear equation in $S(t)$:

$$\frac{d\dot{S}}{dt} + a_1\dot{S} + a_2 S = R_0. \tag{1}$$

for $a_1 = -\left[\frac{B+D}{B}\right] r$, $a_2 = -\left[\frac{Dr}{B}\right]$ and $R_0 = \left[\frac{A - S_0 D}{B}\right] r$. $A - S_0 D$ is rent corresponding to $Q(T) = 0$ and $S(T) = 0$ and must be non-negative. We turn to solve equation (1) in S. Note that the limits of the model require that the values of parameters A, S_0, D and r are such that $R_0 > 0$. Otherwise, for $R_0 = 0$ we have trivial solution $S(t) \equiv 0$ and for $R_0 < 0$ we have negative $S(t)$ for all $t \in (0, T)$.

In the simplest case, the boundary conditions will be $S(T) = 0$ and $\dot{S}(T) = -Q(T) = 0$. Hence the solution involves obtaining the $Q(0)$ which results in the boundary condition being satisfied at end date T.[1] In our case the solution of (1) is

$$S(t) = C_1 e^{\lambda_1 t} + C_2 e^{\lambda_2 t} + F$$

because the determinant of the characteristic equation $\Delta = a_1^2 - 4a_2$ for the corresponding homogeneous equation is always positive in our problem. We have $\lambda_1 = (-a_1 + \sqrt{\Delta})/2$ and $\lambda_2 = (-a_1 - \sqrt{\Delta})/2$ - the roots of the characteristic equation and $F = R_0/a_2$ is a particular solution of the nonhomogeneous equation (1). Using terminal condition $\dot{S}(T) = 0$ we have $C_1 = -C_2 \lambda_2 e^{T(\lambda_2 - \lambda_1)}/\lambda_1$. Substituting it into (1) and using the initial condition $S(0) = S_0 = C_2 \left[1 - \lambda_2 e^{T(\lambda_2 - \lambda_1)}/\lambda_1\right] + R_0/a_2$ we obtain

$$C_2 = \frac{S_0 - R_0/a_2}{1 - \frac{\lambda_2}{\lambda_1} e^{T(\lambda_2 - \lambda_1)}} \text{ and } C_1 = -\frac{S_0 - R_0/a_2}{\frac{\lambda_1}{\lambda_2} e^{T(\lambda_1 - \lambda_2)} - 1}.$$

Then we can use the second terminal condition $S(T) = 0$ in order to define T :

$$C_2(T) e^{\lambda_2 T} + C_1(T) e^{\lambda_1 T} + R_0/a_2 = 0$$

or

$$\frac{S_0 - R_0/a_2}{1 - \frac{\lambda_2}{\lambda_1} e^{T(\lambda_2 - \lambda_1)}} e^{\lambda_2 T} - \frac{S_0 - R_0/a_2}{\frac{\lambda_1}{\lambda_2} e^{T(\lambda_1 - \lambda_2)} - 1} e^{\lambda_1 T} + \frac{R_0}{a_2} = 0$$

which after transformations can be written as follows

$$e^{-\lambda_2 T} = R_1 + \frac{\lambda_2}{\lambda_1} e^{-\lambda_1 T} \tag{2}$$

where $R_1 = (1 - a_2 S_0/R_0)(1 - \lambda_2/\lambda_1)$. Nonlinear equation (2) in T has a unique positive solution (Fig. 2) because $\lambda_2 < 0, a_2 < 0$ which follows $R_1 > 1$. This implies that we have in the left hand side an exponentially increasing function starting from unity and in the right hand side an increasing function starting from the value which is greater than unity but less than R_1 and asymptotically approaching R_1.

[1] A more complicated case will have price rise relatively rapidly and some high cost firms will choose not to extract because their profit would be negative.

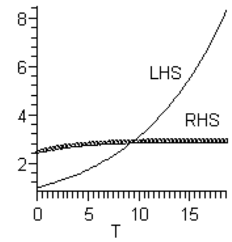

Figure 2: Solution of equation in T.

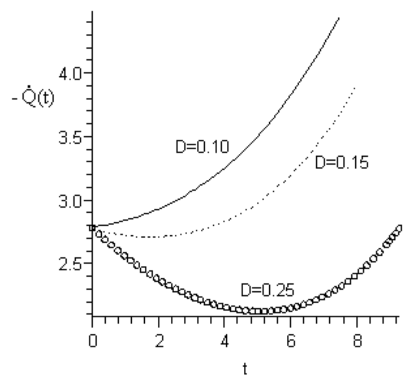

Figure 3: Acceleration of extraction $-\dot{Q}(t)$ for $D = 0.1$ (solid line), $D = 0.15$ (dotted line), and $D = 0.25$ (in circles).

We report on numerical example with $r = 0.1, A = 50$, and $B = 0.9$ that growth in D follows the inverted hump-shaped accelerations $-\dot{Q}(t)$ (Fig. 3)[2]. This indeed implies the decreasing pattern of rent in a neighborhood of initial point $t = 0$ for $D = 0.25$ (Fig. 4).

The corresponding behavior of extraction $Q(t)$ and price $p(t)$ are depicted on figures 5 and 6.

[2]For the values of $D = 0.1$, $D = 0.15$, and $D = 0.25$ equation (2) implies $T_{0.1} = 7.45$, $T_{0.15} = 7.94$, and $T_{0.25} = 9.28$.

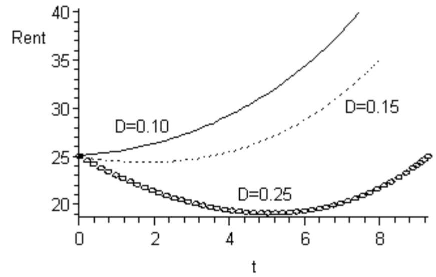

Figure 4: Rent function for $D = 0.1$ (solid line), $D = 0.15$ (dotted line), and $D = 0.25$ (in circles).

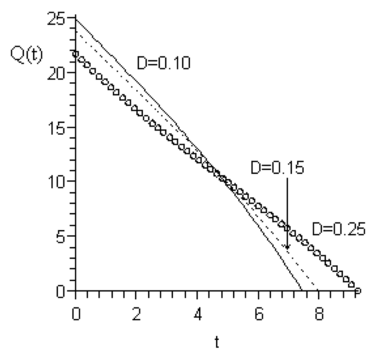

Figure 5: Extractions.

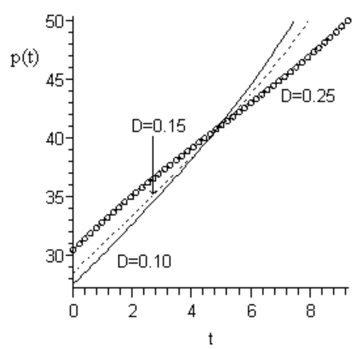

Figure 6: Price.

3 Planning Solution versus Market Solution

The textbook planning solution (Levhari and Leviatan [1975] and Livernois and Martin [2001]) works with an extraction cost function defined on current industry output. The planner has of course perfect foresight and is able to vary current industry output in order to "smooth" extraction costs over periods by varying industry output across periods with a view to maximizing the present value of surpluses. In a sense such planning solutions involve both meeting current demands for extracted output and arranging outputs across periods in such a way that aggregate costs are "minimized" over many periods. Our market solution involves no such long run "optimization" of extraction costs. Our marginal firm at any date simply responds to zero profit arbitrage across "periods" and industry costs evolve without the guidance of a planner. Our "decentralized" solution and the planning solution that is well-known are inherently different formulations of the problem of extraction with steadily rising unit extraction costs. That the planning solution was difficult to represent as a decentralized solution was pointed out by Cummings [1969]. We have not seen what might be referred to as "the Cummings problem" with a heterogeneous stock addressed by the contributors in the Levhari-Leviatan tradition. We are not proposing a general treatment of the Cummings problem. Rather we set out a plausible market outcome in an example with heterogeneous stock and observe that rising rent is not the inevitable outcome.

4 Concluding Remark

We have constructed an example of extraction by many firms with a small holding of oil, each holding distinct, which has Hotelling [1931] as a special case. The solved example fails to exhibit steadily increasing rent over time, the outcome that Livernois and Martin [2001] established as the inevitable outcome in their model, a model in the tradition of Levhari and Leviatan [1977]. Our aim is to establish that in a plausible model of competitive extraction, heterogeneity of stock can lead in an unforced way to periods of declining rent. This we contend is a natural property of an extended Hotelling model, one that a person might well observe in the data on extraction. We contend that the heterogeneous stock models in the Levhari-Leviatan tradition possess the problem of being difficult to translate into market outcomes with many competitive extractive firms (a version of what we refer to as the Cummings problem). Hence our approach above, perhaps imperfect, based on the marginal extractive firm at each date rather than on the industry at each date.

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