Coordination Failure in Technological Progress, Economic Growth and Volatility

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Abstract

Technological progress has long been posited to be crucial in a country’s economic growth. This paper argues that coordination failure in a country’s new technology investment can be one of the barriers in a country’s capital accumulation and economic growth. The global game established by Morris and Shin(2000) is extended to a two-sector Overlapping Generation model where capital goods can be produced by two different technologies. The first is a conventional technology with constant returns, which are perfectly revealed to economic agents. The second is a new technology exhibiting increasing return to scale due to technological externalities, whose returns economic agents only have incomplete information about. Economic agents have to choose which technology to invest. My model reveals that under certain circumstances coordination failure in the capital good sector will occur and be manifested as the under-investment in the new technology. In this way, I explain how coordination failure in a country’s technology updating process leads to slower capital accumulation and economic growth. More interestingly, the model generates a positive correlation between economic growth and volatility through a new channel associated with coordination failure. Policy implications are discussed as well.

Keywords: Economic Growth, Technological externalities, Coordination Failure

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1 Introduction

Technological progress has long been posited to play a key role in economic growth by economists such as Schumpeter dating back to the 1950’s. In the 1990’s, Romer, Aghion, Howitt, as well as others, formalized this idea by introducing it into the mainstream neoclassical growth models and establishing the so-called endogenous growth theory.

Despite the importance of technological progress in economic growth, it is a topic difficult for formal economic modeling. As Dowrick (1995) summarizes, new technology in general exhibits three characteristics which distinguish it from other ordinary goods: first, new technology contains great uncertainty due to its new and unknown nature. Second, technological externalities exist, leading to investment complementarities in new technology investment. Third, information asymmetry exists. Producers of new products have more information than their creditors or users. All these characteristics are difficult to be modeled by standard economic tools.

Global games, first studied by Carlsson and van Damme (1993), turn out to be a useful tool for the study of new technology. Global games model the situations where both uncertainty and strategic complementarities are present. By doing so, they capture two main characteristics in new technology investment: uncertainty and technological externalities. Hence, global games allow us to formally model new technology investment with these two features to gain valuable insights and policy implications.

This paper introduces new technology investment modeled by global games into an Overlapping Generation economy with two sectors: the consumption goods sector and capital goods sector. While consumption goods are assumed to be produced by an exogenously given time-invariant technology, I assume that capital goods can be produced by two different technologies in each period. The first is a conventional one with constant returns that are perfectly revealed to economic agents. The other is a new one exhibiting increasing returns to scale due to technological externalities. In addition, economic agents have only incomplete information about the returns of the new technology due to its unknown nature. The returns of the new technology in
each period are determined by two parts: the first part is a technology shock, which is random and i.i.d over time. This part is called economic fundamental, because it purely depends on the nature of the technology. The second part is the proportion of economic agents investing in the new technology. The returns are higher with a larger proportion of firms investing. Thus, the second part introduces investment complementarities due to technological externalities. Furthermore, economic agents only have incomplete information about the economic fundamental of the new technology. More specifically, they have a prior belief and receive a noisy private signal about the technology shock before they make their investment decisions. This assumption captures the feature of uncertainty in new technology investment.

The above assumptions are a simple way to model technological progress in an economy. I focus on technological progress in the capital goods sector, because it has more scope for technological innovation than the consumption goods sector. By assuming that in each period capital goods can be produced by both conventional and new technologies, I model the dynamic technological updating process in a simplified way. By assuming that the new technology exhibits externalities and uncertainty, I capture the two main features of a new technology. Technological externalities play an important role in the endogenous growth theory. The theoretical literature emphasizing technological externalities and their implications to economic growth includes Romer (1990), Dowrick (1995), and Aghion and Howitt (1998). Meanwhile, empirical work studying technological externalities is abundant. As Dowrick (1995) surveys, this work reinforces the prevalent view that technological externalities are significant. Moreover, the huge uncertainty in new technology investment is self-evident due to its new and unknown nature: the whole process from inventing a new technology to putting it into production is long and unpredictable.

This paper does not intend to model the mechanism through which technological externalities originate. Instead, I use the reduced form to assume their existence, and focus on the implications that they have to capital accumulation and economic growth. Due to technological externalities, economic agents face two kinds of uncertainties about the returns of new technology investment: first, they have incomplete information about the technology shock; second, they are uncertain about the
actions of other agents. My model reveals that the two uncertainties will lead to inefficiency in equilibrium. Coordination failure can occur, which is manifested as under-investment in the new technology. In such a framework, I study when and how coordination failure leads to slower capital accumulation and economic growth. More interestingly, the model generates a positive correlation between economic growth and volatility through a very peculiar mechanism associated with coordination failure. In my model, more investment in the new technology can alleviate coordination failure and lead to higher economic growth. Meanwhile, the new technology is risker by nature and more investment on it leads to more volatility as well. Policy implications are examined as well.

The rest of the paper is organized as follows: Section 2 gives a survey on related literature. A two-sector Overlapping Generation model is established in Section 3 and its equilibrium is characterized. Section 4 analyzes when and how coordination failure leads to slower economic growth. The relationship between economic volatility and growth is also examined. Numerical simulation is given in Section 5. Section 6 explores policy implications, followed by conclusions in Section 7.

2 Literature Survey

This paper is related to three strands of economic literature. The first strand is on global games and their applications to macroeconomics. The second strand is on traditional coordination games with perfect information and their applications to macroeconomics. The third strand is on the endogenous growth theory, which emphasizes the importance of technological progress in economic growth.

Global games were first introduced by Carlsson and van Damme (1993). They incorporate incomplete information into a traditional coordination game with perfect information. In the game each player observes his payoffs with some noise. By iterated elimination of strictly dominated strategies, they prove that when the noise gets infinitely small, there is a unique equilibrium in the game.

Morris and Shin (1998) study currency attacks in a global game setup. They find that when speculators need to coordinate their actions to successfully attack a fixed
exchange rate regime, and meanwhile are only able to observe economic fundamentals with some small noise, there is a unique equilibrium in the game, determined by both economic fundamentals and the beliefs of speculators. This result differs from that of a traditional coordination game with perfect information, where a currency attack is solely determined by the self-fulfilling beliefs of speculators. Successfully overcoming the problem of indeterminacy of multiple equilibria models, their model allows for the analysis of policy implications.

Morris and Shin (2000) summarize the applications of global games in macroeconomic modeling by explaining how global games can be used in the context of bank runs, currency crises, and debt pricing. They argue that global games are a useful approach for the analysis of many macroeconomic issues where players’ payoffs are interdependent. They reckon that global games provide a more solid ground for policy analysis than multiple equilibria models due to their property of unique equilibrium.

How public information influences equilibrium allocation and social welfare in economies with investment complementarities is studied by Angeletos and Pavan (2004). They demonstrate that when coordination is socially desirable, an increase in the precision of public information will always increase social welfare, given that the complementarities are weak so that the equilibrium is unique. When the complementarities are strong, however, so that multiple equilibria are possible, the increase in public information may facilitate the coordination on both “bad” and “good” equilibria.

Finally, Chamley (2004) gives a survey on coordination games and global games. A detailed summary about the theory and applications of global games is also given by Morris and Shin (2003).

The second strand of literature is on macroeconomic complementarities and their implications for the economy. Bryant (1983) uses a special form of production function to study how technological complementarities generate Pareto-ranked multiple equilibria. The business cycle implications of technology complementarities is explored by Baxter and King (1991) in a model whose structure is similar to a standard real business cycle model. In their quantitative work, business cycles are generated by a demand shock and propagated through technological complementarities is quan-
Diamond (1982) studies how trading externalities cause "thick market" effects in the presence of trading frictions. He finds that the return of an individual economic agent will be higher due to reduced search costs if more agents are in the market searching for trading partners.

Cooper (1999) comprehensively surveys macroeconomic complementarities and their implications for macroeconomic behavior. He examines a variety of sources of macroeconomic complementarities, such as technological complementarities, demand spillover effects and trading externalities, and studies their implications.

The third strand of literature is about endogenous economic growth. This literature combines the idea of Schumpeter (1950) that technological progress is crucial in economic growth with standard neoclassical growth models, and studies how technological progress influences a country’s economic growth and consequently its policy implications.

Romer (1990) emphasizes the importance of non-rivalry of technology as a main source both of growth and of potential market failure. He argues that the non-rivalry feature of a technology and the consequent increasing returns to scale in the sector that uses the technology make long-run growth possible. Meanwhile, due to non-rivalry of technology, private motives for investing in new technology are usually sub-optimal. Thus an important policy implication is that governments should take steps to stimulate more new technology investment.

Aghion and Howitt (1998) provides a comprehensive survey on the endogenous growth theory literature. They examine how technological progress, influenced by a variety of factors such as organizations, institutions, market structure, market imperfections, trade, government policy and the legal framework, affects long-term economic growth.
3 The Model

3.1 Environment

The framework of this model follows the two-sector Overlapping Generation model established by Ennis and Keister (2003), who explore the impact of bank runs on capital stock and output. As in their model, I assume that the consumption goods sector exhibits constant capital returns. My model differs from theirs in the capital goods sector, where a global game is applied to the study of new technology investment.

This is a standard overlapping generation model with infinite time horizon, where each generation lives for two periods. There is also an initial old generation endowed with capital \( k_0 \) at the beginning of time.

At the beginning of each period \( t \), a new generation of a continuum of agents with mass 1, denoted by generation \( t \), is born. Each agent is endowed with 1 unit of labor when young, and nothing when old. Labor is supplied inelastically.

Capital goods are produced as follows. At the end of time \( t \), young agents of generation \( t \) with wage income can choose from two ways of investing to produce capital goods. One is to invest in a conventional technology, which transforms 1 unit of consumption goods at the end of time \( t \) into \( R^c_t = r \) units of capital goods at the beginning of time \( t+1 \). Here \( r \) is an exogenously given constant. The other is to invest in a new technology. The return on the new technology, \( R^n_t \), is determined by two factors: the technology shock denoted by \( \theta_t \) (also called the economic fundamental of the technology) and the proportion of the agents who invest in the new technology, \( 1 - \lambda_t \). So \( \lambda_t \) is the proportion of the agents who invest in the conventional technology. If a young agent invests one unit of the consumption goods in the new technology at the end of time \( t \), he will get \( R^n_t = e^{\theta_t - \lambda_t} \) units of capital at the beginning of time \( t + 1 \).

The technology shock \( \theta_t \) is i.i.d over time and is normally distributed with mean \( \bar{\theta} \) and precision \( \alpha \).

In period \( t \), after \( \theta_t \) is realized, it is observed with noise by the young agents. In
particular, each young agent observes his own private signal

\[ x_{it} = \theta_t + \varepsilon_{it}, \]

where \( \varepsilon_{it} \) is normally distributed with mean 0 and precision \( \beta \). It is assumed that \( \{ \varepsilon_{it} \} \) is i.i.d over agents.

Consumption goods are produced as follows. At the beginning of time \( t + 1 \), old agents of generation \( t \) rent their capital produced from the investment at the end of time \( t \) to a continuum of perfectly competitive firms. These firms produce a single consumption good in the economy, using labor and capital according to the production function

\[ Y_t = \bar{K}_t^{1-\mu} K_t^\mu L_t^{1-\mu}, \]  

where \( \bar{K}_t \) is the average capital-labor ratio in the economy at time \( t \). The depreciation rate of capital is assumed to be 1 (In this paper I use capital and small letters to denote variables at aggregate and individual levels respectively).

The utility function for each agent born at period \( t \) is given by

\[ u_t = \beta \log(c_{2,t}), \]

where \( c_{2,t} \) denotes the consumption of an old agent born at period \( t \), and \( \log \) represents natural logarithm function. So we can see that an agent will consume nothing when young and consume all he has when old. Figure 1 gives the timing of the model.

### 3.2 Market Equilibrium

This section characterizes the market equilibrium in this model.

#### 3.2.1 Consumption Goods Market Equilibrium

The consumption goods market is perfectly competitive. Equilibrium labor supply is given by \( L_t = 1 \), since labor is supplied inelastically. Also in equilibrium \( \bar{K}_t = K_t \). Thus, in this competitive equilibrium, capital rent \( r_t \) and wage \( w_t \) are respectively

\[ r_t = \mu \bar{K}_t^{1-\mu} K_t^{\mu-1} = \mu; \]

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Young agents of generation $t$ with wage income decide which technology to invest in to produce capital goods.

Old agents of generation $t-1$ provide capital produced from their investment and hire labor to produce consumption goods.

Young agents of generation $t-1$ consume all they have and die.

Old agents of generation $t$ provide capital and hire labor to produce consumption goods.

Young agents of generation $t+1$ are born and provide labor to earn wage income.

Figure 1: The timeline

$$w_t = (1 - \mu)K_t.$$  

Notice that here I follow the assumption of AK models that capital has constant returns. This assumption will greatly simplify the analysis.

3.2.2 Capital Goods Market Equilibrium

In each period $t$, after observing his own private signal $x_{it}$, a young agent with wage $w_t$ has to decide to invest in the conventional or new technology at the end of period $t$.

Given such a setup, I can prove that there is a unique Bayesian Nash equilibrium in this game. In equilibrium, each agent will invest in the new technology if and only if his private signal $x_{it}$ is greater than a threshold level. Otherwise, he will invest in the conventional technology.

After observing the private signal $x_{it}$, agent $it$ updates his belief about $\theta_t$ according to Bayes’ rule. Since both $\theta_t$ and $x_{it}$ are normally distributed, $(\theta_t|x_{it})$ is also normally distributed.
Moreover, the mean of \((\theta_t|x_{it})\) is

\[\rho_{it} = \frac{\alpha \bar{\theta} + \beta x_{it}}{\alpha + \beta}.\]  (3)

Its precision is simply \(\alpha + \beta\).

Let

\[\gamma = \frac{\alpha^2 (\alpha + \beta)}{\beta (\alpha + 2\beta)}.\]  (4)

**Proposition 1.** Provided that \(\gamma \leq 2\pi\), there is a unique symmetric trigger strategy equilibrium in the capitals good market in each period. In this equilibrium, each young agent chooses to invest in the new technology if and only if \(\rho > \rho^*\), where \(\rho^*\) is the unique solution to

\[\rho^* - \Phi(\sqrt{\gamma}(\rho^* - \bar{\theta})) = \log(r).\]

Otherwise, the young agent chooses to invest in the conventional technology.

**Proof:**

Here I confine attention to symmetric trigger strategy equilibria and prove that there is such a unique equilibrium. In the appendix, I prove that this symmetric trigger strategy equilibrium is the unique equilibrium that survives the iterated elimination of strictly dominated strategies.

Given that attention is confined only to symmetric trigger strategy equilibria, it takes two steps to prove that there is such a unique equilibrium. First, the unique threshold level \(\rho^*\) is pinpointed, given the hypothesis that each young agent follows the strategy of investing in the new technology if and only if his updated belief \(\rho_{it} > \rho^*\). Second, it is proved that this strategy is optimal for every agent.

For \(\rho^*\) to be an equilibrium triggering point, a young agent with the updated belief \(\rho^*\) must be indifferent between investing in the new technology and a conventional one.

Suppose an agent is at the equilibrium triggering point. I will abuse the notation and denote him by \(\rho^*\), his updated belief about the mean of the return from the new technology investment. Then each agent \(it\) is assumed to invest in the new technology if and only if \(\rho_{it} > \rho^*\).
We know that the expected utility of agent $\rho^*$ from investing 1 unit of consumption goods in the new technology is given by

$$E[\log(e^{\theta_t - \lambda_t | x^*})] = E(\theta_t - \lambda_t | x^*) + \log(\mu).$$

Recall that the relationship between $x^*$ and $\rho^*$ is given by Equation (3).

We already know that

$$E(\theta_t | x^*) = \rho^*.$$

Now I need to find $E(\lambda_t | x^*)$. Given the equilibrium strategy that each firm $i t$ will invest in the new technology if and only if $\rho_{it} > \rho^*$,

$$E(\lambda_t | x^*) = \text{Prob}(\rho_{jt} < \rho^* | x^*).$$

Therefore we get:

$$E(\lambda_t | x^*) = \text{Prob}(\rho_{jt} < \rho^* | x^*).$$

From Equation (3), we know that

$$\rho_{jt} = \frac{\alpha \bar{\theta} + \beta x_{jt}}{\alpha + \beta}.$$

Therefore, we get

$$\text{Prob}(\rho_{jt} < \rho^* | x^*) = \text{Prob}(x_{jt} < \rho^* + \frac{\alpha}{\beta}(\rho^* - \bar{\theta}) | x^*).$$

We know that $(x_{jt} | x^*) = (\theta + \varepsilon_{jt} | x^*)$ is normally distributed with mean $\rho^*$ and variance $\frac{1}{\alpha + \beta} + \frac{1}{\beta}$. Therefore, we get

$$\text{Prob}(\rho_{jt} < \rho^* | x^*) = \Phi(\sqrt{\frac{\beta(\alpha + \beta)}{\alpha + 2\beta}((\rho^* + \frac{\alpha}{\beta}(\rho^* - \bar{\theta}) - \rho^*)}).$$

Thus

$$E(\lambda_t | x^*) = \Phi(\sqrt{\gamma}(\rho^* - \bar{\theta})), (7)$$

where $\Phi(.)$ is the CDF of the standard normal distribution with mean 0 and variance 1.
Then we get

$$E(\theta_t - \lambda_t | x^*) = \rho^* - \Phi(\sqrt{\gamma}(\rho^* - \bar{\theta})). \quad (8)$$

The utility of agent $\rho^*$ from investing in the conventional technology is given by $\log(r\mu)$.

Since at the triggering point the agent has to be indifferent between investing in the new and conventional technology, we get

$$\rho^* - \Phi(\sqrt{\gamma}(\rho^* - \bar{\theta})) = \log(r).$$

Given $\gamma < 2\pi$, $\rho^* - \Phi(\sqrt{\gamma}(\rho^* - \bar{\theta}))$ is strictly increasing in $\rho^*$. So there is a unique solution of $\rho^*$ satisfying the above equation.

Now I need to show that, given $\rho^*$, the strategy that an agent $i$ will invest in the new technology if and only if $\rho_{it} > \rho^*$ is optimal for every agent.

For agent $it$ with $\rho_{it} > \rho^*$,

$$E(\theta_t - \lambda_t | x_{it}) = \rho_{it} - \Phi(\sqrt{\gamma}(\rho^* - \bar{\theta} + \frac{\beta}{\alpha}(\rho^* - \rho_{it}))). \quad (9)$$

And its precision is $\alpha + \beta$.

Agent $it$ will invest in the new technology, because

$$E(\theta_t - \lambda_t | x_{it}) - \log(r) = \rho_{it} - \Phi(\sqrt{\gamma}(\rho^* - \bar{\theta} + \frac{\beta}{\alpha}(\rho^* - \rho_{it}))) - \log(r) > 0. \quad (10)$$

The reason that the above function is positive is that it is strictly increasing in $\rho_{it}$, and when $\rho_{it} = \rho^*$, it is equal to 0 by construction. It is easy to show that its first order derivative with respect to $\rho_{it}$ is greater than 0:

$$1 + \sqrt{\gamma} \frac{\beta}{\alpha} \Phi'(\sqrt{\gamma}(\rho^* - \bar{\theta} + \frac{\beta}{\alpha}(\rho^* - \rho_{it}))) > 0.$$

Similarly, I conclude that an agent $it$ with $\rho_{it} < \rho^*$ will invest in the conventional technology, because $E(\theta_t - \lambda_t | x_{it}) - \log(r) < 0$. Note that equilibrium strategy $\rho^*$ is time invariant.

Q.E.D
3.2.3 Law of Motion of the Capital

The law of motion of the capital is given as follows:

\[ K_{t+1} = (1 - \mu)K_t[\lambda tr + (1 - \lambda_t)e^{\theta_t - \lambda_t}], \]  

where

\[ \lambda_t = \Phi(\sqrt{\beta}(x^*(\rho^*) - \theta_t)). \]

The consumption profile of a typical generation \( t \) is:

\[ c_{1,t} = 0; \]  

\[ c_{2,t} = \begin{cases} 
(1 - \mu)k_t r \mu & \lambda_t \text{ of agents investing in conventional technology} \\
(1 - \mu)k_t e^{\theta_t - \lambda_t} \mu & 1 - \lambda_t \text{ of agents investing in new technology}
\end{cases} \]  

The initial old generation has the consumption profile \( \{c_{2,0} = \mu k_0\} \).

4 An Analytical Study on the Model

This section applies the above model to the study of how incomplete information and coordination failure influence new technology investment, economic growth, and volatility.

4.1 Coordination Failure and Economic Growth

First, let us look at the case where economic agents have perfect information about \( \theta_t \). When \( \theta_t < \log(r) \), no agent will invest in the new technology and it is the first best solution. When \( \theta_t > 1 + \log(r) \), every agent will invest in the new technology and it is also the first best solution. The interesting case is \( \log(r) < \theta_t < 1 + \log(r) \). With perfect information, this case has two (stable) equilibria: one is that all the agents invest in the conventional technology; the other is that all the agents invest in the new technology. The latter equilibrium is Pareto superior to the former one.
Thus, under perfect information, coordination failure is manifested as the randomness about which equilibrium is realized.

In my model, I find that given $\gamma < 2\pi$, there is always a unique equilibrium due to the introduction of incomplete information. Moreover, I find that the first best solution under perfect information can never be achieved in this equilibrium. The inefficiency in equilibrium is caused by two kinds of uncertainties: one is the uncertainty about $\theta_t$, the economic fundamentals of the new technology; the other is the uncertainty about the actions of other agents.

A special case with $\beta \to \infty$ can analytically reveal how the uncertainty about the actions of other agents leads to an inefficient equilibrium outcome. We know that $\beta \to \infty$ means that the private signal of agents almost perfectly reveals the new technology shock. That is, the first kind of uncertainty is vanishingly small. I find that this assumption cannot eliminate the uncertainty about other agents’ actions, and the first best equilibrium with perfect information cannot be achieved due to coordination failure among economic agents.

When $\beta \to \infty$, $\gamma$, which is strictly decreasing in $\beta$, goes to zero. In addition, $\rho^* = \frac{\alpha \bar{\theta} + \beta x^*}{\alpha + \beta} \to x^*$. Thus, the equation pinning down $\rho^*$,

$$\rho^* - \Phi(\sqrt{\gamma}(\rho^* - \bar{\theta})) = \log(r),$$

is transformed into

$$x^* - \frac{1}{2} = \log(r).$$

So we get

$$x^* = \rho^* = \log(r) + \frac{1}{2}.$$

The intuition for this result is that when $\beta \to \infty$, each agent believes that his private signal $x$ is exactly the true value of $\theta$ so that there is always half of the agents receiving private signals below his, and half of the agents receiving private signals above his.

Now let us examine economic growth, i.e., how outputs $Y$ grow over time. Note that due to the specific production function in the model, $Y = K$ in each period.

We know that

$$K_{t+1} = (1 - \mu)K_t[\lambda_t r + (1 - \lambda_t)e^{\theta_t - \lambda_t}].$$
So the ex ante expected average gross capital (economic) growth rate is given by

\[ g = E(Y_{t+1}/Y_t) = E(K_{t+1}/K_t) = (1 - \mu)E[\lambda_t r + (1 - \lambda_t)e^{\theta_t - \lambda_t}]. \]

We also know that

\[ \lambda_t = \Phi(\sqrt{\beta}(x^* - \theta_t)). \]

So \( \lambda_t = 1 \) when \( \theta_t < \log(r) + \frac{1}{2} \) since \( \sqrt{\beta}(x^* - \theta_t) \) goes to \( +\infty \) in this case, and \( \lambda_t = 0 \) when \( \theta_t > \log(r) + \frac{1}{2} \) since \( \sqrt{\beta}(x^* - \theta_t) \) goes to \( -\infty \) in this case.

The ex ante expected gross economic growth rate is given by:

\[ g = (1 - \mu)[r\Phi(\sqrt{\alpha}(\log(r) + \frac{1}{2} - \bar{\theta})) + \int_{\log(r)+\frac{1}{2}}^{+\infty} e^{\theta_t}d\Phi(\sqrt{\alpha}(\theta_t - \bar{\theta}))]. \]

It is the expected value of \( K_{t+1}/K_t \), given that each period the agents take the equilibrium strategy described above, and that \( \theta_t \) is normally distributed with mean \( \bar{\theta} \) and precision \( \alpha \). Note that in this infinite time horizon model, the actual average gross economic growth rate will converge to the expected average gross economic growth rate when time goes to infinity.

Assume that the Pareto superior equilibrium with perfect information is always realized, that is, the agents will all invest in the new technology when \( \theta_t < \log(r) \) and all invest in the conventional technology when \( \theta_t > \log(r) \). The ex ante expected average gross economic growth rate is then given by:

\[ g^{FB} = (1 - \mu)[r\Phi(\sqrt{\alpha}(\log(r) - \bar{\theta})) + \int_{\log(r)}^{+\infty} e^{\theta_t}d\Phi(\sqrt{\alpha}(\theta_t - \bar{\theta}))]. \]

It is the expected value of \( K_{t+1}/K_t \) given the Pareto superior equilibrium is realized each period and \( \theta_t \) is normally distributed with mean \( \bar{\theta} \) and precision \( \alpha \).

We get the above equation because it is optimal for all the agents to invest in the conventional technology if \( \theta_t < \log(r) \) and to invest in the new technology if \( \theta_t > \log(r) \).

It is obvious to see that \( g^{FB} - g > 0 \) and

\[ g^{FB} - g = (1 - \mu)\int_{\log(r)}^{\log(r)+\frac{1}{2}} (e^{\theta_t} - r)\sqrt{\alpha}\phi(\sqrt{\alpha}(\theta_t - \bar{\theta}))d\theta_t. \]
The above function indicates that coordination failure will most severely dampen economic growth when $\bar{\theta}$ is around the range $[\log(r), \log(r) + \frac{1}{2}]$. When $\bar{\theta}$ is in this range, $\phi(\sqrt{\alpha}(\theta_t - \bar{\theta}))$ is at its highest values given $\theta_t \in [\log(r), \log(r) + \frac{1}{2}]$. Therefore, $g^{FB} - g$ will be higher, indicating more losses from coordination failure.

The policy implication from this result is that when the new technology returns of an economy fall into the range that is close to that of the conventional technology returns, coordination failure turns to be most severe, and the economy should benefit more from encouraging more investment in the new technology.

Note that this solution is Pareto optimal not only because it maximizes the economic growth rate, but also because it maximizes the ex ante expected utility of an agent, which is given by:

$$EU_t = E\beta \log(c_{t+1}) = \beta(\Phi(\sqrt{\alpha}(x^* - \bar{\theta}))) \log(\mu r w_t) + \int_{x^*}^{+\infty} \log(\mu e^{\theta_t w_t})d\Phi(\sqrt{\alpha}(\theta_t - \bar{\theta})).$$

It is obvious that this expected utility function is maximized when $x^* = \log(r)$.

### 4.2 Implications for Economic Volatility

This model can also be used to study economic volatility and its relationship with economic growth. In this model, economic volatility originates from two kinds of uncertainties. One uncertainty is about the economic fundamentals of the new technology. This uncertainty is exogenously given. The other uncertainty is about the actions of other economic agents. This uncertainty is endogenously generated in an economy where investment complementarities exist, but coordination is not available.

As mentioned before, the gross capital (economic) growth rate is given by

$$g_{t+1} = \frac{K_{t+1}}{K_t} = (1 - \mu)[\lambda_t r + (1 - \lambda_t)e^{\theta_t - \lambda_t}],$$

where

$$\lambda_t = \Phi(\sqrt{\beta}(x^* - \theta_t)).$$

From the above expression, we can see that $\theta_t$, the economic fundamentals of the new technology, represents the first kind of uncertainty. The proportion of agents not investing, $\lambda_t$, represents the second kind of uncertainty.
It is difficult to derive an analytical expression for the volatility. But the later numerical simulation shows that the economy is most stable when $\bar{\theta} < \log(r)$. The economy is moderately volatile when $\bar{\theta} \in [\log(r), \log(r) + 1]$, which I call the coordination zone. Finally, the economy is most volatile when $\bar{\theta} > \log(r) + 1$. So in general the relationship between economic growth and volatility is positive with the increase in $\bar{\theta}$. A detailed examination will be given in Section 5.3.

Another interesting result generated by this model is that economic agents’ risk attitude will influence their choices between two technologies, leading to a positive correlation between economic growth and volatility that is associated with coordination failure. The main difference between the conventional technology and the new one is that the returns of the former are constant, while the returns of the latter are uncertain. As mentioned before, the uncertainty embedded in the new technology investment is twofold: first, it stems from the uncertainty about economic fundamentals, $\theta_t$. Second, it stems from the uncertainty about other agents’ beliefs about $\theta_t$ and their actions based on their beliefs.

My model reveals that the less risk-averse attitude of economic agents can alleviate coordination failure. The intuition is simple: the main consequence of coordination failure is under-investment in the new technology. Since the returns of the new technology are volatile and those of the conventional technology are constant, the less risk-averse attitude will encourage more investment in the new technology, and therefore overcome coordination failure.

Note when an economic agent, say $i$, decides which technology to choose, his expected utility from investing in the new technology is given by

$$u_{it} = EU(e^{\theta_t - \lambda_t \mu w_t} | x_{it}),$$

where $E(\lambda_t | x_{it}) = \Phi(\sqrt{\gamma}(\rho^* - \bar{\theta}))$.

It is obvious to see that utility function forms will influence economic agents’ expected utility derived from the new technology investment. According to Jensen’s inequality, a risk neutral agent will more tend to invest in the new technology than a risk averse agent. That is, $x^*$, the threshold level of private signal above which economic agents will invest in the new technology, is lower for a risk neutral agent than
for a risk averse agent. In this way, the less risk-averse attitude of economic agents helps alleviate the coordination failure problem and leads to higher economic growth. Meanwhile, the new technology is more volatile by nature and more investment in the new technology leads to higher economic volatility as well. This mechanism will be most significant when the returns of the new technology are in the coordination failure zone.

In general, my model generates a positive relationship between economic growth and volatility. It is no surprise because the fundamental idea of this model is that economic growth comes from new technology investment, which is volatile by nature. Therefore, the pursuit of higher economic growth is consequently accompanied by higher economic volatility. Empirical evidence in general finds a negative relationship between economic growth and volatility (Hnatkovska and Loayza (2005)). However, this negative relationship is caused mostly by factors such as institutional and financial development, and fiscal policies, which are missing in my model. Moreover, the negative relationship can be caused by the causal effect of volatility on growth, which is not addressed in my model either. Empirical testing does find a positive relationship between economic growth and volatility (Hnatkovska and Loayza (2005)) among industrious countries, where the factors mentioned above are insignificant.

5 A Numerical Study on the Model

This section gives some numerical examples based on my model. These examples will help explain my model more clearly.

5.1 Economic Growth Paths with Different Levels of $\bar{\theta}$

Suppose that the capital share of income $\mu = 0.4$. The new technology shock $\theta \sim N(\bar{\theta}, 1/10)$ ($\alpha = 10$) and the precision of the agents’ private signal $\beta = 20$. Note that $\gamma = 3 < 2\pi$ such that the condition for a unique equilibrium is held. I also assume $r = 1.7$. Three different levels of $\bar{\theta}$ are given: 0, 1.0 and 1.6. Note that $\log(r) = 0.53$, and $\log(r) + 1 = 1.53$. So $\bar{\theta} = 0 < \log(r) = 0.53$, $\bar{\theta} = 1.0 \in [\log(r), \log(r) + 1]$, and
\[ \bar{\theta} = 1.6 > \log(r) + 1 = 1.53. \]

Given the parameter values above, I can calculate the equilibrium \( \rho^\star \), time series of the proportion of agents investing in the conventional technology \( \lambda_t \), and time series of natural logarithm of capital stock (output), \( \log(K_t) \) (\( \log(Y_t) \)). Each example is simulated for 50 periods. The results are as follows.

It turns out that \( \rho^\star \) is equal to 0.5675, 1.0981 and 1.5265 respectively, given that \( \bar{\theta} \) takes the values of 1.6, 1.0 and 0. The following figures give the time series of realized \( \theta_t \), \( \lambda_t \), and natural logarithm of capital stock (output) \( \log(K_t) \) (\( \log(Y_t) \)), given that \( \bar{\theta} \) is equal to 0, 1 and 1.6.
5.1.1 The Case of $\bar{\theta} = 0$

![Figure 2: Time series of realized $\theta$ when $\bar{\theta} = 0$](image1)

![Figure 3: Time series of $l$ when $\bar{\theta} = 0$](image2)
5.1.2 The Case of \( \theta = 1.0 \)

Figure 4: Time series of \( \log K (\log Y) \) when \( \bar{\theta} = 0 \)

Figure 5: Time series of realized \( \theta \) when \( \bar{\theta} = 1 \)
5.1.3 The Case of $\bar{\theta} = 1.6$
The figures above clearly reveal that coordination failure is closely related to the relative return levels between the conventional technology and new technology.

When $\bar{\theta} < \log(r)$, $\theta_t < \log(r)$ most of the time. We know that the first best solv-
tion when \( \theta_t < \log(r) \) is to invest in the conventional technology, and in equilibrium, \( \lambda_t \) is actually equal to 1 most of the time. Similarly, when \( \theta > \log(r) + 1, \theta_t > \log(r) \) most of the time. We know that the first best solution when \( \theta_t < \log(r) \) is to invest in the new technology, and in equilibrium, \( \lambda_t \) is actually equal to 0 most of the time. However, when \( \log(r) < \theta_t < \log(r) + 1, \log(r) < \theta_t < \log(r) + 1 \) most of the time. In this case coordination failure becomes severe. The first best solution in this case is that \( \lambda_t = 0 \) most of the time. But we see that actually \( \lambda_t \) swings between 0 and 1.

5.2 Coordination Failure and Economic Growth

Given different \( \bar{\theta} \) levels, I am going to compare economic growth rates with and without coordination failure, holding all the other factors the same. In this way, I can explicitly show how coordination failure leads to slower economic growth.

First, given each level of \( \bar{\theta} \), I will simulate the model with 500 periods and simulate it 501 times. Then the average economic growth rate will be calculated. All the parameters take the same values as in Section 5.1.

Next, given the parameter values unchanged, I assume that the Pareto optimal
equilibrium with perfect information is realized in each period. I will simulate this new model with 500 periods and simulate it 501 times. The average net economic growth rates are calculated and compared with those in the case with coordination failure.

Table 1 and Figure 11 give the results:

<table>
<thead>
<tr>
<th>$\bar{\theta}$</th>
<th>0</th>
<th>0.1</th>
<th>0.2</th>
<th>0.3</th>
<th>0.4</th>
<th>0.5</th>
<th>0.6</th>
<th>0.7</th>
<th>0.8</th>
</tr>
</thead>
<tbody>
<tr>
<td>$g^{-1}$</td>
<td>0.0200</td>
<td>0.0200</td>
<td>0.0200</td>
<td>0.0200</td>
<td>0.0200</td>
<td>0.0201</td>
<td>0.0204</td>
<td>0.0233</td>
<td></td>
</tr>
<tr>
<td>$g^{FB^{-1}}$</td>
<td>0.0262</td>
<td>0.0329</td>
<td>0.0450</td>
<td>0.0647</td>
<td>0.0959</td>
<td>0.1404</td>
<td>0.2010</td>
<td>0.2828</td>
<td>0.3822</td>
</tr>
<tr>
<td>$\frac{g^{-1}}{g^{FB^{-1}}}$</td>
<td>0.7626</td>
<td>0.6084</td>
<td>0.4444</td>
<td>0.3089</td>
<td>0.2086</td>
<td>0.1425</td>
<td>0.0998</td>
<td>0.0722</td>
<td>0.0610</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>$\bar{\theta}$</th>
<th>0.9</th>
<th>1.0</th>
<th>1.1</th>
<th>1.2</th>
<th>1.3</th>
<th>1.4</th>
<th>1.5</th>
<th>1.6</th>
</tr>
</thead>
<tbody>
<tr>
<td>$g^{-1}$</td>
<td>0.0440</td>
<td>0.1803</td>
<td>0.5840</td>
<td>0.9354</td>
<td>1.1874</td>
<td>1.4328</td>
<td>1.6889</td>
<td>1.9727</td>
</tr>
<tr>
<td>$g^{FB^{-1}}$</td>
<td>0.5041</td>
<td>0.6472</td>
<td>0.8108</td>
<td>0.9960</td>
<td>1.2016</td>
<td>1.4360</td>
<td>1.6896</td>
<td>1.9729</td>
</tr>
<tr>
<td>$\frac{g^{-1}}{g^{FB^{-1}}}$</td>
<td>0.0873</td>
<td>0.2786</td>
<td>0.7203</td>
<td>0.9391</td>
<td>0.9882</td>
<td>0.9978</td>
<td>0.9996</td>
<td>0.9999</td>
</tr>
</tbody>
</table>

Figure 11: Growth rates with and without a coordinator when $\bar{\theta}$ changes
Both Table 1 and Figure 11 reveal that coordination failure can severely dampen economic growth when $\bar{\theta}$ is in the coordination failure zone. On the other hand, coordination failure will not have significant effects on economic growth when the returns of the new technology are either much lower or higher than those of the conventional technology. We know that the lower $\frac{g-1}{g^{\bar{\theta}}-1}$ is, the more severe the coordination failure is. When $\bar{\theta}$ approaches the coordination zone from below, $\frac{g-1}{g^{\bar{\theta}}-1}$ goes lower and lower, which means that coordination failure is more and more severe. In the extreme case when $\bar{\theta} = 0.8$, the economic growth rate with coordination failure is only 6 percent of that in the Pareto optimal solution. Meanwhile, when $\bar{\theta}$ goes higher and leaves the coordination zone, $\frac{g-1}{g^{\bar{\theta}}-1}$ goes higher and higher, which means that coordination failure is less and less severe.

5.3 Economic Growth and Volatility

First, I will use the same method as in Section 5.2 to find the average growth rates and variances given different levels of $\bar{\theta}$. Then the relationship between economic growth and variance will be checked.

![Figure 12: How economic growth changes in $\bar{\theta}$](image-url)
Figure 13: How economic volatility changes in $\theta$

Figure 14: Relationship between economic growth and volatility when $\theta$ changes

From the above figures we can see that in general, there is a positive relationship between economic growth and volatility with an increase in $\theta$. However, the positive relationship becomes less salient when $\theta$ is in some area of the coordination failure.
zone. We can see that in this range, the economy grows without significant increase in volatility.

The intuition for the above results is as follows. When $\bar{\theta} < \log(r)$, $\lambda$ is relatively stable and equal to 1 most of the time, which means that all the economic agents will choose to invest in the conventional technology most of the time. Since the conventional technology has constant returns, both the uncertainty about the economic fundamentals of the new technology and the uncertainty about the actions of other agents vanish. Thus the economy is the most stable in this case. When $\bar{\theta} \in [\log(r), \log(r) + 1]$, which is in the coordination zone, $\lambda$ swings between 0 and 1. Therefore, both kinds of uncertainties contribute to the overall economic volatility. The reason for the less salient positive relationship between growth and volatility in some area of this range is that the second kind of uncertainty about the actions of other agents kicks in and prevents agents from investing in the new technology. Consequently, even when $\theta$ increases, $\lambda$ does not increase very significantly, depressing the increase in economic volatility. When $\bar{\theta} > \log(r) + 1$, $\lambda$ is relatively stable and equal to zero most of time. In this case the main uncertainty comes from the economic fundamentals of the new technology. The economy is most volatile in this case due to two reasons. First, all the agents will invest in the new technology, which is riskier by nature compared to the conventional technology, leading to the volatility. Second, the returns of the new technology are assumed to be log-normally distributed. So their variance will increase in the mean. This is because for a log-normally distributed variable $X = e^x$, where $x$ is normally distributed with mean $\mu$ and variance $\sigma^2$, the variance of $X$ is given by:

$$V AR(X) = (e^{\sigma^2} - 1)e^{2\mu + \sigma^2}.$$ 

In general, economic growth and variance exhibit a positive correlation with the increase in $\bar{\theta}$ because of the shifting of investment to the new technology and the nature of the new technology modeled in this model.

Next, I will use the same method in Section 5.2 to find the average growth rates and variances given different levels of $x^*(\rho^*)$. Here $\bar{\theta} = 1.0$, which is in the coordination failure zone. Then the relationship between economic growth and volatility is examined.
The figures above reveal that a small decrease in $x^*$ will greatly increase economic growth and generate a significant positive correlation between economic growth and variance. This is because a decrease in $x^*$ leads to both more investment in the
new technology and higher returns of the new technology, both of which contribute to higher growth. Meanwhile, more investment in the new technology results in higher economic volatility. We know that a risk neutral agent will definitely have a lower $x^*(\rho^*)$ than a risk averse agent. So the risk attitude of agents could be a source of a positive correlation between economic growth and volatility associated with coordination failure.

6 Policy Implications

This paper reveals that coordination failure can hamper a country’s new technology investment and economic growth. Therefore, the government needs to play the role of a coordinator to improve social welfare. In summary, there are several implications that I have obtained from the model. First, when the returns of the new technology are close to those of the conventional technology, coordination failure is significant and government intervention is most needed. Second, coordination failure is usually manifested as under-investment in the new technology. Therefore, any government policies stimulating more investment in the new technology will help alleviate coo-
dination failure. Third, public information can play a role in alleviating coordination failure.

Next I will discuss one specific government policy based on my model.

Based on the above example of $\beta \to \infty$, a simple way to achieve the socially optimal solution is to tax the agents investing in the conventional technology or to subsidize those investing in the new technology. This is a natural result due to my assumption of the increasing returns to scale in the new technology investment. In fact, a lot of research emphasizing the public good property of new technology suggests this policy.

Suppose that the government introduces a penalty $\tau$ to the investment in the conventional technology. Now the cutoff level of $x^*$ is determined by:

$$x^* - \frac{1}{2} = \log(r - \tau).$$

Then we get $x^* = \log(r - \tau) + \frac{1}{2}$. Let $\tau = (1 - e^{-\frac{1}{2}})r$, then $x^* = \log(r)$, which is exactly the optimal cutoff level. Suppose this lump-sum tax will be returned to the agents, then the first best solution is achieved. Note that here the government can achieve the first best outcome because this is a very special case with $\beta \to \infty$. In such a case, the first kind of uncertainty about the economic fundamental of the new technology vanishes. The inefficiency is purely caused by the coordination failure. So the government can step in to play a role as a coordinator and correct the inefficiency.

Note that we have to treat this policy implication with caution. We must be aware that it is derived from a model under some highly simplified assumptions about a real economy. New technology is unknown by nature and involves a severe information asymmetry problem, which increases the difficulty for governments to implement targeted intervention in reality. However, the general message that this analysis sends us is that the presence of externalities in new technology investment demands government intervention to stimulate more new technology investment.
7 Conclusions

In this paper I argue that due to the special features of uncertainty and externalities of new technology, coordination failure can occur in a country’s new technology investment and lead to slower capital accumulation and economic growth. Combining a global game into a two-sector Overlapping Generation model, I demonstrate that coordination failure can be manifested as under-investment in the new technology and it is most severe when the returns from the new technology investment are close to those from the conventional technology investment. My model also reveals that the tradeoff between economic growth and volatility can occur because more investment in the new technology will alleviate coordination failure, but meanwhile will lead to higher economic volatility. In addition, policy implications of my model are explored. I find that the government intervention in favor of new technology investment is needed to alleviate coordination failure.
APPENDIX

This appendix proves that there is a unique trigger strategy equilibrium in the model by using the iterated elimination of strictly dominated strategies.

In a typical period $t$, the expected utility of an agent receiving a private signal $\rho$, given that all the others follow the trigger strategy $\hat{\rho}$, denoted by $u(\rho, \hat{\rho})$, is given by

$$u(\rho, \hat{\rho}) = E(\theta - l|\rho) + \log(\mu) = \rho - \Phi(\sqrt{\gamma}(\hat{\rho} - \bar{r} + \frac{\beta}{\alpha}(\hat{\rho} - \rho))) + \log(\mu).$$

Note that $u(\rho, \hat{\rho})$ is increasing in $\rho$, and decreasing in $\hat{\rho}$ (The subscript $t$ is omitted here since the proof can be applied to any period).

When $\rho$ is sufficiently low, investing in the new technology will be the dominant strategy for an agent, no matter what strategies other agents will take. Let us denote it as $\rho_0$. All agents realize this and rule out any strategy for agents to invest in the new technology below $\rho_0$. Then investing in the new technology cannot be optimal for an agent receiving a private signal below $\rho_0$, which solves

$$u(\rho_0, \rho_0) = \log(r) + \log(\mu).$$

This is because the trigger strategy around $\rho_0$ is the best response to the trigger strategy around $\rho_0$, and all agents believe that other agents will invest in the conventional technology when their private signals are below $\rho_0$. Since the agents’ expected return is decreasing in the second argument, this rules out any strategy for agents to invest in the new technology below $\rho_0$. Proceeding this way, we get an increasing sequence:

$$\rho_0 < \rho_1 < \cdots < \rho_k < \cdots,$$

where any strategy of investing in the new technology when $\rho < \rho_k$ does not survive $k$ rounds of deletion of strictly dominated strategies. The sequence is increasing because $u(\cdot, \cdot)$ is increasing in the first argument and decreasing in the second one. The smallest solution $\rho$ to the equation $u(\rho, \rho) = \log(r) + \log(\mu)$ is the least upper bound of this sequence. Any strategy of investing in the new technology below $\rho$ cannot survive iterated dominance.
Similarly we can have an analogous argument beginning with the case that $\rho$ is large enough and the strategy to invest in the new technology is dominant no matter what strategies other agents will take. If $\rho$ is the largest solution to $u(\rho, \rho) = \log(r) + \log(\mu)$, any strategy of investing in the conventional strategy when the signal is higher than $\rho$ cannot survive the deletion of strictly dominated strategies.

Given $\gamma < 2\pi$, there is only a unique solution to $u(\rho, \rho) = \log(r) + \log(\mu)$. So the smallest solution is equal to the largest solution. There is only one strategy surviving the iterated elimination of strictly dominated strategies, which is the unique equilibrium strategy in this game.
References


