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Are Sunnier Cities Denser?

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Abstract

We set out an open, monocentric city with residential structures and reflect on how changes to an amenity index affects the city. On the production side, the shock is represented by a productivity improvemnt and a local wage increase and on the consumption side the shock is represented by an exogenous boost to the utility of a resident's current commodity bundle. In each case the city's population, land rent and footprint expand. In the second case there is an increase in density. (robak_jan07)

- key words: urban amenities, density, wages
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1 Introduction

We investigate the idea that a productivity premium for a city shows up as a wage premium for its workers and in turn as a city size and rent "premium" relative to a comparable city, and in addition that consumption amenities (sunnier location) for workers in a city result in somewhat smaller commodity bundles per worker and higher local land rents. Shapiro [2006; p. 325] takes these ideas as uncontroversial, though back in 1979, Rosen [1979] argued for consumption amenities to be negatively capitalized in local wages as in sunnier cities having lower wages.[1] Here we work with a stylized monocentric city[2] and this allows us to fill in detail on land rent and density changes in comparative statics exercises.[3] Novel is our result that a consumption amenity "jump" for a city leads to a contraction in the size of a local worker's commodity bundle and thus to a denser and larger city, one with a land rent schedule that shifts up at every location. When we proceed to invoke the idea that a larger city should have somewhat higher wages on average (via a little-understood productivity jump[4]), we are arguing for an outcome opposite to that of Rosen [1979]: a city with a climate improvement will ultimately experience a rise, likely small, in the level of its

1 Shapiro [2006] addresses how higher levels of human capital in certain cities feeds into higher rates of growth of local land rent, housing and wages. When local human capital is measured by the number of local college graduates, Shapiro concludes: "evidence from wages and rents implies that, although the majority of the employment growth effect of college graduates operates through changes in productivity, roughly one-third of the effect seems to come from more rapid improvement in the quality of life." A large dimension of "quality of life" are "consumer city" amenities such as bars and restaurants.

2 Our model is essentially that of Mills [1967] with the housing sector tightened up in Brueckner [1982]. Wheaton [1974] is the basic comparative statics work on the Mills monocentric city model. Brueckner and Fansler [1983] performed some econometric tests along the lines of Wheaton [1974]. The Brueckner-Fansler work has been revisited by McGrath [2005] and Spivey [2008] with new data and "angles". Both McGrath and Spivey argue that their "replication" of the Brueckner-Fansler results supports the formulation of a city in Mills [1967]. Black and Henderson [1999] employ an abstract monocentric city as the building block for their model of the growth of a system of cities.

3 Rosen [1979] and his student Roback [1982] worked in an aspatial setting and much subsequent analysis of wage premia in larger cities has been carried out in aspatial models. We contend that an analysis with an explicit spatial structure yields interesting new insights. Black and Henderson [1999] worked with an explicitly spatial model.

4 Rauch [1993], Glaeser and Mare [2001] and Moretti [2004] have explored explanations for higher wages to be present in larger cities. Ciccione and Hall [1996] have explored the productivity premium associated with higher densities of workers with county level data for the US.

wage, in our view.

Our city occupies space and faces an infinitely elastic demand schedule for its export good. Free migration of a representative worker-household between cities in our system means that a worker-household's utility level is parametric to each unit. The capital good rental is also parametric to producers in our model. We derive then the comparative statics result in a standard textbook, open city model: an increase in a consumption disamenity (eg. local crime) for a worker-household in city j induces a larger bundle of directly consumed commodities (composite $c(x)$ and housing $h(x)$ at distance x from the center, where production takes place). Our consumption amenity impact "effect" implies: *ceteris paribus*, cities with a better climate (marginal amenity improvement) are larger in population, area and density.

We report also some other comparative statics results for a seemingly familiar, textbook model. In particular we note that technical progress for city j will induce a small wage increase in city j and that this wage increase will be precisely capitalized in an increase in net local land rent. "Net" here refers to land rent increase net of the increase in land rent induced by the expansion of the geographic edge of the city.[5] This provides a new avenue for measuring the impact of technical progress. An analogous capitalization result holds for a reduction in commuting cost in a city in a system of cities: the dollar value of the reduction in aggregate commuting cost is precisely capitalized in net land rent increase for our city.

Our basic model has the wage for a city emerge from the unit cost of its export good, given an infinitely elastic demand schedule facing the city for its export. The level of the wage is influenced in turn by a location-specific parameter in the production function. Davis and Weinstein [2002] refer to production amenities such as a good port "locational fundamentals" and argue that the spatial distribution of cities is strongly influenced by such amenities while

[5] The Sunday New York Times (August 5, 2007) presented a brief profile of Silicon Valley, comparing employment categories, median incomes and median home prices with those in the rest of the United States, averaged over the whole. The reported existing single family home prices were \$788,000 and \$212,300 respectively and the incomes were \$46,920 and \$30,400 respectively. The difference in home prices multiplied by 2.87% equals the difference in incomes. Broad brush analysis is indicating that the wage advantage of a "worker" in Silicon Valley is fully capitalized in the "extra" cost of a home in Silicon Valley, given a discount rate of 2.87%. Alternatively, the high cost of a home in Silicon Valley is fully accounted for by the high wage prevailing in the area, relative to an alternative, "average" location in the United States (say Peoria with a hypothetical wage of \$30,400 and a home price of \$212,300).

city size can in addition be a positive function of scale economies in production. We buy into the Davis-Weinstein view but in addition interpret our location-specific parameter as a standard technical progress index number, a premium turning on say earlier R&D activity. Matters become complicated when the value of the parameter in question is considered to reflect current city size or current city density and we return to this issue later. We do argue that a city can experience a once-over shock of technical progress and in turn experience a jump in the local wage for a representative worker-resident.

2 The Model

The wage can be explained to emerge from the export sector where export good price $p^e = \zeta w^B i^{1-B}$, p^e and i parameters. "Amenity advantage" (eg. superior port) is a lower ζ and a larger w. Associated with a particular w could be a large city, given good climate, or a smaller city, given a poorer climate. ζ can also be viewed as a "level of technology". Then an decrease in ζ can be interpreted as technical progress.[6] Once again a decline in ζ (technical progress) corresponds to a rise in w and an increase in the size of the city.

We take as given each worker's utility function and utility level, $\overline{U}$ in a city. Migration arbitrage of similar workers insures that each household-worker achieves the same utility in a representative city and between two cities. A shock such as a local wage increase leads to a local jump in the utility being achieved and induces in-migration from surrounding cities. The in-flow of new residents pushes up the local rent schedule (house-price schedule) and this chokes off in-migration. The local jump in utility gets "washed out" by the in-migration. We work with a monocentric city with all production activity at a point in the center and commuting cost is t per unit distance. The rental rate on capital i is exogenous to the city as well. City size emerges from a city edge rent equal to the agricultural land rent, ρ at the edge. That is, given values for w, t, i, ρ and $\overline{U}$, equilibrium is essentially an endogenous edge distance, $\overline{x}$ such that $r(\overline{x}; w, t, i, \overline{U}) = \rho$. Given $\overline{x}$, we can solve for the population (labor force), $N(\overline{x})$ which is the supply of labor to production, given an infinitely elastic demand at the center at wage, w.

Given our representative city with its wage and population in place, we consider the

[6] A more complicated production amenity would not be simply multiplicative.

impact of amenity differences, such as changes in sunshine, traffic congestion, crime, etc. An amenity shift shows up in the local level of utility shifting up for say more sunshine on average. This draws in workers from surrounding cities with their utility levels unchanged and ultimately expands the city with the amenity fillip, pushes local rents upward and brings utility back to the initial level.[7] We move on to consider a shift in the local wage, via a shift in the location parameter in the production function for exports, and a shift in the local commuting cost parameter, t.

We summarize. A city in our system of cities is defined by its export sector

$$p^e = \zeta w^B i^{1-B},$$

with B, i, ζ and p^e as parameters and by its equilibrium size in

$$r(\overline{x}; w, t, i, \overline{U}) = \rho$$

given a utility function for the representative household. The first equation defines the equilibrium wage and the second the equilibrium edge. We will proceed to consider comparative statics effects on a representative city in this framework. We turn to filling in some detail on the internal structure of a representative city.

In the Mills-Brueckner monocentric city a worker commutes from his home at radial distance x to the workplace at the center at roundtrip cost, tx. The wage at the center is w, leaving $w - tx$ to be spent on housing, $h(x)$ and the other consumer good, $c(x)$. That is, the budget constraint for a worker-household is $w - tx = pc(x) + q(x)h(x)$. A home comprises some capital (structure) and some land. That is, we have $h(x) = L(x)^A k(x)^{1-A}$ with i the fixed rental price of a unit of capital and $r(x)$ current land rent. Here $L(x)$ is the land for a house at radial distance x and $k(x)$ is the capital in the house at x. For $q(x)$ the price of a unit of house, we know that $q(x) = Mr(x)^A i^{1-A}$ for $r(x)$ land rent at x and i the rental of a unit of capital at x, with $M = A^{-A}(1-A)^{-(1-A)}$.

7 Rosen and Roback [1982] developed the view that the amenity differences we are focusing on get capitalized in wage differences between cities. For example a location with a very good climate would have relatively low wages in the Rosen-Roback view. Our view is clearly quite different. In our view, wage levels are determined almost entirely in the production side of the economy while consumption amenity differences show up first in changes in the size of a household's commodity bundle currently consumed. Places with a better climate imply a contraction in the household's commodity bundle, *ceteris paribus.*

Each household has Cobb-Douglas utility, $U = c(x)^{\alpha} h(x)^{1-\alpha}$ where $c(x)$ is an amount of composite of other goods available at a constant unit price, p. The residential area is an annulus surrounding the CBD, with x radial distance from the center. Hence at $\widehat{x}$, a household spends income net of roundtrip commuting cost, $w - t\widehat{x}$ on "other", $pc(\widehat{x})$ and housing, $h(\widehat{x})q(\widehat{x})$.[8] Given the Cobb-Douglas form for the utility function, we have

$$\begin{aligned} (1-\alpha)[w - t\widehat{x}] &= h(\widehat{x})q(\widehat{x}), \\ \text{and } \alpha[w - t\widehat{x}] &= pc(\widehat{x}). \end{aligned}$$

We assume that our city in question belongs to a system of cities with equilibrium utility level $\overline{U}$ prevailing for like workers. All households are thus achieving for the moment the same utility level, $\overline{U}$. Hence $h(\widehat{x}) = \left[\frac{\overline{U}}{c(\widehat{x})^{\alpha}}\right]^{\frac{1}{1-\alpha}}$, with $c(\widehat{x}) = \frac{\alpha}{p}[w - t\widehat{x}]$. This gives us, at an arbitrary distance x,

$$\begin{aligned} q(x) &= \frac{(1-\alpha)[w - tx]}{h(x)} \\ &= \frac{(1-\alpha)}{\overline{U}^{\frac{1}{1-\alpha}}}[w - tx]c(\widehat{x})^{\alpha/(1-\alpha)} \\ &= \frac{(1-\alpha)}{\overline{U}^{\frac{1}{1-\alpha}}}\left[\frac{\alpha}{p}\right]^{\frac{\alpha}{1-\alpha}}[w - tx]^{\frac{1}{1-\alpha}}. \end{aligned}$$

This is the distance profile of unit floorspace price. A household's housing expenditure, as a flow, is $h(x)q(x)$. House-price in colloquial parlance would be this flow divided by the market rate of interest.

We can solve for the land rent function by substituting in $q(x)$ above from our expression for $q(x)$ earlier. That is, $Mr(x)^{A} i^{1-A} = (1-\alpha)\left[\frac{\alpha}{p}\right]^{\frac{\alpha}{1-\alpha}}\{[w - tx]/\overline{U}\}^{\frac{1}{1-\alpha}}$ and we obtain the rent-distance function

$$r(x) = \xi \times [w - tx]^{\frac{1}{(1-\alpha)A}} \text{ for } \xi = \left[\frac{(1-\alpha)}{Mi^{1-A}\overline{U}^{\frac{1}{1-\alpha}}}\left[\frac{\alpha}{p}\right]^{\frac{\alpha}{1-\alpha}}\right]^{\frac{1}{A}}.$$

Given rent ρ prevailing at the city edge, $\overline{x}$, we can solve for size $\overline{x}$ in $r(\overline{x}; \overline{U}) = \rho$. We can

[8] We treat each household a commuting to the exact center of the city for technical convenience. The alternative is to have each household commute to the edge of the CBD and this latter approach is slightly more complicated to deal with. A cleaner, more abstract approach is to have a CBD with zero area (eg. Black and Henderson [1999]).

then solve for city size in $N(\overline{x};\overline{U}) = \int_0^{\overline{x}} \frac{2\pi x}{L(x)} dx$ for

$$\begin{aligned} \frac{1}{L(x)} &= \frac{r(x)}{Aq(x)h(x)} \\ &= \frac{r(x)}{A(1-\alpha)[w-tx]}. \end{aligned}$$

Total rent is $R(\overline{x};\overline{U}) = \int_0^{\overline{x}} r(x) 2\pi x dx$.

3 Comparative Statics

We are interested in three particular comparative statics results.

(1) Amenity improvement (more sunshine, lower crime, less traffic congestion, etc.) Formally this is an increase in parameter ψ in the utility function. Since $\overline{U}$ is the open city "opportunity cost" utility level and $\psi U(c(x), h(x)) = \overline{U}$, an increase in ψ (as with more sunshine for our city), $U(c(x), h(x))$ must decline for each distance, x. (In our model above ψ was unity.[9] To obtain a new equilibrium (post amenity shock) we simply redo our calculations above with $\overline{U}$ set at a smaller value.) This yields the revised Rosen-Roback rule: cities with more amenities will "offer" workers there smaller commodity bundles $(c(x), h(x))$. A family is in a sense substituting more sunshine for a smaller home h and less c. Since w is unchanged, the smaller commodity bundles will result in a denser city with increased land rent (higher priced homes). Since a household at the edge experiences no change in prices for c and h, their bundle will have c and h shrink proportionately, given a homothetic utility function like the Cobb-Douglas, and their "extra" income will be directed to marginally more commuting ($\overline{x}$ increases). Every "interior" household will have its $\frac{c(x)}{h(x)}$ ratio rise. Extra income available from the contraction in each $(c(x), h(x))$ bundle for an interior household will, roughly speaking, end up in higher land rents. Hence the result: marginally more amenity on the utility side yields a larger city (larger $\overline{x}$), a larger population with on average a higher density, a rise everywhere except at the edge in the $\frac{c(x)}{h(x)}$ ratio, and a shift out in the rent function at every interior location. Calculations confirm these results. For parameter values,

[9] A more complicated formulation would not have the amenity index Ψ simply multiplicative.

$\alpha = 0.7$, $A = 0.35$, $\rho = 0.1$, $t = 0.1$, $i = 0.06$ and $w = 12.5$, we solved for two cities with distinct values for $\overline{U}$ and have recorded outputs in Table 1.

Table 1

$\overline{U}$	$\overline{x}$	$pop'n$	$r - intercept$
5.5	69.185	1.6086×10^5	215.0
5.4	70.200	1.917×10^5	258.0

The results confirm that with a decline in $\overline{U}$, the edge expands, rent shifts upward and population increases considerably. Density rises. Hence an amenity increase leaves us with a larger, denser city. O'Sullivan [2007; Figure 7-6] reports on population density for thirty-five large cities of the world. Of the densist fifteen, only two have what might be termed "cold climates", namely Moscow and Seoul. A website[10] lists 125 of large, dense cities of the world by density. US cities Los Angeles, San Francisco-Oakland, San Jose, New York, New Orleans, Las Vegas, Denver appear on the list, starting a place 90. New York and possibly Denver would have what we might consider non-sunny climates. If we believe that warm climate is a location amenity then we are seeing denser cities with "more sunshine".

For this Cobb-Douglas case above we have

$$\frac{dR(\overline{x})}{d\overline{U}} = -\frac{R(\overline{x})}{(1-\alpha)A\overline{U}} + \rho 2\pi\overline{x}\frac{d\overline{x}}{d\overline{U}}$$

with $\frac{d\overline{U}}{d\overline{x}} = -t \times \left[\frac{\rho}{\varphi}\right]$ for $\varphi = \left[\frac{(1-\alpha)}{Mi^{1-A}}\left[\frac{\alpha}{p}\right]^{\frac{\alpha}{1-\alpha}}\right]^{\frac{1}{A}}$. Hence with $\frac{dR(\overline{x})}{d\overline{U}} < 0$. With an increase in the local amenity indicator, $d\overline{U} < 0$ and net aggregate rent for the city rises.

(2) "Amenity" improvement on the production side.

On the production side we postulate production for export from the city at the center with a constant returns to scale function, $Q = \gamma F(K, N)$. Here γ is the "amenity" index. Given the rental on capital fixed at i and the price of output fixed "out there" at p^e, the wage is determined via the unit cost function. An increase in γ implies a higher wage, w via

[10] http://www.citymayors.com/statistics/largest
-cities-density-125.htm. This was consulted on April 4, 2008.

the unit cost function, $p^e = \zeta w^B i^{1-B}$. An increase in γ can be interpreted as a reduction in ζ.

Consider now a decrease in ζ in $p^e = \zeta w^B i^{1-B}$ which induces an increase in w. Consider the impact of this wage increase. Observe that if one differentiates $r(x)$ above with respect to w, one gets $\frac{dr(x)}{dw} = \frac{1}{(1-\alpha)A}\xi \times [w - tx]^{[\frac{1}{(1-\alpha)A}]-1}$, which is the same as $1/L(x)$. Hence

$$\frac{dR}{dw} = \int_0^{\overline{x}} \frac{1}{L(x)} 2\pi x dx + \rho 2\pi \overline{x} \frac{d\overline{x}}{dw}.$$

Hence we have the basic result $dR - \rho 2\pi\overline{x}d\overline{x} = N(\overline{x})dw$.[11] The wage increase aggregated over all workers is precisely capitalized in aggregate net land rent increase, where net refers to the netting out of the new rent ascribable to the edge expansion (and population expansion).

A numerical illustration with our above functions (Cobb-Douglas for both utility and production of housing) allows us to get a feel for the impact of not just a marginal increase in the local wage but also a non-marginal increase. Our parameter values for the runs reported in Table 2 are $\alpha = 0.7$, $A = 0.35$, $\rho = 0.1$, $i = 0.06$ and $\overline{U} = 5.7$.

Table 2

	w	t	$\overline{x}$	Pop'n	M	Agg. Rent
base	12.5	0.1	67.1552	114,350.1	2,457,320.0	124,282.6
	12.6	0.1	68.1552	124,386.1	2,695,296.0	136,255.6
	13.6	0.1	78.1552	278,370.35	6,538,746.2	328,856.03

For the first experiment, we increase the wage from 12.5 to 12.6. $\Delta R = 11973.0$ and $N\Delta w$ is 11936.8 (with N the average of the two values, 114,350.1 and 124,386.1). $\rho 2\pi\overline{x}d\overline{x} = 42.49$ (with $\overline{x}$ the average of 67.1552 and 68.1552). Thus $\Delta R - \rho 2\pi\overline{x}d\overline{x} = 11930.51$ which is quite close to $N\Delta w$ at 11936.8.

Our second experiment was a repeat of the first one, except for a "large" wage change, namely 1.1. Our main result is that net rent, $\Delta R - \rho 2\pi\overline{x}d\overline{x}$, falls short of change $N\Delta w$ for the large wage change experiment (204,071.53 compared with 215,996.25). The formula

[11]This result holds for any homogeneous utility function and homogeneous production function for housing. See the Appendix.

which we derived was of course for small changes (infinitesimals) and our "large change" experiment is a rough test of the robustness of our capitalization formula.

(3) Commuting cost reduction

We can repeat the derivation above for the case of a small reduction in commuting cost, t and obtain the capitalization formula, $dR - \rho 2\pi\overline{x}d\overline{x} = -Mdt$, for $-dt$ the result of say an infrastructure improvement. M is total roundtrip commuting miles in the city for all households. That is $M = \int_0^{\overline{x}} x\frac{2\pi x}{L(x)}dx$.We checked on this formula with a numerical example and the same parameters above.

Table 3

	w	t	$\overline{x}$	Pop'n	M	Agg. Rent
base	12.5	0.1	67.1552	114,350.1	2,457,320.0	124,282.6
	12.5	0.096	69.9534	124,077.8	2,777,461.46	134,855.3
	12.5	0.08	83.9440	178,670.0	4,799,453.75	194,191.6

Our first experiment involved a small reduction in t from 0.1 to 0.096. For this $dR - \rho 2\pi\overline{x}d\overline{x} = 10572.7 - 120.47 = 10452.23$ for the case of $\overline{x}$ the average of 67.1552 and 69.9534. $-Mdt = 10469.56$ for the case of M the average of 2,457,320.0 and 2,777,461.46. Hence our formula checks out reasonably well. We proceeded to an experiment with a "large" change in t. That is, $\Delta t = -.02$ for this large-change case. Here we observed that $dR - \rho 2\pi\overline{x}d\overline{x} = 69,112.45$ and $-Mdt = 72,567.74$. Clearly the rent-change under-estimates the value of the change in t for this large-change case. This is qualitatively the same as what we observed for the case of the large change in w above.

This commuting cost capitalization result also holds for general functional forms for utility and housing production. See the steps in the Appendix for the case of a wage increase.

4 Discussion

First, we should note that though our model resembles one for textbook small, open economy, it could be brought closer to the standard case if (a) land rent were returned as an equal lump sum to each household and (b) if the local price of say the composite good, c were endogenized so that the value of aggregate exports from the city equalled the value of the aggregate composite good consumed in the city. The "fix" for land rent return can be interpreted as the local provision for a public good consumed in equal amount by each household. We see no reason for the qualitative nature of our comparative statics results to change, though details would. Thus though what we have above is a rough version of a small, open economy, it lends itself to manipulation leading to useful comparative statics "predictions".

Two familiar shortcomings of our model are (a) only a single income group is present and (b) all jobs are located at the center. It is pretty well understood how to extend the model to accommodate multiple income groups when all workers continue to work at the center. The introduction of multiple groups should not change our qualitative results, though the analysis would become complicated when various boundaries between income groups shift under a comparative statics "shock". Secondly it is fairly straightforward to extend the textbook model to accommodate a fringe of suburban job centers (an annulus) if one accepts the idea that workers employed in lower densities are somewhat less productive (Ciccione and Hall [1996]) than others and thus command somewhat lower wages. One can then set out a suburban ring of employment centers with the workers who worker there commuting radially from farther out. Each of these outer households will achieve the same equilibrium $\overline{U}$ for the city but will have a lower wage than the counterpart worker who works in the center and commutes from inside the suburban job ring. There will be separate land rent functions for the two groups, center workers (inside commuters) and suburban workers (outer commuters) and the difference in wages allows us to join the two rent functions with no jump. If the suburban jobs are at distance $\widetilde{x}$ from the center then inside workers will command wage w and suburban workers wage w^o with $w - t\widetilde{x} = w^o$. The arrangement of wages will assure that the equilibrium $\overline{U}$ is the same for both types of workers and that the land rent function exhibits no jump at $\widetilde{x}$. Thus we argue that one can extend the monocentric city model to accommodate an outer ring of suburban job places without invoking a host of new economic arguments. Given the extended model, we see no reason for our comparative statics results not to remain valid in a qualitative sense.[12]

A fairly large extension of our model would have the location parameter in the production function, γ (in $Q = \gamma F(K, N)$) depend on the size of the city or perhaps on the density of the city. That is, $\gamma(N, \phi)$ might increase with N and say ϕ, where ϕ is both location and technology specific. Such a formulation would incorporate the current view that labor productivity is city-size specific and hence that local wages reflect a city-size productivity indicator. In this formulation, larger cities can offer similar workers higher wages because workers in larger cities are more productive simply because they are working "along side" many more workers. This seems like the proper extension of our formulation, in view of the work of Glaeser and Mare [2001] and others on the subject of higher wages in larger cities. We have not pursued this extension to our model because it complicates our analysis considerably to have a city size feedback onto our local wage in our comparative statics investigation. A positive feedback of city size to the current wage in the city becomes in our view a second order step from our perspective. First the increase in a utility-amenity boosts city size and density and then the increase in city size and density boosts local wages somewhat. This line of thinking runs counter to that of Rosen and Roback. They wanted an increase in an amenity impinging directly on households to lead to lower local wages as in a compensating response.

5 Concluding Remarks

We have developed the comparative statics result: an improvement in a local utility-based amenity index will show up in a contraction in the commodity bundle consumed by a representative local household. Cities with favorable climates should thus be somewhat denser than those with poorer climates since residents of the good-climate city will be living with somewhat higher land rents, *ceteris paribus*. This interesting prediction derives in a fairly straightforward way from a textbook economic model of an urban area (say the Mills model).

[12] McGrath [2005] and Spivey [2008] argue that their econometric results support traditional comparative statics predictions from the textbook Mills-Muth model first reported on by Brueckner and Fansler [1983]. They fail to emphasize that a model with multiple employment locations can exhibit qualitative behavior much like one with a single employment (the textbook Mills-Muth model). On the other hand their line of thought supports the idea that there is a basic textbook model of a modern city with agreed upon characteristics and thus we are not off base in appealling to "the standard textbook model" for our analysis.

In our view it is the size of the local commodity bundle of a representative household that changes with local amenities such as days of sunshine per year, traffic congestion, crime, etc rather than the local wage as Rosen and Roback contended. There is in addition a bundle composition effect of a change in a local amenity index. Urban form thus should reflect local utility-based urban amenities.

Our analysis contributes to the debate about why larger cities have workers with relatively high wages. If we buy into the Ciccione-Hall argument that the DENSITY of workers contributes strongly to local labor productivity, then our result that good local climate leads to a relatively high density for a city might provide an argument for explaining part of the large-city-high-wage link. The line of thinking from a positive "jump" for the local amenity to a higher local wage and city size runs counter to the basic thinking of Rosen and Roback[13], however.

[13] "This study has proven that the conventional wisdom which holds that only land prices are affected by local amenities is incorrect. The theory demonstrated that the value of the amenity is reflected in both the wage and the rent gradient. The precise decomposition depends on the influence of the amenity on production and the strength of consumer preferences." Roback [1982; p. 1275].

6 Appendix: The General Case

Once again, we can obtain our capitalization results for this model with housing stuctures for a general utility function and general production function for housing. At radial distance x, consumer and house "production" equilibrium satisfy the following six equations in $h(x), c(x), q(x), L(x), k(x)$ and $r(x)$:

$$\frac{U_c}{U_h} = \frac{1}{q(x)}, \text{ with the price of } c \text{ at unity,} \tag{1}$$
$$w - tx = c + q(x)h(x) \tag{2}$$
$$U(c(x), h(x)) = \overline{U} \tag{3}$$
$$h = h(L(x), k(x)), \text{ with function } h \text{ homogeneous of degree unity,} \tag{4}$$
$$\frac{h_L}{h_k} = \frac{r(x)}{i} \tag{5}$$
$$\text{and } h(x)q(x) = L(x)r(x) + ik(x). \tag{6}$$

Given constant returns to scale in housing "production", we have also that

$$q(x)h_k = i. \tag{7}$$

Using equations (1), (2) and (3), we can repeat our steps for the general case earlier and obtain

$$\frac{dq(x)}{dw} = \frac{1}{h(x)}. \tag{8}$$

Now from (4) and (5), we have

$$\begin{aligned} dh &= h_L dL + h_k dk \\ &= \{\frac{r(x)}{i} dL + dk\} h_k. \end{aligned} \tag{9}$$

From (6), we get

$$\begin{aligned} hdq + qdh &= rdL + Ldr + idk \\ &= Ldr + \left[\frac{rdL}{i} + dk\right] i \\ &= Ldr + \frac{idh}{h_k} \end{aligned}$$

using (9). This latter becomes

$$hdq - Ldr = \left[\frac{i}{h_k} - q\right] dh.$$

The term in square brackets is zero, given (7). Hence $h(x)dq(x) = L(x)dr(x)$. This in (8) yields

$$\frac{dr(x)}{dw} = \frac{1}{L(x)}.$$

Hence

$$\begin{aligned} \frac{dR}{dw} &= \int_0^{\overline{x}} \frac{dr(x)}{dw} 2\pi x dx + \rho 2\pi \overline{x} \frac{d\overline{x}}{dw} \\ &= N + \rho 2\pi \overline{x} \frac{d\overline{x}}{dw} \end{aligned}$$

and our capitalization result, $dR - \rho 2\pi \overline{x} d\overline{x} = N dw$ is established.

And the same steps yields our result on transportation improvements for the case of a general utility function and a general production function for housing, namely, $dR - \rho 2\pi \overline{x} d\overline{x} = -M dt$ for $M = \int_0^{\overline{x}} \frac{x}{L(x)} 2\pi x dx$. M is total commuting miles of all residents.

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