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Abstract

We find that a discount rate of 3.8% allows us to derive the schedule of "value of life years" in Murphy and Topel [2006] from their schedule of "value of remaining years of life", this latter presumably being based on a "value of statistical life" of $6.3 milliion. We draw on the Makeham function for life expectancy in our calculation.

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- key words: value of life, discount rate, life expectancy

1. Introduction

Murphy and Topel [2006] provide two interesting graphs (Figures 2 and 3) in their paper on the value of health improvements and the extension of human lifespan. In Figure 2, they present "value of life year" estimates for a representative male and female, ages 20 to 110, in say the United States and in Figure 3 they present "value of remaining life" estimates for ages 0 to 110. These latter figures are

*Boyan Jovanovic suggested to me that a "value of life year" should be obtainable from the derivative of the corresponding "value of remaining years of life" and this we have pursued here. Thanks to him and to participants in a seminar at McGill University in March, 2009. Bob Cairns kindly provided a careful critique of an earlier draft of this note.

capital values while the former are flow values per year. The value of life year numbers rise smoothly from \$200,000 per year at age 20 to \$360,000 at age 50 and then decline with a gentle point of inflection to \$70,000 at age 110. The values for the representative female lie slightly above those for the representative male. The value of remaining years of life numbers rise smoothly from \$6,700,000 at age 20 to \$6,900,000 at age 30 and decline smoothly to zero at age 110 with a point of inflection. At age 40 a male's value of remaining years is \$6.3 million, the same as the \$6.3 million "value of a statistical life" favored by Murphy and Topel. There is no explicit "documentation" linking these two figures; in particular no report of a discount rate employed or a life expectancy table or function. Here we tie these two series together by invoking a discount rate of 3.8% and an explicit life expectancy function, namely the Makeham life expectancy function, parameterized by Gavrilov and Gavrilova [1991; p. 76]. The Makeham function is an extension of the Gompertz life expectancy function and has an additional parameter that allows for a decline in the probability of dying after a person passes through infancy. Our exercise provides a complete "theory" and accounting for the claim that 3.8 represents a highly plausible value for a representative agent's consumption discount rate.

We exploit the idea that the capital value for remaining years of life at age x has a time derivative that "breaks out" the current value of a life year. Hence a time series of say life year estimates implies a time series of values of remaining years of life, given a life expectancy function and a discount rate, and *vice versa*. With a discount rate of 3.8%, we are able to generate the graph in Table 3 of Murphy and Topel from age 20 to age 54 from the data in the graph in Table 2. Beyond age 54 our "values of remaining years of life" decline too rapidly relative to those in Murphy and Topel's Figure 3 (to zero at age 76 rather than to zero at age 110). Thus 3.8% appears to emerge as a true discount rate employed by individuals up to age 54 but beyond this age, Murphy and Topel have "value of life years" that are too large for a discount rate of 3.8%, too large relative to the values in the schedule of "value of remaining years of life" (Table 3). Given our recursive equation (from our discrete time derivative), including now the discount rate of 3.8%, we proceed to calculate a schedule of lower value of life years beyond age 54 for our representative agent. These new values bring the revised schedules in Figures 2 and 3 into "compatibility" for a consumption discount rate of 3.8%. (We could instead have solved for the stated capital values beyond age 54, using the stated value of life years, and new discount rates for each year beyond age 54 above 3.8%.)

Though Murphy and Topel develop a fairly complete theory of a life-time consumption profile for a representative agent facing an uncertain span of life, they present few details of their "parameterization" of the model, essentially the process for generating the data in their Figure 3. A first-blush inference is that they sketched a life-time series of values of statistical life for a representative agent, with $6.3 million for age 40 as a reference point, and then backed out a series of value of life years. Precisely how they carried out the numerical backing out is not documented. We provide complete details here for linking their schedules in Figures 2 and 3 together and in that process arrive at a consumption discount rate of 3.8%. Murphy and Topel state that they generated their value of life years schedule first from standard sources and then generated their schedule of values of remaining years of life with the first series. The second series contains that important reference point, namely the value of remaining years of life at age 40 is precisely $6.3 million. So they may well have tried various values of the consumption discount rate until the second schedule passed through the $6.3 million dollar value for "the person" at age 40. To get this fit, they require a discount rate and as we just noted provide no information on this important issue. Here we find that a discount rate of 3.8% works, given the Makeham life expectancy function for the calculations.

2. The Analysis

The life expectancy of a person age z is defined as $A(z) = \int_z^\infty th(t)\frac{S(t)}{S(z)}dt$ for $h(t)$ the hazard function and $S(t)$ the survival function. This latter is a distribution function and $-\frac{dS}{dt}$ is the density function $h(t)S(t)$, with $\int_0^\infty h(t)S(t)dt = 1$. The function $h(t)\frac{S(t)}{S(z)}$ is also a density function with $\int_z^\infty h(t)\frac{S(t)}{S(z)}dt = 1$. If one dies, say in an accident at age 30, one would have attained, on average, age $\int_{30}^\infty th(t)\frac{S(t)}{S(30)}dt$. We investigate, instead of the age one would have attained, the current or present value of the "income" one would have attained. "Income" data are reported in Table 2 of Murphy and Topel. In our exercise, we will make use of the Makeham life expectancy function, $S(t) = \exp\{-Bt - \frac{\alpha}{\beta}\{\exp(\beta t) - 1\}\}$. This corresponds to the density $h(t)S(t)$ with $h(t) = [B + \alpha\exp(\beta t)]$. For a group of Swedish men, Gavrilov and Gavrilova find that $\alpha = 0.0000274$, $\beta = 0.104$, and $B = 0.00374$ yields a good estimate of life expectancy at birth of 71.4 years in 1925.[1] Starting at age 40, life expectancy rises to 80 years. They recommend reducing the value of B to capture an increase in the life expectancy of Swedish men since the 1920's (p. 77). We proceed to work with $\alpha = 0.0000274$, $\beta = 0.104$, and $B = 0.0000006$. The first two parameters are recommended by Gavrilov and Gavrilova (p. 76). We change their value of B to a low number to reflect the longer life expectancy of a Swedish male since the 1920's. Our parameter values yield a life expectancy at birth of 83.31 (standard deviation of 12.30), at age 20 of 83.36 (st. dev. 12.16), at age 30 of 83.44 (st. dev. 11.96), at age 40 of 83.64 (st. dev. 11.58), at age 50

[1] In the literature on life expectancy, observers note that for base years of say 30 to 50, the Gompertz function (Makeham function with $B = 0$) works fairly well. But specialists are always seeking functions that fit the data better (eg. Bebbington, Lai, and Zitikis [2007]).

of 84.06 (st. dev. 10.87), at age 60 of 84.96 (st. dev. 9.71), and at 70 of 86.72 (st. dev. 8.26).

If one dies, say in an accident at age 30, one foregoes the stream of utility to year x, in $\int_{30}^{x} u(c(v)) \exp(-\rho(v-30))dv$ $(= L(x))$ with probability $h(x)\frac{S(x)}{S(30)}$ and discount rate ρ. Hence unexpected death at age 30 results in an average or expected future lifetime loss of $V(30) = \int_{30}^{\infty} L(v)h(v)\frac{S(v)}{S(30)}dv$. Viscusi [2004] and others have provided estimates of the value of a life cut short by an accident. These are the so-called "values of a statistical life". As we noted above, Murphy and Topel select \$6.3 million as their favored estimate for "the value of a statistical life". Of interest then is the possibility of estimating a value for discount rate ρ in $\int_{30}^{\infty} L(v)h(v)\frac{S(v)}{S(30)}dv = \$6,300,000$ if we have dollar values for the $u(c(t))'s$ and a function for $S(t)$. Murphy and Topel provide the required dollar values for the $u(c(t))'s$ in their Figure 2 and Gavrilov and Gavrilova provide the required function $S(t)$, namely the Makeham function, in their monograph.[2] Hence we could solve for the true consumption discount rate ρ, for an individual at age 30 in the above equation.

We are to think of the $u(c(t))'s$ as emerging from utility maximization over an uncertain lifespan by a representative agent. Given these optimized values, $u(c^*(t))'s$, one can proceed to calculate $V(t) = \int_{t}^{\infty} L(v)h(v)\frac{S(v)}{S(t)}dv$ for various values of t. Murphy and Topel set out dollar values for each $u(c^*(t))$, our m_t $'s$, and then present in Figure 3 the derived schedule of $V(t)'s$. They define the "value of life year" (p. 881), $m(t)$ as the representative agent's value of her current "full consumption" weighted by "surplus per dollar of full consumption" plus "full income". On page 880 "full income and consumption" are defined by "adding

[2] The well-known Gompertz function is the Makeham function above with $B = 0$. The positive B parameter captures a somewhat "high" probability of death in infancy.

the shadow value of non-market time to each". In addition lifetime income is smoothed so that the is a flow of income to a person in retirement (post age 65). The value of a life year is then current consumption in dollars plus current value of leisure in dollars. The series is reported to be constructed from actual life-cycle wage and consumption data. However there is no information on the function $S(t)$ used or on the discount rate employed in moving from the schedule of $m_t\ 's$ to the schedule of $V(t)'s$. Here we find that a discount rate of 3.8% works in the "translation" from $m_t\ 's$ to $V(t)'s$ and we draw on the Makeham function for our $S(t)$ in this "translation".

Instead of solving for various distinct values for the $V(t)\ 's$ directly, given the schedule of $m_t\ 's$, we now consider taking the time derivative of $V(t) = \int_t^\infty L(x)h(x)\frac{S(x)}{S(t)}dx$, "breaking out" an expression for m_t for each date, and solving for a schedule of $V(t)'s$ in a recursive fashion. In particular we are interested in arriving at a value for the discount rate that allows us to generate the data in Figure 3, given those in Figure 2. We define $\delta = 1/(1+\rho)$ and use time subscripts to make our expressions more compact. For $u(c(t))$ we now have dollar valued m_t. We consider the time derivative in discrete time.[3]

In discrete time we now have value of remaining years of life in

$$\begin{aligned} V_t &= m_t h_t \frac{S_t}{S_t} + \{m_t + \delta m_{t+1}\} h_{t+1} \frac{S_{t+1}}{S_t} + \{m_t + \delta m_{t+1} + \delta^2 m_{t+2}\} h_{t+2} \frac{S_{t+2}}{S_t} + ... \\ V_{t+1} &= m_{t+1} h_{t+1} \frac{S_{t+1}}{S_{t+1}} + \{m_{t+1} + \delta m_{t+2}\} h_{t+1} \frac{S_{t+2}}{S_{t+1}} \\ &\quad + \{m_{t+1} + \delta m_{t+2} + \delta^2 m_{t+3}\} h_{t+3} \frac{S_{t+3}}{S_{t+1}} + ... \end{aligned}$$

[3]In continuous time, the time derivative of $V(t) = \int_t^\infty \int_t^x m_v \exp(-\rho(v-30)) dv h(x) \frac{S(x)}{S(t)} dx$ is $\frac{dV(t)}{dt} = -m_t + [\rho + h_t] V(t)$.

If we multiply V_{t+1} on both sides by $\delta \frac{S_{t+1}}{S_t}$ and subtract from the first expression, we obtain

$$V_t - \delta \frac{S_{t+1}}{S_t} V_{t+1} = m_t \sum_{z=t}^{\infty} h_z \frac{S_z}{S_t}.$$

The term $\sum_{z=t}^{\infty} h_z \frac{S_z}{S_t}$ approximates $\int_t^{\infty} h_z \frac{S_z}{S_t} dz$, this latter equalling unity. Hence we have the basic recursion

$$V_{t+1} = \{V_t - m_t\} \frac{S_t}{S_{t+1}} \frac{1}{\delta}.$$

This is the equation that we employed to generate a schedule of V_t $'s$, given the schedule of m_t $'s$ in Figure 2 of Murphy and Topel. For S_t we drew on the Makeham function with parameters reported above. We obtained an excellent replication of Murphy and Topel's V_t $'s$ in their Figure 3 with $\rho = 3.8\%$ up to age 54. Beyond age 54, our V_t $'s$ declined too rapidly. We observed that the value of life years in Murphy and Topel's Figure 2 are too large for years beyond age 54. To complete our investigation of a 3.8% discount rate, we simply reversed our "thinking" at age 54 and invoked the Murphy-Topel "value of remaining years of life" schedule and solved for the remaining schedule of m_t $'s$ (the value of life years). At age 65, we worked out $m_t = \$276,351.8$ in place of the Murphy-Topel \$300,000; at age 75, we worked out $m_t = \$185,344.2$ in place of the Murphy-Topel \$214,000; at age 85, we worked out $m_t = \$118,764.3$ in place of the Murphy-Topel $\$140,000$; and at age 95, we worked out $m_t = \$90,000$ in place of the Murphy-Topel \$105,000. Hence with relatively minor changes to the "primary data", we found that a consumption discount rate of 3.8% works well in the "translation" of the data in Murphy and Topel's Figure 2 to those in Figure 3.

Since $S(t+1)/S(t) \cong (1 - h(t))$, our basic recursion can be expressed as

$$\frac{m_t + V_{t+1} - V_t}{V_t} \cong \left[\frac{\rho + h_t}{1+\rho}\right] \frac{V_{t+1}}{V_t}$$

This in continuous time is

$$\frac{m_t + dV_t/dt}{V_t} = \rho + h_t.$$

The capital theory here[4] is that V_t is capturing the uncertain present value of a non-stochastic stream of earnings, m_t. Since h_t is the probability of our representative agent dying in the current period, the current earnings of our asset (the representative agent), namely $\frac{m_t+dV_t/dt}{V_t}$ "requires" a risk premium, above the certain rate, ρ. It is reasonable to argue that h_t rises significantly for our person above say age 70 and in addition that V_t becomes relatively small and unchanging. Thus m_t might not be relatively small for a person above 70 and this we observe for the m_t schedule in Murphy and Topel, as well as in our adjusted m_t schedule. This is something of a paradox. Our representative agent exhibits a very low value of statistical life (measured here by V_t) in old age while also exhibiting a not-so-low value of life year. Beyond age 30, the representative agent in Murphy and Topel ends up with her capital value ever declining. This implies of course that $\frac{dV_t/dt}{V_t}$ is negative for much of the agent's life and this tends to shift m_t $'s$ upward over the latter "stages" of the agent's life. One can argue that the reason that Murphy and Topel have a schedule of m_t $'s$ above ours for our agent beyond age 54 is that they worked with a schedule of larger h_t $'s$ for the agent beyond age 54. Our h_t $'s$ emerge from the Makeham life expectancy function, parameterized by Gavrilov and Gavrilova.

[4] One should resist the temptation to treat $V_{t+1} \cong (1+\rho)V_t$ in the discrete time expression since that would imply that $\frac{V_{t+1}-V_t}{V_t} \cong \rho$. And this latter relationship would imply that $m_t/V_t \cong h_t$, a very special result.

3. Comment

In fact, Murphy and Topel report the use of a discount rate of 3% later in their article and one might infer that this is the value that they used in "translating" their value of a life year estimates into their "value of remaining years of life" estimates. But this remains a conjecture. They also left unclear what function they were using for our $S(t)$ and whatever function they chose may have contributed to the difference we are observing in "the" discount rate (our 3.8 compared with say their 3%). When extra risk is involved later in their article, they suggest using the estimate of 3.5% in calculations. In any case we have linked their two striking plots together with an explicit life expectancy function and a discount rate of 3.8%. Some other calculations we carried out support our view that a discount rate of between 3.5 and 4% works well in linking up the two central series in Murphy and Topel. We think that a dispassionate reader would find the estimates for the values of remaining years of life plausible, given what is claimed for estimates of the values of statistical life. Hence the plausibility of our 3.8% estimate for the discount rate really rests on the matter of the plausibility of the values Murphy and Topel have arrived at for the values of life years. Such series have not been worked out by many other investigators and thus the reader is left to scrutinize what Murphy and Topel have to say on this matter. We have quoted their thinking in brief above.

Finally, we note that a different approach might consider the agent's discount rate declining with age *a priori*, as with hyperbolic discounting (eg. Azfar[1999]). This would require an approach more complicated than ours to linking the two key schedules in Murphy and Topel together.

References

[1] Azfar, O. [1999] "Rationalizing Hyperbolic Discounting" *Journal of Economic Behavior and Organization*, 38, pp. 245-52.

[2] Bebbington, Mark, Chin-Diew Lai and Ricardas Zitikis [2007] "Modeling Human Mortality using Mixtures of Bathtub-shaped Failure Distributions", *Journal of Theoretical Biology*, 245, pp. 528-538.

[3] Gavrilov, Leonid A. and Natalia S. Gavrilova [1991] *The Biology of Life Span: A Quantitative Approach*, Chur: Harwood.

[4] Murphy, Kevin M., Robert H. Topel, [2006] "The Value of Health and Longevity", *Journal of Political Economy*, vol. 114, no. 5, October, pp. 871-904.

[5] Viscusi, W. Kip [2004] "The Value of Life: Estimates with Risks by Occupation and Industry" *Economic Inquiry* 42 (1), January, pp. 29-48.