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# Optimal Unemployment Insurance in GE: a RobustCalibration Approach 

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# Optimal Unemployment Insurance in GE: a Robust Calibration Approach 

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#### Abstract

This paper implements a simple Monte Carlo calibration approach to quantitatively study the Hansen and Imrohoroglu (1992) economy, a GE model with uninsurable employment risk, designed to assess the optimal replacement rate for a public Unemployment Insurance scheme.

The results of this sensitivity analysis are consistent with the original findings, but with several caveats. One novel result in particular is that the sampling distribution of the optimal UI is bimodal. Depending on the calibrated parameters, the optimal UI is in one of two regions: a very generous scheme with high replacement rates, where insurance is mainly provided by the UI scheme, or one with low replacement rates, where insurance is mainly achieved through self-insurance. Even in the absence of moral hazard, it is never optimal to provide full insurance. Moreover, for many plausible parameters' configurations, the welfare maximizing replacement rate does not decrease with the level of MH. The qualitative patterns and quantitative findings are not altered substantially when considering an enlarged labor force, which includes the marginally attached workers. Finally, the parameters representing the hours worked, the leisure share in the households' consumption bundle, and the risk aversion have a first order impact on the average welfare. The determination of the optimal UI scheme depends heavily on them. This finding suggests that extra caution should be paid when calibrating these parameters in similar environments.


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## 1 Introduction

Unemployment Insurance (UI) programs are widespread systems designed to provide insurance to workers experiencing unemployment spells. Public expenditures devoted to UI and assistance programs are sizeable in most OECD Countries, being in the order of $2-3 \%$ of GDP, as reported by Martin (2002). Given their importance, several theoretical and applied studies have been analyzing the welfare consequences of UI schemes, together with their effects on job search efforts and unemployment duration.

A very influential paper in this area is Hansen and Imrohoroglu (1992). Arguably, this contribution started the theoretical and quantitative assessment of UI programs in models with Heterogeneous Agents (HA), later studied by Hopenhayn and Nicolini (1997), Acemoglu and Shimer (1999), Abdulkadiroglu, Kuruscu and Sahin (2002), Young (2004), Pollak (2007), Shimer and Werning (2008), Pavoni (2009), Krusell, Mukoyama, and Sahin (2010), and Boostani, Gervais and Siu (2011), among others.

The purpose of this paper is to use a robust calibration procedure to quantify the optimal UI scheme. To this end, a thorough robustness exercise for the environment proposed by Hansen and Imrohoroglu (1992) will be performed. This global robustness check will implement a Monte Carlo procedure. Relying on the methodology developed by Canova (1994), priors for the parameters' distributions consistent with the available evidence are going to be postulated. The model is going to be solved 1,000 times, each time with a different calibration drawn from the joint distribution of parameters' priors, delivering sampling distributions for the optimal UI program.

The overall message of Hansen and Imrohoroglu (1992) is confirmed, with several caveats: optimal UI replacement rates tend to be quantitatively large, and the presence of Moral Hazard (MH) does play a role in decreasing them. However, there are some additional novel findings.

First, the sampling distribution of the optimal UI appears to be bimodal. This implies that, even in the absence of MH , there are many empirically plausible calibrations such that low replacement rates are optimal. The $60 \%$ threshold seems to be the watershed of two different regions: one region with high replacement rates, low precautionary savings and low consumption inequality, and another one with the opposite outcomes. In the first case insurance is mainly provided by the UI scheme, while in the second one consumption smoothing is achieved mainly through self-insurance.

Second, increasing the level of MH in the economy decreases the optimal UI. But this is only true on average. Just like for the Hansen and Imrohoroglu (1992) calibration, for several cases, the higher the MH, the lower the optimal replacement rate. However, in no less than $26 \%$ of the experiments, the welfare maximizing replacement rate does not decrease below $60 \%$ with the level of MH. Even with pervasive MH , there is still a large fraction of economies for which high replacement rates are optimal.

Third, taking a different stand on the labor force composition leads to similar qualitative patterns and quantitative findings. To this end, an enlarged labor force including also the marginally attached workers is considered in the model economy.

Fourth, even in the absence of moral hazard, it is never optimal to provide full income insurance. The increased taxation stemming from both a more generous scheme and higher unemployment rates makes it too costly to give a $100 \%$ replacement rate. A basic trade-off between insurance and search incentives prevents full insurance to be the welfare maximizing scheme.

Fifth, the number of hours worked, the leisure share in the households' consumption bundle, and the risk aversion have a first order impact on the determination of average welfare, hence on the selection of the optimal UI scheme. This finding suggests that extra caution should be paid when calibrating these parameters in similar environments and calls for some robustness checks with respect to them.

The rest of the paper is organized as follows. Section 2 presents the theoretical model. Section 3 is devoted to the description of the Monte Carlo experiments. Section 4 presents the main results, while Section 5 concludes. A series of appendices provide more details on the methodology together with some additional results.

## 2 The Hansen and Imrohoroglu (1992) Economy

The economy is a GE one with incomplete markets and uninsurable employment risk, designed to quantify the optimal UI program, with and without MH. The UI scheme is specified as a fixed replacement rate of labor earnings, while MH arises because of limited observability of the unemployed agents' actions. This allows a fraction of unemployed workers rejecting employment opportunities to be able to keep collecting UI benefits. Crucially, idiosyncratic employment shocks together with incomplete markets and a no-borrowing constraint give a role for social insurance on top of selfinsurance. Time is discrete. The economy is populated by a measure one of infinitely lived ex-ante identical agents. ${ }^{1}$

### 2.1 Preferences

Agents' preferences are assumed to be represented by a time separable utility function $u($.$) . Their$ utility is defined over stochastic processes for consumption and leisure $\left\{c_{t}, l_{t}\right\}_{t=0}^{\infty}$. Agents choose how much to consume $\left(c_{t}\right)$, whether to work or not (when given the opportunity to choose) a fixed amount

[^1]of hours $0<\widehat{h}<1$ by setting their desired leisure $\left(l_{t}=\{1-\widehat{h}, 1\}\right)$, and how much to save in a non interest bearing asset $\left(m_{t+1}\right)$ in each period of their lives, in order to maximize their objective function. The agents' problem can be defined as:
$$
\max _{\left\{c_{t}, l_{t}, m_{t+1}\right\}_{t=0}^{\infty}} E_{0} u\left(c_{0}, l_{0}, c_{1}, l_{1}, \ldots\right)=\max _{\left\{c_{t}, l_{t}, m_{t+1}\right\}_{t=0}^{\infty}} E_{0} \sum_{t=0}^{\infty} \beta^{t} U\left(c_{t}, l_{t}\right)
$$
where $E_{0}$ represents the expectation operator over the employment opportunity shocks $s \in S=\{u, e\}$, and $\beta \in(0,1)$ is the subjective discount factor. Following Hansen and Imrohoroglu (1992), the perperiod utility function $U(.,$.$) is assumed to be strictly increasing, strictly concave, and to satisfy the$ Inada conditions. The specific functional form is $U\left(c_{t}, l_{t}\right)=\frac{\left(c_{t}^{1-\sigma} l_{t}^{\sigma}\right)^{1-\rho}-1}{1-\rho}$, which has a $R R A=\rho$ that maps into its "consumption only" counterpart that is typically estimated with the transformation $\widehat{\rho}=1-(1-\sigma)(1-\rho)$.

### 2.2 Endowments

Agents can be employed $(e)$, or unemployed $(u)$. Every agent is endowed with the same exogenous efficiency units: when employed they all produce the same amount of output $y$, normalized to $y=1$. If employed, they earn a wage $w=y$ and pay proportional taxes $(\tau)$ to finance the unemployment benefit scheme. If unemployed, agents collect unemployment benefits, which are specified as a constant replacement rate $\theta$ of the going wage, and pay proportional taxes $(\tau)$.

Employment opportunities are stochastic, and follow a two-state first-order Markov process. The transition function of the employment opportunity state is represented by the matrix $\chi_{\left(s^{\prime}, s\right)}=$ $[\chi(i, j)]$, where each element $\chi_{(i, j)}$ is defined as $\chi_{(i, j)}=\operatorname{Pr}\left\{s_{t+1}=i \mid s_{t}=j\right\}, i, j=\{u, e\}$. Once the state $s=e$ is realized, the agent can still reject this employment opportunity, by setting his labor supply to zero. The unemployment rate and the unemployment duration in this economy are a mix of choice and chance. The persistent stochastic process gives the agents the opportunity to work, while at the same time the generosity of the UI scheme potentially gives disincentives for agents to accept these jobs.

Agents can refuse employment opportunities and, because of imperfect monitoring, still receive unemployment benefits, which leads to a form of MH to be present in the economy. The parameter capturing the extent of this feature is $\pi(\eta)$, namely the probability that an agent turning down a job offer is not detected. This probability might depend on the agent's previous employment status $\eta$. Finally, UI eligibility is represented by $\mu$, the workers with $\mu=0$ being the non eligible ones. If eligible agents turning down a job are not detected, they can collect the UI benefits. ${ }^{2}$

[^2]There are no state-contingent markets to insure against the idiosyncratic employment risk, but workers can self-insure by saving into the asset $m_{t}$, while facing a no-borrowing constraint, so that $m_{t+1} \geq 0$.

## 3 (Monte Carlo) Calibration

Following Canova (1994), the model economy is parametrized with Monte Carlo methods. Several combinations of parameters are drawn from a set of prior distributions, and the model is solved for each realization of the parameters vector. Hence, this calibration procedure embeds in itself a global sensitivity analysis. With this methodology it is possible to: 1) provide evidence on extensive robustness checks, 2) have sampling distributions for the outcomes of interest, showing which results are more likely to be relevant and which ones appear to be less plausible, 3) study which parameters affect the determination of the computed equilibria the most, informing other researchers in the field on what parameters should be calibrated more carefully. ${ }^{3}$

## [Table (1) about here]

Table (1) provides the parameters' list in the Hansen and Imrohoroglu (1992) economy, together with their upper and lower values that are going to be used in the simulations. These values are chosen to match some macroeconomic facts and to be consistent with the available empirical evidence. In order to check if the findings are robust to relatively large changes in the parameters, and if they are well behaved with respect to them, the parameters' bounds are chosen to allow for a wide range of possible calibrations. At the same time, an attempt is made to prevent the model to grossly miss some quantitative implications. This could happen in several replications if the parameters' ranges were to be too wide.

There are six independent parameters: $\beta, \sigma, \rho, \widehat{h}$, and the two probabilities in the Markov chain, $\chi_{\left(e^{\prime}, e\right)}$ and $\chi_{\left(e^{\prime}, u\right)}$. Canova (1994) provided an intuitive yet powerful procedure to implement global robustness checks for fully parametrized quantitative models. The methodology acknowledges parameters' uncertainty and assumes prior distributions for the model's parameters. At the simulation stage, it repeatedly solves the model for many different calibrations, rather than computing the quantitative implications of a theory for a unique set of calibrated parameters.

Specifying prior distributions to calibrate the model has some positive implications.

[^3]First, it makes a global sensitivity analysis feasible. An alternative approach would be to discretize the parameters' space and consider all possible combinations of parameters' grid points. Unfortunately, for this class of models, this is not a viable option. With several parameters, six in this case, even considering only 5 points in each dimension would lead to another manifestation of the curse of dimensionality. This would generate 15,625 possible combinations: given the average computing time of an optimal UI scheme, it would take no less than 50 days to complete each experiment on a top of the line workstation. As in other applications, randomization can help in breaking the curse of dimensionality.

The second implication is more grounded in economics. As stressed in the Bayesian literature, for example by DeJong, Ingram, and Whiteman (2000) and An and Schorfheide (2007), specifying a set of prior distributions for the parameters allows the researcher to exploit some of the available estimates, giving more weight to the parameters' values that are more likely to be empirically relevant.

Finally, standard calibration procedures tend to overlook parameters' uncertainty. This can arise from uncertainty on the moments the model should match, from the intrinsic uncertainty of the estimates used to pin down a subset of the parameters, or by the unavoidable abstractions that a model makes. These issues are tackled by the simple Monte Carlo methodology used here.

### 3.1 Calibration and Global Sensitivity Analysis

The calibration procedure consists of performing a series of Monte Carlo experiments, which rely on two steps.

In the first step, the prior distributions from which the parameters are going to be drawn are postulated. In the second step, 1,000 economies are simulated and solved. These economies differ only in the parameters' vector that is drawn at each replication $i$. For every economy, the equilibria for all possible UI replacement rates $\theta$ are computed. Notice that the computation of the equilibrium requires an iterative procedure on the tax rate $\tau$ until the government achieves a balanced budget. Once the sequence of equilibria for the economies with all the different $\theta$ 's are computed, a welfare measure comparison determines which UI scheme $\theta^{*}$ is optimal, namely the one that achieves the highest average steady-state welfare, with a utilitarian functional. ${ }^{4}$

As an additional robustness check, these experiments are repeated several times, with different priors for the parameters' distributions, with different assumptions on the degree of moral hazard (i.e. how likely it is for unemployed workers to reject a job offer while keeping the entitlement to the UI benefits), and with different assumptions on the nature of the labor market.

[^4]
## [Table (2) about here]

Table (2) reports the list of experiments that are going to be considered. More in detail, four sets of experiments ( $B 1-B 4$ ) are going to be reported.

As for the parameters' priors, two independent Beta distributions that vary in their dispersion are specified: a $\operatorname{Beta}(10,10)$ and a $\operatorname{Beta}(2,2) .{ }^{5}$ The choice settled on these distributions for two reasons: 1) with the priors centered around the Hansen and Imrohoroglu (1992) calibration, it is possible to assess if there is a tendency for the outcomes to cluster around their results; 2$)$ the $\operatorname{Beta}(10,10)$ distribution is substantially more concentrated than the $\operatorname{Beta}(2,2)$ one, hence by increasing the dispersion of the priors it is possible to appreciate the effect of relying on more extreme calibrations, that differ more markedly from the benchmark one. Figure (1) shows the shape of the two Beta priors over the $[0,1]$ interval.

## [Figure (1) about here]

As for MH, three cases are going to be considered: a no MH case, with $\pi(u)=0 \%$, a mild MH case, with $\pi(u)=10 \%$, and a more severe MH case, with $\pi(u)=50 \%$. The first case is characterised by perfect monitoring of the unemployed agents, because the UI agency never allows workers turning down jobs to collect UI benefits. In the second and third cases there is a $10 \%$ and a $50 \%$ chance, respectively, that an unemployed worker turning down a job opportunity will still be able to collect UI benefits.

For each prior-MH combination, two different assumptions on the nature of the labor market are going to be made. In the baseline model, the labor force is going to be the same as the one in Hansen and Imrohoroglu (1992). In this case, considered in experiments $B 1$ and $B 2$, the underlying concept of unemployment is the one defined and computed by the relevant statistical agencies, for example in the U.S. by the Bureau of Labor Statistics. The alternative specification, considered in experiments $B 3$ and $B 4$, takes a different stand on the nature of unemployment in the model economy, and on the

[^5]related interpretation of the UI program. Jones and Riddell (1999) show that in the data it is hard to distinguish between individuals who are genuinely unemployed and individuals who are formally not unemployed, but they still are in the labor force. There is some evidence that workers who are marginally attached to the labor force, namely those who are not actively looking for a job, have labor market outcomes that are similar to the ones experienced by the workers that are formally counted as unemployed. With these facts in mind, compared to the baseline model, it seems reasonable to consider an enlarged labor force, where also the discouraged workers and people that are in the labor force but would not be necessarily considered as unemployed workers are included into the labor market figures. Accordingly, this alternative assumption calls for a different interpretation of the UI scheme. In this case, it will implicitly refer to a broader concept of a public assistance program. The main characteristics of this program are: a) its financing, obtained through general taxation, b) the value of the benefits, which is tightly linked to the labor earnings of the employed workers, and c) the recipients of the benefits, who are the workers who do not have a job.

The experiments $B 1$ and $B 3$ assume that all parameters are distributed according to a $\operatorname{Beta}(10,10)$. This is a symmetric and fairly concentrated distribution, whose average for every parameter is picked to coincide with the values used in the calibration by Hansen and Imrohoroglu (1992). Case B1 will deal with the standard definition of the unemployment rate, while case $B 3$ will deal with the alternative definition, and it will target the employment rate instead.

The other experiments, denoted by $B 2$ and $B 4$, consider as the parameters' priors the $\operatorname{Beta}(2,2)$ distribution. The averages of the priors are kept to the values that are consistent with the typical calibrations of these models, the one used by Hansen and Imrohoroglu (1992) in particular.

More precisely: $\beta^{a v g}=\beta^{H I 92}=0.995, \sigma^{a v g}=\sigma^{H I 92}=0.67, \rho^{a v g}=\rho^{H I 92}=2.5, \widehat{h}^{a v g}=\widehat{h}^{H I 92}=$ $0.45, \chi_{\left(e^{\prime}, u\right)}^{\text {avg }}=\chi_{\left(e^{\prime}, u\right)}^{H I 92}=0.5$ and $\chi_{\left(e^{\prime}, e\right)}^{\text {avg }}=\chi_{\left(e^{\prime}, e\right)}^{H I 92}=0.9681$.

The upper and lower values for $\beta,(\bar{\beta}=0.999$ and $\underline{\beta}=0.991)$, imply a discount rate at the annual frequency of $0.87 \%$ and $8.1 \%$, while the average $\beta^{a v g}=0.995$ implies a discount rate of $4.44 \%$. Working with a time period of six weeks opens the issue of how individuals discount future economic quantities over short time horizons. Identification problems lead to a lack of consensus on the point estimates. The available values for labor market environments, for example estimated by van den Berg (1990) and Dey and Flinn (2005), suggest large ranges for the discount rates. This explains why a relatively broad set of values for $\beta$ is considered. ${ }^{6}$

As for $\sigma$ and $\rho$, the main target pinning them down are the estimates of the Intertemporal Elasticity of Substitution (IES), obtained by Attanasio, Banks, Meghir, and Weber (1999), among others. It

[^6]goes without saying that, given a value $\hat{\rho}$ for the $R R A=\frac{1}{I E S}$ coefficient, there are infinite pairs of $\sigma$ and $\rho$ that satisfy it. This consideration supports the view that an extensive sensitivity analysis is needed. The upper and lower values for $\sigma$ and $\rho$ are chosen to satisfy a restriction on the adjusted $R R A(\widehat{\rho})$, which has to lie in the [1.0, 2.5] interval.

## [Figure (2) about here]

Figure (2) shows the distributions of the Relative Risk Aversion that are implied by the values of $\sigma$ and $\rho$ that are used in the simulations. As it can be seen, $\hat{\rho}$ lies always in the [1.0, 2.5] interval, which represents the range of available estimates for this parameter. As pointed out by Hansen and Imrohoroglu (1992), it was also found that the results are quite sensitive to the specific value of the $R R A$ coefficient. This is shown in Figure (2): the outcomes, namely being in a high ( $\theta^{*}>60 \%$ ) vs. low ( $\theta^{*} \leq 60 \%$ ) optimal UI regime, differ systematically with the average and the dispersion of the $R R A$ coefficient. The higher the $R R A$, the lower the likelihood that a low replacement rate is optimal. ${ }^{7}$

As for the utility costs of working, the share of market time is represented by the parameter $\widehat{h}$. Data from the American time use survey for the year 2009 show that on average employed people with children spend (on working days) approximately 7.7 hours sleeping, and 8.7 working. The benchmark calibration considered $\widehat{h}^{H 192}=0.45$. This value was kept as the prior's average, while the lower bound was $\underline{\widehat{h}}=0.35$ (in this case sleeping is included in the definition of leisure, notice that $\frac{8.7}{24}=0.36$ ), and the upper bound was $\overline{\widehat{h}}=0.55$ (in this case only non-sleeping time is considered to be available for market and non market activities, notice that $\frac{8.7}{24-7.7}=0.53$ ).

Finally, the Markov chain parameters in the benchmark calibration $\left(\chi_{\left(e^{\prime}, e\right)}^{H I 92}=0.9681\right.$ and $\left.\chi_{\left(e^{\prime}, u\right)}^{H I 92}=0.5\right)$ imply an unemployment rate of $6 \%$ and an unemployment duration of two model periods, namely 12 weeks. ${ }^{8}$ The model considers a stationary environment, hence the relevant labor market figures are long-run averages. However, 1) labor market statistics vary wildly over the cycle, 2) in the last 20 years the long-run unemployment duration has increased. This explains the ranges that are considered for $\chi_{\left(e^{\prime}, e\right)}$ and $\chi_{\left(e^{\prime}, u\right)}$. The upper and lower bounds for the latter imply an unemployment duration of 9 and 20 weeks. While the upper and lower values for the unemployment rate are $10.1 \%$ and $2.3 \%$.

As argued above, it seems important to check the robustness of the results with respect to the measurements in the labor market. These cases are denoted with $E 3$, for the $\operatorname{Beta}(10,10)$ priors, and with $E 4$, for the $\operatorname{Beta}(2,2)$ priors.

[^7]To achieve this target, a substantially different parametrization is going to be specified for the Markov chain governing the employment opportunities. For these experiments, $\chi_{\left(e^{\prime}, e\right)}^{a v g}=0.87$ and $\chi_{\left(e^{\prime}, u\right)}^{a v g}=0.4$. Although these might seem to be pretty minor changes, quantitatively they imply drastically higher unemployment rates and longer average unemployment durations, even for the cases when no job opportunities are turned down. These values imply an employment rate of $75.7 \%$, which roughly matches the employment rate for the U.S. economy, and an unemployment duration of two and half model periods, namely 15 weeks. The upper and lower bounds for $\chi_{\left(e^{\prime}, u\right)}$ imply a (minimum) average unemployment duration of 10 and 30 weeks. While the upper and lower values for the (potential) employment rate are $96.6 \%$ and $49.6 \%$.

## 4 Results

This Section presents the main results. First, it documents how allowing for very flexible calibrations does quantitatively alter the optimal UI replacement rates. Second, it provides a discussion on how the endogenous variables, namely the aggregate asset holdings, the tax rate, and the variance of consumption behave in this global robustness exercise. ${ }^{9}$

## [Tables (3) and (4) about here]

Tables (3) and (4) report a set of statistics for the outcomes that are the focus of the analysis. The minimum, maximum, mean, median and coefficient of variation are listed for all experiments, which are a mix of parameters' priors, degree of MH, and labor market definition.

Moreover, in order to provide a more intuitive summary of such a rich analysis, several figures are included. Most figures show six panels. In these cases, each panel represents the non-parametric kernel density estimate of an endogenous variable, for a particular prior-MH pair and a given definition of the labor market. The figures presenting more panels show an additional breakdown of the experiments, which are ordered according to the level of their optimal UI replacement rate $\theta^{*}$.

### 4.1 Equilibrium Outcomes

Figures (3) and (4) display the welfare maximizing UI replacement rates $\theta^{*}$ coming from the solution of the model, calibrated 1,000 different times for each experiment B1-B4.

The first striking result is that the overall patterns are shared by all experiments.

[^8]For any prior-MH-labor market triplet, the sampling distribution of the optimal UI is bimodal, a fact that has a natural economic interpretation. Broadly speaking, there are two possible optimal UI regions: one region is characterized by a high replacement rate, low precautionary savings and low consumption inequality. In this region insurance is mainly provided by the UI. When the MH and the utility costs of working are not too severe, it is inexpensive for the government to provide generous UI benefits, which in turn lead to reasonable tax rates. On the other hand, the other optimal regime entails the opposite outcomes. In this case, insurance is achieved mainly through self-insurance, because the incentives to stay unemployed for too long a period, together with the high expenditure per unemployed worker, make a generous UI system too costly in terms of taxation.

The $60 \%$ threshold seems to be the level of the replacement rate that separates the two different regions. What is striking, is that the same threshold persists when moving to economies with a higher level of MH, when moving to economies with more dispersed priors, and to economies with a different underlying labor market. ${ }^{10}$

Quantitatively, the median $\theta^{*}$ across experiments falls in the [ $60 \%, 70 \%$ ] range, dropping sharply to the $[10 \%, 36 \%]$ range only for the high MH cases. Notice that the variability of $\theta^{*}$, as represented by its coefficient of variation, increases monotonically in the degree of MH , a result that is discussed below.

## [Figures (3), (4), (5) and (6) about here]

The results for the optimal UI sampling distributions are mirrored by the behavior of savings, of consumption inequality and of the equilibrium tax rate. Detailed statistics about these endogenous variables are provided in Tables (3) and (4).

Figures (5) and (6) depict the sampling distributions of the average asset holdings. Also this variable has bimodal distributions, because the households compensate with their savings behavior the level of insurance provided by the UI system. A generous system induces low precautionary savings, because being unemployed does not make consumption smoothing too difficult. Differently, high precautionary savings are needed for all UI schemes that do not provide enough funds to achieve a relatively flat consumption profile over all possible states. This also explains why savings are increasing with the level of MH (moving left to right in either figure), and with the number of workers receiving UI payments (case $B 3$ or $B 4$ in Figure 6 vis-à-vis case $B 1$ or $B 2$ in Figure 5). The median of the

[^9]aggregate asset holdings is zero, or close to zero, for most cases. The only exceptions being the cases with the highest degree of MH. Furthermore, the variability of the aggregate asset holdings decreases monotonically in the degree of MH , which, again, highlights the substitution that takes place between public and self-insurance.

Figures (7) and (8) provide the tax rates sampling distributions. For the standard definition of the labor market the distributions are not bimodal, and are fairly concentrated around the $4 \%$ value. Even though for a given parameterization the equilibrium tax rates are monotonically increasing in $\theta$, their actual behavior depends on the optimal $\theta^{*}$. This is why the tax distributions do not shift to the right when a higher MH is considered. The decrease in the optimal $\theta^{*}$ triggered by a more pronounced MH prevents taxes to increase systematically, leading to a compression of the distribution towards lower tax rates. This feature shows how the equilibrium nature of the model is crucial. MH gives unemployed workers the chance to reject job offers. The higher the $\theta$ the more likely for workers to turn down employment opportunities. Potentially, this would lead to a decrease in total output, driving down the pool of resources that can be redistributed, and would drive up taxes. The only way to prevent this from happening is to keep replacement rates relatively low. Quantitatively, for the standard definition of the labor market, tax rates above $15 \%$ (with medians in the [ $1.1 \%, 4.4 \%$ ] range) are very rarely needed to finance the UI scheme. For the alternative definition of the labor market tax rates as high as $42 \%$ (with medians in the $[8.3 \%, 17.3 \%]$ range) are needed to achieve a balanced budget. This could explain why public assistance programs granting high benefits are not observed in reality. They could entail too high distortions and/or not pass a majority voting reform.

Finally, Figures (9) and (10) document the sampling distributions of the variance of consumption. The economy displays a low degree of consumption inequality every time the UI scheme and selfinsurance are able to mitigate the market incompleteness, although the figures suggest that in many instances market incompleteness does bite. Interestingly, more extreme calibrations lead to a lower consumption inequality when MH is not too pervasive, as observed in cases $B 2$ and $B 4$.
[Figures (7), (8), (9) and (10) about here]

### 4.2 Optimal UI Replacement Rates and Moral Hazard

Another way of looking at the previous findings is to recognize that there are several empirically plausible calibrations such that low replacement rates are optimal. This is an interesting outcome, because it takes place even in the absence of MH, something that Hansen and Imrohoroglu (1992) did not find, suggesting that their result stems from their specific calibration.

Increasing the level of MH in the economy does decrease the optimal UI, as Hansen and Imrohoroglu (1992) reported. But this is true only on average. For several calibrations, when increasing the MH there is still a large fraction of economies where high optimal replacement rates, even well above $75 \%$, survive.

Across experiments, a significant difference appears to be the share of economies with high optimal UI schemes (namely with $\theta^{*}>60 \%$ ). By comparing, say, the no MH case with the prior $B 1$ to the corresponding case with the prior $B 2$ in Figure (3), it is clear that the mass of replications with $\theta^{*}>60 \%$ is higher in the former experiment. The same pattern is found when moving from the cases with the concentrated priors $B 1(B 3)$ to the corresponding case with the more dispersed priors $B 2$ (B4).

## [Table (5) about here]

Table (5) provides a related statistic: the fraction of replications that display an optimal UI replacement rate that is above $60 \%$. This share ranges from $76.2 \%$ for the case $B 3$ with no MH, to $26.3 \%$ for the case $B 1$ with the highest MH.

It is also worth stressing that these shares are systematically higher for each case in the experiments B3 (B4) than their counterparts in the experiments B1 (B2). Economies with worse employment opportunities and higher unemployment durations command, on average, a higher $\theta^{*}$. The rationale for this result is as follows. The crucial aspect that allows high replacement rates to stay optimal in these environments is the lack of incentives for unemployed agents to turn down an unemployment offer, when they have one. Rejecting a job is a very risky choice, because the absence of full insurance makes it costly (unless the disutility from working is particularly large). Jobs are relatively rare to obtain, so MH has a lower influence in these cases. The experiments assuming enlarged labor markets represent economies where aggregate welfare can improve by trading-off lower taxation for higher public insurance, because individuals will not often turn down jobs. This can also be seen in the behavior of the employment differential, namely the difference between the potential employment rate (as driven by the Markov chain) and the actual one (as driven by both the Markov chain and the agents' choices): it decreases or remains stable with the degree of MH. ${ }^{11}$

Finally, even in the absence of moral hazard, it is never optimal to provide full income insurance. The increased taxation stemming from higher unemployment rates makes it too costly to give a $100 \%$ replacement rate. A basic trade-off between insurance and search incentives prevents full income insurance to be the welfare maximizing scheme.

[^10]
### 4.3 Labor Force

The nature of the labor market, including the definition of the labor force, and the related labor market risk seem to be key when assessing the welfare effects of UI schemes. Typically, in applied GE, labor market risk is considered as pertaining to the most restrictive definition of the unemployment rate. It is then interesting, in a robustness exercise, to consider the effects of an enlarged labor force, which includes the marginally attached workers, and whose main characterization derives from the employment rate data, as it was discussed above.

Taking a different stand on the labor force composition does lead to similar qualitative patterns. However, comparing the results obtained from the enlarged labor force economies to the restricted labor force ones highlights some differences in the quantitative findings.

Quantitatively, the former cases have higher optimal UI schemes (on average), coupled with higher aggregate assets (on average). It goes without saying that the drawback of higher public insurance is a higher taxation rate.

Higher labor market risk calls for more insurance, which is a combination of UI and asset holdings. Overall, social and private insurance seem to work well, because they achieve relatively similar distributions of consumption inequality in the two substantially different labor markets, as comparing the statistics in Tables (3) and (4) show. Across experiments, the mean and median of the consumption variance are surprisingly similar in the two definitions of the labor market. The range of the mean (median) is $[0.007,0.013]([0.005,0.013])$ in the experiments $B 1-B 2$, while it is $[0.010,0.017]$ ([0.008, 0.019]) in the experiments B3-B4.

### 4.4 Comparative Dynamics

The simulated economies form an artificial dataset that can be exploited to analyse how changes in the parameters affect the endogenous variables. Moreover, it is possible to understand which parameters have a stronger effect on the outcomes: with a Monte Carlo approach, regression analysis, instead of calculus, allows for the study of comparative dynamics.

Table (6) provides the standardized beta coefficients of a Pooled OLS regression with the average steady state utility on the LHS, and the six parameters on the RHS. ${ }^{12}$

## [Table (6) about here]

[^11]There are several results that can be read from Table (6).
First, there are two trends in the explanatory power of this simple linear regressions. More in detail, a substantial drop in the $R^{2}$ of the regression is evident when comparing the experiments with a more concentrated prior to the ones with a more disperse prior. This shows that, in the more extreme calibrations, non linearities become important in the determination of the average steadystate welfare. The second trend in the $R^{2}$ is related to the degree of MH. When moving from the no MH case to the most extreme case, a big drop can be seen (from 0.90 to 0.56 in experiment 11 , for example), possibly because of some effects starting to offset each other (e.g. the utility loss of working vs. the partial loss of public insurance). With a high enough MH non linearities become predominant, which suggests to take the following comments with caution for those cases.

Second, the standardized beta coefficients associated with the hours worked, the leisure share in the households' consumption bundle, and the intertemporal elasticity of substitution document that these parameters have a first order impact on the determination of average welfare, hence on the selection of the optimal UI scheme.

As for $\widehat{h}$, in the regressions with a good fit, its beta coefficient ranges between $[-.735,-.594]$ : this is the parameter that affects the average steady state utility the most. The other two important parameters for the determination of aggregate welfare are $\sigma$, with a range of $[-.613,-.167]$, and $\rho$, which with a range of $[-.414,-.345]$ shows also the most consistent effect.

Both $\chi_{\left(e^{\prime}, e\right)}$, the job retention probability, and $\chi_{\left(e^{\prime}, u\right)}$, the job finding probability, have negligible effects in the experiments $B 1$ and $B 2$, while their quantitative importance in affecting the average utility becomes larger in the experiments $B 3$ and $B 4$. This result could be accounted for by a level effect. Very low probabilities for workers to keep their current job and find one while unemployed mechanically determine a high unemployment rate. This leads a large chunk of the labor force to rely on UI, making a generous system very expensive. Small changes in these probabilities can have first order welfare effects due to the reduced need to save and reduced costs of in terms of taxes. This interpretation seems to be confirmed by the decreasing importance of these parameters with respect to the degree of MH.

Finally, the effect of $\beta$ is always tiny, at least for the subset of the parameter's space considered here.

Further evidence on how the parameters affect the determination of the optimal UI system is given in Figures (11) and (12).
[Figures (11) and (12) about here]

Figure (11) focuses on the parameter $\widehat{h}$, while Figure (12) focuses on $\sigma$, and they show several
distributions. Each Figure displays, at the very left, two panels. These represent the unconditional distributions of a parameter in a given experiment. ${ }^{13}$ As expected, given the high number of replications, they approximate well the prior distributions from which they are sampled.

The other six panels (per experiment) condition the estimated parameters' densities on the computed optimal UI regime found in that replication, namely if $\theta^{*}$ was less than $60 \%$ or if it was above this threshold. Whenever the top distribution differs systematically from the bottom one, there is some selection taking place. This implies that differences in that parameter are likely to induce a different regime in the optimal UI scheme. Put it differently, the values of that parameter have a major impact for the main outcome of the analysis.

For example, Figure (12) clearly shows that the UI schemes with $\theta^{*} \leq 60 \%$ come from a distribution of the parameter $\sigma$ that has a higher mean and a lower variance than the distribution of the same parameter leading to optimal replacement rates $\theta^{*}>60 \%$. Similarly, Figure (11) shows that the distribution of the parameter $\widehat{h}$ such that $\theta^{*} \leq 60 \%$ has consistently a higher mean than its counterpart with $\theta^{*}>60 \%$. These results seem important, because such clear patterns are not detected for the other parameters.

These findings suggest where caution should be paid when calibrating similar environments. Other researchers in the field should calibrate carefully the parameters $\widehat{h}, \sigma$ and $\rho$, and perform robustness checks with respect to them.

## 5 Conclusions

This paper applied Canova (1994) methodology to perform a thorough sensitivity analysis for the Hansen and Imrohoroglu (1992) economy. This is a calibrated GE model with incomplete markets and uninsurable employment risk, designed to quantify the size of the optimal replacement rate for a government-run unemployment insurance scheme.

The results of this global robustness analysis are broadly consistent with the original findings. A novel finding is that the sampling distribution of the optimal UI is bimodal. According to the 1,000 calibrations, for any degree of moral hazard considered, it is optimal for the UI to be in one of two regions: a very generous scheme with a high replacement rate and low savings, or one with a low replacement rate and high savings. In the first region insurance is mainly provided by the UI scheme, while in the second one it is self-insurance that does the job.

One of the main findings of Hansen and Imrohoroglu (1992) is the sharp decrease of the optimal

[^12]replacement rate when the economy is plagued by MH. Here it was found that, in no less than $26 \%$ of the experiments, the welfare maximizing replacement rate did not decrease below $60 \%$ when the level of MH was increased.

The paper shows how the findings are affected particularly by the calibration of three parameters: the number of hours worked, the share of leisure in the households' expenditure, and the curvature of the per-period utility function. This suggests that similar studies based on a calibration approach should pay special attention to the values assigned to these parameters, and should provide some robustness checks with respect to them.

A positive message resulting from this paper, and from Cozzi (2011), is that more robust calibration methods are feasible also for rich HA economies, usually used to perform (ex-ante) policy evaluations. This methodology should be interpreted as complementary to full-blown structural estimation techniques. Notice that an advantage of this analysis is the ability to provide ranges for the outcomes of interest, which not only allows to better understand the mechanics of the model, but also goes along the lines of Manski (1995) and Sims (2004).

A possible extension could be to consider fully Bayesian methods, proposed for example by DeJong, Ingram, and Whiteman (1996), even though computational limits might still be binding for the computation of the likelihood function and the posterior distributions. More general UI policies (specifying both the replacement rate and the maximum duration of the benefits), and a more general GE set-up (for example with market clearing in the asset market as well) could also be studied. I leave these modifications for future work.

| Parameter | Description | Min | Max |
| :--- | :--- | :--- | :--- |
| Model Period | Six weeks |  |  |
| $\beta$ | Rate of time preference | $\underline{\beta}=0.991$ | $\bar{\beta}=0.999$ |
| $\sigma$ | Leisure Share | $\underline{\sigma}=0.37$ | $\bar{\sigma}=0.97$ |
| $\rho$ | Risk Aversion | $\underline{\rho}=1.5$ | $\bar{\rho}=3.5$ |
| $\widehat{h}$ | Share of Market Time | $\underline{\widehat{h}}=0.35$ |  |
| $\chi_{\left(e^{\prime}, e\right)}$ | Job Retention Probability | $\underline{\chi_{\left(e^{\prime}, e\right)}}= \begin{cases}0.937, B 1-B 2 \\ 0.75, B 3-B 4 \\ 0.3, B 1-B 2 \\ 0.2, B 3-B 4\end{cases}$ | $\overline{\chi_{\left(e^{\prime}, e\right)}=}$ <br> $\chi_{\left(e^{\prime}, u\right)}$ |

Table 1: Model parameters and their support

|  | Experiment B1 | Experiment B2 | Experiment B3 | Experiment B4 |
| :--- | :--- | :--- | :--- | :--- |
| Parameters Priors | Beta $(10,10 ; \underline{p}, \bar{p})$ | Beta $(2,2 ; \underline{p}, \bar{p})$ | Beta $(10,10 ; \underline{p}, \bar{p})$ | Beta $(2,2 ; \underline{p}, \bar{p})$ |
| Labor MKT target | Unemployment rate | Unemployment rate | Employment rate | Employment rate |

Table 2: Experiments - Prior Distributions over a generic parameter p and Labor market targets

|  | Min | Max | Mean | Med. | C.V. | Min | Max | Mean | Med. | C.V. |
| :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- |
|  | $1)$ | Opt. | UI |  |  | $2)$ | Assets |  |  |  |
| $B 1_{M H=0}$ | .14 | .90 | .604 | .66 | .234 | 0 | 1.020 | .117 | 0 | 1.181 |
| $B 1_{M H=0.1}$ | .04 | .82 | .518 | .65 | .422 | 0 | 1.849 | .341 | 0 | 1.177 |
| $B 1_{M H=0.5}$ | .02 | .82 | .280 | .10 | 1.056 | 0 | 3.175 | 1.137 | 1.262 | .774 |
| $B \boldsymbol{2}_{M H=0}$ | .02 | .98 | .559 | .66 | .423 | 0 | 1.699 | .154 | 0 | 1.242 |
| $B \mathcal{2}_{M H=0.1}$ | .02 | .90 | .474 | .62 | .626 | 0 | 3.896 | .388 | 0 | 1.323 |
| $B \boldsymbol{2}_{M H=0.5}$ | .02 | .90 | .346 | .12 | 1.001 | 0 | 4.793 | .835 | .611 | 1.112 |
|  | $3)$ | Tax |  |  |  | $4)$ | Var. of | Cons. |  |  |
| $B 1_{M H=0}$ | .018 | .077 | .042 | .041 | .240 | .001 | .030 | .009 | .006 | .205 |
| $B 1_{M H=0.1}$ | .007 | .087 | .038 | .038 | .296 | .001 | .029 | .010 | .007 | .671 |
| $B 1_{M H=0.5}$ | .001 | .073 | .019 | .011 | .909 | .001 | .030 | .013 | .013 | .532 |
| $B \mathcal{2}_{M H=0}$ | .002 | .185 | .045 | .044 | .505 | .0001 | .034 | .007 | .005 | .923 |
| $B \boldsymbol{2}_{M H=0.1}$ | .002 | .310 | .042 | .038 | .635 | .0001 | .035 | .008 | .005 | .936 |
| $B \boldsymbol{2}_{M H=0.5}$ | .001 | .253 | .031 | .022 | .989 | .0001 | .038 | .009 | .006 | .871 |

Table 3: Monte Carlo Results - Optimal UI, Assets, Tax Rate and Consumption Variance

|  | Min | Max | Mean | Med. | C.V. | Min | Max | Mean | Med. | C.V. |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
|  | 1) | Opt. | $U I$ |  |  | 2) | Assets |  |  |  |
| $B 3_{M H=0}$ | . 02 | . 84 | . 595 | . 70 | . 392 | 0 | 3.084 | . 356 | 0 | 1.953 |
| B3 ${ }_{M H}=0.1$ | . 02 | . 84 | . 563 | . 70 | . 469 | 0 | 3.434 | . 546 | 0 | 1.685 |
| B3 ${ }_{M H=0.5}$ | . 02 | . 84 | . 382 | . 35 | . 746 | 0 | 4.395 | 1.380 | 1.124 | . 924 |
|  | . 02 | . 92 | . 524 | . 70 | . 614 | 0 | 5.029 | . 552 | 0 | 1.672 |
| B4 M H $=0.1$ | . 02 | . 92 | . 497 | . 70 | . 685 | 0 | 5.015 | . 709 | 0 | 1.485 |
| B4 MH=0.5 | . 02 | . 94 | . 407 | . 36 | . 866 | 0 | 5.216 | 1.169 | . 736 | 1.113 |
|  | 3) | Tax |  |  |  | 4) | Var. of | Cons. |  |  |
| $B 3_{M H=0}$ | . 011 | . 305 | . 159 | . 173 | . 403 | . 003 | . 031 | . 012 | . 011 | . 500 |
| $B 3_{M H}=0.1$ | . 007 | . 305 | . 151 | . 170 | . 475 | . 003 | . 033 | . 013 | . 011 | . 525 |
| $B 3_{M H}{ }_{\text {M }}{ }^{\text {a }}$ | . 004 | . 280 | . 106 | . 098 | . 720 | . 003 | . 034 | . 017 | . 019 | . 453 |
| B4 MH=0 | . 005 | . 421 | . 142 | . 127 | . 677 | . 0003 | . 042 | . 010 | . 008 | . 776 |
| B4 M H $=0.1$ | . 005 | . 411 | . 135 | . 120 | . 742 | . 0003 | . 042 | . 010 | . 008 | . 787 |
| B4 M H $=0.5$ | . 002 | . 411 | . 113 | . 083 | . 872 | . 0003 | . 043 | . 012 | . 010 | . 760 |

Table 4: Monte Carlo Results - Optimal UI, Assets, Tax Rate and Consumption Variance

| $B 1_{M H=0}$ | $B 1_{M H=0.1}$ | $B 1_{M H=0.5}$ | $B 2_{M H=0}$ | $B 2_{M H=0.1}$ | $B 2_{M H=0.5}$ |
| :--- | :--- | :--- | :--- | :--- | :--- |
| $62.0 \%$ | $54.8 \%$ | $26.3 \%$ | $56.5 \%$ | $51.3 \%$ | $38.9 \%$ |
|  |  |  |  |  |  |
| $B 3_{M H=0}$ | $B 3_{M H=0.1}$ | $B 3_{M H=0.5}$ | $B 4_{M H=0}$ | B4 $_{M H=0.1}$ | $B 4_{M H=0.5}$ |
| $76.2 \%$ | $71.9 \%$ | $31.7 \%$ | $61.1 \%$ | $58.8 \%$ | $42.1 \%$ |

Table 5: Fraction of Experiments with an optimal UI $>0.60$

|  | $B 1_{M H=0}$ | $B 1_{M H=0.1}$ | $B 1_{M H=0.5}$ | $B 2_{M H=0}$ | $B 2_{M H=0.1}$ | $B 2_{M H=0.5}$ |
| :--- | :--- | :--- | :--- | :--- | :--- | :--- |
| $E U$ |  |  |  |  |  |  |
| $\widehat{h}$ | -.622 | -.610 | -.147 | -.606 | -.612 | -.225 |
| $\beta$ | .012 | .009 | -.038 | .021 | .017 | -.054 |
| $\chi_{\left(e^{\prime}, e\right)}$ | .028 | .032 | .037 | .051 | .064 | .053 |
| $\chi_{\left(u^{\prime}, u\right)}$ | .004 | $-.001^{*}$ | $-.001^{*}$ | $-.003^{*}$ | -.012 | -.010 |
| $\sigma$ | -.580 | -.613 | -.219 | -.256 | -.336 | -.189 |
| $\rho$ | -.395 | -.384 | -.148 | -.417 | -.417 | -.227 |
| $R^{2}$ | 0.90 | 0.92 | 0.56 | 0.55 | 0.60 | 0.33 |
|  | $B 3_{M H=0}$ | $B 3_{M H=0.1}$ | $B 3_{M H=0.5}$ | $B 4_{M H=0}$ | $B 4{ }_{M H=0.1}$ | $B 4{ }_{M H=0.5}$ |
| $E U$ |  |  |  |  |  |  |
| $\widehat{h}$ | -.735 | -.622 | -.205 | -.594 | -.501 | -.274 |
| $\beta$ | .012 | .006 | -.023 | $-.000^{*}$ | $-.002^{*}$ | -.053 |
| $\chi_{\left(e^{\prime}, e\right)}$ | .181 | .179 | .017 | .260 | .239 | .054 |
| $\chi_{\left(u^{\prime}, u\right)}$ | -.089 | -.079 | -.012 | -.152 | -.134 | -.018 |
| $\sigma$ | -.379 | -.437 | -.275 | -.167 | -.244 | -.224 |
| $\rho$ | -.414 | -.345 | -.133 | -.406 | -.351 | -.230 |
| $R^{2}$ | 0.92 | 0.78 | 0.57 | 0.72 | 0.61 | 0.41 |

Table 6: Comparative Dynamics on Average Welfare - POLS Regressions


Figure 1: Monte Carlo Experiments: Beta $(2,2)$ and $\operatorname{Beta}(10,10)$ priors.


Figure 2: CRRA - Estimated Densities for the Monte Carlo Experiments B1-B2


Figure 3: Optimal UI - Estimated Densities for the Monte Carlo Experiments B1-B2


Figure 4: Optimal UI - Estimated Densities for the Monte Carlo Experiments B3-B4


Figure 5: Average Asset - Estimated Densities for the Monte Carlo Experiments B1-B2


Figure 6: Average Asset - Estimated Densities for the Monte Carlo Experiments B3-B4


Figure 7: Tax Rate - Estimated Densities for the Monte Carlo Experiments B1-B2


Figure 8: Tax Rate - Estimated Densities for the Monte Carlo Experiments B3-B4


Figure 9: Consumption Inequality - Estimated Densities for the Monte Carlo Experiments B1-B2


Figure 10: Consumption Inequality - Estimated Densities for the Monte Carlo Experiments B3-B4


Figure 11: Parameter $\widehat{h}$ - Estimated Densities for the Monte Carlo Experiments B1-B2


Figure 12: Parameter $\sigma$ - Estimated Densities for the Monte Carlo Experiments B1-B2

## References

Abdulkadiroglu, A., Kuruscu, B., and Sahin, A. (2002). "Unemployment Insurance and the Role of Self-Insurance," Review of Economic Dynamics, Vol. 5, 681-703.

Acemoglu, D., and Shimer, R. (1999). "Efficient Unemployment Insurance," Journal of Political Economy, Vol. 107, 893-928.

An, S., and Schorfheide, F. (2007). "Bayesian Analysis of DSGE Models," Econometric Reviews, Vol. 26, 113-172.

Attanasio, O., Banks, J., Meghir, C., and Weber, G. (1999). "Humps and Bumps in Lifetime Consumption," Journal of Business and Economic Statistics, Vol. 17, 22-35.

Boostani, R., Gervais, M., and Siu, H. (2011). "Optimal Unemployment Insurance in a Directed Search Model," University of Southampton, mimeo.

Canova, F. (1994). "Statistical Inference in Calibrated Models," Journal of Applied Econometrics, Vol. 9, S123-S144.

Cozzi, M. (2011). "Precautionary Savings and Wealth Inequality: a Global Sensitivity Analysis," QED Working Paper n. 1270.

DeJong, D., Ingram, B. and Whiteman, C. (1996). "A Bayesian Approach to Calibration," Journal of Business and Economic Statistics, Vol. 14, 1-9.

DeJong, D., Ingram, B. and Whiteman, C. (2000). "A Bayesian Approach to Dynamic Macroeconomics," Journal of Econometrics, Vol. 98, 201-223.

Dey, M., and Flinn, C. (2005). "An Equilibrium Model of Health Insurance Provision and Wage Determination," Econometrica, Vol. 73, 571-627.

Hansen, G. and Imrohoroglu, A. (1992). "The Role of Unemployment Insurance in an Economy with Liquidity Constraints and Moral Hazard," Journal of Political Economy, Vol. 100, 118-142.

Hopenhayn, H., and Nicolini, J.P. (1997). "Optimal Unemployment Insurance," Journal of Political Economy, Vol. 105, 412-438.

Krusell, P., Mukoyama, T., and Sahin, A. (2010). "Labour Market Matching with Precautionary Savings and Aggregate Fluctuations," Review of Economic Studies, Vol. 77, 1477-1507.

Jones, S. and Riddell, C. (1999). "The Measurement of Unemployment: an Empirical Approach," Econometrica, Vol. 67, 147-162.

Manski, C. (1995). Identification Problems in the Social Sciences, Harvard University Press, Cambridge (MA).

Martin, J. (2002). "What Works among Active Labour Market Policies: Evidence from OECD Countries' Experiences," OECD Economic Studies, Vol. 30, 79-113.

Pavoni, N. (2009). "Optimal Unemployment Insurance, with Human Capital Depreciation, and Duration Dependence," International Economic Review, Vol. 50, 323-362.

Pollak, A. (2007). "Optimal Unemployment Insurance with Heterogeneous Agents," European Economic Review, Vol. 51, 2029-2053.

Shimer, R., and Werning, I. (2008). "Liquidity and Insurance for the Unemployed," American Economic Review, Vol. 98, 1922-1942.

Sims, C. (2004). "Econometrics for Policy Analysis: Progress and Regress," De Economist, Vol. 152, 167-175.
van den Berg, G. (1990). "Nonstationarity in Job Search Theory," Review of Economic Studies, Vol. 57, 255-277.

Young, E. (2004). "Unemployment Insurance and Capital Accumulation," Journal of Monetary Economics, Vol. 51, 1683-1710.

## Appendix A - The Model and its Recursive Representation

## 6 Stationary Equilibrium

In this Section I first define the problem of the agents in their recursive representation, then I provide a formal definition of the equilibrium concept used in this model, the recursive competitive equilibrium.

The individual state variables are asset holdings $m \in \mathcal{M}=[0, \bar{m}]$, labor market status $s \in \mathcal{S}=$ $\{u, e\}$, previous employment status $\eta=\{u, e\}$ and UI eligibility $\mu=\{0,1\}$. The stationary distribution is denoted by $\lambda(m, s, \eta, \mu)$.

### 6.1 Problem of the workers

The value function of an agent whose current asset holdings are equal to $m$, whose current labor status is $s$, and whose previous employment status is $\eta$ is denoted with $V(m, s, \eta)$. The problem of these agents can be represented as follows:

$$
\begin{aligned}
& V(m, s, \eta) \\
& =\left\{\begin{array}{l}
\max _{m^{\prime}}\left\{U\left(m+(1-\tau) \theta y-m^{\prime}, 1\right)+\beta \sum_{s^{\prime}} \chi_{\left(s^{\prime}, u\right)} V\left(m^{\prime}, s^{\prime}, \eta^{\prime}\right)\right\}, s=u \\
\max \left\{\max _{m^{\prime}}\left\{U\left(m+(1-\tau) y-m^{\prime}, 1-\widehat{h}\right)+\beta \sum_{s^{\prime}} \chi_{\left(s^{\prime}, e\right)} V\left(m^{\prime}, s^{\prime}, e\right)\right\},\right. \\
\pi(\eta)\left[\max _{m^{\prime}}\left\{U\left(m+(1-\tau) \theta y-m^{\prime}, 1\right)+\beta \sum_{s^{\prime}} \chi_{\left(s^{\prime}, e\right)} V\left(m^{\prime}, s^{\prime}, u\right)\right\}\right]+ \\
\left.[1-\pi(\eta)]\left[\max _{m^{\prime}}\left\{U\left(m-m^{\prime}, 1\right)+\beta \sum_{s^{\prime}} \chi_{\left(s^{\prime}, e\right)} V\left(m^{\prime}, s^{\prime}, u\right)\right\}\right]\right\}, s=e
\end{array}\right.
\end{aligned}
$$

$$
\text { s.t. } m^{\prime} \geq 0
$$

Agents have to set optimally their consumption/savings and their labor supply. They enjoy utility
from consumption, disutility from working and face some uncertain events in the future. In the next period they could have an employment opportunity that they can turn down, which is going to affect their chances of receiving the UI benefits. The production side of the model is represented by a linear technology which relies on labor to produce the final output. Each employed worker produce the same amount of output $y$. Notice that the interest rate is set to $0 \%$.

### 6.2 Recursive Stationary Equilibrium

Definition 1 For a given UI policy $\theta$, a recursive stationary equilibrium is a set of decision rules $\left\{c(m, s, \eta, \mu), m^{\prime}(m, s, \eta, \mu), \eta^{\prime}(m, s, \eta)\right\}$, value functions $V(m, s, \eta)$, a tax rate $\tau$ and stationary distributions $\lambda(m, s, \eta, \mu)$ such that:

- Given the tax rate $\tau$, the individual policy functions $\left\{c(m, s, \eta, \mu), m^{\prime}(m, s, \eta, \mu), \eta^{\prime}(m, s, \eta)\right\}$ solve the household problem (1) and $V(m, s, \eta)$ are the associated value functions.
- The goods market clears

$$
\sum y \eta^{\prime}(m, s, \eta) \lambda(m, s, \eta, \mu)=\sum c(m, s, \eta, \mu) \lambda(m, s, \eta, \mu)
$$

- The government balances the budget

$$
\sum\left\{(1-\tau) \theta y[\lambda(m, e, \eta, 1)+\lambda(m, u, \eta, 1)]-\tau y \eta^{\prime}(m, e, \eta) \lambda(m, e, \eta, 0)\right\}=0
$$

- The stationary distributions $\lambda(m, s, \eta, \mu)$ satisfy

$$
=\left\{\begin{array}{c}
\lambda\left(m^{\prime}, s^{\prime}, \eta^{\prime}, \mu^{\prime}\right)  \tag{2}\\
\sum_{s, \eta, \mu, m: m^{\prime}(m, s, \eta, \mu)=m^{\prime}} \chi_{\left(s^{\prime}, s\right)} \lambda(m, s, \eta, \mu), \text { if } s^{\prime}=u, \mu^{\prime}=1 \\
\sum_{\left(s^{\prime}, s\right)} \lambda(m, s, \eta, \mu)\left\{\eta^{\prime}\left(.^{\prime}\right)+\left[1-\pi\left(\eta^{\prime}(.)\right)\right]\left[1-\eta^{\prime}(.)\right]\right\}, \text { if } s^{\prime}=e, \mu^{\prime}=1 \\
\sum_{s, \eta, \mu, m: m^{\prime}(m, s, \eta, \mu)=m^{\prime}} \chi_{\left(s^{\prime}, s\right)} \lambda(m, s, \eta, \mu) \pi\left(\eta^{\prime}(.)\right)\left[1-\eta^{\prime}\left(.^{\prime}\right)\right], \text { if } s^{\prime}=e, \mu^{\prime}=0 \\
0, \text { if } s^{\prime}=u, \mu^{\prime}=0
\end{array}\right.
$$

In equilibrium the measure of agents in each state is time invariant and consistent with individual decisions, as given by the functional equation (2) above.

## Appendix B - Computation

- All codes solving the economies were written in the FORTRAN 95 language, relying on the Intel Fortran Compiler, build 11.1.048 (with the IMSL library). They were compiled selecting the O3 option (maximize speed), and without automatic parallelization. They were run on a 64 -bit PC platform, running Windows 7 Professional Edition, with an Intel Q6600 Quad Core 2.4 Ghz processor.
- Typically, each replication takes between 10 and 22 minutes. The actual computing time depends essentially on the discount factor $\beta$ that is drawn, how far the initial guess on the tax rate is from the equilibrium one, and on how many iterations are needed to find the optimal value functions and the stationary measures. This means that, for any prior distribution, the whole Monte Carlo procedure takes between 7 and 15 days to complete. However, the Quad Core processor allows to run at least four cases simultaneously, with a $100 \%$ load, without losing any computational speed. Obviously, more recent Quad and Hexa Core CPU's could handle even more simultaneous simulations, that would be completed in less time. Running some simulations with a faster Intel Core i7 2600-k Quad Core 3.4 Ghz processor approximately halved the computational time.
- In the actual solution of the model the continuous state variable $m$ needs to be discretized (the employment status and the UI eligibility are already discrete). An unevenly spaced grid is allowed for. In order to keep the computational burden manageable, 301 grid points on the asset space are used, the lowest value being the (no) borrowing constraint ( $m_{\min }=0$ ) and the highest one being a value $m_{\max }>\bar{m}$ high enough for the saving functions to cut the 45 degree line $\left(m_{\max }=20\right)$.
- The model is solved with a 'successive approximation' procedure on the set of value functions. Non-differentiabilities in the value functions prevent from using faster Euler equation and interpolation methods. I start from a set of guesses $V(m, s, \eta)_{0}$. For each point in the discretized state space I solve the constrained maximization problem and retrieve the policy function, $m^{\prime}(m, s, \eta, \mu)$. Notice that the agents' asset holdings are restricted to belong to a discrete set, hence there is no approximation between gridpoints. The procedure keeps on iterating until a fixed point is reached, i.e. until two successive iterations satisfy:

$$
\underset{m}{S u p}\left|V(m, s, \eta)_{n+1}-V(m, s, \eta)_{n}\right|<10^{-8}, \forall s, \forall \eta .
$$

- The stationary distributions are computed relying on their definition, iterating until a very low convergence criterion is met:

$$
\operatorname{Sup}_{m}\left|\lambda(m, s, \eta, \mu)_{n+1}-\lambda(m, s, \eta, \mu)_{n}\right|<10^{-10}, \forall s, \forall \eta, \forall \mu .
$$

- As for the UI scheme, an evenly spaced grid with 50 points on the replacement rate $\theta$ is used:

$$
\left[\theta_{\min }=0.02, \theta_{2}=0.04, \ldots, \theta_{\max }=1.0\right]
$$

Compared to Hansen and Imrohoroglu (1992) more than twice the points for the possible UI policies are considered. In some cases the average steady-state utility is pretty flat in the $\theta$ dimension, calling for a finer grid in order to better isolate the maximum. See Figure (13), which plots nine typical behaviors for the average steady-state utility as a function of $\theta$.

## Appendix C - Monte Carlo Algorithm

The computational procedure used to solve the Monte Carlo experiments can be represented by the following algorithm:

1. Draw 1,000 combinations of parameters from their prior distributions and store them.
2. Set the degree of moral hazard $\pi(u)=\{0,0.1,0.5\}$ and the indicator $i=1$.
3. Calibrate the model by reading the vector $i$ of simulated parameters and begin the model solution.
4. Generate a discrete grid over the UI replacement rate $\left[\theta_{\min }, \ldots, \theta_{\max }\right]$.
5. Set the UI replacement rate $\theta_{j}=\theta_{\text {min }}$.
6. Generate a discrete grid over the asset space $\left[0, \ldots, m_{\max }\right]$.
7. Guess on the tax rate $\tau_{0}$.
8. Get the value functions $V\left(m, s, \eta ; \theta_{j}\right)_{i}$ and the policy functions for savings $m^{\prime}\left(m, s, \eta, \mu ; \theta_{j}\right)_{i}$ and labor supply $\eta^{\prime}\left(m, s, \eta ; \theta_{j}\right)_{i}$.
9. Compute the stationary distributions $\lambda\left(m, s, \eta, \mu ; \theta_{j}\right)_{i}$.
10. Get the aggregate unemployment rate.
11. Get the aggregate UI costs.
12. Check the government budget constraint; Get $\tau_{1}$.
13. Update $\tau_{0}^{\prime}=\varpi \tau_{0}+(1-\varpi) \tau_{1}$ (with bisection).
14. Iterate until the government budget constraint is satisfied.
15. Compute the average steady-state utility $\bar{V}\left(. ; \theta_{j}\right)_{i}=\sum_{m, s, \eta, \mu} \lambda\left(. ; \theta_{j}\right)_{i} V\left(. ; \theta_{j}\right)_{i}$.
16. Consider the next point in the UI grid, until $\theta_{j}=\theta_{\max }$.
17. Compute the optimal UI scheme $\theta_{i}^{*}$ as the steady-state utility maximizer, i.e. $\theta_{i}^{*} \equiv \underset{\theta_{j}}{\operatorname{Arg} \max }\left\{\bar{V}\left(. ; \theta_{j}\right)_{i}\right\}$.
18. Save the output, set $i=i+1$ and repeat from step 3 for all the 1,000 combinations of simulated parameters.

## Appendix D - Additional Figures











Figure 13: Average Steady State Utility Vs. UI Scheme - Selected Calibrations


Figure 14: Optimal RR - Histograms for the Monte Carlo Experiments B1-B2


Figure 15: Optimal RR - Histograms for the Monte Carlo Experiments B3-B4


Figure 16: Parameter $\widehat{h}$ - Estimated Densities for the Monte Carlo Experiments B3-B4


Figure 17: Parameter $\sigma$ - Estimated Densities for the Monte Carlo Experiments B3-B4


Figure 18: Parameter $\beta$ - Estimated Densities for the Monte Carlo Experiments B1-B2


Figure 19: Parameter $\beta$ - Estimated Densities for the Monte Carlo Experiments B3-B4


Figure 20: Parameter $\rho$-Estimated Densities for the Monte Carlo Experiments B1-B2


Figure 21: Parameter $\rho$ - Estimated Densities for the Monte Carlo Experiments B3-B4


Figure 22: Parameter $\chi_{\left(u^{\prime}, u\right)}$ - Estimated Densities for the Monte Carlo Experiments B1-B2


Figure 23: Parameter $\chi_{\left(u^{\prime}, u\right)}$ - Estimated Densities for the Monte Carlo Experiments B3-B4


Figure 24: Parameter $\chi_{\left(e^{\prime}, e\right)}$ - Estimated Densities for the Monte Carlo Experiments B1-B2


Figure 25: Parameter $\chi_{\left(e^{\prime}, e\right)}$ - Estimated Densities for the Monte Carlo Experiments B3-B4


Figure 26: Employment Opportunity - Estimated Densities for the Monte Carlo Experiments B1-B2


Figure 27: Employment Opportunity - Estimated Densities for the Monte Carlo Experiments B3-B4


Figure 28: Employment Rate - Estimated Densities for the Monte Carlo Experiments B1-B2


Figure 29: Employment Rate - Estimated Densities for the Monte Carlo Experiments B3-B4


Figure 30: Employment Delta - Estimated Densities for the Monte Carlo Experiments B1-B2


Figure 31: Employment Delta - Estimated Densities for the Monte Carlo Experiments B3-B4


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[^1]:    ${ }^{1}$ In the interest of space, just a sketch of the model is going to be presented. For more details see Appendix A and Hansen and Imrohoroglu (1992).

[^2]:    ${ }^{2}$ In the applied section of the paper we are going to set $\pi(e)=0$. It follows that MH matters only for the unemployed agents.

[^3]:    ${ }^{3}$ For further details on this methodology applied to a standard HA model, and on the related issues, see Cozzi (2011).

[^4]:    ${ }^{4}$ The solution algorithm together with some computational aspects are explained in Appendices B and C.

[^5]:    ${ }^{5}$ More precisely, these distributions have four parameters: two shape parameters together with two location parameters. The location parameters allow to define the support of the distribution in an interval different from the $[0,1]$ one, and they coincide with the upper and lower bounds reported in the last two columns of Table 1. For the other two (shape) parameters that are needed to fully characterize the Beta distribution, two different set-ups are considered. The former relies on the prior $\operatorname{Beta}(10,10)$ for all the parameters, while the latter considers a $B e t a(2,2)$ prior. These specifications deal with Beta distributions which are symmetric around their means, because the two shape parameters are equal. The pair $(10,10)$ implies a distribution which is substantially more concentrated around its mean, when compared to the $(2,2)$ one.

[^6]:    ${ }^{6}$ Notice that, although this is a production economy, capital does not enter the production function. Hence, it is not possible to set $\beta$ for the economy to match the observed real interest rate or, equivalently, the capital/output ratio.

[^7]:    ${ }^{7}$ Figure (2) focuses on the B1-B2 experiments. The $B 3-B 4$ ones have similar graphs. Notice that the figure provides the breakdown with respect to the degree of MH as well.
    ${ }^{8}$ This is true only for the cases where employment opportunities are never turned down.

[^8]:    ${ }^{9}$ In the interest of space, the results related to the employment rate and the difference between the employment opportunity and the actual employment rate are reported in an appendix.

[^9]:    ${ }^{10}$ Figures (14) and (15) in the appendix confirm that this finding is not an artifact of the non-parametric kernel estimation procedure. The very same patterns are present in the related histograms. Understanding why the distributions are bimodal is challenging. A possible explanation could be related to the lumpiness of the labor supply choice.

[^10]:    ${ }^{11}$ See Figures (26)-(31) in the appendix.

[^11]:    ${ }^{12}$ In the table a ${ }^{\prime *}$ " symbol means that a parameter is not significant at the $10 \%$ level, and all regressions include a constant term and the replacement rate as regressors, not reported. Several other specifications were tried, including a Fixed Effect one (the FE being the replacement rate) and a SUR model with all the five endogenous variables as the LHS. The estimated parameters in the expected utility equation were pretty robust.

[^12]:    ${ }^{13}$ The labels hint that the densities refer to the no MH case. This is immaterial, because the other MH cases rely on a fixed seed and have the exact same parameters' draws, hence the same distributions. The Figures for the other cases and parameters are in an appendix.

