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The Law of Urban Growth and the Local Public Sector

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Abstract

We set out a city as a price-taking exporter and importer with its own local structure (housing (land per household) and a local pure public good are produced endogenously). We impove labor efficiency in the export sector, observe a jump in the local wage, and trace the impact, particularly on production of the public good. In one case the population, output of the public good, and residential density expand (the law of urban growth) and in another, population, output of the public good and density contract.

- key words: small, open city; urban public sector; law of urban growth
- R230; H400; F430

1 Introduction

We take up a model of a small, open city exporting q_x at world prices and importing q_c in return, and we report on the city's response to an improvement in labor efficiency in the export sector. The higher wage associated with the efficiency improvement goes along typically with a city that is larger and more densely settled (the law of urban growth; more detail below). We observe this for a city with a small or non-existant local government sector. With a "large" government sector added (involving essentially the production of a local Samuelson public good), we oberve a quite general violation of the law of urban growth (we see a smaller, less dense city; one with the size of the public sector shrinking with the wage increase). Housing (land per worker) and the local public good are endogenously defined, each with an own-city price. The scenario with the public sector shrinking as the city-wide wage increases echoes somewhat Baumol's

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(1967) argument that technical progress in the private sector of a city raises the local wage and makes local government higher cost and in turn makes budget balance for the local government difficult to achieve. In our framework with local government always in budget balance, the higher wage induces a contraction in the quantity of the local government good produced and a compensating rise in the quantity of housing consumed by each household. Migration into and out of our city turns on the current utility level being achieved for a representative worker-household being above or below a reservation level set out in the system of cities surrounding our city.

Empirical work has established quite well that larger cities in many places including the United States are observed to have a higher productivity for labor, controlling for a worker's education and experience, as well as higher wages and average housing prices (Ciccone and Hall (1996), Glaeser and Mare (2001) and Van Nieuwerburgh and Weill (2010)). Settlement density is higher in the larger cities as well. Ciccone and Hall estimated labor productivity in various counties in the United States as a function of the density of local residents. They controlled for worker education and local infrastructure abundance. They observed strong gains in worker productivity with increased local density. Glaeser and Mare present the standard evidence for "the law of urban growth": a positive coefficient on city size in estimations of Mincer equations. The basic Mincer equation "explains" a worker's wage as a weighted combination of her education and experience. Higher wages correlate positively with larger cities, even when worker education and experience are controlled for.[1] Van Nieuwerburgh and Weill (2010) estimate a model of a many-city labor market that includes the local cost of housing. They have worker heterogeneity, housing heterogeneity within a city and city-specific heterogeneity of the industrial structure. They conclude: "House price differentials between metropolitan areas compensate for the income differential of the marginal, lowest ability household in the location, making that household indifferent between staying and moving to the next best metropolitan area. Households also live in smaller and more expensive quarters if they choose to work in higher income metropolitan areas. Lastly, higher in-

[1] Glaeser and Gyourko (2005) note that there is a lag in the productivity improvement exhibited by an in-migrant to a large city.

come metropolitan areas have on average a larger housing stock and a larger workforce." (p. 1568). Part of the positive correlation of city wage and size has obviously to do with (a) larger cities exhibiting higher proportions of better educated residents and (b) the high average wage in a large city discouraging firms in certain "low-wage" industries from operating in a particular city.[2] There are then these tricky labor force and industry composition effects at work in the larger city, higher wage relationship. Our model of a city abstracts from these issues involving composition effects. Our worker-households are homogeneous. We adhere to the idea that the extra productivity a worker exhibits in a larger city is due to some complicated urban-ness factor, such as easier and more productive net-working with fellow workers, when a worker is employed in a higher density arrangement. We remain open-minded on the source of the extra productivity a worker gains from being in a larger city. We simply take the extra productivity as real and explore its implications in a model of a small open city.

Our point of departure for this work was the development of an "international trade" framework for a city, a framework that would have the law of urban growth baked in from the outset. We took for granted that our framework would have a local public good endogenous. It was a surprise that the Law of urban growth turned out to be sensitive to the way we introduced a local public good into our model. We are not claiming that our avenue to the violation of the law of urban growth is unique. There are many parameters in our model to experiment with. However the particular violation which we came upon and which we report on below is we believe worthy of much reflection.

Our model also allows us to work through a boost in the climate amenity enjoyed by citizens of our city. For the case of our public sector small or nonexistant we observe the "free utility" from the boost to climate amenity showing up by making our city larger and more densely settled. For the case of our public sector large, the boost to the amenity makes our city smaller and less densely settled. We also note that the introduction of a local, non-traded goods sector (eg. haircuts) does not change our results. We turn now to the details of our model of a small, open city.

[2] Duranton and Puga (2001) provide evidence for firms doing innovative activity tending to be located in larger centers and firms doing more routine activity to be located in smaller, lower-cost places.

2 The Model

We turn to the details of our model of a small open trading city with a local public good being produced and charged for with Samuelsonian "taxes". A worker (household) consumes some of the import good q_c/N, some housing, L/N, for L the land occupied by the city and the public good q_g. N is number of worker-households in the city. Production of the export good q_x is with fixed coefficients:

$$\begin{aligned} q_x a_N &= N_x \\ \text{and } q_x a_K &= K_x. \end{aligned}$$

The world price is p_x. Consumption good q_c is imported with world price unity and we assume that $q_c = p_x q_x$.

Production of q_g is contant returns to scale in

$$q_g = g(K_g, N_g)$$

and efficiency in production requires

$$\frac{g_{K_g}}{g_{N_g}} = \frac{r}{w}$$

and

$$p_g = rK_g + wN_g.$$

The utility of a worker-household is

$$u(\frac{q_c}{N}, \frac{L}{N}, q_g)$$

and this becomes equal to the "external" utility level, $\overline{u}$, by worker inflow and outflow.[3] Note each resident is consuming the full output of the public good, q_g. A worker's budget constraint is $w + \frac{rK}{N} = \frac{q_c}{N} + \frac{q_g p_g}{N} + \frac{Lp_L}{N}$.[4]

[3] This approach is referred to as "the open city assumption". A better approach would be to have migration determined by a response to lifetime welfare attainable in various places (see for example Eaton and Eckstein (1997) and Lucas (2004)). The better approach would complicate our analysis greatly. In a somewhat stationary world, current utility for a person is a reasonable proxy for lifetime utility.

[4] We assume mutualization of local endowments across local worker-households. When a new worker attaches herself to our city, her new income ends up the same as the income of a current resident. This not an attractive assumption but is standard in many models with costless migration and local public goods (eg Flatters, Henderson and Mieszkowski (1974)).

Land, L is available for residential activity in our city at an increasing price[5], p_L:

$$p_L = A[L]^B, \ A \text{ and } B \text{ positive.}$$

A resident's house is then simply a plot of land. Consumer equilibrium requires

$$\frac{u_{\frac{q_c}{N}}}{u_{\frac{L}{N}}} = \frac{1}{p_L}$$

for $N = N_x + N_g$. Demand for the government good satisfies

$$\frac{N u_{q_g}}{u_{\frac{q_c}{N}}} = \frac{p_g}{1}$$

where $\frac{u_{q_g}}{u_{\frac{q_c}{N}}}$ is the unit charge (Samuelsonian public good "tax" (Samuelson (1954)) per household; i.e. this charge is p_g/N. Recall that all households (workers) are identical. p_g/N works as price a household ends up paying for a unit of q_g. This ends up related to current quantity q_g. A high price goes along with relatively less of q_g produced and *vice versa*.

We can summarize the equilibrium system in a "national" account matrix. The right column entries in Table 1 sum to the value of aggregate production per period and the bottom row entries sum to the value of aggregate primary inputs per period. Entries interior to the matrix capture the value of inputs to our three sectors: exports, public goods, and housing.

Table 1: Account Matrix

rK_x	wN_x	0	$= p_x q_x$ $(=q_c)$
rK_g	wN_g	0	$= p_g q_g$ $(=N q_g \frac{u_{q_g}}{u_{\frac{q_c}{N}}})$
0	0	$Lp_L(L)$	$= Lp_L(L)$
$= rK$	$= wN$	$= Lp_L(L)$	sum

We turn to solving the model. We make the utility function for our representative agent and the production function for the public good each, Cobb-Douglas. The model then can be reduced to a polynomial in N alone. We

[5] Though a household has a residence (plot of land), we abstract from the structure of the internal movement of people and goods. On this latter one might read Lucas and Rossi–Hansberg (2003). The internal movement of people from home to workplace typically generates a particular pattern of demand for land in a city, a so-called rent function. We have a representative worker-household demanding land for and "residence" and positive rent emerges because sites are not available in unlimited supply (more supply of sites corresponds with a higher price per site). We treat local land supply for the current total of sites as rising in local quantity supplied. Our approach allows us to focus on the determinants of the industrial structure of our abstract city, the size of its government's flow of services and the size of its area and population.

proceed with this "reduction". We have:

$$
\begin{aligned}
w &= \frac{p_x - ra_K}{a_N}, \\
q_c &= \frac{(N - N_x)p_x}{a_N}, \\
Lp_L &= AL^{B+1}, \\
q_g &= K_g^{\gamma_1} N_g^{\gamma_2} \\
\text{and } p_g q_g &= rK_g + wN_g.
\end{aligned}
$$

We consider now solving the model as three equilibrium conditions plus the $\overline{u}$ "constraint" in the three unknowns, N, N_g, K_g, and L. The three equilibrium conditions are

$$
\begin{aligned}
\frac{\gamma_1 K_g}{\gamma_2 N_g} &= \frac{r}{w}; && (1) \\
\frac{\alpha_1 p_g q_g}{\alpha_2 q_c} &= \frac{rK_g + wN_g}{\alpha_2 q_c} = 1; && (2) \\
\text{and } \frac{\alpha_1 L p_L}{\alpha_3 q_c} &= 1; && (3)
\end{aligned}
$$

and the fourth equation is

$$
\overline{u} = [\frac{q_c}{N}]^{\alpha_1}[q_g]^{\alpha_2}[\frac{L}{N}]^{\alpha_3} \tag{4}
$$

Equation (1) gives us an expression for K_g. This in equation (2) gives us N_g in terms of N alone. Using the expression for q_c in (3) and our new expression for N_g gives us an expression for L interms of N alone. (Recall that $Lp_L = AL^{B+1}$.) Given the expression for q_g above, we can now substitute into (4) and obtain a polynomial in N alone. We now make use of $\gamma_2 = 1 - \gamma_1$ and $\alpha_3 = 1 - \alpha_1 - \alpha_2$. (homogeneity of degree unity for the production and utility functions). The derived polynomial is a formidable looking object when expressed in terms of the "raw" parameters. We worked with MAPLE software. However, when numerical substitutions are carried out, the polynomial appears quite "regular" and compact. See the graph of the polynomial in Figure 1 for the first experiment reported on below. We proceed to present what we consider to be central numerical runs for the model, runs culled roughly speaking from many different experiments.[6]

[6] Among experiments were varying the elasticity of supply of land and the elasticity of

(a) Large exponent for q_g in the utility function:

The utility function is $[\frac{q_c}{N}]^{0.1}[q_g]^{0.5}[\frac{L}{N}]^{0.4}$ and the production function is $q_g = [K_g]^{0.6}[N_g]^{0.4}$. The export good has coefficients $a_K = K_x/q_x = 0.3$ and $a_N = N_x/q_x = 0.9$ for the base run. $r = 1.3$, $p_x = 1.4$, $A = 0.1$, $B = 0.75$, and $\overline{u} = 3.0$. We solve for N and work back to other variables. We get

w = 1.122222222, N = 6.0088, L = 17.387413, N_g= 3.624094255,
K_g= 4.692737432, q_g= 4.231901462, p_g= 2.402607400,
q_c = 3.7095, q_c/N = 0.6173, L/N =2.895476343, p_g/N =0.4004345667.

With $a_N = 0.8$ (labor more efficient in the export sector) we get

w = 1.2625, N = 4.867, L = 17.732598, N_g= 2.674083592,
K_g =3.895419848, q_g =3.351217038, p_g =2.518510810,
q_c = 3.8376, q_c/N = 0.7884, L/N =3.645707169, p_g/N =0.5182121008.

The export sector gets a boost in labor productivity and the city shrinks in population. The quantity of the public good declines (and p_g/N increases) as well as the density of residential activity. We link the decline in the quantity of the public good produced to Baumolian "cost disease" here since the wage in the city has risen.[7]

(b) Small exponent for q_g in the utility function:

The utility function is now $[\frac{q_c}{N}]^{0.1}[q_g]^{0.05}[\frac{L}{N}]^{0.85}$. Other parameters are unchanged.

a_N = 0.9 for our base case. Solving yields

w = 1.1222, N = 34.478, L = 108.5780002, N_g= 7.078722213, K_g= 9.166037738,
q_g= 8.265914951, p_g=2.402607, q_c = 42.62, q_c/N = 1.247, L/N =3.129048997,

substitution between inputs in the production of the public good. We also experimented with different weights on items in the utility function of a representative agent and with different weights or parameters of production of both the export good and the public good.

[7] Recall that our public sector is always in budget balance. A serious criticism of Baumol's approach to local cost disease is that public sector workers, say fire and police people, take their cues for wage setting from rates in closely related cities (those similar in size and structure), not so much from rates of closely related workers within their own city. This so-called "levelling up" process can still be driven by wage increases in sectors enjoying productivity gains but the linkages between wage levels can be less direct than Baumol suggested.

p_g/N =0.022128.

Labor efficiency rises: $a_N = 0.8$. Solving yields

w = 1.2625, N = 43.864, L = 133.4625232, N_g= 8.669252680, K_g =12.62876712, q_g =10.86448732, p_g=2.51851, q_c = 61.5907, q_c/N = 1.404, L/N =3.040148592, p_g/N =0.0188705.

The increase in labor efficiency in the export sector has yielded a larger city in population and area and one that is denser in residential settlement. q_g has expanded while "price" per unit, p_g/N has declined. A resident is trading off more q_g for less "housing" with the boost in labor efficiency in the export sector. This experiment goes along with "the law of urban growth": a larger, denser city with improved labor productivity and a higher average wage.

3 Capitalization of a Climate Amenity

We can probe a climate amenity premium for a city with our model of a small, open city (a price-taker for goods and capital and a utility-taker for worker-households). We take our city in equilibrium and perturb the utility function so that the change reflects a resident getting a sudden infusion of free utility into the indefinite future, the free utility deriving from say a jump in sunny days per year for our city.[8] In place of our base case with $u(.) = \overline{u}$, we have say $u(.) \times 1.1 = \overline{u}$ as the new condition for the climate enhanced city. When we solve the model for such an experimental perturbation we get results that parallel those above: for a specification with q_g "counting" small in the representative agent's utility function, we observe the amenity premium capitalized in local land rent (local housing prices end up higher and population and density increase; q_g expands and p_g/N contracts); and for a specification with q_g "counting" large in utility, we observe the city to shrink in population and density as the city "digests" a once-over boost to its climate amenity (q_g contracts and p_g/N rises).

[8] Haurin (1980) allows for firms to respond directly to a climate amenity. We only have worker-households responding directly. Firms are assumed to not be climate sensitive. Roback (1982) made use of worker wage data in her pioneering exploration of capitalization of locational characteristics in land rents.

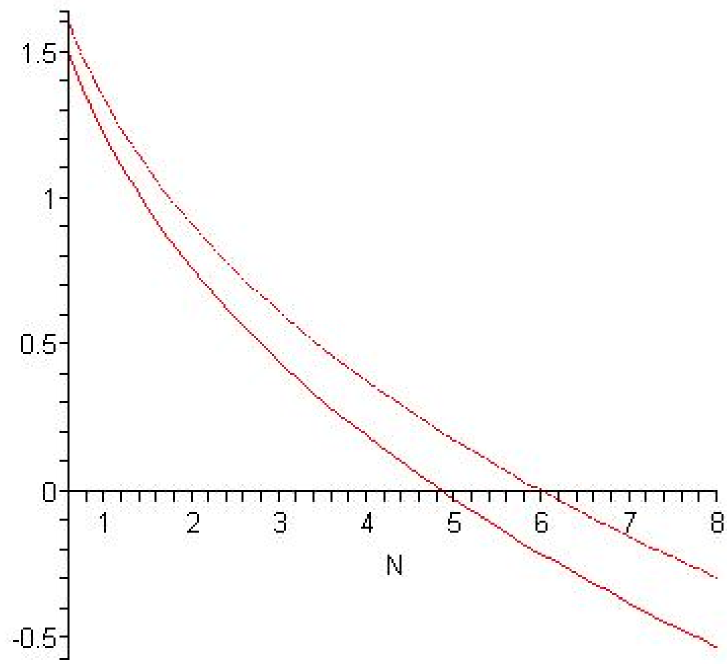

Figure 1: Two solution values (4.867 and 6.0088) for the case of q_g "large" in the utility function. The graph is output from MAPLE.

4 An additional Local Non-traded Good

We have carried out numerical experiments with our model above, expanded to include a private, local non-traded good q_d (eg. haircuts) produced under constant returns to scale. The model with the endogenous haircuts sector has four additional equations to accommodate our additional sector:

$$\begin{aligned} q_d &= h(K_d, N_d) \\ p_d q_d &= rK_d + wN_d \\ \frac{h_{K_d}}{h_{N_d}} &= \frac{r}{w} \\ \frac{u_{\frac{q_d}{N}}}{u_{\frac{q_c}{N}}} &= p_d. \end{aligned}$$

The four new variables to solve for are q_d, p_d, K_d, and N_d. The utility function for a representative worker-household is now $u = [\frac{q_c}{N}]^{\alpha_1}[\frac{q_d}{N}]^{\alpha_2}[q_g]^{\alpha_3}[\frac{L}{N}]^{[1-\alpha_1-\alpha_2-\alpha_3]}$

The account matrix for this four good model is

Table 2: Account Matrix

rK_x	wN_x	0	$= p_x q_x$ $(=q_c)$
rK_d	wN_d	0	$= p_d q_d$
rK_g	wN_g	0	$= q_g p_g$ $(=Nq_g \frac{u_{q_g}}{u_{\frac{q_c}{N}}})$
0	0	$Lp_L(L)$	$= Lp_L(L)$
$= rK$	$= wN$	$= Lp_L(L)$	sum

Row entries sum to the entry on the right and column entries sum to the entry on the bottom. The sum of the entries on the right is the value of "national" product and the sum of the entries on the bottom is value of "national" inputs.

Our basic results concerning an improvement in the efficiency of labor in the export sector re-appear without change: for a specification with q_g "counting" small in the representative agent's utility function, we observe the efficiency and wage boost to result in local housing prices ending up higher and population and density increase (q_g expands and p_g/N contracts); and for a specification with q_g "counting" large in utility, we observe the city to shrink in population and density (q_g contracts and p_g/N rises).

5 Concluding Remarks

We have set out a simple model of a city that has allowed us to investigate "the law of urban growth", related Baumol government cost effects, and capitalization of climate amenities. Some might characterize our model as basic international trade theory (the small open economy) with urban add-ons (space using housing, worker inflow and outflow based on a utility level benchmark, and a local public goods sector). This is a fair characterization. We abstract from the detail of the internal movement in our small, open city of people and goods. We can deal with equilibrium size and structure of our city quite directly. We focused on a seeming paradox. Though our model can produce a simulated city that conforms to the law of urban growth (larger cities being denser and higher-wage entities), we turned up a somewhat anomalous condition for the conventional scenario to work out successfully in our model, namely a small local government sector for the city. Our simple model required some unusual fine-tuning of parameters for the law of urban growth to appear. Our paradoxical outcome turned on us formulating local government as essentially Samuelsonian (there was a local pure public good that was funded by charges in accord with marginal benefits). Since real-world local governments have complicated bundles of local services and "blunt" mechanisms for eliciting payment from consumers of the services, it remains open as to how likely our "model" paradox might show up in real-world data for cities. Baumol's 1967 article is the closest analysis we have of an significant connection between local worker productivity change and local government size and it would be inaccurate to say that our simulations were capturing the essentials of Baumol's argument.

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