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# How High Might the Revenue-maximizing Tax Rate Be? 

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Dan Usher, December 1, 2014


#### Abstract

Through tax evasion, through the labour-leisure choice or in other ways, taxpayers reduce the tax base in response to an increase in the tax rate. The process is commonly-believed to generate a humped Laffer curve with a revenue-maximizing tax rate well short of $100 \%$. That need not be so. In the "new tax responsiveness literature", the revenue-maximizing tax rate is inferred from the observed "elasticity of taxable income". It is shown in this article 1) that the inference is unwarranted because the elasticity of taxable income may vary with the tax rate, 2) that the "new tax responsiveness literature" imposes the implicit assumption that tax revenue falls to 0 when the tax rate rises to $100 \%, 3$ ) that tax revenue may increase together with the tax rate all the way up to $100 \%$ and 4) that the Laffer curve is ill-defined because tax revenue at any given rate may depend upon how tax revenue is spent.


Key Words revenue-maximizing tax rate, Laffer curve
JEL Codes H21, H23

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"In so far as men act rationally, they will at a higher (wage) rate divide their time between wageearning and non-industrial uses in such a way as to earn more money but to work fewer hours." Frank Knight $(1921,117)$ quoted by Robbins (1930)

The most dramatic argument against extensive redistribution of income is that it is impossible. No matter how compassionate we may be, if the revenue-maximizing tax rate is, for example, $60 \%$, so that no higher tax rate yields any extra tax revenue, then expenditure on redistribution cannot exceed the proceeds of a tax of $60 \%$ less whatever portion of revenue is required to finance ordinary public goods, the army, the police and so on. There is a whiff of hypocrisy to the argument. People opposing redistribution would rather claim it to be impossible than to admit to a want of sympathy toward the poor. People favouring redistribution for selfish or altruistic reasons persuade themselves that the revenue-maximizing tax rate is high enough to accommodate policies they support. But there is a real issue here, and much effort has been devoted to the difficult task of figuring out what the revenue-maximizing tax rate might actually be.

This essay is an attempt to clarify the matter not by offering a definitive route to the truth, but by identifying bumps along the way. The essay begins with a critical assessment of the "new tax responsiveness literature" by which estimates of "the elasticity of taxable income" and the revenue-maximizing tax rate are derived without specific reference to the underlying mechanism by which tax revenue and tax rate are connected. The core of this essay is about how the Laffer curve may emerge from tax evasion or from the labour-leisure choice, and how inferences in the new tax responsiveness literature hold up in each case.

Taxation is impeded by the taxpayer's incentive to reduce his tax bill, contracting his observable tax base through outright tax evasion, by working less or by other means. Contraction of the tax base is expensive, for, if that were not so, nobody would pay tax at all. Since taxpayers naturally choose the least expensive way of concealing any given share of the tax base, the marginal cost of contracting the tax base must increase with the proportion of tax base already concealed.

Figure 1: Three Possible Shapes of the Laffer Curve
a) A Humped Curve
b) Upward -sloping
Concave
c) Upward-sloping Convex




Three possible shapes of the Laffer curve are illustrated side by side in figure 1 with tax rate on the horizontal axis and tax base on the vertical axis. It is commonly assumed that the Laffer curve is humped as shown on the left-hand curve with a revenue-maximizing tax rate, $\mathrm{t}^{*}$, at the top of the Laffer curve well short of $100 \%$. The other two curves are upward-sloping, one convex and the other concave, both with a revenue-maximizing tax rate of $100 \%$. It turns our, however that all three shapes are possible. If the tax base remained unchanged regardless of the tax base, the Laffer curve would be an upward-sloping straight line.

## Inferring the Revenue-maximizing Tax Rate from the Elasticity of Taxable Income

As the revenue-maximizing tax rate can never be observed directly, it must be inferred from some observable aspect of taxing at actual rates. The standard procedure in "the new tax responsiveness' literature ${ }^{1}$ is to infer the revenue-maximizing tax rate from the estimated "elasticity of taxable income" defined as the percentage change in the observable tax base in response to a given percentage change in the share of income that the taxpayer retains. Specifically,

$$
\begin{equation*}
\varepsilon=[\delta y / y] /[\delta(1-t) /(1-t)]=-[(1-t) / y] \delta y / \delta t \tag{1}
\end{equation*}
$$

where $\varepsilon$ is the elasticity of taxable income, y is taxable income as observed by the tax collector and $t$ is the tax rate. The elasticity of taxable income is analogous to the elasticity of supply of labour, with y playing the role of labour supply and (1-t) playing the role of the after tax wage of labour. By definition, tax revenue, $R$, is

$$
\begin{equation*}
\mathrm{R}=\mathrm{ty} \tag{2}
\end{equation*}
$$

so that the elasticity of tax revenue with respect to the tax rate, the elasticity of R with respect to t is

$$
\begin{equation*}
(\mathrm{t} / \mathrm{R})(\delta \mathrm{R} / \delta \mathrm{t})=(1 / \mathrm{y})(\mathrm{y}+\mathrm{t} \delta \mathrm{y} / \delta \mathrm{t})=[1+(\mathrm{t} / \mathrm{y})(\delta \mathrm{y} / \delta \mathrm{t})]=1-[\mathrm{t} /(1-\mathrm{t})] \varepsilon \tag{3}
\end{equation*}
$$

ensuring that there is a revenue-maximizing tax rate of less than $100 \%$ as long as $\varepsilon$ is positive and independent of $t$. On these assumptions, the revenue-maximizing tax rate $t^{*}$, the rate at the top of the Laffer curve for which $(t / R)(\delta R / \delta t)=0$, becomes

$$
\begin{equation*}
\mathrm{t}^{*}=1 /(1+\varepsilon) \tag{4}
\end{equation*}
$$

The revenue-maximizing tax rate can be estimated from equation (4) if $\varepsilon$ can be observed. For example, if a rise in the tax rate from $30 \%$ to $34 \%$ causes the tax base to shrink from $\$ 100$

[^0]billion to $\$ 95$ billion, the elasticity of taxable income would be 0.875 , equal to [5/100]/[0.04/(1-0.3)], and the estimated revenue-maximizing tax rate would be about $53 \%$, equal to $1 /(1+0.875)$.

There are at least four difficulties with this procedure:

1) In estimating $\varepsilon$, changes in the tax base due to changes in the tax rate must be distinguished from changes in the tax base due to changes in other factors that would have occurred regardless of the change in the tax rate.
2) the elasticity of taxable income may differ for people in different tax brackets or, more generally, from one group to another.
3) The elasticity of taxable income may not be independent of the tax rate as is required for the estimation of $t^{*}$ from equation (4).
4) The elasticity of taxable income may be negative.

A considerable literature on the first two difficulties will be mentioned briefly here, but the emphasis in this essay is upon the third, that $\varepsilon$ may vary with $t$. The next section presents a list of estimates of the revenue-maximizing tax rate and a brief description of how such estimates are derived. The postulated constancy of the elasticity of taxable income will then be tested with reference to simple models of tax evasion and the labour-leisure choice.

If it is indeed true that $\varepsilon$ is independent of the tax rate, then the impact of tax rate on tax base must be in accordance with the equation ${ }^{2}$

$$
\begin{equation*}
y=(1-t)^{\varepsilon} Y \tag{5}
\end{equation*}
$$

where Y is gross income as it would be in the absence of taxation, and the Laffer curve itself must be

$$
\begin{equation*}
\mathrm{R}=\mathrm{ty}=\mathrm{t}(1-\mathrm{t})^{\varepsilon} \mathrm{Y} \tag{6}
\end{equation*}
$$

which is necessarily humped as illustrated in figure 2 with revenue falling to 0 as $t$ approaches $100 \%$. That need not be so when $\varepsilon$ is not invariant.

Two Laffer curves are shown on the figure for different values of $\varepsilon$. The curve corresponding to the higher $\varepsilon$ has the lower revenue-maximizing tax rate $t^{*}$, but both curves show R falling to 0 as t approaches 1 . The question then arises whether this strong implication of the new tax responsiveness literature is an inescapable fact of society or nothing more than a

[^1]consequence of the assumed invariance of $\varepsilon$, that is, whether the disappearance of all tax revenue when the tax rate rises to $100 \%$ at any given time and place is an empirical fact or a theoretical construct imposed upon the evidence.

Figure 2: Laffer Curves for Two Values of $\varepsilon$


More generally, is it reasonable to suppose that the entire Laffer curve can be identified from observations of t , R and the $\delta \mathrm{R} / \delta \mathrm{t}$ at a single point on the curve? The two curves in figure 3 have the same values of $t, R$ and the slope $\delta R / \delta t$ as indicated by the point A. Should it be obvious which is the true Laffer curve? The new tax responsiveness literature requires that it should.

Figure 3: Two Possible Laffer Curves with the Same Elasticity of Taxable Income


Finally, there is an important difference between tax evasion and the labour-leisure choice as mechanisms generating the Laffer curve. When the mechanism is tax evasion, the elasticity of taxable income must necessarily be positive because tax evasion and the contraction of the tax base as seen by the tax collector are two sides of the very same coin. The elasticity of taxable income need no longer be positive when the mechanism is the labour-leisure choice. With hours of labour as the tax base, a decrease in one's post-tax wage may lead one to contract the tax base by substituting leisure for goods, but may equally-well lead one to expand the tax base by working more to avoid a drastic fall in the proportion of goods to leisure consumed.

## Estimating the Elasticity of Taxable Income

A selection of estimates of the revenue-maximizing tax rate is presented in table 1.
Table 1: Estimates of the Revenue-maximizing Tax Rate
[ These estimates are not really comparable because they are based upon different assumptions about the response of tax base to tax rate.]

| author | revenue- <br> maximizing <br> tax rate |  |
| :--- | :--- | :--- |
| Clark (1945), page 380 | $25 \%$ | postulated a different mechanism constraining the tax rate |
| Stuart (1981), page 1020 | $70 \%$ | estimate for Sweden based up on elasticity of labour supply |
| Feige and McGee, (1983), <br> table 1, page 511 | $32 \%$ to $91 \%$ | depending upon estimated elasticities of supply in the observed and <br> unobserved economy and upon the trade-off between sectors |
| Lindsey (1987), table 8, <br> page 202 | $40 \%$ | introduces "new tax response" estimation |
| Feldstein (1995), table 2, <br> page 565 | $25 \%$ to $50 \%$ | elasticity of taxable income estimated to be between 1 and 3 |
| Diamond and Saez (2011), <br> page 171 | $73 \%$ | estimate for top income bracket based upon the "new tax response" |
| Goolsbee (2000), page 375 | $70 \%$ to $100 \%$ | elasticity of taxable income estimated to be between 0 and 4 |

The principal difficulty in constructing these estimates to distinguish between changes in the tax base in response to changes in the tax rate and changes in the tax base reflecting other changes in the economy destined to occur regardless. It is not sufficient just to observe the tax base before and after a change in the tax rate. To deal with this problem, estimates of the elasticity of taxable income are based upon the comparison of proportional changes in tax bases of two
essentially similar groups, one confronted with a tax increase and the other not. Call the groups J and K. Suppose that, initially, the two groups are taxed at the same rate, $t$, but that, as part of a general change in the tax structure, the tax rate on people in group $J$ is increased to $t+\Delta t$, while the rate on people in group K remains unchanged at t . [Alternatively, $\Delta \mathrm{t}$ may be the difference in the changes in their tax rates.] Both groups' tax bases may change over time, but the two groups are presumed to be sufficiently alike that their tax bases would have changed proportionally if the tax increase on people in group $J$ had not occurred. With percentage changes in the observed tax bases of groups $J$ and $K$ of $x$ and $y$, the percentage change in the tax base of group $J$ attributable to the increase in its tax rate becomes $x-y$.

Suppose that, initially, both groups are taxed at a rate of $40 \%$, that the tax rate on group J alone is increased from $40 \%$ to $44 \%$ while the tax rate on group K remains unchanged at $40 \%$, and that, over a period spanning the change in the tax rate, the tax base of group J is observed to rise by $9 \%$ while the tax base of group K is observed to rise by $12 \%$. From this information, the estimated elasticity of taxable income becomes

$$
\begin{align*}
\varepsilon & =[\Delta \mathrm{y} / \mathrm{y}] /[\Delta(1-\mathrm{t}) /(1-\mathrm{t})]=[.09-.12] /[-.04 /(1-.40] \\
& =[.03] /[.04 / .6]=.45 \tag{7}
\end{align*}
$$

and the estimated revenue-maximizing tax rate becomes

$$
\begin{equation*}
\mathrm{t}^{*}=1 /[1+.45]=69 \% \tag{8}
\end{equation*}
$$

There are many complications in this estimation procedure. Typically, groups J and K are people in two adjacent tax brackets observed at times before and after the moment when tax rates on incomes in these brackets are changed. ${ }^{3}$ Both components - $[\delta y / y]$ and $[\delta(1-t) /(1-t)]$ - of the definition of $\varepsilon$ in equation (4) present complications. These must be approximated by $[\Delta y / y]$ and [ $\Delta(1-\mathrm{t}) /(1-\mathrm{t})]$ where $\Delta \mathrm{y}$ and $\Delta \mathrm{t}$ are actual changes in the data. In measuring $\Delta(1-\mathrm{t}) /(1-\mathrm{t})]$, it must be decided which taxes to take into account. In measuring [ $\Delta y / y$ ], a time period must be chosen over which the change in the tax base is observed. Specify the date of the tax chance as 0 , and suppose the tax rate is observed i years before and i years after the change in the tax rate. With group K as the control group, the estimate of $\Delta y$ for group $J$ becomes

$$
\begin{equation*}
\Delta y_{J}=y_{J, i}-y_{J,-i}\left[y_{K, i} / y_{K,-i}\right] \tag{9}
\end{equation*}
$$

where $y_{J, i}$ is the observed tax base of group J in year $i$,
$y_{J,-i}$ is the observed tax base of group $J$ in year -i,
$y_{K, i}$ is the observed tax base of group K in year i
and $\quad y_{\mathrm{K},-\mathrm{i}}$ is the observed tax base of group K in year -i.
${ }^{3}$ For a detailed description of how the elasticity of taxable income is estimated, see Saez, Slemrod and Giertz (2012).

The choice of i matters because the timing of the impact of tax rate on tax base can occur in several ways with very different impacts upon estimates of $\Delta y_{j}$. Some possibilities are illustrated in table 2. The table is constructed on the assumption that, initially, groups J and K are taxed at the same rate and that their tax bases would have changed proportionally over time in response to economic conditions in society as a whole - specifically, that the tax base in group J would always be twice the tax base of group K - if their tax rates remained the same. Then, on January 1 of the year 1, the tax rate on group $J$ is raised, while the tax rate on group $K$ remains the same as before.

Table 2: Alternative Estimates of the Effect upon the Tax Base of Group J of a Tax Increase at Time 0 on Group J but not Group K

|  |  | Implications of alternative assumptions about the effect of the tax increase at time 0 upon the tax base of group J |  |  |  |
| :---: | :---: | :---: | :---: | :---: | :---: |
| (1) years (i) before $(-)$ or after $(+)$ the tax increase | (2) tax base of group K | (3) no impact of taxation | (4) <br> immediant <br> and <br> permanent contraction of tax base | (5) gradual contraction of the tax base after the year 0 | (6) switching part of the tax base from year 1 to year -1 |
| -3 | 9 | 18 | 18 | 18 | 18 |
| -2 | 8 | 16 | 16 | 16 | 16 |
| -1 | 10 | 20 | 20 | 20 | 23 |
| 1 | 12 | 24 | 22 | 23 | 21 |
| 2 | 11 | 22 | 20 | 20 | 22 |
| 3 | 10 | 20 | 18 | 17 | 20 |
| $\Delta \mathrm{y}_{\mathrm{J}}\{-1$ to +1$\}$ | ---------- | 0 | -2 | -1 | -6 |
| $\Delta \mathrm{y}_{\mathrm{J}}\{-3$ to +3$\}$ | ----- | 0 | -2 | -3 | 0 |

Ignore for the moment the last two rows of the table which will be explained below. The first column of the table identifies years before (-i) or after (i) the tax increase on people in group J. The second column shows the postulated tax base of the control group K in each of these years. Numbers in this column are chosen arbitrarily. The remaining four columns show four alternative responses of the tax base of group J to the tax increase: As shown in column (3), there may be no response, in which case the tax base of group J remains at twice the tax base of group K. As shown in column (4), there may be a sudden and permanent decrease in tax base of group J starting
immediately after the tax increase; specifically, beginning in year 1 , the tax base of group $J$ is assumed to become 2 less than it would otherwise be. As shown in column (5), the tax base of group J may decrease gradually because it is costs less to make changes slowly than to make changes all at once. And as shown in the last column (6), if the tax increase is anticipated, there may be an increase in the observable tax base before the tax increase followed by a decrease afterwards, as taxpayers divert declared taxable income from a time when the tax rate is relatively high to a time when the tax rate is relatively low.

This final possibility is illustrated on the supposition that the tax increase has no effect upon the tax base except in the years immediately before and immediately after the tax increase. In year 1, the observable tax base rises from 20 to 23. Then, in year 1, tax base falls from 24 to 21 , bringing it back to what it would be with no tax increase at all. Movement of part of the observable tax base from one year to another may be undertaken in several ways, among them by realizing capital gains or by cashing in stock options before rather than after a tax increase. Such opportunities are likely to be greater for the wealthy than for the poor.

The last two rows in the table show how the estimate in equation (10) of the change, $\Delta y_{J}$, in the tax base of group $J$ responds to the duration of the span over which the tax bases in groups $J$ and K are observed. In the second to last row, the observations are over adjacent years, -1 and 1 . In the last row, the observations are over six years from year -3 to year 3 . By definition, $\Delta y_{J}$ is 0 whenever the tax increase has no impact on the tax base. A immediate and permanent reduction in the tax base causes the estimated reduction in the tax base ( -2 ) to be the same regardless of the time span over which it is observed. A gradually increasing impact of the tax increase causes the estimated reduction in the tax base to be less when observed from year - 1 to year 1 than when observed from year -3 to year 3. A substitution of the declared tax base from a year when tax is high to a year when tax is low is just the opposite. Observed from year -1 to year 1 , the declared tax base of group $J$ falls by 6 , i.e. $\Delta B_{J}=-6$. Observed over a six year period from year -3 to year 3 , there is no change in the declared tax base at all. The difference between Goolsbee's high estimate of the revenuemaximizing tax rate in the last row of table 1 and the lower estimates in the rest of the table is due to this phenomenon. ${ }^{4}$ Additional complications in the estimation of $\Delta B_{J}$ arise from the possibility that, even without the tax increase, forces in the economy might have caused the tax bases of groups J and K to change at different rates.

The New Tax Responsiveness literature is a black box generating estimates ungrounded in any specific mechanism connecting tax base to tax rate. The estimates are what they are without specific reference to the reason why the base responds to rate. The tax-induced contraction of the tax base may be by the labour-leisure choice, increased do-it-yourself activity, illegal tax evasion, legal tax avoidance, out-migration of highly-taxed people, or some combination of these. Only the total outcome matters.
${ }^{4}$ By contrast, Buchanan and Lee (1982) argue that the tax induced contraction in the tax base is likely to occur gradually, as shown here in column (6) of table 2 , so that the revenuemaximizing tax rate at the top of the Laffer curve diminishes over time.

This is a considerable advantage, but there are disadvantages too. Appropriate public policy may depend on why taxable income is responds to changes in the tax rate. Tougher enforcement of the tax code might be especially warranted when tax evasion is the major influence on the elasticity of taxable income. An understanding of the innards of the black box might be helpful in deciding which of the estimates of the revenue-maximizing tax rate in Table 1 is more nearly correct. Also, as the sample of tax changes is small and as several arbitrary assumptions are required in the estimation of the elasticity of taxable income, it would be helpful to know whether the direction of the influence of tax rate on tax base is a necessary consequence of rational behaviour or just an observed fact at some given time and place.

The next two sections of this essay deal with the constancy or variability of $\varepsilon$ as a reflection of tax evasion and of the labour-leisure choice. It will be shown that a) when the mechanism connecting base to rate is tax evasion, equation (4) might be misleading because the elasticity of taxable income, $\varepsilon$, is not independent of the tax rate, and $b$ ) when the mechanism connecting base to rate is the labour-leisure choice, an increase in the tax rate need not induce taxpayers to diminish their tax base.

## Tax Evasion and the Dependence of the Elasticity of Taxable Income on the Rate of Tax

Imagine an economy with a given distribution of people's gross incomes $Y$ where public revenue is acquired by a proportional income tax at a rate $t$ and where people evade tax in so far as it is advantageous to do so. By definition, the tax base, $y$, of a person with income Y who conceals a portion $\tau$ of his income from the tax collector is

$$
\begin{equation*}
y=(1-\tau) Y \tag{10}
\end{equation*}
$$

Suppose, for convenience that i) tax evasion is costly but completely undetectable once the appropriate cost of concealment is borne and ii) the marginal cost of concealment is proportional to the share, $\tau$, of income concealed. Suppose specifically that the marginal cost per dollar of gross income of concealing a share $\tau$ of one's income is $\beta \tau$ where $\beta$ is a parameter interpretable as the "efficiency of tax collection" is a technically-given constant. The full cost of concealing a portion $\tau$ of one's income, Y, becomes $\beta \tau^{2} / 2$.

On these assumptions, a person confronted with a tax rate $t$ conceals a portion $\tau$ of his income Y to minimize the total burden of taxation, equal to the sum of

$$
\begin{equation*}
\text { tax paid and cost of tax evasion }=t(1-\tau) Y+\beta \tau^{2} Y / 2 \tag{11}
\end{equation*}
$$

from which it follows - by minimizing this expression with respect to $\tau$ - that regardless of Y and for any given $t$ and $\beta$, the taxpayer chooses $\tau$ such that

$$
\begin{equation*}
\mathrm{t}=\beta \tau \tag{12}
\end{equation*}
$$

Income is concealed from the tax collector up to the point where the extra cost of concealment, $\beta \tau$, is just equal to the extra cost, t , of the tax that would otherwise be paid.

The tax base, y , becomes

$$
\begin{equation*}
y=(1-\tau) Y=(1-t / \beta) Y \tag{13}
\end{equation*}
$$

and tax revenue becomes

$$
\begin{equation*}
\mathrm{R}=\mathrm{ty}=\mathrm{t}(1-\tau) \mathrm{Y}=\mathrm{t}(1-\mathrm{t} / \beta) \mathrm{Y} \tag{14}
\end{equation*}
$$

Since, by assumption, everybody's proportional response to taxation is the same, the term Y in this equation can be reinterpreted as national income and the equation itself can be reinterpreted as the Laffer curve for an economy where the source of shrinkage of the tax base in response to the tax tate is tax evasion.

Figure 4: The Tax Payer's Decision about How Much Tax to Evade by Concealing Income from the Tax Collector


The taxpayer's behaviour is illustrated in figure 4. The height of the flat line is the constant marginal cost of not concealing one's income and paying tax instead. The height of the diagonal line is the marginal cost of concealing income from the tax collector. The taxpayer's choice, $\tau^{0}$, of the proportion of income to conceal is where marginal costs are equalized as indicated in equation (12). The area of the box at the bottom right-hand side of the figure is tax revenue as a proportion
of income, the triangular area to the right of the box is the corresponding deadweight loss, and their sum is the full cost to society per dollar of income. Absence of Y in equation (12) means that, for any given $\beta$, everybody's chosen value of $\tau$ must be the same.

It follows at once from equation (14) that the slope of the Laffer curve becomes

$$
\begin{equation*}
\delta \mathrm{R} / \delta \mathrm{t}=\mathrm{Y}[1-2 \mathrm{t} / \beta] \tag{15}
\end{equation*}
$$

and that the revenue-maximizing tax rate, $\mathrm{t}^{* *}$, becomes

$$
\begin{equation*}
\mathrm{t}^{* *}=\beta / 2 \tag{16}
\end{equation*}
$$

No higher rate could ever be advantageous to anybody because the revenue would be diminished, but, as is evident from inspection of figure 1 , the greater the efficiency of tax collection, the higher the revenue-maximizing tax rate would be.

Figure 5: How the Location of the Laffer Curve
Depends upon the Efficiency of Tax Collection


Figure 5 is a comparison of Laffer curves for four possible values of $\beta: 1 / 2,1,2$ and $\infty$. The curves are derived from equation (15) on the assumption that $Y=1$. For any given value of $\beta$, tax revenue, $R$, begins 0 when $t=0$, rises to a maximum of $\beta / 4$ when $t=\beta / 2$ (because $\delta R / \delta t=0$ implies that $2 t / \beta=1$ ), and declines thereafter. If $\beta=1 / 2$, total revenue is maximized at a tax rate of $25 \%$. If $\beta=1$, total revenue is maximized at a rate of $50 \%$. If $\beta=2$, total revenue is maximized at a rate of $100 \%$. The greater the efficiency of tax collection, the higher the tax rate at which revenue is maximized and the larger maximal revenue must be. The highest of the three curves, that for which $\beta=$ 2 , is truncated because the tax rate cannot exceed 1 . If $t$ is set equal to 1 , tax revenue becomes half the national income, wastage resources in concealing income from the tax collector becomes a quarter of
the national income and the remaining quarter is left with the tax payer to be consumed. Note also, that if $\beta=$ infinity, tax evasion is always prohibitively expensive so that the Laffer curve becomes an upward-sloping straight line and the entire gross, pre-tax income can be appropriated by the tax collector with a tax rate of $100 \%$.

An interesting feature of this economy is that, depending on the efficiency of tax collection, the Laffer curve need not peak at a tax rate of less than $100 \%$. The base shrinks as the rate increases, but not necessarily by enough to reduce the revenue acquired. Any value of $\beta$ equal to or greater than 2 yields a steadily upward-sloping Laffer curve. To be sure, the value of $\beta$ - or, more generally, the cost of diminishing one's observed tax base no matter how it is done - is an empirical matter. Perhaps the truth is that the revenue-maximizing tax rate is well short of $100 \%$. Perhaps, as has been alleged, it is in the order of $50 \%$, in which case there is little room for redistribution once ordinary public goods have been acquired. The most that can be said on the strength of this model is that an entirely upwardsloping Laffer curve cannot be ruled out a priori. On the other hand, to say that society can maximize tax revenue at very high tax rates is not to say that it would wish to do so. That depends on the will of the electorate.

From equations (12) and (13), it follows that the elasticity of taxable income is

$$
\begin{equation*}
\varepsilon=[(1-\mathrm{t}) / \mathrm{y}][\delta \mathrm{y} / \delta(1-\mathrm{t})]=[(1-\mathrm{t}) /\{(1-\mathrm{t} / \beta) \mathrm{Y}\}][\mathrm{Y} / \beta]=(1-\mathrm{t}) /(\beta-\mathrm{t}) \tag{17}
\end{equation*}
$$

which is clearly dependent on $t$ and which falls to 0 as $t$ approaches $100 \%$.
There is here a marked discrepancy between the inferred variability of the tax-dependent elasticity of taxable income in the tax evasion story and the assumed constancy of the elasticity of taxable income in the new tax responsiveness literature. Revenue-maximizing tax rates differ accordingly: $\mathrm{t}^{* *}=\beta / 2$ in equation (16) in the tax evasion story, and $\mathrm{t}^{*}=1 /(1+\varepsilon)$ in equation (4) in the new tax responsive literature. Suppose, for example, that $\beta=2$ so that the revenue-maximizing tax rate of $100 \%$ as implied by equation (16) and may be read off figure 3 , and suppose the economy is observed when the actual tax rate is $20 \%$. In accordance with equation (17), the observed value of $\varepsilon$ would have to be .44 [equal to $(1-.2) /(2-.2)$ ] implying, if $\varepsilon$ were invariant, a revenue-maximizing tax rate of about $70 \%$ [equal to $1 /(1+.44)$ ]. In an economy where deadweight loss is due to tax evasion, the new tax responsive literature would mistakenly infer that the revenue-maximizing tax rate is $70 \%$ when, in fact, tax increases would generate additional revenue all the way up to $100 \%$.

An important feature of this model of tax evasion in an economy with given pre-tax incomes as distinct from the model of the labour-leisure choice to be discussed below - is that the shape of the Laffer curve and the revenue-maximizing tax rate are independent of the distribution of income and of how tax revenue is spent. Instead, the distribution of income and the usage of public revenue determine each person's preferred point on the Laffer curve and society's choice of the tax rate.

A particular case is of special interest. Ignore ordinary public goods and suppose all tax revenue is redistributed through a negative income tax, a fixed payment to everybody (the demogrant) financed by a proportional income tax at some rate $t$. Though everybody conceals the same proportion of income
in response to any given tax rate, each person has his own preferred degree of redistribution which can be represented unambiguously by his preferred tax rate. One's preferred tax rate is the rate at which one's net income, I, is as large as possible, where net income under a negative income tax takes account not just of tax paid and demogrant received, but of expenditure to hide a part of one's income from the tax collector and of one's estimate other people's tax evasion. On the assumptions made so far, everybody's chosen $\tau$ as a function of $t$ is the same. One's preferred tax depends on one's own gross income, Y , the average gross income, $\mathrm{Y}^{\mathrm{AV}}$, the efficiency of tax collection, $\beta$, and the common rate of tax evasion $\tau$. Specifically, one's preferred tax rate maximizes one's net income where
net income $=$ gross income - tax paid + demogrant - cost of tax evasion.

$$
\begin{equation*}
\mathrm{I}=\mathrm{Y} \quad-\mathrm{t}(1-\tau) \mathrm{Y}+\mathrm{t}(1-\tau) \mathrm{Y}^{\mathrm{AV}}-\quad \beta \tau^{2} \mathrm{Y} / 2 \tag{18}
\end{equation*}
$$

or, equivalently, using equation (3),

$$
\begin{equation*}
I\left(t, Y, Y^{A V}, \beta\right)=y-t(1-t / \beta) Y+t(1-t / \beta) Y^{A V}-t^{2} Y / 2 \beta \tag{19}
\end{equation*}
$$

For given values of $\mathrm{Y}, \mathrm{Y}^{\mathrm{AV}}$ and $\beta$, one's preferred tax rate is identified by

$$
\begin{equation*}
\delta \mathrm{I} / \delta \mathrm{t}=-\mathrm{Y}+(\mathrm{t} / \beta) \mathrm{Y}+\mathrm{Y}^{\mathrm{AV}}-(2 \mathrm{t} / \beta) \mathrm{Y}^{\mathrm{AV}}=0 \tag{20}
\end{equation*}
$$

Thus, as long as $\mathrm{Y}<\mathrm{Y}^{\mathrm{AV}}$, the preferred t of a person with a gross income Y becomes

$$
\begin{equation*}
\mathrm{t}(\mathrm{Y})=\beta\left\{\mathrm{Y}^{\mathrm{AV}}-\mathrm{Y}\right\} /\left\{2 \mathrm{Y}^{\mathrm{AV}}-\mathrm{Y}\right\} \tag{21}
\end{equation*}
$$

Several things follow from equation (21). First, the preferred tax rate of a person with no income at all, with $\mathrm{Y}=0$, is the revenue-maximizing tax rate at the top of the Laffer curve as indicated by equation (16). Such a person wants the largest possible demogrant regardless of the tax rate, t . Second, the larger one's income, the smaller one's preferred tax rate. Third, strictly speaking, a person with above average income would prefer a negative tax rate, a subsidy proportional to income, financed by a lump sum tax, but, where that is not feasible, would settle for a tax rate of 0 .

The story can be told in a different way. To maximize net income, I in equation (18), is to choose the tax rate $t$ for which $\Delta I=0$ or, equivalently, for which

$$
\begin{equation*}
\Delta(\text { tax paid })+\Delta(\text { deadweight loss })=\Delta(\text { demogrant }) \tag{22}
\end{equation*}
$$

where $\Delta$ refers to the change resulting from a slight increase in the tax rate, where deadweight loss in this case is expenditure to conceal tax evasion, and where, with proportional income taxation,

$$
\begin{equation*}
\Delta(\text { demogrant }) / \Delta(\text { tax paid })=\mathrm{Y}^{\mathrm{AV}} / \mathrm{Y} \tag{23}
\end{equation*}
$$

so that

$$
\begin{equation*}
\{\Delta(\text { tax paid })+\Delta(\text { deadweight loss })\} /\{\Delta(\text { tax paid })\}=\mathrm{Y}^{\mathrm{AV}} / \mathrm{Y} \tag{24}
\end{equation*}
$$

where the ratio on the left-hand side of this equation is the marginal cost of public funds, MCPF, defined as the full cost to the taxpayer per dollar of public revenue acquired. The marginal cost of public funds plays a central role in benefit-cost analysis where a project is deemed desirable if and only if benefits exceed costs to people whomsoever they may be. If benefits are measured as values to people and costs are measured as expenditures by the government, then the required benefit-cost ratio must be raised from 1 to MCPF.

The marginal cost of public funds is the same for everybody because income, Y, cancels out in the numerator and denominator on the left-hand side of equation (24). The marginal cost of public funds is an increasing function of the tax rate, $t$, so that the preferred tax rate, $t$, of a person with income Y may be identified by the equation

$$
\begin{equation*}
\operatorname{MCPF}(\mathrm{t})=\mathrm{Y}^{\mathrm{Av}} / \mathrm{Y} \tag{25}
\end{equation*}
$$

where

$$
\begin{align*}
\operatorname{MCPF}(\mathrm{t}) & =\{\Delta[\operatorname{tax} \text { paid }]+\Delta[\operatorname{cost} \text { of tax evasion }]\} /\{\Delta[\text { tax paid }]\} \\
& \left.=\{\delta[\mathrm{t}(1-\mathrm{t} / \beta) \mathrm{Y})] / \delta \mathrm{t}+\delta\left[\left(\mathrm{t}^{2} / 2 \beta\right) \mathrm{Y}\right] / \delta \mathrm{t}\right\} /\{\delta[\mathrm{t}(1-\mathrm{t} / \beta) \mathrm{Y})] / \delta \mathrm{t} \\
& =\left\{\delta\left[\mathrm{t}-\mathrm{t}^{2} / \beta+\mathrm{t}^{2} / 2 \beta\right] / \delta \mathrm{t}\right\} /\left\{\delta\left[\mathrm{t}-\mathrm{t}^{2} / \beta\right] / \delta \mathrm{t}\right\}=\{1-\mathrm{t} / \beta\} /\{1-2 \mathrm{t} / \beta\} \\
& =\{\beta-\mathrm{t}\} /\{\beta-2 \mathrm{t}\} \tag{26}
\end{align*}
$$

Figure 6: Determining a Person's Preferred Tax Rate


As illustrated in figure 6, the marginal cost of public funds begins at 1 when $t=0$ and rises steadily to infinity as $t$ approaches $\beta / 2$ which is the revenue-maximizing tax rate. ${ }^{5}$ A person with
${ }^{5}$ Equations (21), (25) and (26) are entirely consistent. Setting $\{\beta-\mathrm{t}\} /\{\beta-2 \mathrm{t}\}=\mathrm{Y}^{\mathrm{AV}} / \mathrm{Y}$, yields precisely the function $\mathrm{t}(\mathrm{Y})$ in equation (21). Equation (25) is discussed in greater detail in
income $Y$ chooses the tax rate $t(Y)$ at which MCPF has risen to $\mathrm{Y}^{\mathrm{AV}} / \mathrm{Y}$.
When public decisions are in accordance with the preference of the median voter, the extent of redistribution and the tax rate to finance it are chosen where the ratio of mean to median income is just equal to the marginal cost of public funds.

Essentially the same story can be told with reference to the Laffer curve because the elasticity of the Laffer curve is the inverse of the marginal cost of public funds. Specifically, since a person's preferred tax rate is where the marginal cost of tax avoidance is equal to the tax that would otherwise be paid, since $\mathrm{dy} / \mathrm{dt}<0$, and using the definition of the elasticity of taxable income, $\varepsilon$, in equation (3) above,

$$
\begin{align*}
\mathrm{MCPF} & =\{\mathrm{dR} / \mathrm{dt}-\mathrm{tdy} / \mathrm{dt}\} /\{\mathrm{dR} / \mathrm{dt}\} \\
& =\{\mathrm{y}+\mathrm{tdy} / \mathrm{dt}-\mathrm{tdy} / \mathrm{dt}\} /\{\mathrm{y}+\mathrm{tdy} / \mathrm{dt}\}=1 /\{1+(\mathrm{t} / \mathrm{y}) \mathrm{dy} / \mathrm{dt}\} \\
& =1 /\{1-\mathrm{t} /(1-\mathrm{t}) \varepsilon\}=1 /\{(\mathrm{t} / \mathrm{R})(\mathrm{dR} / \mathrm{dt})\} \tag{27}
\end{align*}
$$

The lower one's income, the higher the marginal cost of public funds at which redistribution is deemed sufficient and the farther along the Laffer curve one wants society to go.

Bear in mind that the revenue-maximizing tax rate is significantly higher than anybody with any income at all would actually favour. The impression that the revenue-maximizing tax rate is lower than is actually the case may nevertheless influence the public's choice of tax rates.

Three features of the tax evasion story should be emphasized before the labour-leisure story is introduced. First, the elasticity of taxable income is necessarily positive, for the tax base always contracts when the tax rate is increased. Second, the Laffer curve may, but need not, be humped. Depending on the efficiency of tax collection, the Laffer curve may instead rise gradually all the way to a tax rate of $100 \%$ as illustrated in figure 1 b . Third, the impact of tax rate on tax revenue is independent of how tax revenue is spent. Whether, tax revenue is redistributed as assumed in the preceding essay or spent on public goods such as the army and the police, has no impact at all on the shape of the Laffer curve. All this changes in the model of the labour-leisure choice.

## The Labour-leisure Choice

Can a humped Laffer curve with a revenue-maximizing tax rate well below $100 \%$ be restored when tax evasion is replaced by the labour-leisure choice as the mechanism by which tax revenue responds to the tax rate? This is sometimes thought to be so - with people switching from work to

Usher (2006).
leisure in response to a rise in the tax rate - but it is false not in the sense that it is never so, but in the sense that it need not be. It will be argued later on that there are indeed reasons why tax rates cannot rise to $100 \%$, but it is important to determine which influences on the behaviour of the tax payer constrain tax rates and which do not. It turns out that the labour-leisure choice does not necessarily impose such a constraint.

When a fixed supply of time is devoted to goods and leisure, an income tax, being a tax on goods alone, may reduce the supply of labour as taxpayers substitute less expensive leisure for more expensive goods, or may increase the supply of labour as taxpayers work more to compensate for the tax-induced reduction in the proportion of goods to leisure consumed. The first of these outcomes generates the concave Laffer curve in figure 1a. The second generates the convex Laffer curve in figure 1 b . Both outcomes are possible depending on the elasticity of substitution in use between goods and leisure.

Discussion of the labour-leisure choice will proceed in two stages: i) with L-shaped indifference curves implying perfect complementarity in use between goods and leisure, and ii) with indifference curves generated by utility functions with constant elasticity of substitution in use between goods an labour, which may be anywhere from 0 to infinity. It will at first be assumed that the labourleisure choice is affected by the tax rate but independent of how public revenue is used, whether for public goods or for redistribution. This assumption was plausible when the elasticity of taxable income was a reflection of tax evasion, for the given efficiency of tax collection rendered the choice between paying pay tax and evading tax independent of everything but the amount of tax evaded. The assumption is less plausible in the context of the labour-leisure choice as will be shown below.

Perfect Complementarity between Goods and Leisure: Consider a person with a fixed supply of time, designated as 1 , to be allocated between work H and leisure L measured not in hours, but as a proportion of total available time, so that

$$
\begin{equation*}
\mathrm{L}(\mathrm{t})+\mathrm{H}(\mathrm{t})=1 \tag{28}
\end{equation*}
$$

Figure 7: Indifference Curves with No Substitution Between Goods and Leisure


Perfect complementarity between goods and leisure means that indifference curves are Lshaped as illustrated in figure 7, with goods, G , on the vertical axis and leisure, L , on the horizontal axis. The path of the vertices of indifference curves, called the "wasteless combinations" curve, is upward-sloping. The wasteless combinations curve shows all combinations of goods and leisure for which neither more goods nor more leisure would increase the person's utility unless combined with more of the other.

The response to taxation is illustrated in figure 8, an extension of figure 7 with the same "wasteless combinations" curve but with the addition of the person's budget constraints in the absence of taxation and when a tax rate of $t$ is imposed. To avoid cluttering the diagram, the indifference curves are not shown. In the absence of taxation, the budget constraint of a person with a wage w is the diagonal line with slope w originating at the point 1 on the horizontal axis. The highest attainable indifference curve is at the crossing of the budget constraint and the wasteless combinations curve, yielding a combination, $\mathrm{G}(0)$ and $\mathrm{L}(0)$, of goods and leisure as indicated by the point A . It is immediately evident that one's allocation of time between labour and leisure is dependent on the tax rate. When a tax $t$ is imposed, the taxpayer's net wage falls from $w$ to $w(1-t)$ causing a counter-clockwise swing in the budget constraint. Once again, the taxpayer, seeking to maximize utility, chooses a combination of goods and leisure at the crossing of the (new) budget constraint and the wasteless combination curve, yielding a combination, $G(t)$ and $L(t)$, of goods and leisure as shown by the point C .

Figure 8: A Person's Response to Taxation


An increase in the tax rate, $t$, represented by a counter-clockwise rotation of the budget constraint, leads to a decrease in leisure, $\mathrm{L}(\mathrm{t})$ and a corresponding increase in work, $\mathrm{H}(\mathrm{t})$, ensuring that $\delta \mathrm{H} / \delta \mathrm{t}>0$. Since revenue acquired from this person is

$$
\begin{equation*}
\mathrm{R}(\mathrm{t})=\mathrm{twH}(\mathrm{t}) \tag{29}
\end{equation*}
$$

The Laffer curve becomes steadily upward-sloping.

$$
\begin{equation*}
\delta \mathrm{R} / \delta \mathrm{t}=\mathrm{w}[\mathrm{H}(\mathrm{t})+\mathrm{t} \delta \mathrm{H} / \delta \mathrm{t}]>0 \tag{30}
\end{equation*}
$$

regardless of the value of $t$ as shown in figure 1 b and1c. Tax rate and tax base rise or fall together, guaranteeing that tax revenue increases with the tax rate all the way to $100 \%$, verifying Frank Knight's claim in the quotation at the beginning of this essay that people work more hours, not less, in response to an increase in the tax rate.

Consider the special case where the wasteless combinations curve is an upward-sloping straight line with slope $\theta$. Along any such line,

$$
\begin{equation*}
\theta \mathrm{L}=(1-\mathrm{t})(1-\mathrm{L}) \mathrm{w} \tag{31a}
\end{equation*}
$$

because $\mathrm{G}=\theta \mathrm{L}$ along the wasteless combinations curve, and $\mathrm{G}=(1-\mathrm{t})(1-\mathrm{L}) \mathrm{w}$ where $(1-\mathrm{L})$ is the labour required to procure an amount of goods $G$ when tax at a rate $t$ is imposed.

Equivalently, $\quad \mathrm{L}=(1-\mathrm{t}) \mathrm{w} /[\theta+(1-\mathrm{t}) \mathrm{w})$
so that the Laffer curve becomes

$$
\begin{equation*}
\mathrm{R}=\mathrm{t}(1-\mathrm{L}) \mathrm{w}=\theta \mathrm{tw} /[\theta+(1-\mathrm{t}) \mathrm{w}] \tag{32}
\end{equation*}
$$

This Laffer curve is not humped. It starts at $R=0$ at $t=0$ and then rises steadily as $t$ increases, reaching a maximum of $w$ - which is as large as output can ever be when the endowment of time is set equal to $1-$ at $\mathrm{t}=100 \%$, as illustrated in figure 1 c .

This result is subject to an important qualification. Tax revenue is shown to increases steadily with the tax rate on the assumption that the taxpayer's labour-leisure choice is unaffected by how tax revenue is spent, as might be the case when revenue is to finance foreign aid, the army and the police. There is a very different outcome when tax revenue is redistributed.

Without abandoning the assumption that goods and leisure are perfect complements, suppose all public revenue is redistributed, increasing consumption of goods without affecting consumption of leisure except in so far as the taxpayer chooses to work more, or to work less, in response to a taxfinanced transfer of goods. A distinction is required here between a person's tax paid, R , and subsidy received, S , where the two may be but are not necessarily the same.

With tax revenue is completely redistributed and with perfect complementarity between goods and leisure, the supply of labour becomes invariant as illustrated in figure 6 . In the absence of any tax or subsidy, a person with a pre-tax wage w consumes $\underline{L}$ units of leisure and $\underline{G}$ units of goods as represented by the point A. Provision of a subsidy, S , raises the initial endowment from 1 unit of labour and no goods to 1 unit of labour and $S$ goods as represented by the point $B$ which is a distance $S$ above the horizontal axis. Now suppose that there is imposed a tax at a rate just high enough to pay for the subsidy. Tax revenue, R , must be such $\mathrm{R}=\mathrm{S}$. Being no better off and no worse off on account of the tax and subsidy together, the person's behaviour is unchanged. The upward shift in the
person's budget constraint brought about the subsidy is exactly matched by the downward shift brought about by the imposition of the tax to finance it.

For any subsidy, S, there is some tax rate - call it $\mathrm{t}(\mathrm{S})$ - that leaves leisure, goods, labour, $\mathrm{L}, \mathrm{G}$ and $1-\mathrm{L}$, exactly as they would be in the absence of all tax and subsidy. The required tax rate is such that the slope of the post-subsidy budget constraint - the line starting at B and with slope $\mathrm{w}(1-\mathrm{t})$ passes through the point $A$. When goods and leisure are prefect complements, tax-financed redistribution has no effect upon the welfare of the taxpayer for whom the subsidy is just equal to the tax paid. Otherwise, with some substitutability on use between goods and leisure, a tax-financed subsidy in a society of identical people makes everybody worse off because everybody bears the cost of actions - working less or hiding income from the tax collector - to reduce one's tax bill.

Figure 9: Taxation and Redistribution Leave Labour Supply Unchanged ( $\mathrm{R}=\mathrm{CA}=$ the height of the point B above the horizontal axis)


In a society where people's wages differ, where total tax revenue is redistributed equally as in a negative income tax and where goods and leisure are perfect complements, the net beneficiaries of redistribution - people with less than average wage - are induced by the system to work less, and the net contributors are induced to work more. If, in addition, the income distribution is skewed in the usual way so that the median wage is less than the average wage and as long as the prospect of starvation has no effect upon the marginal product of labour, the median voter (the person with the median wage) favours a tax rate of $100 \%$.

A uniform tax rate of $100 \%$ may be impossible without redistribution when a minimal consumption of goods and leisure is required, but a top marginal rate of as much as $100 \%$ remains feasible as illustrated in figure 10.The figure compares the effects of a simple form of progressive taxation upon two taxpayers with identical L-shaped indifference curves but with different wages. One taxpayer has a high wage, $\mathrm{w}^{\mathrm{H}}$, and the other has a low wage, $\mathrm{w}^{\mathrm{L}}$. The tax schedule has three components: a uniform subsidy, S , a uniform tax rate t on all income less than some specified amount and a tax rate of $100 \%$ on all income above that amount. The figure is drawn on the
assumption that only the person with the high wage ever earns enough for any of his income to be subject to the higher tax rate but that the entire income of the person with the low wage is taxed at the low rate $t$. Thus, each person has two budget constrains, a pre-tax constraint represented by the upward-sloping straight lines through the point 1 on the horizontal axis, and a post-tax budget constraint represented by the kinked heavy lines. Each person`s chosen supply of labour is where the post-tax budget constraints cuts the wasteless combinations curve, placing him on the highest attainable indifference curve. ${ }^{6}$

Figure 10: Progressive Taxation with a Top Marginal Tax Rate of 100\%


The figure could easily have been drawn with more than two tax brackets and with a top rate

[^2]of less than $100 \%$. The reason for the special assumption about the top bracket is to demonstrate that, though a tax rate of $100 \%$ is virtually impossible when all income is taxed at one flat rate, a top rate of $100 \%$ becomes possible when goods and leisure are perfect complements. ${ }^{7}$ In fact, a top rate of more than $100 \%$ could be revenue-maximizing on the extreme assumptions that have been made so far. The rationale for these assumptions is not that they are likely to be valid in practice, but to add weight to the argument that the revenue-maximizing top rate may be higher than is often supposed and to serve as preface to the discussion of the substitutability between goods and leisure.

Before proceeding to the more general case with substitutability in use between goods and leisure, note that the tax evasion story and the simple labour-leisure story as told so far have completely opposite implications about the elasticity of taxable income. In the tax evasion story, the elasticity of taxable income is necessarily positive; $\varepsilon>0$ almost be definition. In the labour-leisure story wioth perfect complementarity, the elasticity of taxable income is necessarily negative; $\varepsilon<0$ as people work more to preserve their desired balance between goods and leisure. The contrast may have empirical implications. Any observed value of the elasticity of taxable income pertains to a group of people within which the elasticity might be positive for some and negative for others. The observed $\varepsilon$ can only be an average which may or may not be independent of the tax rate.

The Elasticity of Substitution in Use between Goods and Leisure: The question then arises of how large the substitutability between goods and leisure can be before the revenue-maximizing tax rate falls below $100 \%$. Substitutability is introduced by means of a utility function with a constant elasticity of substitution in use between goods and leisure.

$$
\begin{equation*}
\mathrm{u}=\mathrm{u}(\mathrm{G}, \mathrm{~L})=\left\{\mathrm{mG}^{\rho}+\mathrm{nL}^{\rho}\right\}^{1 / p} \tag{33}
\end{equation*}
$$

where, to keep matters simple, $m$ and $n$ are both set equal to 1 , where $\rho$ is a transformation of the elasticity of substitution in use, $\sigma$, between goods and leisure ${ }^{8}$ and where, with no redistribution of
${ }^{7}$ This result is in sharp contrast to the proposition, derived on very different assumptions, that the appropriate top marginal tax rate is $0 \%$. The proposition is derived on the assumption that the income for which the top rate applies can be set so high that it only applies to the additional income earned by the very richest person if and only if that income is not subject to tax. Diamond and Saez (2011, page 173) dismiss this result as irrelevant in practice.
${ }^{8}$ The elasticity of substitution, $\sigma$, is defined as $\sigma=[\%$ change in $\mathrm{G} / \mathrm{L}] /[\%$ change in $-\delta \mathrm{G} / \delta \mathrm{L}]=\{\mathrm{d}(\mathrm{G} / \mathrm{L}) /(\mathrm{G} / \mathrm{L})\} /\left\{\mathrm{d}\left(\mathrm{u}_{\mathrm{L}} / \mathrm{u}_{\mathrm{G}}\right) /\left(\mathrm{u}_{\mathrm{L}} / \mathrm{u}_{\mathrm{G}}\right)\right\}$
where $\mathrm{u}_{\mathrm{G}}=\left\{\mathrm{mG}^{\rho}+\mathrm{nL}^{\rho}\right\}^{1 / \rho-1}\left\{\mathrm{a}^{\rho} \mathrm{G}^{\rho-1}\right\} \quad$ and $\quad \mathrm{u}_{\mathrm{L}}=\left\{\mathrm{mG}^{\rho}+\mathrm{nL}^{\rho}\right\}^{1 / \rho-1}\left\{\mathrm{~b} \rho \mathrm{~L}^{\rho-1}\right\}$
so that $\quad u_{L} / u_{G}=(n / m)(G / L)^{1-\rho} \quad$ and $\quad d\left(u_{L} / u_{G}\right) / d(G / L)=(n / m)(1-\rho)(G / L)^{-p}$
Rearranging the components of the definition of the elasticity, we see that
$\sigma=\left\{1 / \mathrm{d}\left(\mathrm{u}_{\mathrm{L}} / \mathrm{u}_{\mathrm{G}}\right) / \mathrm{d}(\mathrm{G} / \mathrm{L})\right\}\left\{\left(\mathrm{u}_{\mathrm{L}} / \mathrm{u}_{\mathrm{G}}\right) /(\mathrm{G} / \mathrm{L})\right\}=\left\{1 /\left[(\mathrm{n} / \mathrm{m})(1-\rho)(\mathrm{G} / \mathrm{L})^{-\mathrm{p}}\right]\right\}\left\{(\mathrm{n} / \mathrm{m})(\mathrm{G} / \mathrm{L})^{1-\rho} /(\mathrm{G}, \mathrm{L})\right\}=1 /(1-\rho)$
income, the attainable $G$ depends on the wage rate, $w$, the tax rate, $t$, and the supply of labour, H ,

$$
\begin{equation*}
\mathrm{G}=\mathrm{wH}(1-\mathrm{t})=\mathrm{w}(1-\mathrm{L})(1-\mathrm{t}) \tag{34}
\end{equation*}
$$

Maximizing utility with respect to the budget constraint yields a first order condition

$$
\begin{equation*}
(\mathrm{G} / \mathrm{L})^{\rho-1}=1 / \mathrm{w}(1-\mathrm{t}) \tag{35}
\end{equation*}
$$

Replacing G with $\mathrm{wH}(1-\mathrm{t})$ from equation (34), replacing L with $1-\mathrm{H}$, and recognizing that $\sigma=1 /(1-\rho)$ as shown in the preceding footnote, equation (35) is converted to

$$
\begin{equation*}
[\mathrm{H} /(1-\mathrm{H})]^{\rho-1}[\mathrm{w}(1-\mathrm{t})]^{\rho}=1 \tag{36}
\end{equation*}
$$

from which, recognizing that $\sigma=1 /(1-\rho)$, it follows that

$$
\begin{equation*}
\left.\mathrm{H}=1 /[\mathrm{w}(1-\mathrm{t})]^{1-\sigma}+1\right] \tag{37}
\end{equation*}
$$

so that $\quad \delta \mathrm{H} / \delta(1-\mathrm{t})=(\sigma-1) \mathrm{H}^{2} \mathrm{w}^{1-\sigma}(1-\mathrm{t})^{-\sigma}$
which is positive or negative depending on whether $\sigma>1$ or $\sigma<1$. The same is true of the elasticity of taxable income.

$$
\begin{equation*}
\varepsilon=[(1-\mathrm{t}) / \mathrm{H}][\delta \mathrm{H} / \delta(1-\mathrm{t})]=(\sigma-1) \mathrm{Hw}^{1-\sigma}(1-\mathrm{t})^{1-\sigma} \tag{39}
\end{equation*}
$$

If $\sigma>1$ indicating that goods and leisure are quite substitutable in use, then $\delta \mathrm{H} / \delta(1-\mathrm{t})>0$ and $\varepsilon>0$ as well so that the response to a tax increase is to decrease the supply of labour, substituting leisure for goods because goods have become more expensive. If $\sigma<1$, indicating that goods and leisure are less substitutable in use, then $\delta \mathrm{H} / \delta(1-\mathrm{t})<0$ and $\varepsilon<0$ as well so that the response to a tax increase is to increase the supply of labour, working more to make up the lost income as Knight claimed in the quotation at the beginning of this essay and generating a convex Laffer curve as was the case with perfect complementarity between goods and leisure. In either case, as shown in equation (39), the elasticity of taxable income depends on $t$ so that revenue-maximizing tax rate cannot be predicted from $\varepsilon$ in accordance with equation (4) above. If $\sigma=1$, then $\varepsilon=0$ meaning that the tax base is independent of the tax rate so that the Laffer curve is an upward-sloping straight line. The C.E.S. production function in equation (33) reduces to a Cobb-Douglas function in that case. ${ }^{9}$

Equation (39) shows $\varepsilon$ as a function of $t$ regardless of whether tax revenue is maximized, but it follows immediately from equation (4) above that

$$
\begin{equation*}
\varepsilon=(1-\mathrm{t}) / \mathrm{t} \tag{40}
\end{equation*}
$$

[^3]when tax revenue is maximized. Thus, from equation (39) and (40) together, it follows that tax revenue is maximized when
\[

$$
\begin{equation*}
(1-t) / t=(\sigma-1) \mathrm{Hw}^{1-\sigma}(1-t)^{1-\sigma} \tag{41}
\end{equation*}
$$

\]

which holds for some $t$ less than $100 \%$ as long as $\sigma>1$. Other things equal, the greater the elasticity of substitution, the lower the revenue-maximizing $t$ must be.

From equation (37), it follows that the taxpayer's Laffer curve - tax payment denominated in goods as a function of the tax rate - becomes

$$
\begin{equation*}
\mathrm{R}=\mathrm{Htw}=\mathrm{tw} /\left[\mathrm{w}(1-\mathrm{t})^{1-\sigma}+1\right] \tag{42}
\end{equation*}
$$

from which tax paid may be computed as a function of the tax rate, the elasticity of substitution and the wage rate.

Table 3 shows tax paid as a proportion of a person's income as computed from equation (42) for a selection of tax rates, $t$, from 0 to $100 \%$, for a selection of elasticities of substitution, $\sigma$, from 0 to 5 and for the special case where $\mathrm{w}=1$. Combinations of t and $\sigma$ indicate by $*$ indicate the revenue-maximizing tax rates for each $\sigma$.

Table 3: Tax Revenue as a Function of the Elasticity of Substitution and the Tax Rate
(The revenue-maximizing tax rate for each elasticity of substitution is indicated by *)

|  | $\sigma=0$ | $\sigma=0.5$ | $\sigma=1$ | $\sigma=1.5$ | $\sigma=2$ | $\sigma=3$ | $\sigma=5$ |
| :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- |
| $\mathrm{t}=0$ | 0 | 0 | 0 | 0 | 0 | 0 | 0 |
| $\mathrm{t}=.1$ | .0526 | .0513 | .05 | .0487 | .0474 | .0448 | .0396 |
| $\mathrm{t}=.2$ | .1111 | .1056 | .1 | .0944 | .0889 | .0780 | $.05812 *$ |
| $\mathrm{t}=.3$ | .1765 | .1633 | .15 | .1367 | .1235 | .0987 | .05808 |
| $\mathrm{t}=.4$ | .25 | .2254 | .2 | .1746 | .15 | $.1059 *$ | .0459 |
| $\mathrm{t}=.5$ | .3333 | .2929 | .25 | .2071 | .1667 | .1 | .2071 |
| $\mathrm{t}=.6$ | .4286 | .3675 | .3 | .2325 | $.1714 *$ | .0828 | .0150 |
| $\mathrm{t}=.7$ | .5385 | .4523 | .35 | $.2477 *$ | .1615 | .0578 | .0056 |
| $\mathrm{t}=.8$ | .6667 | .5528 | .4 | .2472 | .1333 | .0308 | .0013 |
| $\mathrm{t}=.9$ | .8182 | .6838 | .45 | .2162 | .0818 | .0089 | .0001 |
| $\mathrm{t}=1$ | $1 *$ | $1 *$ | $.5 *$ | 0 | 0 | 0 | 0 |

Several features of the table should be noted.

- The numbers in the table show the Laffer curve to be humped - maximizing tax revenue at a rate of less than $100 \%$ - if and only if the elasticity of substitution in use between goods and leisure is greater than 1 .
- From equation (37), it follows immediately that, regardless of the tax rate, exactly half of the taxpayer's available time is devoted to labour when the elasticity of substitution between goods and leisure is exactly 1 , so that $\mathrm{R}=\mathrm{Ht}=\mathrm{t} / 2$. The Laffer curve is an upward-sloping straight line with a maximal revenue of $1 / 2$.
- When the elasticity of substitution is greater than 1 , tax revenue is always maximized at a tax rate of less than $100 \%$, a rate beyond which the revenue loss from the diversion from labour to leisure caused by an increase in the tax rate exceeds the revenue gain from the higher tax on what is left of the original base. For each value of $\sigma$, the maximal revenue is indicated by *.
- The higher the elasticity of substitution, the lower the revenue-maximizing tax rate. At $\sigma=$ 1 , the revenue-maximizing tax rate is $100 \%$. At $\sigma=1.5$, the revenue-maximizing tax rate falls to $70 \%$. At $\sigma=2$, the revenue-maximizing tax rate falls to $60 \%$. At $\sigma=3$, the revenue-maximizing tax rate falls to $40 \%$. At $\sigma=5$, the revenue-maximizing tax rate falls to $20 \%$.
- There is an imposed symmetry in the utility function of equation (33); the rate of substitution between goods and leisure is assumed to depend on their ratio alone and is unaffected by a proportional increase or decrease in both together. That restriction does not apply to the L-shaped indifference curves in figure 7 .

The Elasticity of Taxable Income as a Generalization of the Elasticity of Supply of Labour: When the source of the Laffer curve is the labour-leisure choice, the elasticity of taxable income and the elasticity of supply of labour are one and the same. The supply of labour, $H$, is the tax base and the wage as seen by the worker is $\mathrm{w}(1-\mathrm{t})$ where w is treated as a constant that might as well be set equal to 1 . A positive value of the elasticity of taxable income arises from the usual upward-sloping supply curve of labour. A negative value of the elasticity of taxable income arises from a backward-bending supply curve.

Upward-sloping and backward-bending supply curves of labour are shown side by side on figure 11 with hours of labour, H , on the horizontal axis and net wage, $\mathrm{w}(1-\mathrm{t})$, on the vertical axis. Both sides of the figure show effects on H of a small increase, $\Delta \mathrm{t}$, in the tax rate, lowering net wage from $\mathrm{w}(1-\mathrm{t})$ to $\mathrm{w}(1-\mathrm{t}-\Delta \mathrm{t})$.

With an upward-sloping curve of labour as shown on the left-hand figure, taxation at a rate $t$ yields revenue, $R(t)$, equal to $t w H(t)$ and represented by the area $A+C$. When the tax rate rises to $t$ $+\Delta t$, tax revenue becomes $R(t+\Delta t)$ equal to $(t+\Delta t) w H(t+\Delta t)$ and represented by the area $\mathrm{A}+\mathrm{B}$. The change in tax revenue, $\Delta \mathrm{R}$, brought about by the increase in the tax rate, $\Delta \mathrm{t}$, is [area B area C] which, as is evident from the figure, becomes progressively smaller and eventually turns
negative as t increases.
Figure 11: Upward-sloping and Backward-bending Supply Curves of Labour
a) An Upward-sloping
Supply Curve

b) A Backward-bending

Supply Curve


Tax revenue is maximized when area $B=$ area $C$ where

$$
\begin{align*}
& \text { area } B=w \Delta t H  \tag{43}\\
& \text { area } C=w t|\Delta H| \tag{44}
\end{align*}
$$

and $|\Delta \mathrm{H}|$ is the absolute value of $\Delta \mathrm{H}$. The elasticity of supply of labour is

$$
\begin{equation*}
\varepsilon=[|\Delta \mathrm{H}| / \mathrm{H}] \div[\Delta \mathrm{t} /(1-\mathrm{t})]>0 \tag{45}
\end{equation*}
$$

which is essentially the elasticity of taxable income as defined in equation (1) with H as the tax base. It follows that

$$
\begin{equation*}
|\Delta \mathrm{H}|=\varepsilon \mathrm{H} \Delta \mathrm{t} /(1-\mathrm{t}) \tag{46}
\end{equation*}
$$

so that $\mathrm{B}=\mathrm{C}$ implies

$$
\begin{equation*}
\mathrm{w} \Delta \mathrm{tH}=\mathrm{wt} \mathrm{t} H \Delta \mathrm{t} /(1-\mathrm{t}) \tag{47}
\end{equation*}
$$

or

$$
\begin{equation*}
1=\varepsilon t /(1-t) \tag{48}
\end{equation*}
$$

implying that $\mathrm{t}^{*}=1 /(1+\varepsilon)$ as in equation (4) above, where $t^{*}$ is the revenue-maximizing tax rate, guaranteed to be less than 1 as long as $\varepsilon>0$ as must be the case when there is an upward-sloping supply curve of labour. Equation (4) above has been reproduced in a roundabout but instructive way. ${ }^{10}$ Notice, however, that there is no escape here from the difficulty discussed at the beginning of

[^4]this essay that the elasticity of taxable income need not be invariant, that $\varepsilon$ observed at one tax rate need not remain the same at another.

By contrast, tax revenue increases steadily with the tax rate - with no revenue-maximizing rate of less than $100 \%$ - when supply curve of labour is backward-bending as shown on the righthand side of the figure. Now, ignoring the little rectangle between area B and area C, a tax increase from $t$ to $t+\Delta t$ leads to an increase in tax revenue from $w t H(t)$ represented by the area $A$ to $w(t+$ $\Delta t) H(t+\Delta t)$ represented by the sum of areas A, B and C. The change in revenue is

$$
\begin{align*}
\Delta \mathrm{R} & =\operatorname{area} \mathrm{B}+\operatorname{area} \mathrm{C}=\mathrm{w} \Delta \mathrm{tH}+\mathrm{wt} \Delta \mathrm{H} \\
& =\mathrm{w} \Delta \mathrm{tH}[1+(\mathrm{t} / \mathrm{H})(\Delta \mathrm{H} / \Delta \mathrm{t})]=\mathrm{w} \Delta \mathrm{tH}[1-\mathrm{t} /(1-\mathrm{t})] \tag{49}
\end{align*}
$$

where $\varepsilon=[(1-\mathrm{t}) / \mathrm{H}][\delta \mathrm{H} / \delta(1-\mathrm{t})]$ is at once the elasticity of taxable income and the elasticity of supply of labour. By definition, $\delta \mathrm{H} / \delta(1-\mathrm{t})<0$ when the supply curve is backward-bending so that $\Delta \mathrm{R}$ must be positive regardless of the value of $t$. The Laffer curve is upward-sloping rather than humped in this case.

Note finally that an elasticity of substitution in use between goods and labour of less than 1 , a negative elasticity of taxable income, a backward-bending supply curve of labour and a convex Laffer curve with a revenue-maximizing tax rate of $100 \%$ are, as it were, four sides of the same coin. When taxation influences the labour-leisure choice, the Laffer curve is humped if the elasticity of substitution in use between labour and leisure is greater than land is uniformly upward-sloping if the elasticity of substitution is less than $1 .{ }^{11}$
income tax. See Peter Diamond and Emmanuel Saez, (2011), page 170. As it stands, equation (45) is for a proportional tax at a uniform rate. To focus on the top bracket in progressive taxation, let $\mathrm{Y}^{\mathrm{T}}$ be the income at which the top rate first takes effect, let $\mathrm{Y}^{*}$ be the average income in excess of $\mathrm{Y}^{\mathrm{T}}$ for only those taxpayers whose tax base exceeds $\mathrm{Y}^{\mathrm{T}}$ and on whom the top rate is imposed, and let $t$ refer to the top marginal tax rate. The major analytical change in equation (45) is that the elasticity of base to rate is constructed for the marginal tax rate and the average tax base $\left(\mathrm{Y}^{\mathrm{T}}+\mathrm{Y}^{*}\right.$ rather than $\mathrm{Y}^{*}$ alone). The elasticity in equation (45) is converted to

$$
\varepsilon=\left[\Delta \mathrm{Y}^{*} /\left(\mathrm{Y}^{\mathrm{T}}+\mathrm{Y}^{*}\right)\right] \div[\Delta \mathrm{t} /(1-\mathrm{t})]
$$

$B$ and $C$ in equations (43) and (44) become $B=Y^{*} \Delta t$ and $C=t \Delta Y^{*}$. Representing $Y^{*} /\left(\mathrm{Y}^{\mathrm{T}}+\mathrm{Y}^{*}\right)$ by " a " and setting $\mathrm{B}=\mathrm{C}$ yields the revenue-maximizing tax rate

$$
\mathrm{t}^{*}=1 /(1+\mathrm{a} \varepsilon)
$$

which is the Diamond-Saez formula.
${ }^{11}$ Scraps of evidence suggest that the supply curve of labour is backward-bending. The shape of the supply curve of labour should be reflected in the historical statistics of hours of work and the real wage. Between 1909 and 1999, real wage per hour (expressed in 1999 dollars) in the United States rose from $\$ 3.80$ in to $\$ 13.90$, but work per week fell from 53 hours to 42

## The Disintegration of the Laffer Curve

The story so far can be looked upon as a search for the Laffer curve and for the revenuemaximizing tax rate at its peak on the working assumption that there is a well-specified Laffer curve out there to be found. That may not be so, for the shape of the Laffer curve may depend upon public decisions about how tax revenue is spent and how tax revenue is collected. When the Laffer curve is a reflection of the response to taxation of the labour-leisure choice, the shape of the Laffer curve depends upon the composition of public expenditure. When the Laffer curve is a reflection of tax evasion, the shape of the Laffer curve depends upon laws and regulations about tax collection These mechanisms will be discussed separately in turn, even though both are likely to be operative at once.

In the special case described in figure 7 with no substitutability in use between goods and leisure, the shape of the Laffer curve depends on whether public revenue is used for public goods, such as foreign aid, with no impact on the labour-leisure choice or is redistributed in such a way that goods lost when the tax rate is increased are, as it were, found again when public revenue is redistributed. The net impact of taxation on labour supply and the shape of the Laffer curve is illustrated in figure 12 with goods shown on the vertical axis, leisure shown on the horizontal axis, one unit of available time to be allocated between labour and leisure, and a subsidy of S goods financed by tax at a rate $t$.

Figure 12: Redistribution and Labour Supply


With subsidy as well as tax, a person's chosen leisure time is $L(t, S)$ where $t$ is the tax rate and S is the subsidy denominated in goods. With neither tax nor subsidy, consumption of goods and leisure, $\mathrm{L}(0,0)$, is as indicated by the point a where utility is maximized (putting the person on the highest attainable indifference curve) subject to the person's budget constraint beginning at the point 1 on the horizontal axis and rising with slope equal to the wage rate, w . The supply of labour is 1 $\mathrm{L}(0,0)$. When income is taxed but no subsidy is provided, the budget constraint swings counterclockwise and utility is maximized at the point $b$ where an indifference curve is tangent to the new
lower budget constraint with slope $\mathrm{w}(1-\mathrm{t})$. Leisure is now $\mathrm{L}(\mathrm{t}, 0)$ which, as the figure is drawn is less than $L(0,0)$, but could be greater or less than $L(0,0)$ depending on the elasticity of substitution between goods and leisure. Provision of a subsidy, S, raises the starting point of the budget constraint from 0 to $S$ as indicated by the point $d$, shifting the entire budget constraint upward by the amount of the subsidy and giving rise to a new combination of goods and leisure consumed at the point c . Leisure becomes $\mathrm{L}(\mathrm{t}, \mathrm{S})$ which is greater than $\mathrm{L}(\mathrm{t}, 0)$ as long as an increase in income leads one to consume more of both goods and leisure.

Note that, as the figure is drawn, the tax revenue is more than sufficient to cover the subsidy. The tax revenue is the distance from e to $f$. The subsidy is the distance from e to c which is somewhat less. This must so on average, but subsidy may exceed tax paid for some people.

The story in the figure is that, as the subsidy is denominated in goods and is, in effect, a gratuitous increase in the quantity of goods over and above what one acquires by labour, the taxpayer is provided with an incentive to reset the balance between goods and leisure, reducing the supply of labour at any given tax rate, diminishing the tax base beyond whatever the effect of taxation alone might be and altering the shape of the Laffer curve.

In preceding sections, it was assumed that revenue is separable in use from goods and leisure so that a person's welfare, W, can be described as

$$
\begin{equation*}
\mathrm{W}=\mathrm{W}(\mathrm{u}(\mathrm{G}, \mathrm{~L}), \mathrm{X}) \tag{50}
\end{equation*}
$$

where $u$ is utility as described in equation (33) above and $X$ is financed by tax revenue. The effect of an increase in X is to augment the impact of any and every combination of G and L upon the citizen's well-being, but without changing the impact of the tax rate upon labour supply.

Redistribution of income conforms badly to this pattern because X and G are essentially the same stuff. When an amount $S$ is redistributed, a person's value of $G$ becomes

$$
\begin{equation*}
\mathrm{G}=[1-\mathrm{L}] \mathrm{w}(1-\mathrm{t})+\mathrm{S} \tag{51}
\end{equation*}
$$

so that utility of a person with pre-tax wage w becomes

$$
\begin{equation*}
\mathrm{u}=\left\{\mathrm{a}[(1-\mathrm{L}) \mathrm{w}(1-\mathrm{t})+\mathrm{S}]^{\rho}+\mathrm{bL}^{\rho}\right\}^{1 / \rho} \tag{52}
\end{equation*}
$$

With t and S looked upon as invariant, the person chooses L to maximize u , setting $\delta \mathrm{u} / \delta \mathrm{L}=0$ so that,

$$
\begin{equation*}
\left\{(1 / \rho) u^{-1}\right\}\left\{\rho a[(1-L) w(1-t)+S]^{\rho-1}(-w(1-t))+\rho b L^{\rho-1}\right\}=0 \tag{53}
\end{equation*}
$$

implying that ${ }^{12}$

[^5]$$
\rho a[(1-L) w(1-t)+S]^{\rho-1}(w)(1-t)=\rho b L^{\rho-1}
$$
\[

$$
\begin{equation*}
\mathrm{dL} / \mathrm{dS}=1 /\left\{\mathrm{w}(1-\mathrm{t})+\left[(\mathrm{b} / \mathrm{w}(1-\mathrm{t}) \mathrm{a})^{1 /(\rho-1)}\right]\right\}>0 \tag{54}
\end{equation*}
$$

\]

At any given tax rate, an increase in the transfer, $S$, causes the taxpayer to consume more leisure and to decrease hours of work accordingly. Similarly, a tax increase devoted to redistribution reduces the supply of labour from what it would be if the tax increase had instead been devoted to the acquisition of ordinary public goods.

There is a parallel effect upon the elasticity of taxable income when aditional tax revenue is, at least in part, redistributed. Suppose the supply of labour, H , depends not just on the tax rate, t , as has been assumed so far, but on the subsidy, S , so that $\mathrm{H}=\mathrm{H}(\mathrm{t}, \mathrm{S})$. A distinction can now be drawn between total and partial elasticities of taxable income. The total elasticity, $\varepsilon$, is the effect on the supply of labour of a fall in the tax rate inclusive of the indirect effect of the reduction in the subsidy made necessary by the fall in tax revenue. The partial elasticity, $\varepsilon_{\mathrm{H},(1-t)}$, is the effect on the supply of labour of a fall in the tax rate as it would be if the subsidy had not changed accordingly. Total and partial elasticities are connected in equation (55) in which d refers to the total derivative and $\delta$ refers to the partial derivative

$$
\begin{align*}
\varepsilon & =[(1-\mathrm{t}) / \mathrm{H}][\mathrm{dH} / \mathrm{d}(1-\mathrm{t})]=[(1-\mathrm{t}) / \mathrm{H}]\{\delta \mathrm{H} / \delta(1-\mathrm{t})+(\delta \mathrm{H} / \delta \mathrm{S})(\delta \mathrm{S} / \delta \mathrm{R})(\delta \mathrm{R} / \delta(1-\mathrm{t}))\} \\
& =\varepsilon_{\mathrm{H},(1-\mathrm{t})}+[(1-\mathrm{t}) / \mathrm{H}]\{(\delta \mathrm{H} / \delta \mathrm{S})(\delta \mathrm{S} / \delta \mathrm{R})(\delta \mathrm{R} / \delta(1-\mathrm{t}))\} \tag{55}
\end{align*}
$$

where the expression in curly brackets is positive as long as an increase in the subsidy reduces the supply of labour, extra revenue is in part devoted to increasing the subsidy, and a reduction in the tax rate reduces the revenue acquired, that is, as long as $(\delta \mathrm{H} / \delta \mathrm{S})<0,(\delta \mathrm{~S} / \delta \mathrm{R})>0$ and $(\delta \mathrm{R} / \delta(1-\mathrm{t}))<0$. If so, it must be the case that

$$
\begin{equation*}
\varepsilon>\varepsilon_{\mathrm{H},(1-\mathrm{t})} \tag{56}
\end{equation*}
$$

meaning that, by diminishing the net wage (1-t), a tax increase used, all or in part, to increase the per capita subsidy reduces the supply of labour by more than if tax revenue were confined to some other purpose unrelated to the labour-leisure choice.

This inserts a huge ambiguity into both the elasticity of taxable income and the Laffer curve, for the revenue attainable at any given tax rate depends on how that revenue is spent. If extra revenue is to be spent supplying ordinary goods or close substitute for goods, then the elasticity of taxable income should be relatively high and the revenue-maximizing tax rate low. If extra revenue is spent on something with no impact on the rate of trade-off in use between goods and leisure, then the elasticity of taxable income should be quite low and the revenue-maximizing tax rate relatively high. Similarly, the observed elasticity of taxable income may depend on how extra tax revenue is to be spent or on which public services are to be cut back in the event of a tax cut. A tax increase to finance a war may have less impact on the tax base than a tax increase to finance a demogrant, to
or $(24-L) w(1-t)+S=\left[(b / w(1-t) a)^{1 /(\rho-1)}\right] L$
so that $\quad-w(1-t) d L+d S=\left[(b / w(1-t) a)^{1 /(\rho-1)}\right] d L$
from which equation (54) follows immediately.
raise unemployment insurance or to increase the old age pension.
As illustrated in figure 13, there may be many Laffer curves depending on how public revenue is spent. It is at least possible that, with low elasticity of substitution between goods and leisure and with non-redistributive public expenditure, an increase in the tax rate increases tax revenue all the way up to $100 \%$, while, with a higher elasticity and with substantially redistributive public expenditure, a tax rate of $100 \%$ yields no revenue at all.

Figure 13: Laffer Curves Dependent on the Content of Public Expenditure


One should not make too much of this point. That $\varepsilon>\varepsilon_{\mathrm{H},(1-\mathrm{t})}$ as indicated in equation (56) suggests that the Laffer curve is more concave than would otherwise be the case but does not rule out the possibility of a revenue-maximizing tax rate of as high as $100 \%$. A markedly backward bending supply curve of labour might be sufficient to outweigh the effect of redistributive expenditure. Also, the negative income tax may be seen as representative of all redistributive tax and public expenditure - progressivity, the old age pension, unemployment insurance, welfare, socialized medicine and so on - rather than as a tax system one would actually want to impose. Socialized medicine, for example, is certainly redistributive in its impact, but it is more likely to increase rather than decrease the labour supply by keeping people healthy.

The shape of the Laffer curve can be influenced by public policy in another way as well. When the Laffer curve is a reflection of tax evasion, the concavity of the Laffer curve can be reduced and the revenue maximizing tax rate increased by a stricter enforcement of the tax code. Enforcement of the tax code was ignored in the exposition of the tax evasion story on the assumption that tax evasion is costly but undetectable once the required cost of tax evasion has been borne. Suppose instead that there a chance that tax evasion will be detected and that the tax evader will be punished accordingly.

To keep matters simple, suppose once again that evasion will surely be detected if the appropriate cost is not borne, but there remains a small chance, say $1 \%$, that tax evasion is detected no matter how much or how little of one's income is concealed. Suppose that, if evasion is detected, a fine is imposed equal to a multiple $\lambda$ of the cost of concealment where the government is free to make the cost as small or as large as it pleases. To the risk averse taxpayer, the situation is as though the efficiency of tax collection had increased from $\beta$ to $\beta^{*}$ where

$$
\begin{equation*}
\beta^{*}=\beta(1+(.01) \lambda) \tag{57}
\end{equation*}
$$

It is difficult to say what the appropriate penalty for tax evasion should be. One might invoke the well-known Becker paradox that we should hang people for parking violations, for with such severe a punishment there would be no parking violations and no hangings! The analogy is to set $\lambda$ at infinity, turning the Laffer curve into an upward-sloping straight line as illustrated in figure 5. Taken at face value, the paradox is nonsense, but it is helpful nonsense because articulation of reasons why it is nonsense is a first step in the process of deciding what the severity of punishment ought to be. ${ }^{13}$ The essential point here is that the government chooses the Laffer curve through its choice of a fine for tax evasion.

Recognition that the government must choose the severity of punishment for tax evasion and that, one way or another, severity is probably costly, suggests that a distinction might be drawn between gross and net Laffer curves, the former connecting tax revenue to the tax rate as discussed above, the latter replacing "tax revenue" with the portion of tax revenue left over once the cost of administering the tax system is borne. The location of the gross Laffer curve depends on the government's behaviour. Identification of the net Laffer requires estimates of the full cost to the government and the citizen of measures to confine tax evasion.

## Starvation, Exhaustion, Migration and Progressivity

It has been argued so far that the revenue-maximizing tax rate may well rise to $100 \%$, due in the tax evasion story to the high efficiency of tax collection or in the labour-leisure story to the backward-bending supply curve of labour. One may well protest that this cannot be so. At a tax rate of $100 \%$, all of one's income is appropriated by the tax collector with nothing left over for the taxpayer at all. There would be nothing to eat. There would be no point in working. The Laffer curve must surely be humped after all.

There are several possible responses to this objection. One response is to reinterpret the $100 \%$ as a limit that the tax rate can approach without causing revenue to diminish. Thus, with perfect complementarity between goods and leisure and as long as the rate never quite gets to $100 \%$, a rise in the tax induces the taxpayer to consume less of both goods and leisure, increasing labour and increasing revenue accordingly. Another response is that, when the Laffer curve is a reflection of tax evasion, the difference between to amount of income hidden and the cost of hiding income from the tax collector maybe sufficient to save the taxpayer from starvation, so that a progressive tax with a top rate of $100 \%$ may leave nobody in abject poverty. Alternatively, a tax increase may provoke taxpayers to switch not from labour to leisure, but from paid work to do-ityourself activities. There may be a steadily rising Laffer curve if each additional hour devoted to do-it-yourself activities yields progressively less benefit, the additional benefit eventually falls to zero but there is always some return to paid work. People may or may not be able to subsist on do-it-yourself activities. Hiding income from the tax collector and diverting time from work for pay to

[^6]do-it-yourself activities are alike in that a residue of consumption is preserved no matter how high the tax rate happens to be.

High taxation may deter enterprise and innovation. Better take a safe and steady job if a disproportionate share of the benefit of successful innovation will be taxed away. This is a questionable argument because it takes no account of how tax revenue is spent. Inherently risky innovation may be more encouraged by the infrastructure and social safety net that taxation supplies than discouraged by the taxation required to finance these programs. The would-be innovator who might earn $\$ 50$ million from his innovation but who might equally-well go bankrupt may not be deterred on balance by a tax bill of $\$ 30$ million in the event that the innovation is successful when the chance of success is increased by tax-financed roads, bridges and scientific research or when protected from utter destitution in the event of failure by a taxfinanced old age pension and socialized medicine. The innovator may work harder to procure a target income when part of gross income is taxed away. Taxation may deter entrepreneurship, but it may equally-well have the opposite effect. ${ }^{14}$

There may be migrational externalities. High and progressive taxation in one jurisdiction may drive wealthy people to other jurisdictions where taxes are lower and less progressive. A distinction may be drawn in this context between the Laffer curve as it would be in the absence of migration and as it would be for the remaining residents of a jurisdiction if high taxation drives out many of the principal contributors to a system of redistribution. High taxation alone need have no such effect if combined with desirable and expensive public services, but rich people may be driven away when taxation is to finance a transfer from rich to poor. That is the standard argument for assigning redistributive powers to high levels of government, to provincial governments rather than to municipalities, to the federal government rather than to the provinces.

High top marginal tax rates may be counter-productive because high income earners may avoid tax in ways that are not available to the rest of society, by complex financial manoeuvres that are excessively expensive unless one has a great deal of income to hide. More-taxed ordinary income may be converted to less-taxed capital gains. Money may be shielded from the tax collector in trust funds or in off-shore accounts. The relevant $\varepsilon$ in equation (1) may be higher for the rich than for the poor. On the other hand, to oppose high tax rates on the grounds that wealthy people are especially adept at tax evasion is like the patricide who appeals for mercy on the grounds that he is an orphan. The argument is not altogether wrong but it adds weight to the case for closing loopholes as well as to the case for moderate top tax rates.

More generally, arguments about the shape of the Laffer curve have been based on models with given parameters; gross income and the efficiency of tax collection in the model of tax evasion, and the wage rate in the model of labour-leisure choice. That parameters like Y or w are invariant may be a reasonable assumption as long incomes are well above some subsistence level, but not otherwise. There may be a minimal post-tax, post transfer income below which the postulated pre-tax income, Y , in the tax evasion story, or the pretax wage, w , in the labour-leisure story, cannot be sustained. Such modifications to the assumptions would be more than sufficient to

[^7]keep the revenue-maximizing tax, $t^{*}$, below $100 \%$, but they are radically different from the assumptions on which $t^{*}$ was estimated.

Instead of postulating a fixed wage w , it might be supposed that one's capacity to work is affected by one's standard of living, that $w$ is invariant as long as $G>G^{\min }$ and $L>L^{\text {min }}$ where $G^{\text {min }}$ and $\mathrm{L}^{\mathrm{min}}$ are minimal requirements for a person to work well, but that, otherwise, the wage rate is a diminishing function, $\mathrm{w}(\mathrm{G}, \mathrm{L})$, of G and L , where $\mathrm{w}(\mathrm{G}, \mathrm{L})<\mathrm{w}, \delta \mathrm{w} / \delta \mathrm{G}>0$ and $\delta \mathrm{w} / \delta \mathrm{L}>0$ unless $\mathrm{G}>\mathrm{G}^{\text {min }}$ and $\mathrm{L}>\mathrm{L}^{\text {min }}$.

The argument that a top rate of $100 \%$ is impossible is somewhat different for the rich than for the poor. For the poor, it is that the tax base would crumble because people cannot work if they do not eat. For the rich, it is that a top rate of $100 \%$ in a progressive income tax would eliminate all incentive to earn income above the base of the top rate. That is possible but questionable. What is really meant by a top rate of $100 \%$ is a rate approaching that limit, say $95 \%$. As discussed above, an increase in the tax rate may induce the taxpayer to work less because goods have become expensive relative to leisure, or to work more because goods have become scarce, the balance depending on the substitutability in use between goods and leisure. Consider a person making $\$ 300,000$ per year. In the absence of taxation, the person may have little incentive to earn, say, an extra $\$ 50,000$ because his time is quite valuable relative to goods. But if the tax system has reduced his net income to, say, $\$ 100,000$ and if his marginal tax rate is $95 \%$, the extra $\$ 2,500$ of net income may be worth the loss of leisure required to earn the additional $\$ 50,000$.

## A Final Word

The new tax responsiveness literature has the virtue of identifying the effect of taxation upon the tax base regardless of why that effect takes place. The corresponding vices are that the elasticity of taxable income is difficult to measure accurately and that an observed elasticity of taxable income may be consistent with a wide range of revenue-maximizing tax rates depending on how exactly tax rate and tax base are connected. Prediction of the revenue-maximizing tax rate requires the elasticity of taxable income to remain unchanged along the entire Laffer curve, but the elasticity does change substantially in the tax evasion story and the labour-leisure story, with predictions of the revenue-maximizing tax rate varying accordingly. In particular, constancy of the elasticity of taxable income generates the prediction that the Laffer curve is humped with tax revenue falling to 0 when the tax rate rises to $100 \%$, but both the tax evasion story and the labourleisure story contain the possibility that tax revenue does not fall at all. Constantly upward-sloping curves as in figure 1 a and 1 b are not ruled out. The labour-leisure story allows for the possibility that the shape of the Laffer curve depends on how tax revenue is spent, that the Laffer curve is more likely to be humped when tax revenue is redistributed than when tax revenue is spent on public goods with no effects on the labour-leisure choice. There may be no unique Laffer curve at all.

The moral of the story is that the revenue-maximizing tax rate ought not to be taken too seriously. There may be such a rate out there, but it is likely to be far higher than any majority of voters is likely to favour. To say that the revenue-maximizing rate may reach $100 \%$ is really to say that it is likely to be well above the electoral equilibrium rate and to be no impediment to redistribution. Redistribution is constrained because it is expensive rather than impossible.

## References

Allen, R.D.G., Macro-Economic Theory, Macmillan, 1967
Buchanan, J. M. and Lee, D.R., "Politics. Time and the Laffer Curve", Journal of Political Economy, vol 90 (4), 1982, 816-816.

Clark, Colin, "Public Finance and Changes in the Value of Money", The Economic Journal, vol 55, 1945, 371-389

Diamond, P. And Saez, E., "The Case for Progressive Tax: From Basic Research to Policy Recommendations", The Journal of Economic Perspectives, vol 25, 2011, 165-190.

Feige, E. L. And McGee, R.T., "Sweden's Laffer Curve: Taxation and the Unobserved Economy" Scandinavian Journal of Economics, vol. 85 (4), 1983.

Feldstein, M, "The Effects of Marginal Tax Rates on Marginal Income: a Panel Study of the 1986 Tax Reform Act", Journal of Political Economy, vol 103, 1995, 351-72.

Fisk, Donald, M., "American Labor in the Twentieth Century", Compensation and Working Conditions, US Bureau of Labour Statistics, Fall 2001.

Goolsbee, A.,"What Happens When You Tax the Rich? Evidence from Executive Compensation", Journal of Political Economy, vol 108, 352-378.

Knight, F. H., Risk, Uncertainty and Profit, 1921.
Lin, C., 2003, A Backward-bending Labour Supply Curve without an Income Effect", Oxford Economic Papers, vol 55, 336-343.

Lindsey, L., 1987, "Individual Taxpayer Response to Tax Cuts, 1982-1984, with Implications for the Revenue-maximizing Tax Rate", Journal of Political Economy, vol 33, 173-206.

Mankiw, Gregory N., "Optimal Taxation in Theory and Practice", Journal of Economic Perspectives, vol 23(4), 147-174.

Robbins, L., "On the Elasticity of Demand for Income in Terms of Effort", Economica, 1930, 123-129.
Saez, E., Slemrod, J. and Giertz, S., "The Elasticity of Taxable Income with Respect to Marginal Tax Rates", Journal of Economic Literature, 2012, vol L, 3-50.

Stuart, C.E., "Swedish Tax Rates, Labour Supply and Tax Revenue" Journal of Political Economy, vol 89, 1981, 1020-38.

Usher, Dan, "The Marginal Cost of Public Funds is the Ratio of Mean to Median Income", Public Finance Review, vol. 34, 2006, 687-711.

Yitzaki, S., "On the Excess Burden of Tax Evasion", Public Finance Review, Vol 15(2), 1987.


[^0]:    ${ }^{1}$ For a proper account of the new tax responsiveness literature, see Feldstein (1995) and Diamond and Saez (2011)

[^1]:    ${ }^{2}$ When $\mathrm{y}=\mathrm{Y}(1-\mathrm{t})^{\varepsilon}$ as in equation (5), it follows that the elasticity of B with respect to $(1-\mathrm{t})$ becomes $[(1-\mathrm{t}) / \mathrm{y}] /[\delta \mathrm{y} / \delta(1-\mathrm{t})]=\left[(1-\mathrm{t}) / \mathrm{Y}(1-\mathrm{t})^{\varepsilon}\right]\left[\varepsilon \mathrm{Y}(1-\mathrm{t})^{\varepsilon-1}\right]=\varepsilon$

[^2]:    ${ }^{6}$ A top marginal tax rate of $100 \%$ may not be quite as silly as it may at first appear. First, reference to a top marginal rate of $100 \%$ may be no more than code for a very high top marginal tax rate of, say, $90 \%$ or $95 \%$. Retention of as little as five percent of earned income in excess of a million dollars may be incentive enough to keep potentially high incomes from, in effect, being thrown away. Second, incentives may be preserved by uncertainty. An entrepreneur may be confronted with a risky prospect: a chance of making a great fortune, together with a chance, to be dramatic about it, of bankruptcy and destitution, where the probabilities depend on the entrepreneur's diligence and inventiveness. He works hard to avoid bankruptcy tomorrow. It may be too late to slack off by the time he knows whether he is earning enough to the subject to the top marginal tax rate. Third, the principal incentive of a highly-paid executive may be the risk of losing his job. With a top marginal tax rate of $100 \%$ on income above, say, a million dollars, the market might fix top incomes at that amount, leaving more money left over for shareholders. The executive may even be induced to work harder than when his income in any year is enough to support him for the rest of his life.

[^3]:    ${ }^{9}$ See R.G.D. Allen, Macro-Economic Theory, Macmillan, 1967, page 51.

[^4]:    ${ }^{10}$ With appropriate changes in the assumptions, equation (45) is easily converted into the Diamond-Saez formula for the revenue maximizing tax rate in the top bracket of a progressive

[^5]:    ${ }^{12}$ From equation (53), it follows that

[^6]:    ${ }^{13}$ Among the reasons, discussed in detail in textbooks of "law and economics", are to avoid massive injustice in the event of mistaken conviction, that juries would not convict, and to preserve the honesty of the police force by limiting the size of bribes that criminals would be willing to offer,

[^7]:    ${ }^{14}$ Back in 1975, a would-be entrepreneur was considering whether to start a firm to be called Microsoft. Deterred by the marginal tax rate of $75 \%$ at that time, he decided against it and took an ordinary job instead.

