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# Daily Share Price-changes for Eleven D-J Companies over Five Three-month Periods and Short-term Profit-seeking 

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# Daily Share Price-changes for Eleven D-J Companies over Five Three-month Periods and Short-term Profit-seeking 

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#### Abstract

We link daily price-changes for 11 Dow-Jones companies for 5 three month periods to the prevailing interest rate. In "normal times", the interest carrying cost of owning a share exceeds average price change. In periods of high volatility (eg. August to November, 2008), average price change exceeds the interest carrying cost. We also observe considerable regularity across companies of volumes of shares traded per day and percentage price changes. The intercept differs across time periods (1995, 2006, 2007, 2008, 2010). Larger volumes yield larger price changes, on average. - JEL classification: G100; G120 - key words: interest carrying-cost; average share price changes


## 1 Introduction

It only makes sense to buy a share today at price $p_{t}$ if gains $p_{t+z}-p_{t}$ plus dividend income covers the interest cost of holding the share over the period $z$, plus any tax costs, and "brokerage costs" of purchase and sale. In the short run, dividend income will probably be insignificant. Costs are known pretty well but anticipated revenue, $p_{t+z}-p_{t}$ is uncertain at time of purchase in both size and "dating". Future price $p_{t+z}$ is uncertain. What we observe in the data below is that average price increases over a day (or separately, average price decreases over a day for those shorting the stock) do not cover the interest cost of holding

[^0]a share for a day during normal trading periods, where normal refers to periods not impacted by the financial crisis of 2007 and 2008 in the United States. We consider averages calculated over three month periods, starting on August 20, for $1995,2006,2007,2008$ and 2010, and over 11 Dow-Jones companies selected somewhat randomly, and observe that for 1995 and 2006, daily interest carrying charges for owning a share exceed on average daily average price increases per share (or separately daily average price declines). For 2008, the opposite is true: average price increases (or separately, decreases) exceed daily interest carrying costs. For 2007 and 2010, we get results similar to those for 2008 but of a smaller magnitude. We infer that trading in shares for the short run, intervals in which dividends are not considered in income, in times of "normal" stock market volatilty is not on average profitable. ${ }^{1}$ In other words, only in times of unusually high volatility does it pay to be a day-trader, large or small. Thus rational short-term traders, those who make profits on average, must be more active in times of unusually high volatility.

One is always interested in relative sizes of volumes of trade for those with short "planning horizons" (say day-traders) and those trading or purchasing for longer terms, terms in which the dividend payouts become significant in a plan. Below we turn to daily trading volumes per se for our eleven companies and inquire about how closely daily volume correlates with the size of daily price changes for our eleven companies and 5 three month periods. Our regression results support the view that there is considerable stability in the link between daily volume in a company's shares traded and in the size of the corresponding day's price change: more trading correlates positively with larger price changes. We do not unfortunately have a division of daily volume into trades for the short term and trades for the long term. ${ }^{2}$

[^1]
## 2 Some Detail

The buy-the-share-today inequality is then

$$
p_{t+z}-p_{t}+d>c_{p t}+c_{s, t+z}+\tau+i * p_{t} * z
$$

for $p_{t+z}-p_{t}$ the selling price at $t+z$ minus the purchase price at $t, d$ dividend income, $c_{p t}$ the brokerage cost of purchase at $t, c_{s, t+z}$ the brokerage cost of selling at $t+z, \tau$ the tax cost of owning and selling the share, and $i * p_{t} * z$ the interest cost of holding the share over the interval of length $z$. The tricky part of deciding is of course whether a high enough selling price will be realized and how long one must wait for a "good price" to be realized. The interest cost of waiting increases with the waiting interval, $z$.

If one were somehow locked into trading between consecutive closings (over a day), our inequality can be reduced to

$$
\frac{p_{t+1}-p_{t}}{p_{t}}>\frac{c_{p t}+c_{s, t+1}+\tau}{p_{t}}+i
$$

for $i$ the interest rate per day. ${ }^{3}$ At purchase time, the purchaser must expect a sufficiently high price $p_{t+1}$ to get the required inequality. Otherwise it not profitable to buy for this day. (An agent can be in for expected price declines over the day and purchase a shorting contract, and anticipate a sufficiently large price decline so that she ends up in the black, over the day.) The costs are known to the purchaser but the left side is in fact a random variable here. At the time of purchase, the future price $p_{t+1}$ is anticipated.

We turn to our empirical work. We separate values of $\frac{p_{t+1}-p_{t}}{p_{t}}$ into groups of positive and negative values for a particular three month period and averaged over our 11 Dow-Jones companies, for each of 5 different years. Our calculations indicate that average values of daily $\frac{p_{t+1}-p_{t}}{p_{t}}$ over about 65 consecutive trading days (a three month period starting August 20) and over 11 Dow-Jones companies fall short of $i$, for 1995 and 2006; and exceed $i$ or 2007, 2008 and 2010. We ignore the transactions and tax cost term $\frac{c_{p t}+c_{s, t+1}+\tau}{p_{t}}$ in our calculations. Since Bear-Sterns approached bankruptcy in 2007 and Lehman Brothers entered

[^2]bankruptcy in 2008, we consider our three month periods for these years to be ones of "high price volatilitiy". ${ }^{4}$ Our three month periods for 2007 and 2008 exhibited large average price changes per day relative to interest costs. Trading was potentially profitable, on average. The year 2010 exhibited relatively large price changes relative to the interest rate. We should probably infer that average percentage price changes were "large" because interest rates had be driven down by the Federal Reserve in the United States. In this interpretation, our three month period for 2010 was not one of "high price volatility".

We turn to our calculations.

## 3 Results

A three month period of 2008 was our starting interval or reference period, since this marked the arrival of the so-called "financial crisis". Our typical interval was August 20, to November 20. We took up the same three month periods for four other years for comparison (1995, 2006, 2007, and 2010). Our eleven companies were Bank of America (BAC), General Electric (GE), Companhia Vale (VALE), Pfizer (PFE), Ford Motor (F), Att (T), Cemex (CX), Citigroup (C), Merck (MRK), Coca Cola (KO), and Potash Saskatchewan (POT).

The results in Table 1 are a summary of some results from Table 2 below. Consider the first entry (cell BAC and 95d) in Table 1. The plus sign indicates that when we ADDED the negative percentage price change for (BAC, 1995) from the bond rate for the three month period from 1995, we got a positive result, meaning that the negative percentage price change was LESS than the value of the daily bond rate. ${ }^{5}$ The corresponding average positive price change for the same period, when the bond rate was SUBTRACTED from the price

[^3]change value, we obtained a negative results, indicating that the percentage price change was SMALLER THAN the corresponding bond rate. Hence for 1995 and all companies, the percentage price change on average, up or down, tended to be smaller in absolute value than was the value of the bond rate for that same period. We refer then to our three month period for 1995 as one of low volatility, one with the value of the bond rate tending to be higher on average than either the corresponding average up percentage price change or down percentage price change. Results are somewhat mixed for 2006, with a few large average percentage change down entries (for Vale, Ford, At\&t) and a few large percentage change up entries (Vale, Ford, Cemex). We still label the three month period for 2006 as one of not-high volatility (more on this below in Table 2). In contrast, for the three month period for 2008, we observe consistent large average percentage down movements and consistent large average percentage up movement in share prices for our 11 companies. These price movements have reference point the value of the 10 year bond rate for the corresponding period.

Table 1 (deviations of $\%$ price-chanege from bond rate)

|  | 95 d | 1995 u | 06 d | 2006 u | 07 d | 2007 u | 08 d | 2008 u | 10 d | 2010 u |
| :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- |
| BAC | + | - | + | - | + | - | - | + | - | + |
| GE | + | - | + | - | + | - | - | + | - | + |
| Vale | + | - | - | + | + | + | - | + | - | + |
| PFE | + | + | + | - | + | - | - | + | - | + |
| F | + | - | - | + | - | + | - | + | - | + |
| T | + | - | - | - | + | - | - | + | + | - |
| Cemex | x | x | + | + | + | + | - | + | - | + |
| Citi | + | - | + | - | - | + | - | + | - | + |
| MRK | + | - | + | - | + | - | - | + | - | + |
| KO | + | - | + | - | + | - | - | + | + | - |
| POT | - | + | + | + | - | + | - | + | - | + |

* not present in the Dow Jones set of companies for 1995

For the year 2010, the results in Table 2 indicate considerably volatility but not as much as for the three month period of 2008 for our set of companies. The sum of the values for the first column for 2010 is negative and for the second column is positive, an indicator of large price changes relative to the 10 year bond rate. This metric is giving the result of relatively high volatility in share price changes for the three month period of 2010 . Overall then, the
results reported in Table 2 are indicating relatively low volatility for our three month period for 1995 and 2006, and rising volatility in 2007, peaking in 2008. Volatility has declined somewhat for the three month period of 2010.

With regard to companies over the period 1995 to 2010 (in our chunks of 3 month periods), Ford and Vale can be labelled as volatile. Bank of America, GE, Pfizer, Cemex and Citigroup become more volatile over our period 1995 to 2010. Merck and Coca-Cola might be labelled as not volatile. Table 2 contains numerical values for the average percentage price changes for the relevant three month periods by company, AVERAGED over the trading days. We separate "up" or positive price-change days from "down" or negative price-change days. We then record the values of the deviations from the corresponding 10 year bond rate, taken in its daily form. For example for BAC for our three month period in 1995, the average over trading days for negative price changes, when ADDED to the bond rate, yielded 0.006 ; for the average of positive price change days, when the bond rate was subtracted from the average percentage value, one obtains -0.0025 . Hence we infer that the average percentage price-change was LOW (was near the value of the 10 year bond rate). Hence we infer that for our three month period of 1995, BAC price-changes were "not volatile", on average. In contrast, we observe for each company in 2008, over our three month period, percentage price changes were large, relative to the corresponding bond rate, and we infer that share prices were volatile over this period.

Table 2: Detailed Results by Company and Specific Three-month Interval

| 1995 | neg \% ch. | pos \% ch. | dev-neg | dev-pos |
| :--- | :--- | :--- | :--- | :--- |
| BAC | -.0109 | .0145 | .006 | -.0025 |
| GE | -.0072 | .01197 | .0098 | -.0050 |
| Vale $^{*}$ | x | x | x | x |
| PFE | -.01442 | .01441 | .0026 | -.0026 |
| F | -.01342 | .01724 | .0036 | .00024 |
| T | -.00865 | .0109 | .0084 | -.0061 |
| Cemex | x | x | x | x |
| Citi | -.01522 | .01536 | .0018 | -.0016 |
| MRK | -.00722 | .00299 | .0098 | -.014 |
| KO | -.00729 | .00366 | .0097 | -.0133 |
| POT | -.01762 | .02265 | -.0006 | .0057 |
| .. sum | .. | $\ldots$ | .0511 | -.0392 |

[^4]| 2006 | neg \% ch. | pos \% ch. | dev-neg | dev-pos |
| :--- | :--- | :--- | :--- | :--- |
| BAC | -.00527 | .00485 | .0076 | -.0081 |
| GE | -.00423 | .00550 | .0087 | -.0074 |
| Vale $^{*}$ | -.01783 | .01718 | -.0049 | .0043 |
| PFE $^{2}$ | -.00739 | .00639 | .0055 | -.0055 |
| F | -.01645 | .02026 | -.0036 | .00736 |
| T | -.00907 | .009195 | -.0038 | -.0037 |
| Cemex* | -.01226 | .013246 | .0006 | .00035 |
| Citi | -.00388 | .00476 | .0090 | -.0081 |
| MRK | -.00909 | .00896 | .0038 | -.0039 |
| KO | -.00334 | .00591 | .0096 | -.007 |
| POT | -.01122 | .01666 | .0017 | .0038 |
| $\ldots$ sum | $\ldots$ | $\ldots$ | .0342 | -.028 |
| 2007 | neg \% ch. | pos \% ch. | dev-neg | dev-pos |
| BAC | -.0021 | .01047 | .0095 | -.0012 |
| GE | -.00895 | .00931 | .0027 | -.0023 |
| Vale* | -.04603 | .03483 | .0034 | .023 |
| PFE | -.00967 | .00836 | .0020 | -.008 |
| F | -.0198 | .02346 | -.0082 | .0182 |
| T | -.01149 | .01121 | .0002 | -.0004 |
| Cemex | -.01104 | .02145 | .0006 | .0098 |
| Citi | -.01878 | .01547 | -.0071 | .0038 |
| MRK | -.00849 | .01106 | .0031 | -.0006 |
| KO | -.00701 | .008611 | .0046 | -.003 |
| POT | -.02939 | .024941 | -.0178 | .0133 |
| $\ldots$ sum | $\ldots$ | $\ldots$ | -.007 | .0526 |
| 2008 | neg \% ch. | pos \% ch. | dev-neg | dev-pos |
| BAC | -.07167 | .054939 | -.0617 | .045 |
| GE | -.04014 | .032338 | -.0301 | .022 |
| Vale* | -.06741 | .068285 | -.0574 | .058 |
| PFE | -.02539 | .026014 | -.0159 | .016 |
| F | -.05649 | .036793 | -.04649 | .027 |
| T | -.03058 | .036003 | -.02058 | .026 |
| Cemex | -.060139 | .05083 | -.05139 | .041 |
| Citi | -.06984 | .067422 | -.0598 | .057 |
| MRK | -.02991 | .03698 | -.0199 | .027 |
| KO | -.02273 | .027545 | -.0127 | .018 |
| POT | -.06506 | .04879 | -.0551 | .039 |
| $\ldots$ sum | $\ldots$ | $\ldots$ | -.4312 | .376 |
|  |  |  |  |  |


| 2010 | neg \% ch. | pos \% ch. | dev-neg | dev-pos |
| :--- | :--- | :--- | :--- | :--- |
| BAC | -.01696 | .018195 | -.0097 | .0109 |
| GE | -.00875 | .012374 | -.0015 | .0051 |
| Vale $^{*}$ | -.01247 | .01703 | -.0052 | .0097 |
| PFE | -.00829 | .010709 | -.001 | .0034 |
| F | -.01333 | .019549 | -.006 | .0123 |
| T | -.00686 | .003794 | .0004 | -.0035 |
| Cemex | -.01898 | .020331 | -.0117 | .013 |
| Citi | -.01357 | .018353 | -.0063 | .0111 |
| MRK | -.01783 | .009356 | -.0005 | .0021 |
| KO | -.00508 | .005554 | .0022 | -.0017 |
| POT | -.00929 | .009885 | -.002 | .0026 |
| $\ldots$ sum | $\ldots$ | .. | -.0413 | .065 |

The sums in Table 2 represent a different sort of average. Obviously when the entries in the sum have one sign the sums then come out with the sign in question as for example is the case for 2008. 1995 works like that also. However when the entries in the sum have different signs, we can appeal to the sign of the sum as indicating a preponderance of relative deviations. Thus for 1995, 2006 and 2008, the sums become short hand for communicating the values within each sum. For 2007 and 2010, the entries in the sums have different signs but from the sign of the sum we infer that each of 2007 and 2010 were years of relative volatility. The sign of the sums for 2007 and 2010 are the same as those for 2008 , and this latter year is our unambiguous case of volatile share prices.

Implicit has been our view of an agent buying at opening on one day and selling or holding at closing that day. A rigid one day "planning horizon" is not crucial to our main point about earning profit from short term trading. Any short horizon will operate in the same way. The option a buyer or holder has to select the date of transacting is an option with a positive value. Still, there is simply not enough price-change, rise or decline, on average to cover the interest cost of carrying the original cost of a share over any short horizon, during "normal times". This is ON AVERAGE. Just as there are winners for lotteries or at betting at Las Vegas, there will be isolated buyers of shares who make profit in the short run. ${ }^{6}$ Nevertheless, our point is that expected profit is negative

[^5]for a purchase and sale in the short run. However, once dividends become a significant part of returns to the purchase of a share, then presumably expected profit can be positive, but by our definition, this is a "long run" phenomenon.

Many observers believe that large well-funded traders make profit from short run trading employing computer algorithms and the techniques of HFT (high frequency trading). HFT involves very high volumes, sophisticated computer algorithms and very short trading horizons. ${ }^{7}$ There is some dimension of pricesetting power by large traders in these environments. Some observers contend that profits in HFT settings derive mostly from "front-running", essentially grabbing information about current trades and becoming the current supplier of the shares being sought. ${ }^{8}$ Spoofing is another practice associated with HFT. ${ }^{9}$

That's why we want at least a one-point average daily range in stocks we trade short-term, especially intraday. Stocks that trade less than a point provide a too-slim-to-bother-with profit potential." (p. 22).

It is taken for granted that large traders can earn profits in the short run employing computers, speed, and sophisticated algorithms. Buying or selling a stock that does not have much volume can move it up or down. Small investors have little effect but large mutual funds and hedge funds can determine the minute-to-minute pricing of stocks through supply and demand (Cramer, 2005, p. 96). Cramer, James (2009). Jim Cramer's Real Money: Sane Investing in an Insane World. New York: Simon \& Schuster.

7 "While there is no single definition of HFT, among its key attributes are highly sophisticated algorithms, specialized order types, co-location, very short-term investment horizons, and high cancellation rates of orders. HFT can be viewed as a primary form of algorithmic trading in finance. Specifically, it is the use of sophisticated technological tools and computer algorithms to rapidly trade securities. HFT uses proprietary trading strategies carried out by computers to move in and out of positions in seconds or fractions of a second... HFT firms make up the low margins with incredibly high volumes of trades, frequently numbering in the millions. It has been argued that a core incentive in much of the technological development behind high-frequency trading is essentially front running, in which the varying delays in the propagation of orders is taken advantage of by those who have earlier access to information.... Algorithmic and high-frequency traders were both found to have contributed to volatility in the Flash Crash of May 6, 2010, when high-frequency liquidity providers rapidly withdrew from the market." Wikipedia, October 4, 2015.
${ }^{8}$ An illustration of front-running: Agent A contracts to buy 100,000 shares in company i. This order is automatically divided into tranches say 10,000 shares each which get transmitted to 10 different trading companies electronically. Agent B has a computerized system that detects the large order when a 10,000 share order arrives at a company. The 9 tranches remaining for filling are moving infinitesimally slower to their 9 dealers. B's computers jump in from of these remaining 9 orders, and buy the shares desired by A, BEFORE A's orders reach the dealers. When A's one of A's orders reaches the dealer, B is there selling at a price a small amount above what A was expecting (agreeing to) to pay. The order gets filled and A has paid $B$ the higher than expected price and $B$ pockets the profit at the 9 other dealers. B has front-run A's original order. This activity requires that B have extemely fast computers and very sophisticated algorithms that allow B to get in front of A's remaining orders, once the first tranche asked for reaches its dealer.

The practice of front-running is an outgrowth of computerized trading and was a focus of the book, Flash Boys, by Michael Lewis.

9 "Spoofing" involves a trader placing a large order for stocks, bonds or futures to nudge the

Neither front-running or spoofing fit into what economists would consider standard practices of competitive trading. Both activities are being investigated by regulators and legislators. ${ }^{10}$ It is probably not an exaggeration to refer to front-running and spoofing as a form of price manipulation by insiders. ${ }^{11}$ Our results on negative expected profit in general in normal times suggest that professional traders are obliged to turn to non-orthodox ways fo making profits. Front-running and spoofing can be said to be such practices. Earning profit with short-run trading in any way is more difficult when expected or average price movements up or down are small relative to the prevailing interest rate.

Note that we have framed our analysis in terms of a buyer entering the market. The same difficulty, negative expected profit, faces current owner of shares, if they expect to be trading over short runs. This suggests that persistent holders of shares, if rational, are holding shares for the long-term, when dividend income matters. Holding a share for only the short run is generally a negative profit exercise. We turn to a brief overview of price change and related trading volume. The matter of the prevailing interest rate is left aside in this investigation.

## 4 Moving Current Share Price with a Quantity of Trading

Above, we linked the current interest rate to a current percentage price change per day. Obviously what moves price is current selling and buying of shares. We left that link open in our investigation of the link between the interest rate and
market price up or down. Just before the order is scheduled to be executed, the spoofer cancels the order and actually trades in the opposite direction, taking advantage of the temporary shift in price. Navinder Sarao has been charged by the US Justice Department with this practice. He employed a program known as "the Matrix" and had the assistance of an expert programmer in getting his spoofing going. He is alleged to have precipitated the flash crash of May 6, 2010. (Wall Street Journal, September 5-6, 2015, p B2.)
10 "The most commonly cited statistics suggest that high-frequency traders are making, at most, a few billion dollars a year in the stock markets. That take has been shrinking in recent years, according to research from Tabb Group, and is much smaller than what was captured by the old middlemen in the stock market whom the high-speed traders largely replaced: bank and brokerage employees on the floor of the New York Stock Exchange." Nathaniel Popper, "Are High-frequency Traders Really such Villains?" p. A3, The New York Times, Tuesday, October 13, 2015.
11 "Investing has always been, to at least some degree, about finding and using information that others do not have. Insider-trading cases are typically full of grey areas." The Economist, p. 74 , October $10,2015$.
the price change. Now we report on current trade volume ${ }^{12}$ and current price change. We argued above that the amount of selling and buying will in part be mediated by the value of the current interest rate. We turn to a reduced form relationship between the current quantity traded over a day and the current change in the price of the shares in company i. We are interested in how a particular volume is linked to a particular price change over one day of trading. This depends of course on the willingness to sell of current holders of equities and on the strength of demand of prospective buyers. If the current value of all equities in a company is $p_{t} * F_{t}$, we can observe current trade volume $Q_{t}$ and trade value $Q_{t} * p_{t}$, that at closing changes the value of the company by $\Delta p_{t} * F_{t}$, with $\Delta p_{t}=p_{t}-p_{t-1} . F_{t}$ stands for total equities or Float and $p_{t}$ is the current price of a share in the company under consideration.

We can think then of a reduced-form price movement coefficient

$$
\frac{\Delta p_{t} * F_{t}}{Q_{t} * p_{t}}=\frac{\Delta p_{t} / p_{t}}{Q_{t} / F_{t}}=m_{t} .
$$

In other words, fraction $Q_{t} / F_{t}$ of shares get traded and share price ends up moving by $\Delta p_{t} / p_{t} . Q_{t} / F_{t}$ measures trading volume and $\Delta p_{t} / p_{t}$ measures current equity price change. We take then $Q_{t}$ as trades per day and $p_{t}$ and $p_{t-1}$ being distinct closing prices at day's end for consecutive trading days. Our expression is clearly a reduced form formulation since we are abstracting from details on willingness to sell of current holders and willingness to buy of current bidders, and how the value of the current interest rate affects these decisions. Be that as it may, if one plots $Q_{t} / F_{t}$ on the vertical axis and $\Delta p_{t} / p_{t}$ on the horizontal axis for DAILY price and quantity per company on the NYSE, one generally gets a U-shaped scatter of points, centered roughly on $\Delta p_{t}=0$, with $Q_{t} / F_{t}$ at this point, significantly above zero. These scatters are then taking account of $\Delta p_{t}$ being positive or negative at the end of a day. In a representative scatter we are observing that there are many trades per day which result in almost no change in the current price of a share at the end of the day. That is, in a scatter plot, most observations cluster in something of a cloud centered on the vertical axis. Outliers correspond to relatively large percentage price changes and these

[^6]typically are linked in plots to relatively large volumes of shares trades. Hence the scatter, with "outliers", is generally U-shaped with a "body" of points in the center.

Such a scatter-plot is probably not surprising to even the casual observer. For a typical day, there is a moderate volume of trading in a company's shares and a moderate price change. Then there are somewhat rare days with large volumes traded in the shares of company i and these days display relatively large price changes from one close to the next. We turn to investigating such scatters across days for a company in detail. In particular, (a) how similar are the coefficients characterizing the U-shaped function "driven through" our scatter, and (b) do these functions change systematically across different time periods? Three months of 2008 became our reference period, since this marked the arrival of the so-called "financial crisis". Our typical interval was August 20, to November 20. We took up the same three month periods for four other years for comparison (1995, 2006, 2007, and 2010). We worked with the same eleven companies as above, namely Bank of America (BAC), General Electric (GE), Companhia Vale (VALE), Pfizer (PFE), Ford Motor (F), Att (T), Cemex (CX), Citigroup (C), Merck (MRK), Coca Cola (KO), and Potash Saskatchewan (POT).

We tried regressions first of the form

$$
F * \Delta p=a+b_{2}\left[p_{t-1} * Q_{t}\right]
$$

leading to

$$
\frac{\Delta p}{p_{t-1}}=\frac{a}{T p_{t-1}}+b_{1} * D u m+b_{2} * \frac{Q_{t}}{F}
$$

for $F$ the total shares in company i, $\mathrm{Q}_{t}$ total trades per day (one trade is one buyer transacts with one seller), $\Delta p=p_{t}-p_{t-1}$, and Dum is a dummy for $\Delta p$ positive or negative. Some regressions had a significant coefficient on $b_{2}$ but not consistently. Scatter plots indicated that "outliers" were not being "processed" satisfactorily, typically large price declines but occasionally large price rises.

To capture some non-linearity in the basic relationship, we investigated this alternative regression equation:

$$
F * \Delta p=a+b_{2}\left[p_{t-1} * Q_{t}\right]+b_{3}\left[p_{t-1} * Q_{t}\right]^{2}
$$

leading to

$$
\frac{\Delta p}{p_{t-1}}=\frac{a}{F p_{t-1}}+b_{1} * D u m+b_{2} * \frac{Q_{t}}{F}+b_{3} * \frac{Q_{t}^{2} p_{t-1}}{F}
$$

This form also failed to yield significant coefficients regularly.
We turned to $a d$ hoc forms that captured the U-shaped scatter of $\frac{\Delta p}{p_{t-1}}$ linked to trade intensity, $Q_{t} / T$, namely

$$
\frac{Q_{t}}{F}=a+b_{1} \frac{\Delta p}{p_{t-1}}+b_{2}\left[\frac{\Delta p}{p_{t-1}}\right]^{2}
$$

This is trade intensity as a function of current price-change allowing for the nonlinear effect of "outlier" price changes. Given $\Delta p$ having different signs over our typical interval, $b_{1}$ turned out to be generally very small and $b_{2}$ "carried" the explanation of the link between current price change and current trade intensity. We also estimated

$$
\frac{Q_{t}}{F}=a+b_{1} * D u m+b_{2} \frac{\Delta p}{p_{t-1}}+b_{3}\left[\frac{\Delta p}{p_{t-1}}\right]^{2}
$$

with Dum capturing the sign difference for $\Delta p$. Our estimates of $b_{3}$ turned out to be very close to the estimate of $b_{2}$ for the similar form of the equation immediately above. ${ }^{13}$ We tend to observe that relatively small pairs of values of $\Delta p$ and $Q_{t} / F$ tended to cluster about $\Delta p=0$, often in something of a cloud in our scatter diagrams, while relatively large values of pairs of $\Delta p$ and $Q_{t} / F$ strongly influenced the relationship of $\Delta p$ and $Q_{t} / F$. A rough summary is that large volumes of trade correlate positively with relatively large changes in current share price, based on daily observations.

We proceeded then to estimate

$$
Q_{t} / F_{t}=a+b_{1} *\left[\Delta p_{t} / p_{t-1}\right]+b_{2} *\left[\Delta p_{t} / p_{t-1}\right]^{2}
$$

for our 11 companies for three month intervals at five different dates. With our 11 companies we had 55 OLS regressions with 63 observations (days) each. ${ }^{14}$ See Table 5 below. Table 3 contains an overview: we combined the 11 companies for each period in a fixed effects regression.

[^7]Table 3: Combined regression per period (each year) with a fixed effect for each company.

|  | intercept | $b_{1}$ | $b_{2}$ | $\mathrm{R}^{2}$ | av q | av pr | av sq |
| :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- |
| 1995 | .0027 | $.0011 \_$ns | 1.675 | .512 | .0026 | .00254 | .00026 |
| 2006 | .0022 | $-.0156 \_$ns | 6.335 | .801 | .0069 | .00175 | .0002 |
| 2007 | .0061 | $.00524 \_$ns | .012 | .763 | .01057 | -.00019 | .00085 |
| $2007 \mathrm{a}^{*}$ | .0056 | $.0143 \_\mathrm{ns}$ | 1.922 | .668 | .00823 | -.00028 | .00042 |
| 2008 | .018 | -.023 | 0.834 | .726 | .025 | -.0111 | .00428 |
| 2010 | .0148 | -.059 | 10.22 | .649 | .014 | .0014 | .000269 |
| $2010 \mathrm{a}^{* *}$ | .015 | -.0393 | 9.518 | .716 | .0134 | .0017 | .000281 |

* This regression has VALE removed because of its "deviation" when regressed on its own. Vale acquired nickel mining giant INCO in October 2006.
** This regression has POTASH SASK removed because of its "deviation" when regressed on its own. In August 2010, PotashCorp became the subject of a hostile takeover bid by Anglo-Australian mining giant BHP Billiton. The takeover was disallowed by the Government of Saskatchewan with the cooperation of the Government of Canada.

In Table 3 we observe that $b_{2}$ is small for the crisis period, fall of 2008, reflecting large price changes and, relatively large quantities traded per day. For the same periods for 2006 and 2010, the large values for $b_{2}$ reflect relatively small price changes relative to say normal volumes of trading. "av sq" is average $\left[\Delta p_{t} / p_{t}\right]^{2}$ and we see this term to be unusally large for the 3 months (August to October) of 2008. "av q" for average quantity of daily trades is relatively large for this period of 2008 also. For a law of crisis trading we get: daily volumes traded rise more than daily values of $\sum\left[\Delta p_{t} / p_{t}\right]^{2}$. "av pr" is an average over $\Delta p / p$. We also observe that $\sum \Delta p_{t} / p_{t}$ is negative for the three month period of 2008 , as well as for 2007 . Outside of the crisis intervals, we observe $b_{2}$ relatively large for 2006 and 2010 but small for 1995. Clearly in terms of the key coefficient, $b_{2}$, the intervals for years 2006 and 2010 are outliers, given their relatively large magnitudes. Average quantities traded per day (av q) rise to .025 for the crisis year 2008 and decline after that. Not only was the three month period for 2008 one of relatively large quantities of shares traded but it was also one of relatively large price changes, mostly negative. "ns" is indicating not statistically significant.

Table 4 summarizes results for each company and three different three-month periods.

Table 4: Estimates of $b_{2}$ for different Time Periods, by Company

|  | $06 ; b_{2}$ | $06 ; R^{2}$ | $08 ; b_{2}$ | $08 ; R^{2}$ | $10 ; b_{2}$ | $10 ; R^{2}$ |
| :--- | :--- | :--- | :--- | :--- | :--- | :--- |
| BofAmerica | 4.507 | 0.159 | 0.593 | 0.243 | 8.53 | 0.322 |
| GE | 3.523 | 0.145 | 1.585 | 0.296 | 4.33 | 0.710 |
| Vale | 3.985 | 0.224 | 0.606 | 0.089 | 4.48 | 0.165 |
| Pfizer | 8.146 | 0.587 | 0.900 | 0.201 | -0.551 | 0.006 |
| Ford | 6.653 | 0.703 | 0.049 | 0.045 | 15.71 | 0.357 |
| At\&t | 4.039 | 0.082 | 0.259 | 0.118 | 5.092 | 0.270 |
| Cemex | 3.398 | 0.365 | 0.837 | 0.392 | 10.71 | 0.602 |
| Citigroup | 4.575 | 0.069 | 1.091 | 0.687 | 9.20 | 0.434 |
| Merck | 4.852 | 0.479 | 0.322 | 0.099 | 3.08 | 0.132 |
| Coca-cola | 5.205 | 0.626 | 0.681 | 0.263 | 3.41 | 0.222 |
| Potash-Sask | 8.455 | 0.458 | 1.405 | 0.397 | 37.58 | 0.370 |

If we are willing to ignore the values for $b_{2}$ for Pfizer, Ford and PotashSask in 2010 in Table 4, we can say that the average value for the remaining $b_{2}^{\prime} s$ is similar to that for 2006. Two periods of "normal" financial transacting are yielding fairly similar values for $b_{2}$ for a selection of blue chip companies for the NYSE. This leaves the relatively low values for $b_{2}$ for our companies for 2008 explainable by relatively large price changes, mostly negative. These relatively large price changes for 2008 were in fact accompanied by relatively large trading volumes also. $b_{2}$ for 2008 gets its low value because the average size of the price changes was unusually large.

Table 5: Detailed Results by Company and Specific Three-month Interval

| 1995 | intercept | $b_{1}$ | $b_{2}$ | $\mathrm{R}^{2}$ | av q | av pr | av sq |
| :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- |
| BAC | .0029 | .012 | .969 | .157 | .0031 | .0027 | .00023 |
| GE | .001 | -.0029 | 1.004 | .137 | .0011 | .0023 | .00012 |
| Vale $^{*}$ | x | x | x | x | x | x | x |
| PFE | .002 | -.0037 | .829 | .096 | .0021 | .0032 | .00027 |
| F | .003 | -.0014 | 1.884 | .437 | .0032 | .00056 | .00031 |
| T | .0009 | .0067 | .640 | .116 | .0010 | .0016 | .00013 |
| Cemex* | x | x | x | x | x | x | x |
| Citi | .002 | -.0067 | $.365 \_\mathrm{ns}$ | .042 | .0023 | .0022 | .00032 |
| MRK | .002 | .0023 | 1.67 | .243 | .0020 | .0030 | .00013 |
| KO | .001 | -.0079 | 1.21 | .135 | .0012 | .0028 | .00011 |
| POT | .006 | .0059 | 1.88 | .173 | .0075 | .0046 | .00070 |

* These companies were not reporting in 1995 as DowJones "members"

| 2006 | intercept | $b_{1}$ | $b_{2}$ | $\mathrm{R}^{2}$ | av q | av pr | av sq |
| :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- |
| BAC | .00224 | .00213 | 4.51 | .159 | .0024 | .00074 | .000038 |
| GE | .00208 | .00314 | 3.53 | .144 | .0022 | .00092 | .000038 |
| Vale $^{*}$ | .0158 | .0493 | 3.98 | .224 | .0177 | .0027 | .00046 |
| PFE | .00358 | -.0337 | 8.15 | .587 | .0043 | .00028 | .000091 |
| F | .0142 | -.0292 | 6.65 | .703 | .0181 | .0027 | .00061 |
| T | .00508 | -.0252 | 4.04 | .082 | .0056 | .00134 | .00013 |
| Cemex | .00019 | -.0448 | 3.40 | .365 | .0063 | .0017 | .00028 |
| Citi | .00282 | -.0169 | 4.58 | .069 | .0030 | .00071 | .000033 |
| MRK | .00411 | -.0090 | 4.85 | .479 | .0047 | .0019 | .00013 |
| KO | .00262 | .0244 | 5.21 | .626 | .0029 | .0010 | .000053 |
| POT | .00100 | -.0707 | 8.46 | .458 | .0087 | .0053 | .00035 |

* Vale acquired Canadian minining giant Inco in October, 2006.

| 2007 | intercept | $b_{1}$ | $b_{2}$ | $\mathrm{R}^{2}$ | av q | av pr | av sq |
| :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- |
| BAC | .0055 | -.0135 | 1.70 | .252 | .0061 | -.0027 | .0003 |
| GE | .0032 | -.041 | 1.58 | .166 | .0034 | .00004 | .00014 |
| Vale | .034 | $-.025 \_$ns | $-.076 \_$ns | .020 | .034 | .0007 | .0052 |
| PFE | .0052 | -.022 | 1.74 | .167 | .0054 | -.0009 | .00012 |
| F | .0187 | $.035 \_$ns | 5.177 | .511 | .022 | -.0012 | .0007 |
| T | .0034 | -.0052 | 1.08 | .141 | .0037 | -.00014 | .0002 |
| Cemex | .0087 | -.0076 | 1.62 | .143 | .0097 | -.0019 | .00062 |
| Citi | .0097 | -.010 | 3.36 | .302 | .0123 | -.0062 | .00059 |
| MRK | .0044 | .028 | 3.38 | .293 | .005 | .0023 | .0002 |
| KO | .0033 | $.011 \_$ns | 2.89 | .249 | .0036 | .0022 | .0001 |
| POT | .0090 | .048 | 1.16 | .212 | .0108 | .0057 | .0012 |


| 2008 | intercept | $b_{1}$ | $b_{2}$ | $\mathrm{R}^{2}$ | av q | av pr | av sq |
| :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- |
| BAC | .0202 | .017 | .593 | .243 | .024 | -.013 | .007 |
| GE | .0013 | -.0008 | 1.48 | .311 | .012 | -.011 | .0023 |
| Vale | .062 | $-.048 \_$s | .606 | .089 | .067 | -.013 | .0079 |
| PFE | .0080 | -.014 | .900 | .449 | .009 | -.0028 | .0011 |
| F | .0297 | .018 | .540 | .117 | .032 | -.017 | .0047 |
| T | .0059 | -.0116 | .244 | .133 | .006 | -.0025 | .0019 |
| Cemex | .01314 | -.025 | .837 | .392 | .018 | -.0217 | .0055 |
| Citi | .0189 | $-.026 \_\mathrm{s}$ | 1.09 | .687 | .028 | -.016 | .0083 |
| MRK | .0074 | -.0027 | .340 | .099 | .008 | -.0052 | .00168 |
| KO | .0061 | $-.028 \_\mathrm{s}$ | .681 | .263 | .007 | -.0034 | .001 |
| POT | .0520 | -.039 | 1.405 | .397 | .061 | -.017 | .006 |


| 2010 | intercept | $b_{1}$ | $b_{2}$ | $\mathrm{R}^{2}$ | av q | av pr | av sq |
| :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- |
| BAC | .0154 | $-.145 \_$s | 8.53 | .322 | .0200 | -.0013 | .00048 |
| GE | .0047 | $-.0228 \_$s | 4.32 | .710 | .0055 | .0013 | .0002 |
| Vale | .0267 | $.0087 \_$ns | 4.48 | .166 | .0285 | .0025 | .00038 |
| PFE | .0056 | $-.0061 \_n s$ | $-.551 \_$ns | .006 | .0055 | .00091 | .00012 |
| F | .0179 | $-.0801 \_n s$ | 15.71 | .357 | .0243 | .0053 | .00044 |
| T | .00379 | $-.1451 \_$s | 5.092 | .271 | .0041 | .0011 | .00007 |
| Cemex | .0159 | $-.0096 \_n s$ | 10.71 | .602 | .0225 | .0019 | .00062 |
| Citi | .0131 | $-.0355 \_n s$ | 9.203 | .434 | .0163 | .0022 | .00035 |
| MRK | .00376 | $-.028 \_n s$ | 3.080 | .132 | .0041 | .00047 | .00010 |
| KO $_{\text {POT }}$ | .0037 | $.0503 \_$s | 3.41 | .222 | .0040 | .0024 | .00045 |
| POT $^{2}$ | .0156 | $-.358 \_$s | 37.58 | .370 | .0215 | -.0009 | .00015 |

*In August 2010, PotashCorp became the subject of a hostile takeover bid by Anglo-Australian mining giant BHP Billiton. The bid was not successful.

For the most part in Table 5 , the coefficient $b_{1}$ was not statistically significant (we can take it as zero), and the coefficient $b_{2}$ was positive and statistically significant. Roughly speaking we infer that the U-shaped scatter of points yielded a "true" U-shaped function, "capturing" the scatter. The value of $R^{2}$ for these regressions varied considerably, being low typically when the scatter of points formed a cloud around $\Delta p_{t}=0$ in a scatter plot.

For the 2006 period, the eleven values of $b_{2}$ ranged from $3.4\left(\mathrm{CX}, R^{2}=0.365\right)$ to 8.45 (POT, $R^{2}=0.458$ ) and the value of $R^{2}$ ranged from $0.07(\mathrm{C})$ to $0.70(\mathrm{~F})$. $b_{1}$ was negative and significant for PFE and POT and positive and significant for VALE. In each case the values of $b_{1}$ estimated were close to zero. We infer that the term $\left[\Delta p_{t} / p_{t}\right]^{2}$ is closely connected with $Q_{t} / F_{t}$.

For the 2008 period, the eleven values of $b_{2}$ ranged from $0.049\left(\mathrm{~F}, R^{2}=\right.$ 0.0447 ) to $1.58\left(\mathrm{GE}, R^{2}=0.296\right)$. The values of $R^{2}$ ranged from $0.05(\mathrm{~F})$ to $0.69(\mathrm{C}) . b_{1}$ was significant and negative at about -0.027 for C and KO. In short the coefficient capturing the U-shape, namely $b_{2}$, was much smaller uniformly for the 3 months of 2008 compared with the 3 months of 2006 .

So far, we seem to be seeing some regularity in the quadratic connection between $\Delta p_{t} / p_{t}$ and $Q_{t} / F_{t}$. We did observe however that some six or eight scatter plots had an outlier that went into the regressions. These outliers were mostly of the form "large value of $\Delta p_{t} / p_{t}$ linked to large value of $Q_{t} / F_{t}$ ".

For the same three months in 2010, there were two anomalies. Pfizer's two
non-intercept coefficients were not significant and its $\mathrm{R}^{2}$ was 0.006; and Potash Saskatchewan's value for $b_{2}$ was 37.577 , essentially "off-the-chart" positive and significant. In addition the value for $b_{1}$ was significant with a value of -0.359 for POT. It turns out that on average about $2 \%$ of POT's float was traded per day compared with about $0.4 \%$ for Merck's and At\&t's floats respectively. Hence our measure of daily trading (trades over float) was relatively large for POT. Hence a relatively large value for $b_{2}$ was required to get the value of the variable $\left[\Delta p_{t} / p_{t}\right]^{2}$ up to the value of daily trading. The anomaly of a very high fraction of the float traded per day gets translated into the anomaly of a very large value for $b_{2}$. Ford also has a relatively large value for its $b_{2}$ and it turns out, like PotashSask, exhibits a relatively large number of shares traded each day, relative to its float. Hence the relatively large value of Ford's $b_{2}$ is explained in the same way that we did for POT. POT had very little price variation over the three months with a slight negative tilt in price on average. POT had a relatively high fraction of its total shares traded each day. Both its $b_{1}$ and $b_{2}$ coefficients are significant and each is large in absolute value relative to the $b_{1}^{\prime} s$ and $b_{2}^{\prime} s$ of the other ten companies. Volumes traded were small in 2006 period; larger in 2010 and quite large in 2008. Average price-change squared is large in 2008, while 2006 and 2007 have values similar to each other. Hence 2008 period had large price declines on average, with large price changes downward and relatively large fractions of float traded.

We end up with two anomalies. (1) A few companies experience a very high fraction of their total shares (float) traded each day and (2) Pfizer displayed considerable trading in its shares over 3 months in 2010 and the price-change each day was independent of the volume of its shares traded. Potash Saskatchewan displayed a very high fraction of its total shares traded each day, especially in the three-month period of 2010. This large av q for POT in part pulls up the sq coef for 2010 to the relatively high value of 10.22 . Potash Sask 2010 is a clear anomaly because during the period in question it was being bid for by BHP Billiton in a takeover attempt. This take-over was "rejected" by governments in Canada. Also for this period, BkofAmeric, Vale, Ford and Cemex exhibited relatively high fractions of their floats traded. Hence August to October, 2010 was a period of large volumes, relative to floats, for five of our eleven companies.

## 5 Concluding Remarks

We have looked into the daily price-changes for shares of 11 Dow-Jones companies in 5 different three month periods, over 1995 to 2010 . We observed that averages of price change percentages in "normal" times are smaller than the interest rate prevailing in the periods in question; for times of considerable price volatility, the opposite is true. We infer that short term trading cannot be profitable during normal periods, "on average". We suggest then, that the prevalence of short-term trading in these periods, must be based on some exotic trading procedure, turning most likely on a variant of procedures associated with high frequency trading. Our data for these companies and periods also displayed significant positive correlation between trade volume per day and the size of the price-change per day, whether the price-change was positive or negative. Though our samples are not large (11 companies and 5 distinct three month periods), we see no reason to expect that our results would be be overturned in similar analyses with much larger samples.

## APPENDIX:

A SERIES OF REGRESSIONS FOR LARGE PRICE-CHANGE AGAINST
LARGE QUANTITY TRADED, WITH A DUMMY FOR POSITIVE VERSUS
NEGATIVE PRICE CHANGE. Earlier regression involved a change in sign a priori for a difference in direction of price movement. Results are very similar for alternative formulations of estimating equation.

2010
...........
Bank of America 10

| Rsq .3425 | Coefficients | Standard Error | t Stat | P-value |  |
| :--- | :---: | :---: | :---: | :---: | :---: |
| Intercept | 0.018299137 | 0.002575253 | 7.105763882 | $1.65 \mathrm{E}-09$ |  |
| X Variable 1 | -0.005565248 | 0.004064864 | -1.369110282 | 0.176068345 |  |
| X Variable 2 | -0.244817188 | 0.0927489 | -2.639569724 | 0.010562835 |  |
| X Variable 3 | 8.629745064 | 1.701882303 | 5.070706152 | $4.08 \mathrm{E}-06$ |  |
| GE 10 |  |  |  |  |  |
| Rsq .7587 | Coefficients | Standard Error | t Stat | P-value |  |
| Intercept | 0.00548751 | 0.00027608 | 19.87649035 | $4.3929 \mathrm{E}-28$ |  |


| X Variable 1 | -0.001384785 | $5 \quad 0.000399848$ | - -3.463282156 | 0.000990518 |
| :---: | :---: | :---: | :---: | :---: |
| X Variable 2 | -0.059612623 | $3 \quad 0.01422695$ | -4.190119602 | $9.29315 \mathrm{E}-$ |
| 05 |  |  |  |  |
| X Variable 3 | 4.109208744 | 0.341289271 | 12.04025175 | $1.19505 \mathrm{E}-$ |
| 17 |  |  |  |  |
| Vale 10 |  |  |  |  |
| Rsq . 1659 | Coefficients | Standard Error | r t Stat P-value |  |
| Intercept | 0.026882115 | 0.001563402 | $17.19462717 \quad 7.30 \mathrm{E}-25$ |  |
| X Variable 1 | -0.000327724 | $4 \quad 0.002546118$ | - -0.128715252 | 0.898013456 |
| X Variable 2 | 0.001732011 | 0.071580432 | 0.024196706 | 0.980776001 |
| X Variable 3 | 4.524390118 | 1.520440846 | 2.975709401 | 0.004208448 |
| Pfizer 10 |  |  |  |  |
| Rsq . 0064 | Coefficients | Standard Error | r t Stat P-value |  |
| Intercept | 0.005629885 | 0.00046534 | $12.09842657 \quad 9.72397 \mathrm{E}-18$ |  |
| X Variable 1 | $-9.34769 \mathrm{E}-05$ | $5 \quad 0.000718986$ | -0.130012038 | 0.8969918 |
| X Variable 2 | -0.009885054 | $4 \quad 0.033249917$ | $7-0.297295581$ | 0.767267736 |
| X Variable 3 | -0.549117922 | 21.451618461 | -0.378279787 | 0.706557872 |
| Ford 10 |  |  |  |  |
| Rs . 3728 | Coefficients | Standard Error | t Stat P-value |  |
| Intercept | 0.020740771 | 0.003022888 | $6.861243694 \quad 4.31 \mathrm{E}-09$ |  |
| X Variable 1 | -0.006599647 | $7 \quad 0.005315882$ | - -1.241496188 | 0.219254117 |
| X Variable 2 | -0.221778298 | $8 \quad 0.145895433$ | - -1.520118172 | 0.133733872 |
| X Variable 3 | 16.97298748 | - 3.055741977 | 5.554457021 | $6.72 \mathrm{E}-07$ |
| AT\&T 10 |  |  |  |  |
| Rsq . 2780 | Coefficients | S Standard E | Error t Stat P-value |  |
| Intercept | Intercept 0.00 |  | 000220917.840 | 74096 1.14016E- |
| 25 |  |  |  |  |
| X Variable 1 | X Variable 1 | -0.000336664 | $4 \quad 0.00038218$ | -0.880904815 |
| 0.381884439 |  |  |  |  |
| X Variable 2 | X Variable 2 | -0.032870152 | 20.022654881 | -1.450908162 |
| 0.152013762 |  |  |  |  |
| X Variable 3 | X Variable 3 | 5.103174149 | 1.14044697 | 4.47471411 |
| $3.47545 \mathrm{E}-05$ |  |  |  |  |

Cemex 10

| Rsq .6041 | Coefficients | Standard Error | t Stat | P-value |  |
| :--- | :---: | :---: | :---: | :---: | :---: |
| Intercept | 0.016706252 | 0.001892698 | 8.826684555 | $1.93395 \mathrm{E}-12$ |  |
| X Variable 1 | -0.001833284 | 0.003362062 | -0.545285628 | 0.5875785 |  |
| X Variable 2 | -0.038602352 | 0.070045342 | -0.551105191 | 0.583608031 |  |
| X Variable 3 | 10.87322409 | 1.198585154 | 9.071715979 | $7.47732 \mathrm{E}-$ |  |

13
Citigroup 10

| Rsq . 4388 | Coefficients | Standard Error | t Stat | P-value |  |
| :--- | :---: | :---: | :---: | :---: | :---: |
| Intercept | 0.013805883 | 0.001262156 | 10.93833486 | $6.42 \mathrm{E}-16$ |  |
| X Variable 1 | -0.001609697 | 0.002269862 | -0.709160677 | 0.480971413 |  |
| X Variable 2 | -0.073494193 | 0.067488484 | -1.088988644 | 0.28051361 |  |
| X Variable 3 | 9.589036655 | 1.540230874 | 6.225713832 | $5.14 \mathrm{E}-08$ |  |

Merck 10

| Rsq .1326 | Coefficients | Standard Error | t Stat | P-value |  |
| :--- | :---: | :---: | :---: | :---: | :---: |
| Intercept | 0.003833869 | 0.000386649 | 9.915620076 | $2.94 \mathrm{E}-14$ |  |
| X Variable 1 | -0.000126854 | 0.000583661 | -0.217341973 | 0.828679251 |  |
| X Variable 2 | -0.0328675 | 0.02863969 | -1.147620664 | 0.255680589 |  |
| X Variable 3 | 3.042163565 | 1.294051384 | 2.350883128 | 0.022032765 |  |

Coke-KO 10

| Rsq .2230 | Coefficients | Standard Error | t Stat | P-value |  |
| :--- | :---: | :---: | :---: | :---: | :---: |
| Intercept | 0.003696973 | 0.000244281 | 15.13407075 | $3.76 \mathrm{E}-22$ |  |
| X Variable 1 | -0.000118821 | 0.000544355 | -0.218278279 | 0.827953013 |  |
| X Variable 2 | 0.042837578 | 0.043611729 | 0.982249019 | 0.32992147 |  |
| X Variable 3 | 3.65320606 | 2.090407601 | 1.747604657 | 0.08564859 |  |
| Potask Sask 10 |  |  |  |  |  |
| Rsq . 4362 | Coefficients | Standard Error | t Stat | P-value |  |
| Intercept | 0.022611114 | 0.003205322 | 7.054240716 | $2.02 \mathrm{E}-09$ |  |
| X Variable 1 | -0.013294652 | 0.005009951 | -2.653649081 | 0.010177209 |  |
| X Variable 2 | -0.787492479 | 0.205912686 | -3.824400017 | 0.00031449 |  |
| X Variable 3 | 38.10354538 | 6.422321087 | 5.932986667 | $1.59 \mathrm{E}-07$ |  |
| ........ |  |  |  |  |  |
| 2008 |  |  |  |  |  |


| Bank of America 08 |  |  |  |  |
| :---: | :---: | :---: | :---: | :---: |
| Rsq . 2436 | Coefficients | Standard Error | t Stat P-v | value |
| Intercept | 0.019831473 | 0.002915145 | 6.802910951 | $5.05 \mathrm{E}-09$ |
| X Variable 1 | 0.000613701 | 0.00465896 | 0.131724849 | 0.895635443 |
| X Variable 2 | 0.020285646 | 0.030780864 | 0.659034322 | 0.512353934 |
| X Variable 3 | 0.598090629 | 0.14123236 | 4.234798801 | 7.84E-05 |
| GE 08 |  |  |  |  |
| Rsq .3157 | Coefficients | Standard Error | $t$ Stat P-v | value |
| Intercept | 0.009900926 | 0.001813134 | 5.460669867 | $9.21 \mathrm{E}-07$ |
| X Variable 1 | -0.001809684 | 0.002755362 | -0.656786387 | $7 \quad 0.513788594$ |
| X Variable 2 | -0.01626615 | 0.030185676 | -0.538869841 | 0.591937945 |
| X Variable 3 | 1.450258443 | 0.303414306 | 4.779795866 | $1.15 \mathrm{E}-05$ |
| Vale 08 |  |  |  |  |
| Rsq . 0992 | Coefficients | Standard Error | t Stat | lue |
| Intercept | 0.066822098 | 0.007359098 | 9.080202324 | $6.25 \mathrm{E}-13$ |
| X Variable 1 | -0.009012476 | 0.010949585 | -0.823088341 | $1 \quad 0.413664003$ |
| X Variable 2 | -0.086997236 | 0.061163246 | -1.422377669 | - 0.160010264 |
| X Variable 3 | 0.596302743 | 0.284667988 | 2.094730595 | 0.04035552 |
| Pfizer 08 |  |  |  |  |
| Rsq . 1579 | Coefficients | Standard Error | t Stat P-v | value |
| Intercept | 0.002178077 | 0.000130098 | 16.74176997 | $2.76 \mathrm{E}-24$ |
| X Variable 1 | -0.000204916 | 0.000210219 | -0.974771581 | $1 \quad 0.333587301$ |
| X Variable 2 | -0.010045255 | 0.017519266 | -0.573383344 | $4 \quad 0.568528129$ |
| X Variable 3 | 3.625915439 | 1.150299029 | 3.152150309 | 0.002529842 |
| Ford 08 |  |  |  |  |
| Rsq . 1227 | Coefficients | Standard Error | t Stat P-v | value |
| Intercept | 0.031292049 | 0.003443593 | 9.087034535 | $6.09 \mathrm{E}-13$ |
| X Variable 1 | -0.003267441 | 0.00545062 | -0.59946227 | 0.551084818 |
| X Variable 2 | 0.001049002 | 0.042014406 | 0.02496767 | 0.98016225 |
| X Variable 3 | 0.530093858 | 0.189972309 | 2.790374344 | 0.00701681 |
| AT\&T 08 |  |  |  |  |
| Rsq .1557 | Coefficients | Standard Error | t Stat P-v | value |


| Intercept | 0.005359059 | 0.000492862 | 10.87335595 | $6.57 \mathrm{E}-16$ |
| :---: | :---: | :---: | :---: | :---: |
| X Variable 1 | 0.001059198 | 0.000820797 | 1.290450886 | 0.201764238 |
| X Variable 2 | -0.001477573 | 0.010546172 | -0.140105113 | 0.889038538 |
| X Variable 3 | 0.204743908 | 0.084700144 | 2.417279338 | 0.018641322 |
| Cemex 08 |  |  |  |  |
| Rsq . 4021 | Coefficients | Standard Error | t Stat P | lue |
| Intercept | 0.015344191 | 0.002636074 | 5.820849238 | 2.33E-07 |
| X Variable 1 | -0.003836629 | 0.003851983 | -0.996013909 | 0.323178084 |
| X Variable 2 | -0.044865074 | 0.026873351 | -1.669500575 | 0.100142909 |
| X Variable 3 | 0.807252025 | 0.153928249 | 5.244339702 | $2.08 \mathrm{E}-06$ |
| Citigroup 08 |  |  |  |  |
| Rsq . 6908 | Coefficients | Standard Error | t Stat |  |
| Intercept | 0.016651389 | 0.002949405 | 5.645676428 | $4.76 \mathrm{E}-07$ |
| X Variable 1 | 0.003754613 | 0.004253819 | 0.88264523 | 0.380950681 |
| X Variable 2 | -0.009971239 | 0.023728344 | -0.420224808 | 0.675822386 |
| X Variable 3 | 1.1111033 | 0.104057711 | 10.67776031 | $1.68 \mathrm{E}-15$ |
| Merck 08 |  |  |  |  |
| Rsq . 1280 | Coefficients | Standard Error | t Stat P | lue |
| Intercept | 0.006573727 | 0.000675234 | 9.735478626 | 84E-14 |
| X Variable 1 | 0.00140732 | 0.000995804 | 1.413249802 | 0.162749365 |
| X Variable 2 | 0.010898361 | 0.012260645 | 0.888889687 | 0.377612284 |
| X Variable 3 | 0.311274589 | 0.12482731 | 2.493641736 | 0.015418575 |
| coke-KO 08 |  |  |  |  |
| Rsq . 2761 | Coefficients | Standard Error | t Stat |  |
| Intercept | 0.006541358 | 0.000594064 | 11.01119162 | $4.91 \mathrm{E}-16$ |
| X Variable 1 | -0.000947971 | 0.000904073 | -1.048556198 | 0.298587867 |
| X Variable 2 | -0.038449695 | 0.014373921 | -2.674962197 | 0.009617854 |
| X Variable 3 | 0.706179799 | 0.155292346 | 4.547421789 | $2.69 \mathrm{E}-05$ |
| Potash Sask 08 |  |  |  |  |
| Rsq . 4048 | Coefficients | Standard Error | t Stat P-v | value |
| Intercept | 0.056293603 | 0.005715679 | 9.848979449 | $3.79 \mathrm{E}-14$ |
| X Variable 1 | -0.007831828 | 0.008802606 | -0.889717064 | 0.377171344 |
| X Variable 2 | -0.083014627 | 0.063526849 | -1.306764444 | 0.196279314 |



| X Variable 3 | 4.969734789 | - 0.778257501 | 6.385720386 | $2.60 \mathrm{E}-08$ |
| :---: | :---: | :---: | :---: | :---: |
| AT\&T 07 |  |  |  |  |
| Rsq . 1603 | Coefficients | Standard Error | t Stat P-va |  |
| Intercept | 0.003672242 | 0.000255324 | 14.38266705 | $2.90 \mathrm{E}-21$ |
| X Variable 1 | -0.000471219 | $9 \quad 0.000398084$ | $4-1.183717059$ | 0.241117472 |
| X Variable 2 | -0.017032357 | $7 \quad 0.013379232$ | - -1.273044412 | 0.207832678 |
| X Variable 3 | 1.040000409 | 0.360091856 | 2.888153095 | 0.005355948 |
| Cemex 07 |  |  |  |  |
| Rsq . 1435 | Coefficients | Standard Error | t Stat P-va | value |
| Intercept | 0.008592435 | 0.000930268 | 9.236511144 | $3.40 \mathrm{E}-13$ |
| X Variable 1 | $9.63 \mathrm{E}-05$ | 0.001387619 | 0.069406076 | 0.944893419 |
| X Variable 2 | -0.006112429 | $9 \quad 0.027748118$ | - -0.220282657 | 0.826386609 |
| X Variable 3 | 1.616870518 | 0.505906886 | 3.195984404 | 0.002208149 |
| Citigroup 07 |  |  |  |  |
| Rsq . 3053 | Coefficients | Standard Error | t Stat |  |
| Intercept | 0.010568614 | 0.001955985 | 5.403217066 | $1.14 \mathrm{E}-06$ |
| X Variable 1 | -0.00153582 | 0.002852613 | -0.5383905 | 0.592266739 |
| X Variable 2 | -0.122503618 | $8 \quad 0.06066035$ | -2.019500689 | 0.047834762 |
| X Variable 3 | 3.284888572 | 0.880253669 | 3.731752209 | 0.00041965 |
| Merck 07 |  |  |  |  |
| Rsq . 2934 | Coefficients | Standard Error | t Stat P-va | value |
| Intercept | 0.004400245 | 0.000382003 | 11.51886919 | $6.06 \mathrm{E}-17$ |
| X Variable 1 | 8.44E-06 | 0.000617884 | 0.013651542 | 0.989152536 |
| X Variable 2 | 0.028280962 | 0.025538934 | 1.107366571 | 0.272483067 |
| X Variable 3 | 3.375246992 | 0.831628337 | 4.05860027 | 0.000142777 |
| Coke-KO 07 |  |  |  |  |
| Rsq . 2491 | Coefficients | Standard Error | t Stat P-valut |  |
| Intercept | 0.00334368 0 | 0.0001914071 | $17.4689334 \quad 2.0$ | 2E-25 |
| X Variable 1 | -5.06E-05 | 0.000335946 | -0.150638524 | 0.880757899 |
| X Variable 2 | 0.008770678 | 0.017634994 | 0.497345093 | 0.620733731 |
| X Variable 3 | 2.922220863 | 0.792212931 | 3.688681096 | 0.000482037 |
| Potask Sask 07 |  |  |  |  |
| Rsq . 2150 | Coefficients | Standard Error | t Stat P-va | value |



| Intercept | 0.01534117 | 0.001891728 | 8.109608406 | E-11 |
| :---: | :---: | :---: | :---: | :---: |
| X Variable 1 | -0.002106452 | 20.002901265 | $5-0.72604599$ | 0.470633384 |
| X Variable 2 | -0.064913579 | $9 \quad 0.066207414$ | $4-0.980457841$ | 0.330797155 |
| X Variable 3 | 6.455054948 | 0.702002122 | 9.195207174 | $4.64 \mathrm{E}-13$ |
| AT\&T 06 |  |  |  |  |
| Rsq . 3230 | Coefficients | Standard Error | r t Stat P-v | -value |
| Intercept | 0.005626456 | 0.000612586 | 9.18476253 4.83E-13 |  |
| X Variable 1 | -0.001226602 | $2 \quad 0.00102938$ | $-1.191593481$ | 0.238112838 |
| X Variable 2 | -0.066145105 | $5 \quad 0.044327989$ | $39 \quad-1.492174729$ | 0.140893 |
| X Variable 3 | 4.210526 | 1.823632137 | 2.308868064 | 024411898 |
| Cemex 06 |  |  |  |  |
| Rsq . 3718 | Coefficients | Standard Error | P -value |  |
| Intercept | 0.005058306 | 0.00059488 | $8.503062148 \quad 6.83 \mathrm{E}-12$ |  |
| X Variable 1 | 0.00073433 | 0.000922082 | 0.796381939 | $0.428949947$ |
| X Variable 2 | -0.02757109 | 0.027999028 | -0.984715958 | 0.328717937 |
| X Variable 3 | 3.467299768 | 0.696385572 | 4.978994265 | 66 |
| Citigroup 06 |  |  |  |  |
| Rsq . 0719 | Coefficients | Standard Error | t Stat P-v | -value |
| Intercept | 0.002890111 | 0.000206376 | 14.00408677 | $1.44 \mathrm{E}-20$ |
| X Variable 1 | -0.000151631 | $1 \quad 0.000370031$ | $1-0.409778697$ | 0.683427684 |
| X Variable 2 | -0.029153713 | $3 \quad 0.037674167$ | $7-0.773838313$ | 0.442064858 |
| X Variable 3 | 4.990446463 | 2.434285903 | 2.050065877 | 0.044735315 |
| Merck 06 |  |  |  |  |
| Rsq . 4879 | Coefficients | Standard Error | t Stat P-v | -value |
| Intercept | 0.004381028 | 0.000319913 | 13.69499821 | $4.01 \mathrm{E}-20$ |
| X Variable 1 | -0.000525345 | $5 \quad 0.000507488$ | $8-1.035185597$ | 0.304736564 |
| X Variable 2 | -0.027973567 | $7 \quad 0.023230103$ | $3-1.204194706$ | 0.233243334 |
| X Variable 3 | 4.640942938 | 0.750882512 | -6.180651251 | $6.12 \mathrm{E}-08$ |
| Coke-KO 06 |  |  |  |  |
| Rsq . 6270 | Coefficients | Standard Error | - Stat P |  |
| Intercept | 0.002715343 | 0.000251439 | 10.79922612 | 07E-15 |
| X Variable 1 | -0.000183677 | $7 \quad 0.000417542$ | $2-0.439901647$ | 0.6615889 |
| X Variable 2 | 0.008690241 | 0.044801925 | 0.193970267 | 0.84685438 |


| X Variable 3 | 5.542618715 | 1.201464018 | 4.613220729 | $2.13 \mathrm{E}-05$ |
| :---: | :---: | :---: | :---: | :---: |
| Potask Sask 06 |  |  |  |  |
| Rsq . 4877 | Coefficients | Standard Error | t Stat P-v | value |
| Intercept | 0.007692057 | 0.001080888 | 7.116426027 | $1.58 \mathrm{E}-09$ |
| X Variable 1 | -0.003228276 | 0.00172965 | -1.866432896 | 0.066869955 |
| X Variable 2 | -0.141316597 | 0.049559788 | -2.851436699 | 0.005959885 |
| X Variable 3 | 8.720190546 | 1.163570059 | 7.494340782 | $3.58 \mathrm{E}-10$ |
| 1995 |  |  |  |  |
| Bank of America 95 |  |  |  |  |
| Rsq . 1580 | Coefficients | Standard Error | $t$ Stat P-v | value |
| Intercept | 0.002922974 | 0.000258072 | 11.32618118 | $1.55 \mathrm{E}-16$ |
| X Variable 1 | -9.02E-05 | 0.000419294 | -0.215103176 | 0.830416364 |
| X Variable 2 | 0.009559634 | 0.01378697 | 0.693381817 | 0.490745534 |
| X Variable 3 | 0.966006727 | 0.319476592 | 3.023716765 | 0.003670487 |
| GE 95 |  |  |  |  |
| Rsq .1685 | Coefficients | Standard Error | t Stat |  |
| Intercept | 0.000904125 | $9.63 \mathrm{E}-05 \quad 9.38$ | 3876714172.2 | E-13 |
| X Variable 1 | 0.000244412 | 0.000160896 | 1.519068118 | 0.133997579 |
| X Variable 2 | 0.006710701 | 0.008179282 | 0.820451165 | 0.415206667 |
| X Variable 3 | 0.871918722 | 0.347640424 | 2.508105107 | 0.014860121 |
| Vale 95 |  |  |  |  |
| Pfizer 95 |  |  |  |  |
| Rsq . 1520 | Coefficients | Standard Error | t Stat P |  |
| Intercept | 0.002152254 | 0.000187927 | $11.4526219 \quad 9$. | . $82 \mathrm{E}-17$ |
| X Variable 1 | -0.000809731 | 0.000405337 | -1.997675386 | 0.050291607 |
| X Variable 2 | -0.025331605 | 0.012698585 | -1.99483685 | 0.05060889 |
| X Variable 3 | 1.159806742 | 0.361433267 | 3.208909767 | 0.002140068 |
| Ford 95 |  |  |  |  |
| Rsq . 4544 | Coefficients | Standard Error | t Stat P-v | value |
| Intercept | 0.002338838 | 0.000265178 | 8.820156444 | .98E-12 |
| X Variable 1 | 0.000680306 | 0.000491998 | 1.382740797 | 0.17186925 |
| X Variable 2 | 0.016712657 | 0.015658615 | 1.067313924 | 0.290105824 |


| X Variable 3 | 1.662128896 | $6 \quad 0.327903857$ | $7 \quad 5.06895196$ | 4.11E-06 |
| :---: | :---: | :---: | :---: | :---: |
| AT\&T 95 |  |  |  |  |
| Rsq . 1202 | Coefficients | Standard Error | - Stat P-va | value |
| Intercept | 0.000863059 | 0.000107903 | 7.998479754 | $4.93 \mathrm{E}-11$ |
| X Variable 1 | 0.000106237 | 0.000196628 | 0.540293322 | 0.59099474 |
| X Variable 2 | 0.010348809 | 0.008933369 | 1.158444193 | 0.251273016 |
| X Variable 3 | 0.592140429 | 0.324829645 | 1.822926074 | 0.073297769 |
| Cemex 95 |  |  |  |  |
| Citigroup 95 |  |  |  |  |
| Rsq . 0831 | Coefficients | Standard Error | t Stat P |  |
| Intercept | 0.00242541 | 0.000183256 | 13.23511001 | 89E-19 |
| X Variable 1 | -0.000589543 | -0.00035989 | -1.638121076 | 0.10663069 |
| X Variable 2 | -0.020955008 | - 0.010355491 | $1 \quad-2.023564877$ | 0.04747582 |
| X Variable 3 | 0.524321474 | 0.2770709 | 1.892372942 | 0.063268821 |
| Merck 95 |  |  |  |  |
| Rsq . 2430 | Coefficients | Standard Error | t Stat P-valut |  |
| Intercept | 0.00176031 | 0.000156971 | 11.21424364 | 4E-16 |
| X Variable 1 | $5.18 \mathrm{E}-05$ | 0.000250858 | 0.206580143 | 0.837037204 |
| X Variable 2 | 0.004164059 | 0.011591008 | 0.359249036 | 0.720670029 |
| X Variable 3 | 1.662645306 | 0.421902611 | 3.940827255 | 0.00021459 |
| Coke-KO 95 |  |  |  |  |
| Rsq . 1581 | Coefficients | Standard Error | t Stat P |  |
| Intercept | 0.001154555 | $9.86 \mathrm{E}-05 \quad 11$. | 1.71167563 3.86 | E-17 |
| X Variable 1 | -0.00024357 | 0.000190102 | -1.281259669 | 0.205032347 |
| X Variable 2 | -0.019091438 | - 0.010583504 | $4 \quad-1.803886377$ | 0.076269598 |
| X Variable 3 | 1.442984368 | 0.43135317 | 3.345250404 | 0.001421936 |
| Potash Sask 95 |  |  |  |  |
| Rsq . 1770 | Coefficients | Standard Error | t Stat P-va | value |
| Intercept | 0.005614486 | 0.001309045 | 4.288995069 | $6.62 \mathrm{E}-05$ |
| X Variable 1 | 0.001393955 | 0.002478719 | 0.562369323 | 0.575959554 |
| X Variable 2 | 0.030916816 | 0.056229738 | 0.549830339 | 0.584476717 |
| X Variable 3 | 1.68236057 | 0.749021334 | 2.246078308 | 0.028393093 |

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[^0]:    *Thanks for comments to participants in a seminar in the McGill Economics Department, April, 2015.

[^1]:    ${ }^{1}$ Additional costs of short-term trading that can figure significantly are brokerage fees associated with purchasing and selling the shares.
    ${ }^{2}$ It is estimated that as of 2009 , HFT accounted for $\mathbf{6 0 - 7 3 \%}$ of all US equity trading volume, with that number falling to approximately $50 \%$ in 2012. (Wikipedia, October ...., 2015 under High Frequency Trading). And John Fullerton: "High Frequency Trading is a Blight on Markets that the Tobin Tax Can Cure", The Guardian, April 4, 2014: "... algorithmic trading... practices in a sector that accounts for about half of all trades on the New York and London stock exchanges."

[^2]:    ${ }^{3}$ This problem is artificial because in reality the purchaser retains the option of selling at any date after her purchase, an option to sell of positive value to the owner. More on this below.

[^3]:    ${ }^{4}$ Schwert (2011) investigates "excess volatility" for 2008.
    ${ }^{5}$ We use the US Government 10 year bond rate for Sept 1, "the year" as the reference interest rate, divided by 365 to get a daily version. For 1995, we have the 6.20 annual rate and 0.017 as the daily rate; for 2006 we have 4.72 and 0.0129 ; for 2007 we have 4.25 and 0.01164 ; for 2008 we have 3.10 and 0.0085 ; and for 2010 we have 2.65 and 0.0073 . For each company and each year's three month interval, we take the average value of percentage price decline days and average value of price increase days. These two values are compared with the corresponding representative 10 year bond rate of the US government for the three month period.

    Given an annual interest rate of 0.04 , we have the corresponding compound rate $x$ in $x^{365}=0.04$ with $x=0.99122$. We do not make use of daily compound rates.

[^4]:    * These companies were not reporting in 1995 as DowJones "members"

[^5]:    6 "Trading in the short run is viewed as one of two things: gambling or the possibility of strategic investing." (Wikipedia: "Short Term Trading, October 4, 2014). Turner, Toni (2005) Short-Term Trading in the New York Stock Market, New York: St. Martins Press:
    "As traders, we crave volatility. Rapid price movements provide our bread and butter.

[^6]:    ${ }^{12}$ A trade is the selling by the owner or her agent of one share to a buyer or her agent. Volume is the sum of such trades over an interval. We are dealing with the interval: close on day $t-1$ to close on day $t$.

[^7]:    ${ }^{13}$ See the Appendix for details. Results in Table 5 below are closely related to those in the Appendix.
    ${ }^{14}$ Each of Cemex and Vale were not part of the Dow-Jones average in 1995 and thus we only had 9 companies for 1995. Some three month periods had 64 trading days.

