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Sources of Variation in International Real Interest Rates

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Abstract

This paper analyses the effects of inflation on ex-post real interest rates in an international framework. A dynamic factor model is estimated in which real interest rates are influenced by real interest and inflation factors that are common to all the countries, and by country-specific factors. We find that the source of domestic inflation is an important determinant of the effect of inflation on real interest rates. A common inflation factor has a negative effect on ex-post real interest rates lending support to a form of the Mundell-Tobin effect in international real interest rates, and that a country-specific inflation factor tends to have a positive effect.

Key Words: real interest rates, world components, Kalman filter, scoring

JEL Classification Numbers: E43, F40

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Doug Purvis – Econometrician

People familiar with Doug's research often overlook his econometric insights into economic problems. I would like to relate a short story that I think illustrates this strength. Doug Purvis was a member of my Ph.D. thesis committee at Queen's University in the early 80's and took part in my oral defense. My thesis was on Canadian money demand (M1) and the implementation of what was at the time, fairly high-tech econometrics. It is customary at Queen's University to allow members of the thesis committee to ask questions, so after about an hour Doug's turn came. My defense had been going swimmingly, and I thought I was handling all kinds of technical questions adroitly. Home free so to speak. Doug began with a rather long preamble about how I was the resident expert on money demand in Canada having spent a little over a year and half on the subject and that there really wasn't very much that he could ask me. Nevertheless he thought he should ask something. "What is the current level of M1 in Canada?" he inquired. Like many young applied econometricians, I had treated the data as something that is loaded into a file, transformed, and then forgotten. I had no idea what the current level was but I replied "I know its logarithm!" This was obviously not satisfactory, but Doug let me off the hook and supplied the answer. I have remembered this valuable econometric lesson and often teach it to my students.

For Doug, applied econometrics had to be relevant and had to answer economic problems. I hope this study of international real interest rates meets this standard. By the way, if anyone should ask, the ex-post real interest rate in Canada in June 1990 was 4.67 percent (annualized).

Allan W. Gregory

1. Introduction

The short-run relationship between interest rates and monetary phenomena has been an active field of research. There are many models that attempt to explain the negative relationship between changes in the supply of money and interest rates or the "liquidity effect" both theoretically (see Christiano (1991) and Christiano and Eichenbaum (1991)) and empirically (see Cochrane (1989) and Leeper and Gordon (1992)). A companion relationship is that between inflation and real interest rates.

Mundell (1963) and Tobin (1965) have presented models that yield a negative relationship between real returns and inflation. Fried and Howitt (1983) also develop a model that captures the negative effects of inflation on real interest rates. There is also some empirical support for the hypothesis (Fama and Gibbons (1982) and Lee (1992)). Chan (1994) solves and estimates an asset-pricing model to detect the effect of inflation uncertainty on the time series behavior of real interest rates, where the uncertainty takes the form of a covariability risk premium in interest rates.

These studies typically look at one country (usually the U.S.) in a closed economy setting [1]. However, as Blanchard and Summers (1984) have argued, interest rates are determined for a large part worldwide rather than domestically, since capital flows toward nations with high real rates, thus tending to equalize rates around the world. This argument suggests that in order to model the relationship between inflation and real interest rates in any country, one must allow for the international channels that are driving real rates as well as the potential effects of inflation.

This paper estimates a model for international ex-post real interest and inflation rates with the aim of uncovering a particular kind of structural relationship between real rates and inflation. We use monthly data from nine countries: Canada, the U.S., Japan, Germany, France, the U.K., the Netherlands, Switzerland, and Belgium. Dynamic factor

[1] Mishkin (1984) is an exception.

analysis and Kalman filtering techniques are used to analyze the stochastic structure of short term ex-post real interest and inflation rates by decomposing the observed rates for each country into unobservable orthogonal stochastic components.

Real interest rates are decomposed into four unobserved factors:

(1.) A common (world) real interest rate factor.

(2.) A common (world) inflation rate factor.

(3.) An idiosyncratic (country) real interest rate factor.

(4.) An idiosyncratic (country) inflation factor.

International real interest rates and inflation rates together are modeled as a multivariate system, with cross-equation links via the common factors. From these decompositions we determine if there is a negative or a positive influence on real interest rates arising from the common inflation factor or the country specific factors. If we find a negative relationship, this would lend support the to the ideas of Mundell and Tobin within an international setting that allows for joint determination of real rates.

The first and third factors capture the effects on real rates of real factors. The first factor affects all the real rates in the system and is that part of a countries' real interest rate that is determined contemporaneously with all foreign real interest rates. The third factor represents the part of the real interest rate attributable solely to real domestic factors. The estimation of the model sheds light on whether or not there are statistically significant international links in real interest rates, and gives the quantitative contribution of each of the factors to the variation in real rates.

The second and fourth factors are the channels by which the effects of inflation impact on the real rates. The discussion is similar to that for real rates. The second factor measures the effect on a countries' real rate of factors that jointly affect the rates of inflation in each country (a world factor). The fourth factor measures those domestic factors that affect domestic inflation, and their potential effect on the real interest rate. By introducing two inflation factors to the real interest rate equations, we allow for potentially different effects arising from the two sources of inflation on the real interest rates. Specifically, by imposing no restrictions on the parameters that measure the effects on real interest rates, we are able to uncover any differences from an inflation factor that affects all economies versus a domestic inflation factor.

The econometric set-up is similar to Stock and Watson (1991) and Gregory, Head and Raynauld (1994), where dynamic factor models are used to study various macroeconomic phenomena. These models specify that some observed series are functions of a common and unobserved variable, or factor. In Stock and Watson, the observed variables are the elements of the Index of Coincident Economic Indicators which are jointly related to the unobserved component which is interpreted as 'economic activity' and individually related to variable specific components. In Gregory, Head and Raynauld, the observed series are several international macroeconomic variables and the unobserved series are a world (common) fluctuation interpreted as a world business cycle and country specific components for each of the variables.

We find evidence for a special form of the Mundell-Tobin effect with respect to the common world inflation factor since there is indeed a significant negative relationship with the real interest rate. On the other hand, there is typically a significant positive relationship between the country-specific inflation factor and that countries' real interest rate. This indicates that the source of inflation is an important determinant of the relationship between inflation and real interest rates. An increase in inflation common to all countries leads to a decrease in all of the countries real interest rates. However, if the inflation increase is due to domestic factors, then the domestic real rate rises with no contemporaneous change in any of the foreign real rates. To our knowledge this dichotomy has not been documented empirically in the literature. We think this also raises an important challenge to explain this transmission process in a dynamic general equilibrium model.

The organization of this paper is as follows. Section 2 outlines the dynamic factor model for the real interest rates and inflation rates, and discusses the data. Since the stationarity of the data is crucial, in Section 3 we examine in some detail this question as well as present some preliminary correlation analysis. Section 4 presents the results from the estimation of the state-space model, and Section 5 concludes with some final remarks.

2. Dynamic Factor Representation and Data

The multivariate dynamic factor analysis based upon the Kalman filter is used to estimate the parameters of the equations for the real interest rates and inflation rates, and also to estimate the unobservable factors. Campbell and Clarida (1987), Fama and Gibbons (1982), and Hamilton (1985) have all estimated state-space models involving ex-post real interest rates. Campbell and Clarida are primarily concerned with the movements of real exchange rates as related to real interest rates. Fama and Gibbons model the negative relation between the expected real return component of interest rates and the expected inflation component, and Hamilton attempts to uncover financial market expectations of inflation within a bivariate system consisting of realized inflation and ex-post real interest rates. The approach here is quite different and we commence with outlining the dynamic factor model.

The Dynamic Factor Model

For country i ($i = 1, \ldots, 9$) the ex-post real interest rate, r_{it}, is decomposed as:

$$r_{it} = \phi_i r_t^w + \delta_i \pi_t^w + \alpha_i \eta_{it}^\pi + \eta_{it}^r, \tag{1}$$

and inflation rates, π_{it}, as:

$$\pi_{it} = \beta_i \pi_t^w + \eta_{it}^\pi. \tag{2}$$

r_t^w and π_t^w are the common world factors to real interest rates and inflation rates respectively. η_{it}^r and η_{it}^π are the country-specific factors affecting real interest rates and inflation rates for country i respectively. The system (1) and (2) are the *measurement equations* (see Harvey (1989)) and comprise 18 equations.

Equations (1) indicates that there are four contemporaneous effects driving real interest rates. Two factors are common to all real interest rates and two country-specific effects. The first factor is the world real interest rate factor r_t^w and captures any cross-country linkages in the real interest rates. The magnitude of its influence on the real rate of interest for country i is given by ϕ_i. The second common factor is the world inflation factor (π_t^w). The magnitude of the effect on the real interest rate in country i of the common inflation factor is given by the parameter δ_i. The idiosyncratic inflation factor (η_{it}^{π}) appear as a separate source of variation for r_{it}, with α_i measuring the sensitivity of the country's real rate to country-specific inflation movements. We would expect both δ_i and α_i to be negative if there is a Mundell-Tobin effect on real interest rates from inflation regardless of the source (although the magnitudes could be different). We place no restrictions on these parameters, allowing the sources of inflation to have differing effects on the real rates.

In equations (2), we decompose the rate of inflation for country i into two factors, the common world inflation factor and the country-specific factor, both of which also affect the real interest rate. The parameter β_i captures the effect of the common inflation factor on the rate of inflation in country i. We have not introduced any feedback mechanism of real rates on inflation, although that is possible.

We assume that each of the unobserved factors in (1) and (2) follow univariate first-order autoregressive processes.

$$
\begin{aligned}
r_t^w &= \rho_r r_{t-1}^w + \epsilon_t^r \\
\pi_t^w &= \rho_\pi \pi_{t-1}^w + \epsilon_t^\pi \\
\eta_{it}^r &= \rho_{\eta i}^r \eta_{it-1} + \epsilon_{rit} \\
\eta_{it}^\pi &= \rho_{\eta i}^\pi \eta_{it-1} + \epsilon_{\pi it}.
\end{aligned}
\tag{3}
$$

Each of the ϵ's are assumed to be normally distributed and contemporaneously uncorrelated, with zero mean and constant variance. Equations (3) are referred to as the *transition equations* and with nine countries there are 20 of them.

The estimation problem is a formidable one in which the state vector at each time period as well as the parameters of the dynamic factor model are estimated. In order to identify the model we assume that the variances of ϵ_t^r and ϵ_t^π are equal to one, leaving 20 unobserved components with 74 parameters to estimate. The model is cast in the usual state-space form with the associated log-likelihood. We maximize this likelihood using a Gauss-Newton scoring algorithm written by Raynauld, Simonato, and Seqouin (1993). The program was written in GAUSS and all calculations were done on an IBM RS/6000 Model 355 workstation.

Data

We use short-term Euromarket interest rates for 9 OECD countries (the U.S., U.K., Canada, Japan, Germany, France, Netherlands, Switzerland and Belgium) for the period October 1975 to June 1990. The interest rates are monthly from the Harris Banks, and for the inflation data, we use the CPI for each of the countries from the IMF International Financial Statistics. Eurodollar rates are used since they are market-clearing, offshore and have similar risk characteristics (see Mark (1986) and Mishkin (1981,1984)). Similar data has been used in studies done by Backus, Gregory and Telmer (1993), Mishkin (1984) and Campbell and Clarida (1987). Both the real and inflation rates are calculated as annualized monthly rates, and are deseasonalized using seasonal dummies and standardized to a sample mean and variance of zero and one respectively.

3. Stationarity and some Correlations

In the estimation of the state-space model, the data are assumed to be stationary processes. On the basis of the estimated autocorrelation functions, one could safely conclude both the real interest and inflation rates are stationary. Indeed, if we look solely at the first-order autocorrelation estimates (Table 1, Panel b), we see no compelling evidence in favour of a unit root. However, other authors (Rose (1988), Siklos and Wohar (1993), and Katsimbris and Miller (1993)) using more formal tests of the unit root hypothesis, give a somewhat different picture.

Rose (1988), King, Plosser, Stock, and Watson (hereafter KPSW, 1991) and King and Watson (1992) give conflicting evidence on the stationarity of (quarterly) real interest and inflation rates. Rose concludes that the real interest rate has a unit root and that inflation does not. KPSW on the other hand, give evidence that the ex-post real interest rate may or may not have a unit root and that the inflation rate does not have a unit root. King and Watson present evidence that does not reject the null of a unit in either the inflation rate, the nominal interest or the real interest rate. Mishkin (1992), using monthly data, finds unit roots in nominal interest rates but not in inflation.

The studies by Mishkin (1992) and King and Watson (1992) also indicate that their results would also not be able to reject a null of stationarity. They further find that the Fisher effect does not hold in the short-run, while there conclusions regarding the long-run conflict. Mishkin finds support for a long-run Fisher effect whereas King and Watson do not.

Siklos and Wohar (1993) perform unit root tests on nominal Eurorates and inflation rates. They find unit roots in nominal interest rates at the one month frequency for the U.S., Canada, Japan, France, Belgium and Switzerland. They reject the null of a unit root for the U.K., Germany and Netherlands. With respect to inflation, they conclude that the only unit root at the one month frequency can be found in France.

Table 2 presents the results of unit root tests performed on the ex-post real interest rates and inflation rates in this study. Two popular unit root testing procedures are used, the augmented Dickey-Fuller (ADF) test and the Phillips and Perron t-test using nonparametric corrections. Two lag length selection procedures are used for the ADF tests. The first uses a general to specific testing procedure for the significance of the last lag at the 5% level of significance following Campbell and Perron (1993), while the other is an arbitrary choice of four lags. From the critical values at the bottom of the table, we find that the ADF tests, using the Campbell and Perron procedure, do not reject the null of a unit root in any of the series tested, at the 5% or even the 10% level of significance. This would suggest that the real interest and inflation rates are nonstationary processes. The Phillips and Perron version of the τ-test for nonstationarity, offers a different picture. These tests reject the null hypothesis of a unit root at the 5% level of significance for all the series.

One difference in these test procedures is the number of lags used in the tests. For both the real interest rates and the inflation rates a large number of lags are included in the ADF test regressions, except for the French real interest rates. The Phillips-Perron test includes 4 lags, since the sample size $T = 177$ and the lag length for the nonparametric correction is set as $T^{\frac{1}{4}}$ (see Banerjee et al (1993)).

If a small number of lags is included the ADF tests, we see that all but three of the series would reject the unit root null hypothesis at the 5% level of significance, these being the U.S. real rate, the Canadian inflation rate and the French inflation rate (the Canadian and French inflation rates can reject at the 10% level). Hence we have conflicting evidence on the presence of unit roots in both the real interest rates and the inflation rates, depending on the lag correction. In lieu of strong evidence to the contrary, we therefore assume that the real interest rates and inflation rates are stationary processes.

In Table 1 (Panel a) we record the cross-country correlations of real rates for the 9 countries. The first column is the estimated world real factor that will be discussed in the next section. A striking feature from this table is that all of the correlations are positive and range from .10 to .51. The largest correlation is between the U.S. and Germany, with the U.S. and France the next largest. With respect to the European countries, we notice that France has larger correlations with the U.S. real rate than with the German rate. Whereas the U.K. and the smaller European countries, the Netherlands, Switzerland and Belgium have larger correlations with the German real rate than with the U.S. This accords with evidence presented in Karfakis and Moschos (1990) who examine interest rate linkages in the European Monetary System using a vector autoregressive analysis, but differs from evidence presented in Katsimbris and Miller (1993) who perform cointegration tests using U.S. interest rates and various European rates. Katsimbris and Miller find stronger relations between the U.S. rates and the European rates than between the German rate and other European rates.

Table 3 presents a summary of some simple linear regressions of the real rates on the inflation rates for each country, using the regression equation

$$r_{it} = \mu + \theta\pi_{it} + u_{it}.$$

The estimated $\hat{\theta}$'s are all negative, and are generally significant at the 5% level of significance (against the normal distribution), and would appear to support a Mundell-Tobin inverse inflation effect on ex-post real interest rates.

4. Results from the Dynamic Factor Model

Table 4 presents the impact coefficients from the estimation of equations (1) and (2). Note that the common world real factor has a positive effect on all of the interest rates and strengthens the interpretation that the high correlation amongst international real interest rates as due to common elements. Except for Switzerland and Belgium, the estimated coefficients are significant at conventional levels. The common inflation factor is estimated to have a significant negative effect and the idiosyncratic inflation factor typically has a significantly positive effect on the real interest rate. Only the U.S., the U.K. and France have estimated country-specific impact coefficients on the inflation component which are insignificant. These results show that the source of the measured inflation rate in a country is important. Factors that tend to affect the inflation rates of all countries will have a negative effect on real rates in country i, while factors that only affect country i's inflation rate will tend to have a positive effect.

We also test the restrictions that the coefficients on the country-specific inflation factors are jointly equal to zero (α_i, $i = 1, \ldots, 9$). The calculated statistics for a Wald and Likelihood ratio tests are 132.5 and 107.3 respectively, both implying marginal significance levels of 0.00 (located on a χ^2 distribution with 9 degrees of freedom). These tests show that even though individual t-statistics for some of the countries real interest rates are not be significantly affected by domestic inflation factors, the joint restriction that none of the countries inflation component is relevant is overwhelmingly rejected. Finally, the joint exclusion restriction of both of the inflation factors in the real interest rate equations is strongly rejected using a Wald test (again a marginal significance level of 0.00, with 18 degrees of freedom). We can safely conclude from these tests that the inflation factors, regardless of the source, do have significant effects on the domestic real interest rates of these nine countries.

These results may also be interpreted in the context of the literature discussing the effects of inflation on growth. The models used by Mundell and Tobin imply that through a real-balance effect, inflation increases the level of investment (through a fall in the real interest), and stimulates growth in the economy. In light of results in Table 4, this interpretation requires that most of the variation in inflation rates be due to common factors. Stockman (1981) on the other hand, uses a cash-in-advance constraint applied to consumption and investment to illustrate that inflation causes a fall in investment and thus has a negative effect on growth. This argument implies that most of the variation in inflation rates is due to country-specific factors since these factors have been found to be positively related to real rates.

The larger is the estimated impact coefficient on the world components, the larger will be the influence of the world factors on the level of the real interest rate in that country. In Table 4 we see that the three countries that have the largest estimated $\hat{\phi}_i$'s are the U.S., Germany and Japan. While the smallest estimated $\hat{\phi}_i$'s are from the Netherlands, Switzerland and Belgium.

With respect to the world inflation factor, France, Germany and the U.K. have the largest estimated impact coefficients, while Canada, Netherlands, Switzerland and Belgium have the smallest impact coefficients. Both of the world factors have the smallest effects on the smaller economies (Canada, Netherlands, Switzerland and Belgium) and the largest impact on the larger economies (the U.S., Germany, Japan, France and the U.K.). The two countries with the largest impact coefficients with respect to the country specific inflation shock are the Netherlands and Canada.

The common world inflation factor is estimated to have a positive and significant effect on each country's rate of inflation. France and the Netherlands are the countries with the largest and the smallest estimated impact coefficients of the common world inflation factor on their domestic inflation rates respectively. The impact coefficients for the rest of the countries are fairly uniform and fall in the range .11 to .17. With the country-specific inflation component having a positive effect on a countries inflation rate by construction, we have that all of the countries' inflation rates respond positively to inflation factors regardless of the source.

Table 5 presents the estimated autocorrelation parameters from transition equations (3). Interestingly, we see that the two estimated common world components are highly autocorrelated, in fact a test for a unit root would not be rejected. The strongly persistent world factors contrasts markedly with the far less autocorrelated actual series (Table 1, Panel b). All of the country-specific real interest rate factors have estimated autocorrelation parameters that are generally positive and significant. The Canadian real rate factor has an estimated autocorrelation coefficient of 0.465 and is the largest of those estimated for the real rates. This us despite the fact that from a univariate analysis, the Canadian rate is not the most autocorrelated. Only Japan and the U.K. have autocorrelation parameters for the real rates that are insignificantly different from zero.

The country-specific inflation factors have estimated autocorrelation parameters that can differ sharply. The autocorrelation coefficient for the estimated Canadian inflation factor is positive but insignificant, while for the U.S., the autocorrelation coefficient for the inflation factor is large and significant. The estimated coefficients for Japan and the Netherlands are negative, though only that for the Netherlands is significantly different from zero at the 5% level of significance.

Figure 1 plots the estimated world real interest rate component, the estimated world inflation component and the Canadian ex-post real interest rate. In the figure, the world components are weighted by their impact coefficients from Table 4. The impact coefficient for the world real interest rate factor is positive while that for the world inflation factor is negative. Hence, when the weighted world real factors are above (below) zero, they tend to push up (down) the Canadian ex-post real interest rate. The world inflation factor is somewhat more complicated to interpret due to the estimated negative coefficient in the real interest rate equation. When the weighted world inflation factor is above (below) zero, this indicates a below (above) average world inflation factor. The 1970's and early 1980's thus can be characterized as higher world inflation environments compared to the late 1980's and early 1990's. We see clearly that the world inflation in the 1970's exerted downward pressure on the real interest rates.

The figure illustrates that the world factors are characterized by long slow swings and are above or below zero for long periods of time. This is, of course, owing to the persistence in these series due to their near unit-root estimates (Table 5). This figure also indicates that the ex-post real rate for Canada is a much more volatile series than either of the two world factors and suggests that the high frequency movements in the real rates is principally domestic in origin.

A particularly interesting feature from this decomposition is that domestic real interest rates are on some occasions high (low) with either high (low) world real factors and/or low (high) world inflation factors. These world factors are sometimes reinforcing and are sometimes opposing. For instance, the low real rates of the 1977-1979 correspond to the reinforcing movements of below average weighted world real and inflation factors (i.e. a low world real interest factor and a high world inflation factor). In contrast, the high real rates in the early 1980's are accompanied by positive and negative weighted world real and inflation factors respectively. Thus the world real interest rate factor was positive and the world inflation factor was positive, each exerting an opposing force on the Canadian ex-post real interest rate. Finally, we observe that the rather high ex-post real rates of the late 1980's are mostly due to other domestic factors since we see that the levels of the world factors are low compared to the level of the Canadian ex-post real rate. This lends support to the interpretation of a made-in-Canada interest rate, perhaps as a result of the tight monetary policy at the Bank of Canada to combat domestic inflation.

Figure 2 displays the same information with the U.S. ex post real rate. In this figure, it is apparent that the world real interest rate component and the U.S. real interest rate are very closely related. Again in the early 1980's with the high real rates, we see the complementary association of high weighted world real and low weighted world inflation factors. There also appears to be episodes (for example, late 1985) where the real ex-post interest rates are determined solely by U.S. factors.

An interesting comparison is between the impact coefficients of the world real interest rate factor on the domestic real interest rates with the GDP weights used by Barro and Sala-i-Martin (1990). Their paper attempts to estimate a world real interest rate by creating a global capital market where the various countries enter weighted by the ratio of the size of their GDP to the world GDP. In Table 6 we present the estimated impact coefficients from the world real factor and the GDP ratio form Barro and Sala-i-Martin (1990). For each we also give the rank (from highest to lowest). These ranks display a very close correspondence. The U.S. is first according to both measures, with the rankings of Japan and Germany reversed. Japan is ranked third in terms of the estimated impact coefficient but second using the GDP ratios. For the other countries that are in both studies the rankings are identical.

Variance Decompositions

Given the orthogonal decomposition of the real rate into four factors and using the estimated parameters from the Kalman filter, we can decompose the variance of the real rates into that accruing to each of the factors:

$$\hat{V}[r_{it}] = \hat{\phi}_i^2 \left[\frac{1}{1-\hat{\rho}_r^2}\right] + \hat{\delta}_i^2 \left[\frac{1}{1-\hat{\phi}_\pi^2}\right] + \hat{\alpha}_i^2 \left[\frac{\hat{\sigma}^2_{\epsilon_{\pi it}}}{1-(\hat{\rho}^{\pi}_{\eta i})^2}\right] + \left[\frac{\hat{\sigma}^2_{\epsilon_{rit}}}{1-(\hat{\rho}^2_{\epsilon^r_{\eta i}})}\right].$$

Table 7 (Panel a) presents the results for the variance decompositions together with their standard errors (obtained from the Delta method) for the real interest rates. We see that the countries that have the largest amount of variation attributable to the world real interest rate factor are the U.S., Germany and Japan. The U.S. is the only country in excess of 50% of the variation in its real rate coming from the world real interest rate factor. The U.K. and France are the two countries that have the greatest amount of variation attributable to the world inflation factor, while the Netherlands is the only country with greater than 10% of the variation in real rates coming from its idiosyncratic inflation factor. All of the countries, except the U.S., can attribute in excess of 50% of the variation in their real rates to idiosyncratic real rate factors. This indicates a potential for policy to influence the variation in real rates of a country. We can also note that while the idiosyncratic inflation factor had a positive effect on the level of the real rates in most countries, we find that it contributes very little to the variation in the real rates, except for Canada and the Netherlands.

In comparing the U.S. and Canada, we see that for the U.S., 50.9% of the variation in the real rate is due to the world real rate factor, 33.5% to the idiosyncratic real rate factor, 15.7% to the world inflation factor and just 2 tenths of one percent from the idiosyncratic inflation factor. The comparable numbers for Canada are 12.9%, 66.0%, 11.2%, and 10.0%. Thus for Canada, while the largest contributing factor to the variation in the real interest rate are domestic real interest rate factors, these other factors can account for sizeable fractions of that variation.

Table 7 (Panel b) presents the results of the variance decomposition of the inflation rates according to:

$$\hat{V}[\pi_{it}] = \hat{\beta}_i^2 \left[\frac{1}{1-\hat{\rho}_\pi^2}\right] + \frac{\hat{\sigma}^2_{\epsilon_{\pi it}}}{1-(\hat{\rho}^{\pi}_{\eta i})^2}.$$

The countries with the largest fraction of variation in the inflation rates from the world inflation factor are France, Germany and the U.S., with 68.5%, 45.8% and 42.3% respectively. Those with the lowest contributions from the world inflation factor are Switzerland and Japan with 8.1% and 20.4% respectively. Except for France, more than 50% of the variation in the inflation rates is attributable to the country-specific inflation factor and again indicates a prominent role played by domestic considerations in determining inflation.

We estimated another dynamic factor model in which we introduced additional region-specific factors (a North American and European factor) that could affect real interest rates in place of the country-specific inflation factors. The equations that are comparable to equations (1) and (2) are:

$$r_{it} = \phi_i r_t^w + \delta_i \pi_t^w + \theta_i r_t^{reg} + \eta_{it}^r \tag{4}$$

and

$$\pi_{it} = \beta_i \pi_t^w + \eta_{it}^\pi. \tag{5}$$

where the region-specific components are indexed by $reg = North\ American,\ European$, respectively. The North American factor comprises Canada and the U.S., and the European factor includes Germany, the U.K., France, the Netherlands, Switzerland, and Belgium.

These results are presented in Table 8 and we see some important changes with regard to Table 7 (Panel a). We note that the fraction of the variation in the Canadian real rate attributable to the North American factor is 64.5%, and the country-specific interest rate factor's contribution is only 23.2%. In Table 7 (Panel a), the country-specific interest rate factor contributed 66%. On the other hand, the results for the U.S. are very similar to those of Table 7. In Table 8 we see that only 3.4% of the variation in the U.S. real interest rate is attributable to the North American component. This seems to indicate that the U.S. has a large influence over the variation in Canadian real interest rates, while Canada has little reciprocal effect on the variation in U.S. real interest rates. The results with the European factor are also enlightening. France and the Netherlands can both attribute in excess of 50% of the variation in their real interest rates to the European component. On the other hand, the U.K. only attributes 2.0% of the variation in its real interest rate to the European factor. The German real interest has only 20.5% of its variation coming from the European component, but now 48.5% of the variation in its real interest rate is attributable to the common world real interest rate factor.

While these regional estimates are interesting, clearly much more work needs to be done to isolate and define appropriate regional decompositions. We think this is a useful first step.

5. Final Remarks

This paper has used a dynamic factor model to decompose international real interest rates and inflation into various world and domestics factors. We have found that there are significant common factors that affect real interest rates and inflation rates, and that the common inflation factor has a negative effect on real interest rates for all the countries in the study. This would lend support to the theory that there is a negative relationship between inflation and real interest rates in an international setting. The results presented also indicate that the source of inflation may matter. Typically we find the effect on real interest rates from domestic inflation factors are positive. This implies that if common world inflation is the prevailing source of inflation variation then we would see a negative relationship between inflation and real rates. While if domestic inflation is the prevailing source, we would see a positive relationship between inflation and real rates. With regard to the inflation and growth debate the results also indicate that if world inflation factors are predominant we should expect to see a positive relationship between inflation and growth. If domestic inflation factors are the more prominent we should expect to see a negative relationship between inflation and growth. We believe these difference arising from the source of inflation are worthy of further study both theoretically and empirically.

We have also illustrated that there are significant common factors that are affecting international real interest rates. This supports Blanchard and Summers (1984) argument that interest rates are to some extent determined in international financial markets. Despite these common components we have found predominant domestic influences. The exception is the U.S. This would suggest domestic factors, perhaps through monetary policy initiatives, can have direct and large effects on domestic real interest rates.

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Table 1: Cross-Country Correlations

Panel a: Cross Country Correlations of Real Interest Rates

	*World**	*Canada*	*U.S.*	*Japan*	*France*	*Germany*	*U.K.*	*Netherlands*	*Switzerland*
$\rho(r_i, r_j)$									
Canada	.413								
U.S.	.783	.469							
Japan	.414	.244	.342						
France	.476	.402	.499	.303					
Germany	.579	.309	.514	.352	.311				
U.K.	.375	.164	.380	.331	.395	.431			
Netherlands	.231	.191	.135	.351	.272	.391	.188		
Switzerland	.199	.208	.271	.155	.210	.406	.201	.096	
Belgium	.305	.138	.299	.242	.205	.476	.166	.322	.197

Notes:

The contemporaneous correlation of the real interest rate in country i with the real interest rate in country j is denoted $\rho(r_i, r_j)$. When "country j" refers to "*World*" the correlation is that of the real interest rate in country i with the world common real interest factor.

Panel b: First-Order Autocorrelations of Real Interest Rates

*World**	*Canada*	*U.S.*	*Japan*	*France*	*Germany*	*U.K.*	*Netherlands*	*Switzerland*	*Belgium*
.963	.368	.703	.106	.630	.528	.378	.134	.357	.302

Panel c: First-Order Autocorrelations of Inflation Rates

*World**	*Canada*	*U.S.*	*Japan*	*France*	*Germany*	*U.K.*	*Netherlands*	*Switzerland*	*Belgium*
.970	.357	.747	.075	.788	.523	.483	.210	.332	.423

Notes:

The first-order coefficients for the World Real Interest Factor, and the World Inflation Factor are taken from Table 5 where the standard errors can be found.

Table 2: Unit Root Tests

Panel a: Unit Root Tests of Real Interest Rates

	ADF Test	*Lag*	*ADF 4 Lags*	*Phillips-Perron* (τ)
Canada	-1.129	15	-2.903	-10.4078
U.S.	-1.972	24	-2.737	-5.8091
Japan	-1.884	23	-4.707	-11.9873
France	-2.068	5	-3.602	-7.0801
Germany	-1.755	23	-4.887	-7.7959
U.K.	-1.633	23	-4.734	-10.4956
Netherlands	-2.391	11	-5.097	-10.5736
Switzerland	-2.249	13	-5.235	-10.4558
Belgium	-1.881	17	-4.949	-10.7807

Panel b: Unit Root Tests of Inflation Rates

	ADF Test	*Lag*	*ADF 4 Lags*	*Phillips-Perron* (τ)
Canada	-1.459	18	-2.510	-10.3998
U.S.	-1.621	20	-3.045	-5.3426
Japan	-2.164	23	-5.081	-12.5503
France	-0.373	17	-2.220	-5.5369
Germany	-1.178	23	-4.707	-7.9946
U.K.	-1.639	23	-4.092	-9.7851
Netherlands	-2.055	13	-3.841	-10.1604
Switzerland	-1.961	13	-5.010	-10.8711
Belgium	−1.544	15	-3.099	-9.8475

Notes:

The critical values for the above tests at the 1%, 5% and 10% significance levels are -3.43, -2.86 and−2.57 respectively. The number of lags for the ADF test were selected using a general to specific testing procedure suggested by Campbell and Perron (1993). Four lags were included in the nonparametric correction for the Phillips-Perron tests.

Table 3: Regressions of Real Rates on Inflation

	$\hat{\theta}$	$se(\hat{\theta})$	*Student t*	*Q(36)*	*DW*	R^2
Canada	-.0692	.076	-0.905	198.229	1.532	0.0047
U.S.	-.3531	.067	-5.246	509.840	0.995	0.1359
Japan	-.0379	.075	-0.507	227.595	1.843	.0015
France	-.3143	.076	-4.147	243.199	1.130	.0895
Germany	-.0231	.075	-3.093	170.486	1.254	.0518
U.K.	-.2216	.069	-3.211	100.183	1.991	.0556
Netherlands	-.1397	.0742	-1.883	190.313	1.805	.0199
Switzerland	-.0901	.079	-1.146	67.491	1.633	0.0074
Belgium	-.0870	.073	-1.889	56.788	1.619	0.0080

Notes:

The results are based upon the estimated equation:

$$r_{jt} = \alpha + \theta\pi_{jt} + u_{jt}$$

Student t is the t-test for the null hypothesis $\theta = 0$ in each equation. Q(36) is a portmanteau $\chi^P 2$ test for residual autocorrelation including 36 lags, and a 5% critical value of 50.998. DW is the Durbin-Watson statistic, and R^2 is the coefficient of determination.

Table 4: Impact Coefficients

	Interest Rate Factors: World Real $\hat{\phi}_i$	Interest Rate Factors: World Inflation $\hat{\delta}_i$	Interest Rate Factors: Country Inflation $\hat{\alpha}_i$	Inflation Rate Factors: World Inflation $\hat{\beta}_i$
Canada	.108	-.091	.439	.150
	(.038)	(.037)	(.070)	(.026)
U.S.	.207	-.104	.017	.160
	(.047)	(.045)	(.076)	(.034)
Japan	.136	-.104	.293	.111
	(.033)	(.033)	(.071)	(.022)
France	.128	-.137	.200	.204
	(.037)	(.036)	(.108)	(.032)
Germany	.161	-.114	.246	.168
	(.040)	(.040)	(.075)	(.029)
U.K.	.111	-.127	.007	.129
	(.031)	(.030)	(.076)	(.029)
Netherlands	.080	-.096	0.527	.150
	(.033)	(.032)	(.074)	(.025)
Switzerland	.058	-.074	0.153	.070
	(.033)	(.032)	(.073)	(.027)
Belgium	.009	-.064	0.202	.148
	(.034)	(.033)	(.085)	(.029)

Notes:

The following equations are estimated:

$$r_{it} = \phi_i r_t^w + \delta_i \pi_t^w + \alpha_i \eta_{it}^\pi + \eta_{it}^r \qquad r_t^w = \rho_r r_{t-1}^w + \epsilon_t^r$$

$$\pi_{it} = \beta_i \pi_t^w + \eta_{it}^\pi \qquad \pi_t = \rho_\pi \pi_{t-1}^w + \epsilon_t^\pi$$

$$\eta_{it}^k = \rho_\eta^k \eta_{it-1}^k + \epsilon_{kit}$$

The nine countries are indexed by $i = 1, ..., 9$. Interest rates, and inflation are indexed by $k = r$, π, respectively. Standard errors are in parentheses.

Table 5: Autoregressive Coefficients

	World Factors $\hat{\rho}_k$	*Real Interest Factors* $\hat{\rho}^r_{\eta i}$	*Inflation Factors* $\hat{\rho}^\pi_{\eta i}$
Real Interest	.964 (.02)		
Inflation	.970 (.02)		
Canada		.475 (.07)	.001 (.06)
U.S.		.313 (.10)	.587 (.06)
Japan		.029 (.08)	-.131 (.08)
France		.436 (.07)	.393 (.08)
Germany		.411 (.08)	.179 (.08)
U.K.		.127 (.08)	.307 (.07)
Netherlands		.369 (.07)	-.199 (.080)
Switzerland		.401 (.07)	.292 (.08)
Belgium		.374 (.07)	.200 (.08)

Notes:

The following equations are estimated:

$$r_{it} = \phi_i r^w_t + \delta_i \pi^w_t + \alpha_i \eta^\pi_{it} + \eta^r_{it} \qquad r^w_t = \rho_r r^w_{t-1} + \epsilon^r_t$$

$$\pi_{it} = \beta_i \pi^w_t + \eta^\pi_{it} \qquad \pi_t = \rho_\pi \pi^w_{t-1} + \epsilon^\pi_t$$

$$\eta^k_{it} = \rho^k_{\eta i} \eta^k_{it-1} + \epsilon_{kit}$$

The nine countries are indexed by $i = 1, ..., 9$. Interest rates, and inflation are indexed by $k = r$, π, respectively. Standard errors are in parentheses.

Table 6: Impact Coefficients and GDP Weights

	Impact Coefficients		*GDP Weights*	
Canada	.108	6	.0433	6
	(.038)		(.002)	
U.S.	.207	1	4528	1
	(.047)		(.025)	
Japan	.136	3	.1315	2
	(.033)		(.004)	
France	.128	4	.0815	4
	(.037)		(.004)	
Germany	.161	2	.1002	3
	(.040)		(.004)	
U.K.	.111	5	0806	5
	(.031)		(.008)	
Netherlands	.080	7	.0202	7
	(.033)		(.001)	
Belgium	.009	8	.0147	8
	(.034)		(.000)	

Notes:

The GDP weights are those reported in Barro and Sala-i-Martin (1990). The Impact coefficients are those reported in Table 4. The number beside the impact coefficients and the GDP weights are the respective rankings of the values in the studies. The numbers reported below the impact coefficients and the GDP weights are the standard errors.

Table 7: Variance Decompositions

Panel a: Real Interest Rate Variance Decompositions

	World Factors		*Country Factors*	
	Interest Rates	*Inflation*	*Inflation*	*Interest Rates*
Canada	.129	.112	.100	.660
	(.09)	(.10)	(.04)	(.11)
U.S.	.509	.157	.000	.335
	(.17)	(.16)	(.00)	(.11)
Japan	.214	.154	.058	.574
	(.10)	(.12)	(.03)	(.11)
France	.195	.276	.011	.517
	(.12)	(.17)	(.01)	(.14)
Germany	.287	.177	.027	.508
	(.14)	(.15	(.02)	(.13)
U.K.	.150	.243	.000	.607
	(.09)	(.15)	(.00)	(.13)
Netherlands	.069	.121	.139	.671
	(.06)	(.10)	(.04)	(.10)
Switzerland	.041	.083	.019	.858
	(.05)	(.08)	(.02)	(.09)
Belgium	.092	.061	.024	.824
	(.07)	(.07)	(.02)	(.10)

Notes:

The following equations are estimated:

$$r_{it} = \phi_i r_t^w + \delta_i \pi_t^w + \alpha_i \eta_{it}^\pi + \eta_{it}^r \qquad r_t^w = \rho_r r_{t-1}^w + \epsilon_t^r$$

$$\pi_{it} = \beta_i \pi_t^w + \eta_{it}^\pi \qquad \pi_t = \rho_\pi \pi_{t-1}^w + \epsilon_t^\pi$$

$$\eta_{it}^k = \rho_{\eta i}^k \eta_{it-1}^k + \epsilon_{kit}$$

The nine countries are indexed by $i = 1, \ldots, 9$. Interest rates, and inflation are indexed by $k = r, \pi$, respectively. Standard errors are in parentheses. Variance decompositions are calculated as:

$$V[r_{it}] = \hat{\phi}_i^2 \left[\frac{1}{1-\hat{\rho}_r^2}\right] + \hat{\delta}_i^2 \left[\frac{1}{1-\hat{\rho}_r^2}\right] + \hat{\alpha}_i^2 \left[\frac{\hat{\sigma}_{\epsilon_{\pi it}}^2}{1-(\hat{\rho}_{\eta i}^\pi)^2}\right] + \left[\frac{\hat{\sigma}_{\epsilon_{rit}}^2}{1-(\hat{\rho}_{\eta i}^r)^2}\right].$$

Numbers may not add up to 1.00 due to rounding.

Table 7: Variance Decompositions

Panel b: Inflation Rate Variance Decompositions

	World Factor	*Country Factor*
Canada	.368	.632
	(.15)	(.15)
U.S.	.423	.571
	(.17)	(.17)
Japan	.204	.796
	(.11)	(.11)
France	.685	.315
	(.14)	(.14)
Germany	.458	.542
	(.16)	(.16)
U.K.	.277	.723
	(.14)	(.14)
Netherlands	.372	.629
	(.15)	(.15)
Switzerland	.081	.919
	(.07)	(.07)
Belgium	.356	.644
	(.15)	(.15)

Notes:

The following equations are estimated:

$$r_{it} = \phi_i r_t^w + \delta_i \pi_t^w + \alpha_i \eta_{it}^\pi + \eta_{it}^r \qquad r_t^w = \rho_r r_{t-1}^w + \epsilon_t^r$$

$$\pi_{it} = \beta_i \pi_t^w + \eta_{it}^\pi \qquad \pi_t = \rho_\pi \pi_{t-1}^w + \epsilon_t^\pi$$

$$\eta_{it}^k = \rho_{\eta i}^k \eta_{it-1}^k + \epsilon_{kit}$$

The nine countries are indexed by $i = 1, ..., 9$. Interest rates, and inflation are indexed by $k = r$, π, respectively. Standard errors are in parentheses. Variance decompositions are calculated as:

$$V[\pi_{it}] = \hat{\beta}_i^2 \left[\frac{1}{1-\hat{\rho}_\pi^2}\right] + \left[\frac{\hat{\sigma}^2_{\epsilon_{\pi it}}}{1-(\hat{\rho}_{\eta i}^\pi)^2}\right].$$

Numbers may not add up to 1.00 due to rounding.

Table 8: Real Rate Variance Decompositions

	World Factors		Regional Factors		Country Factors
	Interest Rates	*Inflation*	*North American*	*European*	*Interest Rates*
Canada	.081	.042	.645		.232
	(.07)	(.05)	(.14)		(.08)
U.S.	.521	.092	.034		.352
	(.18)	(.13)	(.04)		(.12)
Japan	.186	.114			.700
	(.10)	(.10)			(.11)
France	.130	.060		.537	.273
	(.07)	(.06)		(.05)	(.04)
Germany	.485	.175		.205	.135
	(.23)	(.16)		(.10)	(.19)
U.K.	.152	.202		.020	.627
	(.09)	(.14)		(.02)	(.13)
Netherlands	.035	.058		.522	.385
	(.03)	(.05)		(.04)	(.04)
Switzerland	.024	.071		.433	.473
	(.03)	(.07)		(.08)	(.07)
Belgium	.084	.110		.124	.683
	(.06)	(.10)		(.08)	(.11)

Notes:

The following equations are estimated:

$$r_{it} = \phi_i r_t^w + \delta_i \pi_t^w + \theta_i r_t^{reg} + \eta_{it}^r \qquad r_t^w = \rho_r r_{t-1}^w + \epsilon_t^r$$

$$\pi_{it} = \beta_i \pi_t^w + \eta_{it}^\pi \qquad \pi_t = \rho_\pi \pi_{t-1}^w + \epsilon_t^\pi$$

$$\eta_{it}^k = \rho_{\eta i}^k \eta_{it-1}^k + \epsilon_{kit} \qquad r_t^{reg} = \rho_{reg} r_{t-1}^{reg} + \epsilon_t^{reg}$$

The nine countries are indexed by $i = 1, ..., 9$. Interest rates, and inflation are indexed by $k = r, \pi$, respectively. Regions are indexed by $reg = na, eur$, respectively. Standard errors are in parentheses. Variance decompositions are calculated as:

$$V[r_{it}] = \hat{\phi}_i^2 \left[\frac{1}{1-\hat{\rho}_r^2}\right] + \hat{\delta}_i^2 \left[\frac{1}{1-\hat{\rho}_r^2}\right] + \hat{\theta}_i^2 \left[\frac{1}{1-(\hat{\rho}_{reg})^2}\right] + \left[\frac{\hat{\sigma}_{\epsilon_{rit}}^2}{1-(\hat{\rho}_{\eta i}^r)^2}\right].$$

Numbers may not add up to 1.00 due to rounding.

Figure 1: Canada

Decomposition of Ex Post Real Rate

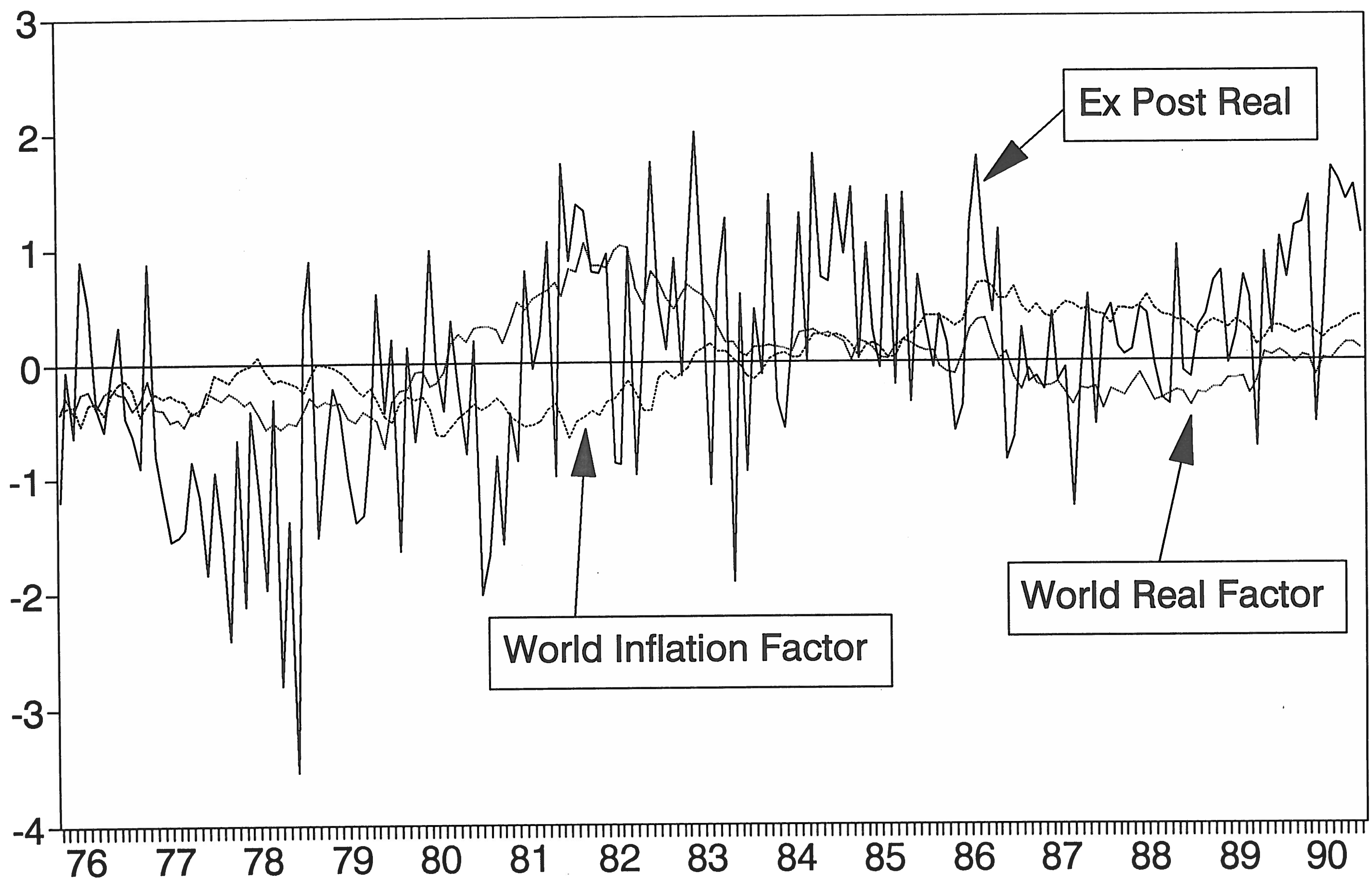

Figure 2: United States
Decomposition of Ex Post Real Rate