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# Roller Coastering Up and Down the Demand Curve of a Durable Goods Monopolist

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**ABSTRACT**

This paper considers a durable goods monopolist who can commit to prices at each date, total output, and possibly release dates for stock. The monopolist faces a finite number of arbitrarily patient consumers. Surprisingly, if the monopolist can commit only to prices and total output, she earns only those profits that a static monopolist would earn. When the monopolist can also commit to release dates for stock, we show how the optimal pricing rule can be characterized by a programming problem. The monopolist sets high prices in odd periods and low prices in even periods, releasing one good in every odd period. Sufficient conditions are determined for the monopolist's total output to exceed that of a static monopolist.

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## 1. Introduction

Since the pioneering work of *Coase* [1972], a number of economists have considered the problem faced by a monopolist selling a durable–good. The standard result in the literature (e.g. *Bulow* [1982], *Gul, Sonnenschein and Wilson* [1986] and *Stokey* [1981]) is that without the power to commit, the monopolist will marginal–cost price and saturate demand almost immediately. This is a direct consequence of intertemporal competition the monopolist engages in with herself.

Relatively little attention has been paid to the issue of commitment, despite the observation of *Sobel and Takahashi* [1983] that endogenizing the cost of commitment might be of interest when analyzing bargaining between a single buyer and seller. In their setting, commitment power was beneficial to the seller, but few other general conclusions could be drawn. In this paper we analyze a durable goods monopolist who can commit to total quantity, prices and possibly the dates at which to make stock available.

Usually when commitment is considered in the context of a durable–goods monopolist, it refers to the ability to credibly set an upper bound on output. An often–cited example is that of an artist who, after making the desired number of prints, publically destroys the lithograph. This output commitment allows the monopolist to price above marginal cost, but without further powers to commit, she may still only levy a single price, depending on the discount factor of the consumers and the monopolist.

*Bagnoli, Salant and Swierzbinski* [1989] derive conditions when there are a finite number of consumers that permit the monopolist to do better than merely setting price equal to the valuation of the marginal consumer. They show that for high enough discount factors there is a subgame perfect equilibrium in which the firm does better by playing a 'Pacman' strategy. The monopolist makes a myopic 'take–it–or–leave–it' offer in each period equal to the valuation of the highest remaining consumer, and consumers adopt a 'get–it–while–you–can' strategy. For discount factors approaching one the profits tend to those of a perfectly discriminating monopolist.

*Al-Najjar* [1993] shows that the optimality of any 'Pacman' strategy is potentially undermined if the monopolist has any uncertainty about consumer valuations: it is no longer necessarily optimal for the monopolist to make a take–it–or–leave–it offer, since with positive probability along the equilibrium path, no sale will be made. Then since the monopolist would anticipate the possibility that no consumer had the conjectured valuation, she will lower her price, and the equilibrium unravels. Such a problem is not encountered in our setting.

In our paper we also allow the monopolist to commit to prices and possibly to quantities of stock shipped at each date, but go out of our way to make it difficult for the monopolist to extract additional profits from these commitment powers. We preclude 'Pacman' pricing strategies by assuming that the monopolist cannot make prices conditional on sales to that point. In particular, the monopolist commits to prices for all future periods at the outset of the game. Further, consumers are assumed perfectly patient. As a consequence the monopolist can only entice consumers to pay higher prices by the threat of rationing.

Despite stacking the deck against the monopolist by assuming perfectly patient consumers, we show that when the monopolist can commit both to a price path and to the dates at which she will make additional stock available, the monopolist can earn greater profits than those she would obtain were she to always charge the static monopoly price. We show how the monopolist's profit maximization problem can be formalized as a simple programming problem, and then characterize its solution. This solution features a "roller–coastering" sequence of prices in which the monopolist successively alternates between importing new stock and setting a high price, and then setting a low price. The effect of this commitment is to convince high–valuation consumers to purchase the good early at higher prices, rather than risk being rationed if they try holding out for a lower price. If a high–valuation consumer fails to purchase at the high price, then there is an over–subscription for the good the next period at the low price, so the high–valuation customer is unlikely to get the good. Indeed, this over–subscription will carry on as he competes with other higher–valuation consumers on future shipments.

We contrast this result with that obtained when the monoplist can only commit to prices and total output; she has to make all stock available at the beginning of the game. We show that this reduced commitment power has a dramatic effect: when the monopolist can only commit to a price at each date, she is unable to extract greater profits than a 'static' monopolist. The threat to ration high valuation consumers loses all bite.

Section two of the paper outlines the model. Section three characterizes the monopolist's actions when she can commit to prices, but has to release all stock in the first period. Section four demonstrates that commitment to prices and shipments of stock allows the monopolist earn greater profits. We characterize the monopolist's actions and derive conditions under which the monopolist will serve at least as many customers as a static monopolist. Section five concludes. All proofs are left to the appendix.

## 2. The Model

There is a single profit–maximizing monopolist who sells a durable good that can be produced at constant marginal cost, $c > 0$. The monopolist can sell the good in any of periods $t = 1, 2, \ldots$. To simplify the notation we refer to prices, $p_t$, as net of the monopolist's marginal cost. Hence, it is possible that the price may be negative, although the gross price is always positive.

Mindful of the results of *Bagnoli, Salant and Swierzbinski* [1989] and *Levine and Pesendorfer* [1992],[1] we assume that the monopolist faces demands from a finite number of positive net reserve valuation consumers. Consumer $\ell$'s valuation of the good, net of the marginal cost $c$, is denoted $v_\ell$. There are $L$ consumers with positive reserve valuations, ordered without loss of generality such that $v_1 > v_2 > \ldots > v_\ell > \ldots > v_L > 0$. It is assumed that a consumer's valuation of a second unit of the good is zero; no–one demands more than one unit of the good. There is also an arbitrarily large pool of individuals with positive gross valuations, but net valuation $v_\infty \leq 0$. The introduction of these consumers allows for simplified expressions, but similar qualitative results could be generated without them. The presence of this pool of consumers with valuation $v_\infty \leq 0$ means that any particular consumer has a zero probability of purchasing the good at a price less than $v_\infty$. Were there not this pool of low–valuation consumers, identical equilibrium outcomes would still obtain if the monopolist could credibly commit to destroying output if necessary.

Denote the initial distribution of net valuations by $\Omega_1$. The monopolist knows this initial distribution of net valuations,[2] but is unable to determine which consumer has which valuation, preventing her from discriminating against a consumer who seeks to make a purchase. Therefore if, in any period $t$, $n$ consumers seek a purchase when there are only $q$ goods available, where $q < n$, the probability any given potential consumer makes a purchase that period is given by $\frac{q}{n}$; the probability is independent of the consumer's valuation. One can think of this probability as reflecting a first–come, first–served approach to sales in any period, where arrival time is randomly determined by nature. Let $\Omega_t$ be the distribution of net valuations of unserved consumers at the beginning of period $t$. We assume that consumers know $\Omega_t$.[3]

Let $p_t$ denote the price charged by the monopolist at each date, $t$, and let $q_t \geq 0$ represent the additional quantity of the durable good the monopolist makes available to the market at period $t$. Once the stock is delivered, it remains available until sold. A strategy for the monopolist is a vector, $S = \{p_t, q_t\}_{t=1}^{\infty}$. If $N = \sum_{t=1}^{\infty} q_t$ is the total amount of output, the restriction that the monopolist must ship all units at the first date amounts to a restriction that $q_1 = N$, and $q_t = 0, t > 1$.

1 These papers show how results may differ significantly when there is a large, but finite number of agents, and when there is an infinite number of agents. Our analysis only holds when there are a finite number of positive valuation consumers because it is not possible in our model to selectively ration agents of measure zero.

2 This is not crucial to the analysis.

3 This does not affect equilibrium outcomes.

The seller and all buyers have lexicographic preferences with regard to time and expected payoff; they always prefer a high expected payoff to a low expected payoff, but if expected payoffs in two periods are the same then an agent prefers receiving the payoff sooner rather than later. So, too, if $p_t \geq v \; \forall t$, then if $p_t = v$, an unfulfilled consumer with valuation $v$ attempts to make a purchase at $t$. In this sense, consumers and the monopolist effectively act as arbitrarily patient, risk–neutral agents.

The monopolist chooses $S$ to maximize profits

$$\Pi = \sum_{t=1}^{\infty} p_t s_t,$$

where $s_t$ represents sales in period $t$, and $\sum_{\tau \leq t} s_\tau \leq \sum_{\tau \leq t} q_\tau$, $t = 1, 2, \ldots$, captures the fact that consumers cannot purchase more goods than are made available.

Given the sequence of prices and shipments to which the monopolist has committed and the past purchases by other consumers, each consumer $\ell$ who has yet to purchase must decide whether to attempt to purchase at each period $t$. A pure strategy for a consumer $\ell$ is a vector of decision rules, $h_t^\ell(S, \delta_t^\ell, \Omega_t) \rightarrow \{0, 1\}$. If an attempt to purchase is made $h_t^\ell = 1$, otherwise $h_t^\ell = 0$. $\delta_t^\ell$ details whether consumer $\ell$ has made a purchase by period $t$. Clearly, if $\delta_t^\ell = 1$ then $\delta_{t+i}^\ell = 1$ and $h_{t+i}^\ell = 0$, $i > 0$. Consumers maximize expected period payoff, where $U_\ell = v_\ell - p_t$ when a purchase is made in period $t$ and is zero if the consumer fails to make a purchase. It makes sense to talk about expected period payoff because for certain consumers there is a positive probability of not being served in the period(s) in which they seek to make a purchase.

## 3. Pricing Commitment Only

In this section we assume that the monopolist must make all stock available in the first period; shipments of the good have to be made at the first date. The monopolist can still commit to a binding pricing strategy, as well as commit to not producing more of the good at a later date. One might think that the monopolist can earn greater profits than those of a static monopolist by probabilistically rationing consumers; a high–valuation consumer might be willing to pay more in the first period, in order to avoid the possibility of rationing in subsequent periods. We now show that this is not the case.

With all goods available from period one, rationing can only occur in one period, and once it has occurred there can be no more sales. Consequently, if the monopolist wants consumer $\ell$ to purchase at a high price to avoid rationing, she has to set the same price for the $\ell - 1$ higher–valuation consumers even though they would have been willing to pay an even higher price. This is because if consumer $\ell$ is going to be served with probability one in period $t$, then there is no incentive for any consumer to pay a price greater than $p_t$ in any period before $t$. Therefore, only in period one can there be purchases at a price higher than $v_N$, where $N$ is the total number of goods the monopolist ships to the market.

In period two, the monopolist has to set $p_2 \leq v_{N+1}$ if she is to make the threat of rationing credible, the necessary condition for consumers to be persuaded to purchase at a high price in period one. The second–period price must be set sufficiently low that any potential consumer not purchasing in the first period faces the possibility of being rationed.

Let $N^*$ be the quantity sold and $\Pi^*$ be the profits earned by a 'static monopolist', a seller who only sells stock in the first period. $N^*$ is the solution to $\max_N \; N v_N$.

**PROPOSITION 1:** In the subgame perfect equilibrium, the monopolist sells $N^*$ goods in the first period. There are no sales in subsequent periods.

An immediate corollary is that the firm earns the same profits as a static monopolist:

**COROLLARY 1:** A monopolist with the powers to set total quantity and commit to a price path makes the same profits as a static monopolist, serving the $N^*$ highest–valuation consumers in the first period at price $v_{N^*}$. At each future date, $p_t \geq v_{N^*}$.

The monopolist is unable to do better than a static monopolist. To do so would require sales in both periods, with a price greater than $v_{N^*}$ in period one. However, profits are a linear function of the proportion of consumers still seeking a purchase in period two. This linearity means that the proportion of those seeking a purchase in period two who are served should either be zero or one. If everyone is to be served in period two, then no–one would buy in period one when $p_1 > p_2$, but we know that the monopolist's lexicographic preferences ensure she would prefer to sell in period one.

If $p_t > v_{N^*}$, $t > 1$, the $N^*$ highest–valuation consumers will pay any price up to, and including, their valuation in period one. Thus, the problem for the monopolist is identical to the one faced by a static monopolist; maximize total net revenue in a single period, $N v_N$.

It also follows immediately that if the monopolist can only commit to quantities in each period, but not prices, she can do no better than a static monopolist. By committing to the static monopoly output level, she can ensure static monopoly profits. However, she can do no better because of the time preferences of consumers, since they will wait until the monopolist reduces price to that which a static monopolist would charge. Thus, all equilibria are (outcome) equivalent to that where the monopolist releases $N^*$ goods in period one and charges $v_{N^*}$ in all periods, and the $N^*$ highest–valuation consumers all buy at date one.

## 4. Choosing When to Release Stock

We now increase the commitment powers of the firm; more specifically, we allow the monopolist to delay the release of stock. However, we assume that the firm cannot withdraw stock from the market once it is made available, and that her commitment to make stock available in a certain period is binding.

Clearly, the monopolist can at least obtain the static monopoly level of profits. We now show the firm can do better by staggering the release of goods, and adopting a high–low pricing strategy, as outlined in the following proposition.

**PROPOSITION 2:** In equilibrium, the monopolist releases one good every other period, charging a high price in the odd periods, when she releases the good, and a price no greater than $v_\infty$ in the even periods. Given the subgame perfect equilibrium output level, $N$, the prices charged in periods $t = 1, 3, \ldots, 2N-3$, are given by

$$P_{2(N-j)-1} = v_N + \sum_{i=1}^{j} 0.5^i (v_{N-i} - v_{N-i+1}) \quad , j = 1, ..., N-1.$$

In period $2N-1$ the price is $v_N$. After period $2N$, the monopolist charges $p_t \geq v_1$. The firm's profit maximization problem can be rewritten as

$$\max_{N} \quad N v_N + \sum_{i=1}^{N-1} 0.5^i (v_{N-i} - v_{N-i+1})(N-i).$$

Along the equilibrium path, consumer $\ell$ seeks a purchase in all even periods, and in all odd periods from period $2\ell-1$ on. She is served in period $2\ell-1$. In equilibrium, the $N$ highest–valuation consumers make a purchase: $\delta^{\ell}_{2N-1} = 1, \ \ell = 1, ...N$.

This result is a consequence of the firm's ability to exploit consumers' fears of being rationed. The pool of $v_\infty$ consumers means that in all even periods, when the price is set low, any unsold goods are almost never purchased by positive net valuation consumers. Consequently, the $L$ positive net–valuation consumers almost always have to purchase the good in odd periods.

By just releasing one good every other period the monopolist takes fullest advantage of her ability to ration consumers. The highest–valuation consumer is willing to pay more to avoid rationing than any other unserved consumer. Therefore, every period the monopolist releases a good, she targets the highest–valuation consumer remaining to be served, setting the greatest price she can such that the consumer does not anticipate a better expected period payoff by delaying seeking a purchase.

Clearly this tactic results in a higher price being charged than if more than one good became available; with more than one good available the firm has to set a price attractive to the lowest–valuation consumer to whom she is selling that period, or allow some stock to sell in the even periods at a price no greater than marginal cost. Since the monopolist has lexicographic preferences, favouring an early sale to a later sale only if the price is at least as large today as could be charged at a later date, she prefers to sell a single good and receive the highest price she can from the highest–valuation consumer remaining. Therefore, she delays sales to other consumers until future odd periods.

The prices charged to each consumer $\ell$ differ as a function of the number of units, $N$, that the monopolist sells. The more units the monopolist sells, the less she can get for each of them. This reflects the intertemporal competition that the monopolist provides against herself. The greater is $N$, the smaller is the probability that a high–valuation consumer who delays a purchase is rationed: she has more opportunities to purchase a good at a later date if she does not purchase in the period the monopolist intends to sell to her in. This offsets the advantage, in terms of additional sales, that accrues from making more goods available.

The optimal number of goods to make available can be calculated by solving the maximization problem detailed in the proposition. The expression depends only on the valuations and $N$, because this is sufficient information to determine the price each consumer pays. This programming problem reflects the fact that the higher are subsequent prices, the more a higher–valuation consumer is willing to pay rather than defer and be stochastically rationed. As when the monopolist must release all stock at date one, it never pays to stochastically ration the last consumer: the linearity of profits in the rationing probability implies a corner solution.

Hence, to calculate the price to charge each consumer if a total of $N$ goods are to be sold, the monopolist solves recursively, calculating the largest price she can charge the $N$–th valuation consumer, and working backwards from there, solving for the price to charge the $(N-1)$–th consumer, and then the $(N-2)$–th, and so on until the appropriate price to charge in each of the $N$ odd periods has been calculated.

Denote the optimal quantity sold by a monopolist who can delay the release of stock and credibly commit to prices by $N^d$, and denote her profits by $\Pi^d$.

**PROPOSITION 3:** For different distributions of consumer valuations it is possible that a monopolist able to commit to price and release stock in separate periods will sell to more than, to less than or to the same number of consumers as she would were she a static monopolist, *i.e.*, $N^d \gtreqless N^*$.

For a static monopolist, selling to $N+j$ consumers, instead of $N$ consumers, yields extra revenue of $jv_{N+j}$ from the additional $j$ consumers served, but the $N$ highest–valuation consumers collectively pay $N(v_N - v_{N+j})$ less. Consequently, if the difference between $v_N$ and $v_{N+j}$ is large enough, the static monopolist will sell $N$ goods instead of $N+j$ goods. Conversely, if $v_N - v_{N+j}$ is small, the static monopolist has an incentive to sell more stock.

In contrast, the monopolist who delays the release of stock and commits to prices will not necessarily want to sell more stock when $v_N - v_{N+j}$ is small. This is because the extra stock reduces the probability that one of the $N$ highest–valuation consumers will not be served if they delay attempting to make a purchase. Because of this, the rent the monopolist is able to extract from consumers concerned about being rationed is reduced.

Therefore, if there is a large group of consumers with similar valuations, it is possible that a monopolist with the ability to commit to price and delay the sale of goods will choose not to sell to these consumers, preferring instead to extract greater rents from consumers with higher valuations. In contrast, because a static monopolist is unable to discriminate when selling to the highest valuation consumers, she faces a lower opportunity cost, in terms of the revenue she has to forego from the highest–valuation consumers, if she does sell to the group of consumers with a similar valaution.

Returning to the case when $(v_N - v_{N+j})$ is large, the static monopolist's inability to price discriminate is the reason for her to favour selling $N$. However, a firm that extracts rents by delaying the release of stock may decide to sell the extra $j$ goods. The monopolist need not reduce the prices she would charge the top $N$ valuation consumers by the margin $(v_N - v_{N+j})$. Hence, she may prefer to sell the extra $j$ goods.

Proposition three demonstrated that it is necessary to know something about the distribution of consumers' valuations before comparing $N^*$ with $N^d$. Proposition four details sufficient conditions for $N^d \geq N^*$.

**PROPOSITION 4:** There exists a $\sigma > 0$ such that, if $(v_i - v_{i+1}) - (v_{i-1} - v_i) < \sigma, \quad i \leq N^*$, then the monopolist able to commit to price and delay the release of stock serves at least as many consumers as the static monopolist. A special case for which this is true is when valuations decline at a constant rate, *i.e.*, $v_i - v_{i+1} = k \quad \forall i$.

The quantity a static monopolist sells, $N^*$, maximizes $Nv_N$, a term that also appears in the maximization problem of the monopolist able to commit to price and delay the release of stock. The additional term in the durable–goods monopolist's payoffs, $\sum^N 0.5^i(v_{N-i} - v_{N-i+1})(N-i)$, captures those profits that accrue because of her ability to price discriminate. As $N$ increases, the price a consumer is willing to pay to purchase immediately falls, because the danger of being rationed is less. However, there are more consumers for the monopolist to price discriminate against, so the profits resulting from price discrimination may rise or fall as $N$ increases.

The more positive net–valuation consumers served, the more efficient is the outcome in the sense that total surplus is increased. Therefore proposition three offers conditions that ensure a monopolist with the ability to commit to prices, total output and release dates is more efficient than a static monopolist. If we were to link the consumer demands to form a 'demand curve,' we find the discriminating monopolist is more efficient so long as the curve is not too convex; concave and linear demand curves satisfy this requirement.

While the durable–goods monopolist who can commit may sell either more or less than the static monopolist, there is an unconditional characterization of the relationship between $\Pi^*$ and $\Pi^d$.

**PROPOSITION 5:** Whenever more than one consumer is served, the profits of the monopolist able to commit to price and delay the release of stock strictly exceed those she would earn were she a static monopolist. When only one consumer is served, profits are the same.

This result just reflects the fact that the monopolist able to commit to prices and delay the release of stock can price discriminate. Of course, when the optimal number of stock to release is one, no discrimination occurs: the price charged is $v_1$ (followed by $p_t \geq v_1$), the same as the price charged by a static monopolist. This is the only case where the two types of monopolists earn the same profit.

More generally, when we drop the assumption of an arbitrarily large pool of $v_\infty$ consumers (and the monopolist cannot destroy unsold output), the monopolist with the ability to commit to prices and delay the release of stock never earns lower profits than a static monopolist, and may do better if there are a sufficient number of low–valuation consumers. In sharp contrast, without the ability to delay the release of stock the durable–goods monopolist can do no better than a static monopolist.

## 5. Conclusion

This paper analyzes the durable–goods monopolist when the monopolist can commit both to release dates for stock and to prices at each date. We show that if the monopolist can only commit to prices and total output, that she does no better than a static monopolist. A monopolist with the additional ability to commit to stock release earns strictly greater profits than a static monopolist provided she sells more than one unit. Throughout the paper we assume that the monopolist is perfectly patient. However, it is clear that all the results would hold for a range of seller's discount factors close enough to unity. So in contrast to *Wang* [1994], the introduction of commitment makes it possible for a monopolist with a smaller discount factor than the buyer to earn profits greater than those of a static monopolist.

The result that the monopolist who can commit to both prices and stock release dates does better than a static monopolist depends on there being sufficiently many low–valuation consumers who increase the threat of rationing to high–valuation consumers. As this pool of low–valuation consumers shrinks, the monopolist increases the level of intertemporal competition with herself, so that her profits decline. The qualitative features of our analysis extend. However, if there are sufficiently few low–valuation consumers, the power of commitment may not be sufficient for the dynamic durable–goods monopolist to do better than the static monopolist. If the monopolist can commit to destroying output that is not purchased, then it does not matter how small the pool of low–valuation consumers is: the equilibrium outcome is always the one characterized here. In this case, only assumption of a finite number of high–valuation consumers is key: discriminating profits are only possible when there are a finite number of consumers since it is impossible in our economy to selectively ration agents of measure zero.

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## APPENDIX

### Proof of Proposition 1 and Corollary 1

We first argue that the lexicographic preferences of consumers and the monopolist mean that only prices in the first two periods matter. Because the firm prefers revenue today to revenue tomorrow, in equilibrium prices $p_t$ will be set so that there are no periods without sales preceding periods with sales: were there no sales in equilibrium in period $t$ at price $p_t$, and the next sales were at some future date $t+k$ at price $p_{t+k}$, then the monopolist could set the revised price $p_t' = p_{t+k}$, receive the same revenues as before, and receive the revenues earlier. If the firm prices so that she expects to make sales in period $t+1$, but not period $t$, she would be better off bringing all prices from period $t+1$ on forward one period, eliminating the original price she proposed for period $t$. Consequently, it must be that there is no "gap" between sales dates.

An optimizing consumer, given the choice of two prices in two different periods, purchases in the period with the higher price *only* if there is a positive probability of being rationed in the lower–priced period. But this implies that all stock must be sold in at most two periods — were there three periods in which stock were purchased along the equilibrium path, then consumers are not rationed in the first two sale periods because there is stock remaining for the third period. But then they all want to purchase in the lower–price period, and, if the prices are the same, they want to purchase at the earlier date. Since there can be no gaps of time between when stock is sold, it must be that stock is sold only at the first two dates.

Suppose the monopolist decides to sell $N$ goods in period one, and wants to determine $Q$, the number of goods to sell in period two, and prices $p_1$ and $p_2$. Let $v_{N+J}$ be the valuation of the marginal consumer who seeks a purchase in period two, so $p_2 = v_{N+J}$. The valuation of the marginal consumer buying in period one is $v_N$, so

$$v_N - p_1 = \frac{Q+1}{J+1}(v_N - v_{N+J}).$$

Therefore, the problem for the monopolist is to maximize

$$Np_1 + Qp_2,$$

where $Q \leq J$. Re–arranging, the objective function of the monopolist becomes:

$$N\left[v_N - \frac{Q+1}{J+1}(v_N - v_{N+J})\right] + Qv_{N+J}.$$

Since the function is linear in $Q$, the optimum number of second–period sales is either $Q = 0$ or $Q = J$. However, if $Q = 0$ then the monopolist's problem is maximize $Nv_N$, while if $Q = J$, the problem reduces to maximize $(N + J)v_{N+J}$. Both problems are identical to the profit–maximizing problem for the static monopolist.

The lexicographic preferences of the monopolist then ensure that all sales occur in period one at $v_{N^*}$, with future prices merely set to force consumers to buy in period one. ■

**Proof of Proposition 2.**

The lexicographic preferences of the monopolist ensure that she will not choose prices such that there is more than one period between sales. Hence, $p_t \leq v_1$.

We first verify that given the prices specified in the proposition, consumers optimize. We then show that the monopolist cannot earn greater profits with some other sequence of prices and shipments.

Along any path where no low–valuation consumer $j$ with valuation $v_j > v_{N+1}$ purchases before period $2j-1$, consumer $j$'s strategy specifies that he attempt to buy in any even period at a price of $v_\infty$, and that he seek to buy in all odd periods $2j-1, 2j+1, ...2N-1$, until successful.

Since price is never lower than $v_\infty$, all consumers seek a purchase in an even period until they have been served, but positive–valuation consumers are almost surely not served in even periods.

Clearly, $h_t^\ell = 0$ for all $p_t > v_\ell$, so consumers $N+1, ..., L$ only seek a purchase in even periods, and the only odd period ($t \leq 2N$) in which consumer $N$ seeks a purchase is period $2N-1$. The stategy outlined for consumer $N-1$ is also optimal. The payoff from seeking a purchase in $2N-3$,

$$v_{N-1} - p_{2N-3} = \frac{1}{2}v_{N-1} - v_N,$$

the payoff from not seeking a purchase in an odd period until $2N-1$. The payoff from seeking a purchase in an odd period prior to $2N-3$, given the equilibrium strategies of all other players, is no greater than

$$q(v_{N-1} - p_{2N-5}) + (1-q)(v_{N-1} - p_{2N-3}),$$

where $q$ is the probability $\delta_{2N-2}^{2N-1} = 1$. Since $p_{2N-5} > p_{2N-3}$ the consumer should not seek a purchase in odd periods prior to $2N-3$. Lexicographic preferences imply the consumer does not defer seeking a purchase in an odd period until $2N-1$. Working recursively, we can show that no consumer has an incentive to deviate in equilibrium from the strategies detailed in the proposition.

Now consider what the optimal response of a consumer should be if other consumer(s) deviate from their equilibrium strategy. Clearly for consumers with valuation $v_\ell < v_N$, they optimally continue to only seek a purchase in even periods and are almost surely not served. Consumer $N$ also has no incentive to deviate from her equilibrium strategy, since it is never an optimal to purchase when $v_\ell < p_t$.

For consumer $N-1$, suppose $J$ higher–valuation consumers were not served in the period they were supposed to be in equilibrium, and that the units were purchased by agents with valuations $v < v_N$. Then the payoff to consumer $N-1$ if she continues to attempt to purchase in $2N-3$ is

$$\begin{aligned}
&\frac{1}{J+1}(v_{N-1} - p_{2N-3}) + \frac{J}{J+1}\frac{1}{J+1}(v_{N-1} - p_{2N-1}) \\
&= \frac{1}{J+1}\left[\frac{1}{2}(v_{N-1} - p_{2N-1}) + \frac{J}{J+1}\frac{1}{J+1}(v_{N-1} - p_{2N-1})\right] \\
&= \frac{1}{J+1}\left[\frac{3J+1}{2J+2}(v_{N-1} - p_{2N-1})\right] \\
&> \frac{1}{J+1}(v_{N-1} - p_{2N-1}),
\end{aligned}$$

which is the payoff to waiting until $2N-2$ before seeking a purchase. Hence, the payoff from seeking a purchase in odd period $2N-3$ is at least as great as the payoff from waiting until $2N-1$ before first seeking a purchase.

Consider now the expected payoff to $2N-3$ to seeking a purchase in an odd period prior to $2N-3$. Let $q$ denote the probability that consumer is served prior to $2N-3$. The lowest odd period price prior to $2N-3$ is $p_{2N-5}$. Therefore the payoff to seeking a purchase earlier than $2N-3$ can be no greater than

$$q(v_{N-1} - p_{2N-5}) + (1-q)\frac{3J+1}{2J+2}(v_{N-1} - p_{2N-1}).$$

Since $(v_{N-1} - p_{2N-5}) < (v_{N-1} - p_{2N-1})$ and $\frac{3J+1}{2J+2} \geq 1$ it is not optimal for the $(N-1)$–th consumer to attempt purchases in odd periods prior to $2N-3$ even when $J$ higher–valuation consumers failed to get served in the periods they were supposed to in equilibrium. So too, one can recursively determine that if $J$ higher–valuation consumers were not served in the period they were supposed to be in equilibrium, and the units were purchased by agents with valuations $v < v_N$, a consumer $\ell > 2N-1$ should continue to attempt to purchase in even periods and all periods $2\ell-1, 2\ell...$ until successful.

Now suppose $\delta^k_{2\ell-1} = 1$ for $\ell < k < N$. This is a zero probability event off the path where a high–valuation consumer failed to purchase when he should have. If it did occur one subgame perfect equilibrium would be for all consumers to pursue the same strategy as before, with the exception of consumer $k-1$ who now does not attempt to buy in all odd periods prior to $2k-1$.

Finally, suppose that there are $M > 0$ agents for whom $\delta^k_{2\ell-1} = 1$ for $\ell < k < N$, and $Z > 0$ agents for whom $\delta^J_{2\ell-1} = 1$ for $J > N$. Then one subgame perfect equilibrium set of strategies would be for all consumers to pursue the same strategy as before: consumer $\ell$ attempts to purchase in even periods and odd periods $2\ell-1, 2\ell+1, ...$

We have now specified consumer strategies off the equilibrium path and shown that the strategies are subgame perfect.

Suppose that $N$ has been chosen optimally, but that the pricing strategy is not optimal. Then it must be that at least one consumer would be willing to pay more. Clearly consumer $N$ will not pay more than $v_N$, so only in period $2N-1$ will he seek a purchase with positive probability of being served. Therefore the $N-1$ highest–valuation consumers each have a probability no greater than 0.5 of being served in period $2N-1$.

Consequently consumer $N-1$ is willing to pay no more than $p_{2N-3}$, since at this price she is just indifferent between buying or waiting a period, i.e.

$$v_{N-1} - p_{2N-3} = \pi(v_{N-1} - p_{2N-2}) + (1-\pi)0.5(v_{N-1} - v_N).$$

Re–arranging,

$$p_{2N-3} = 0.5(v_{N-1} + v_N) - \frac{\pi}{2}(v_{N-1} - v_N) + \pi(v_{N-1} - p_{2N-2}).$$

Notice that $p_{2N-2}$ cannot exceed $v_N$, because if it were then either $p_{2N-3}$ or $p_{2N-2}$ would be redundant. Therefore $p_{2N-3}$ is maximized when $\pi = 0$, which is achieved by setting $p_{2N-2} = v_\infty$, and corresponds to the price stated in the proposition.

Working back inductively, similar reasoning shows that the first $N-2$ odd period prices stated in the proposition correspond to the highest prices the $N-2$ higher–valuation consumers are willing to pay, and that to set prices this high requires that even period prices be no greater than $v_\infty$. Since, in equilibrium, a single sale is made in each of the odd periods, summing over the prices in the odd periods from 1 to $2N-1$ gives the firm's profit maximization problem as a function of $N$ and $\Omega_t$. This assumes that the prices charged later consumers should not be reduced, i.e in equilibrium none of the $N$ highest–valuation consumers are probabilistically rationed.

For a given $N$, suppose the firm were to probabilistically ration at the final date with positive sales. Set price $p_{2N-1}$ such that $v_{N-1+J} \geq p_{2N-1}$, *some* $J > 0$. Suppose that in period $2N-1$, $Q \in \{0, ..., J\}$ units arrive for sale. Along the equilibrium path the $N-1$ highest–valuation consumers have been sold to in the first $2N-3$ dates. Given $Q$ and $p_{2N-1}$ the prices generating the greatest revenues can be calculated inductively as before:

$$p_{2N-3} = v_{N-1} - \frac{Q+1}{J+1}(v_{N-1} - p_{2N-1}), \ etc.$$

Given these prices as a function of $Q$ and $p_{2n-1}$, the monopolist's problem is

$$\max_{p_{2N-1} \leq v_{N-1+J}, Q \in \{0,1,...J\}} \sum_{i=1}^{2N-3} p_i s_1 + Q p_{2N-1}.$$

Observe that fixing $p_{2N-1}$, profits are linear in $Q$, implying $Q = 0$ or $J$. Therefore the assumption of no probabilistic rationing is correct. ■

**Proof of Proposition 3.**

This is by example.

Suppose there are two consumers with valuations 50 and 20, and an infinite pool of zero valuation consumers. Then $N^d = 2$ (goods are released in periods one and three and sell at prices $p_1 = 35$ and $p_3 = 20$). In contrast $N^* = 1$ (price is 50.) If the second consumer had a valuation of 30, instead of 20, $N^d = N^* = 2$.

To complete the proof, the following gives an example of when $N^d < N^*$. Suppose two consumers have valuations 60 and 40 respectively, 80 have valuation 1[4] and there is infinite pool of zero valuation consumers. $N^d = 2$ with prices $p_1 = 50, p_2 = 0$, and $p_3 = 40$. In contrast $N^* = 82$ and price is 1. ∎

**Proof of Proposition 4.**

Suppose not. Then there must be some $j > 0$ such that

$$N^* v_{N^*} + \sum_{i=1}^{N^*-1} 0.5^i (v_{N^*-i} - v_{N^*-i+1})(N^* - i)$$
$$< (N^* - j)v_{N^*-j} + \sum_{i=1}^{N^*-j-1} 0.5^i (v_{N^*-j-i} - v_{N^*-j-i+1})(N^* - j - i),$$

where $N^*$ is the quantity that maximizes $Nv_N$. By definition, $N^* v_{N^*} - (N^* - j)v_{N^*-j} \geq 0$. Therefore if the proposition is incorrect it must be that

$$\sum_{i=1}^{N^*-j-1} 0.5^i [(v_{N^*-i} - v_{N^*-i+1})(N^* - i) - (v_{N^*-j-i} - v_{N^*-j-i+1})(N^* - j - i)]$$
$$+ \sum_{i=N^*-j}^{N^*-1} 0.5^i (v_{N^*-i} - v_{N^*-i+1})(N^* - i) < 0.$$

This rearranges to

$$\sum_{i=1}^{N^*-j-1} 0.5^i (N^* - i)[(v_{N^*-i} - v_{N^*-i+1}) - (v_{N^*-j-i} - v_{N^*-j-i+1})]$$
$$+ \sum_{i=1}^{N^*-j-1} 0.5^i (v_{N^*-j-i} - v_{N^*-j-i+1})j + \sum_{i=N^*-j}^{N^*-1} 0.5^i (v_{N^*-i} - v_{N^*-i+1})(N^* - i) < 0.$$

However, by assumption $(v_\ell - v_{l+1}) > 0$. Therefore the only term that can be negative on the left hand side of the inequality is

$$\sum_{i=1}^{N^*-j-1} 0.5^i (N^* - i)[(v_{N^*-i} - v_{N^*-i+1}) - (v_{N^*-j-i} - v_{N^*-j-i+1})],$$

but it must be that if $[(v_{N^*-i} - v_{N^*-i+1}) - (v_{N^*-j-i} - v_{N^*-j-i+1})]$ is small enough the inequality can no longer hold. Hence the supposition at the start of this proof does not hold. ∎

**Proof of Proposition 5.**

Immediate. For any $N$, including $N^*$, the static monopolist's profits are $Nv_N$, which can never exceed $Nv_N + \sum_{i=1}^{N} 0.5^i (N - i)(v_{N-i} - v_{N-i+1})$. Since $\Pi^*(N^*) \leq \Pi^d(N^*)$, it must be true that $\Pi^*(N^*) \leq \Pi^d(N^d)$. ∎

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[4] This is not strictly in keeping with the assumption that $v_\ell > v_{l+1}$, but the example holds if $v_3 = v_{82} + \epsilon$, $\epsilon$ sufficiently small.