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Screening, Bidding, and the Loan Market Tightness

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Abstract

Bank loans are more available and cheaper for new and small businesses in the U.S. in areas with highly concentrated banks than in areas with highly competitive banks. To explain this fact, we analyze banks' decisions to screen the project and their subsequent competition in loan provisions. It is shown that, by increasing a negative informational externality to an informed winner, an increase in the number of banks in the market can reduce banks' screening probability sufficiently, reduce the number of banks that actively compete in loan provisions and increase the expected loan rate. This occurs when the screening cost is not very high, in which case all active bidders are informed. The opposite outcome occurs when the screening cost is high, in which case there are sufficiently many uninformed banks in bidding to attenuate the negative informational externality.

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JEL classifications: G21, D44, L15.

Preliminary; Commetns welcome.

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1. Introduction

Bank loans are more available and cheaper for new and small businesses in the U.S. in areas with highly concentrated banks than in areas with highly competitive banks. This empirical finding by Petersen and Rajan (1995) is paradoxical when viewed with the standard price theory, which predicts that the cost of the loan increases rather than declines with the degree of bank concentration.[1] Since bank loans account for roughly two thirds of the debt of small businesses, it is important to explain why the loan market behaves in such a way. The explanation may also suggest policies that can improve the functioning of the loan market.

One explanation, by Petersen and Rajan (1995), argues that banks in highly concentrated markets expect to use their monopoly power to extract surplus from the firm in the future relationship to compensate for the low profit at the beginning of the relationship. This important insight is supported by their evidence that the average loan rate in a highly concentrated market declines more slowly with firms' age than in a highly competitive market. Despite the plausibility of and the empirical support for this story, there are good reasons to construct an alternative explanation. First, the observed age-pattern of loan rates can be contaminated by differences in firms' survivorship in different markets, making it a less reliable indication for banks' intertemporal trade-off. Second, it is difficult and risky for a bank to forge a relationship with a new and small business. When a business just gets started, there is great uncertainty on whether it will succeed. Since the foreseeable future surplus to be extracted by the lending bank is small from such a business, there is not much room for the bank to provide a lower current loan rate and a greater current loan availability. Rather, it is with a time-tested business that a bank can expect more from the continued relationship and, in exchange for this future benefit, the bank can offer a currently lower loan rate and provide more available loans. Thus, the monopoly power story suggests that the differences in the loan rate and loan availability between a highly concentrated market and a highly competitive market should be more pronounced for the not-so-young firms

[1] The measure of loan market concentration in Petersen and Rajan (1995) is the Herfindahl index, which is the sum of the banks' shares (squared) of deposits in the area.

than for young firms. This is at odds with the evidence in Petersen and Rajan (1995).

These reasons motivate us to construct a complementary theory on why the observed differences between differently concentrated markets can be consistent with rational decisions. Our theory does not rely on any intertemporal trade-off that the bank makes based on its monopoly power. In fact, to abstract from the intertemporal considerations, we deliberately restrict our model to a one-period financing problem. We instead focus on the informational problem that banks encounter in screening a project and consequently bidding on it. In particular, there is one entrepreneur with a project whose quality is unknown to the banks. Each bank decides the probability with which to screen the project with a costly resource. The screening activity yields an inaccurate signal about the true quality of the project. The signal is more likely to be right than wrong. Without knowing other banks' signals, each bank (including those that did not screen) decides whether to submit a sealed bid and how much to bid on the loan rate. The entrepreneur takes the lowest bid and carries out the project to reveal the outcome.

The screening, participation and bidding decisions are affected by the number of banks in the market. It is shown that banks that receive a good signal about the project bid, banks that receive a bad signal do not bid, and (uninformed) banks that do not screen participate in bidding only when each bank screens with a low probability. When there are only a few banks in the market, each bank screens with a high probability and so all active bidders are informed ones who receive good signals about the project. Competition among these banks produces a low loan rate and low loan market tightness (i.e., great loan availability). When the number of banks in the market increases, it makes each bank less likely to screen and hence generates fewer informed bidders. In particular, there is a negative informational externality to an informed winner and such an externality increases with the number of potential bidders. Together with usual increased competition brought about by the increased number of banks, this informational externality reduces the screening probability down below what is needed to offset the increased number of banks. The end result is that the number of active banks in bidding falls, the loan market becomes tighter, and the expected loan rate increases.

That a decrease in bank concentration leads to a higher loan rate and a tighter loan market is not an inevitable outcome in our model. It occurs when the screening cost is not very high. When the screening cost is very high, almost all active bidders are uninformed and bids are sufficiently contaminated by uninformed ones. In this case, the negative informational externality to an informed winner is weak and increases only slightly with the increase in the number of banks. The dominating effect of an increase in the number of banks is to increase the number of uninformed banks in bidding, which reduces the loan market tightness and the average loan rate. Thus, it is possible that a not-so-concentrated market has higher loan rates and less available loans than both a highly concentrated and a highly competitive market.

We propose our theory as a complementary rather than a competing one to that by Petersen and Rajan (1995). By constructing such a theory, we hope to bring two issues to the forefront. First, screening decisions are important factors that determine loan rates and loan availability. This is perhaps an obvious point, given that lending institutions often spend sizable resources, if not more, in screening projects than in other activities such as monitoring. Unfortunately, it is lenders' ex post decisions that have received much more attention in the literature. Second, information revealed through loan market competition is important for explaining the behavior of loan rates and loan market tightness. Loan markets seem to behave competitively when there are either very few uninformed bidders or sufficiently many uninformed bidders. In contrast, when informed and uninformed bidders are moderately mixed, the loan market reveals very little about private information and hence behaves less competitively. The second issue distinguishes our analysis from other models of loan market imperfections, e.g., Stiglitz and Weiss (1981) and Wang and Williamson (1998), that have also examined banks' screening decisions but have not paid attention to the informational contents that the market reveals from bidding.

A natural framework that can be used to examine loan market competition is one in which banks obtain private information about the unknown value of the project through screening before competing in loan provisions. For this reason, our analysis is generally related to the literature on common-value auctions (see McAfee and McMillan (1987) for a survey). The specific informational structure employed here is similar to that in Wang (1991), where bidders' information is coarse, represented by random draws from a discrete space. In contrast to Wang's work, where bidders' information is exogenous and bidders always participate in bidding, we examine banks' decisions on whether to obtain a signal and whether to participate in bidding. These decisions are obviously vital to loan market competition. Some other models, such as Harstad (1990) and Levin and Smith (1994), also examine agents' choices of obtaining a costly signal in common-value auctions. The general feature that our model shares with those models is that a market with a thicker potential supply does not necessarily produce lower prices. Our focus on the loan market and the use of a discrete instead of a continuous signal space clearly differ from theirs. More importantly, these authors assume that paying the cost for obtaining a signal is a precondition for bidding. This is unrealistic for the loan market, where it is difficult to know the effort which a bidding bank has put into screening. Allowing uninformed banks also to bid both achieves better reality and generates novel results.

In the remainder of this paper, Section 2 examines the simple case where banks have exogenous information; Section 3 endogenizes banks' decisions to obtain a costly signal; Section 4 details the effects of the screening cost and the number of banks on loan rates and loan market tightness; Section 5 concludes and the appendixes provide proofs.

2. Bidding with Exogenous Information

2.1. The Environment and the Equilibrium

There are $n \geq 2$ banks, indexed by i, and one entrepreneur. Both the banks and the entrepreneur are risk-neutral, living for one period. At the beginning of the period, the entrepreneur has one project to be financed with an investment normalized to one unit of goods but does not have any internal funds and must resort to outside financing from the banks. If financed, the project yields output at the end of period that can be consumed. Output is publicly observable and depends on the quality of the project. The quality, denoted q, is either good ($q = g$) or bad ($q = b$). Output is $a(q)y$ where $a(q) = 1$ if $q = g$ and 0 if $q = b$.

Banks do not know the true quality of the project. All banks have the following common prior on the quality:

$$q = \begin{cases} g, & \text{with prob. } \alpha \in (0,1); \\ b, & \text{with prob. } 1-\alpha. \end{cases}$$

The entrepreneur may or may not know the true quality of the project. This is not important in the current setting. Since the entrepreneur has no internal funds, under limited liability the entrepreneur always likes to go ahead with the project if financing is obtained. The entrepreneur contacts all banks in the market separately in an attempt to obtain the fund (see Section 5 for a discussion).

Banks may want to screen the project to find information, a signal, about the quality of the project before providing a loan to the entrepreneur. This screening activity is central to our discussion and will be examined in Section 3. To illustrate how information affects bidding and loan rates, in this section we assume that each bank receives a signal exogenously. The signal of bank i, denoted s_i, can be either g (good) or b (bad). Conditional on the true quality of the project, different banks' signals are independent draws from the same distribution:

$$s|q = \begin{cases} q, & \text{with prob. } \gamma \in (1/2,1); \\ q' \neq q, & \text{with prob. } 1-\gamma. \end{cases}$$

That is, with probability γ the signal is right and with probability $1-\gamma$ the signal is wrong. The information is thus not accurate but, since $\gamma > 1/2$, the signal is more likely to be right than wrong. The above conditional distribution of the signal has the monotone likelihood ratio property or, according to Milgrom and Weber (1982), the signal and the true quality are affiliated.[2] The discrete signal space not only simplifies the analysis but also reflects the reality that information is usually coarse.

Although all banks are ex ante identical, a bank is called a bank G if it receives signal g and bank B if it receives signal b. A bank does not observe other banks' signals. After receiving

[2]A density f has the monotone likelihood ratio property if for all $s' > s$ and $q' > q$, $f(s'|q')/f(s|q') > f(s'|q)/f(s|q)$. In the current case, the ranking is such that $g > b$ and so the property is equivalent to $\gamma > 1/2$.

the signal, each bank submits a sealed bid to the entrepreneur. There is no cost for submitting a bid.[3] A bid specifies the percentage of output, denoted r, that the entrepreneur gives to the bank if the project is successful. If the project is not successful, nothing can be given to the bank. Because of limited liability, a bank cannot ask the entrepreneur to give more than what the project yields and so a bid is *feasible* if and only if $r \leq 1$. The loan rate implied by r is $ry - 1$ and, for brevity, we will simply refer to r as the loan rate. A winning bid r generates a profit $a(q)yr - 1$ to the bank and $(1 - r)a(q)y$ to the entrepreneur. Clearly, the entrepreneur will choose the lowest bid (the most aggressive bid). If there are two or more identical bids that are the lowest, one is chosen randomly with equal probability. Given the signal s, a bank chooses a bid so as to maximize the expected profit, which is denoted $m_s(r)$.

The above loan competition among banks is a common-value, first-price auction with sealed bids, where "first price" means that the lowest (most aggressive) loan rate wins. Different from most common-value auction models, here both the value of the project and the signals are distributed in discrete spaces. This type of auction has been analyzed by Wang (1991) and we adapt his results for the current context. We will consider only symmetric equilibria in which banks with the same signal bid with the same strategy.

First, a bank G bids lower (more aggressively) than a bank B. This is because receiving signal g makes the expected prospect of the project higher than receiving signal b; if a bank B's bid makes a non-negative profit, a bank G can always increase the winning probability and profit by bidding lower than banks B. To see this, note that the unconditional (marginal) distribution of a signal is

$$\Pr(s) = \sum_{q=g,b} \Pr(s|q) \cdot \Pr(q) = \begin{cases} \gamma\alpha + (1-\gamma)(1-\alpha), & \text{if } s = g; \\ (1-\gamma)\alpha + \gamma(1-\alpha), & \text{if } s = b. \end{cases}$$

By Bayes' rule, the posterior for the project's success after observing s alone is

$$\Pr(q = g|s) = \begin{cases} \frac{\gamma\alpha}{\gamma\alpha + (1-\gamma)(1-\alpha)}, & \text{if } s = g; \\ \\ \frac{(1-\gamma)\alpha}{(1-\gamma)\alpha + \gamma(1-\alpha)}, & \text{if } s = b. \end{cases} \tag{2.1}$$

[3]The results are robust to the introduction of a small cost of submitting a bid, see footnote 4.

The posterior for $q = b$ can be calculated similarly. Because $\gamma > 1/2$, banks G indeed have a more optimistic assessment on the project than banks B:

$$\Pr(q = g|s = g) > \alpha > \Pr(q = g|s = b).$$

Also, banks' signals are unconditionally dependent on each other, although they are independent conditional on the true quality. In particular, conditional on that bank 1 receives a signal, the probability for bank 2 to receive the same signal is higher than the unconditional probability.

Second, winning convenes information (Wilson (1977)) and so a bank may suffer from the so-called "winner's curse" if it bids according to its own signal alone, irrespective of the informational content of winning. In particular, if a bank, say bank 1, receives signal b and bids according to this signal alone, the bid that makes a zero expected profit is r_1 such that:

$$r_1 y \cdot \Pr(q = g|s_1 = b) = 1.$$

If this bid wins, the bank realizes that all other banks must have received signal b, since the bank would not have won if there were any bank G. Thus, after winning, bank 1's information is $I_1 = \{s_1 = ... = s_n = b\}$ and the expectation on the project's success will be much lower than the original estimation based on s_1 alone. That is, $\Pr(q = g|I_1) < \Pr(q = g|s_1 = b)$. The bank's expected profit conditional on I_1 is negative under r_1. Anticipating the informational content revealed by winning, a bank will calculate the project's success probability conditioning on both the bank's own signal and the outcome that the bid r wins. Precisely, the expected profit from bidding r with a signal s can be written in the following well-known form:

$$m_s(r) = \Pr(\text{bid } r \text{ wins } |s) \cdot [ry \cdot \Pr(q = g|s; \text{ bid } r \text{ wins}) - 1]. \tag{2.2}$$

This can be rewritten as:[4]

$$\begin{aligned} m_s(r) &= (ry-1)\cdot \Pr(q=g|s)\cdot \Pr(r \text{ wins } |q=g;\, s) \\ &\quad - \Pr(q=b|s)\Pr(r \text{ wins } |q=b;\, s). \end{aligned} \tag{2.3}$$

Third, there is no equilibrium where all banks bid with pure strategies on the loan rate. The reason is that banks G have incentive to under-bid each other, which drives their expected profit to zero; but if the expected profit is indeed zero then any such bank can make a positive expected profit by bidding slightly below the bid of banks B. To see this, suppose that all banks G bid r_g and all banks B bid r_b, where $r_g < r_b$ and $r_g, r_b \in [0,1]$. Clearly, $m_g(r_g) \geq 0$ and $m_b(r_b) \geq 0$. Since a bank G always wins against banks B, it competes only with other banks G. If $m_g(r_g) > 0$, such a bank can lower the bid slightly to increase the winning probability to 1 and hence increase the expected profit. Thus, $m_g(r_g) = 0$. But if $m_g(r_g) = 0$, a bank G can choose to bid $r_b - \varepsilon$, where ε is an arbitrarily small positive number. Although this bid will not win against other banks G, it guarantees winning when all other banks have signal b. Since the latter event occurs with a strictly positive probability, the bid makes a positive profit rather than zero profit.

The only equilibrium in this environment (with symmetry in each type) is such that each bank B bids with pure strategy on a rate r_b and each bank G bids according to a cumulative distribution function (cdf for short) $F(.)$ over a support $[r_L, r_H]$ (see Wang (1991) for a closely related version). A bank B's bid can be determined by examining the bank's expected profit. The bid r_b wins only when all other banks receive signal b, in which case it wins with probability $1/n$. That is,

$$\Pr(r_b \text{ wins } |s_1 = b;\, q) = \frac{1}{n}[\Pr(s=b|q)]^{n-1}.$$

[4] The fact used for the manipulation is that, for any events A, B, C,

$$\Pr(C|B \cap A)\Pr(B|A) = \Pr(C \cap B|A).$$

In particular, for $q^* \in \{g, b\}$,

$$\begin{aligned} &\Pr(\text{bid } r \text{ wins } |s)\Pr(q=q^*|s;\ \text{bid } r \text{ wins}) \\ &= \Pr(q=q^*;\ r \text{ wins } |s) = \Pr(q=q^*|s)\cdot \Pr(r \text{ wins } |q=q^*;\, s). \end{aligned}$$

By (2.3), the expected profit of a bank B is

$$m_b(r_b) = \frac{1/n}{(1-\gamma)\alpha + \gamma(1-\alpha)}[\alpha(yr_b - 1)(1-\gamma)^n - (1-\alpha)\gamma^n]. \quad (2.4)$$

For a bank B to bid, $m_b(r)$ must be positive for the maximum feasible bid $r = 1$, as required by the following assumption (later in this section we will discuss the cases where this assumption is violated):

Assumption 1. *The level y satisfies:*

$$y > 1 + \frac{1-\alpha}{\alpha}\left(\frac{\gamma}{1-\gamma}\right)^n.$$

Under this assumption, all banks have incentive to bid. By submitting a bid close to but lower than 1 a bank cannot do worse than not submitting a bid; the worst it might happen to the bid is that it does not win, in which case the bank gets nothing. The bid r_b cannot make a positive expected profit, either: if it did, a deviation to a slightly lower bid increases the probability of winning from $\frac{1}{n}[\Pr(s=b|q)]^{n-1}$ to $[\Pr(s=b|q)]^{n-1}$ and hence increases the expected profit. Thus, $m_b(r_b) = 0$ and this solves for r_b.

For a bank G, the bid distribution F and the support $[r_L, r_H]$ are found by invoking the mixed-strategy requirement that the expected payoff be the same for all bids in $[r_L, r_H]$. To see how this works, note that the requirement implies that F does not have any mass point over the support: If F had a mass point at r_m, say, then by moving the mass slightly up or down around r_m a bank G could do better than bidding according to F. Since there is a positive mass of bidders at r_b, this argument also shows that $r_H \le r_b$: If $r_H > r_b$, the expected payoff of a bank G would have discrete changes rather than remaining constant when r increases from below r_b to above r_b.

With these properties of F, a bid $r \in [r_L, r_H]$ by bank 1 (with signal g) loses against a randomly chosen competitor if and only if this competitor received a signal g and bid below r, which occurs with probability $\Pr(s=g|q) \cdot F(r)$. Put differently, a bid $r \in [r_L, r_H]$ wins against a randomly chosen competitor with probability $1 - \Pr(s=g|q) \cdot F(r)$. Since there are

$n-1$ competitors and bank 1 wins only when it wins against all $n-1$ competitors, its winning probability with the bid r is

$$\Pr(r \in [r_L, r_H] \text{ wins } | s_1 = g; \, q) = [1 - \Pr(s = g|q) \cdot F(r)]^{n-1}. \tag{2.5}$$

Substituting this and (2.1) into (2.3) yields:

$$m_g(r) = \frac{(ry-1)\alpha\gamma[1-\gamma F(r)]^{n-1} - (1-\alpha)(1-\gamma)[1-(1-\gamma)F(r)]^{n-1}}{\gamma\alpha + (1-\gamma)(1-\alpha)}. \tag{2.6}$$

We have the following proposition (see appendix A for a proof):

Proposition 2.1. *When every bank receives a signal exogenously and y satisfies Assumption 1, the equilibrium is such that every bank with signal b bids r_b and every bank with signal g bids according to a continuous and differentiable cdf $F(.)$ over the support $[r_L, r_H]$ where*

$$r_H = r_b = \frac{1}{y}\left[1 + \frac{1-\alpha}{\alpha}\left(\frac{\gamma}{1-\gamma}\right)^n\right], \tag{2.7}$$

$$r_L = \frac{1}{y}\left[1 + \frac{1-\alpha}{\alpha\gamma}\left(1 - \gamma + \frac{2\gamma-1}{1-\gamma}\gamma^{n-1}\right)\right], \tag{2.8}$$

and the inverse of $F(.)$, denoted by H, is

$$r = H(F) = \frac{1}{y}\left\{1 + \frac{1-\alpha}{\alpha\gamma}\left[\frac{2\gamma-1}{1-\gamma}\left(\frac{\gamma}{1-\gamma F}\right)^{n-1} + (1-\gamma)\left(\frac{1-(1-\gamma)F}{1-\gamma F}\right)^{n-1}\right]\right\}. \tag{2.9}$$

A bank with signal b makes a zero expected profit conditional on his own signal; a bank with signal g makes a positive expected profit conditional on his own signal, which is

$$M_g = \frac{(1-\alpha)\gamma^{n-1}}{\gamma\alpha + (1-\gamma)(1-\alpha)} \cdot \frac{2\gamma-1}{1-\gamma}. \tag{2.10}$$

Some properties of this equilibrium are noteworthy. First, the bid r_b by a bank B makes a positive expected profit when the true quality is g and a loss when the true quality is b; the two sides exactly cancel out and so a bank B makes a zero expected profit overall.[5] In contrast, a

[5]Despite the zero expected profit, the strategy is robust to the introduction of a small cost of participating in bidding. With a small bidding cost, a bank b will participate in bidding with a probability strictly less than 1 and use a mixed strategy to bid over the support $[r_b^*, 1]$, where r_b^* is close to r_b. This strategy earns an expected profit from bidding that exactly covers the bidding cost. When the bidding cost approaches zero, the participation probability approaches one and the cdf of a bank b's bids degenerates into a mass point at r_b.

bank G makes a positive expected profit conditional on its signal, even when the bid is arbitrarily close to but lower than a bank B's bid. This is simply because a bank G's assessment on the project's success is higher than a bank B's and the expected profit is conditional on the bank's own signal. More precisely, when a bank B wins, the bank's assessment is that all n banks have received signal b. When a bank G wins with a bid arbitrarily close to but lower than r_b, the bank's assessment is that $n-1$ banks have received signal b. The latter assessment gives a higher probability for the project's success than the former and so r_b makes a positive expected profit for a bank G, even though it makes a zero expected profit for a bank B. Indeed, when the signal becomes uninformative (i.e., when $\gamma \to 1/2$), the difference between the two assessments vanishes and a bank G's expected profit goes to zero.

Second, r_L is higher than the bid that a bank G would bid if it were known that other banks all had signal g. The latter is

$$\underline{r} = \frac{1}{y}\left[1 + \frac{1-\alpha}{\alpha}\left(\frac{1-\gamma}{\gamma}\right)^n\right].$$

$r_L > \underline{r}$ because a bank does not know other banks' signals and the bid $\underline{r}$ makes a negative expected profit if any other bank has signal b (in which case the posterior for the project's success is lower than that required to make a zero profit under $\underline{r}$). In fact, bidding r_L makes a positive expected profit M_g and it is the supremum among such bids that bidding below them, which guarantees winning, will make an expected profit less than M_g.

Third, the density $F'(r)$ is a decreasing function, which can be verified from (2.9) by showing $H''(F) > 0$. That is, a bank G's bids are concentrated at low bids. This is because a higher winning bid makes a higher profit and so, for the mixing strategy to be rational, the winning probability for a higher bid must be lower in order to make the expected profit equal to those of lower bids.

2.2. Loan Rates and the Number of Bidders

We now examine how the loan rate depends on the number of banks. First, the highest bid, r_b (or r_H), increases with the number of banks. This is because the winner's curse is more severe for banks B when n is larger and so, to minimize the winner's curse, banks B bid more conservatively. To elaborate, recall that r_b is the bid by a bank with signal b that makes a zero expected profit when all other banks also receive signal b. When there are more banks and all of them receive signal b, the posterior for the project's success is lower and so a higher loan rate is necessary for making a zero expected profit.

Second, the expected profit of a bank G, M_g, decreases when the number of banks increases. This result is the net of two conflicting effects of the increased extent of the winner's curse generated by an increase in the number of banks. On the one hand, a larger n pushes up the bid by banks B and so raises the highest bid by a bank G, as explained above. This effect increases a bank G's expected profit. On the other hand, for any given r_b, if a bid $r_b - \varepsilon$ by a bank G wins when there are more bidders then the winner's assessment of the project's success will be lower. This reduces a bank G's expected profit. The second effect dominates because, as explained before, a bank G's expected profit from bidding $r_b - \varepsilon$ relies on the difference between a bank G's and a bank B's assessment of the project's success, which diminishes when n increases.

Third, the lowest bid, r_L, decreases when the number of banks increases. This is because r_L is the lowest bid that can yield M_g as the expected profit and so, when M_g falls with n, a lower bid can make such a profit.

Therefore, it is imprecise to state that increasing the number of banks increases competition in the loan market. Although the lowest bid becomes more aggressive and each bank's expected profit becomes smaller with a larger n, the highest (most pessimistic) bid also increases. As the support of the bids widens on both ends, loan rates do not decrease with the number of competitors in the sense of first-order stochastic dominance. This is also clear from examining (2.9). For any given $F \in [0, 1]$, H does not depend on n monotonically and so its inverse, F, does not depend on n monotonically either.

Given this ambiguity, we can try to determine the influence of n on the expected loan rate. Let R_q be the expected loan rate that an entrepreneur with a quality q project gets, defined as the expected value of $ry - 1$ over the winning bids. Then,

$$R_g = y\left[(1-\gamma)^n r_b + n\gamma \int_{r_L}^{r_b} r[1-\gamma F(r)]^{n-1} dF(r)\right] - 1. \tag{2.11}$$

The first term in [.] is the expected value of the winning bid when r_b wins. The second term in [.] deals with the case where the winning bid is lower than r_b. To explain this term, note that each bid is lower than or equal to a level $r \in [r_L, r_b)$ with probability $\gamma F(r)$ and so, with probability $[1-\gamma F(r)]^n$, there is no bid lower than or equal to r. Thus, the winning bid is lower than or equal to r with probability $1 - [1-\gamma F(r)]^n$, and the second term in (2.11) is obtained with this cdf of the winning bid.

The expected loan rate when $q = b$ can be calculated similarly by replacing γ with $1-\gamma$. Substituting $r = H(F)$ from (2.9), we have:

Proposition 2.2. *Expected loan rates for an entrepreneur with a quality q project are:*

$$R_g = \frac{1-\alpha}{\alpha}\left[1 + \frac{2\gamma - 1}{1-\gamma} n\gamma^{n-1}\right]; \tag{2.12}$$

$$R_b = y\left[\gamma^n r_b + (1-\gamma)n \int_0^1 H(F)[1-(1-\gamma)F]^{n-1} dF\right] - 1. \tag{2.13}$$

The rate R_g increases with n if and only if $\gamma > e^{-1/n}$. The rate R_b increases with n for both $\gamma \to 1/2$ and $\gamma \to 1$.

Proof. Substituting (2.9) for r in (2.11) and integrating yields (2.12). It is clear from (2.12) that R_g increases with n if and only if $n\gamma^{n-1}$ is an increasing function of n, which is equivalent to $\gamma > e^{-1/n}$. A similar substitution into R_b yields (2.13), but the integration cannot be analytically computed. Nevertheless, when $\gamma \to 1/2$ or $\gamma \to 1$, it can be verified that R_b increases with n. ■

The most interesting feature is that the expected loan rate can be higher when there are more banks. The expected loan rate increases with the number of banks if the accuracy of the signal is sufficiently high. This can be explained by recalling that an increase in the number of banks increases the extent of the winner's curse to a bank B. When the accuracy of the signals is sufficiently high but not perfect, the potential winner's curse is sufficiently strong. That is, if information is fairly accurate and yet a non-aggressive bid like r_b wins, it reveals that the project's success must be extremely unlikely and, to be rational, the highest bid must increase significantly with the number of bidders in order to break even for a bank B. The rising highest bid can dominate the falling lowest bid and so the expected loan rate can increase with the thickness of the market. Notice that, in contrast to Levin and Smith (1994), this result emerges here when all banks participate in bidding with probability one.

The critical level of γ for a positive dependence of R_g on n increases with n, suggesting that the dependence can be non-monotonic. In particular, if $\gamma \geq e^{-1/3}$, the dependence of R_g on n will be hump-shaped. Increasing n from 2 increases R_g but, when n passes the level $1/(-\ln \gamma)$, further increases in n reduce the expected loan rate. Therefore, reducing the concentration of banks increases the competition in the sense of reducing the expected loan rate only if the concentration is reduced sufficiently.

The above argument that the expected loan rate may positively depend on n seems to apply as well to the case where the true quality of the project is bad. Unfortunately, the form of the expected loan rate in this case is too complicated to permit a clear-cut analysis. Nevertheless, the rate R_b does increase with n when either $\gamma \to 1/2$ or $\gamma \to 1$.

To conclude this section, we remark on the fragility of the above equilibrium. The equilibrium relies on Assumption 1. When n is large or γ is close to one, the assumption is violated. In this case $r_b > 1$ and there is no feasible bid for banks B to break even. Such banks drop out of bidding. When n and γ are such that

$$y < 1 + \frac{1-\alpha}{\alpha}\left(\frac{\gamma}{1-\gamma}\right)^{n-2}, \tag{2.14}$$

Banks G also drop out of bidding as the expected profit from bidding is negative. The equilibrium described in Proposition 2.1 does not exist. Note that the right-hand side of (2.14) increases very rapidly with n and so the equilibrium will disappear very quickly as n increases.

The existence problem arises because the signals are exogenously (and freely) obtained by banks. When there are many banks, the expected profit becomes negative if every bank G bids with probability one. If signals are costly obtained, instead, banks will choose not to obtain them. The interaction between screening and competition in bidding will be analyzed next. The analysis will also yield predictions about the loan market tightness.

3. Screening and Competitive Bidding

Now consider the case where a cost $c > 0$ must be incurred in order to get a signal and we refer to the action of getting such a costly signal as screening. For simplicity, assume that each bank can get at most one signal. The screening probability is denoted p. Banks that do not screen are called banks U. After screening, banks decide whether to participate in bidding. We assume that screening is an action unobservable by outsiders. This is realistic since an entrepreneur is likely to contact each bank separately for funds. The assumption also simplifies the analysis: Since banks bid without knowing the actual number of banks that have screened, which is a random variable, bids do not directly depend on the realization of this random variable but rather depend on the screening probability. Obviously, screening is *not* a precondition for bidding, since an uninformed bank can always pretend to be informed and participate in bidding. Thus, the screening cost is different from the participation cost in Harstad (1990) and Levin and Smith (1994).

Assumption 2. *The smallest number of banks considered, n_L, is at least 3 and the following condition holds:*

$$\frac{1}{\alpha} < y < 1 + \frac{1-\alpha}{\alpha}\left(\frac{\gamma}{1-\gamma}\right)^{n_L-2}.$$

The part $y > 1/\alpha$ requires that, if all banks are uninformed, there are feasible loan rates to finance a project and yield non-negative profit. The second part of the assumption is imposed for us to focus on banks' non-trivial decisions on the screening probability. The cases where y is sufficiently large to violate the above assumption have already been analyzed in the last section. The restriction $n_L \geq 3$ is made to ensure that the interval for y in the above assumption is non-empty for all $n \geq n_L$.[6]

We will focus on symmetric equilibria where all banks choose the same screening probability and all banks with the same information use the same bidding strategy. Given the upper bound on y, the project may not generate a positive expected profit for a bank B or a bank U if all banks participate in bidding. To be general and for the continuity of the equilibrium, we allow these banks to choose the probability of participating in bidding after observing their own signals. The participation choice and the bidding strategies are summarized below.

Banks' bidding choices			
type	G	U	B
participation probability	1	u	b
cdf of bids	F_g	F_u	F_b
support of cdf	$[r_{gL}, r_{gH}]$	$[r_{uL}, r_{uH}]$	$[r_{bL}, r_{bH}]$

Similar to the argument in the last section, it can be shown that each bid distribution has no mass point over the corresponding support if the support is not a singleton. To find more properties of the bid distributions, let us expand the notation s to include $s = u$, which means that a bank is uninformed. Accordingly, $\Pr(q|s = u) = \Pr(q)$. Let $m_s(r)$ be the expected profit for a bank of type s with a bid r, where $s = g, u, b$. For $s = g, b$, denote

$$Em_s(r) = m_s(r)\Pr(s); \quad EM_s = \int_{r_{sL}}^{r_{sH}} Em_s(r) dF_s(r). \tag{3.1}$$

For $s = u$, denote $Em_u(r) = m_u(r)$. Let us calculate $m_s(r)$ from (2.3) without additional knowledge of the relative location of the supports of the three bid distributions.

First, let us calculate the probability for a bid to win when the true quality of the project is g. For any bid $r \in [0, 1]$, it loses to an arbitrary competitor in three cases: (i) The competitor screened, received a signal g and bid below r, the probability of which is $\gamma p F_g(r)$; (ii) The competitor did not screen, participated in bidding and bid below r, the probability of which is $(1-p)uF_u(r)$; (iii) The competitor screened, received signal b, participated in bidding and bid below r, the probability of which is $(1-\gamma)pbF_b(r)$. Thus, a bid r loses to an arbitrary competitor

[6] We can replace the restriction by $n_L \geq 2$ and change the upper bound for y in the assumption to $1 + \frac{1-\alpha}{\alpha}(\frac{\gamma}{1-\gamma})^{n_L-1}$. This introduces additional cases that do not seem particularly interesting.

with probability $\gamma p F_g(r) + (1-p)uF_u(r) + (1-\gamma)pbF_b(r)$. For the bid to win over $n-1$ potential competitors, the probability is

$$W(r|q=g) \equiv [1 - \gamma p F_g(r) - (1-p)uF_u(r) - (1-\gamma)pbF_b(r)]^{n-1}. \tag{3.2}$$

Similarly, when the true quality of the project is b, the winning probability of a bid r is

$$W(r|q=b) \equiv [1 - (1-\gamma)pF_g(r) - (1-p)uF_u(r) - \gamma pbF_b(r)]^{n-1}. \tag{3.3}$$

Next, $m_s(r)$ can be calculated using (2.3) and so $m_s(r)$ is given as follows:

$$m_s(r) = \boxed{\Pr(q=g|s)}(yr-1)W(r|q=g) - \boxed{\Pr(q=b|s)}W(r|q=b). \tag{3.4}$$

The boxes highlight the terms where $m_s(r)$ differs across s for the same bid r.

The important feature is that the ratio $\Pr(q=g|s)/\Pr(q=b|s)$ is increasing in s, where s is ranked according to $g > u > b$. That is, the assessment of the project's success is higher for a bank G than for a bank U, which in turn is higher than for a bank B. This implies that a bank G's bids are higher than a bank U's, which in turn are higher than a bank B's, as stated below (see Appendix B for a proof).

Lemma 3.1. *$r_{gH} = r_{uL}$ if $u > 0$; $r_{uH} = r_{bL}$ if $b > 0$.*

In fact, under Assumption 2 there is no feasible bid for a bank B to break even and so those banks do not bid. Similarly, it is not feasible for a bank U to participate in bidding with probability one. Uninformed banks participate in bidding with a positive probability when the screening probability p is small enough. Formally, define

$$p_A(n) = \left[\gamma + \frac{2\gamma - 1}{\left(\frac{\alpha(y-1)}{1-\alpha}\right)^{1/(n-1)} - 1}\right]^{-1}. \tag{3.5}$$

Assumption 2 ensures $p_A(n) \in (0,1)$. We have the following proposition (see Appendix C for a proof):

Proposition 3.2. *Assume $p > 0$. Banks B do not participate in bidding. Uninformed banks participate in bidding if and only if $p < p_A(n)$, in which case they participate in bidding with probability $u(p,n) < 1$ and bid according to the cdf F_u over the support $[r_{uL}, 1]$, where*

$$u(p,n) = \frac{1-(p/p_A)}{1-p}, \tag{3.6}$$

$$r_{uL}(p,n) = \frac{1}{y}\left[1 + \frac{1-\alpha}{\alpha}\left(\frac{1-(1-\gamma)p}{1-\gamma p}\right)^{n-1}\right], \tag{3.7}$$

and the inverse of F_u, $H_u(F)$, is given by

$$r = H_u(F) = \frac{1}{y}\left[1 + \frac{1-\alpha}{\alpha}\left(\frac{1-(1-\gamma)p-(1-p)uF}{1-\gamma p-(1-p)uF}\right)^{n-1}\right]. \tag{3.8}$$

Uninformed banks make zero expected profit conditional on their signals. Their participation probability decreases with p and their lowest bid increases with p.

The reasons why a bank U's participation rate decreases with p and why their lowest bid increases with p are similar. When each bank screens with a higher probability, it is more likely that at least one bank receives signal g and hence less likely that an uninformed bank wins. The expected profit of banks U from bidding is lower for any given participation rate and so, to break even, a lower participation rate is necessary. If an uninformed bank wins despite the low likelihood, this bank should rationally believe that the prospect of the project's success is low. Anticipating this, uninformed banks bid more pessimistically when p is higher in order to break even.

If no bank screens, uninformed banks will surely participate in bidding but, if all other banks screen, uninformed banks will not bid. Equilibrium screening probability may exceed or fall short of the critical level $p_A(n)$. To find out, let us examine the bidding decision by a bank G, given that all other banks screen with probability p. Since $b = 0$ and $F_u(r_{gH}) = 0$, the expected payoff to a bank G from bidding $r \in [r_{gL}, r_{gH}]$ is $m_g(r) = EM_g/\Pr(s = g)$, where

$$\begin{aligned} EM_g &= (yr-1)\alpha\gamma[1-\gamma p F_g(r)]^{n-1} - (1-\alpha)(1-\gamma)[1-(1-\gamma)pF_g(r)]^{n-1} \\ &= (yr_{gH}-1)\alpha\gamma(1-\gamma p)^{n-1} - (1-\alpha)(1-\gamma)[1-(1-\gamma)p]^{n-1}. \end{aligned}$$

This can be used to solve for the inverse of the bid distribution F_g. Also, the payoff is an increasing function of r_{gH}. Thus, $r_{gH} = 1$ if banks U do not bid ($p \geq p_A(n)$) and $r_{gH} = r_{uL}$ if banks U bid ($p < p_A(n)$). For a bank G to participate in bidding, the above profit must be non-negative, which requires $p \leq p_H(n)$ where

$$p_H(n) = \left[\gamma + \frac{2\gamma - 1}{\left(\frac{\alpha\gamma(y-1)}{(1-\alpha)(1-\gamma)}\right)^{1/(n-1)} - 1} \right]^{-1}. \tag{3.9}$$

Assumption 2 ensures $p_H(n) \in (0,1)$ and $p_H(n) > p_A(n)$. With this discussion, the following proposition can be established and the proof is omitted:

Proposition 3.3. *Suppose that all other banks screen with probability $p \in [0,1]$. A bank G participates in bidding if and only if $p \leq p_H(n)$. If a bank G bids, the expected profit is $m_g = EM_g / \Pr(s = g)$; the bid distribution is F_g with an inverse H_g; and the support is $[r_{gL}, r_{gH}]$. These characteristics of bids are given as follows:*

$$EM_g(p,n) = \begin{cases} (1-\alpha)(2\gamma-1)[1-(1-\gamma)p]^{n-1}, & \text{if } p < p_A(n) \\ (y-1)\alpha\gamma(1-\gamma p)^{n-1} \\ \quad -(1-\alpha)(1-\gamma)[1-(1-\gamma)p]^{n-1}, & \text{if } p \geq p_A(n), \end{cases} \tag{3.10}$$

$$r_{gH}(p,n) = \begin{cases} r_{uL}(p,n), & \text{if } p < p_A(n), \\ 1, & \text{if } p \geq p_A(n). \end{cases} \tag{3.11}$$

$$r_{gL}(p,n) = \frac{1}{y}\left\{1 + \frac{(1-\alpha)(1-\gamma) + EM_g(p,n)}{\alpha\gamma}\right\}, \tag{3.12}$$

$$r = H_g(F) = \frac{1}{y}\left[1 + \frac{EM_g(p,n)}{\alpha\gamma(1-\gamma pF)^{n-1}} + \frac{(1-\alpha)(1-\gamma)}{\alpha\gamma}\left(\frac{1-(1-\gamma)pF}{1-\gamma pF}\right)^{n-1}\right]. \tag{3.13}$$

The expected profit of a bank G is positive for all $p < p_H(n)$ and is a decreasing function of p.

As before, a bank G makes a positive expected profit because its signal gives a higher assessment of the project's success than other signals do. Also, as every bank increases the screening probability, there are more banks that receive signal g and the profit for each such bank falls.

Now we determine the screening probability. To do so, let us calculate the unconditionally expected profit of a bank that screens (i.e., before the signal is revealed). Since receiving signal

b yields zero expected profit, the unconditionally expected profit is $\Pr(s = g) \cdot m_g = EM_g$. An individual bank's screening probability, say p^*, is the following best response to other banks' decisions:

$$p^*(p,n) \begin{cases} = 1, & \text{if } EM_g(p,n) > c \\ = 0, & \text{if } EM_g(p,n) < c \\ \in [0,1], & \text{if } EM_g(p,n) = c. \end{cases} \tag{3.14}$$

Equilibrium screening probability is such that $p^*(p,n) = p$. Since $EM_g(p,n)$ is a decreasing function of p, equilibrium screening probability is unique, as shown in Figure 1 by point E.

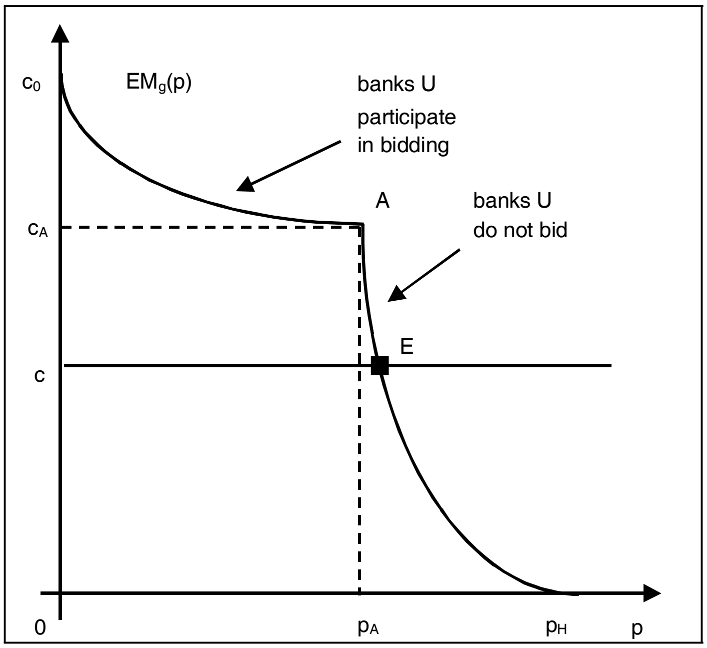

Figure 1 Equilibrium screening probability

Define $c_A = EM_g(p_A(n), n)$ and $c_0 = EM_g(0, n) = (1 - \alpha)(2\gamma - 1)$. Note that $p < p_A(n)$ if and only if $c > c_A$. The following proposition becomes evident.

Proposition 3.4. *Under Assumption 2, a unique equilibrium exists as follows:*

(i) If $c \geq c_0$, then $p = 0$ and all banks bid $1/(\alpha y)$, making zero expected profit;

(ii) If $0 < c < c_0$, then $p \in (0, p_H(n))$ and satisfies $EM_g(p,n) = c$. The screening probability is a decreasing function of the screening cost. Banks G always bid and make a positive expected profit. Uninformed banks bid with probability $u(p,n) > 0$ if and only if $c > c_A$, and make a zero expected profit when they bid. Banks B do not bid. The bids are characterized in Propositions 3.2 and 3.3.

4. Loan Rates and Loan Market Tightness

The effects of the screening cost and the number of banks on the loan market can now be examined. Let us restrict attention to $0 < c < c_0$ and so $p > 0$.

4.1. Definitions

A change in the screening cost or the number of banks affects both the loan rate and the tightness of the loan market. As in Section 2, the effects on loan rates are complicated and so we focus on the expected loan rate conditional when $q = g$. The loan market tightness can be measured by the probability with which the project fails to get financed. Denote the tightness as T_q for a given project quality. A higher value of T_q means a tighter loan market.

To calculate T_q, note that it is the probability that no bid is submitted. There are two cases where a randomly selected bank bids: when it screens and receives signal g, the probability of which is $p \cdot \Pr(s = g|q)$, or when it does not screen and chooses to bid, the probability of which is $(1-p)u$. Thus, a randomly selected bank will bid with probability $p \cdot \Pr(s = g|q) + (1-p)u$. For every bank not to bid, the probability is:

$$\begin{aligned} T_q &= [1 - p \cdot \Pr(s = g|q) - (1-p)u]^n \\ &= \begin{cases} [1 - \gamma p - (1-p)u]^n, & \text{if } q = g \\ [1 - (1-\gamma)p - (1-p)u]^n, & \text{if } q = b. \end{cases} \end{aligned} \tag{4.1}$$

The loan market is tighter for bad projects than for good projects, i.e., $T_b > T_g$, even though banks do not know the project's quality. This is because the signals are more likely to be right than wrong and so screening helps the banks to finance good projects more often than bad ones, although the banks' ability of doing so is limited.

The expected loan rate R_g can be calculated similarly to (2.11). The important difference is that a project now does not always receive a bid here and so, to interpret the expected loan rate as the average of observed rates, it must be calculated conditionally on that at least one bid is received. Let us first consider the case $p < p_A(n)$ and calculate the joint probability with which the project receives at least one bid and the winning bid does not exceed a level $r_1 \in [r_{gL}, r_{gH}]$.

Given $q = g$, a randomly selected bank's bid is lower than or equal to r_1 with probability $\gamma p F_g(r_1)$. Then $[1 - \gamma p F_g(r_1)]^n$ is the probability with which no bid below or equal to r_1 is received and $1 - [1 - \gamma p F_g(r_1)]^n$ is the probability with which the project receives at least one bid below or equal to r_1. Since a project receives at least one bid with probability $1 - T_g$ when $q = g$, the probability for the winning bid not to exceed $r_1 \in [r_{gL}, r_{gH}]$ conditional on receiving at least one bid is

$$\frac{1}{1-T_g}\{1 - [1 - \gamma p F_g(r_1)]^n\}.$$

The similar conditional probability for the winning bid not to exceed $r_1 \in [r_{uL}, 1]$ is

$$\frac{1}{1-T_g}\left\{1 - [1 - \gamma p - (1-p) u F_u(r_1)]^n\right\}.$$

Therefore, the expected loan rate when $q = g$ (defined as the expectation of $yr - 1$ rather than that of r) is

$$\begin{aligned} R_g &= -1 + \frac{y}{1-T_g}\left\{\int_{r_{gL}}^{r_{uL}} r \cdot d\{1 - [1 - \gamma p F_g(r)]^n\}\right. \\ &\quad \left. + \int_{r_{uL}}^{1} r \cdot d\{1 - [1 - \gamma p - (1-p) u F_u(r)]^n\}\right\}. \end{aligned}$$

Substituting $r = H_g(F)$ for the first integral and $r = H_u(F)$ for the second integral, one can integrate to obtain:

$$R_g = \frac{cnp + (1-\alpha)\{1 - [1 - (1-\gamma)p - (1-p)u]^n\}}{\alpha\{1 - [1 - \gamma p - (1-p)u]^n\}} \tag{4.2}$$

If $p \geq p_A(n)$, a similar derivation can be used to show that R_g is given by (4.2) with $u = 0$.

4.2. Effects of a Higher Screening Cost

The expected loan rate and loan market tightness depend on the screening cost as follows (see Appendix D for a proof):

Proposition 4.1. *The loan market tightness is an increasing function of the screening cost when $c < c_A$ and a decreasing function of the screening cost when $c > c_A$. The expected loan rate R_g increases with the screening cost when $c < c^*$, where $c^* \in (c_A, c_0)$, and decreases with the screening cost when c is close to c_0.*

The loan market tightness has a hump-shaped dependence on the screening cost. To explain, let us start from a low screening cost and try to gradually increase it. When the screening cost is small, i.e., $c < c_A$, only banks G bid. The number of such banks falls when the screening cost increases, because the screening probability falls. When c increases passing the level c_A, both banks G and banks U bid. Increasing the screening cost further has two effects on the expected number of bids. First, it increases uninformed banks' participation probability and this effect always increases the number of bids. Second, by reducing the screening probability, the higher screening cost shifts some banks that would otherwise choose to be informed to the uninformed group. Since an uninformed bank bids with probability u and an informed bank bids with probability γ, the shift changes the number of bids by $(u - \gamma)(-dp)$. When u is large, the second effect is also positive and so the expected number of bids increases. When u is small, the second effect is negative but in this case the marginal increase in u is large enough to make the first effect dominate the second, again increasing the expected number of bids.

The dependence of the expected loan rate on the screening cost has a similar hump shape, although the peak of the effect occurs at a higher level $c^* > c_A$. It is easy to understand why the expected loan rate increases with the screening cost when $c < c_A$. In this case, uninformed banks do not bid and so the highest bid by banks G is fixed at 1. The lowest bid must increase with c in order to produce a higher expected profit for banks G to cover the increased screening cost. In fact, since bids are concentrated near low levels, the higher screening cost increases bids in the sense of first-order stochastic dominance. When $c > c_A$, the lowest bid by banks G continues to increase with n. But, since banks U participate in bidding and their bids fall when n increases, the high end of the winning bid distribution gets thinner and the low end moves up, creating the bulge in the middle. The overall effect of a higher n on the expected loan rate is ambiguous. When c is close to c_A, there are few uninformed banks in bidding and so the effect of the rising lowest bid by banks G dominates, generating a rising expected loan rate. When c is close to c_0, almost all banks in bidding are uninformed and so the effect of the falling bids by banks U

dominates, generating a lower expected loan rate.

The effects of the screening cost on the loan market tightness and the loan rate are illustrated in the following example:

Example 4.2. $\alpha = 0.65$, $\gamma = 0.7$, $y = 1.89$, $n = nn \equiv 6$. *These parameters satisfy Assumption 2. In this case, the highest screening cost that induces positive screening is* $c_H = 0.14$. *Figure* $2a$ *depicts the screening probability of each bank and bank* U*'s participation probability; Figure* $2b$ *depicts the loan market tightness and the expected loan rate. As discussed above, when* c *increases,* p *falls,* u *increases and the graphs of* (T_g, R_g) *both have a hump shape.*[7]

These effects of the screening cost show that, for any given number of banks, two economies can be quite different in the screening cost and yet exhibit similar loan market characteristics such as the average loan rate and the loan market tightness. In one economy, the active bidders are all informed and have high valuations of the project. In the other economy, most active bidders are uninformed and whose chance of winning is often spoiled by a few informed bidders. Although these two economies have similar market characteristics, they have opposite responses to policies that reduce the screening cost. With informed bidders (the first economy), the policy reduces the market tightness and lowers the loan rate by increasing the number of informed bidders. With mostly uninformed bidders (the second economy), the policy also increases the number of informed bidders but it has a much greater adverse effect on the number of uninformed bidders, leading to a tighter market and a higher expected loan rate. The economy with informed bidders has more dispersed loan rates than the economy with mostly uninformed bidders.

Therefore, it is ambiguous whether a policy that only reduces the screening cost moderately can improve competition among banks in providing cheaper and more available loans. An unambiguous measure of increased competition when the screening cost is lower is that the expected profit is lower for banks, but this would be difficult to measure empirically. There is a sense, however, that a significant reduction in the screening cost can increase competition by reducing

[7] Although expected loan rates appear large in the figure, they are clearly reasonable if each period is interpreted as 5 years.

the loan rate, since the expected loan rate is lower when the screening cost is close to zero than when the cost is high (see Figure 2*b*).

4.3. Effects of Reducing the Concentration of Banks

We now turn to the influence of the number of banks on the loan rate and the loan market tightness. The following Proposition can be established (see Appendix E for a proof):

Proposition 4.3. *For any fixed $c \in (0, c_0)$, equilibrium screening probability is a decreasing function of the number of banks. The loan market tightness is an increasing function of n if and only if $c < c^{**}$, where $c^{**} \geq c_A$.*

Equilibrium screening probability decreases when n increases because, for any given $p \in [0, p_H]$, the expected screening profit of a bank G is a decreasing function of n. To cover the screening cost, each bank's screening probability must fall when n increases.

The loan market can become tighter when there are more banks, as in the case $c < c_A$. This seemingly paradoxical result arises because the reduction in the screening probability induced by an increase in the number of banks is more than offsetting the increase in the number of banks itself. The dominating intensive effect is a manifestation of the negative informational externality to an informed winner. In particular, when the true quality is bad, fewer banks receive signal g and hence fewer banks bid than when the true quality is good. For any given bid, if it has won it must have done so more often when the true quality is bad than when the quality is good. This rational inference by the winner reduces the expected profit. The increase in the number of banks exacerbates this negative informational externality of winning and reduces a bank G's expected profit beyond the conventional competition effect. This calls for a large reduction in the screening probability that increases the market tightness.

To be more precise, let us consider the case $c < c_A$ and rewrite

$$EM_g(p, n) = (1 - \gamma p)^{n-1} \cdot \alpha\gamma \left\{ (y - 1)\alpha\gamma - \frac{(1 - \alpha)(1 - \gamma)}{\alpha\gamma} \left[\frac{1 - (1 - \gamma)p}{1 - \gamma p} \right]^{n-1} \right\}. \tag{4.3}$$

This is the expected profit of a bank G with a bid $r_{gH} = 1$, which is the same as the expected profit generated by any other bid in the support of F_g. The conventional competition effect of an increased n is captured by the term $(1-\gamma p)^{n-1}$, which is the probability that the bid r_{gH} wins when the true quality is good. An increase in n reduces this winning probability for any given $p > 0$ and hence calls for a reduction in p to cover the screening cost. The informational externality of winning to a bank G is captured by the term $[.]^{n-1}$, which is the relative likelihood of winning when the true project is bad as opposed to when the quality is good. For any given $p > 0$, an increase in n increases this relative likelihood and hence reduces the expected profit. This additional effect calls for a further reduction in the screening probability and an increased loan market tightness.

The negative informational externality to an informed winner is stronger when each bank screens with a higher probability. This is because, as the screening probability increases, the expected number of banks that receive signal g when the true quality is good increases more quickly than the expected number of banks that receive signal g when the true quality is bad. That is, for any given bid, the chance of winning against a randomly selected bank when the true quality is bad rises relative to that when the true quality is good. Thus, the gap between the two is more responsive to changes in n, leading to a stronger negative informational externality. This can be confirmed by showing that the derivative of the term $[.]^{n-1}$ in (4.3) with respect to n is an increasing function of p.

An implication is that the negative informational externality to an informed winner is weaker when uninformed banks also participate in bidding than when they do not, making the screening probability less responsive to further increases in n in the former case. Put differently, the informational content of winning to an informed winner is contaminated by uninformed bids and the degree of contamination increases with n. Since winning by a bank G against uninformed banks does not reveal anything new about the project quality in addition to the bank's own signal and since an increase in n increases the number of uninformed banks in bidding when $u > 0$, an increase in n does not increase the negative informational externality by as much as when $u = 0$. Moreover, the increase in the number of uninformed bidders itself eases the loan market tightness. Therefore, when $c > c_A$, increases in n may not increase the market tightness. In fact, when c is sufficiently large, almost all bidders are uninformed and so the negative informational externality to an informed winner is dominated by other forces described above. In this case, the loan market becomes less tight when n increases.

In comparison to the effect of an increase in the number of banks on the market tightness, the effects on the expected loan rate are more difficult to detail analytically. To illustrate, we give three numerical examples. The three examples differ among themselves in the level of the screening cost but have the same values of (α, γ, y) as in Example 4.2. The lowest value of n is $n_L = 3$, the highest value is $n_H = 15$, and $c_0 = 0.14$.

Example 4.4. *Low screening cost:* $c = cc = 0.056$. *Figure* $3a$ *shows how equilibrium screening probability, the market tightness and the expected loan rate vary with* n*; Figure* $3b$ *shows the distribution of the winning bids for* $n = n_L$ *and* n_H*, denoted* $FWg(\cdot, n, c)$ *for given* (n, c).

In this example, the screening cost is sufficiently low that only banks G bid for all $n \in [n_L, n_H]$. As the number of banks increases, each bank reduces the screening probability and the loan market gets tighter. The lowest bid r_{gL} does not change with n since it is pinned down by the screening cost (setting $EM_g = c$ in (3.12)). The highest bid does not change with n either since it is fixed at 1. However, the density of bids is less concentrated at low bids when n is large since fewer banks bid. Shown in Figure $3b$, the winning bid distribution with a higher n dominates that with a smaller n (see Figure $3b$). The expected loan rate increases (see Figure $3a$).

Example 4.5. *High screening cost:* $c = cc = 0.119$. *Figure* $4a$ *shows how equilibrium screening probability, the participation probability of a bank* U*, the market tightness and the expected loan rate vary with* n*; Figure* $4b$ *shows the distribution of the winning bids for* $n = n_L$ *and* n_H.

In this example, the screening cost is sufficiently high that banks U also participate in bidding for all $n \in [n_L, n_H]$. Again, the increase in the number of banks reduces each bank's screening probability by reducing the expected profit from screening. The increased number of banks also increases the competition among uninformed banks and so the participation rate u falls. Despite this reduction in the participation probability, the total number of uninformed banks in bidding increases as a result of the increased total number of banks, leading to a less tight loan market.

On loan rates, the reduction in the screening probability increases bids by banks G as in the previous example. Shown in Figure $4b$, the distribution of the winning bids submitted by banks G tilts toward higher bids when n is higher. However, bids by uninformed banks decrease as there are more uninformed banks in bidding. The distribution of the winning bids submitted by banks U tilts toward lower bids, producing the bulging shape of the distribution in the middle. The overall effect of the increase in n on the expected loan rate is ambiguous in general but, for the current example, is negative as shown in Figure $4a$. Since the winning bids are more concentrated in the middle with a higher n, the standard deviation of the winning bids is likely to fall.

Example 4.6. *Moderate screening cost:* $c = cc = 0.098$. *Figure* $5a$ *shows how equilibrium screening probability, the participation probability of a bank* U*, the market tightness and the expected loan rate vary with* n*; Figure* $5b$ *shows the distribution of the winning bids for* $n = n_L$ *and* n_H.

In this example, increases in the number of banks change the nature of the equilibrium. For $n < 9$, the screening probability is high, which deters uninformed banks from participating in bidding. In this case, the loan market tightness and the expected loan rate behave very like those in Example 4.4. For $n > 9$, the screening probability is sufficiently low that uninformed banks can participate in bidding without making a loss. For $n > 9$, the loan market becomes less tight with the increase in n, as in Example 4.5. In contrast to Example 4.5, the participation probability by uninformed banks increases with n. Also, the expected loan rate increases with n, because the number of banks U in bidding is not significant. The distribution of the winning bids when n is higher dominates the one when n is lower (see Figure 5b).

Several results emerge from these numerical examples. First, a market with fewer banks can provide both a lower loan rate and a higher availability of loans than a market with more banks, as shown in Figure 3. The result is consistent with empirical findings by Petersen and Rajan (1995). Different from the explanation by Petersen and Rajan (1995), which relies on the bank's monopoly power to extract gains from a continued relationship, our result comes from banks' screening decisions and it is not an inevitable one. Key to our result is the effect of the number of banks on the negative informational externality to an informed winner, which depends on the composition of the bidders. When most bidders are informed, a more concentrated loan market produces a lower average loan rate and more available loans. When there are sufficiently many uninformed bidders in the market, a more concentrated market produces a higher average loan rate and less available loans, as conventional price theory suggests.

Second, in most cases the increase in the number of banks has a greater effect on the market tightness than on the expected loan rate. In Examples 4.4 and 4.5, the market tightness responds to n significantly but the expected loan rate remains almost flat. Only in Example 4.6 and only for n large enough does the increase in n have a sizable effect on the expected loan rate. The small magnitude in the response of the expected loan rate occurs here because, when a bank responds to an increase in n, it changes both the participation probability and the bids; these two have opposite effects on the expected loan rate. This might explain why the degree of bank concentration empirically has a stronger effect on loan availability than on loan rates (Petersan and Rajan (1994)).

Third, a lower expected loan rate does not always indicate a less tight loan market. In Example 4.6 when n increases above 9, the probability of obtaining a loan increases (as T_g falls) but the expected loan rate increases. Despite this disagreement, the two measures are consistent in most cases. They respond to changes in the screening cost in the same way and move in the same direction for either high or low c when n changes.

4.4. Discussion

In this subsection we further discuss the result that increasing the number of banks can reduce loan availability and increase the average loan rate. We do so by comparing this result with the similar one in Petersen and Rajan (1995).

First, as in Petersen and Rajan (1995), the result implies that the "quality" of projects financed in areas with a high bank concentration is lower than in areas with a low concentration, where the meaning of "quality" differs from that in previous sections and refers to the prior probability of project success, α. That is, a combination of a lower α and a smaller n generates the same market tightness as a combination of a higher α and a larger n. This can be shown by establishing that the iso-tightness condition, $T_g =$ constant, generates a positive relationship between n and α when $c < c_A$. Intuitively, when α increases, a bank G's expected profit increases for any given screening probability and so the zero net-profit condition implies that the screening probability must increase. This reduces the market tightness for any given n. Since the tightness is an increasing function of n in this case, to maintain a constant tightness the number of banks must increase.

Second, in contrast to Petersen and Rajan (1995), the positive relationship between n and the market tightness relies on there being sufficiently many informed bidders and hence is not an inevitable prediction of the current theory. Also, for suitable parameters such as the ones in Example 4.6, it is possible that loans are less available in the not-so-competitive market than in both the highly concentrated and the highly competitive markets. This result, although documented in Petersen and Rajan (1995) (pp428-431), is inconsistent with their story that relies on ex post monopoly power. It is consistent with the screening decision here. In the highly concentrated market, every bank screens and so each participates in bidding with a high probability, making loans highly available. In the highly competitive market, almost every bank chooses to be uninformed but it is possible that every bank participates in bidding with a high probability, again making loans highly available.

Third, in contrast to Petersen and Rajan (1995), our result does not rely on the intertemporal trade-off that banks make via their monopoly power. In particular, the project described here yields output once and there is no further stage of financing involved. In this sense the result is more robust and suitable for loan markets that involve new and small businesses, from whom banks cannot expect much from a continued relationship since the firms' expected survivorship is low. Despite the absence of an intertemporal structure, our model can be used to indirectly check whether it is roughly consistent with the empirical finding by Petersen and Rajan (1995) that loan rates decline more slowly with firms' age in a highly concentrated market than in a highly competitive market. This can be done by interpreting an increase in α, the prior probability of the project's success, as a proxy for the firm's age. This interpretation is reasonable since a firm's good quality is increasingly revealed to the public when the firm's survivorship increases. When α increases, our model is capable of producing the age-pattern of loan rates described by Petersen and Rajan (1995) if the increase in α sufficiently increases the participation rate of uninformed banks. This is because increases in the participation rate drive down the average loan rate more quickly in a market with more banks than in a market with fewer banks.[8]

5. Conclusion

The mere presence of more banks in a market does not imply more available and cheaper loans. The active participation of more banks in loan provisions does. Whether a bank chooses to actively compete in providing a loan for a project is determined by the expected profit to banks from such participation. When banks can obtain private information about the project's quality by screening, loan market competition generates a negative informational externality to the winning bank, which can reduce banks' incentive to screen the project and to participate in competition in the first place. This externality arises because, for any given loan rate, a bank's chance of winning is greater when the project's quality is bad than when the quality is good; thus winning

[8] A numerical example is as follows: $\gamma = 0.7$, $y = 2.19$, $c = 0.072$, $n1 = 4$, and $n2 = 10$. As α increases from 0.7 to 0.8, the expected loan rate is first lower with $n1$ firms than with $n2$ firms and then the pattern is reversed.

increases the expectation that the quality is bad.

The informational externality is exacerbated by the increase in the number of banks in the market. Therefore, in a market with more banks, each bank's screening probability can be much lower and the number of active competitors can be smaller, making loans less available and the expected loan rate higher than in a market with just a few banks. This is the case when the screening cost is not very high, in which case each bank's screening probability is high and elastic with respect to changes in the expected profit from competition. In contrast, when the screening cost is high, each bank's screening probability is low which allows uninformed banks to bid as well. In this case, the negative informational externality to an informed winner is weak and an increase in the number of banks in the market increases the number of uninformed banks in bidding, making loans more available and cheaper.

There are two policy implications. First, policies that reduce the barrier for banks to enter a market can make loans more expensive and less available to new and small businesses when the screening cost is not very high. Second, markets that have similar loan rates and tightness may differ substantially in the screening cost. Policies that marginally reduce the screening cost can achieve opposite effects in these markets. When the screening cost is low, reducing it further lowers the loan rate and the market tightness; when the screening cost is high, reducing it marginally increases the loan rate and the market tightness. To create an unambiguous benefit, the policy must reduce the screening cost substantially.

There are two important questions that we plan to address in future researches. First, How do loan rates and loan availability evolve dynamically? In particular, in a continuing relationship between an entrepreneur and a lending bank, the realization of the project's output in each period reveals information about the "common value" of the project, which will affect future loan rates and availability in the ensuing bidding. Examining the time-pattern of loan rates and availability is interesting but such an examination must await the development of a dynamic auction theory. Second, What is the optimal mechanism for an entrepreneur to obtain external financing? For example, it appears that the entrepreneur can improve the loan terms when the screening cost is low by simply restricting the number of banks to contact. Although this is true in the current context, it is perhaps not an interesting venue to explore the optimal mechanism since the loan market consists of many rather than one entrepreneur. With many entrepreneurs, it is less clear whether a single entrepreneur would want to limit the contacts – there might be a chance to be left out. Examining the competition in optimal mechanisms when there are many banks and entrepreneurs should lead us to a new theory of loan market competition.

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Appendix

A. Proof of Proposition 2.1

Proof. For banks G, imposing $m_g(r) = m_g(r_H)$ for all $r \in [r_L, r_H]$ solves the inverse of F. It can be verified that $m_g(r_H)$ is an increasing function of r_H. Then, $r_H = r_b$: If $r_H < r_b$, a bank G could gain by deviating to a bid slightly higher than r_H. Substituting $r_H = r_b$ into the expected profit function yields $m_g(r_H) = M_g$ as given by (2.10). Setting $F(r_L) = 0$ solves r_L as in (2.8). The function F implicitly defined by (2.9) is continuous and differentiable over the support. Also, $F'(r) = 1/H'(F(r)) > 0$ and so F is indeed a cdf. There is no incentive for a bank G to deviate from the described strategy. Deviations to bids lower than r_L earn a profit smaller than M_g and deviations to bids higher than r_b earn a zero profit.

For banks B, the argument for a zero-profit at the bid r_b has already been made in the text. We show that there is no incentive for a bank B to deviate from r_b. Clearly, deviations to bids higher than r_b will never win and hence will always generate a zero payoff. Consider deviations to bids below r_b, say at a level r^*. The expected profit is

$$m_b(r^*) = \frac{(r^*y-1)\alpha(1-\gamma)[1-\gamma F(r^*)]^{n-1} - (1-\alpha)\gamma[1-(1-\gamma)F(r^*)]^{n-1}}{(1-\gamma)\alpha+\gamma(1-\alpha)}.$$

If $r^* < r_L$, then $F(r^*) = 0$ and it can be shown that $m_b(r^*) < m_b(r_L) < 0$. If $r_L \leq r^* < r_b$, then we can substitute the term $(r^*y-1)\alpha[1-\gamma F(r^*)]^{n-1}$ from (2.6) to rewrite

$$m_b(r^*) = \frac{(1-\gamma)[\gamma\alpha+(1-\gamma)(1-\alpha)]M_g - (1-\alpha)(2\gamma-1)[1-(1-\gamma)F(r^*)]^{n-1}}{\gamma[(1-\gamma)\alpha+\gamma(1-\alpha)]}.$$

This is an increasing function of r^* and so the best deviation is $r^* = r_b - \varepsilon$, where $\varepsilon > 0$ is arbitrarily small. In this case, however, substituting M_g from (2.10) and setting $F(r^*) = 1$ yields $m_b(r^*) < 0$. ■

B. Proof of Lemma 3.1

First, we show that the supports of the three bid distributions, (F_g, F_u, F_b), do not overlap except for the endpoints. Since the proof is similar, we show the result only for the pair (F_g, F_u). Suppose, to the contrary, that $[r_{gL}, r_{gH}] \cap [r_{uL}, r_{uH}] = [r_1, r_2]$, with $r_1 \neq r_2$. Then $m_s(r) = m_s(r_2)$ for all $r \in [r_1, r_2]$ and $s = g, u$. Using (3.4), we can rewrite these requirements as

$$\begin{bmatrix} \Pr(q=g|s=g), & -\Pr(q=b|s=g) \\ \Pr(q=g|s=u), & -\Pr(q=b|s=u) \end{bmatrix} \begin{bmatrix} (yr-1)W(r|q=g) - (yr_2-1)W(r_2|q=g) \\ W(r|q=b) - W(r_2|q=b) \end{bmatrix} = 0.$$

The above coefficient matrix is invertible and so $W(r|q=b) = W(r_2|q=b)$ for all $r \in [r_1, r_2]$. This cannot hold, since $F_g(r)$ or $F_u(r)$ is strictly increasing in r for some $r \in (r_1, r_2)$, implying that $W(r|q=b)$ is strictly decreasing in r for some $r \in (r_1, r_2)$. Thus, F_g and F_u do not have overlapping supports except for the endpoints.

Next, we show that $r_{uL} = r_{gH}$ if $u > 0$. The proof for $r_{bL} = r_{uH}$ when $b > 0$ is similar. Suppose, to the contrary, that $u > 0$ but $r_{uL} \neq r_{gH}$. We have four cases.

Case (i): $r_{uH} = r_{gL} \equiv r^*$. In this case the support of F_b does not cover (r_{uL}, r_{gH}), since the supports of any two bid distributions cannot overlap. That is, $F_b(r)$ is constant (either 0 or 1) for all $r \in (r_{uL}, r_{gH})$ and we use F_b^* to denote it. When $u > 0$, the payoff to a bank U in this case can be obtained from (3.4) as follows:

$$\begin{aligned} Em_u(r \in [r_{uL}, r^*]) &= (yr-1)\alpha[1-(1-p)uF_u(r) - (1-\gamma)pbF_b^*]^{n-1} \\ &\quad -(1-\alpha)[1-(1-p)uF_u(r) - \gamma pbF_b^*]^{n-1}. \end{aligned}$$

Since Em_u is constant over the support, then $Em'_u(r) = 0$ for all $r \in [r_{uL}, r^*]$. In particular, $Em'_u(r^*) = 0$, which yields

$$F'_u(r^*) = \frac{yA_1^{n-1}}{(n-1)(1-p)u}\left[(yr^*-1)A_1^{n-2} - \frac{1-\alpha}{\alpha}A_2^{n-2}\right]^{-1}, \tag{B.1}$$

where

$$A_1 = 1-(1-p)u-(1-\gamma)pbF_b^*; \;\; A_2 = 1-(1-p)u-\gamma pbF_b^*.$$

There is a profitable opportunity for a bank G to deviate. In particular, consider a deviation by a bank G to bidding $r^* - \varepsilon$, where $\varepsilon > 0$ is arbitrarily small. The payoff is

$$\begin{aligned} Em_g(r^*-\varepsilon) &= [y(r^*-\varepsilon)-1)]\alpha\gamma[1-(1-p)uF_u(r^*-\varepsilon) - (1-\gamma)pbF_b^*]^{n-1} \\ &\quad -(1-\alpha)(1-\gamma)[1-(1-p)uF_u(r^*-\varepsilon) - \gamma pbF_b^*]^{n-1}. \end{aligned}$$

For the deviation to be non-profitable, the limit $\lim_{\varepsilon\to 0}[Em_g(r^*-\varepsilon)-Em_g(r^*)]/\varepsilon$ must be non-positive, which requires

$$F_u'(r^*) \le \frac{yA_1^{n-1}}{(n-1)(1-p)u}\left[(yr^*-1)A_1^{n-2}-\frac{(1-\alpha)(1-\gamma)}{\alpha\gamma}A_2^{n-2}\right]^{-1}.$$

This is violated for $r > 1/2$, given (B.1).

Case (ii): $r_{uH} < r_{gL}$ and the support of F_b is not a subset of $[r_{uH}, r_{gL}]$. In this case, $F_b(r)$ is again constant (either 0 or 1) for all $r \in (r_{uL}, r_{gH})$. The payoff to a bank U from bidding $r \in [r_{uL}, r_{uH}]$ is

$$Em_u = (yr_{uH}-1)\alpha[1-(1-p)u-(1-\gamma)pbF_b^*]^{n-1}-(1-\alpha)[1-(1-p)u-\gamma pbF_b^*]^{n-1}.$$

This is an increasing function of r_{uH}. That is, if a bank U deviates from the distribution F_u and bids slightly higher than r_{uH}, the bid has the same winning probability as r_{uH} does and yet receives a higher profit when it wins. Thus, $r_{uH} < r_{gL}$ cannot be an equilibrium in this case.

Case (iii): $r_{uH} < r_{gL}$ and the support of F_b is a subset of $[r_{uH}, r_{gL}]$. In this case, $r_{bH} \le r_{gL}$, but this cannot hold in equilibrium, as the same arguments in Cases (i) and (ii) can be repeated for the pair (F_g, F_b) to yield a contradiction.

Case (iv): $r_{uH} > r_{gL}$. In this case, we must have $r_{uL} \ge r_{gH}$, since the supports of F_g and F_u cannot overlap except for the endpoints. Since $r_{uL} \ne r_{gH}$ by the supposition, then $r_{uL} > r_{gH}$. If the support of F_b is not a subset of $[r_{gH}, r_{uL}]$, then the argument in Case (ii) shows that there is incentive for a bank G to deviate to a bid slightly higher than r_{gH}. If the support of F_b is a subset of $[r_{gH}, r_{uL}]$, then $r_{bH} \le r_{uL}$, but this cannot be consistent with an equilibrium since the arguments in Cases (i) and (ii) can be repeated for the pair (F_u, F_b) to yield a contradiction.

Therefore, $r_{gH} = r_{uL}$. This completes the proof of the Lemma. ■

C. Proof of Proposition 3.2

We first show $b = 0$. Suppose, to the contrary, $b > 0$. Then the expected profit for a bank B must be non-negative which, by (3.4), requires

$$(yr_{bL} - 1)\alpha[1 - \gamma p - (1-p)u]^{n-1} \geq \frac{\gamma}{1-\gamma} \cdot \frac{1-\alpha}{\alpha}[1 - (1-\gamma)p - (1-p)u]^{n-1}.$$

Substituting this into (3.4) for $s = u$ one can show that $m_u(r_{uH}) > 0$ (note $r_{uH} = r_{bL}$). Thus, $u = 1$. But, when $u = 1$ (and $p > 0$), the above non-negative profit condition for a bank B requires

$$r_{bL} \geq \frac{1}{y}\left[1 + \frac{1-\alpha}{\alpha}\left(\frac{\gamma}{1-\gamma}\right)^n\right],$$

which is infeasible under Assumption 2. Thus, $b = 0$.

Next, we can show $u < 1$ for all $p > 0$: If $u = 1$, one can calculate the expected profit for a bank U from (3.4) and show that it is negative for any $p > 0$ under Assumption 2.

Now if a bank U participates in bidding, it must be indifferent between bidding and not bidding (since $u < 1$). The payoff from bidding must be zero. Substituting $b = 0$ into (3.4) for $s = u$ and setting $m_u(r) = 0$ yields (3.8). Setting $F_u(r_{uL}) = 0$ in (3.8) gives (3.7). Also, since banks B do not bid, $r_{uH} = 1$ by the proof of Lemma 3.1 (Case (ii) there). Setting $r = 1$ in (3.8) and solving for u yields (3.6). Then $u > 0$ if and only if $p < p_A(n)$. When $p < p_A(n)$, the cdf F_u defined implicitly by (3.8) has a positive derivative for all $r \in (r_{uL}, r_{uH})$ and so has a positive density. Moreover, it can be verified that $u(p, n)$ decreases with p and $r_{uL}(p, n)$ increases with p.

Finally, we need to show that there is no incentive for a bank U to deviate from the bid distribution F_u, given that other banks U use F_u and that other banks G use F_g described later in proposition 3.3. The proof for this part follows a similar procedure to that used in Case (i) in the proof of Lemma 3.1. ■

D. Proof of Proposition 4.1

We first verify that T_g has the described dependence on c. The same property can be confirmed for T_b in the same way. Recall that $u > 0$ iff $c > c_A$. When $c > c_A$, substituting u from (3.6) into (4.1) yields $T_g = p^n(\frac{1}{p_A} - \gamma)^n$, which is an increasing function of p and hence a decreasing function of c. When $c < c_A$, substituting $u = 0$ into (4.1) yields $T_g = (1 - \gamma p)^n$, which is a decreasing function of p and hence an increasing function of c.

For the effect of c on R_g, examine first the case $c < c_A$. In this case, setting $u = 0$ and substituting $c = EM_g(p, n)$ into (4.2) yields

$$\begin{aligned} R_g &= \frac{1-\alpha}{\alpha}\left\{1 + \left[\frac{(1+\gamma z)^n - 1}{z}\right]^{-1} \times \right. \\ &\left. \left[n(y-1)\frac{\alpha\gamma}{1-\alpha} - n(1-\gamma)[1+(2\gamma-1)z]^{n-1} - \frac{[1+(2\gamma-1)z]^n - 1}{z}\right]\right\}, \end{aligned}$$

where $z = 1/(p^{-1} - \gamma)$ is an increasing function of p. It can be verified that $[(1+x)^n - 1]/x$ is an increasing function of x for any $x > 0$. Then R_g is a decreasing function of z and hence a decreasing function of p. Since p decreases with c, R_g is an increasing function of c.

For the case $c > c_A$, substituting u from (3.6) into (4.2) and replacing c by the expression for $EM_g(p, n)$ yields

$$R_g = \frac{1-\alpha}{\alpha}\left\{1 + \frac{n(2\gamma-1)(x-1+\gamma)^{n-1} + (x_A-\gamma)^n - (x_A-1+\gamma)^n}{x^n - (x_A-\gamma)^n}\right\},$$

where $x = 1/p$ and $x_A = 1/p_A(n)$. The derivative of R_g with respect to x has the same sign as that of $h(x)$ where

$$\begin{aligned} h(x) &= n(1-\gamma) - x - (n-1)x^{1-n}(x_A-\gamma)^n \\ &+ \frac{(x-1+\gamma)^{2-n}}{2\gamma-1}\left[(x_A-1+\gamma)^n - (x_A-\gamma)^n\right]. \end{aligned}$$

This is clearly negative when $x \to \infty$. Thus, $dR_g/dc < 0$ when c is close to but lower than c_0. When $c \downarrow c_A$, substitute $x = x_A = \gamma + (2\gamma-1)/(Y-1)$, where $Y = [\alpha(y-1)/(1-\alpha)]^{1/(n-1)} > 1$:

$$\begin{aligned} h(x) &= \frac{(2\gamma-1)}{Y-1}\left(\frac{\gamma}{2\gamma-1}Y + 1\right)^{1-n} \times \\ &\left\{\left[\frac{1-\gamma}{2\gamma-1}(n-1)(Y-1)^2 + Y - Y^{2-n}\right]\left(\frac{\gamma}{2\gamma-1}Y+1\right)^{n-1} - (n-1)(Y-1)\right\}. \end{aligned}$$

The expression in $\{.\}$ is a decreasing function of γ for any given $Y > 1$ and so

$$h(x) > \frac{(2\gamma-1)}{Y-1}\left(\frac{\gamma}{2\gamma-1}Y+1\right)^{1-n}\left[(Y - Y^{2-n})(Y+1)^{n-1} - (n-1)(Y-1)\right]$$

The expression in [.] is an increasing function of Y and has value 0 when $Y = 1$. Thus, $h(x) > 0$, i.e., $dR_g/dc > 0$ when c is close to c_A. Thus, there exists $c^* \in (c_A, c_0)$ such that $dR_g/dc > 0$ for all $c < c^*$. This completes the proof of Proposition 4.1.

E. Proof of Proposition 4.3

To prove the proposition, we first establish the following lemma:

Lemma E.1. *The following relations hold for all* $p > 0$*:*

$$\gamma[1-(1-\gamma)p]\ln[1-(1-\gamma)p] - (1-\gamma)(1-\gamma p)\ln(1-\gamma p) < 0, \tag{E.1}$$

$$\frac{d}{dp}\left(\frac{\ln[1-(1-\gamma)p]}{\ln(1-\gamma p)}\right) < 0. \tag{E.2}$$

Proof. The left-hand side of (E.1) is a decreasing function of p and has a value zero when $p = 0$. Thus (E.1) is evident. Computing the derivative in (E.2) shows that it has the same sign as that of the left-hand side of (E.1) and so it is negative. ■

We now show that equilibrium screening probability is a decreasing function of n. Differentiating the equation $EM_g(p, n) = c$, where $EM_g(p, n)$ is given by (3.10), yields

$$\frac{dp}{dn} = \frac{1-(1-\gamma)p}{(n-1)(1-\gamma)}\ln[1-(1-\gamma)p], \text{ if } c > c_A \text{ (i.e. } p < p_A), \tag{E.3}$$

$$\frac{dp}{dn} = \frac{1}{(n-1)\Delta}\left\{\begin{array}{l}(y-1)(1-\gamma p)\ln(1-\gamma p) \\ -\frac{(1-\alpha)(1-\gamma)}{\alpha\gamma}\cdot\frac{[1-(1-\gamma)p]^{n-1}}{(1-\gamma p)^{n-2}}\ln[1-(1-\gamma)p]\end{array}\right\}, \text{ if } c < c_A, \tag{E.4}$$

where

$$\Delta \equiv (y-1)\gamma - \frac{(1-\alpha)(1-\gamma)^2}{\alpha\gamma}\left(\frac{1-(1-\gamma)p}{1-\gamma p}\right)^{n-2}.$$

Clearly, $dp/dn < 0$ for $c > c_A$. When $0 < c < c_A$, $EM_g > 0$ implies

$$y > 1 + \frac{(1-\alpha)(1-\gamma)}{\alpha\gamma}\left(\frac{1-(1-\gamma)p}{1-\gamma p}\right)^{n-1}. \tag{E.5}$$

With (E.1) one can show that $-dp/dn$ is an increasing function of y and so (E.5) implies

$$-\frac{dp}{dn} > \frac{(1-\gamma p)[1-(1-\gamma)p]}{(n-1)(2\gamma-1)}\{\ln[1-(1-\gamma)p] - \ln(1-\gamma p)\} > 0. \tag{E.6}$$

Thus, $dp/dn < 0$ for $c \in (0, c_A)$ as well.

To show the dependence of T_g on n, examine first the case $c < c_A$. Differentiate $T_g = (1-\gamma p)^n$ and substitute (E.6) to obtain:

$$\begin{aligned}\frac{dT_g}{dn} &> \frac{n\gamma(1-\gamma p)^n}{(n-1)(2\gamma-1)}[-\ln(1-\gamma p)] \times \\ &\quad \left\{\frac{n-1}{n}\left(\frac{1}{\gamma}-2\right)-[1-(1-\gamma)p]\left[\frac{\ln[1-(1-\gamma)p]}{\ln(1-\gamma p)}-1\right]\right\}.\end{aligned}$$

Using (E.2) it can be verify that the expression in $\{.\}$ is an increasing function of p and that its value at $p = 0$ is positive. Thus, $dT_g/dn > 0$ for $c < c_A$.

For $c > c_A$, let $\xi = 1/(n-1)$ and $\sigma = \alpha(y-1)/(1-\alpha) > 1$. Solving p as a function of c from $EM_g = c$ and substituting p_A yields

$$\ln T_g = \left(1+\frac{1}{\xi}\right)\left\{\ln\left[1-\left(\frac{c}{c_0}\right)^{\xi}\right]-\ln(\sigma^{\xi}-1)-\ln\left(\frac{1-\gamma}{2\gamma-1}\right)\right\}.$$

The following two properties can be verified: (i) $d\ln T_g/d\xi$ is an increasing function of c/c_0; (ii) $d\ln T_g/d\xi > 0$ when $c \to c_0$. Thus, there exists some $c_1 < c_0$ such that T_g is an increasing function of ξ and hence a decreasing function of n if and only if $c > c_1$. The level c_1 may or may not be greater than c_A. Since the current case is restricted to $c > c_A$, let $c^{**} = \max\{c_1, c_A\}$. Then T_g is an increasing function of n if and only if $c < c^{**}$. This completes the proof of Proposition 4.3. ■

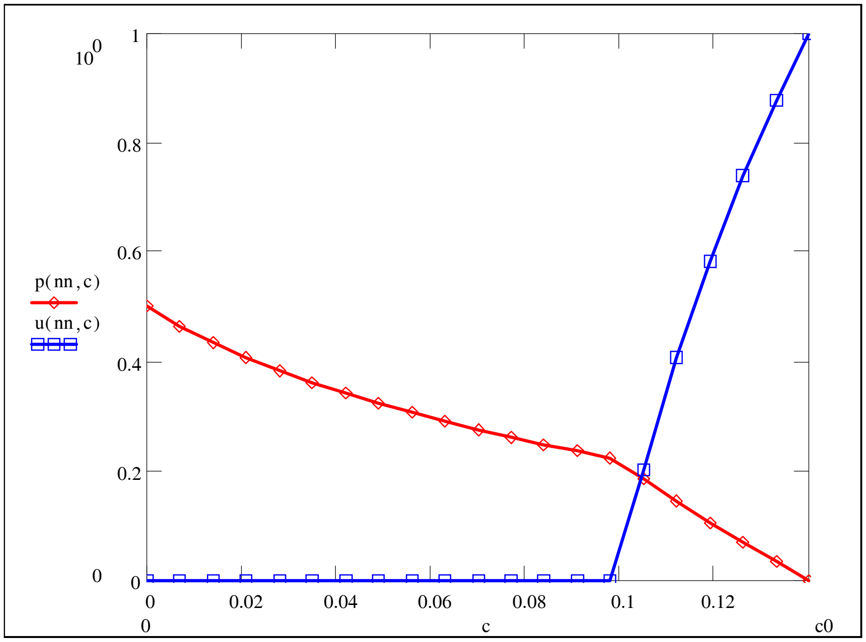

Figure 2a Effects of the screening cost on the screening probability and uninformed banks' participation

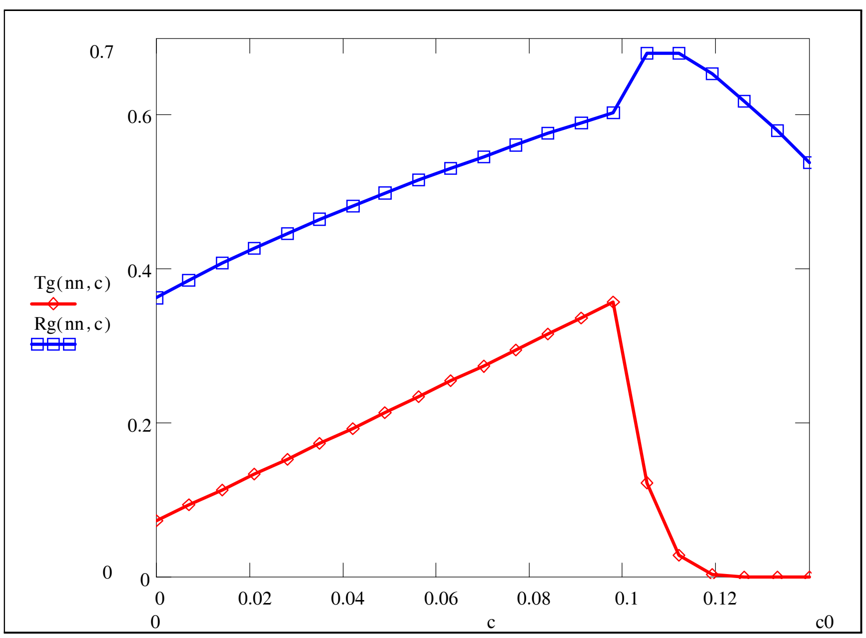

Figure 2b Effects of the screening cost on loan market tightness and the expected loan rate

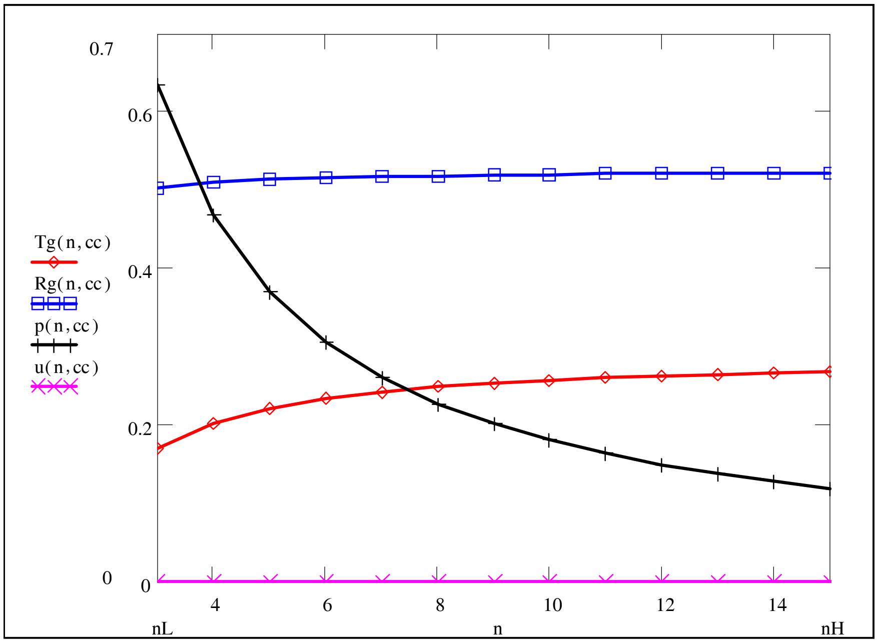

Figure 3a Effects of the number of banks on tightness, the expected loan rate and screening probability – low c

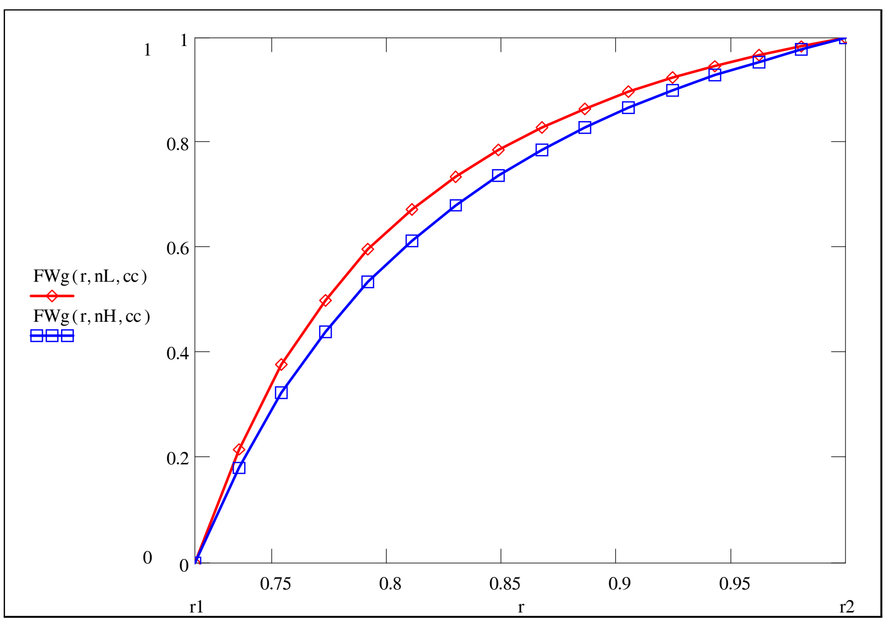

Figure 3b Effects of the number of banks on the winning bid distribution – low c

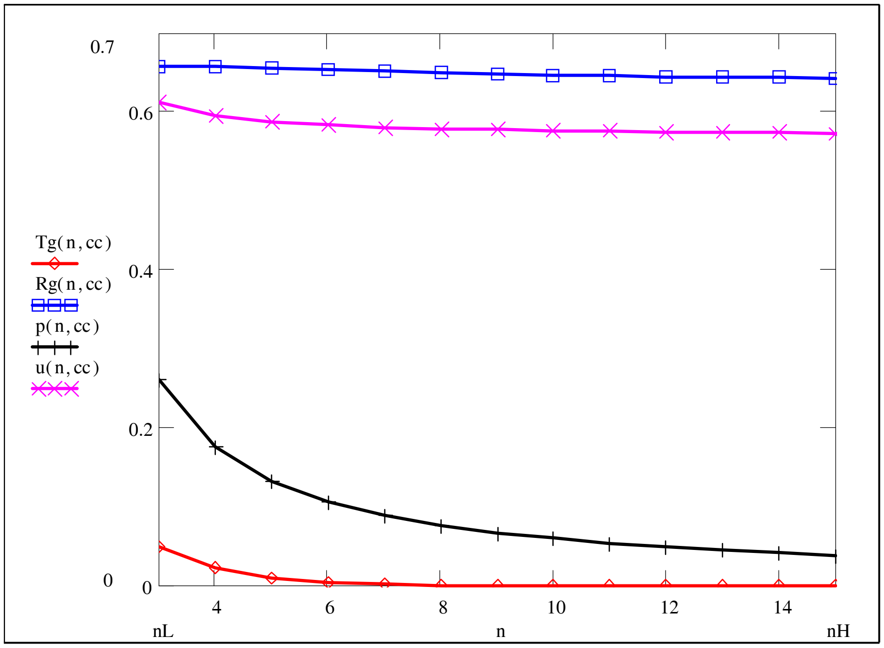

Figure 4a Effects of the number of banks on tightness, the expected rate, screening probability and participation–high c

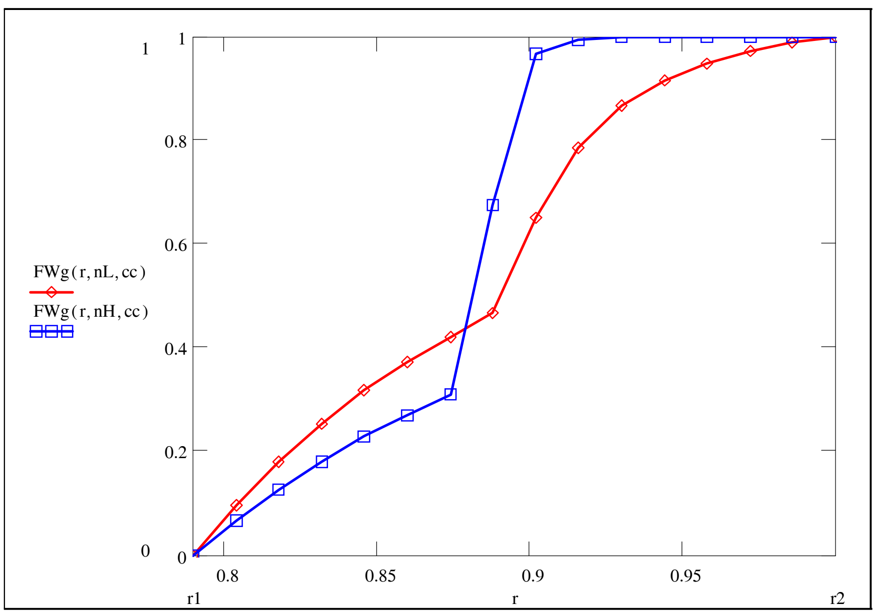

Figure 4b Effects of the number of banks on the winning bid distribution – high c

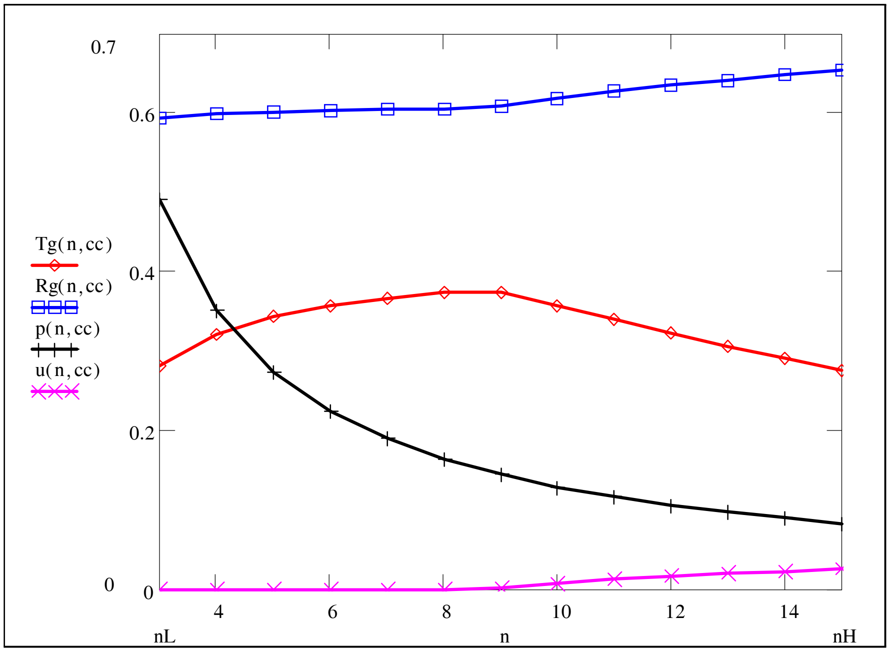

Figure 5a Effects of the number of banks on tightness, the expected rate, screening probability and participation – moderate c

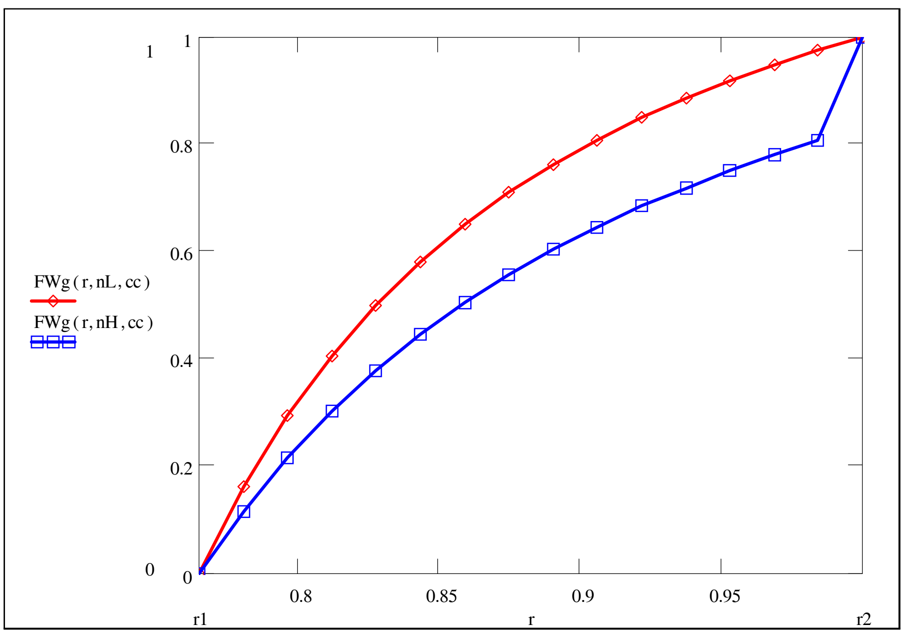

Figure 5b Effects of the number of banks on the winning bid distribution – moderate c