A Search Model of Market Segmentation: Implications of International Trade for the Two 'Fundamental' Questions of Modern Monetary Economics

by

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Abstract

Modern studies in monetary economics have been centered around the following two interrelated questions: (1) what is the optimal monetary policy?; and (2) what is the welfare cost of deviating from this optimal policy? Search theoretical models of money have studied these two questions in a variety of environments. In spite of the different nature of the environments in which these two questions have been answered, all of them have a common property. This property is that none of them includes international trade in both currency and commodities. To address this issue, this paper first extends the Lagos and Wright (2004) model to a two-country, two-currency framework, and then uses this framework to answer these two important questions. This paper finds that in this framework though the Friedman's rule is still the optimal monetary policy, it is unattainable. In addition, it also finds that the measurement of welfare loss of deviating from the Friedman's rule is, at least in theory, no smaller, and possible higher than the measurement in the Lagos and Wright (2004) model.

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As usual, all errors and omissions are mine alone.

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"Nature, by giving a diversity of geniuses, climates and soils, to different nations, has secured their mutual intercourse and commerce, as long as they all remain industrious and civilized. Nay, the more the arts increase in any state, the more will be the demand from its industrious neighbours [neighbors]. The inhabitants, having became more opulent and skillful, desire to have every commodity in the utmost perfection, and as they have plenty of commodities to give in exchange, they make large importations from every foreign country..." (Hume in Rotwein 1945)

1 Introduction: Search Theory of Money and International Trade

Fundamental to most analysis in the modern literature of monetary economics are the following two questions: (1) what is the optimal monetary policy (Friedman 1969, Lagos and Wright 2004, Arouba *et.al.* 2007[1], Lucas 2000)?, and (2) what is the welfare cost of deviating from the optimal monetary policy (Arouba *et.al.* 2007[1], [2], Chiu and Molico 2007, Craig and Rocheteau 2007, Friedman 1969, Bailey 1954, Kiyotaki and Wright 1993, Lagos and Wright 2004, Lucas 2000, Trejos and Wright 1995)? [1] In answering these two 'fundamental' questions of modern monetary economics, the aforementioned analysis propose a variety of models with different environments. The aim of the various environments proposed in these models is to explain the microeconomics foundations of money. In other words, these environments try to depict the decision making of rational economic agents and their interaction within monetary economies.

Notwithstanding the differences, there is one common characteristic among these environments. The common characteristic among these environments is that none of them explicitly model trading patterns between agents in domestic and agents in foreign markets (i.e. international trade). Instead, they are mainly concentrated in

[1] I consider modern monetary economics to the literature that started with Bailley (1954) and Friedman (1969).

assessing the role of money only at the domestic level. As a consequence, each monetary economy is treated as an 'isolated' island without external contact with other economies. The exclusion of this large sector of the economy from these environments is particularly harmful when employing these models to answer the two 'fundamental' questions of modern monetary economics.

Consequently, in this paper, I firstly, propose a model whose trading environment includes both a foreign and a domestic market; and then, I employ this model to theoretically analyze these two important questions.[2] To that end, I extend the Lagos and Wright (2004) model (LW, hereafter) to a two-country, two-currency model. I choose the LW model because of both its analytical tractability and simplicity in analyzing these two central questions of monetary economics. Furthermore, it is easy to extend to cover more difficult models that include numerous trading frictions (examples of extensions to LW to include more trading frictions are Arouba et.al. 2007[1],[2], Chiu and Molico 2007, Craig and Rocheteau 2007, Camera *et.al.* 2004). The construction of Search Theoretical models of two-currency, two-country is not new to monetary economics. Head and Shi (2003) proposed a two-currency, two-country model and Camera (*et.al.* 2004) as well as Kiyotaki and Wright (1993) analyzed a two-currency model. In addition, Obstfeld and Rogoff (1995, 1996) as well as Champ and Freeman (2001) showed a two-country, two-currency money cash-in-advance and overlapping generations models respectively. What it is new in my analysis is the inclusion of foreign trade and currencies in an environment that is simple enough to serve as a general "[f]Framework for [m]Monetary [t]Theory and [p]Policy [a]Analysis"(Lagos and Wright 2004).

The addition of this foreign sector to the LW model, however, is not without problems. Since Kareken and Wallace (1981) seminal paper, it has been common knowledge in monetary economics that without any restrictive assumption the ex-

[2]It is important to note that, though this paper is mainly theoretical, the answers to the two 'fundamental' questions of modern monetary economics are as much an empirical issue as they are theoretical.

change rate between two intrinsically worthless objects is undefined. This is the case when we consider the exchange rate between two fiat monies. In the short history of Search Theoretical models of monetary economics there have been different restrictions to force the exchange rate to be determined (see Head and Shi 2003; Camera *et.al.* 2004). In this paper, I impose a cash-in-advance constraint in which different currencies are needed to participate in markets located in different geographical locations. That is to say, buyers need to pay sellers in the currency of their (the seller's) own country. [3]

The result of this model in terms of the answers to the two 'fundamental' questions of modern monetary economics is very interesting. This paper demonstrates that though the optimal monetary policy is the same as in the LW model and other similar models (Arouba *et.al.* 2007[1],[2], Chiu and Molico 2007, Craig and Rocheteau 2007), it is unattainable under a representative agent's solution. That is to say that without a world's planner, it is not possible to achieve the Friedman's rule. Instead, the steady state equilibrium of the model proposed in this paper is a sub-optimal Nash equilibrium. The reason for this is that in maximizing the average utility of a representative agent, governments have the incentive to set the inflation rate above the one consistent with the Friedman's rule. With respect to the second 'fundamental' question of welfare economics, this paper theoretically shows that the welfare cost of deviating from the Friedman's rule in this model is greater or equal than the welfare cost in the LW model. However, in proving this qualitative property, this paper employs very strict assumptions of the form of the model. Therefore, an empirical quantitative analysis of this property is necessary before a final conclusion can be reached.

Even before inquiring into the assumptions and results of this model, it is constructive to review a short and instrumental history of the development of ideas in monetary economics. It is instrumental as it is by no means exhaustive, but just an

[3]The first one to introduce a cash-in-advance constraint like this was Lucas (1982)However, his model was not a Search Theoretical model of money, but a cash-in-advance model.

instrument to understand the subsequent analysis. Consequently, what follows is in four parts. This paper, firstly, investigates this short history of thought in monetary economics. Secondly, it presents the model, its assumptions and results. Subsequently, this paper inquires into the implications of this model in answering the two 'fundamental' questions of modern monetary economics, and finally, it provides some concluding remarks. In addition, the appendix contains very important technical results

2 Toward a value theory of money with applications to monetary policy and social welfare

The aim of this section is to review the current literature in monetary economics and how the following analysis fits into it. However, in reviewing this literature, it is impossible not to mention past analysis in monetary theory. The reason for this is that modern monetary economics comes from a long standing line of thought that dates back to Pre-'Smithian' (1776) economic thought. The concern of both modern and past monetary theories is twofold. Firstly, they are concerned with the construction of a theory that gives value to an otherwise intrinsically worthless object such as fiat money. Secondly, they are interested in analyzing the consequences of fluctuations in money supply and demand to society (Shi 1999).

As a consequence of this continuum of ideas in monetary economics, this section includes two different sub-sections directed at reviewing the literature in past and modern periods respectively. To keep the analysis simple, the first section is just a short investigation into past monetary thought that provides the reader with some insight to understand the subsequent analysis. The second sub-section, on the other hand, is a more comprehensive inquire into modern literature in monetary economics.

2a. The 'Classical' and 'Neo-Classical' Periods

Though there were many advances in monetary economics during the 'Classical' and 'Neo-Classical' periods, only one in particular played a big role in shaping modern research in monetary economics. The distinction between short run adjustments and long run equilibrium was the discovery that played this big role in monetary theory. This is not to say that other advances had little or no impact in the development of current ideas in this sub-discipline, but that their importance is of second order.

Smith (1776) was the first one to state that unlike any other commodity which has a value in use and a value in exchange, money has only value in exchange. It was Hume (in Rotwein 1945), however, the first to demonstrate, in his price-specie-flow mechanism, that the value of money in terms of goods adjusts to offset any excess in supply or demand.

Given that the value of money adjusts to excesses of supply and demand, Thornton (in Backhouse 1985) became very interested in explaining the mechanisms behind these adjustment and the consequences for the real economy. His major contribution was the distinction between short run adjustments and long run equilibriums (Backhouse 1985). Thornton discovery could be observed in the following equation of exchange of the quantity theory of money:[4]

$$M \cdot V = P \cdot Q \tag{1}$$

where M is the total amount of money in circulation in the economy, V is the velocity of money, P is the price level and Q is an index of expenditures. Thornton (in Backhouse 1985) stated that an increase in the supply of money M will cause all the variables to change in the equation of exchange during a short run transition period. However, in the long run when the economy reaches a new equilibrium both V and Q return to their previous level (before the increase in M) and there is a change in P proportional to the change in M. In other words, an increase in the money supply causes, in the long run, a proportional change in the nominal price of all commodities, leaving the velocity of money and the quantities traded constant (Backhouse 1985).

The ‘Neo-Classical’ economists expanded Thornton idea of the differences between

[4]Walsh 2003

the short run and long run. In addition, Fisher (in Schumpeter 1954), who was responsible for the equation of exchange previously shown, was the first to identify the difference between the real and nominal interest rates. According to Fisher (in Backhouse 1985), the real interest rate in the long run is determined as follows:

$$r = [(1 + i)/(1 + \pi^e)] - 1 \tag{2}$$

or[5]

$$r \approx i - \pi^e \tag{3}$$

where π^e is the expected inflation rate, r and i are the real and nominal interest rates respectively. The importance of the 'Fisher' equation lies in that it identifies the real opportunity cost (after adjusting for nominal changes) of holding money as opposed to holding other productive assets.

Fisher used his theory to show that one time price changes do not have any impact in the real money holdings of economic agents. The reason for this is that price changes do not affect the opportunity cost of holding real money balances. However, sustained long run price changes produce changes in the real economy *via* changes in inflation expectations. The transmission mechanism by which sustained changes in prices affects the real output is simple. Facing a higher opportunity cost, rational economic agents would reduce their real money holdings. Reduced real money holdings translate into less money being use for trade purposes and therefore, demand and subsequently production falls.

Hitherto, economists in the 'Classical' and 'Neo-Classical' periods were mostly concerned with long run equilibriums. However, with the ascendancy of Keynesian macroeconomics after the Second World War this changed. Keynesian macroeconomics was the economics of short run demand stabilization and monetary policy became a powerful tool for regulating the real economy. Thereby, monetary theory had an unexpected turn which resulted in economists investigating the short run

[5] Assuming that $r \cdot \pi^e \approx 0$

effects of different monetary policies. Given this unexpected turn in monetary economics, the long run nature of monetary equilibriums was not revisited until the rise of the 'New Classical' economics.

2b. Modern Monetary Economics:The 'New Classical Period'

The importance that long run macroeconomic analysis regained in the second half of the 1950's did not contribute to expand the understanding of the role that fiat money plays in monetary economies. Moreover, in assuming a frictionless economy, the general equilibrium model (or the Arrow-Debreu model), the most important model developed in that time, ignored the value of fiat money altogether (see Shi 1999). This model consists in agents maximizing an intertemporal utility function alongside a production constraint which is composed of perishable consumption goods and productive real assets. The solution to this model yields an asset pricing equation in which assets are valued in real terms according to a measure of intertemporal marginal rate of substitution and the consumption commodities that they yield after a specific period of time.[6]That is, the solution to this model is a pricing equation that gives value to assets according to a measure of their value in use adjusted by a time preference parameter. Accordingly, assets with no value in use because they are intrinsically worthless and do not produce consumption goods, such as fiat money, have no value in this model.

Even though Friedman (1969) accepted the pre-conditions of the Arrow-Debreu economy, to name frictionless complete commodity markets, he acknowledged that financial markets are far from inclusive. Therefore, for the majority of households in an economy the only means of smoothing consumption through time is not by holding productive assets, but fiat money. In addition, he recognized that if holding money is the only means of smoothing consumption through time, then whenever an economy possesses a positive nominal interest rate regime, trades and production are inefficiently low. The reason for this inefficiency lies in the fact that there is a

[6]For a complete explanation of the asset pricing equation in the Arrow-Debreu model see Lucas 1978, Shi 1999.

difference between the private marginal nominal cost of holding money and the social marginal nominal cost of printing money. On the one hand, for society as a whole there is a zero marginal nominal cost of printing one more unit of fiat money. On the other hand, as long as the nominal interest rate is positive there is a positive private marginal nominal cost of holding money balances. As a consequence, in maximizing utility, economic agents hold less than optimal real money balances which as aforementioned results in lower than optimal output (see the previous sub-section). The solution he proposes is to deflate at the rate of the real interest rate so as to equate social and private marginal costs of money (from equation 2 with a constant inflation rate [i.e. $\pi^e = \pi = 1 - \frac{1}{1+r}$] this translates into i=0).[7]

Friedman's analysis resulted in the two 'fundamental' questions of modern monetary economics mentioned in the introducing section of this paper. To answer the first question, Friedman (1969) defined the optimal rule of zero interest rate as the Friedman's rule. The first 'fundamental' question then evolved into whether the Friedman's rule is still optimal under more general and realistic conditions than the ones assumed by Friedman (1969). In this context, it is simple to observe the relevance of the second 'fundamental' question. In the case that zero nominal interest rate is optimal, it is not always possible to attain because of short run demand management considerations (Judd and Rudebusch in Rabin and Stevens 2002) or seignorage revenue requirements (Haslag in Rabin and Stevens 2002). What is, then, the welfare cost of deviating from this rule?

Taking the optimality of the Friedman's rule as given, Bailey (1956) and Friedman (1969) proposed empirically matching a semi-log real money demand function whose only argument is the real interest rate to answer this question. Under this framework

[7]Though Friedman never mentioned, it is interesting to note that this equates the real social marginal benefit of printing money to the private real marginal cost of holding money (in this case a benefit given that it is a negative cost). This is because inflation affects the value of all money, the one that it was held before printing and the one being print. On the other hand, the interest rate only affects the opportunity cost of the money that is being held. So, by eliminating the nominal interest rate, the real interest (the private marginal real cost of holding money) is equal the inflation rate (the social real marginal cost of printing money).

(known as the welfare triangle), the welfare cost of inflation consists of the area under the demand curve (the consumer's surplus) between the value of the function at a real interest rate consistent with the Friedman's rule and the value of the function at the chosen real interest rate. In revisiting this framework, Lucas (2000) discovered, however, that the use of a log-log function provides a more accurate description of the welfare cost of inflation. Nonetheless, both the semi-log and the log-log functions lacked micro-foundations to explain the behavior of agents in monetary economies, and therefore, they were not an adequate framework to measure the cost of deviating from the Friedman's rule. For instance, Friedman (1969) discovered that the behavior of individuals is different under low interest/low inflation rates regimes than under high interest/high inflation rates regimes. So, matching a unique money demand function leads to biased measurements of welfare loss.

To address this problem, economists proposed a methodology based on theoretical models with micro-foundations. The first micro-founded model of monetary economics proposed, was the so called representative agent model or money-in-utility model (Lucas 2000; Walsh 2003). The only difference between this model and the Arrow-Debreu one is that it includes an additively separable function from consumption as an argument in the utility function. When money is generally accepted by all agents in the economy, the exchange of money solves the double coincidence of wants problem associated with barter trade. In solving this problem, money reduces the time spent shopping and thus, increases utility. This increase in utility caused by holding real money balances is captured by this additional argument in the utility function (Walsh 2003).

There are also two other similar models where money has a positive value in equilibrium introduced to solve the lack of micro-foundations. The first model is the model of cash-in advance in which money is not introduced in the utility function, but in the budget constraint (see Walsh 2003, for models of cash in advance at an international level see Obstefeld and Rogoff 1995, 1996). In this type of models, people have to use money to purchase commodities and commodities to purchase money. In equilibrium, fiat money has a positive value because it is the only way in which individuals are able to smooth consumption through time (there are no other types of assets and barter is not allowed). The other model is the overlapping generations model in which the main function of money is to continuously transfer income from a productive generation (young) to an unproductive generation (old) (Champ and Freeman 2001).

Though these models introduce micro-foundations to justify people's holdings of fiat money, there are still issues that remained unaddressed by them. In both the overlapping generations and the cash-in-advance models, agents can only transfer consumption between periods by holding fiat money. Hence, fiat money is only valued in equilibrium as it is the only asset that allows individuals to 'store' value in order to delay consumption. However, in reality there are various assets that compete with money as storage of value and unlike money they bear interest in the subsequent periods. The existence of these assets creates a problem for these models as they do not explain why people still hold fiat money to smooth consumption. In addition, none of the models provide any explanation of how agent's monetary holdings evolve through time. And, most importantly, the positive value of money was added as an *ad hoc* result to modify a model that given its assumptions renders any type of non-productive asset worthless.

In order to model monetary economies more adequately, economists had to look for a different type of models that are able to depict the frictions that in the real economy makes money essential. The place where they found this type of models was in the Search Theory of Labour Economics. [8] In adapting Search Theoretical models to monetary economics, economists introduced the following assumptions to make money essential (i.e. valued in equilibrium): (i) bilateral trades (to rule out competitive markets); (ii) lack of double coincidence of needs (to rule out barter trade); (iii)

[8] For a complete review of models of search theory in labour economics see Pissarides 2000.

informational and commitment problems (to rule out credit); (iv) search frictions (to make individuals hold money for precautionary purposes); (v) random matching (to rule out gift giving).[9]

Kiyotaki and Wright (1989) provided the first model of Search Theory in monetary economics called commodity money because one of the commodities produced in the economy is used as a medium of exchange. The model consists on infinitely lived agents grouped in three categories wherein individuals produce and consume the same commodity. [10] Each of these categories or groups produces a commodity which is not consumed by any of their members but by the members of one of the other two groups. In addition, there is a continuation of random bilateral matching among these three groups and the groups can hold only one type of commodity at a time. It is almost needles to say that in maximizing their utility each group will trade the commodity they produce for the commodity the other group produces only if they expect to gain utility from this trade. Kiyotaki and Wright (1989) discovered that a special type of solution is the fundamental solution in which the commodity with the lowest storage cost is used as a medium of exchange. However, this is not the only solution. There are other solutions in which people use other commodity as a medium of exchange, even though its storage cost is not the lowest (Kiyotaki and Wright (1989) called this a speculative equilibrium).

Although the previous model is able to provide a more accurate description of the frictions that in equilibrium give value to money, it still fails to explain why fiat money is essential. [11] To explain this issue Kiyotaki and Wright (1993) extended their model to include fiat money as an indivisible commodity. In their model, individuals are only able to store either money or the commodity they produce. As

[9]Models with these assumptions are: Arouba *et.al.* 2006, 2007; Berensten *et.al.* 2007[5][6]; Camera *et.al.* 2004; Chiu and Molico 2007; Craig and Rocheteau 2008; Head and Shi 2003; Kiyotaki and Wright 1989, 1993; Lagos and Wright 2004; Rocheteau and Wright 2005; Shi 1997; Trejos and Wright 1995.

[10]To rule out barter we need an odd number of categories.

[11]Note that although commodity money has only value in exchange for the group that uses as a medium to obtain their consumption good, it has use value for one of the other groups.

in the previous model, the authors discovered two different solutions to this model. The first solution consists on a monetary solution in which people believe that fiat money will be accepted as a medium of exchange and therefore, money is valued in equilibrium (a self-fulfill prophecy). On the other hand, the second solution consists in a non-monetary solution in which money is worthless because individuals do not believe that it will be accepted as medium of exchange.

A different Search Theoretical model of money is the LW model. This model comprises two different markets, one centralized (or Walrasian) and one decentralized. In the centralized market, people behave as in the Arrow-Debreu general equilibrium model. That is, they maximize their utility of consumption subject to a production constraint by producing their own consumption commodity. However, there is a difference which is that people can choose to hold real money balances whose price is taken as given for trading purposes in the decentralized market. [12] In the decentralized market people are subject to idiosyncratic shocks which with certain probability transform them into either sellers (produce but cannot consume) or buyers (consume but cannot produce). Buyers and sellers are matched according to a random matching function and the terms of trade are given by a Nash General Bargain Solution (see Arouba et.al. 2007 for an exposition of different game theoretical bargaining solutions). In equilibrium, individuals hold money for both precautionary purposes and for trading in the decentralized market.

The distinctive property of this model with respect to other Search Theoretical models is that in equilibrium the distribution of money is degenerate making monetary policy assessments easy to perform. In other words, since the distribution of money across individuals is the same every period, it is possible and relatively simple to compare the impact of different monetary policies on the welfare of individuals. In assessing different monetary policies, Lagos and Wright (2004) concluded that the Friedman's rule is optimal in general but only efficient when buyers get the entire sur-

[12] The centralized market of this model is very similar to the cash-in-advance model as people can exchange their production for other assets, in this particular case, money.

plus.[13] That is, the solution is efficient whenever buyer's market power allows them to make take-it-or-leave-it offers to the sellers. On the other hand, whenever buyers have less than full market power, they under-save real money balances in the centralized market and therefore, quantities traded in the decentralized market are less than efficient. The reason for this inefficiency is that the surplus from trade is shared by both buyer and sellers, but the cost of holding money between markets is only particular to buyers. This problem is known to the monetary economics literature as the hold-up problem (Lagos and Wright 2004; Arouba et.al. 2007[1]). It is worth noting that the hold-up problem is not particular to the bargaining solution included in the LW. Similar Search Theoretical models with different bargaining solutions also suffer from this problem whenever buyers do not get the entire surplus from trade (see Arouba et.al. 2007[1]).

In recent years there have been many extensions to the LW model in which economists have tried to model an increasing amount of monetary frictions. Arouba (*et.al.*2007[2]) added agents capital investment decisions to the decentralized market. The main result of this addition to the LW model is that there is a double hold-up problem in which either consumers under save, producers under invest or both. Given this double hold-up problem, the planner's solution is unattainable even under take-it-or-leave-it offers (for which there is no closed form solution in this model) and therefore, the Friedman's rule is only optimal but not efficient. In addition, Berensten (*et.al.*2006) extended the LW model by introducing financial intermediation (i.e. commercial banks). The most relevant result of this analysis is that when there is no other way of enforcing debt repayment than the exclusion of agents from financial markets, deviating from the Friedman's rule is optimal.

Moreover, Berensten (*et.al.*2007), Chiu and Molico (2007), Rocheteau and Wright (2005) investigated search models of divisible money in which agent's choice to participate in either the centralized market (Chiu and Molico 2007) or decentralized

[13]Efficiency is only attained when private economy is able to reach the same equilibrium as the planner's solution.

market (Bernesten *et.al.*2007; Rocheteau and Wright 2005) is endogenous. Unlike in the LW model in which decisions to participate in either market are exogenous, Berensten (*et.al.*2007) indicated that endogenous decisions models have two sources of frictions. The first source of friction is the intensive one and it is shared between these two types of models (see Rocheteau and Wright 2005). This friction consists of the quantity of commodities that are traded in each bilateral matching between buyers and sellers and depends on the quantity of money that buyers bring to the decentralized market. When the interest rate is not at the Friedman's rule, agents bring sub-optimal amount of real money balances to the decentralized market and thus, quantities traded are less than optimal.

The second source of inefficiency, the extensive source of friction, on the other hand, is only particular to the endogenous decision models and consists of the amount of matching trades in the decentralized market. Consumers search for trade until the surplus they get from the last trade is equal to the marginal cost of searching for that trade. As it is the case with other sources of frictions, the economy reaches an efficient outcome whenever the private economy is able to replicate the planner's solution. In these models, the efficient amount of trades could be reached by adjusting the policy variable which is the interest rate.[14]However, it is worth noting that there are two sources of frictions and one policy variable and therefore, efficiency in the extensive source of friction could mean inefficiency in the intensive source of friction and vice-versa. The only time efficiency is achieved in the model is whenever the Friedman's rule replicates the planner's solution in both the extensive and intensive sources of friction.

With respect to the second 'fundamental' question of monetary economics, the one related to the welfare cost of inflation, these models also give heterogeneous answers. Craig and Rocheteau (2008) surveyed the differences in the measurements of the welfare cost of inflation among all these different above-mentioned models.

[14]The inflation rate which is consistent with the interest rate that replicates the efficient amount of trade is called the Hosios' rule (see Berensten *et.al.* 2007).

They discovered that in the models with exogenous participation decisions the measurement of welfare cost is bigger than under the welfare triangle methodology and highly dependent on the bargaining solution utilized. For instance, in models with proportional bargaining solutions (see Arouba et.al.2007; Craig and Rocheteau 2008) the welfare triangle methodology could be adjusted by the proportion of surplus that goes to the seller to measure the social cost of inflation. In models with Nash General Bargain solutions, it is not as simple since the surplus is non linear in the quantities traded and therefore, the welfare triangle has to be adjusted by a non linear function to measure the society's welfare cost. Furthermore, in models with endogenous decisions and with capital investment decisions, there is no rule and numerical analysis is required. For instance, Arouba (*et.al.*2007) found that the welfare cost of inflation is bigger than in the previous literature (including under the welfare triangle methodology) but Chiu and Molico (2007) discovered it to be smaller.

Even though all of these Search Theoretical models of monetary economics cover a big range of frictions and different assumptions, none of them either investigates the optimal monetary policy or measures the welfare cost of inflation in an international monetary framework. Modern economies are based on a free international flow of both commodities and money. As a consequence of this free flow, households are able to substitute domestic money holdings and commodities for their foreign counterparts. This free substitution is of uttermost importance when modeling monetary economies and ignoring them could potentially lead to false conclusions about the consequences of monetary policy. Therefore, the following section extends a linear version of the LW model to include international exchange in both fiat money and commodities in order to create a more 'general' framework in which to model monetary economies.

3 A basic Search Theoretical model of money in an international framework

To investigate this monetary model of international trade, this section is organized as follows. Sub-section 3a describes the model's environment and defines its framework. Sub-section 3b solves for the planner's solution (i.e. the efficient solution). Finally, sub-section 3c solves for the representative agent's solution.

3a. The Environment

There are two [0, 1] continuum of infinitely lived agents that live in two separate countries, country A and country B. As in the LW model, time is discrete and agents' discount factor is $\beta \in (0, 1)$. Each period is divided into two different sub-periods. The first sub-period is composed of two different decentralized markets (DM), a domestic and a foreign one and the second sub-period comprises a centralized market (CM).

In the first sub-period, agents are subjected to idiosyncratic shocks in which with probability σ^{ii} an agent of the i^{th} country is a consumer in the domestic market, with probability σ^{ij} an agent of the i^{th} country is a consumer in the foreign market, with probability $\sigma^{ii} + \sigma^{ji}$ an agent of the i^{th} country is a producer and with probability $1 - 2\sigma^{ii} - \sigma^{ij} - \sigma^{ji}$ an agent of the i^{th} country is a non trader, for all $i, j = a, b,\ i \neq j$.[15] In this sub-period a consumer in the i^{th} country gets utility $u(x^a, x^b)$ from consuming x^a, x^b quantities of each countries' perishable commodities where $u'_{x^i}(x^a, x^b) > 0, u''_{x^i x^i}(x^a, x^b) < 0, u'_{x^a}(0, x^b) = \infty, u'_{x^b}(x^a, 0) = \infty, u'_{x^a}(\infty, x^b) = 0, u'_{x^b}(x^a, \infty) = 0, u''_{x^i x^j}(x^a, x^b) = 0, \forall i, j = a, b; i \neq j$.[16] Moreover, assuming that one hour of work produces one unit of commodity in both countries (i.e. $h^i = x^i, \forall i = a, b$, where h^i is a measurement of hours worked in the i^{th} country's DM), a producer in the i^{th} country gets a desutility of production $c(x^i)$, where $c'_{x^i}(x^i) > 0$,

[15] In what follows, without loss of generality, we assume that $\sigma^{ij} = \sigma^{ji}$. Also, if $\sigma^{ij} = \sigma^{ji} = 0$, we are back at the LW model.

[16] The importance of these assumptions will become evident to the reader in appendix B wherein I prove the existence and uniqueness of 'the' interior solution.

$c''_{x^i x^i}(x^i) > 0, \forall i = a, b.$

In addition, to make money essential in the DMs we need to assume that individuals are randomly bi-laterally matched and that trading histories are private information. These assumptions have two very important consequences for the DMs. Firstly, sellers require immediate compensation in terms of a widely accepted medium of exchange. Given that the two different DMs are located in different countries, this medium of exchange is not the same for both markets.[17] Moreover, financial markets in the DMs are closed and agents are only able to exchange money for commodities and commodities for money. Consequently, in this model agents need to have a portfolio comprised of country A's and B's currency to be able to participate in either market. It is worth noting that individuals do not *a priori* know in which market they will have to participate or whether they are consumers so they must hold both currencies for trading and precautionary purposes. The coexistence of these two different currencies in agents' portfolio holdings is what differentiates this model from the LW model and other similar models existing in the literature (Arouba et.al.2007; Craig and Rocheteau 2008; Chiu and Molico 2007; Rocheteau and Wright 2005; Berensten *et.al.*2006). The second consequence of these assumptions for the DMs is that, because trade is not centralized, the terms of trade are not given by a 'Walrasian auctioneer' but by a bargaining solution. The inclusion of only one foreign market in this sub-period is only a simplification and the analysis can easily be extended to include multiple external markets. Or, more generally the foreign market could be thought as an index of multiple international markets.

The second sub-period is comprised of a Walrasian general equilibrium market where all agents produce and consume a general non-storable good and exchange in terms of exogenously given prices. In addition, the foreign commodities market is closed but financial domestic and foreign markets are open. That is, agents are able to consume the domestic commodity or exchange their production

[17] This is the cash-in-advance assumption indicated in the introducing section.

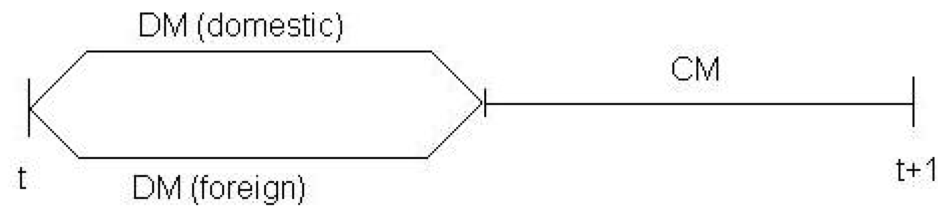

Figure 1: The i^{th} country per period market structure

for either countries' currency. A representative agent in the i^{th} country gets $U(X^i)$ of utility from consumption where $U'(X^i) > 0, U'(0) = \infty, U'(+\infty) = 0$ and $U''(X^i) < 0$.[18] Assuming a linear technology (i.e. $C(H^i) = H^i$, where C is the cost function and H^i is an index of hours worked in the i^{th} country's CM), a representative agent in the i^{th} country gets H^i desutility from production. The assumption of a linear cost function is important as it simplifies the analysis by forcing the real money holdings distribution to be degenerate across individuals every period (see proposition 1 and its proof in appendix B).

There are also two types of taxes in the CM. The first tax consists on a transfer of domestic fiat money from the Central Bank of the i^{th} country to the agents of that country.[19]This transfer of money is given by $M^i_{t+1} = \tau^i_{1t} \cdot M^i_t$ where $\tau^i_{1t} = \gamma^i_t - 1$ denotes the rate of money transfer (note that it could either be positive -a transfer- or negative -a tax), M^i_t denotes the total money stock and $\gamma^i_t > 0$ denotes the rate of money growth in the i^{th} country in period t. The second tax is on individual holdings of foreign currency and is given by $T^i_{2t} = \tau^i_{2t} \cdot b^i_t$ where T^i_{2t} denotes the total tax, $\tau^i_{2t} \geq 0$ denotes the tax rate and $b^i_t \geq 0$ denotes the individual holdings of foreign currency in the i^{th} country in period t. Unlike the transfer (or tax) of domestic money, the tax on private holdings of foreign currency is distortionary whenever $\tau^i_{2t} > 0$.[20]

[18]As before this is true $\forall i = a, b$.

[19]It is important to understand that only agents in the i^{th} country receive a money transfer from the i^{th} country's Central Bank.

[20]The reason for this distortion is that **private** money holdings are taxed and thus, the tax affects individual decisions at the margin.

Figure 1 shows the per period market structure of both countries. From that figure, we can observe that both decentralized markets happen simultaneously while the centralized market takes place in the following sub-period. Both sub-periods together are part of a representative agent's per-period instantaneous utility function which is given as follows:

$$\Omega(x_t^i, x_t^j, h_t^i, X_t^i, H_t^i) = u(x_t^i, x_t^j) - c(x_t^i) + U(X_t^i) - H_t^i, \forall i, j = a, b; i \neq j \qquad (4)$$

where all the components were explained above. There are two important properties to note from this utility function. The first property is that the DM markets and the CM market are additively separable in this utility function. This assumption is not crucial for the existence and uniqueness of an interior solution but it greatly simplifies the model allowing us to solve each sub-period separately (see Lagos and Wright 2004; Arouba *et.al.*2007[1]).[21] The second property is that the instantaneous utility function, the utility functions for each sub-period as well as cost functions are the same for all the individuals in both countries. This indicates that individuals are not only homogeneous within a country, but also across countries.

This finalizes the description of the per-period market structure and trading environment. Though we are ready to define the equilibrium conditions and solve for the agent's maximization problem, it is constructive to first solve for the planner's solution so as to observe in which scenario the representative agent's solution is able to replicate it. Solving for this solution and investigating its properties is the task of the next sub-section.

3b. The planner's solution

In solving for the efficient solution, we are most interested in finding a stationary equilibrium so we can drop the subscript t from our maximization problem. In addition, under the World planner's solution money is not essential. Therefore, assuming

[21] The utility function for the two DM is also additively separable as indicated by the following aforementioned condition: $u''_{x^i x^j}(x^a, x^b) = 0 \forall x^a, x^b \in \Re^+$. The simplifying importance of this assumption will not become clear to the reader until I prove the existence and uniqueness of the interior solution in appendix B.

that J is the value function, the planner's maximization problem is as follows:

$$J(\cdot) = \max_{X^a, X^b, q^a, q^b, H^a, H^b} U(X^a) - H^a + U(X^b) - H^b + (\sigma^{aa} + \sigma^{ba}) \cdot u(q_c^a, 0)$$

$$+(\sigma^{bb} + \sigma^{ab}) \cdot u(0, q_c^b) + (\sigma^{aa} + \sigma^{ba}) \cdot [-c(q_s^a)] + (\sigma^{bb} + \sigma^{ab}) \cdot [-c(q_s^b)] (5)$$

S.T.

$$\text{X}^a = H^a - G^a (6)$$

$$\text{X}^b = H^b - G^b (7)$$

where q_c^i is the total quantity of the i^{th} country's commodities consumed by both domestic and foreign consumers (c denotes consumers) and q_s^i is the total quantity of goods produced by sellers in that country. Furthermore, $G = G^a + G^b$ is the World planner's net revenue requirement which is composed of lump-sum taxes in country A (G^a) and in country B (G^b).

Substituting both equations 6 and 7 as well as the following market clearing condition $q_c^i = q_s^i = q^i, \forall i = a, b$, into the value function (5) and solving, I obtain:[22]

$q^a : (\sigma^{aa} + \sigma^{ba}) \cdot u'(q^a) = (\sigma^{aa} + \sigma^{ba}) \cdot c'(q^a)$

$q^{a^{FB}}$ solves $u'(q^a) = c'(q^a)$

[22] At this stage of the analysis it is important to introduce a a subtle but nonetheless relevant distinction that it is required to solve the planner's solution. As above mentioned, both $u'_{x^a}(0, x^b) = \infty$ and $u'_{x^b}(x^a, 0) = \infty$ are important assumptions for the existence and uniqueness of an interior solution. The reader should note that these assumptions consists in first taking the derivative of the function with respect to either x^a or x^b and then setting the value of the corresponding variable equals to zero. However, if either variable is first set to zero and then the derivative with respect to that variable is taken, the value of the derivative of the function is not infinity, but zero. The utility function that Lagos and Wright (2004) presented in their numerical analysis is an example of an utility function with this characteristic (p.477). Here, I extend their function to include the second DM and provide an example of a multi variable function with this characteristic:

$$u(x^a, x^b) = \frac{(x^a - b)^{1-\eta} - b^{1-\eta}}{1 - \eta} + \frac{(x^b - r)^{1-\gamma} - r^{1-\gamma}}{1 - \gamma} \Rightarrow u'_{x^a}(x^a, x^b) = (x^a - b)^{-\eta}$$

$$\Rightarrow u'_{x^a}(0, x^b) = \frac{1}{b^{-\eta}} \Rightarrow \lim_{b^{-\eta} \to 0} \frac{1}{b^{-\eta}} \to \infty$$

However,

$$u(x^a, x^b) = \frac{(x^a - b)^{1-\eta} - b^{1-\eta}}{1 - \eta} + \frac{(x^b - r)^{1-\gamma} - r^{1-\gamma}}{1 - \gamma} \Rightarrow u(0, x^b) = \frac{(x^b - r)^{1-\gamma} - r^{1-\gamma}}{1 - \gamma} \Rightarrow u'_{x^a}(0, x^b) = 0$$

$q^b : (\sigma^{bb} + \sigma^{ab}) \cdot u'(q^a) = (\sigma^{bb} + \sigma^{ab}) \cdot c'(q^b)$

$q^{b^{FB}}$ solves $u'(q^b) = c'(q^b)$

$X^a : U'(X^a) - 1 = 0$

$X^{a^{FB}}$ solves $U'(X^a) = 1$

$X^b : U'(X^b) - 1 = 0$

$X^{b^{FB}}$ solves $U'(X^b) = 1$ (8)

where FB denotes the First Best solution.

The solution to the planner's maximization problem is the benchmark to which other solutions without a central authority are compared. The purpose of this comparison is to observe whether efficiency could be achieved in the absence of a social planner. It is easy to see both mathematically and intuitively why the planner's solution is efficient. In terms of mathematics, we can observe from the previous solution to the model that in both sub-periods commodities are consumed until the marginal utility of consumption equals the marginal cost of production.[23] Intuitively, a 'benevolent' social planner is able to force consumers and producers to consume and produce until efficiency is attained. Attaining efficiency by solving the model from a representative agent's perspective requires an extra set of assumptions since agents can not be forced to either produce or consume. The following sub-section explores the representative agent's steady state equilibrium and its implications for efficiency.

3c. The representative agent's solution

The first step to solve for the equilibrium is to define it in the context of this model. Assuming that $V(\hat{m}, \hat{b})$ denotes the agent's value function at the beginning of the DMs and $W(m, b)$ denotes the agent's value function at the beginning of the CM from entering each market with $\hat{m}$ and m units of domestic currency, $\hat{b}$ and b units of foreign currency respectively, an equilibrium in this model is a choice of $(X_t^i)_{t=1}^{\infty}, (H_t^i)_{t=1}^{\infty}, (\hat{m}_t^i)_{t=1}^{\infty}, (m_t^i)_{t=1}^{\infty}, (\hat{b}_t^i)_{t=1}^{\infty}, (b_t^i)_{t=1}^{\infty}, (q_{ct}^{ii})_{t=1}^{\infty}, (q_{ct}^{ij})_{t=1}^{\infty}, (db_t^{ii})_{t=1}^{\infty}, (db_t^{ij})_{t=1}^{\infty}, \forall i, j = a, b, i \neq j$ such that:

[23]Recall that the cost of production in the CM is linear, thus the marginal cost of producing one more unit is simply one.

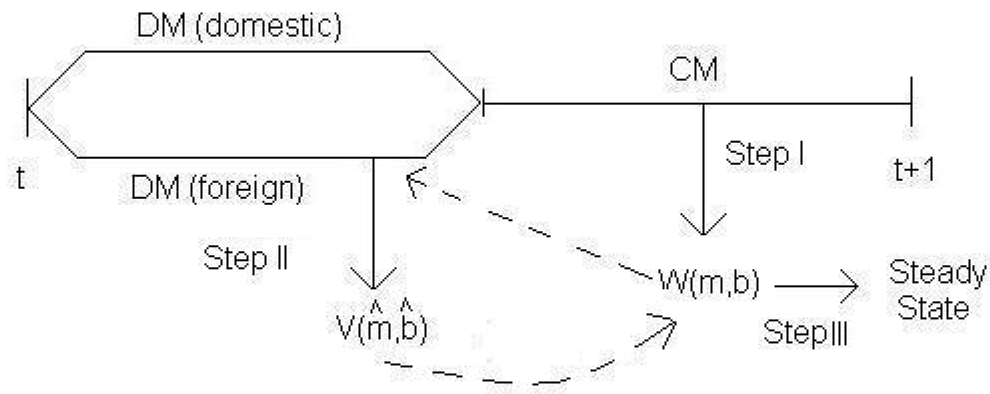

Figure 2: Solution to the agent's problem by backward induction

i) it solves the agent's maximization problem defined below, and
ii) it fulfills the following market clearing conditions: both $M_t^i = \int_0^1 (\hat{m}_t^i + \hat{b}_t^j) dF(\hat{M_t^i})$ and $M_t^i = \int_0^1 (m_t^i + b_t^j) dF(M_t^i)$, $q_{ct}^{ii} + q_{ct}^{ji} = q_{st}^i$ and $ds_t^i = db_t^{ii} + db_t^{ji}$, $\forall i, j = a, b; i \neq j; t = 1, \cdots, \infty$
where q_{ct}^{ii} denotes the quantities of domestic goods and q_{ct}^{ij} denotes the quantities of foreign goods consumed by people; db_t^{ii} is the amount of money given in exchange for those domestic goods and db_t^{ij} is the amount of money given in exchange for those foreign commodities in the second sub-period in the i^{th} country at time t. Moreover, $F(\hat{M_t^i})$ and $F(M_t^i)$ are functions indicating the distribution of the i^{th} country's currency in each sub-period respectively at time t.

The market clearing conditions simple indicate that the supply of money, $(M_t^i)$and commodities (q_{st}^i) in the DMs must equal the demand for them. In addition, it also indicates that the total amount of money that consumers pay for the commodities that they consume in the second sub-period $(db_t^{ii} + db_t^{ji})$ has to be the same as the total amount received by producers (ds_t^i). These implications are of utmost importance to rule out other solutions to the problem that are not sustainable through time. To solve for this equilibrium solution, I am going to use backward induction.

That is, as figure 2 shows, I am going to first solve for the CM, then for the DMs and subsequently re-substitute the solution in the CM and obtain the steady state equilibrium.

3c.i. The Centralized Market

A representative agent in the i^{th} country entering the CM has the following value function:

$$W_t(m,b) = \max_{X_t^i, H_t^i, \hat{m}_{t+1}^i, \hat{b}_{t+1}^i} U(X_t^i) - H_t^i + \beta \cdot V_{t+1}(\hat{m}_{t+1}^i, \hat{b}_{t+1}^i) \quad (9)$$

S.T

$$X_t^i = H_t^i + (\frac{m_t^i - \hat{m}_{t+1}^i}{P_t^i}) + \tau_{1t}^i \cdot \frac{M_t^i}{P_t^i} + \frac{1}{P_t^i \cdot \epsilon_t^i} \cdot [b_t^i(1-\tau_{2t}^i) - \hat{b}_{t+1}^i], \forall i = a, b; \tau_{1t}^i, \tau_{2t}^i \geq 0$$

where $\epsilon_t^i = \frac{1/P_t^j}{1/P_t^i} = \frac{P_t^i}{P_t^j}$ is the nominal exchange rate (the price of j's currency per unit of i's currency)[24] at time t and the other notations were explained before.

The budget constraint indicates that a representative individual is not able to consume (the left side of the equality) more than his income (the right side of the equality). It is also worth noting that in maximizing his utility, a representative agent will never consume less than his income and therefore, the budget constraint is not an inequality but an equality. Substituting this equality in the value function (9) and solving for the first order conditions, I obtain:

$X_t^i : U'(X_t^i) = 1$

$\hat{m}_{t+1}^i : \frac{1}{P_t^i} = \beta \cdot V'_{t+1\hat{m}_{t+1}^i}(\hat{m}_{t+1}^i, \hat{b}_{t+1}^i)$

$\hat{b}_{t+1}^i : \frac{1}{P_t^i \cdot \epsilon_t^i} = \frac{1}{P_t^j}\beta \cdot V'_{t+1\hat{b}_{t+1}^i}(\hat{m}_{t+1}^i, \hat{b}_{t+1}^i)(10)$

There are several important points to note from the solution to the representative agent's CM. First of all, in comparing this solution to the planner's solution, we can observe that the individual consumption of the CM's commodity is efficient. This efficiency relies on the fact that the CM is a 'Walrasian' frictionless market where

[24]Note that an appreciation of i^{th} country's currency is equivalent to an increase in ϵ_t^i

agents are able to choose the hours worked (H_t^i) so as to maximize their utility. The second important point is that neither the choice of X_t^i depends on t, nor the choice of $\hat{m}_{t+1}^i$ and $\hat{b}_{t+1}^i$ depend on their past values. These three conditions together with the existence and uniqueness of an interior solution to $H_t^i, \forall t = 1, \cdots, \infty$ proved in appendix B result in a degenerate distribution of real money holdings at the beginning of the first market across time. In addition, these conditions indicate that the value function is linear in both parameters. Therefore, I can write the envelope conditions and rewrite the value function as follows:

$W'_{tm_t^i}(m_t^i, b_t^i) = \frac{1}{P_t^i}$

$W'_{tb_t^i}(m_t^i, b_t^i) = \frac{1-\tau_{2t}^i}{P_t^i \cdot \epsilon_t^i} = \frac{1-\tau_{2t}^i}{P_t^j}$

$W_t(m_t^i, b_t^i) = \bar{W} + (\frac{1-\tau_{2t}^i}{P_t^j}) \cdot b_t^i + \frac{1}{P_t^i} \cdot m_t^i$ (11)

where $\bar{W}$ denotes any constant term.

3c.ii. The Decentralized Markets

At the beginning of the DMs, agents are under a 'veil of ignorance' with respect to which type they are (i.e. sellers, buyers in the domestic market, buyers in the foreign market or non-traders). The only things agents know are in which country they reside and the probabilities assigned to each type in each country. Consequently, an agent in either country has an expected and not a certain lifetime value function. As follows, I only analyze a representative agent in country A, however, the reader should keep in mind that the exact same analysis applies to a representative agent in country B. As a matter of fact, when analyzing the foreign market, I need to introduce a result from the representative agent maximizing problem in country B which is equivalent to the one obtained in country A's domestic market. The following is the expected lifetime utility of a representative agent in country A:[25]

[25] For simplicity, I rule out the possibility of barter trade (i.e. double coincidence of wants) (see Arouba 2007)

$$V_t(\hat{m}_t^a,\hat{b}_t^a) = \sigma^{aa}\cdot V_t^{ca}(\hat{m}_t^a,\hat{b}_t^a) + \sigma^{ab}\cdot V_t^{cb}(\hat{m}_t^a,\hat{b}_t^a) + (\sigma^{aa}+\sigma^{ba})\cdot V_t^{s}(\hat{m}_t^a,\hat{b}_t^a)$$

$$+\ (1\text{-}2\cdot\sigma^{aa}-\sigma^{ab}-\sigma^{ba})\cdot W_t(\hat{m}_t^a,\hat{b}_t^a)(12)$$

The various superscripts denote the different agent's value function after the idiosyncratic shocks has occurred which are given as follows:

. Consumer in the domestic market$\Rightarrow V_t^{ca} = u(q_{ct}^{aa}) + W_t(\hat{m}_t^a - db_t^{aa},\hat{b}_t^a)(13)$

. Consumer in the foreign market $\Rightarrow V_t^{cb} = u(q_{ct}^{ab}) + W_t(\hat{m}_t^a,\hat{b}_t^a - db_t^{ab})(14)$

. Seller $\Rightarrow V_t^s = [-c(q_{st}^a)] + W_t(\hat{m}_t^a + ds_t^a,\hat{b}_t^a)(15)$

To solve for the DMs, we must first determine the quantities traded as well as the terms of trade in both decentralized markets (the domestic and the foreign one). To that end, we need to solve for the bargain solution after matching has occurred. In this case, the bargain solution is given by a General Nash Bargain Solution (for different bargain solutions and their impact in the equilibrium solution of search models, see Arouba *et.al.* 2007[1]). Given that both markets are additively separable (see above), I can solve for each DM separately.

3c.ii.a. The Domestic Decentralized Market

From (12) and (13), we can observe that the surplus of an agent that trades in the domestic market over an agent that does no trade is as follows:

$$u(q_{ct}^{aa}) + W_t(\hat{m}_t^a - db_t^{aa},\hat{b}_t^a) - W_t(\hat{m}_t^a,\hat{b}_t^a)$$

Given the linearity of the CM solution,(11), and the market clearing conditions, I can rewrite this surplus as follows:

$$u(q_{ct}^{aa}) - \frac{1}{P_t^a}\cdot db_t^{aa}(16)$$

Similarly, I can write the sellers' surplus as follows (from (11),(12),(15) and market clearing conditions):

$$[-c(q_{ct}^{aa}+q_{ct}^{ba})]\frac{1}{P_t^a}\cdot(db_t^{ii}+db_t^{ji})(17)$$

Combining both (16) and (17), I obtain the General Nash Bargain problem:

$$\max_{q_{ct}^{aa},db_t^{aa}}[u(q_{ct}^{aa})-\frac{1}{P_t^a}\cdot db_t^{aa}]^{\theta_1}\cdot[-c(q_{ct}^{aa}+q_{ct}^{ba})+\frac{1}{P_t^a}\cdot(db_t^{aa}+db_t^{ba})]^{1-\theta_1} \quad (18)$$

S.T.

$$db_t^{aa}\leq \hat{m}_t^a$$

where θ_1 represents the buyer's relative market power.

The budget constraint simply states that a consumer is not able to spend more money that she brings from the previous period's centralized market. Lemma 1 in appendix A show that an individual will never spend less than she brings and therefore, the budget constraint becomes an equality. Substituting the equality into (18) and solving for the first order condition, we obtain:

$$q_{ct}^{aa}:\theta_1\cdot(N)^{\theta_1-1}\cdot u'(q_{ct}^{aa})\cdot(L)^{1-\theta_1}=(N)^{\theta_1}\cdot(1-\theta_1)\cdot(L)^{-\theta_1}\cdot c'(q_{ct}^{aa})$$

where $N=u(q_{ct}^{aa})-\frac{\hat{m}_t^a}{P_t^a}$ and $L=[-c(q_{ct}^{aa})]+\frac{\hat{m}_t^a}{P_t^a}$.[26]

The quantities of domestic commodities that maximize the bargain solution are implicitly given by the following equation:

$$Z_t^d(q_{ct}^{aa})=\frac{\hat{m}_t^a}{P_t^a}=\frac{\theta_1\cdot u'(q_{ct}^{aa})\cdot c(q_{ct}^{aa})+(1-\theta_1)\cdot c'(q_{ct}^{aa})\cdot u(q_{ct}^{aa})}{\theta_1\cdot u'(q_{ct}^{aa})+(1-\theta_1)\cdot c'(q_{ct}^{aa})} \quad (19)$$

This equation also indicates the domestic demand for real money balances in country A (this is indicated by the superscript d). It is worth noting that q_{ct}^{aa} and $\hat{m}_t^a$ are not jointly determined since $\hat{m}_t^a$ is pre-determined in the previous period's CM.

3c.ii.b.The Foreign Decentralized Market

The analysis of the foreign DM is similar to the previous analysis of the domestic DM. There are, however, two distinctions worth mentioning. First, as aforementioned, the existence of a tax on individual holdings of foreign currency (τ_{2t}), unlike the transfer of domestic currency (τ_{1t}), affects the decision of agents to hold real money balances at the margin (see the CM envelope condition for b_t^i). To simplify

[26]Note that in this market $q_{ct}^{ba}=0$

this problem without loss of generality, I assume this tax to be zero so as to not distort individual decisions at the margin. However, it is important to note that in reality the taxation of foreign notes is a significant device that governments use to both collect revenue and curve undesirable economic behavior (see for instance Camera *et.al.* 2004).

The second distinction are the components of the General Nash Bargain Solution which is given as follows:

$$\max_{q_{ct}^{ab}, db_t^{ab}} [u(q_{ct}^{ab}) - \frac{1}{P_t^b} \cdot db_t^{ab}]^{\theta_2} \cdot [-c(q_{ct}^{bb} + q_{ct}^{ab}) + \frac{1}{P_t^b} \cdot (db_t^{bb} + db_t^{ab})]^{1-\theta_2} \quad (20)$$

S.T.

$$db_t^{ab} \leq \hat{b}_t^a$$

where θ_2 represents the buyer's relative market power. The differences in the components of the bargain solution arise because buyers must hold country's B money ($\hat{b}_t^a$) to participate in the foreign market and they must bargain with foreign, not domestic producers.

Following the same procedures applied to the DM domestic market, the quantities of commodities that solve the bargain problem is implicitly given by the following equation:[27]

$$Z_t^f(q_{ct}^{ab}) = \frac{\hat{b}_t^a}{P_t^b} = \frac{\theta_2 \cdot u'(q_{ct}^{ab}) \cdot c(q_{ct}^{ab}) + (1-\theta_2) \cdot c'(q_{ct}^{ab}) \cdot u(q_{ct}^{ab})}{\theta_2 \cdot u'(q_{ct}^{ab}) + (1-\theta_2) \cdot c'(q_{ct}^{ab})} \quad (21)$$

As before, this identity also indicates the demand for real foreign money balances in country A (this is denoted by the superscript f).

After solving for the quantities traded and the terms of trade in both DMs separately, we are ready to combine them to solve for the agent's expected value function at the beginning of the first sub-period.

[27] Appendix A, lemma 1 shows that the constraint for this General Nash Bargain problem is also binding

3c.ii.c. The DMs revisited

In combining the solutions to the domestic and foreign DMs (equations 19 and 21), I first of all obtain the total demand for real money balances in each country given by the following function:

$$R^i(q_{ct}^{ii}, q_{ct}^{ji}) = Z_t^d(q_{ct}^{ii}) + Z_t^f(q_{ct}^{ji}) = \frac{\hat{m}_t^i}{P_t^i} + \frac{\hat{b}_t^j}{P_t^i} =$$

$$\frac{\theta_1 \cdot u'(q_{ct}^{ii}) \cdot c(q_{ct}^{ii}) + (1-\theta_1) \cdot c'(q_{ct}^{ii}) \cdot u(q_{ct}^{ii})}{\theta_1 \cdot u'(q_{ct}^{ii}) + (1-\theta_1) \cdot c'(q_{ct}^{ii})} + \frac{\theta_1 \cdot u'(q_{ct}^{ji}) \cdot c(q_{ct}^{ji}) + (1-\theta_1) \cdot c'(q_{ct}^{ji}) \cdot u(q_{ct}^{ji})}{\theta_1 \cdot u'(q_{ct}^{ji}) + (1-\theta_1) \cdot c'(q_{ct}^{ji})} \quad (22)$$

$\forall i, j = a, b; i \neq j$

The total demand for real money balances for each country is only important for future references and not for finding a workable identity of the value function at the beginning of the first sub-period. Instead, in solving for this value function, I will use the demand for domestic and foreign real money balances separately as they were first stated in the General Nash Bargain Solutions.

From these solutions, it is also important to note that the quantities traded ultimately depend on the amount of each country's currency that buyers bring to the first sub-period. This is observed from the equality of the budget constraint in both bargaining problems (the proofs of these equalities are shown in appendix A, lemma 1). Consequently, after the idiosyncratic shocks have occurred, the quantities sold by sellers do not depend on their but on the buyer's real money holdings. Nevertheless, a careful inspection of the intertemporal problem reveals that agents do not *a priori* know which type they will be in the DMs, so they still account for the cost function in deciding the amount of each currency to bring to the DMs (this is indicated by the marginal cost term that appears in both Z_t^d and Z_t^f). To include these two properties and the General Nash Bargain solutions into the analysis, the representative agent's value function, given that everybody else is choosing $m_t'^i$ and $b_t'^i$, can be rewritten as follows:

$V_t(\hat{m}_t^a, \hat{b}_t^a) = \sigma^{aa} \cdot [u(q_{ct}^{aa}(\hat{m}_t^a)) + W_t(0, \hat{b}_t^a)] + \sigma^{ab} \cdot [u(q_{ct}^{ab}(\hat{b}_t^a)) + W_t(\hat{m}_t^a, 0)] + (\sigma^{aa} + \sigma^{ba}) \cdot [[-c(q_{st}^a(m_t'^a, b_t'^b)] + W_t(\hat{m}_t^a + ds_t^a(m_t'^a, b_t'^b), \hat{b}_t^a)] + (1 - 2 \cdot \sigma^{aa} - \sigma^{ab} - \sigma^{ba}) \cdot W_t(\hat{m}_t^a, \hat{b}_t^a) \quad (23)$

Substituting equation (11) into equation (23):[28]

$V_t(\hat{m}_t^a, \hat{b}_t^a) = \sigma^{aa} \cdot [u(q_{ct}^{aa}(\hat{m}_t^a)) + \bar{W}_t + \frac{\hat{b}_t^a}{P_t^b}] + \sigma^{ab} \cdot [u(q_{ct}^{ab}(\hat{b}_t^a)) + \bar{W}_t + \frac{\hat{m}_t^a}{P_t^a}] + (\sigma^{aa} + \sigma^{ba}) \cdot [[-c(q_{st}^a(m_t'^a, b_t'^b)] + \bar{W}_t + \frac{\hat{m}_t^a + ds_t^a(m_t'^a, b_t'^b)}{P_t^a} + \frac{\hat{b}_t^a}{P_t^b}] + (1 - 2 \cdot \sigma^{aa} - \sigma^{ab} - \sigma^{ba}) \cdot [\bar{W}_t + \frac{\hat{m}_t^a}{P_t^a} + \frac{\hat{b}_t^a}{P_t^b}] \quad (24)$

Since by construction this value function is at least twice continuously differentiable in both variables, I can calculate the marginal value of bringing one more unit of each country's currency to the DMs. This is given by the following first derivatives of equation (24):

$$V'_{t\hat{m}_t^a}(\hat{m}_t^a, \hat{b}_t^a) = \sigma_{aa} \cdot [\frac{u'(q_{ct}^{aa})}{P_t^a \cdot Z^{d\prime}(q_{ct}^{aa})}] + \frac{(1 - \sigma_{aa})}{P_t^a} \quad (25)$$

$$V'_{t\hat{b}_t^a}(\hat{m}_t^a, \hat{b}_t^a) = \sigma_{ab} \cdot [\frac{u'(q_{ct}^{ab})}{P_t^b \cdot Z^{I\prime}(q_{ct}^{ab})}] + \frac{(1 - \sigma_{ab})}{P_t^b} \quad (26)$$

where:

$Z^{d\prime} = [\sigma_{aa} \cdot [u''(q_{ct}^{aa}) \cdot c(q_{ct}^{aa}) + u'(q_{ct}^{aa}) \cdot c'(q_{ct}^{aa})] + (1 - \sigma_{aa}) \cdot [u(q_{ct}^{aa}) \cdot c''(q_{ct}^{aa}) + u'(q_{ct}^{aa}) \cdot c'(q_{ct}^{aa})]] \cdot [\sigma_{aa} \cdot u'(q_{ct}^{aa}) + (1 - \sigma_{aa}) \cdot c'(q_{ct}^{aa})] - [\sigma_{aa} \cdot u'(q_{ct}^{aa}) \cdot c(q_{ct}^{aa}) + (1 - \sigma_{aa}) \cdot c'(q_{ct}^{aa}) \cdot u'(q_{ct}^{aa})] \cdot [\sigma_{aa} \cdot u''(q_{ct}^{aa}) + (1 - \sigma_{aa}) \cdot c''(q_{ct}^{aa})]] \cdot [\sigma_{aa} \cdot u'(q_{ct}^{aa}) + (1 - \sigma_{aa}) \cdot c'(q_{ct}^{aa})]^{-2}$

and

$Z^{I\prime} = [\sigma_{ba} \cdot [u''(q_{ct}^{ba}) \cdot c(q_{ct}^{ba}) + u'(q_{ct}^{ba}) \cdot c'(q_{ct}^{ba})] + (1 - \sigma_{ba}) \cdot [u(q_{ct}^{ba}) \cdot c''(q_{ct}^{ba}) + u'(q_{ct}^{ba}) \cdot c'(q_{ct}^{ba})]] \cdot [\sigma_{ba} \cdot u'(q_{ct}^{ba}) + (1 - \sigma_{ba}) \cdot c'(q_{ct}^{ba})] - [\sigma_{ba} \cdot u'(q_{ct}^{ba}) \cdot c(q_{ct}^{ba}) + (1 - \sigma_{ba}) \cdot c'(q_{ct}^{ba}) \cdot u'(q_{ct}^{ba})] \cdot [\sigma_{ba} \cdot u''(q_{ct}^{ba}) + (1 - \sigma_{ba}) \cdot c''(q_{ct}^{ba})]] \cdot [\sigma_{ba} \cdot u'(q_{ct}^{ba}) + (1 - \sigma_{ba}) \cdot c'(q_{ct}^{ba})]^{-2}$

This set of equations finalizes the analysis of the first sub-period composed by both the foreign and domestic DMs. To summarize the analysis, I firstly derive the *a priori* representative agent's expected value function (equation 12). It is expected because individuals at the beginning of the DMs do not know their type until after the idiosyncratic shock has occurred. This expected value function comprises the probability distribution assigned to each type as well as each type's after shock-value

[28] Assuming, as before, that $\tau_{2t}^i = 0$

function (equations 13, 14 and 15). Thereafter, I derived the form of these after shock-value function by calculating the General Nash Bargain solution after matching has occurred (equations 19, 21) which I subsequently substitute in the expected value function (equation 23). In calculating the General Nash Bargain solution, not only did I obtain an implicit function for the closed form solution to the quantities traded and the terms of trade in each DM market (equation 19, 21), but also I obtained the domestic (equation 19), foreign (equation 21) and total (equation 22) demand for real money balances in each country. Finally, I substitute the envelope conditions of the CM in the expected value function (equation 24) and I took the first derivatives of this function to obtain the marginal value of an extra dollar of each currency brought to the DMs (equations 25 and 26). The next steps are to substitute the results obtained in the DMs into the CM to obtain an identity from which we can derive the steady state equilibrium of the model (see figure 2).

3c.iii. The Steady State Monetary Equilibrium

Substituting equations 25 and 26 into the first order conditions for $\hat{m}_t^a$ and $\hat{b}_t^a$ in the CM (equation 10), I get the following equations:[29]

$$\frac{1}{P_{t-1}^a} = \beta \cdot \sigma_{aa} \cdot [\frac{u'(q_{ct}^{aa})}{Z^{d\prime}(q_{ct}^{aa}) \cdot P_t^a}] + \beta \cdot (1 - \sigma_{aa}) \cdot \frac{1}{P_t^a}$$

$$\frac{1}{P_{t-1}^b} = \beta \cdot \sigma_{ab} \cdot [\frac{u'(q_{ct}^{ab})}{Z^{I\prime}(q_t^{ab}) \cdot P_t^b}] + \beta \cdot (1 - \sigma_{ab}) \cdot \frac{1}{P_t^b}$$

In solving for this model, we are most interested in steady states equilibriums of both real variables and the rate of growth of nominal variables. Therefore, eliminating the time subscript and rearranging, I obtain:

$$\frac{1+\pi^a}{\beta} = \sigma_{aa} \cdot [\frac{u'(q_c^{aa})}{Z^{d\prime}(q_c^{aa})}] + (1 - \sigma_{aa}) (27)$$

$$\frac{1+\pi^b}{\beta} = \sigma_{ab} \cdot [\frac{u'(q_c^{ab})}{Z^{I\prime}(q_c^{ab})}] + (1 - \sigma_{ab}) (28)$$

[29] I am actually substituting into the previous period CM, see time subscript

Proposition 1:*There exists a steady state monetary equilibrium in which agents choose* H_t^a *and* H_t^b *every period so as to make the monetary distributions* $F(\hat{M}_t^a)$ *and* $F(\hat{M}_t^b)$ *degenerate and* $q_c^{aa^{ss}}, q_c^{bb^{ss}}, q_c^{ab^{ss}}, q_c^{ba^{ss}}$ *solve equations 27, 28 (ss denotes the steady state solution).*[30] *That is, agents choose* H_t^a *and* H_t^b *such that* $\frac{1}{P_t^a}, \frac{1}{P_t^b} \in (0, +\infty), \forall t = 1, \cdots, \infty$ *and* $\sum_{t=1}^{\infty}(\gamma^a)^{t-1} \cdot (\hat{m}_1^a + \hat{b}_1^b) = \sum_{t=1}^{\infty} \hat{M}_t^a$ *and* $\sum_{t=1}^{\infty}(\gamma^b)^{t-1} \cdot (\hat{m}_1^b + \hat{b}_1^a) = \sum_{t=1}^{\infty} \hat{M}_t^b$*,where* γ^i *denotes the constant rate of money growth in the* i^{th} *country and the subscript 1 denotes the value assigned to the choice variables in the first period of the intertemporal problem.* Appendix B shows a thorough proof of this proposition.

As in the LW model, proposition 1 shows the flexibility of this model to explain the complex individual microeconomic decision process in an environment that is simple enough to investigate the consequences of monetary policy. This simplicity is best reflected in the fact that the distribution of real money holdings, real variables as well as the rate of growth of nominal variables are constant through time. As a consequence of the simplicity of this model's environment, the two aforementioned 'fundamental' questions of modern monetary economics can be easily analyzed while providing strong microeconomic foundations to the analysis. This property is in great contrast with most of the pre-LW monetary economics literature which provides difficult and intractable models of indivisible money (most notable Shi 1997; Trejos and Wright 1995 for which there is not even a closed form solution to the model except under very strict assumptions of the form of this solution).

Even though the environment in which the previous model operates is similar to the LW, the addition of one more decentralized market with a different medium of exchange could potentially lead to different answers to the two questions that modern monetary economics aims to answer. Therefore, the following section analyzes the implications, assumptions and results of the Search Theoretical model of market segmentation for the two 'fundamental' questions of modern monetary economics.

[30] They also solve the same equations for country B.

4 Implications of the model of market segmentation

To address the two 'fundamental' questions of modern monetary economics, this section is divided into two different parts. The first part deals with the question of whether the Friedman's rule of zero interest rate is still the optimal monetary policy under this new framework. Afterward, the second part theoretically compares the measurement of welfare loss of deviating from this optimal policy between this and the LW model.

The first step in using this model to derive the optimal monetary policy, is to investigate equations 27 and 28. Substituting equation 2, assuming that agent's private discounting factor equals the market discounting factor (i.e.$\beta = \frac{1}{1+r}$) and that q_c^{aa}, q_c^{ba} solve the steady state equilibrium, I can rewrite them as follows:

$$\frac{i}{\sigma_{aa}} + 1 = \frac{u'(q_c^{aa^{ss}})}{Z^{d\prime}(q_c^{aa^{ss}})} (29)$$

$$\frac{i}{\sigma_{ab}} + 1 = \frac{u'(q_c^{ab^{ss}})}{Z^{I\prime}(q_c^{ab^{ss}})} (30)$$

From both equations (29) and (30), we can observe that whenever interest rates in both countries are equal zero (the Friedman's rule), the model achieves optimality (i.e. $u'(q_c^{aa^{ss}}) = Z^{d\prime}(q_c^{aa^{ss}})$ and $u'(q_c^{ab^{ss}}) = Z^{I\prime}(q_c^{ab^{ss}})$). Efficiency, however, requires two additional assumptions. These assumptions are that consumers have all the market power both domestically and internationally (i.e. both $\sigma_1 = 1$ and $\sigma_2 = 1$). Whenever this is the case we can observe that $u'(q_c^{aa^{ss}}) = c'(q_c^{aa^{ss}}), u'(q_c^{ab^{ss}}) = c'(q_c^{ab^{ss}})$ and $q_c^{aa^{ss}} = q^{a^{FB}}, q_c^{ab^{ss}} = q^{b^{FB}}$ solves the agent's maximization problem.The previous analysis demonstrated that the Friedman's rule is still the optimal monetary policy rule in the model of market segmentation. However, as the following lemma will demonstrate it is impossible to achieve.

Lemma 2: *The Friedman's rule is impossible to achieve in the model of market segmentation proposed in this paper.*

Proof: Let me first define $\pi^{a*} = \beta - 1$ as the inflation rate consistent with the Friedman's rule. In addition, let me define the *a priori* expected welfare of a representative individual in country A as follows:

$$E[W](\pi^a, \pi^b) = U(X^{a^{FB}}) - \bar{H}^a + \sigma_{aa} \cdot [u(q^{aa^{ss}}(\pi^a)) - c(q^{aa^{ss}}(\pi^a))] +$$

$$\sigma_{ab} \cdot [u(q^{ab^{ss}}(\pi^b))] - \sigma_{ba} \cdot [c(q^{ba^{ss}}(\pi^a))] (31)$$

where $\bar{H}^a = X^{a^{FB}} - \bar{\kappa} \cdot (\pi^a - \pi^b)$ is derived from equations B2, B3, B4 and B5 in Appendix B.

A benevolent government in country A will, *ceteris paribus*, choose the country's inflation rate so as to maximize the average welfare function as follows:

$$\Psi(\pi^b) = \max_{\pi^a} U(X^{a^{FB}}) - \bar{H}^a + \sigma_{aa} \cdot [u(q^{aa^{ss}}(\pi^a)) - c(q^{aa^{ss}}(\pi^a))] +$$

$$\sigma_{ab} \cdot [u(q^{ab^{ss}}(\pi^b))] - \sigma_{ba} \cdot [c(q^{ba^{ss}}(\pi^a))] (32)$$

S.T.

$$\pi^b = \bar{\pi}^b$$

Substituting the constraint into the value function 32 and solving for the first order conditions, I obtain:

$$\pi^a : \bar{\kappa} + \sigma_{aa} \cdot [u'(q^{aa^{ss}}) \cdot \frac{dq^{aa^{ss}}}{d\pi^a} - c'(q^{aa^{ss}}) \cdot \frac{dq^{aa^{ss}}}{d\pi^a}] + \sigma_{ba} \cdot [-c'(q^{ba^{ss}}) \cdot \frac{dq^{ba^{ss}}}{d\pi^a}] = 0 (33)$$

Substituting for the derivatives and rearranging:

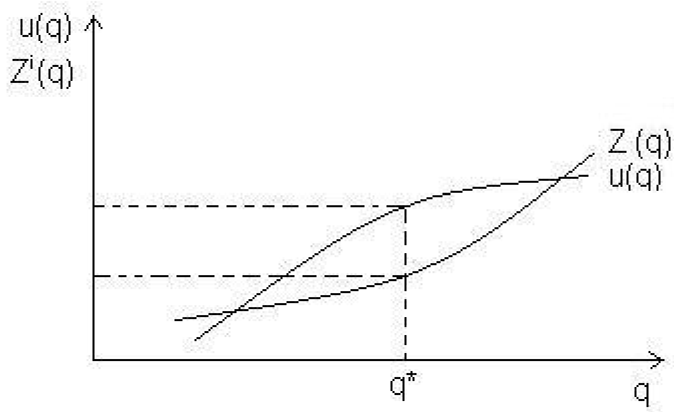

Figure 3: Unique $q * \forall q \in (-\infty, +\infty) \Rightarrow u'(q), Z^{i\prime}(q), Z^{i\prime\prime}(q) > 0; u''(q) < 0$

$$\pi^{a} : \bar{\kappa} + [\frac{u'(q^{aa^{ss}})}{\beta} \cdot \frac{[Z^{d\prime}(q^{aa^{ss}})]^2}{u''(q^{aa^{ss}}) \cdot Z^{d\prime}(q^{aa^{ss}}) - u'(q^{aa^{ss}}) \cdot Z^{d\prime\prime}(q^{aa^{ss}})} - \frac{c'(q^{aa^{ss}})}{\beta} \cdot$$

$$\frac{[Z^{d\prime}(q^{aa^{ss}})]^2}{u''(q^{aa^{ss}}) \cdot Z^{d\prime}(q^{aa^{ss}}) - u'(q^{aa^{ss}}) \cdot Z^{d\prime\prime}(q^{aa^{ss}})}] - \sigma_{ba} \cdot [\frac{c'(q^{ba^{ss}})}{\beta} \cdot$$

$$\frac{[Z^{I\prime}(q^{ba^{ss}})]^2}{u''(q^{ba^{ss}}) \cdot Z^{I\prime}(q^{ba^{ss}}) - u'(q^{ba^{ss}}) \cdot Z^{I\prime\prime}(q^{ba^{ss}})}] = 0 (34)$$

Let's denote π^{a*}_{ms} the argument that maximizes the value function 32 and thus, solves equation 34.[31]

To see how the per country maximizing inflation rate compares to the one consistent with the Friedman's rule , we note the following. From LW, we know that the Friedman's rule is optimal, so $\pi^{a*} = \beta - 1$ solves the following equation which is equivalent to equation 34 without the foreign DM market (i.e. the LW model):

$$[\frac{u'(q^{aa^{ss}})}{\beta} \cdot \frac{[Z^{d\prime}(q^{aa^{ss}})]^2}{u''(q^{aa^{ss}}) \cdot Z^{d\prime}(q^{aa^{ss}}) - u'(q^{aa^{ss}}) \cdot Z^{d\prime\prime}(q^{aa^{ss}})} - \frac{c'(q^{aa^{ss}})}{\beta} \cdot$$

$$\frac{[Z^{d\prime}(q^{aa^{ss}})]^2}{u''(q^{aa^{ss}}) \cdot Z^{d\prime}(q^{aa^{ss}}) - u'(q^{aa^{ss}}) \cdot Z^{d\prime\prime}(q^{aa^{ss}})}] = 0 (35)$$

[31] The subscript ms denotes market segmentation.

The first difference between these two equations is the constant term $\bar{\kappa}$ which is greater than zero in any monetary equilibrium (see Appendix B, equations B1,B2,B3,B4). The second difference is the rightmost term in equation 34 (from σ_{ba} to the end). The sign of this term is determined by the sign of the following equation:

$$\frac{dq^{ba^{ss}}}{d\pi^a} = \frac{[Z^{I\prime}(q^{ba^{ss}})]^2}{u''(q^{ba^{ss}}) \cdot Z^{I\prime}(q^{ba^{ss}}) - u'(q^{ba^{ss}}) \cdot Z^{I\prime\prime}(q^{ba^{ss}})}$$

The sings of $u'(q^{ba^{ss}}) > 0$ and $u''(q^{ba^{ss}}) < 0$ are exogenously determined in this model, therefore, I only need to determine the signs of $Z^{I\prime}(q^{ba^{ss}})$ as well as $Z^{I\prime\prime}(q^{ba^{ss}})$. A close inspection of the model reveals their sign without deriving them. Suffice is to note that the function $Z^{I\prime}(q^{ba^{ss}})$ can be interpreted as the marginal cost of bringing one more unit of currency to the DMs and given both the utility functions assumed in the DMs as well as the uniqueness of the interior solution of the model (Appendix B) we know that: $Z^{I\prime}(q^{ba^{ss}}) > 0$, $Z^{I\prime\prime}(q^{ba^{ss}}) > 0$(see figure 3). Therefore, $\frac{dq^{ba^{ss}}}{d\pi^a} < 0$ which, in conjunction with equation 32 and 33, demonstrates that $\pi^{a*}_{ms} > \pi^{a*}_{=}\beta - 1$. To summarize, the previous analysis showed that $\pi^{a*}_{ms} \in (\beta - 1, \bar{S}) \Rightarrow i \in (0, \bar{T})$, where $\bar{S} < +\infty, \bar{T} < +\infty$ are arbitrary upper boundaries.

QED

In the case shown above, country A has an incentive to inflate beyond the inflation rate consistent with the Friedman's rule as this increases the expected welfare of its citizens. This incentive to inflate is not particular to country A, but shared by both countries (recall that they are symmetric). When both countries inflate, the advantage of inflating above $\pi^{i*}, \forall i = a, b$, dissapears. The result of this process is a sub-optimal steady state Nash Equilibrium in which $q^{ba^{ss}} < q^{ba^{ss}}_{FR}, q^{ba^{ss}} < q^{ba^{ss}}_{FR}, q^{ba^{ss}} < q^{ba^{ss}}_{FR}, q^{ba^{ss}} < q^{ba^{ss}}_{FR}, q^{ba^{ss}} < q^{ba^{ss}}_{FR}$, where the subscript FR (Friedman's rule) denotes the optimal solution. Needless to say is that the representative agent's welfare is also lower in the sub-optimal steady state Nash Equilibrium than in the optimal one.

Because the model fails to achieve optimality, analyzing the second 'fundamental' question of modern monetary economics in this context is very important. To that end, lemma 3 shows that this model provides, in theory, a bigger or equal welfare cost of deviating from the Friedman's rule than the previous literature.

Lemma 3: *The welfare cost of deviating from the optimal monetary policy is, at least in theory, bigger or equal to the welfare cost in the LW model whenever inflation rates of both countries move together at the same rate (i.e.*$d\pi = d\pi^a = d\pi^b$*) and the probabilities of being a buyer are equal (i.e.* $\sigma_=\sigma_{aa} = \sigma_{ab} = \sigma_{ba}$*).*

Proof: To prove this lemma, I need to firstly derive the total differential of both equation 31 as well as its counterpart for the LW model, and subsequently, compare them. Taking the total differential of equation 31, I obtain:

$$dE[W](\pi^a, \pi^b) = [\bar{\kappa} + \sigma_{aa} \cdot [u'(q^{aa^{ss}}) \cdot \frac{dq^{aa^{ss}}}{d\pi^a} - c'(q^{aa^{ss}}) \cdot \frac{dq^{aa^{ss}}}{d\pi^a} -$$

$$\sigma_{ab} \cdot c'(q^{ba^{ss}}) \cdot \frac{dq^{ba^{ss}}}{d\pi^a}]] \cdot d\pi^a + [-\bar{\kappa} + \sigma_{ab} \cdot [u'(q^{ab^{ss}}) \cdot \frac{dq^{ab^{ss}}}{d\pi^b}]] \cdot d\pi^b (36)$$

Moreover, taking total differential of the same function for the LW model, I obtain:

$$dE[W](\pi) = [\sigma \cdot [u'(q^{ss}) \cdot \frac{dq^{ss}}{d\pi} - c'(q^{ss}) \cdot \frac{dq^{ss}}{d\pi}]] \cdot d\pi (37)$$

Substituting the assumptions of lemma 3 in equation 36, I obtain:

$$dE[W](\pi^a, \pi^b) = [\sigma \cdot [u'(q^{aa^{ss}}) + u'(q^{ab^{ss}}) \cdot \frac{dq^{ss}}{d\pi}] - [c'(q^{aa^{ss}}) + c'(q^{ba^{ss}})] \cdot \frac{dq^{ss}}{d\pi}] \cdot d\pi (38)$$

Given the shapes of $u'(.)$ and $c'(.)$, we know that: $u'(q^{aa^{ss}}) + u'(q^{ab^{ss}}) \geq u'(q^{aa^{ss}} + q^{ab^{ss}})$ and $c'(q^{aa^{ss}}) + c'(q^{ba^{ss}}) \leq c'(q^{aa^{ss}} + q^{ba^{ss}})$. Therefore, $dE[W](\pi^a, \pi^b)$(equation 36-38)$\geq dE[W](\pi)$ (equation 37).

QED

It is very important to understand the limitations of this lemma. Though the mathematical proof indicates the veracity of the lemma, its foundations are based on very strict assumptions. These assumptions may not be true as parameters possible change from one model to the other. Therefore, lemma 3 only provides a qualitative result of what future empirical researches into the welfare cost of inflation utilizing this model of market segmentation may expect.

5 Conclusion

Modern studies into monetary economics have been, for the most part, centered around two 'fundamental' questions. To answer these questions, models have been investigating an increasing variety of frictions that make money essential (i.e. valued in equilibrium). Despite the increasing complexity of models' environments and frictions, none of them has studied these questions in an international framework. The exclusion of this large sector of the economy from monetary analysis may lead to false conclusions about monetary theory and policy. In particular, it can provide misguided answers to the two 'fundamental' questions of modern monetary economics.

To avoid this problem, this paper has presented a new framework that consists of extending the LW model to include both a foreign market and currency. The LW was chosen as it provides an environment that is both tractable and able to model the agent's microeconomic decision process. The addition of this foreign market to the LW, however, was not without problems. This extension requires a set of assumptions to force the solution to be determined. The key assumption here is the cash-in-advance constraint imposed on the DMs which required consumers to pay for the commodities they buy in the seller's own currency. Once I imposed this restriction, I obtained a tractable model of Search Theory of money that allowed me to explore the two 'fundamental' questions of modern monetary economic at an international level.

In investigating the two central questions of monetary economics in this new environment, this paper found the following properties. Though the Friedman's rule is still optimal, it is unattainable in the model proposed here. The reason for this, is that in maximizing the representative agent's average utility, each government has an incentive to set the inflation rate above the inflation rate consistent with the Friedman's rule. As a consequence, the result of this model is a sub-optimal Nash Equilibrium in which individuals' consumption and welfare are lower than under the Friedman's rule. In this context the measurement of the welfare cost of deviating from the optimal monetary policy is very important. Theoretically and under very strict assumptions, the welfare cost of inflating above the optimal inflation rate in the LW model serves as a lower boundary for the cost in the model proposed here. The previous result, however, greatly depends on the assumptions made. Therefore, future research should be directed at calibrating the model proposed here and measuring this cost empirically. In addition, future research should also investigate the consequences of adding a foreign market to models with more frictions than the LW model.

6 Appendix

i. Appendix A

Lemma I: *The constraints in the General Nash Bargain Solution for both DMs are binding (i.e. $\hat{m}_t^i = db_t^{ii}, \hat{b}_t^i = db_t^{ij}, \forall i, j = a, b; i \neq j$). That is, people bring only enough money to buy q_{bu}^{ii} and q_{bu}^{ij} quantities of domestic and foreign commodities respectively in the decentralized markets.*

Proof:[32] The Lagrangean for the maximization problem in equation 18 is as follows:

$$L(q_{ct}^{aa}, db_t^{aa}, \lambda) = [u(q_{ct}^{aa}) - \frac{1}{P_t^a} \cdot db_t^{aa}]^{\theta_1} \cdot [-c(q_{ct}^{aa} + q_{ct}^{ba}) + \frac{1}{P_t^a} \cdot (db_t^{aa} + db_t^{ba})]^{1-\theta_1} +$$

$$\lambda \cdot [\hat{m}_t^a \cdot db_t^{aa}](A1)$$

Obtaining the first derivative of A1 and rearranging we obtain the following first order condition for db_t^a:

$$db_t^a : u'(q_{ct}^{aa}) = \frac{\bar{N} \cdot (1 - \sigma_1)}{\sigma_1 \cdot \bar{L}} \cdot [c'(q_{ct}^{aa}) - 1] + \frac{\lambda}{\sigma_1 \cdot \bar{L}} \cdot P_t^a \cdot \bar{L}^{\sigma_1} \cdot \bar{N}^{(-\sigma_1+1)} + 1(A2)$$

where $\bar{N} = u(q_{ct}^{aa}) - \frac{1}{P_t^a} \cdot db_t^{aa}$ and $\bar{L} = -c(q_{ct}^{aa}) + \frac{1}{P_t^a} \cdot db_t^{aa}$. Let q_{ct}^{*aa} be the quantity of domestic good purchased by people in country A that maximizes the unconstrained problem. Then, whenever $q_{ct}^{aa} = q_{ct}^{*aa} \rightarrow \lambda = 0$. By assumption we know that $u''(q_{ct}^{aa}) < 0$. In addition, from A2 we know that whenever $\lambda > 0 \rightarrow u'(q_{ct}^{*aa}) \leq u'(q_{ct}^{aa})$.[33]Therefore, $q_{ct}^{aa} \leq q_{ct}^{*aa}$ and we have the following three situations:

Either

i)$q_{ct}^{aa} \leq q_{ct}^{*aa}, \lambda > 0$ and $db_t^{aa} = \hat{m}_t^a$

or,

ii)$q_{ct}^{aa} = q_{ct}^{*aa}, \lambda = 0$ and $db_t^{aa} < \hat{m}_t^a$

or,

[32]As before, though I only consider the proof of lemma 1 in the context of country A, it is the same analysis for country B.

[33]This is true whenever $\bar{L} \geq 0$ which is always true in any equilibrium with trade, otherwise the sellers would be getting a negative surplus and they will not produce.

iii)$q_{ct}^{aa} = q_{ct}^{*aa}, \lambda = 0$ and $db_t^{aa} = \hat{m}_t^a$

Let's assume that a consumer starts in a situation like i (with $\lambda > \frac{1}{P_t^a}$) , then the consumer will bring more money and spend proportionally more in the second market because the marginal benefit of bringing more money and spending it in the DM exceeds its marginal cost. Then, let's assume that a consumer reach a situation like iii. In this case, the consumer will stop bringing money to the DMs because its marginal cost exceeds the marginal benefit of bringing one more unit in any monetary equilibrium ($\lambda = 0 < \frac{1}{P_t^a}$). Therefore, situation ii could never be reached and the constraint is always binding. A similar analysis apply to the foreign DM.

QED

ii. Appendix B

In this appendix I provide a proof for proposition 1 which indicates that in the interior solution to the monetary model the distributions of real money holdings are degenerate. This appendix is separated into two different parts. The first part shows that there is an interior solution that solves the monetary model presented in the previous analysis. This part is an extension of a similar proof by Berensten (*et.al.* 2007), extended to cover the differences between these two models. However, I believe that this part alone does not prove the proposition as it only states that there is an interior solution to the problem but does not demonstrate that the interior solution is unique. Consequently, in the second part of this appendix, I show that this interior solution is unique.

ii.a. Existence of an interior solution

Let's consider the case where $\sigma_1 = \sigma_2 = 1$ (the buyer make take-it-or-leave-it offers) and the model is in steady state (i.e. $M_{t+i} = \sum_{s=0}^{i} \gamma^s \cdot M_t$, $q_t = q^{ij^{ss}}, \forall i, j = a, b; t = 1, ..., \infty$ and that real variables are constant through time). The take-it-or-leave-it offers means that the seller's surplus equals zero or equivalently that buyers pay exactly the cost of production. In addition, let's assume that a representative individual in country A starts the first sub-period with $\hat{m}_t^a = \bar{m}_t^a$ and $\hat{b}_t^a = \bar{b}_t^a$ units of each country's currency respectively, where the quantities $\bar{m}_t^a, \bar{b}_t^a$ solves the model at some period t.[34] Also, by market clearing conditions $M_t^a = \bar{m}_t^a + \bar{b}_t^b$ and $M_t^b = \bar{m}_t^b + \bar{b}_t^a$. Then, at the beginning of the CM, a representative agent in country A is holding the following amount of currency from each country:

. with probability σ_{aa} an individual is a buyer in the domestic market: $m_t^a = 0$, $b_t^a = \bar{b}_t^a$

. with probability σ_{ab} an individual is buyer in the foreign market: $m_t^a = \bar{m}_t^a$, $b_t^a = 0$

. with probability σ_{aa} an individual is a seller in the domestic market: $m_t^a = 2\bar{m}_t^a$, $b_t^a = \bar{b}_t^a$

. with probability σ_{ba} an individual is a seller in the foreign market: $m_t^a = \bar{m}_t^a + \bar{b}_t^b, b_t^a = \bar{b}_t^a$

. with probability $1 - 2\sigma_{aa} - \sigma_{ab} - \sigma_{ba}$ an individual is a non-trader: $m_t^a = \bar{m}_t^a, b_t^a = \bar{b}_t^a$

Assuming that in maximizing intertemporal utility, individuals consume the optimal amount in the CM, the hours worked in the CM are as follows:

. a buyer in the domestic market works:

$$H_{ca} = X^{FB} - (\frac{0 - \hat{m}_{t+1}}{P_t^a}) - \tau_1^a \cdot \frac{M_t^a}{P_t^a} - \frac{1}{P_t^b} \cdot [\bar{b}_t^a - \hat{b}_{t+1}^a]$$

By S.S, $\sigma_1 = 1$ and substituting for the expressions of both τ_1^a, M_t^a and $\frac{\bar{m}_t^a}{P_t^a} = \frac{\hat{m}_{t+1}^a}{P_{t+1}^a} \Rightarrow \gamma^a \cdot \bar{m} = \hat{m}_{t+1}^a$ (the same for other nominal variables) , I obtain:

$$H_{ca} = X^{FB} + \gamma^a \cdot c(q^{aa^{ss}}) - (\gamma^a - 1) \cdot [c(q^{aa^{ss}}) + \frac{\bar{b}_t^b}{P_t^a}] - [\frac{\bar{b}_t^a}{P_t^b} - \frac{\gamma^b \cdot \bar{b}_t^a}{P_t^b}]$$

By S.S and the fact that representative individuals from both countries are identical we have that $\frac{\bar{b}_t^b}{P_t^a} = \frac{\bar{b}_t^a}{P_t^b} = \bar{\kappa}$, where $\bar{\kappa} > 0$(in a monetary equilibrium) is any constant, I obtain:

$$H_{ca} = X^{FB} + c(q^{aa^{ss}}) - \bar{\kappa} \cdot (\gamma^a - \gamma^b)(B1)$$

. a buyer in the foreign market works:

$$H_{cb} = X^{FB} - (\frac{\bar{m}^a - \hat{m}_{t+1}}{P_t^a}) - \tau_1^a \cdot \frac{M_t^a}{P_t^a} - \frac{1}{P_t^b} \cdot [0 - \hat{b}_{t+1}^a]$$

[34]The same assumptions apply for country B.

By the same analysis as before, I obtain:

$$H_{cb} = X^{FB} + c(q^{ab^{ss}}) - \bar{\kappa} \cdot (\gamma^a - \gamma^b)(B2)$$

. a seller in the domestic market works:

$$H_{sa} = X^{FB} - (\frac{2\bar{m}^a - \hat{m}_{t+1}}{P_t^a}) - \tau_1^a \cdot \frac{M_t^a}{P_t^a} - \frac{1}{P_t^b} \cdot [\bar{b}^a - \hat{b}_{t+1}^a]$$

By the same analysis as before, I obtain:

$$H_{sa} = X^{FB} - c(q^{aa^{ss}}) - \bar{\kappa} \cdot (\gamma^a - \gamma^b)(B3)$$

. a seller in the foreign market works:

$$H_{sb} = X^{FB} - (\frac{\bar{m}^a + \bar{b}^b - \hat{m}_{t+1}}{P_t^a}) - \tau_1^a \cdot \frac{M_t^a}{P_t^a} - \frac{1}{P_t^b} \cdot [\bar{b}^a - \hat{b}_{t+1}^a]$$

By the same analysis as before, I obtain:

$$H_{sb} = X^{FB} - c(q^{ba^{ss}}) - \bar{\kappa} \cdot (\gamma^a - \gamma^b)(B4)$$

. a non-trader works:

$$H_{nt} = X^{FB} - (\frac{\bar{m}^a - \hat{m}_{t+1}}{P_t^a}) - \tau_1^a \cdot \frac{M_t^a}{P_t^a} - \frac{1}{P_t^b} \cdot [\bar{b}^a - \hat{b}_{t+1}^a]$$

By the same analysis as before, I obtain:

$$H_{nt} = X^{FB} - \bar{\kappa} \cdot (\gamma^a - \gamma^b)(B5)$$

Let's $q^{aa^{ss}*}$ be the most efficient quantity of country A's commodities and $q^{ab^{ss}*}$ be the most efficient quantity of country B's commodities consumed by buyers in country A in the DMs. Also, let's $q^{ba^{ss}*}$ be the most efficient amount of country A's commodities consumed by foreigners in the DMs. From the World Planner's solution, we can observe that these most efficient quantities are achieved in a frictionless model. The introduction of search frictions to the model only force consumers to demand a lower quantity of commodities (see above). That is to say, that since no consumer would ever demand more quantities that under the World Planner's solution, but possible less according to the type of search friction introduced in the problem, the quantities demanded obtained under this solution form the upper boundary for the set of all possible solutions. Therefore, and since quantities consumed can not be negative we can say the following: $q^{ii^{ss}} \in [0, q^{ii^{ss}}*]$,$q^{ij^{ss}} \in [0, q^{ij^{ss}}*]$ and $q^{ji^{ss}} \in [0, q^{ji^{ss}}*]; \forall i, j = a, b; i \neq j$. I can rescale $X^{FB} = U'(1)$ to make $H_{ca}, H_{cb}, H_{sa}, H_{sb}, H_{nt} > 0$ whenever $q \in [0, q^*]$.

QED.

ii.b. Uniqueness of the interior solution

Now that I showed that there is an interior solution to the representative agent's problem, I am ready to show that F^a and F^b are degenerate because the solution is unique. The agent's value function could be re-written as follows:

$$V = \max_{\hat{m}^a_{t+1}, \hat{b}^a_{t+1}} \sum_{t=0}^{\infty} \beta^t \cdot [[U(X^{FB}) - X^{FB} + (\frac{m^a_t - m^{\hat{a}}_{t+1}}{P^a_t}) + \tau^a_1 \cdot \frac{M_t}{P^a_t} + \frac{1}{P^b_t} \cdot [b^a_t - \hat{b}^a_{t+1}]] +$$

$$\beta^{t+1} \cdot [\sigma_{aa} \cdot [u(q^{aa^{ss}}(\hat{m}^a_{t+1})) + \bar{W}_{t+1} + \frac{\hat{b}^a_{t+1}}{P^b_{t+1}}] + \sigma_{ab} \cdot [u(q^{ab^{ss}}(\hat{b}^a_{t+1})) + \bar{W}_{t+1} + \frac{\hat{m}^a_{t+1}}{P^a_{t+1}}] +$$

$$\sigma_{aa} \cdot [-c(q^{aa^{ss}}(\hat{m}^{a\prime})) + \bar{W}_{t+1} + \frac{\hat{m}^a_{t+1} + da(\hat{m}^{a\prime})}{P^a_{t+1}} + \frac{\hat{b}^a_{t+1}}{P^a_{t+1}}] + \sigma_{ba} \cdot [-c(q^{ba^{ss}}(\hat{m}^{b\prime})) + \bar{W}_{t+1} +$$

$$\frac{\hat{m}^a_{t+1} + db(\hat{m}^{b\prime})}{P^a_{t+1}} + \frac{\hat{b}^a_{t+1}}{P^a_{t+1}}] + (1 - 2\sigma_{aa} - \sigma_{ab} - \sigma_{ba}) \cdot [\bar{W}_{t+1} + \frac{\hat{m}^a_{t+1}}{P^a_{t+1}} + \frac{\hat{b}^a_{t+1}}{P^b_{t+1}}]](B6)$$

The Euler equation for $\hat{m}^a_{t+1}$is as follows:

$$\hat{m}^a_{t+1} : \beta^t \cdot [\frac{1}{P^a_t}] = \beta^{t+1} \cdot [\sigma^{aa} \cdot u'(q^{aa^{ss}}) \cdot \frac{\partial q^{aa^{ss}}}{\partial \hat{m}^a_{t+1}} + \sigma_{ab} \cdot \frac{1}{P^a_{t+1}} + \sigma_{aa} \cdot \frac{1}{P^a_{t+1}} + \sigma_{ba} \cdot \frac{1}{P^a_{t+1}} +$$

$$(1 - 2\sigma_{aa} - \sigma_{ab} - \sigma ba) \cdot \frac{1}{P^a_{t+1}}] + \frac{1}{P^a_{t+1}} \cdot \beta^{t+1}(B7) \qquad (67)$$

Rearranging equation B3 and substituting for the Fisher equation (equation 3) with a constant inflation rate as well as for $\beta = \frac{1}{1+r}$, I obtain:

$$\frac{i-1}{\sigma^{aa}} + 1 = e(\hat{m}^a_{t+1})$$

where as in Berensten (*et.al.* 2007[1]) and LW $e(\hat{m}^a_{t+1}) = \frac{u'(q^{aa^{ss}}(\hat{m}^a_{t+1}))}{Z^{d\prime}(q^{aa^{ss}}(\hat{m}^a_{t+1}))}$. It is straight forward to prove that given our previous assumptions, $e''(\hat{m}^a_{t+1}) < 0$ (i.e. the function is strictly concave) and therefore, that the solution is unique. A similar analysis applies to $\hat{b}^a_{t+1}$ and to the variables in country B.

QED

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