SIGNALLING IN DEFENDING A FIXED EXCHANGE RATE

by

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Abstract

The Hong Kong dollar, a pegged currency to the US dollar, had encountered massive speculative attacks during the 1997 to 1998 Asian financial crisis. Merton Miller had advised for the Hong Kong Monetary Authority to issue structured notes or currency put options to restore the confidence problem of the Hong Kong dollar. The key theoretical prediction put forward is the existence of separating equilibrium, where only a government that is determined to defend the currency will issue such put options. This paper will present a game theoretical model that demonstrates the existence of separating equilibrium and mixed strategy equilibrium. According to the model, the economy in a signalling equilibrium will save the entire or part of the cost of defending the currency by issuing some amounts of currency put options.

1 Introduction

During the 1997-98 Asian financial crisis, the Hong Kong dollar fixed exchange rate had suffered in the confidence problem. Merton Miller (1998a) advised for the Hong Kong Monetary Authorities (HKMA) to issue currency put options, in order to signal its willingness in defending the currency. In a speech delivered in Hong Kong, Miller (1998b) indicated that

> ”if the Hong Kong Monetary Authority can provide insurance (currency put options) to the public against Hong Kong dollar’s devaluation and make it clear to the public that the Authority will suffer a great loss in case the Hong Kong dollar is devalued afterwards, then the public will be convinced of the Authority’s determination...”

He argued that it was the way to defend the fixed exchange rate without the involvement of raising the interest rate and tightening liquidity. Thus, the Hong Kong economy may had suffered less from the prolong recession following the crisis. If the HKMA had issued the put option, it would hurt the HKMA when the Hong Kong dollar undergoes devaluation. Then, it would be less likely for the HKMA to devalue the currency, and the public confidence about the peg would be restored. When the market is confident in the value of the Hong Kong dollar, the fixed exchange rate will be less likely to experience the pressure of devaluation. The key theoretical prediction put forward is the existence of separating equilibrium, in which only a government that is willing to defend the currency will issue the put options.

The main purpose of this paper is to build a game theoretical model that demonstrates the existence of signalling equilibria, separating and mixed strategy. The issuance of currency put options can serve as a signal to the market about the willingness and ability of a government to defend its fixed exchange rate. The model not only applies to a currency board arrangement, but also to any fixed exchange rate regime that faces a confidence problem. The structure of this model is based on a classic model that was constructed by Spence (1973). He had demonstrated that a worker's level of education signals the productivity level of that worker. Similarly, in my model, the amount of put options that were issued signals a government's willingness and ability to defend its currency. The existences of the signalling equilibria are the major results of the model. These results provide theoretical justifications for the put options policy.

In the model, there are two periods and two players, the government and the speculator. The economy starts out with a fixed exchange rate. It is structured as a sequential game where the informed party, the government, moves first to send a signal to the speculator about its government type. The government can either be a strong type or a weak type. Different government types have different incentives when it face speculative attacks. In the first period of the model, after nature had determined a government's type, the government decides whether to issue currency put options in an attempt to alter the speculator's actions in the second period. In the second period, the speculator responds by either attack or not attack the pegged currency after observing the action of the government. Since the weak government may mimic the strong government by issuing an amount of put options that the strong government would issue, separating equilibrium may not necessary be the result. However, the separating equilibrium and the mixed strategy equilibrium will be the final outcome only if some conditions are satisfied.

The next section of the paper will discuss some of the background information of the 1997-98 Asian financial crisis and some of the previous related literatures. Section III will go through the details of the model environment. Section IV of the paper will provide detail analysis of the model equilibria. Section V will conclude.

2 Background

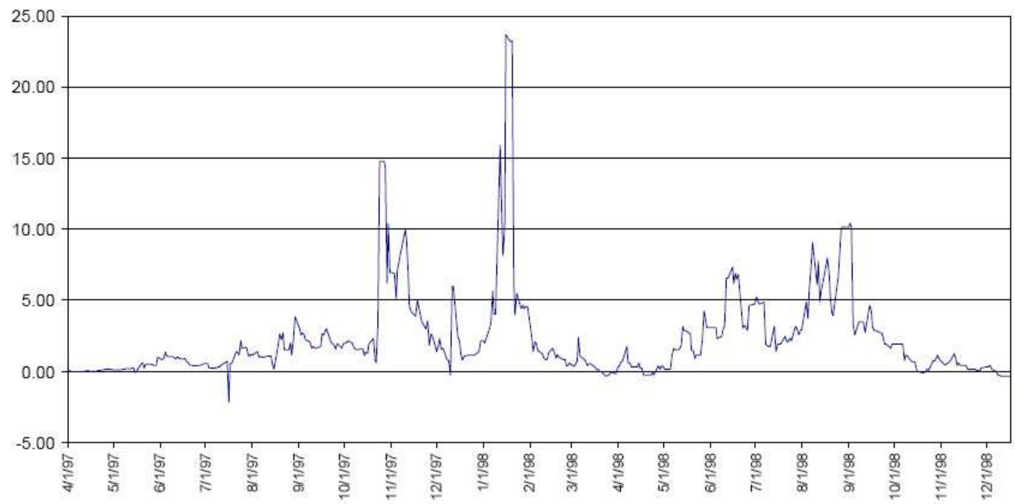
In the 1997-98 Asian financial crisis, numbers of Far East countries including Hong Kong had suffered from speculative attacks against their pegged currency. The Asian financial crisis officially began on July 2, 1997 when the Thai baht was allowed to float and depreciate. Within days, the Philippine peso, the Malaysian ringgit, and the Indonesian rupiah were also subjected to speculative attacks. All these currencies ended up floating. The crisis quickly spread northward to South Korea, Singapore, and Taiwan. The Taiwanese central bank initially widened the band of the target zone. However, on October 17, the Taiwanese central bank suddenly allow the New Taiwan Dollar (NTD) to float without intervention. The NTD had depreciated from the original NTD27 per U.S. dollar to more than NTD30 per U.S. dollar (Jao, 2001). The depreciation of the NTD gave tremendous encouragement to speculators to attack the Hong Kong dollar. Hong Kong had shared many similar characteristics as Taiwan before the outbreak of the crisis. Both economies had sound fundamentals relative to the Southeast Asian developing countries, and both Taiwan and Hong Kong had large amounts of foreign reserves. Speculators believed that the HKMA was also not determined to defend the domestic currency like the Taiwanese central bank.

Moreover, a lot of market participants, politicians, exporters and economics commentators had suggested that Hong Kong should give up the peg. Cheng et al. (1999a) argued that the Hong Kong economy had already lost its international competitiveness after many years of high inflation and the devaluation of other countries currency during the crisis. As a result, the Hong Kong dollar should be devalued.

Despite the fact that Hong Kong had adopted the Currency Board arrangement and had foreign exchange reserves that were 7.1 times the monetary base at the end of 1997 (Liew and Wu, 2002), the Hong Kong dollar had suffered from four major speculative attacks during 1997 and 1998. At that time, the sole defending mechanism of the HKMA was to raise interest rate and contract liquidity in the banking system. On October 20, 1997, the speculators launched their first massive onslaught on the Hong Kong dollar by selling it short or initiating short position on the Hong Kong dollar three to six months forward. Commercial banks not only financed the speculators, but also joined the bandwagon to short the Hong Kong dollar. To defend the Hong Kong dollar, the HKMA conducted non-sterilized intervention to purchase Hong Kong dollars. When these transactions came into settlement on Thursday, 23 October, 1997, the aggregated clearing balance of the banking system went negative. Banks had found themselves short of Hong Kong dollars. The markets ended up scrambling for Hong Kong dollars. At 10:00am on the same day, the HKMA released memorandums to all banks indicating that the HKMA would charge penalty interest for "repeated borrowers" to discourage the borrowing of Hong Kong dollars from the HKMA. There were rumors spreading that the interest rate maybe as high as 1000%. The overnight Hong Kong Interbank Offer Rate (HIBOR) went up to as high as 280% and the major stock index, the Hang Seng Index, slumped 10.4%. This day is known as the "Black Thursday".

After the "Black Thursday", there were three more subsequent major attacks during 1997 and 1998. Compared to other periods, the risk premium was substantially higher dur-

Figure 1: HK$/US$ 1-month Forward Premium % (From 1997:4:1 to 1998:12:16)



ing the periods of major onslaughts. Figure (1) depicts the 1-month forward premium from April 1, 1997 to December 16, 1998. The four spikes of the 1-month forward premium reflect the four major attacks against the Hong Kong dollar. Even though the HKMA had successfully defended all these attacks, the interest rate sensitive Hong Kong economy took a serious toll since interest rate shot up during the period of the crisis. During the crisis the confidence in maintaining the peg had deteriorated since the economy had suffered greatly when the HKMA attempts to defend the currency. Market participants had doubted the willingness of the HKMA to defend the fixed exchange rate and to bear the cost of economic recession. Before the outbreak of the crisis, the HKMA had gradually increased its discretionary power. It differed from the traditional currency board arrangement in adhering to fixed rules of operations. As a result, the question lied in the willingness of the HKMA to defend the peg. The fixed exchange rate faced serious confidence problem during the crisis.

Very few past literatures focused on using currency put options to restore the confidence of the fixed exchange rate or to signal the market about the determination of the government to defend its currency. Cheng et al. (1999a) and Chen (2001) argued that the put options can be a costly signal. Chiu (2003) had constructed a theoretical model to study the desirability of the currency put options. He found the breakthrough of the works of Morris and Shin (1998) an ideal approach, hence, he had built a model that demonstrated the time inconsistency problem of the government in the case of defending the currency. Cheng et al. (1999b) had conducted an empirical study to test the credibility of the Hong Kong currency board before and after the issuance of structured notes. They had found that the issuance helped regain credibility. Cheng et al. (1999c) had also conducted a study which illustrated that the Hong Kong dollar risk premium had increased during the crisis. They suggested that merely increasing the interest rate may not be an effective way to defend a fixed currency. The model that I have constructed in this paper will hopefully contribute to the related literatures.

3 The Model Environment

In the first period, $t = 1$, nature determines the characteristics that the government possesses. It could either be strong or weak. The government type is private information to the government, which is not known to the speculator. In the second period, $t = 2$, speculator decides whether or not to attack the pegged currency. The speculator has some ex ante beliefs on the government type. Without extra information about the government type, the speculator believes that the government is a strong one with a probability q_1, or a weak one with a probability $1-q_1$. The government may issue currency put options in $t = 1$ to signal to the market about its type and consequently, to alter the speculator's strategy in $t = 2$.

In $t = 2$, if there is an attack against the pegged currency, the government weighs between the costs and the benefits of defending the fixed exchange rate. The benefit variable and the cost variable are exogenous. The variables are different for each government type. V_i, $i = s$(strong), w(weak), denotes the monetary benefits of maintaining the fixed exchange rate. The model assumes $V_s > V_w$, in which there are more benefits for the strong government to maintain the peg than the weak government. For instance, if the country has a lot of foreign trade, the country enjoys greater benefits from having a stable exchange rate than a country with less foreign trade. $\alpha_i c(\theta^j)$, $j = h$(high), l(low), $i = s, w$, represents the monetary costs of defending the currency. Notice that the defending costs are a function of θ^j, which denotes the economic fundamental. The economic fundamental could either be high or low, with probability π_h and π_l respectively. This fundamental is a shock that is realized in the second period, $t = 2$. We can interpret the fundamental as the realized economics environment or the real output of the economy in the second period. It is natural to assume that the costs of defending the currency are negatively related to the fundamental, that is $\partial c(\theta^j)/\partial\theta < 0$. Furthermore, the strong and the weak governments have different values for the parameter, α_i, $i = s, w$. It is assumed that $\alpha_s < \alpha_w$; therefore, the weak government incur greater defending costs than the strong government for any given level of fundamental, θ.

Assumptions 1: $V_s > V_w, \quad \alpha_s < \alpha_w, \quad \partial c(\theta^j)/\partial\theta < 0.$

Combining above assumptions would result in the following, $V_s - \alpha_s c(\theta^j) > V_w - \alpha_w c(\theta^j)$. The strong government has strictly greater net benefits than the weak government in the event of defending the currency at any given level of θ. It also means that the ability and the willingness of the strong government to defend the currency is greater than the weak government.

The model makes further assumptions about the net benefits of defending the currency with different levels of fundamental, θ, for both the strong and the weak government. The model assumes that the net gains for a weak government from defending the currency are negative, whether the fundamental is high or low. The weak government has no incentive to defend whatsoever. That is, $V_w - \alpha_w c(\theta^l) < V_w - \alpha_w c(\theta^h) < 0$. On the other hand, the net gains for a strong government from defending the currency are strictly positive when the fundamental is high; that is $V_s - \alpha_s c(\theta^h) > 0$. Therefore, the strong government has the incentive to defend when the fundamental is high. However, the net gains are negative when the fundamental is low, $V_s - \alpha_s c(\theta^l) < 0$. When the fundamental turns out to be low, even the strong government will not have the incentive to defend the currency. In general,

$$\textit{Assumptions 2: } V_w < \alpha_w c(\theta^h) < \alpha_w c(\theta^l), \quad \alpha_w c(\theta^h) < V_s < \alpha_s c(\theta^l).$$

This model is interesting in a sense that the strong and the weak government have different incentives when the fundamental turns out to be high. The strong government may want to send a signal to the public in the first period to discourage the speculator to initiate attack if the fundamental reveals to be high in the second period. The strong government could therefore avoid paying the cost of defending, $\alpha_s c(\theta^h)$. The main focus of this model will be on the case when the fundamental is high.

If the government failed to defend the currency, the currency begins to float and the subsequent floating exchange rate will depend on the level of fundamental, θ^j. $m(\theta^j)$ denotes the subsequent floating exchange. In other words, $m(\theta^j)$ is the shadow exchange rate. In the model, $m(\theta^j)$ is interpreted as the value of the anchor currency per domestic currency. In the case of the fixed exchange rate of the Hong Kong dollar, $m(\theta^j)$ is interpreted as USD/HKD, which is the inverse of the conventional quote we see in the currency market. It is convenient to quote in this way because the value of $m(\theta^j)$ reduces when the domestic currency depreciates. According to the monetary model of floating nominal exchange rates, the fundamental has positive relationship with the value of domestic currency, so $\partial m(\theta^j)/\partial\theta > 0$ or $m(\theta^l) < m(\theta^h)$. However, the model does not distinguish the shadow exchange rate between the strong and the weak governments. Both government types have the same shadow exchange rate at any given level of fundamental.

4 The Attack Game: Equilibrium analysis

Given that the fundamental, θ, reveals to be high in $t = 2$, the speculator relies on its ex ante beliefs to determine his optimal strategy if no further information was provided. In the second period, the expected payoff of the speculator to attack when the fundamental is high is

$$q_1(-t) + (1 - q_1)(e^* - m(\theta^h)), \tag{1}$$

where q_1 is the ex ante beliefs of the government is strong; t is the transaction costs the speculator bears if the government successfully defend the currency attack; e^* is the pegged exchange rate; and $e^* - m(\theta^h)$ is the payoff that the speculator gains, in terms of the anchor currency, for each domestic dollar attacked. If the speculator attacks a strong government when the fundamental is high, the speculator will lose and pay the transaction costs, t, as the strong government has the incentive to defend, $V_s - \alpha_s c(\theta^h) > 0$. If the fundamental is high and the speculator faces a weak government, the speculator will win and earn $e^* - m(\theta^h)$ for each domestic dollar attacked, in which the weak government has no incentive to defend, $V_w - \alpha_w c(\theta^h) < 0$. In contrast, if the speculator does not initiate an attack, the speculator simply gets zero payoff whether the fundamental is high or low. As a result, if equation (1) is strictly greater than zero,

$$q_1(-t) + (1 - q_1)(e^* - m(\theta^h)) > 0, \tag{2}$$

the speculator will attack for sure when the fundamental turns out to be high. If we rearrange equation (2), it becomes

$$\frac{e^* - m(\theta^h)}{e^* - m(\theta^h) + t} > q_1. \tag{3}$$

Equation (3) holds when the ex ante beliefs of having a strong government, q_1, is low. This model only examines the cases where equation (3) holds. If the initial beliefs of having a strong government, q_1, is large enough to make equation (3) not hold, the speculator will not initiate any attack even without any signal sent by the government. This case is not as interesting to study since the model analyzes signalling equilibrium. As a result, the model assumes equation (3) will hold and there is room for the government to send out signals to strengthen the speculator's beliefs.

If the fundamental, θ, reveals to be low, the speculator would definitely attack. That is because both the strong and the weak government have no incentives to defend the currency, $V_s - \alpha_s c(\theta^l) < 0$ and $V_w - \alpha_w c(\theta^l) < 0$ respectively. The speculator simply earns $e^* - m(\theta^l)$ for each domestic dollar attacked.

4.1 payoff functions and strategic choices

Since the model assumes equation (3) will hold, for the case of a high fundamental, the speculator always initiates an attack if the ex ante beliefs are the only information that is known to the speculator. If the government is actually a strong one, the government will then incur the costs of defending, $\alpha_s c(\theta^h)$. However, this defending costs can be avoided if the strong government is able to send a credible signal that informs the speculator of the true government's type. The speculator could also save the transaction costs, t. The strong government can issue any positive level of currency put options in period $t = 1$, denoted as D_s, to serve as the signal. If the signal serves its purpose, it would be a separating equilibrium in which only the strong government issues D_s and the weak government issues $D_w \neq D_s$. In this case, the speculator will be able to distinguish between a strong government and a weak government by looking at the level of D that were issued. Nevertheless, the creditability of the signal may be hindered since the weak government may mimic and issue $D_w = D_s$. In order for the signal to serve its purpose, some conditions must be held to reduce the incentive for the weak government to mimic. We leave the analysis of these conditions and the possible equilibria of the model to the next subsections. Before that, let's turn to the analysis of the government and the speculator's payoff functions.

If the government failed to defend the currency, the issuer of the currency put options is obligated to compensate the holder of the put options. Any kind of put options are similar to an insurance that guarantees the holder the value of the corresponding underlying assets. Since the government is the issuer of the currency put options, the government incurs compensating costs in the case of currency depreciation. The compensating costs affect both the strong and the weak's expected payoff functions. Before the discussion of the payoff function of the government, the variable, γ, should be introduced. This variable is the probability of the speculator to initiate an attack given that the government issues $D = D_s$ and the fundamental is high; that is,

$$\gamma = prob(\text{Attack}|D = D_s, \theta^h). \tag{4}$$

γ is the choice variable of the speculator. The speculator determines the probability to attack after the speculator has observed the government's issuance of $D = D_s$. Since the weak government may mimic and issues $D_w = D_s$, γ may not necessarily be zero. The value of γ is crucial to the government. γ is directly added into the government's expected payoff function since the government needs to take the speculator's decision into account, even though the government makes its decision before the speculator does.

Consequently, the expected payoff function of the strong government to issue $D = D_s$ in the first period, denoted as W_s, is

$$W_s = \pi_h[(1-\gamma)V_s + \gamma(V_s - \alpha_s c(\theta^h))] - \pi_l(D_s)(e^* - m(\theta^l)). \tag{5}$$

The two terms that are inside the square bracket of W_s represent the expected payoff of the strong government when the fundamental is high with probability π_h. It is a weighted average of V_s and $V_s - \alpha_s c(\theta^h)$. The strong government enjoys V_s, given that there is no attack with probability 1-γ. The strong government receives $V_s - \alpha_s c(\theta^h)$, given that there is an attack with probability γ. The second term of W_s represents the expected losts incurred by the government as it compensates the holder of the put options. When the fundamental turns out to be low in $t = 2$ with probability π_l, the government has no incentive to defend the currency; thus the currency will float. The exchange rate then becomes, $m(\theta^l)$. The government is obligated to compensate the amount of $e^* - m(\theta^l)$ for each put option that was issued in the first period. Therefore, the total compensations would be $D_s(e^* - m(\theta^l))$. Equation (5) altogether would be the expected payoff function of the strong government issuing $D = D_s$ in the first period.

On the other hand, the expected payoff function of the weak government to mimic in the first period is,

$$W_w = \pi_h[(1-\gamma)V_w - \gamma(D_s)(e^* - m(\theta^h))] - \pi_l(D_s)(e^* - m(\theta^l)). \quad (6)$$

Inside the square bracket is the expected payoff when the high fundamental is in place. $1-\gamma$ is the probability that the weak government enjoys V_w. The speculator attacks with probability γ. The weak government does not have incentive to defend since $V_w - \alpha_w c(\theta^h) < 0$. The weak government will then incur the costs of compensating the holders of the put options, which is $D_s(e^* - m(\theta^h))$. The second term of W_w represents the total amounts of compensations when the fundamental is low. Equation (6) altogether is the expected payoff of the weak government when it issues $D_w = D_s$ in the first period.

From the payoff functions of both the strong government, W_s, and the weak government, W_w, we can recognize that issuing put options is costly for both types. However, the weak government will incur greater costs to issue the same amount of put options, as compared to the strong government. To initiate a valid signal, it is a necessary condition that the weak government suffers higher costs to issue put options than the strong government does. This condition is consistent with a famous paper written by Spence (1973), who had shown that education serves as the signal for workers' productivity. Spence (1973) indicated that the costs of education, including psychological costs, must be negatively correlated with the productivity of workers in order to make education be a valid signal. In my model, since it is more costly for the weak government to issue put options compared to the strong government, it is more likely that the strong government issues greater amounts of put options. If the amount of put options that were issued, D, is large, it is more likely that the speculator is facing a strong government. If D is low, then it is less likely that the government is a strong one. The speculator reorganize his beliefs about the government type after observing the amount of put options that were issued. q_2 denotes the updated probability that the speculator believes the government is a strong one given that the speculator has observed $D = D_s$ and the fundamental is high. q_2 is the posterior beliefs, which can be written as

$$q_2 = prob(S|D = D_s, \theta^h). \tag{7}$$

To better understand the determination of q_2, from Bayes' rule, it can be rewritten as,

$$q_2 = \frac{prob(S \text{ and } D = D_s)}{prob(D = D_s)}. \tag{8}$$

From the properties of conditional probability, we know that, $prob(S \text{ and } D = D_s) = prob(D = D_s|S)prob(S)$. $prob(S)$ is the unconditional probability of S, which is the ex ante beliefs of having a strong government, q_1. Therefore, $prob(S \text{ and } D = D_s) = prob(D = D_s|S)q_1$. The denominator of equation (8) is the unconditional probability of $D = D_s$. This probability cannot be observed immediately, but it can be written as, $prob(D = D_s|S)q_1 + prob(D = D_s|W)(1 - q_1)$. Therefore,

$$q_2 = \frac{prob(D = D_s|S)q_1}{prob(D = D_s|S) \cdot q_1 + prob(D = D_s|W)(1 - q_1)}. \tag{9}$$

Since the model analyzes signalling equilibrium, the strong government always have the incentive to issue $D = D_s$; thus, $prob(D = D_s|S) = 1$. Equation (9) can be simplified to

$$q_2 = \frac{q_1}{q_1 + prob(D = D_s|W)(1 - q_1)}. \tag{10}$$

$prob(D = D_s|W)$ represents the probability that the weak government mimics. If we denote $prob(D = D_s|W)$ as p_1, it becomes

$$q_2 = \frac{q_1}{q_1 + p_1(1-q_1)}. \tag{11}$$

When p_1 = 1, meaning the weak government mimics for sure, then $q_2 = q_1$. The speculator does not learn anything new about the government's type from observing the government policy. The updated beliefs would then be the same as the ex ante beliefs. A special case is that there are separating equilibria in which only the strong government issues D_s and the weak government always issues D_w, where $D_w \neq D_s$. The result would be $p_1 = 0$ and $q_2 = 1$. This means that the speculator will always believe that the government is a strong one when the speculator observes $D = D_s$. From equation (11), we can see that $q_2 \geqslant q_1$. The issuance of put options will enhances the speculator's beliefs of facing a strong government, rather than reduces it. Since more information is provided to the speculator, the speculator will no longer rely on the ex ante beliefs to determine his strategy. Instead, the speculator considers the updated beliefs, q_2, and the speculator's expected payoff function when the fundamental is high becomes

$$q_2(-t) + (1-q_2)[(e^* - m(\theta^h) + D_s(e^* - m(\theta^h)]. \tag{12}$$

Compared to equation (1), $D_s(e^* - m(\theta^h))$ is added to the function. Since there are only two players in the model, the speculator is the party holding all the put options that the government had issued. When the currency depreciates, the speculator receives the compensations from the government; therefore, it will be part of the speculator's payoff function.

The speculator determines its strategy, γ, by comparing equation (12) to the payoff of not initiating an attack, which is zero. The speculator's strategies can be listed as

$$\gamma = \begin{cases} 0 & when \;\; q_2 = \frac{q_1}{q_1+(1-q_1)p_1} > \frac{(e^*-m(\theta^h))(1+D_s)}{(e^*-m(\theta^h))(1+D_s)+t} \\ 1 & when \;\; q_2 = \frac{q_1}{q_1+(1-q_1)p_1} < \frac{(e^*-m(\theta^h))(1+D_s)}{(e^*-m(\theta^h))(1+D_s)+t} \\ \in (0,1) & when \;\; q_2 = \frac{q_1}{q_1+(1-q_1)p_1} = \frac{(e^*-m(\theta^h))(1+D_s)}{(e^*-m(\theta^h))(1+D_s)+t}. \end{cases} \tag{13}$$

Equation (13) specifies the strategic choices of the speculator. The probability to attack, γ, is zero when the updated belief, q_2, is greater than $[(e^*-m(\theta^h))(1+D_s)]/[(e^*-m(\theta^h))(1+D_s)+t]$. In this case, the expected payoff is strictly negative if the speculator attacks. When q_2 is smaller than $[(e^*-m(\theta^h))(1+D_s)]/[(e^*-m(\theta^h))(1+D_s)+t]$, the probability to attack, γ, is one since the expected payoff to attack is strictly positive. In the third case, when $q_2=[(e^*-m(\theta^h))(1+D_s)]/[(e^*-m(\theta^h))(1+D_s)+t]$ the speculator randomizes the probability to attack since he is indifferent between attack or stay put.

The choice variable for the weak government, p_1, is worth our attention to study. The left hand side of equation (14) is the expected payoff of mimicking, and the right hand side is the expected payoff of not mimicking.

$$\pi_h[(1-\gamma)V_w-\gamma(D_s)(e^*-m(\theta^h))]-\pi_l(D_s)(e^*-m(\theta^l)) \gtreqless \pi_h(0)+\pi_l(0). \tag{14}$$

In $t=1$, if nature determines that the government is a weak one, the weak government continues to decide on the amount of put options to issue. The weak government decides to mimic the strong government when the corresponding expected payoff is greater than not mimicking, that is when the left hand side of equation (14) is strictly greater than the right hand side. However, the weak government chooses not to mimic if the left hand side of equation (14) is strictly smaller than the right hand side. The weak government randomizes its strategy, p_1, if equation (14) holds with equality. The weak government's strategic choices are listed in the following

$$p_1=\begin{cases} 0 & when \;\; D_s>\frac{\pi_h V_w(1-\gamma)}{\pi_h\gamma(e^*-m(\theta(^h))+\pi_l(e^*-m(\theta^l))} \\ 1 & when \;\; D_s<\frac{\pi_h V_w(1-\gamma)}{\pi_h\gamma(e^*-m(\theta(^h))+\pi_l(e^*-m(\theta^l))} \\ \in(0,1) & when \;\; D_s=\frac{\pi_h V_w(1-\gamma)}{\pi_h\gamma(e^*-m(\theta(^h))+\pi_l(e^*-m(\theta^l))}. \end{cases} \tag{15}$$

From equation (15), we can see that D_s and γ have a negative relationship. It makes intuitive sense that when the level of put options is high, it is less likely that the weak government mimics; therefore, it is less likely that the speculator initiates an attack.

Now, the strategies of the speculator and the weak governments have been analyzed.

If the government turns out to be a strong one in the first period, it is required that the strong government has the incentive to issue $D = D_s$ in the first period in order to have a signalling equilibrium. Issuing $D = D_s$ is the only choice of strategy for the strong government in the signalling equilibrium. However, the level of D_s has not yet been determined. We will discuss the equilibrium level of D_s when we analyze different possible equilibria in the next subsections.

4.2 Separating Equilibrium

There are infinite numbers of possible signalling equilibria in the model. The signalling equilibria can be classified into two groups. The first group is the separating equilibria which the weak government never mimics and does not issue any put options. Only the strong government issues $D_s > 0$. The second group is the mixed strategy equilibrium which both the weak government and the speculator randomize their strategies. This subsection of the paper will provide an in depth analysis of the separating equilibria. The mixed strategy equilibrium will be discussed in the next subsection.

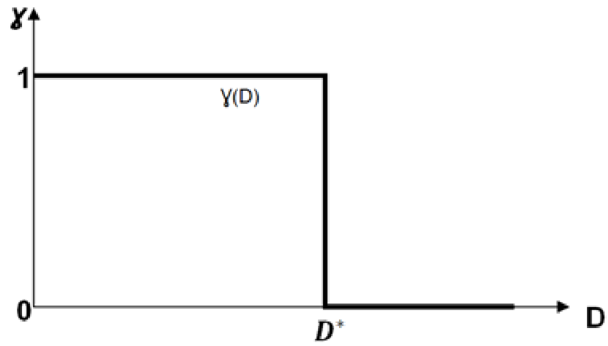
In a separating equilibrium, the strong and the weak government both issue different amounts of put options, $D_s \neq D_w$. In other words, the weak government sets $p_1 = 0$ and does not mimic the strong government. Since the government type is self revealing in this equilibrium, the speculator has no reason to initiate an attack when the fundamental is high and when the government issues $D = D_s$; thus, $\gamma = 0$.

Supposed that the speculator forms some beliefs that there is some level of put options that were issued, D^*, such that if $D < D^*$, then the speculator believes it is a weak government. If $D \geqslant D^*$, then the speculator believes it is a strong government. Equation (16) summarizes these beliefs.

$$\begin{aligned} b(S|D \geqslant D^*) &= b(S|D = D^*) &= 1 \\ b(S|D < D^*) &= b(S|D = 0) &= 0 \end{aligned} \tag{16}$$

If these are the speculator's conditional beliefs, his best response function, $\gamma(D)$, can be shown in Figure 2.

Figure 2: Speculator Best Response Function



Given the best response function of the speculator, the government maximizes its expected payoff by choosing the level of D. Considered that the government issues $D < D^*$, it will simply issue $D = 0$ because put options are costly to the issuer. Therefore, there is no benefits to issue D, which $D^* > D > 0$. Similarly, if the government decides to send the signal and issues $D \geqslant D^*$, it will simply issue $D = D^*$. There are only costs but no benefits to set $D > D^*$. To confirm the speculator's beliefs, the strong government must issue $D_s = D^*$, while the weak government must issue $D_w = 0 < D^*$. Otherwise, it is not optimal for the speculator to form such beliefs.

To confirm this is an equilibrium, the weak government, the strong government and the speculator must be incentive compatible with the conjectures stated in the previous paragraphs. There must be incentive for the weak government not to mimic. After replacing D_s with D^* in the weak government's expected payoff function as stated in equation (6), the expected payoff of mimicking needs to be strictly less than the expected payoff of not mimicking, therefore,

$$\pi_h[(1-\gamma)V_w - \gamma(D^*)(e^* - m(\theta^h))] - \pi_l(D^*)(e^* - m(\theta^l)) < \pi_h(0) + \pi_l(0). \quad (17)$$

The right hand side of equation (17) is the expected payoff of the weak government not mimicking. When the weak government does not mimic, then $D_w = 0$. The speculator will then initiate an attack to the weak government. The weak government simply gives up the peg and receives zero payoff. On the left hand side of equation (17), the weak government mimics and set $D_w = D^*$. The speculator does not attack, as based his belief when $D_i = D^*$; therefore, $\gamma = 0$. After rearranging equation (17), it becomes

$$D^* > \frac{\pi_h V_w}{\pi_l(e^* - m(\theta^l))}. \tag{18}$$

In order for the weak government to set $D_w = 0$, D^* must be large enough to exhaust all the incentives for the weak government to mimic. Equation (18) shows the range of D^* that eliminates the incentive for the weak government to mimic.

On the other hand, there must be greater incentive for the strong government to issue $D_s = D^*$ than to issue $D_s = 0$. After replacing D^* with D_s in the strong government's expected payoff function, as stated in equation (5), it needs to be greater than the expected payoff of not issuing the put options, therefore,

$$\pi_h[(1-\gamma)V_s + \gamma(V_s - \alpha_s c(\theta^h))] - \pi_l(D^*)(e^* - m(\theta^l)) > \pi_h[V_s - \alpha_s c(\theta^h)] + \pi_l(0). \tag{19}$$

The right hand side of equation (19) represents the expected payoff when the strong government issues $D_s = 0$. Given the speculator's beliefs, he initiates an attack when $D < D^*$. As a result, the strong government faces a currency attack and incurs the costs of defending when the fundamental is high. When the fundamental is low and the strong government issues $D_s = 0$, there is no incentive for the government to defend; therefore, its payoff is zero. After simplifying equation (19) and set $\gamma = 0$, it becomes

$$D^* < \frac{\pi_h \alpha_s c(\theta^h)}{\pi_l(e^* - m(\theta^l))}. \tag{20}$$

Equation (20) indicates D^* can not be too large. Otherwise, it would drive out the incentive for the strong government to issue $D_s = D^*$. After combining equation (18) and equation (20), it becomes the condition stated in equation (21),

$$\frac{\pi_h V_w}{\pi_l(e^* - m(\theta^l))} < D^* < \frac{\pi_h \alpha_s c(\theta^h)}{\pi_l(e^* - m(\theta^l))}. \tag{21}$$

In other words,

$$V_w < \alpha_s c(\theta^h). \tag{22}$$

Equation (21) is the range of D^* that is needed to form a separating equilibrium. At the same time, equation (22) is also required. In other words, the benefits of maintaining the peg for the weak government has to be lower than the costs of defending the currency for the strong government when the fundamental is high. If equation (22) does not hold, the range of D^* is vanished; thus, the separating equilibrium is not possible to form.

Figure 3: Optimizing Choice of Put Options for Both types of Government

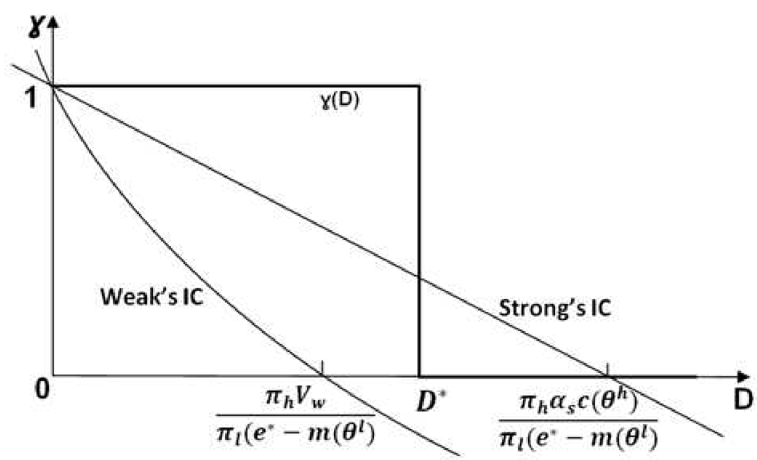


Figure (3) is a graphical approach to the problem. The best response function of the speculator, and the indifference curve for both the weak and the strong government are depicted in figure (3). The weak government's indifference curve goes through the point, $(0, 1)$, and it intersects the x-axis when $D = \pi_h V_w / \pi_l(e^* - m(\theta^l))$. The weak government is indifferent between not issuing put options and issuing $D_w = \pi_h V_w / \pi_l(e^* - m(\theta^l))$. The strong government's indifference curve also goes through $(0, 1)$ but it intersects the x-axis when $D = \pi_h \alpha_s c(\theta^h) / \pi_l(e^* - m(\theta^l))$. The necessary condition for the separating equilibrium, which is stated in equation (21), is simply indicating that the strong government's indifference curve intersects the x-axis on the right of the weak government's indifference curve. If this condition holds, the strong government has incentive to issue $D_s = D^*$. Its expected payoff is maximized when it sets $D_s = D^*$, since the expected payoff increases as the indifference curve move towards southwest.

Before we can conclude that it is the separating equilibria, the speculator's beliefs must be confirmed. Given that the weak government has no incentive to mimic, the speculator has no incentive to deviate from setting $\gamma = 0$. From the speculator's strategies that are listed in equation (13), the speculator sets $\gamma = 0$ when $q_2 = 1$ since $1 > [(e^* - m(\theta^h))(1 + D^*)]/[(e^* - m(\theta^h))(1 + D^*) + t]$; thus, the speculator's beliefs are confirmed. As a result, when equation (21) and (22) hold and the speculator forms beliefs according to equation (16), there is a separating equilibrium. Note that there are infinite numbers of possible equilibrium values for D^*; therefore, there are infinite numbers of separating equilibria.

4.3 Mixed Strategy Equilibrium

The previous sub-section has shown that there are infinite numbers of separating equilibria. This sub-section is going to show that there is a unique mixed strategy equilibrium that both the weak government and the speculator randomize their strategies, $p_1 \in (0, 1)$ and $\gamma \in (0, 1)$ respectively. In other words, the strong government issues $D = D_s$, and the weak government issues

$$D = \begin{cases} D_s & \text{with probability } p_1 \\ D_w \neq D_s & \text{with probability } 1 - p_1. \end{cases} \tag{23}$$

To support these strategic choices of the government, the speculator has to form beliefs that are consistent with the government's actions. The speculator's beliefs become

$$\begin{aligned} b(S|D \geqslant D^*) &= b(S|D = D^*) &= q_2 &= \tfrac{q_1}{q_1 + (1-q_1)p_1} \\ b(S|D < D^*) &= b(S|D = 0) &= 0. & \end{aligned} \tag{24}$$

When the speculator observes $D = D^*$ the speculator believes the government is a strong one with a probability of q_2. Since the weak government may mimic, $p_1 > 0$, then $q_2 < 1$. Moreover, the speculator will definitely not believe that the government is a strong one if $D < D^*$. We had argued in the previous subsection that there is no incentive to issue $D > D^*$ and D, which $D^* > D > 0$. As a result, $D_s = D^*$ and $D_w = 0$ will result in this equilibrium.

To confirm this is an equilibrium, the weak government, the strong government and the speculator must be incentive compatible with the conjectures stated in the previous paragraphs. For the weak government, it must be indifferent between mimicking and not mimicking. After replacing D_s with D^* in equation (6) of the weak government's expected payoff function, the function must be equalized to the expected payoff of not mimicking. Therefore,

$$\pi_h[(1-\gamma)V_w - \gamma(D^*)(e^* - m(\theta^h))] - \pi_l(D^*)(e^* - m(\theta^l)) = \pi_h(0) + \pi_l(0). \quad (25)$$

After simplifying, it becomes

$$D^* = \frac{\pi_h V_w (1-\gamma)}{\pi_h \gamma (e^* - m(\theta^h)) + \pi_l (e^* - m(\theta^l))}. \quad (26)$$

D^* is a decreasing function with γ since $\partial D^*/\partial\gamma < 0$. It makes intuitive sense that as the equilibrium level of put options increases, it is less likely that the weak government mimics. Consequently, it is less likely that the speculator initiates an attack. On the other hand, there must be greater incentive for the strong government to issue $D_s = D^*$ than to issue $D_s = 0$. After replacing D^* with D_s in equation (5) of the strong government's expected payoff function, W_s, the function needs to be greater than the expected payoff of not issuing the put options. At the end,

$$\pi_h[(1-\gamma)V_s + \gamma(V_s - \alpha_s c(\theta^h))] - \pi_l(D^*)(e^* - m(\theta^l)) > \pi_h[V_s - \alpha_s c(\theta^h)] + \pi_l(0). \quad (27)$$

Equation (27) is the same as equation (19). After simplifying and rearranging the equations, it becomes

$$D^* < \frac{\pi_h \alpha_s c(\theta^h)(1-\gamma)}{\pi_l(e^* - m(\theta^l))}. \tag{28}$$

Combining equation (26) and (28), we have

$$\begin{aligned} \frac{\pi_h \alpha_s c(\theta^h)(1-\gamma)}{\pi_l(e^* - m(\theta^l))} &> \frac{\pi_h V_w (1-\gamma)}{\pi_h \gamma (e^* - m(\theta^h)) + \pi_l (e^* - m(\theta^l))} \\ \frac{\alpha_s c(\theta^h)}{\pi_l(e^* - m(\theta^l))} &> \frac{V_w}{\pi_h \gamma (e^* - m(\theta^h)) + \pi_l (e^* - m(\theta^l))}. \end{aligned} \tag{29}$$

This condition is needed to be satisfied in order to have mixed strategy equilibrium. Since the term $\pi_h \gamma (e^* - m(\theta^h))$ is added to the denominator of the right hand side of equation (29), it is a weaker condition than the condition in the separating equilibrium in equation (22).

From equation (26), we know the relationship between D^* and γ. After rearranging it, we can get an expression of γ in terms of D^*, which is

$$\gamma = \frac{\pi_h V_w - D^* \pi_l (e^* - m(\theta^l))}{\pi_h [D^*(e^* - m(\theta^h)) + V_w]}. \tag{30}$$

We can then plug γ into the expected payoff function of the strong government in equation (5), and take the first order condition to solve for the optimal level of D^* of the strong government. The optimal level of D^* is,

$$D^* = \frac{\sqrt{\frac{V_w \alpha_s c(\theta^h)[\pi_h (e^* - m(\theta^h)) + \pi_l (e^* - m(\theta^l))]}{\pi_l (e^* - m(\theta^l))}} - V_w}{e^* - m(\theta^h)}. \tag{31}$$

Noticed that the numerator of equation (31) may not be strictly positive, but it is necessary that the amount of put options being issued is strictly positive. Therefore, the following condition is needed to restrict the range of D^*.

$$\sqrt{\frac{V_w \alpha_s c(\theta^h)[\pi_h (e^* - m(\theta^h)) + \pi_l (e^* - m(\theta^l))]}{\pi_l (e^* - m(\theta^l))}} - V_w > 0$$

After simplifying the equation, the above inequality can be reduced to

$$\frac{\alpha_s c(\theta^h)}{\pi_l(e^* - m(\theta^l))} > \frac{V_w}{\pi_h(e^* - m(\theta^h)) + \pi_l(e^* - m(\theta^l))} \tag{32}$$

Both equation (32) and equation (29) are needed to be satisfied to have a mixed strategy equilibrium. Equation (29) is a stronger condition than equation (32) since $\gamma \in (0, 1)$. If equation (29) holds, equation (32) is automatically satisfied.

Since D^* is known from equation (31), γ^* can also be determined from equation (30). The only choice variable that is not yet known is the probability that the weak government mimics, p_1. When $\gamma \in (0, 1)$, and after replacing D^* with D_s, equation (13) becomes

$$q_2 = \frac{q_1}{q_1 + (1 - q_1)p_1} = \frac{(e^* - m(\theta^h))(1 + D^*)}{(e^* - m(\theta^h))(1 + D^*) + t}.$$

Since D^* is known from equation (31), we can then solve for p_1, which is

$$p_1 = \frac{q_1}{1 - q_1}\left[\frac{t}{(e^* - m(\theta^h))(1 + D^*)}\right]. \tag{33}$$

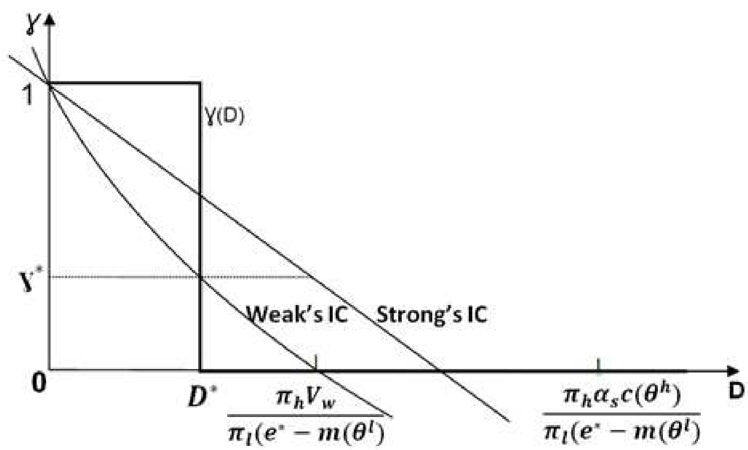
In equation (33), p_1 is not bounded between zero and one. However, if it assumes

$$\frac{(e^* - m(\theta^h))(1 + D^*)}{(e^* - m(\theta^h))(1 + D^*) + t} > q_1 \tag{34}$$

holds, p_1 would then be bounded between zero and one. Equation (34) is a valid assumption since the model assumes equation (3) will hold. Notice equation (3) is a stronger condition than equation (34), so equation (34) holds immediately.

A graphical approach of the mixed strategy equilibrium is shown in figure (4). Given the weak government randomizes p_1, the equilibrium D^* and equilibrium γ^* must lie on the weak government's indifference curve. Meanwhile, the strong government chooses the optimal level of D^* according to its first order condition. The speculator forms beliefs that is consistent with the government's strategy, so the best response function, $\gamma(D)$, cut through the weak government's indifference curve at the optimal level of D^*. In figure (4), the corresponding γ^* can be determined; thus p_1 is also pinned down by equation (33). The necessary condition in equation (29) indicates that at γ^*, the strong government's indifference curve must be on the right of the weak government's indifference. The condition that is needed in the mixed strategy equilibrium, as noted in equation (29), is a weaker condition than the condition in the separating equilibrium, equation (22). In the mixed strategy equilibrium, it is not necessary that the strong government's indifference curve is on the right of the weak government's indifference curve when $\gamma = 0$. The mixed strategy equilibrium is possible as long as the strong government's indifference curve is on the right of the weak government's indifferent curve at the equilibrium level γ^*. At the end of the day, it is better off for the strong government to issue $D_s = D^*$ than to issue $D_s = 0$ if the condition has been met.

Figure 4: Optimizing Choice of Put Options in Mixed Strategy Equilibrium



5 Conclusion

By using Spence (1973)'s job market signalling model as the framework, I have constructed a model that shows the existence of the separating and mixed strategy equilibria as a result of signalling. In the cases where there is no signal, the speculator initiates an attack regardless of whether the government is a strong type. However, in the separating equilibria, the speculator does not initiate an attack when the fundamental is high and the government is a strong type. Compared to the case where there is no signal, society would save the costs of defending and the transaction costs in the scenario where the fundamental is high. In the mixed strategy equilibrium, the probability that the speculator attacks when the fundamental is high is reduced to γ^*. Therefore, the expected costs of defending and the expected transaction costs are also reduced. These results provide justification to Miller's suggestion in that the use of put options as a signal might have restored the confidence of the Hong Kong dollar and discouraged the speculator to initiate currency attacks.

References

Chen, N. F. (2001). The Hong Kong currency board during the 1997-8 crisis: Problems and soluations. *International Review of Finance*, 2:99–112.

Cheng, L. K., Kwan, Y. K., and Lui, F. T. (1999a). An alternative approach to defending the Hong Kong dollar. Finance Working Papers 172, East Asian Bureau of Economic Research.

Cheng, L. K., Kwan, Y. K., and Lui, F. T. (1999b). Credibility of Hong Kong's currency board: The role of institutional arrangements. Finance Working Papers 198, East Asian Bureau of Economic Research.

Cheng, L. K., Kwan, Y. K., and Lui, F. T. (1999c). Risk premium, currency board, and attacks on the Hong Kong dollar. CED Working Papers 9901, HKUST Institutional Repository.

Chiu, Y. W. S. (2003). Signaling Versus Commitment Strengthening: Exchange Rate Insurance Against Currency Attacks. *SSRN eLibrary*.

Jao, Y. C. (2001). *The Asian Financial Crisis and The Ordeal of* H*ong* K*ong*. Quorum Books.

Liew, L. H. and Wu, H. X. (2002). Not all currency traders believe in unfettered free markets: Currency speculation and market intervention in Hong Kong. *The China Quarterly*, 170(-1):441–458.

Miller, M. (1998a). Asian financial crisis. *Japan and the World Economy*, 10(3):355–358.

Miller, M. (1998b). H*ong* K*ong Dollar Crisis Revisited*. Informwide: Hong Kong. edited by Alex Chan and Fred Kwan (in Chinese).

Morris, S. and Shin, H. S. (1998). Unique equilibrium in a model of self-fulfilling currency attacks. *American Economic Review*, 88(3):587–97.

Spence, A. M. (1973). Job market signaling. *The Quarterly Journal of Economics*, 87(3):355–74.