Asymmetric Tax Competition, Agglomeration Economies, and Natural Resources

by

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Abstract

The Canadian constitution entitles the provinces to fully benefit from natural resources found in their territories; thus, rents accrued from natural resources are usually used by the respective provinces for their own development. However, due to reasons of agglomeration economies, it would be more efficient to have the federal government redistribute the rents to places that could make better use of them. This theoretical paper is an attempt to account for that phenomenon; in effect, it extends the standard asymmetric tax competition theory, firstly by considering asymmetry in infrastructure levels, and secondly by adding a natural resource sector. Initially, a simple model of two asymmetric regions that differentiate solely by their population size is constructed. The model predicts the effects of adding a natural resource to the smaller region, especially in the context of possible agglomeration in either of the regions. By comparing between the outcomes under cooperative and non-cooperative settings, the model shows how inefficiency arises from the improper usage of the resource rents. In a tax competition setting, the natural resource gives the smaller region an advantage, so that most of the nation's capital might be drawn to it, given a large enough resource. Therefore, due to the natural resource, the nation only benefits from sub-optimal welfare level, and faces the possibility of reaching an undesired Nash Equilibrium outcome where agglomeration occurs in the smaller region.

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Contents

1 Introduction

Canada is a resource abundant country. Its resources, composed mainly of energy, minerals and forests, are spread across its varied regions. The Canadian economy relies quite heavily on these resources — according to Statistics Canada, in 2007 the natural resource sector was responsible for approximately 12% of Canada's total GDP. However, perhaps more importantly, these natural resources play a key role in one of Canada's main political issues nowadays — regional imbalances.

According to the Canadian Constitution Act of 1982,[1] the regions are entitled to fully benefit from the natural resources found in their territories. Thus, having several regions without natural resources, and a few with, creates an imbalance in the federation. One aspect of this imbalance is derived from the fact that regions with profitable natural resources invest the rents accrued in the development of their own territory, while (supposedly) neglecting the interest of the federation.

A question then arises — what is the interest of the federation? Indeed, this is quite debatable. However, in this paper we view this interest to be maximizing the welfare of the residents of the entire nation. Is this welfare maximized in the case where each region exploits its natural resources to its own good? From the perspective of the federation, are the rents accrued from natural resources invested in the most efficient manner? We look into one specific aspect of this issue, to try and form answers to these questions; this aspect relates to urban increasing returns, also known as agglomeration economies.

Referring to theories on agglomeration economies, initially laid by Marshall (1920), it is suggested that firm productivity is not only dependent on how production is organized within a firm, but also on its location. A firm located in an area where there is concentrated economic activity can benefit from positive externalities in the form of knowledge spillovers, shared inputs, and labor market pooling; thus, potentially

[1]Specifically, by footnote (23) of the consolidated Constitution Act of 1982 that refers to Section (92a) of the Constitution Act of 1867.

exhibiting increasing returns to scale in production. Indeed, this was confirmed by Baldwin et al., in their study for Statistics Canada (2007).

That said, we note that regions with more significant economic agglomeration present greater potential for increased productivity, and for higher returns on investment. Therefore, we ask — should a federation benefit from agglomeration economies by redistributing natural resource rents to regions with higher concentration of economic activity? Does the nation suffer from allowing the regions to fully benefit from the natural resources found in their territories, instead of directing their rents to where the return on investment would be maximized?

In this paper we look into the specific case of Ontario and Alberta to better understand the general phenomenon. Ontario, according to Statistics Canada, is the most populous region in Canada, with around 12.5 million residents (as of 2009); moreover, it has the largest economy in the nation — almost twice the size (in GDP terms) of the second largest one (Quebec). Albeit having the highest economic concentration in Canada, Ontario has limited natural resources; it is, in fact, influenced mostly by its service sector as well as by its large manufacturing sector which accounts for around half of the country's manufacturing output. On the other hand, Alberta is a smaller region in both population and economic terms. It has approximately 3.5 million residents (as of 2009), and an economy that is half the size (in GDP terms) of Ontario's. However, as opposed to Ontario, Alberta benefits from many natural resources found in its territory; namely, oil and gas, forests, and large farm lands. Alberta is, in fact, the largest oil and gas producer in Canada, and the second largest gas exporter in the world, with oil reserves that are second only to Saudi Arabia's in size. To better understand their economic influence — in 2008, revenues from the energy sector alone surpassed $100 billion; note that the aforementioned Constitution Act makes Alberta their sole beneficiary.

Adopting Zodrow and Mieszkowski's basic capital tax competition model (1986), and Bucovetsky's addition of asymmetry in population size (1991), we construct a simple asymmetric tax competition model that tries to mimic the situation seen in Alberta and Ontario. In its basic form the model has two regions (members of the same nation) that differ solely by their population size. The nation has fixed supply of capital and labor, the former being perfectly mobile across the nation and the latter immobile. Regional governments want to maximize the welfare of their residents; thus, they compete for the national capital by means of tax competition. The larger region (which has more population) does not have natural resources, while the smaller (less populated) one does (although not initially). We extend the theory by adding asymmetry in infrastructure levels, to account for possible agglomeration effects in either of the regions (thus, adopting concepts from the *New Economic Geography* literature). Thereafter, the model is extended further, as we add a natural resource sector to the smaller region. We aim to realize how resources are allocated across the nation given this setting, and perhaps more importantly, how the natural resource affects that allocation, especially with regard to the advantages it gives the smaller region in the national tax competition, and in the possibility of causing unwanted agglomeration in that region (as opposed to having it in the larger region, to the benefit of the entire nation). Ultimately, through this model, and the analysis to follow, we try to answer the questions posed earlier in this section. Note that there has been much discussion over natural resources in the economic literature; however, no serious attempt was made at examining the role of natural resources in the context of fiscal competition.

The paper is structured as follows — in the following section we go through a literature review of the two main phenomena seen in the model; namely, capital tax competition and agglomeration economies. Thereafter, the basic model is outlined. Section 3 presents the analysis. That section is divided to two sub-sections; the first goes through the analysis under cooperative environment (deriving the social optimum), while the second looks into the non-cooperative case (deriving the Nash Equilibrium outcome). Each sub-section follows three stages. The first stage goes through the analysis of the basic model, having neither infrastructure nor a natural resource sector in the smaller region. The second stage adds infrastructure to each of the regions so that we get increasing returns to scale in production; thus, potentially having agglomeration in either of the regions. The third and final stage adds a natural resource sector to the smaller region. A comparison is then made between the cooperative and non-cooperative outcomes. Following that, in section 5 we consider possible extensions and limitations to the model, and in section 6 we conclude.

2 Literature Review

Two main topics represent the building blocks of the model — the first being capital tax competition, and the second agglomeration economies. There is a large body of literature on each; therefore, this section is divided to two separate sub-sections, each covering the relevant literature on one of these topics.

2.1 Capital Tax Competition

Let us firstly look into the definition of a tax competition, as it was suggested by Wilson and Wildasin (2004). In a broad sense, a tax competition may be defined as any form of non-cooperative tax setting by independent governments. Narrowing this definition to be more applicable to what is seen in this paper, a tax competition would be regarded as non-cooperative tax setting by independent governments, where each government's policy choices influence the allocation of a mobile tax base of the other regions. Note that in our case this mobile tax base is capital.

Early contributions to this field were initially made by Oates (1972) as well as by Tiebout (1956). Starting with Tiebout (1956), who in some sense adopted the more general definition of tax competition, we see the first concept of tax competition arising, though in a different and simpler context from what is seen in recent related literature. In his paper, Tiebout (1956) presented a model of many regions, where each is controlled by its landowners who seek to maximize the after tax value of the region's land by attracting more people to reside in it. This was done by presenting competitive tax levels, which brought up a tax competition between the regions. Since in his model regions were utility takers, making it similar to models of competitive markets for private goods, the equilibria was pareto efficient so that the tax competition for households led to efficient provision of public goods (referred to as the *Tiebout Hypothesis*). At a later stage this result was extended by Fischel (1975) and White (1975) to include mobile firms.

Oates (1972), on the other hand, adopted the narrower definition of tax competition. In his paper, which looked into the efficiency problems associated with competition for capital by local governments, it was hypothesized that regions are, in fact, interdependent, so that actions taken by one regional government, in order to increase the welfare of its residents, affected (or rather decreased, given scarce national resources) the welfare of the residents in a different region. In the literature this is referred to as *Fiscal Externality*; presenting this idea formed the main difference between this line of thought, and Tiebout's. As opposed to the latter, this new approach resulted in inefficient equilibria where taxes and public expenditure were set at suboptimal levels, due to this fiscal externality.

It was only in the 80s that Oates's intuition was formalized; firstly by Beck (1983), then by Wilson (1986) and Zodrow and Mieszkowski (1986). Beck's model is regarded as being less general, while Wilson's presents cumbersome production structure; thus, it is Zodrow and Mieszkowski's (1986) that is considered the basic tax competition model (Wilson 1999). Drawing on Pigou's proposition (1947) that public services are undersupplied when distortionary taxes are used, as well as on Atkinson and Stern's analysis of that proposition (1974), Zodrow and Mieszkowski formalized the concept of fiscal externality. This is, in fact, the model on which this paper is largely based on; therefore, it is important to discuss it more thoroughly.

In their model there are n identical regions, in a nation with a fixed amount of capital. Capital is perfectly and costlessly mobile across regions, while labor is not.

Each region levies a head tax on residents, and a source-based per unit tax on capital. Residents have identical preferences over private consumption and a pure public good, and each owns an equal share of the nation's capital. Firms use capital and labor to produce a single output that is used partially by the government to produce a pure public good (whereas in a later stage this pure public good turns to business services, and at the final stage both the public good and the business services are considered concurrently). Each region acts strategically, and tries to derive the optimal tax rate that maximizes the utility of its residents, subject to the regional budget constraint; thus, a tax competition is modelled along Cournot-Nash lines. Under the case where the head tax is constrained, the following key result is derived:

$$MB_{G_i} = \frac{MC_{G_i}}{1 + \frac{T_i}{K_i}\frac{dK_i}{dT_i}} > 1 \quad , \quad \forall i \in n$$

The above outcome is given a *Modified Samuelson Condition* (Batina 1990) interpretation, which shows the discrepancy between the social value of an additional unit of capital and the social opportunity cost of this unit, measured from the single region's viewpoint. Consequently, we see that in equilibrium public services would be undersupplied in each of the regions. This is an important outcome that will be replicated in the analysis made in this paper.

Note that in both Wilson's (1986) and Zodrow and Mieszkowski's (1986) models, the regions are assumed so small as to have no influence on national variables. Thus, the equilibrium outcome must equalize the utility levels of all residents across the nation as well as have all regions undertake similar policies; the outcome is, thus, symmetric. As opposed to these two models, in Wilson (1987) the terms of trade are not exogenously given, and so the equilibrium outcome dictates that different regions have different policies; however, since regions are still assumed small, the utility of the residents remain to be equal across the nation, and the result is still symmetric in that sense. Wildasin (1988) presents a model with finite number of jurisdictions, where each can potentially influence national variables; nonetheless, he assumes equal population in each region, and restricts his attention to symmetric Nash Equilibria only. In general, Hoyt (1991) shows that as the number of regions drops, the tax rates as well as the public good levels rise; nevertheless, we take note that all the above models present inefficient outcomes.

Departing from these symmetric settings, Bucovetsky (1991) and Wilson (1991) advanced Zodrow and Mieszkowski's model by considering an asymmetric environment. In both papers the asymmetry is derived solely from differences in population size. Both present a model that resembles Zodrow and Mieszkowski's, with the exception of having only two regions, a larger (more populated) one, and a smaller (less populated) one. The result is similar in both papers. It is shown that differences in population size imply differences in the perceived elasticity of capital supply to each region. Specifically, the result shows that the supply of capital to the larger region is less responsive to tax changes, so that an unilateral tax increase in the larger region makes it lose less capital per capita than the capital per capita that would have been lost in the smaller region had it had undertaken a similar move. This, in turn, implies that in equilibrium the larger region may levy higher tax rates, which is the key outcome derived from these two models.

Note that the benchmark case of this paper follows Bucovetsky's (1991) setting. This is because we initially take Zodrow and Mieszkowski's model, restrict it to two identical regions, and add the sole asymmetry in the regional population sizes. However, thereafter we extend this theory by adding infrastructure that brings about increasing returns in production (thus, presenting the possibility of having agglomeration in either of the regions) as well as by introducing a natural resource sector in the smaller region.

2.2 Agglomeration Economies

The concept of external economies, i.e. that positive externalities may cause agglomeration, was first introduced by Alfred Marshall (1920). Marshall identified three main causes for having economic integration —

- *Labor Market Pooling*: localized firms benefit from a greater selection of industry-specific skilled workers which, thus, ensures lower probability of labor shortage.
- *Shared Inputs*: localized industries benefit from the production of nontradable specialized inputs, especially given the short proximity to each other (which may potentially present lower transportation and trading costs).
- *Knowledge Spillovers*: clustered firms benefit from improved production functions, compared to isolated firms.

This concept was later formalized in several seminal papers by Krugman (1991), Krugman and Venables (1995), and Venables (1996). These papers form the basis for the *New Economic Geography* literature; they analyze the relationship between trade integration and industrial location, or in other words, they introduce and formalize the concept of agglomeration economies. Specifically, it is Krugman's paper (1991) that presents the basic model in the field. Building on Dixit and Stiglitz's framework (1977) of increasing returns to scale and monopolistic competition, Krugman develops a two-region, two-good model involving labor mobility, planet-level scale economies, and trade costs. In this model, both firms and consumers decide where to locate. It is found that firms choose to locate at the larger market, because it is where their trading costs are minimized; conversely, consumers choose to locate where there is a larger number of firms because this offers them greater access to manufactured goods. Thus, these two effects, which Krugman identifies as *backward* and *forward* linkages, produce agglomeration of economic activity in a manufacturing *core* and an agricultural *periphery.* That said, this paper, as well as the other seminal ones, show how economic integration could lead to increased concentration of economic activity.

One of the main results of these models is that once activity is agglomerated in a certain region, it gets stuck there, because of demand and supply linkages. Thus, a consequence of this is that mobile factors may not respond to marginal changes in taxes once they are locked in an industrial cluster (Forslid and Andersson 2003);

nonetheless, Hartman (1985), Boskin and Gale (1987) and Slemrod (1990) show that taxes affect the concentration of industries.

Following that, the connection between tax competition and agglomeration economies was firstly investigated by Persson and Tabellini (1992), who considered the effects of integration on tax competition. In their model capital is the mobile factor, with its ownership distributed across the population, and taxes are set by the median voter. It is shown that economic concentration intensifies tax competition, so that tax rates decrease; also, they show that capital becomes more responsive to tax incentives, but yet the median voter shifts to the left so that the tax reduction is mitigated.

After Persson and Tabellini (1992), there have been few attempts to address issues of tax competition in an economic-geography framework. Three such attempts were made by Ludema and Wooton (2000), Kind et al. (2000), and Baldwin and Krugman (2004). Ludema and Wooton (2000) examine the tendency of tax competition between national governments to influence the location of manufacturing activity. Working under the framework of a homogeneous-good oligopoly and moving costs, where both factor-mobility and trade costs are varied (meaning that agglomeration can occur due to each), they mainly focus on the effects of integration on the intensity of tax competition. They find that agglomeration occurring due to decreased trade costs attenuates tax competition, while that which occurs due to increased labor mobility has mixed effects.

Kind et al. (2000) use a new economic geography model to analyze tax competition between two countries trying to attract internationally mobile capital; specifically, they investigate how spatial agglomeration of economic activity affects the outcome of capital tax competition. They present a model of two identical countries, where capital, goods, and firms are internationally mobile. Here also the tax competition depends on the interaction between two forces of agglomeration; however, as opposed to Ludema and Wooton (2000), in this case the two acting forces are trade costs and pecuniary externalities. They show that a country hosting an agglomeration may find it optimal to levy a source based tax on capital income.

Finally, Baldwin and Krugman (2004) present a two-nation, two-sector, and two-factor model, having capital mobile and labor immobile. Their paper looks at the impact of tighter goods market integration on tax competition when agglomeration economies are significant. In their model, they set one sector to have constant returns technology, while the other has increasing returns in its production, to bring about the possibility of agglomeration; the model presented in our paper follows a similar setting. Their analysis shows that firms choose to locate at the agglomerated region, allowing the government of that region to set higher tax rates compared to those of the other region. In practical terms, they explain why, for instance, in the European economy it has not been the case that integration has led to a narrowing of tax differentials, but rather the opposite. This result is relevant to our paper, where we make an attempt to undertake a parallel discussion, but with an application to the Canadian economy.

While there are differences in the equilibrium concepts used in these papers, they all provide examples where tax competition does not lead to equal tax rates between regions. More importantly however, these papers show that the inertia resulting from concentration of economic activity in one region gives rise to a rent that is taxable, so that a region hosting an agglomeration may charge higher taxes without losing capital or its industrial base.

In our paper we follow the above models since we, as well, look into the connection between agglomeration economies and capital tax competition. However, we extend the ideas presented in these models, as we try to understand how the addition of a natural resource sector affects that connection.

3 The Benchmark Model

Let us use the framework of the basic capital tax competition model developed by Zodrow and Mieszkowski (1986), in its simplest form — using 2 regions, which are members of a common nation. Each region has a fixed supply of immobile population (denoted by L_1 and L_2; where $L_1 + L_2 = L$, which is the nation's population size), and the nation has a fixed supply of perfectly and costlessly mobile capital (denoted by K^*); thus, making K^*, L_1, and L_2 exogenously determined. Adopting then the framework presented by Bucovetsky (1991), the two regions are identical in all respects, except for their population size. Region 1 is larger than region 2, such that $L_1 > L_2$. This asymmetry aspires to capture the main difference —in its simplest form— between the region without the natural resource (region 1), and the one with (region 2) [even though this natural-resource-addition will only be noted at a later stage].

Each member of the nation works and provides one unit of labor, as well as owns an equal amount of the nation's capital which amounts to $(K^*/L) = k^*$.[2] Let us denote the share of the nation's population in each region by S_i; therefore, we have:

$$S_i = \frac{L_i}{L} \tag{1}$$

Thus, the average capital-labor ratio in the nation is $S_1k_1 + S_2k_2$, such that the following holds:[3]

$$S_1k_1 + S_2k_2 \leq k^* \tag{2}$$

In each region, production is undertaken in a manufacturing sector[4] by capital and labor, through a production function, $F(K_i, L_i)$ [$i \in (1, 2)$, for each region],[5] that is concave and has constant returns to scale in both capital and labor, is twice differentiable in both factors, and follows the Inada Conditions.[6] In its per capita form,

[2]Throughout the paper capital letters would represent level variables, while small letters would represent per capita amounts.

[3]Note that when capital is fully employed [such that $\rho > 0$, or otherwise that the unit tax on capital is not large enough to make the returns on capital negative] then the following equation holds with an equality.

[4]Note that currently there is a single sector in each region (manufacturing); in a later stage a second sector will be added to region 2 (natural resource).

[5]This notation will be used similarly throughout the paper.

[6]Such that:

$$\lim_{K\to\infty} F_K(K, L) = 0 \ , \ \lim_{L\to\infty} F_L(K, L) = 0 \ , \ \lim_{K\to 0} F_K(K, L) = \infty \ , \ \lim_{L\to 0} F_L(K, L) = \infty$$

$f(k)$,[7] we therefore have $f_k > 0, f_{kk} < 0$.

In each region, output can be transformed into private consumption good, X, or pure public good, G, such that:

$$X_i + G_i = F(K_i, L_i) \tag{3}$$

Each region levies a per-unit, source-based, tax on capital (denoted as T_i) to fund its expenditure on the public good. Therefore, the regional government's budget constraint is as follows:

$$G_i = T_i K_i \tag{4}$$

The after-tax rate of return on capital is ρ; although determined endogenously (by the free capital mobility condition[8]), ρ is taken as given by each region. Following that, the pre-tax rate of return on capital would be $\rho + T_i$. There are many firms (each being a price taker) that operate in each of the regions, and there is free entry to the market. Capital markets are competitive such that profit maximization by each firm yields:[9]

$$f_{k_i}(k_i) = \rho + T_i \tag{5}$$

Also, the free entry condition yields:[10]

$$w_i = f(k_i) - f_{k_i} k_i \tag{6}$$

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[7]Given the constant returns to scale property of $F(\cdot)$:

$$\frac{F(K, L)}{L} = f(k)$$

[8]Capital moves freely and costlessly between regions so that ρ is equalized across the nation; thus, the following free capital mobility condition must hold in equilibrium:

$$f_{k_1} - T_1 = f_{k_2} - T_2$$

Note that the reason for having each of the sides of this equation to be equal to ρ is explained by equation (5).

[9]Profit of a representative firm in either of the regions is $\pi_i = L_i[f(k_i) - (\rho + T_i)k_i - w_i]$. Therefore, profit would be maximized at $d\pi_i/dk_i = 0$.

[10]The free entry condition imposes $\pi = 0$, for all firms in the nation.

Residents of this nation have identical preferences, represented by a strictly quasi-concave utility function, $U(X, G)$, with the following properties: $U_X, U_G > 0$, $U_{XX}, U_{GG} < 0$, and $U_{XG} > 0$;[11] in addition, they own equal shares of the firms (in their respective regions). Therefore, given that residents spend all their income on private consumption, a representative resident's budget constraint would be:

$$x_i = f(k_i) - (\rho + T_i)k_i + \rho k^* \tag{7}$$

4 Analysis

This section is divided to two parts — the first looks into the social optimum analysis and goes through the national social planner's decision making process, and the second explores the Nash Equilibrium outcome, where each region acts non-cooperatively, and a tax competition is modelled along Cournot-Nash lines. These two parts derive the allocation of resources in the nation under cooperative and non-cooperative means, respectively; in addition, each of these parts is further divided into three sub-sections, as shall be seen later on.

A comparison is undertaken between the outcomes of the cooperative and non-cooperative cases, to better understand the effects of a natural resource endowment under the asymmetric tax competition and agglomeration economies environment, when having two regions with different population sizes. Also, a numerical example accompanies each of the sub-sections, to show how the model and its results work in practice. A key assumption that holds in all parts of this section is that an interior solution is feasible.[12]

[11] Note that X and G are assumed to be normal goods. Also, the following is assumed to hold:

$$\lim_{X\to\infty} U_X(X,G) = 0 \ , \ \lim_{G\to\infty} U_G(X,G) = 0 \ , \ \lim_{X\to 0} U_X(X,G) = \infty \ , \ \lim_{G\to 0} U_G(X,G) = \infty$$

[12] So that whenever an interior solution is discussed, it is assumed that there are sufficient levels of capital, taxes, and public goods in the nation, to make that solution feasible.

4.1 Social Optimum Analysis

As mentioned earlier, this section will be divided to three sub-sections. The first sub-section follows Bucovetsky's formulation (1991), as presented in the former section;[13] it represents the benchmark case. The following two sub-sections extend that theory. The second sub-section replaces the pure public good with a manufacturing-related infrastructure that brings about increasing returns to scale in output and therefore shows the effects of agglomeration economies. Thereafter, the third and final sub-section adds a natural resource sector to region 2, and explores its effects given the former addition of infrastructure.

4.1.1 Benchmark Case

There exists a benevolent social planner in the nation. The objective of this social planner is to find the levels of capital, public goods, and taxes that would maximize the welfare of the nation's residents,[14] by considering the nation as a whole. In effect, we try to derive the socially optimal division of capital in the nation. Using the formulation presented in the former section, equation (3) can be re-written, as follows:

$$L_i x_i = L_i f(k_i) - G_i \tag{8}$$

Thus, the planner's problem would be expressed as follows:

$$\max_{\{k_1,k_2,G_1,G_2,x_1,x_2\}} L_1 U(x_1, G_1) + L_2 U(x_2, G_2)$$

subject to:

$$L_1 x_1 + L_2 x_2 = L_1 f(k_1) + L_2 f(k_2) - G_1 - G_2$$

$$k^* = S_1 k_1 + S_2 k_2$$

$$0 < k_1, k_2, G_1, G_2, x_1, x_2$$

[13]With the difference of having the regions take ρ as given, as opposed to them taking account of their influence on ρ (as was done in Bucovetsky's paper (1991)). Ignoring this terms of trade effect simplifies the analysis and allows us to focus on agglomeration and natural resources more clearly.

[14]Note that in the problem to follow, determination of regional tax rates is implicitly done.

The social planner would maximize the aggregate welfare of the nation's residents (with respect to the regional capital, public goods, and private consumption) subject to the national budget constraint (which implicitly assumes transfers are made freely across the nation), the national per-capita capital constraint (which implicitly assumes capital is traded freely and costlessly between the regions), as well as subject to having strictly positive choice variables.[15]

Lemma 1. *There exists a unique social optimum, at which* $k_1 = k_2$, $K_1 > K_2$ *(due to* $L_1 > L_2$*), and* $L_i(MRS_{G_i x_i}) = 1$.

Proof. See Appendix 1.

Therefore, in this benchmark case there would be a unique social optimum where the social planner would divide the nation's capital so that the regional per-capita capital level would be equal across the nation, but the total capital level would be higher in region 1 because of its higher population size. Also, at that optimum public goods will be supplied efficiently in each of the regions; in fact, the widely recognized *Samuelson Condition* (Samuelson 1954) would hold, as in each of the regions we would have the following condition:[16]

$$\sum_{i=1}^{L_i} MRS_{G_i x_i} = MRT_{G_i x_i} = 1$$

Example - Benchmark Case A numerical example is accompanied to each of the sections to follow. This would help to better illustrate the conclusions from the general analysis. Note that the same functional forms are used in the numerical examples

[15]Note that both the national budget constraint and the national per-capita capital constraint appear in equality in the problem. The former appears in equality due to the nation's residents experiencing local non-satiation in x and G. The latter appears in equality due to the assumption that taxes are not large enough to have negative returns on capital, so that $\rho > 0$ and capital is fully employed. Also, all choice variables are strictly positive because an interior solution is forced by the properties of the production and utility functions. Lastly, note that in the problem G_1 and G_2 are implicitly bounded and determined by the summation of the regional budget constraints, presented previously.

[16]As was proved in Appendix 1, at the optimum we have $L_i(MRS_{G_i x_i}) = 1$, which is equivalent to the *Samuelson Condition* presented.

throughout the paper to better emphasize the differences between the sections.

Let us assign the following functions and figures:[17]

$$U(x_i, G_i) = (x_i)^{0.5}(G_i)^{0.5} \quad , \quad F(K_i, L_i) = (K_i)^{0.6}(L_i)^{0.4}$$

$$K^* = 5,\ L = 5, \text{ such that } L_1 = 3,\ L_2 = 2$$

Therefore, the problem to be solved by the planner would be as follows:[18]

$$\max_{\{k_1,k_2,G_1,G_2,x_1,x_2\}} 3(x_1)^{0.5}(G_1)^{0.5} + 2(x_2)^{0.5}(G_2)^{0.5}$$

subject to:

$$\begin{aligned} 3x_1 + 2x_2 &= 3(k_1)^{0.6} + 2(k_2)^{0.6} - G_1 - G_2 \\ 1 &= \frac{3}{5}k_1 + \frac{2}{5}k_2 \\ 0 &< k_1, k_2, G_1, G_2, x_1, x_2 \end{aligned}$$

$$\mathcal{L} = 3(x_1)^{0.5}(G_1)^{0.5} + 2(x_2)^{0.5}(G_2)^{0.5} - \lambda(3x_1 + 2x_2 - 3(k_1)^{0.6} - 2(k_2)^{0.6} + G_1 + G_2) + \delta(1 - \frac{3}{5}k_1 - \frac{2}{5}k_2)$$

<u>First-order conditions:</u>

(i) $\mathcal{L}_{k_1}:\ 3\lambda 0.6(k_1)^{-0.4} = \delta 3/5 \ \Rightarrow\ 0.6\lambda(k_1)^{-0.4} = \delta/5$

(ii) $\mathcal{L}_{k_2}:\ 2\lambda 0.6(k_2)^{-0.4} = \delta 2/5 \ \Rightarrow\ 0.6\lambda(k_2)^{-0.4} = \delta/5$

(iii) $\mathcal{L}_{G_1}:\ 3(x_1)^{0.5}(G_1)^{-0.5}/2 - \lambda = 0$

(iv) $\mathcal{L}_{G_2}:\ (x_2)^{0.5}(G_2)^{-0.5} - \lambda = 0$

(v) $\mathcal{L}_{x_1}:\ 3(x_1)^{-0.5}(G_1)^{0.5}/2 - 3\lambda = 0$

(vi) $\mathcal{L}_{x_2}:\ (x_2)^{-0.5}(G_2)^{0.5} - 2\lambda = 0$

From FOCS (iii)–(vi), we get:

$$\frac{L_i U_{G_i}}{U_{x_i}} = 1 \Rightarrow \left[\sum_{i=1}^{L_i} MRS_{G_i x_i} = 1\right]$$

[17]Note that each follows its respective assumptions, as they were presented earlier.

[18]Given that all choice variables must be positive at the optimum, their Kuhn-Tucker multipliers would be 0; thus, we ignore them when solving the problem.

From FOCS (i) and (ii), we get:

$$\left[\left(\frac{k_2}{k_1}\right)^{0.4} = 1\right]$$

Thus, at the unique social optimum $k_1 = k_2 = 1$, $K_1 = 3 > K_2 = 2$ (by $L_1 = 3 > L_2 = 2$), and $\sum_{i=1}^{L_i} MRS_{G_i x_i} = 1$ — as expected.

In this benchmark case public goods are supplied efficiently in each of the regions, and per-capita capital is equalized across the nation; thus, we get a symmetric outcome (in per capita levels). This will be changed as we introduce the option of having agglomeration economies, as we are about to see in the following sub-section.

4.1.2 Infrastructure Stage

In this sub-section, we depart from the basic formulation of the model, and extend the theory to make a better resemblance to the situation seen in Canada, using concepts from the *New Economic Geography* literature. The model is similar in every respect to what has been presented previously, except for having the government investing in manufacturing-related infrastructure,[19] instead of in a pure public good.

Having no public goods, output in each region can now be transformed into private consumption good, X, or manufacturing-related infrastructure, P, such that $X + P =$ Output. However, to account for the possible agglomeration effects (due to the differences in population sizes), the infrastructure is added as a multiplicative factor on the manufacturing production function,[20] such that together with the manufacturing production, output exhibits increasing returns to scale in P, L, and K. To keep the setting general this multiplicative factor would be a function of P, denoted as $I(P)$, which is concave in P, twice differentiable,[21] and follows the Inada

[19] Infrastructure is referred to as being manufacturing-related because at a later stage, when a natural resource sector is added to region 2, this specification would be important.

[20] And thus does not enter into the residents' utility function, as is the case in Zodrow and Mieszkowski (1986).

[21] Such that:

$$I_P > 0, I_{PP} < 0$$

Conditions.[22] Thus, output in each region would be:[23]

$$I(P_i)F(K_i, L_i) \tag{9}$$

In addition, since now the government spends solely on infrastructure, the regional government's budget constraint becomes:

$$P_i = T_i K_i \tag{10}$$

Also, equations (5)–(7) now become:[24]

$$I(P_i)f_{k_i} = \rho + T_i \tag{11}$$

$$w_i = I(P_i)[f(k_i) - f_{k_i}k_i] \tag{12}$$

$$x_i = I(P_i)f(k_i) - (\rho + T_i)k_i + \rho k^* \tag{13}$$

The residents' utility function remains the same, only now since there is no public good agents get utility from private consumption only, and so a representative resident's utility function becomes $U(x)$.[25]

Since residents get utility solely from private consumption, and because the social planner (who still maintains the same objective as before) is able to equalize consumption among all residents nationwide, the aim of the planner's problem would, thus, be to maximize the residents' private consumption $(X_1 + X_2)$.[26] Therefore, the

[22]Such that:

$$\lim_{P\to\infty} I_P = 0 \ , \ \lim_{P\to 0} I_P = \infty$$

[23]Given the constant returns to scale property of $F(\cdot)$:

$$I(P_i)F(K_i, L_i) = L_i I(P_i) f(k_i)$$

[24]All of which derived using the same reasoning presented previously.

[25]Having the same properties applied to it, as before.

[26]Given the current formulation, and due to having local non-satiation in x, we know that $X_1 + X_2 = L_1 I(P_1) f(k_1) + L_2 I(P_2) f(k_2) - P_1 - P_2$.

social planner's problem would now be as follows:[27]

$$\max_{\{k_1,k_2,P_1,P_2\}} L_1 I(P_1) f(k_1) + L_2 I(P_2) f(k_2) - P_1 - P_2$$

subject to:

$$k^* = S_1 k_1 + S_2 k_2$$

$$0 \leq k_1, k_2, P_1, P_2$$

Here also the national budget constraint and the national per-capita capital constraint implicitly assume that, firstly, transfers are made freely across the nation, and secondly, capital is traded freely and costlessly between the regions.

Lemma 2. *The social planner's problem may have two local, asymmetric, maxima at either $k_1 > k_2$ or $k_2 > k_1$; however, the global maximum (viewed as the social optimum in this case) would be at $k_1 > k_2 \geq 0$, $K_1 > K_2 \geq 0$ (due to $L_1 > L_2$), $P_1 > P_2 \geq 0$, and $MB_{P_i} = MC_{P_i}$ (at regions where $k_i > 0$ at the optimum).*

Proof. See Appendix 2.

Therefore, this case emphasizes the effects of possible agglomeration economies. Unlike the outcome in the benchmark case, here, due to the output having increasing returns to scale in K,L, and P, the social planner could possibly direct most (or all) of the nation's capital into either of the regions; however, due to having larger population in region 1 the social optimum would be where $k_1 > k_2$. Thus, we see that the social planner takes advantage of the agglomeration occurring in region 1 (due to its larger population size), by directing most (or all) of the nation's resources there. This is a key outcome, since as we shall see in a later stage, under non-cooperative behavior

[27] As reasoned previously, here also the national per-capita capital constraint appears in equality due to the assumption that capital is fully employed. In addition, note that in the given problem P_1 and P_2 are both implicitly bounded and determined by the summation of the regional budget constraints, presented previously. Lastly, as opposed to the base case, here all choice variables are non-negative, because as we shall see later, corner solutions are a possibility (due to the output having increasing returns to scale in K, L, and P).

the nation does not benefit from the agglomeration economies, and the welfare level of the residents of the federation is, consequently, sub-optimal.

This outcome presents another notable result; since there are no distortions, and no tax or infrastructure competitions presented, manufacturing-related infrastructure in each region reaches its efficiency level (up to where the marginal cost of producing another unit of infrastructure equals the marginal benefit from that unit).[28] Again, we shall see at a later stage how this result is changed under non-cooperative environment.

Example - Infrastructure Stage Let us assign the following functions and figures:[29]

$$I(P_i) = (P_i)^{0.5} \quad , \quad F(K_i, L_i) = (K_i)^{0.6}(L_i)^{0.4}$$

$$K^* = 5, \ L = 5, \text{ such that } L_1 = 3, \ L_2 = 2$$

Note that now output exhibits increasing returns to scale (in P, L, and K). We first consider the interior-solution case; thereafter, corner solutions will be considered and compared to the interior case to determine the global maximum. The problem to be solved would be as follows:[30]

$$\max_{\{k_1,k_2,P_1,P_2\}} Z(\cdot) = 3(P_1)^{0.5}(k_1)^{0.6} + 2(P_2)^{0.5}(k_2)^{0.6} - P_1 - P_2$$

subject to:

$$1 = \frac{3k_1}{5} + \frac{2k_2}{5}$$

$$0 < k_1, k_2, P_1, P_2$$

$$\mathcal{L} = 3(P_1)^{0.5}(k_1)^{0.6} + 2(P_2)^{0.5}(k_2)^{0.6} - P_1 - P_2 \ + \ \lambda(1 - \frac{3}{5}k_1 - \frac{2}{5}k_2)$$

[28] Only in regions that have a positive amount of capital at the optimum.

[29] Following the assumptions made earlier on the functional forms.

[30] Given that all choice variables must be positive in this case, their Kuhn-Tucker multipliers would be 0; thus, we ignore them when solving the problem.

First-order conditions:

(i) $\mathcal{L}_{k_1}:\ 3(P_1)^{0.5}0.6(k_1)^{-0.4} = \lambda 3/5$

(ii) $\mathcal{L}_{k_2}:\ 2(P_2)^{0.5}0.6(k_2)^{-0.4} = \lambda 2/5$

(iii) $\mathcal{L}_{P_1}:\ 3(P_1)^{-0.5}(k_1)^{0.6}/2 = 1$

(iv) $\mathcal{L}_{P_2}:\ (P_2)^{-0.5}(k_2)^{0.6} = 1$

Solving through these FOCS we get that $k_1 = 0.2961$, $k_2 = 2.056$. Let us denote this as point **a**. Thus, $Z(\mathbf{a}) = 2.8971$.

Conjecture: This problem has two local maxima at the corners; one at $k_1 = 0$, and another at $k_2 = 0$. The global maximum (and thus the social optimum) is found at $k_2 = 0$.[31]

Proof: From FOCS (iii) and (iv), we get $P_1 = 9(k_1)^{1.2}/4$, $P_2 = (k_2)^{1.2}$. Therefore, substituting this to $Z(\cdot)$, we get:

$$Z(\cdot) = \frac{9(k_1)^{1.2}}{4} + (k_2)^{1.2}$$

$\forall\ i, j$ (such that $i \neq j$, and $i, j \in (1,2)$):

$$\left.\frac{dZ(\cdot)}{dk_i}\right|_{k_i=0} = 0 \qquad\qquad \left.\frac{dZ(\cdot)}{dk_j}\right|_{k_j>0} > 0$$

Therefore, If we start at $k_i = 0$ and transfer capital from region j to region i, social welfare would be decreased. Thus, there are two local maxima at the corners; one at $k_2 = 0$ (denoted as point **b**), and another at $k_1 = 0$ (denoted as point **c**). Substituting these to $Z(\cdot)$, we get:[32]

$$Z(\mathbf{b}) = 4.1534 > Z(\mathbf{c}) = 3.003 > Z(\mathbf{a}) = 2.8971$$

[31] In case output would not exhibit increasing returns to scale in K, L, and P, then the interior case would, in fact, be the global maximum (and thus the social optimum); however, it is beyond the scope of this paper to prove this.

[32] Note that the interior case (point **a**) is, in fact, a local minimum.

Therefore, social welfare is maximized at point **b**, where $k_2 = 0$.[33]

$\square$

This example emphasizes the effects of agglomeration economies. As opposed to the benchmark case, here there are two local maxima, each representing a case of agglomeration in either of the regions. Thus, in this extreme case the social planner would prefer to place all of the nation's capital in one of the regions; however, given its higher population size, it would be most beneficial for him to do so in region 1.

4.1.3 Natural Resource Stage

Let us extend the theory further, and add a natural resource sector to region 2. Again, this extension is adopted to make a better resemblance to what is currently seen in Canada (although this can basically be applied to any federation with similar environment).

This new natural resource sector attracts capital and labor (from within the amounts allocated to region 2), is equally owned by the residents of region 2, and uses a fixed factor, Q,[34] (together with labor and capital) to produce output, through the following production function:[35]

$$H(K_2^2, L_2^2, Q) \tag{14}$$

Similarly to the production function in the manufacturing sector, we assume this one is concave and exhibits constant returns to scale in K, L, and Q, is twice differentiable in

[33] Note that at this point $MB_{P_1} = MC_{P_1} = 1$, as expected.

[34] This fixed factor represents the natural resource endowment.

[35] Note that superscript '1' refers to the manufacturing sector, while superscript '2' refers to the natural resource sector. This notation will be used similarly throughout the paper.

all factors, and follows the Inada Conditions.[36] Also, in its per capita form, $h(k_2^2, q)$,[37] we assume to have $h_{k_2^2}(k_2^2, q) > 0$, $h_{k_2^2 k_2^2}(k_2^2, q) < 0$, $h_q(k_2^2, q) > 0$, and $h_{qq}(k_2^2, q) < 0$.

Given this new production sector, labor is now divided between the two sectors in region 2 as follows:

$$L_2^1 + L_2^2 = L_2 \tag{15}$$

Also, the updated capital-labor ratio in the nation is $S_1 k_1 + S_2^1 k_2^1 + S_2^2 k_2^2$.[38] Therefore, the per-capita capital constraint in the nation becomes:[39]

$$S_1 k_1 + S_2^1 k_2^1 + S_2^2 k_2^2 \leq k^* \tag{16}$$

In this new sector the region collects tax on capital, denoted as T_2^2;[40] however, it may also collect a non-distorting lump sum tax on the natural resource rents, denoted as z. For simplicity, we assume that z is chosen freely, subject to having the resource rents as an upper bound. This may not be critical for the social optimum analysis,[41] but will play an important part at a later stage (in the non-cooperative behavior section).

There are many firms that operate in the natural resource sector. As before, because each of them is a price taker, and since capital markets are competitive,

---

[36]Such that:

$$\lim_{K\to\infty} H_K(K, L, Q) = 0 \ , \ \lim_{L\to\infty} H_L(K, L, Q) = 0 \ , \ \lim_{K\to 0} H_K(K, L, Q) = \infty \ , \ \lim_{L\to 0} H_L(K, L, Q) = \infty$$

Note that the same is not considered for Q because under the current formulation (which follows throughout the remaining of the paper) it is considered an exogenously determined fixed factor.

[37]Due to having constant returns to scale in K, L, and Q, we have:

$$\frac{H(K_2^2, L_2^2, Q)}{L_2^2} = h(k_2^2, q)$$

[38]As we have $S_2^i = L_2^i / L_2$, $i \in (1, 2)$.

[39]Once again, assuming that capital is fully employed, this constraint holds in equality.

[40]Such that:

$$T_2^1 + T_2^2 = T_2$$

[41]As it is expressed implicitly in the problem.

profit maximization by firms yields:[42]

$$h_{k_2^2}(k_2^2, q) = \rho + T_2^2 \tag{17}$$

There is a free entry condition in the natural resource sector;[43] thus, the wage given at that sector is as follows:[44]

$$w_2^2 = h(k_2^2, q) - h_{k_2^2} k_2^2 - h_q q \tag{18}$$

Therefore, residents who are employed in the natural resource sector will now have the following budget constraint:[45]

$$x_2^2 = h(k_2^2, q) - h_{k_2^2} k_2^2 - h_q q + \rho k^* \tag{19}$$

Once again, the benevolent social planner would like to maximize the welfare of the residents; thus, he will find the optimal allocation of resources so that $X_1 + X_2$ would be maximized. Since in each region output can either go to private consumption or to manufacturing-related infrastructure,[46] the following must hold:[47]

$$X_1 + X_2 = L_1 I(P_1) f(k_1) + L_2^1 I(P_2) f(k_2^1) + L_2^2 h(k_2^2, q) - P_1 - P_2 \tag{20}$$

Therefore, the social planner's problem in this case would be as follows:

$$\max_{\{k_1, k_2^1, k_2^2, P_1, P_2, L_2^1, L_2^2\}} L_1 I(P_1) f(k_1) + L_2^1 I(P_2) f(k_2^1) + L_2^2 h(k_2^2, q) - P_1 - P_2$$

[42] Profit of a representative firm in the natural resource sector is $\pi_2^2 = L_2^2[h(k_2^2, q) - (\rho + T_2^2)k_2^2 - w_2^2 - h_q q]$; therefore, profit would be maximized at $d\pi_2^2/dk_2^2 = 0$, while firms take the price of capital, labor, and natural resource as given. Also, note that equation (11), for the other sectors (and other region), still holds.

[43] So that $\pi_2^2 = 0$ for all firms in the natural resource sector of region 2.

[44] Note that equation (12), which represents the wage level at the other sectors (and other region), still holds.

[45] Note that the budget constraint for residents who are employed in the manufacturing sector of region 2, or for residents of region 1, remains to be equation (13).

[46] Note that we take the simplifying assumption that the only infrastructure that can be invested in by the regions is manufacturing-related. This is not to say that the natural resource sector does not use infrastructure; however, it simplifies the analysis to operate under the extreme case of having only manufacturing-related infrastructure, whereas it would not change the qualitative results if we assumed otherwise.

[47] This constraint holds in equality due to having local non-satiation in x.

subject to:

$$k^* = S_1 k_1 + S_2^1 k_2^1 + S_2^2 k_2^2$$
$$L_2 = L_2^1 + L_2^2$$
$$0 \leq k_1, k_2^1, k_2^2, P_1, P_2, L_2^1, L_2^2$$

As before, the national budget constraint allows for free transfers between the regions, and the national per-capita capital constraint allows for free mobility of capital across the nation. In addition, the labor constraint in region 2 allows for free labor mobility within that region.[48]

Lemma 3. *The social planner's problem may have two local, asymmetric, maxima at either* $k_1 > k_2^1$ *or* $k_2^1 > k_1$*; however, the global maximum (viewed as the social optimum in this case) would be at* $k_1 > k_2^1 \geq 0$, $K_1 > K_2^1 \geq 0$ *(due to* $L_1 > L_2^1$*),* $k_2^2 > 0$, $P_1 > P_2 \geq 0$, *and* $MB_{P_i} = MC_{P_i}$ *(at regions that have positive capital in their manufacturing sector).*

Proof. See Appendix 3.

Therefore, we see the effects of adding a natural resource to the smaller region, from the social planner's perspective. In effect, not much is changed from the former case with infrastructure and agglomeration economies. The natural resource sector will attract some of the nation's capital, and some of the region's labor; however, since the social planner would still take advantage of the agglomeration economies in the larger region (due to its higher population size), it would still direct most (or even all, when it comes to manufacturing-related capital) of the nation's resources to region 1, including rents accrued from the natural resource. The social planner would, in fact, use the natural resource for the benefit of the nation as a whole, investing its profits

[48]Once again, it is assumed that capital is fully employed so that the national per-capita capital constraint appears in equality; region 2's labor constraint appears in equality as well, by definition. Also, all choice variables are non-negative due to the possibility of having corner solutions. Lastly, note that P_1 and P_2 are implicitly bounded in the problem by the summation of the regional budget constraints, presented previously.

in the places that would make the best use of it; having a larger population, with more influential agglomeration, makes region 1 that place. Again, as shall be seen at a later stage, this outcome will be inherently changed under the non-cooperative behavior environment, where the natural resource will largely benefit the region in which it is located, instead of the entire nation.

As before, here also we see that under the cooperative case each region is supplied with its efficient level of manufacturing-related infrastructure;[49] again, under non-cooperative behavior, where tax and infrastructure competitions take place, this will be changed, as will be seen in the following section.

Example - Natural Resource Stage Let us assign the following functions and figures:[50]

$$I(P_i) = (P_i)^{0.5} \quad , \quad F(K_i^1, L_i^1) = (K_i^1)^{0.6}(L_i^1)^{0.4} \quad , \quad H(K_2^2, L_2^2, Q) = (K_2^2)^{0.6}(L_2^2)^{0.2}Q^{0.2}$$

$$K^* = 5,\ Q = 0.8368,\ L = 5, \text{ such that } L_1 = 3,\ L_2 = 2$$

Note that output exhibits increasing returns to scale (in P, L, and K) in each of the regions' manufacturing sectors, as well as exhibits constant returns to scale in region 2's natural resource sector, as required. As was done previously, we first consider the interior-solution case; thereafter, corner solutions will be considered and compared to the interior case to determine the global maximum. The problem to be solved would be as follows:[51]

$$\max_{\{k_1,k_2^1,k_2^2,P_1,P_2,L_2^1,L_2^2\}} Z(\cdot) = 3(P_1)^{0.5}(k_1)^{0.6} + L_2^1(P_2)^{0.5}(k_2^1)^{0.6} + L_2^2(k_2^2)^{0.6}q^{0.2} - P_1 - P_2$$

[49] As long as it has positive amounts of capital in its manufacturing sector.

[50] Following from the previous example, in addition to adding the natural resource sector. Also note that Q was chosen so that eventually we get $q = 1$.

[51] Given that all choice variables must be positive in this case, their Kuhn-Tucker multipliers would be 0; thus, we ignore them when solving the problem.

subject to:

$$
\begin{aligned}
1 &= \frac{3k_1}{5} + \frac{k_2^1 L_2^1}{5} + \frac{k_2^2 L_2^2}{5} \\
2 &= L_2^1 + L_2^2 \\
0 &< k_1, k_2, P_1, P_2
\end{aligned}
$$

$\mathcal{L} = 3(P_1)^{0.5}(k_1)^{0.6} + L_2^1(P_2)^{0.5}(k_2^1)^{0.6} + L_2^2(k_2^2)^{0.6}q^{0.2} - P_1 - P_2 \ + \ \lambda(1 - \frac{3}{5}k_1 - \frac{L_2^1}{5}k_2^1 - \frac{L_2^2}{5}k_2^2) \ + \ \delta(2 - L_2^1 - L_2^2)$

First-order conditions:

(i) $\mathcal{L}_{k_1}: \ 3(P_1)^{0.5}0.6(k_1)^{-0.4} = \lambda 3/5$

(ii) $\mathcal{L}_{k_2^1}: \ L_2^1(P_2)^{0.5}0.6(k_2^1)^{-0.4} = (\lambda L_2^1)/5$

(iii) $\mathcal{L}_{k_2^2}: \ L_2^2 0.6(k_2^2)^{-0.4}q^{0.2} = (\lambda L_2^2)/5$

(iv) $\mathcal{L}_{P_1}: \ 3(P_1)^{-0.5}(k_1)^{0.6}/2 = 1$

(v) $\mathcal{L}_{P_2}: \ L_2^1(P_2)^{-0.5}(k_2^1)^{0.6}/2 = 1$

(vi) $\mathcal{L}_{L_2^1}: \ (P_2)^{0.5}(k_2^1)^{0.6} - (\lambda k_2^1)/5 = \delta$

(vii) $\mathcal{L}_{L_2^2}: \ (k_2^2)^{0.6}q^{0.2} - (\lambda k_2^2)/5 = \delta$

Solving through these FOCS we get that $k_1 = 0.021627$, $k_2^1 = k_2^2 = 2.4676$, $L_2^1 = 1.1632$, $L_2^2 = 0.8368$, $q = 1$. Let us denote this as point **a**. Thus, $Z(\mathbf{a}) = 2.4613$.

Conjecture: This problem has two local maxima at the corners; one at $k_1 = 0$, and another at $k_2^1 = 0$. k_2^2 would have an interior solution at each of the points. The global maximum (and thus the social optimum) is found at $k_2^1 = 0$.[52]

Proof: From FOCS (iv) and (v), we get $P_1 = 9(k_1)^{1.2}/4$, $P_2 = (L_2^1)^2(k_2^1)^{1.2}/4$. Therefore, plugging this to $Z(\cdot)$, we get:

$$
Z(\cdot) = \frac{9(k_1)^{1.2}}{4} + \frac{(L_2^1)^2(k_2^1)^{1.2}}{4} + L_2^2(k_2^2)^{0.6}q^{0.2}
$$

[52] As before, in case output in the manufacturing sectors in each of the regions would not exhibit increasing returns to scale in K, L, and P, then the interior case would, in fact, be the global maximum (and thus the social optimum); however, it is beyond the scope of this paper to prove this.

$\forall\ i, j$ (such that $i \neq j$, and $i, j \in (1,2)$):

$$\left.\frac{dZ(\cdot)}{dk_i^1}\right|_{k_i^1=0} = 0 \qquad\qquad \left.\frac{dZ(\cdot)}{dk_j^1}\right|_{k_j^1>0} > 0$$

Also, $\forall\ k_2^2$:

$$\frac{dZ(\cdot)}{dk_2^2} > 0$$

Therefore, If we start at $k_i^1 = 0$ and transfer capital from region j to region i, social welfare would be decreased. Also, from the above condition we see that there must be an interior solution in k_2^2 at any optimum.[53] Thus, there are two local maxima at the corners; one at $k_2^1 = 0$ (denoted as point **b**), and another at $k_1 = 0$ (denoted as point **c**). Plugging these to $Z(\cdot)$, we get:[54]

$$Z(\mathbf{b}) = 4.5368 > Z(\mathbf{c}) = 2.4657 > Z(\mathbf{a}) = 2.4613$$

Therefore, social welfare is maximized at point **b**, where $k_2^1 = 0$.[55]

□

This example emphasizes the effects of adding a natural resource sector to the smaller region under the given setting. Given the agglomeration economies, we see how the social planner directs most of the nation's resources, including the natural resource rents, to one of the regions (given the two local maxima); specifically, we see how the planner puts the resource rents into their best use, by investing them completely in region 1 (given the global maximum), where the agglomeration is most influential due to its higher population size. In fact, in this extreme case, the planner supplies the efficient level of capital to the new natural resource sector in region 2; however, he directs all of the manufacturing-related capital to region 1, and consequently invests in infrastructure in region 1 only. In effect, all of the resource rents go to region 1.

Let us now continue to the non-cooperative behavior part of the analysis, where we shall see how some of the results derived in this part change.

[53] Due to the Inada Conditions imposed on $H(\cdot)$, which are being effective by not imposing increasing returns to scale as in the manufacturing sectors.

[54] Note that as in the previous example, the interior case (point **a**) is in fact a local minimum.

[55] Note that at this point $MB_{P_1} = MC_{P_1} = 1$, as expected.

4.2 Nash Equilibrium Analysis

In this part we have no central planner that looks for the benefit of the federation. Here, the two regions act separately, where each aims to maximize the welfare of its own residents. Under this non-cooperative setting, tax and infrastructure competitions arise as each region sets its tax and infrastructure levels taking as given those set by the other region. Consequently, each region best reacts to the actions of the other, and a Nash Equilibrium outcome is pursued. Having no central planner, note that under this non-cooperative setting natural resource rents accrue to the regional government (although this would only become relevant once a natural resource is added to the model), as opposed to what was seen under the former, cooperative, setting.

As in the former section, here also we have three sub-sections, for each stage of the model. In the first sub-section we return to the benchmark setting, having a pure public good. In the second sub-section, manufacturing-related infrastructure replaces the pure public good to take account of possible agglomeration economies. Thereafter, in the third sub-section, a natural resource sector is added to region 2 (on top of the infrastructure addition). In each of these cases we aim to better understand the non-cooperative outcome, and compare it to the results we derived previously under the cooperative setting.

4.2.1 Benchmark Case

Following the setting of the benchmark model, as was presented previously, let us now look into the behavior of each of the regions, with an aim of characterizing the Nash Equilibrium outcome that might arise. Note that in this case the regional governments set T_i and G_i, so there is only tax competition (as opposed to the following sub-sections, where the regions also choose infrastructure levels to attract capital).

By equation (5) each region derives $k_i(T_i)$ so that it can vary k_i by its choice of

T_i. Totally differentiating equation (5) with respect to k_i and T_i, we get:

$$\frac{dk_i}{dT_i} = \frac{1}{f_{k_i k_i}} < 0 \tag{21}$$

By equation (4), we get the following:

$$\frac{dG_i}{dT_i} = L_i k_i + T_i L_i \frac{dk_i}{dT_i} \tag{22}$$

Also, by substituting equation (5) to equation (7), we get:

$$\frac{dx_i}{dT_i} = f_{k_i k_i}(k^* - k_i)\frac{dk_i}{dT_i} - k^* \tag{23}$$

By substituting equation (21) to equation (23), we now get:

$$\frac{dx_i}{dT_i} = -k_i \tag{24}$$

Each region aims to set the tax level that would maximize the welfare of its residents. Keeping this objective in mind, each region would, thus, maximize the utility of a representative resident, subject to the budget constraints of the region and the resident. Therefore, in its simplest form the problem of each of the regions would be expressed as follows:[56]

$$\max_{\{T_i\}} \ U(x_i, G_i)$$

Let us denote U_{G_i}/U_{x_i} by $m(x_i, G_i)$; thus, we get:[57]

$$\frac{dx_i}{dT_i} + m(x_i, G_i)\frac{dG_i}{dT_i} = 0 \tag{25}$$

Substituting equations (22) and (24) to equation (25), and rearranging, we get:[58]

$$L_i m(x_i, G_i) = \frac{1}{1 + \frac{T_i}{k_i}\frac{dk_i}{dT_i}} > 1 \tag{26}$$

In equilibrium, equations (4), (21), and (26) (for each of the regions), as well as the national per-capita capital constraint ($k^* = S_1 k_1 + S_2 k_2$) and the free capital mobility condition ($f_{k_1} - T_1 = f_{k_2} - T_2$) — must hold.

[56]Note that given the assumptions made on the utility function, as well as based on the setting of the problem — there would be an interior solution to the given problem, in each of the regions, such that $T_i, k_i, G_i, x_i > 0$. Therefore, corner solutions are not considered in this case (though this may not apply to the equilibrium outcome).

[57]This was derived by totally differentiating $U(x_i, G_i)$ with respect to x_i and G_i.

[58]Note that by equation (21) $L_i m(x_i, G_i) > 1$.

Lemma 4. *Under the benchmark setting, there exists a unique and symmetric (in per capita terms) Nash Equilibrium outcome, in which* $k_1 = k_2$*,* $T_1 = T_2$*,* $K_1 > K_2$*, and* $G_1 > G_2$ *(due to having* $L_1 > L_2$*).*[59]

Proof. See Appendix 4.

Thus, we see that despite having different population sizes in the two regions, the asymmetric tax competition that arises still brings about a unique and symmetric outcome (in per capita terms). This will be changed once we introduce the possibility of having agglomeration economies, in the next sub-section.

Also note that as opposed to the social optimum outcome under the benchmark case, here (by equation (26), which must hold in equilibrium for each region) we get:

$$\sum_{i=1}^{L_i} MRS_{G_i x_i} > MRT_{G_i x_i} = 1$$

Thus, in this case the marginal rate of transformation between private and public goods is smaller than the sum (across residents of the region) of the marginal rates of substitution between the two, which means that the *Samuelson Condition* (Samuelson 1954) does not hold; this, in turn, implies that the pure public good is now undersupplied in each of the regions, due to the non-cooperative behavior.

Moreover, following the standard definition of the Marginal Cost of Public Funds (MCPF),[60] presented by Browning (1976), we can assign result (26) such an interpretation. Meaning, in equilibrium each of the regions will face excess costs when raising an additional unit of revenue,[61] caused by the distortionary taxes and the tax competition. This further emphasizes the difference from the social optimum's

[59] Albeit being insightful, we do not examine whether this Nash Equilibrium outcome is stable or not; this goes beyond the scope of this paper. Note that this remark holds true for the remaining of the paper.

[60] The definition refers to the MCPF as the social cost of financing an increment of public spending. It divides that social cost to two main portions; the first being the reduction in private spending (which equals to one), and the second being the change in excess burden required to raise the additional unit of revenue, induced by the distortionary taxes.

[61] This excess cost amounts to $\frac{1}{1+\frac{T_i}{k_i}\frac{dk_i}{dT_i}} - 1$.

benchmark case, where the MCPF was equal to one (so that there was no extra cost for raising an additional unit of revenue, as taxes were not distortionary).[62]

Example - Benchmark Case Let us assign the following functions and figures:

$$U(x_i, G_i) = (x_i)^{0.5}(G_i)^{0.5} \quad , \quad F(K_i, L_i) = (K_i)^{0.6}(L_i)^{0.4}$$

$$K^* = 5,\ L = 5, \text{ such that } L_1 = 3,\ L_2 = 2$$

Then, given the analysis presented in the previous part, the equilibrium conditions would be as follows —

By equation (4):

$$G_1 = 3T_1k_1 \quad , \quad G_2 = 2T_2k_2$$

By equation (21):

$$\frac{dk_1}{dT_1} = \frac{1}{-0.24(k_1)^{-1.4}} \quad , \quad \frac{dk_2}{dT_2} = \frac{1}{-0.24(k_2)^{-1.4}}$$

By equation (26):

$$\frac{3x_1}{G_1} = \frac{1}{1 - \frac{T_1(k_1)^{0.4}}{0.24}} \quad , \quad \frac{2x_2}{G_2} = \frac{1}{1 - \frac{T_2(k_2)^{0.4}}{0.24}}$$

By the national per-capita capital constraint:

$$5 = 3k_1 + 2k_2$$

By the free capital mobility condition:

$$0.6(k_1)^{-0.4} - T_1 = 0.6(k_2)^{-0.4} - T_2$$

Solving through these conditions, we get a unique and symmetric (in per capita terms) Nash Equilibrium outcome at $k_1 = k_2 = 1$, $T_1 = T_2 = 0.185$, $K_1 = 3 > K_2 = 2$, and $G_1 = 0.555 > G_2 = 0.37$ — as expected.

Comparing this result to the one derived under the social optimum's benchmark case, we see that the nation's capital will be divided between the two regions in the

[62] This result is presented in footnote (16).

same manner; however, in the social optimum case we got that $G_1 + G_2 = 2.5$, while in this Nash Equilibrium outcome the parallel result is $G_1 + G_2 = 0.925$. This follows the previous explanation over the higher MCPF and undersupply of public goods under the non-cooperative behavior. Indeed, we see how in this case the nation is supplied with lower level of the public good, compared to what was supplied under the social optimum case.

4.2.2 Infrastructure Stage

Continuing to the second stage of the model, the regional governments now invest solely in manufacturing-related infrastructure (instead of in the pure public good). The setting of the model follows that which was presented previously (in the second sub-section of the social optimum analysis), only in this case we operate under a non-cooperative environment where each regional government sets T_i and P_i to maximize the welfare of its residents, so that tax and infrastructure competitions arise.

By equation (11) each region derives $k_i(T_i, P_i)$, so that it can vary k_i by its choice of T_i and P_i. Totally differentiating equation (11) with respect to k_i, T_i, and P_i, we get the following:[63]

$$\left.\frac{dk_i}{dT_i}\right|_{P_i=c} = \frac{1}{I(P_i)f_{k_ik_i}} < 0 \tag{27}$$

$$\left.\frac{dk_i}{dP_i}\right|_{T_i=c} = \frac{-I_{P_i}f_{k_i}}{I(P_i)f_{k_ik_i}} > 0 \tag{28}$$

Given that under the current setting there is no pure public good, and since the regional governments can equalize private consumption across the residents in their respective regions, each regional government would, thus, aim to maximize X_i, subject to the regional budget constraint. Therefore, by equations (13) and (10), the regional

[63] Note that dk_i/dT_i and dk_i/dP_i are partial derivatives; in each, the second variable remains constant. This notation holds for the remaining of the paper.

problem would be as expressed as follows:[64]

$$\max_{\{T_i, P_i\}} L_i[I(P_i)f(k_i) - (\rho + T_i)k_i + \rho k^*]$$

subject to:

$$P_i = T_i k_i L_i$$

Solving through this problem, we get:

$$\mathcal{L} = L_i[I(P_i)f(k_i) - (\rho + T_i)k_i + \rho k^*] - \lambda(P_i - T_i k_i L_i)$$

<u>First order conditions:</u>

$$\mathcal{L}_{T_i} : -L_i k_i + \lambda(k_i L_i + T_i L_i \frac{dk_i}{dT_i}) \tag{29}$$

$$\mathcal{L}_{P_i} : L_i I_{P_i} f(k_i) - \lambda(1 - T_i L_i \frac{dk_i}{dP_i}) \tag{30}$$

Rearranging equation (29), we get:

$$\lambda = \frac{1}{1 + \frac{T_i}{k_i}\frac{dk_i}{dT_i}} \tag{31}$$

By dividing equation (29) by equation (30) and rearranging, we get the following result:

$$L_i I_{P_i} f(k_i) = \frac{1 - T_i L_i \frac{dk_i}{dP_i}}{1 + \frac{T_i}{k_i}\frac{dk_i}{dT_i}} \tag{32}$$

Both equations, (31) and (32), must hold in equilibrium for each of the regions. To interpret these results — equation (31) is assigned a standard MCPF interpretation (Browning 1976). The shadow price of government revenue (λ) is equal to the social cost of financing an incremental unit of infrastructure. Given result (27), we can conclude that this social cost is higher than one (or in other words, higher than the reduction in private spending); thus, as was seen in the benchmark case, here also the

[64] Note that given the assumptions made on the production and infrastructure functions, as well as based on the setting of the problem — there would be an interior solution to the given problem, in each of the regions, such that $k_i, P_i, T_i > 0$. Therefore, corner solutions are not considered in this case (though this may not apply to the equilibrium outcome). Also, note that the regional budget constraint appears in equality because we have $I_{P_i} > 0$.

distortionary taxes and the tax competition setting bring about higher social costs for financing an additional unit of infrastructure.[65]

Equation (32) expresses the relationship between MC_{P_i} and MB_{P_i}. In the social optimum cases (in the stages that had infrastructure) we realized that efficiency implies having the two equal. In this case, we see how the non-cooperative environment prevents the two regions from reaching efficiency in that sense. For that, we need to follow Zodrow and Mieszkowski's *Stability Condition* (1986).

Let us assume that the marginal cost of diverting a unit of output to infrastructure (which is equal to unity) is greater than the associated increase in output due to an increase in the marginal productivity of capital. This assumption ensures that raising taxes will drive out capital. This happens because raising taxes increases the marginal productivity of capital (through the increase in infrastructure), which increases output; however, since the cost of adding infrastructure is greater than this associated increase in output, higher taxes will, in fact, drive capital away rather than the opposite. This, in turn, ensures that the model is stable (because otherwise taxes would always be raised by the two regions). That said, the *Stability Condition* would be as follows:

$$1 - L_i k_i I_{P_i} f_{k_i} > 0 \tag{33}$$

Conjecture: Given the *Stability Condition* we get that in equilibrium $MB_{P_i} > 1$ for each of the regions.

Proof: Let us assume that $MB_{P_i} > 1$, so that by equation (32) we have:

$$\frac{1 - T_i L_i \frac{dk_i}{dP_i}}{1 + \frac{T_i}{k_i}\frac{dk_i}{dT_i}} > 1 \;\Rightarrow\; -L_i \frac{dk_i}{dP_i} > \frac{1}{k_i}\frac{dk_i}{dT_i} \;\Rightarrow\; -k_i L_i \frac{dk_i}{dP_i} > \frac{dk_i}{dT_i}$$

Substituting equations (27) and (28) to this result, we get:

$$k_i L_i I_{P_i} f_{k_i} \frac{dk_i}{dT_i} > \frac{dk_i}{dT_i}$$

Because $dk_i/dT_i < 0$, we get:

$$1 - k_i L_i I_{P_i} f_{k_i} > 0$$

[65]This extra social cost amounts to $\lambda - 1$.

This result is equivalent to equation (33).

□

Therefore, we see that unlike the outcome under the social optimum, in this case we have $MB_{P_i} > MC_{P_i} = 1$ (in equilibrium), which means that infrastructure is undersupplied in each of the regions.

In equilibrium, equations (10), (27), (28), (31), (32), and (33) (for each of the regions), as well as the national per-capita capital constraint ($k^* = S_1 k_1 + S_2 k_2$) and the free capital mobility condition ($I(P_1) f_{k_1} - T_1 = I(P_2) f_{k_2} - T_2$) — must hold.

Lemma 5. *In the infrastructure stage there exists no symmetric Nash Equilibrium outcome in both per-capita capital and infrastructure levels.*

Proof. In case we have $k_1 = k_2$ and $P_1 = P_2$ in equilibrium, then by the free capital mobility condition, we must also have $T_1 = T_2$. However through equation (10), having $k_1 = k_2$ and $P_1 = P_2$ means we must then have $T_2 > T_1$ in equilibrium. Therefore, we can not have $k_1 = k_2$ and $P_1 = P_2$ in equilibrium.

□

Thus, we see that under non-cooperative environment, the addition of manufacturing-related infrastructure that brings about possible agglomeration economies in one of the regions, prevents the regions from reaching a symmetric equilibrium in both per-capita capital and infrastructure. This, in turn, means that agglomeration would indeed occur in any possible equilibrium outcome, as we are about to see.

Lemma 6. *In the infrastructure stage there exist two possible Nash Equilibria outcomes. The first has agglomeration in region 1 (such that $k_1 > k_2$, and $P_1 > P_2$), and the second has agglomeration in region 2 (such that $k_2 > k_1$, and $P_2 > P_1$).*

Proof. See Appendix 5.

Therefore, similar to the social optimum result, here also the outcome implies that there could be two possible cases of agglomeration, in either region 1 or region 2. The main difference between these two results is that under the social optimum case the planner could choose to reach the preferred optimum of agglomeration in region 1; however, in this case, this is not controllable. The unwanted outcome of agglomeration in region 2 is a viable possibility, as will be evident through the numerical example to follow.

Example - Infrastructure Stage Let us assign the following functions and figures:

$$F(K_i, L_i) = (K_i)^{0.6}(L_i)^{0.4} \quad , \quad I(P_i) = (P_i)^{0.5}$$

$$K^* = 5,\ L = 5, \text{ such that } L_1 = 3,\ L_2 = 2$$

Then, given the analysis presented in the previous part, the equilibrium conditions would be as follows —

By equation (10):

$$P_1 = 3T_1k_1 \quad , \quad P_2 = 2T_2k_2$$

By equation (27):

$$\frac{dk_1}{dT_1} = \frac{1}{-0.24(k_1)^{-0.9}(3T_1)^{0.5}} \quad , \quad \frac{dk_2}{dT_2} = \frac{1}{-0.24(k_2)^{-0.9}(2T_2)^{0.5}}$$

By equation (28):

$$\frac{dk_1}{dP_1} = \frac{5}{12T_1} \quad , \quad \frac{dk_2}{dP_2} = \frac{5}{8T_2}$$

By equation (32):

$$\frac{(k_1)^{0.1}}{2(3T_1)^{0.5}} = \frac{-0.25}{1 - \frac{(T_1)^{0.5}}{(k_1)^{0.1}0.24(3)^{0.5}}} \quad , \quad \frac{(k_2)^{0.1}}{2(2T_2)^{0.5}} = \frac{-0.25}{1 - \frac{(T_2)^{0.5}}{(k_2)^{0.1}0.24(2)^{0.5}}}$$

By equation (33):

$$1 - 0.9(k_1)^{0.6}(P_1)^{-0.5} > 0 \quad , \quad 1 - 0.6(k_2)^{0.6}(P_2)^{-0.5} > 0$$

By the national per-capita capital constraint:

$$5 = 3k_1 + 2k_2$$

By the free capital mobility condition:

$$0.6(k_1)^{-0.4}(P_1)^{0.5} - T_1 = 0.6(k_2)^{-0.4}(P_2)^{0.5} - T_2$$

Solving through these conditions, we get an asymmetric Nash Equilibrium outcome at $k_2 = 1.992 > k_1 = 0.339$, $T_2 = 0.086 > T_1 = 0.075$, $K_2 = 5.976 > K_1 = 1.695$, and $P_2 = 0.343 > P_1 = 0.076$.

This shows that the undesired outcome of having agglomeration in the smaller region is indeed a possible result. Comparing this outcome to the one derived under the social optimum's infrastructure case, the difference is quite significant. While the social planner would direct all of the nation's capital to region 1 (such that $k_2 = 0$), the non-cooperative outcome (as seen above) not only lets region 2 hold positive amounts of capital (such that $k_2 > 0$), but in fact directs most of the nation's stock to it (such that $k_2 > k_1$ and $K_2 > K_1$).

In addition, we see how infrastructure is undersupplied across the nation, as was conjectured previously. In the social optimum's infrastructure case we had $P_1 + P_2 = 2.8971$, while in this Nash Equilibrium outcome we have $P_1 + P_2 = 0.419$, which is lower. This shows some of the previously discussed effects of the distortionary taxes and the tax competition setting.

Continuing to the following sub-section, we will see how adding a natural resource to the smaller region worsens the situation as it makes it easier for region 2 to compete for capital and infrastructure, and thus increases the probability of reaching the undesired result of having agglomeration in that region.

4.2.3 Natural Resource Stage

Let us now add a natural resource sector to region 2. Again, this model follows that which was presented previously (in the third sub-section of the social optimum analysis), except for having a non-cooperative environment in this case, such that each region sets its levels of taxes and infrastructure to maximize the welfare of its residents, and natural resource rents accrue to the regional government instead of to a central planner. Once again, given the simultaneous movement, tax and infrastructure competitions arise, which will have significant effects on the division of resources in the nation.

Since in region 1 nothing has changed from the former section, all the analysis done previously still holds for that region. Therefore, this section will focus its analysis mainly on region 2 (up to the point where interpretation is given, so that the nation as a whole is considered).

By equation (17) region 2 determines $k_2^2(T_2^2, q)$, so that it can vary the level of k_2^2 by its choice of T_2^2, L_2^1, and L_2^2. Because lump sum tax is levied on the natural resource rents, region 2's budget constraint would now be:

$$P_2 = T_2^1 L_2^1 k_2^1 + T_2^2 L_2^2 k_2^2 + z \tag{34}$$

Also, we allow for free mobility of labor between the two sectors in region 2, so that in equilibrium marginal productivities are equal across the region, such that the following holds:[66]

$$I(P_2)[f(k_2^1) - f_{k_2^1} k_2^1] = h(k_2^2, q) - h_{k_2^2} k_2^2 - h_q q \tag{35}$$

Since region 2 can equalize private consumption across its residents, and because we have no pure public good under this setting, then region 2's government would aim to maximize X_2 subject to its regional budget constraint and the free labor mobility condition. Given equations (13), (15), and (19), region 2's problem would be expressed as follows:[67]

$$\max_{\{P_2, T_2^1, T_2^2, L_2^1, z\}} \; L_2^1[I(P_2)f(k_2^1) - (\rho + T_2^1)k_2^1] + (L_2 - L_2^1)[h(k_2^2, q) - (\rho + T_2^2)k_2^2 - h_q q] + L_2 \rho k^* - z$$

subject to:

$$P_2 = T_2^1 L_2^1 k_2^1 + T_2^2 (L_2 - L_2^1) k_2^2 + z$$

$$I(P_2)[f(k_2^1) - f_{k_2^1} k_2^1] = h(k_2^2, q) - h_{k_2^2} k_2^2 - h_q q$$

[66] By equations (12) and (18).

[67] Note that equation (15) is substituted into the problem, instead of L_2^2. Also note that the comment made in footnote (64) is relevant to this problem as well.

Solving through this problem, we get:

$\mathcal{L} = L_2^1[I(P_2)f(k_2^1) - (\rho + T_2^1)k_2^1] + (L_2 - L_2^1)[h(k_2^2, q) - (\rho + T_2^2)k_2^2 - h_q q] + L_2\rho k^* - z -$

$\lambda_2(P_2 - T_2^1 L_2^1 k_2^1 - T_2^2(L_2 - L_2^1)k_2^2 - z) - \delta(I(P_2)[f(k_2^1) - f_{k_2^1}k_2^1] - h(k_2^2, q) + h_{k_2^2}k_2^2 + h_q q)$

First order conditions:

$$\mathcal{L}_z : -1 + \lambda_2 = 0 \tag{36}$$

$$\mathcal{L}_{P_2} : L_2^1 I_{P_2} f(k_2^1) - \lambda_2(1 - T_2^1 L_2^1 \frac{dk_2^1}{dP_2}) - \delta(I_{P_2} f(k_2^1)) = 0 \tag{37}$$

$$\mathcal{L}_{T_2^1} : -L_2^1 k_2^1 + \lambda_2(L_2^1 k_2^1 + T_2^1 L_2^1 \frac{dk_2^1}{dT_2^1}) + \delta k_2^1 = 0 \tag{38}$$

$$\mathcal{L}_{T_2^2} : -(L_2 - L_2^1)k_2^2 + \lambda_2((L_2 - L_2^1)k_2^2 + T_2^2(L_2 - L_2^1)\frac{dk_2^2}{dT_2^2}) - \delta k_2^2 = 0 \tag{39}$$

$$\mathcal{L}_{L_2^1} : I(P_2)[f(k_2^1) - f_{k_2^1}k_2^1] - h(k_2^2, q) + h_{k_2^2}k_2^2 + h_q q + \lambda(T_2^1 k_2^1 - T_2^2 k_2^2) = 0 \tag{40}$$

By substituting equation (36) to equation (38), we get:

$$\delta = -\left(\frac{T_2^1 L_2^1 \frac{dk_2^1}{dT_2^1}}{k_2^1}\right)$$

Furthermore, by substituting equation (36) to equation (39), we get:

$$\delta = \left(\frac{T_2^2 (L_2 - L_2^1)\frac{dk_2^2}{dT_2^2}}{k_2^2}\right)$$

Thus, because all choice variables are by definition non-negative, and since equations (38) and (39) must hold in equilibrium simultaneously, then we have:

$$\delta = 0 \tag{41}$$

By result (41), equations (37)–(39) now become:

$$\mathcal{L}_{P_2} : L_2^1 I_{P_2} f(k_2^1) - \lambda_2(1 - T_2^1 L_2^1 \frac{dk_2^1}{dP_2}) = 0 \tag{42}$$

$$\mathcal{L}_{T_2^1} : -L_2^1 k_2^1 + \lambda_2(L_2^1 k_2^1 + T_2^1 L_2^1 \frac{dk_2^1}{dT_2^1}) = 0 \tag{43}$$

$$\mathcal{L}_{T_2^2} : -(L_2 - L_2^1)k_2^2 + \lambda_2((L_2 - L_2^1)k_2^2 + T_2^2(L_2 - L_2^1)\frac{dk_2^2}{dT_2^2}) = 0 \tag{44}$$

Assuming that z is unrestricted so that lump sum tax can be levied on the natural resource rents such that $\lambda_2 = 1$,[68] we have the following results from equations (42)–(44):[69]

$$T_2^1 = T_2^2 = 0 \tag{45}$$

$$L_2^1 I_{P_2} f(k_2^1) = 1 \tag{46}$$

Then, by equation (45), we get the following result from equation (40):

$$I(P_2)[f(k_2^1) - f_{k_2^1} k_2^1] = h(k_2^2, q) - h_{k_2^2} k_2^2 - h_q q \tag{47}$$

Firstly, by result (47) we see that having free labor mobility within the region requires that the marginal productivities in each of the sectors be equal, as was imposed initially. Secondly, adopting the same interpretations presented previously, we see the effects of having a natural resource in an extreme case where there is no restriction on the amount of lump sum tax that can be levied on its rents. The shadow price of region 2's government revenue (λ_2) is equal to one, as opposed to that of region 1, which is higher than one (by result (31), as was explained previously). In MCPF terms, this means that in region 2 the social cost of financing an incremental unit of infrastructure is equal to the reduction in private spending (which is equal to one), so that the regional government bears no extra cost when financing infrastructure. This happens because in this case region 2 faces no distortionary taxes; the profits from the natural resource and the unrestricted lump sum tax levied on them do not affect the residents' decisions (as otherwise occurs under distortionary environment) so that the regional government can, in fact, supply an efficient level of infrastructure. This is seen by result (46); due to having MCPF$_2 = 1$, we thus get that $MB_{P_2} = MC_{P_2} = 1$.

In region 1 the results remain as before. The MCPF is higher than one, the region uses distortionary taxes (such that $T_1 > 0$), and $MB_{P_1} > MC_{P_1}$ so that infrastructure

[68] Note that the case where z is restricted is considered in the *Extensions and Limitations* section of the paper.

[69] These results hold because we have $\lambda_2 = 1$, $dk_2^i/dT_2^i < 0$, $L_2^i > 0$. Also, note that results (31)–(33) still hold for region 1, as before.

in the region is undersupplied. Comparing this to the outcome in region 2, the effects of adding a natural resource to the smaller region become clearer. Unlike the outcome under the social optimum, here the natural resource rents stay within region 2, and are used strictly for the development of that region.

In equilibrium, equations (10), (31), (32), and (33) hold for region 1, equations (34), (36), (45), (46), and (47) hold for the natural resource sector in region 2, and equations (27), and (28) hold for the manufacturing sectors in both regions. In addition, the following two conditions must hold as well:[70]

$$k^* = S_1 k_1 + S_2^1 k_2^1 + S_2^2 k_2^2 \tag{48}$$

$$h_{k_2^2} = I(P_1) f_{k_1} - T_1 = I(z) f_{k_2^1} \tag{49}$$

From this we see that having the natural resource in the smaller region lets it compete aggressively in a tax competition setting, to the point where it does not need to impose any capital tax to reach its efficient level of infrastructure. This is a key outcome, because albeit being extreme (given that z is unrestricted in this case) it nevertheless illustrates how having a natural resource under non-cooperative setting provides the region with an advantage in the national competition over capital. Moreover, it shows how a natural resource worsens the situation we reached in the previous case (having only infrastructure, with no natural resource), as it significantly increases the probability of reaching the unwanted outcome of having agglomeration in the smaller region, in equilibrium.

Also, because we assume that z is chosen freely subject to having the resource rents as an upper bound, and since the natural resource rents largely depend on the size of the resource (as the resource rents are equal to $L_2^2 h_q q$), we can conclude that the higher Q is the higher would the upper bound on z be. Thus, if we assume that region 2 takes full advantage of these rents, so that it sets z to be equal to its upper bound, it would necessarily mean that a larger natural resource (or in other words, a

[70]The first being the national per-capita capital constraint, and the second being the free capital mobility condition (note that by result (45), equation (34) becomes $P_2 = z$).

higher Q) entails a larger z.

Lemma 7. *If z is set to be equal to its upper bound, the larger that upper bound would be the more capital will region 2 attract, to the point where if $z \geq P_1$ then $k_2^1 > k_1$.*

Proof. As presented in the proof in Appendix 5, by equation (32) we get $T_1(k_1)$, so that $dk_1/dT_1 > 0$. That said, in equilibrium the free capital mobility condition must hold, so that we have the following:

$$I(P_1)f_{k_1} > I(z)f_{k_2^1}$$

Thus, if $z \geq P_1$ then by the above and because $f_{k_i^1 k_i^1} < 0 \ \forall k_i^1$, we must have $k_2^1 > k_1$ in equilibrium. Also, we see that as $z \to \infty$, then $k_1 \to 0$.

$\square$

Therefore, in this case where we have z being unrestricted, we see how adding a large enough natural resource to the smaller region brings about an undesired outcome. Not only do the resource rents not get redistributed more efficiently across the nation (as occurred in the social optimum case), but they in fact cause agglomeration in the smaller region, having higher levels of infrastructure and capital in that region than in the larger one.

Example - Natural Resource Stage In this case two examples are presented. The two have an identical setting, with the sole difference of having a different size of a natural resource (hence, a different size of Q). This will emphasize the propositions that have been raised in the previous discussion.

Example 1 - A Small Natural Resource

Let us assign the following functions and figures:

$$F(K_i^1, L_i^1) = (K_i^1)^{0.6}(L_i^1)^{0.4} \quad , \quad I(P_i) = (P_i)^{0.5} \quad , \quad H(K_2^2, L_2^2, Q) = (K_2^2)^{0.6}(L_2^2)^{0.2}Q^{0.2}$$

$$K^* = 10,\ Q = 3.7,\ L = 10, \text{ such that } L_1 = 9,\ L_2 = 1$$

Then, given the analysis presented in the previous part, the equilibrium conditions would be as follows —

By equations (10) and (34):

$$P_1 = 9T_1 k_1 \quad , \quad P_2 = z$$

By equations (27) and (28):

$$\frac{dk_1}{dT_1} = \frac{1}{-0.24(k_1)^{-0.9}(9T_1)^{0.5}} \quad , \quad \frac{dk_1}{dP_1} = \frac{5}{36T_1}$$

By equations (32) and (46):

$$\frac{3(k_1)^{0.1}}{2(T_1)^{0.5}} = \frac{-0.25}{1 - \frac{(T_1)^{0.5}}{(k_1)^{0.1}0.72}} \quad , \quad L_2^1(k_2^1)^{0.6}(2z^{0.5})^{-1} = 1$$

By equation (47):

$$0.4(P_2)^{0.5}(k_2^1)^{0.6} = 0.2(k_2^2)^{0.6}q^{0.2}$$

By equation (48):

$$10 = 9k_1 + S_2^1 k_2^1 + S_2^2 k_2^2$$

By equation (49):

$$0.6(k_1)^{-0.4}(P_1)^{0.5} - T_1 = 0.6(k_2^1)^{-0.4}(P_2)^{0.5} = 0.6(k_2^2)^{-0.4}q^{0.2}$$

Solving through these conditions, we get an asymmetric Nash Equilibrium outcome at $k_1 = 1.153847 > k_2^2 = 0.45 > k_2^1 = 0.225$, $T_1 = 0.689 > T_2^1 = T_2^2 = 0$, and $P_1 = 7.15 > P_2 = z = 0.574$.

Therefore, we see from these results that agglomeration occurs in region 1, as under the current setting region 1 attracts more capital than the two sectors of region-2 combined, and it has a higher level of infrastructure. Let us now look into the case of having a larger natural resource.

Example 2 - A Large Natural Resource

This example uses the same setting presented in the previous case, only now we have a larger natural resource so that $Q = 86$.

The equilibrium conditions remain as before; solving them through with the updated size of the natural resource we get an asymmetric Nash Equilibrium outcome at $k_2^2 = 2.45 > k_2^1 = 1.225 > k_1 = 1.2$, $T_1 = 0.7 > T_2^1 = T_2^2 = 0$, and $P_1 = 7.5 > P_2 = z = 2.3$.

Thus, we see the effects of having a larger natural resource. In this case, the per-capita capital is higher in the smaller region; specifically, in per capita terms the manufacturing sector in region 2 now attracts more capital than that of region-1. The infrastructure level remains higher in region 1; however, comparing this to example one, we see that the infrastructure level in region 2 has increased more than four times, while that of region 1 remained approximately the same. The result is clear — a larger natural resource creates agglomeration in the smaller region, as was conjectured previously. In this case the agglomeration is only partial; however, choosing yet a higher Q would continue this trend, and would present higher levels of per-capita capital and infrastructure in region 2 compared to those in region 1.

Through this example we see how the natural resource affects the tax and infrastructure competitions. Adding the natural resource to region 2 lets it compete more aggressively against region 1 on the national capital and infrastructure, and thus increases the probability of achieving the undesired outcome of having agglomeration in the smaller region (as occurred in example two).

5 Extensions and Limitations

This paper presented a simple model of asymmetric tax competition, with an addition of a natural resource. Given the complexity of the subject matter, there are numerous extensions and refinements that are worth noting. Some of these extensions were considered and eventually left out of the model due to reasons of length and time constraints; however, note that the final insights and conclusions would not have changed qualitatively even if these extensions were to be included.

In the model we assume that labor is fixed in each of the regions; this simplifying assumption contributed much to the analysis, yet it was largely unrealistic, as in Canada as well as in other western multi-regional federations, labor mobility and population migration is free and unconstrained. Therefore, an important extension to consider is having free labor mobility between the regions. More specifically, one could consider such mobility with an attachment-to-home component where residents get utility from being close to home (and thus migrate only in case the utility gained elsewhere surpasses that which gained at home with the attachment-to-home component included), as was presented by Mansoorian and Myers (1993).

Another potential extension is the addition of multiple periods to the model. Such an extension, as was initially presented in a similar context by Hotelling (1931), could provide insights on several important issues that concern the main topic of this paper. Namely, how does the natural resource industry divert resources from other sectors across time? This is especially interesting when having other sectors that are more innovative. Furthermore, in a case where the natural resource is exhaustible (as seen in Canada), then should resource rents be saved, and if so, how? This important topic of intergenerational equity relates strongly to the discussion presented in this paper, as the usage of the rents concerns such issues as capital tax competition, investments in infrastructure, and fiscal competition. These rents may also affect other Macro-related factors such as, for instance, the real exchange rate, which in turn gives rise to the *Resource Curse* phenomena (Sachs and Warner 2001) where it is conjectured that natural resources impede economic growth. Looking into this in a multi-regional setting may provide new and interesting insights on the effects of adding a natural resource to a small region, especially in the context of possible agglomerations.

This paper follows Zodrow and Mieszkowski's work (1986) quite closely; that said, there are actually two additional exercises considered by Zodrow and Mieszkowski in their paper which may add further insights to our case, if applied to the current setting. The first exercise goes through the case where z (the lump sum tax on the natural resource rents) is restricted,[71] so that the smaller region needs to use some distortionary taxes. In that case we see that despite having the natural resource, region 2 bears higher social costs when financing infrastructure, so that $\text{MCPF}_2 > 1$; in addition, $MB_{P_2} > MC_{P_2}$ such that infrastructure is undersupplied, in equilibrium. It would be interesting to look into this case, and see how z affects the equilibrium outcome, and whether it can still cause agglomeration in the smaller region, when it is restricted. The second exercise combines the benchmark case with that of the infrastructure stage; under that setting the government invests in both infrastructure and a pure public good. Going through this analysis could show more clearly how adding a natural resource affects the supply of public goods (in addition to the effects on tax and infrastructure competitions as well as on possible agglomeration effects); also, it may bring new insights on the composition of public spending (between infrastructure and the pure public good) as was done under similar setting (only without a natural resource) by Keen and Marchand (1997).

Finally, another important extension to consider is the role of the government in correcting the inefficiency resulting by the natural resource. As was seen in the analysis part, under non-cooperative environment, the natural resource rents are invested in region 2, so that agglomeration may occur in the smaller region, although efficiency requires that these rents, as well as most of the nation's capital and infrastructure, be addressed to the more populated jurisdiction. Thus, the government has room to intervene. Boadway and Flatters (1982) suggest the option of equalization payments as a viable correction mechanism. Hindriks et al. (2008) show that such a mechanism is beneficial, when having infrastructure competition between two heterogeneous regions, both for the federation and for each of the regions. The equalization payments mechanism is, in fact, used in-practice in several federations, including Canada. In

[71] Note that in Zodrow and Mieszkowski (1986) there was no natural resource considered, and so the parallel exercise in their paper had the head tax constrained.

the context of this paper, such a correction can be made by making transfer payments between the regions. In an extreme case, such a transfer can reach the amount of the resource rents, so that the effect of adding a natural resource to the smaller region is completely reversed. Theoretically, it would be possible to reach efficiency this way, despite the non-cooperative behavior. Indeed, this could be an interesting exercise to investigate, as it provides a potential solution to the problem raised in this paper.

6 Conclusions

This theoretical paper touched on a vital issue in the Canadian economy. Having a resource-rich economy, Canada faces the question of whether it makes the best economic use of these resources by letting the regions to fully benefit from the natural resources located in their jurisdictions.

Although being a broad question that addresses many topics, we try in this paper to take one perspective of this issue to better assess the current situation. That perspective is the welfare level of the residents of the federation. In a case where the natural resource is located in a smaller region (which is less populated), will the nation benefit from taking advantage of the possible agglomeration economies in the more populated region by investing the resource rents there?

To be able to look into that more carefully, we tried to mimic Alberta and Ontario's case by adopting Zodrow and Mieszkowski's (1986) capital tax competition model (for the two-region case), in conjunction with Bucovetsky's (1991) addition of asymmetry in labor. The standard asymmetric tax competition theory was extended by considering asymmetry in infrastructure levels (thus, adopting concepts from the *New Economic Geography* literature), and at a later stage by adding a natural resource sector to the smaller region. Two routes were then investigated, one which followed the social optimum, and another that followed a more realistic setting of non-cooperative behavior. Each route was sub-divided to three stages, to be able to better realize the effects of possible agglomeration economies and the addition of a natural resource to that.

Although each stage of the model provided its own insights, it is the final results that are most interesting. Under the social optimum analysis we saw that the nation's welfare is maximized when we have agglomeration in the more populated region (directing most, if not all, of the nation's manufacturing-related capital to region 1), and also when the natural resource rents do not fully stay in region 2, but are in fact invested in the development of the larger region. This is in contrast, and in complete opposition, to the outcome derived under the non-cooperative behavior analysis. In that case we saw that the natural resource rents do not reach region 1; they stay in the smaller region, and are inefficiently invested there. In addition, we realized that the natural resource gives region 2 an advantage in the national tax and infrastructure competitions, to the point where having a large enough natural resource may in fact create agglomeration in the less populated region, which is, in efficiency and welfare terms, an undesired outcome.

Indeed, we see this outcome emerging in practice. According to Statistics Canada, Alberta currently has the lowest personal and business tax rates in Canada. In 2008 its corporate income tax general rate was set at 10%, making it the lowest in the nation; in addition, it had no capital, employer payroll, or sales tax at all. For comparison, in the parallel period in Ontario the corporate income tax general rate was set at 14%, and its capital, employer payroll, and sales tax rates were set at 0.225%, 1.95%, and 8% respectively. Thus, we see the outcome of the tax competition between the two regions. Also, we see clear indicators for emerging agglomeration economies in Alberta. Firstly, in recent years Alberta consistently had the highest investment per capita among the provinces; in 2008, for instance, its investment per capita level was more than double the national average. Secondly, its manufacturing base is rapidly growing; in the period between 2007 and 2008 (being a representative period for recent years) Alberta's manufacturing shipments increased by 6.9%, which was the highest increase in the nation. Thirdly, Alberta's annual GDP growth rate has been the highest in Canada for the past two decades. In addition, its GDP per capita is consistently the highest in the country as well; in fact, in 2008 it was approximately 60% above the Canadian average. All of this to show that what our simple model predicted is actually occurring (or perhaps in the process of occurring) in practice. Nonetheless, we can not completely attribute the above figures to the abundance of Alberta's natural resources; however, given that the energy sector alone is responsible for more than third of Alberta's total GDP (and since Ontario has limited natural resources), the link between these economic indicators and Alberta's resources becomes clear, and even necessary.

In conclusion, this paper showed how the current situation brings about an inefficiency that negatively affects the Canadian economy (and could potentially do the same in any other multi-regional federations), and which merits governmental intervention (possibly in the form of equalization payments) for correcting that. Nevertheless, this topic is far from being exhausted. Several main questions remain open for further study, some of which were raised in the previous section.

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A Appendix 1

Proof of Lemma 1. Let us assume that $\mathbf{m} = (k_1, k_2, G_1, G_2, x_1, x_2)$ is the social optimum point, where $k_1 = k_2$, $L_i(MRS_{G_i x_i}) = 1$, and $G_1, G_2, x_1, x_2, k_1, k_2 > 0$.

The problem to be solved is as follows -

$$\max_{\{k_1,k_2,G_1,G_2,x_1,x_2\}} Z(\cdot) = L_1 U(x_1, G_1) + L_2 U(x_2, G_2)$$

subject to:

$$\begin{aligned} L_1 x_1 + L_2 x_2 &= L_1 f(k_1) + L_2 f(k_2) - G_1 - G_2 \\ k^* &= S_1 k_1 + S_2 k_2 \\ 0 &< k_1, k_2, G_1, G_2, x_1, x_2 \end{aligned}$$

Stage 1 - $Z(\cdot)$ is continuous and defined over a closed and bounded set $S \in R^n$. Therefore, by the *Extreme-Value Theorem*: $\exists$ a maximum $\mathbf{a} = (k_1, k_2, G_1, G_2, x_1, x_2)$, in S, s.t: $Z(\mathbf{x}) \leq Z(\mathbf{a}) \; \forall \; x \in S$. Since $Z(\cdot)$ is continuous and differentiable, **a** would be either a stationary or a boundary point, and would thus be located by the Kuhn Tucker method.

Stage 2 - Showing that point **m** is a potential maximum.

From the above problem, it is possible to derive the lagrangian expression, as follows:

$$\mathcal{L} = L_1 U(x_1, G_1) + L_2 U(x_2, G_2) - \lambda(L_1 x_1 + L_2 x_2 - L_1 f(k_1) - L_2 f(k_2) + G_1 + G_2) + \delta(k^* - S_1 k_1 - S_2 k_2) + \alpha(k_1) + \beta(k_2) + \gamma(G_1) + \varsigma(G_2) + \varphi(x_1) + \xi(x_2)$$

FOCS:
(1) $\mathcal{L}_{k_1} : \; \lambda L_1 f_{k_1} - \delta S_1 + \alpha = 0$
(2) $\mathcal{L}_{k_2} : \; \lambda L_2 f_{k_2} - \delta S_2 + \beta = 0$
(3) $\mathcal{L}_{G_1} : \; L_1 U_G - \lambda + \gamma = 0$
(4) $\mathcal{L}_{G_2} : \; L_2 U_G - \lambda + \varsigma = 0$
(5) $\mathcal{L}_{x_1} : \; L_1 U_x - \lambda L_1 + \varphi = 0$
(6) $\mathcal{L}_{x_2} : \; L_2 U_x - \lambda L_2 + \xi = 0$

Kuhn-Tucker Conditions:
(7) $\lambda(L_1 x_1 + L_2 x_2 - L_1 f(k_1) - L_2 f(k_2) + G_1 + G_2) = 0$
(8) $\delta(S_1 k_1 + S_2 k_2 - k^*) = 0$
(9) $\alpha(-k_1) = 0$
(10) $\beta(-k_2) = 0$
(11) $\gamma(-G_1) = 0$
(12) $\varsigma(-G_2) = 0$
(13) $\varphi(-x_1) = 0$

(14) $\xi(-x_2) = 0$

The point - ($k_1 = k_2,\ G_1, G_2, x_1, x_2, \lambda, \delta > 0,\ \alpha, \beta, \gamma, \varsigma, \varphi, \xi = 0$) follows conditions (1)–(14),[72] as well as all constraints in S. Therefore, by Stage 1 and the above, point **m** is a *potential* maximum of $Z(\cdot)$.

Stage 3 - Showing that point **m** is the maximum of $Z(\cdot)$, and is in fact the only maximum.

Proof by contradiction:
Let us assume that $\exists$ a maximum **m'** $\neq$ **m** s.t $Z(\mathbf{x}) \leq Z(\mathbf{m'})\ \forall\ x \in S$.
This would mean that there could be two options:

Option 1:
In **m'** $\exists\ k_1^{'}$ and $k_2^{'}$, s.t: $k_2^{'} > k_1^{'}$.
Then, since $(f_{k_1}/f_{k_2}) \neq 1$, then FOCS (1) and (2) can not hold together.
$\therefore$ [Option 1 is not a maximum of $Z(\cdot)$] (I)

Option 2:
In **m'** $\exists\ k_1^{'}$ and $k_2^{'}$, s.t: $k_2^{'} < k_1^{'}$.
Then, since $(f_{k_1}/f_{k_2}) \neq 1$, then FOCS (1) and (2) can not hold together.
$\therefore$ [Option 2 is not a maximum of $Z(\cdot)$] (II)

$$\Downarrow$$

By (I) and (II), **m'** is not the maximum of $Z(\cdot)$.
$\therefore$ The initial supposition is contradicted.
$\therefore$ Point **m** is the single, and thus global, maximum of $Z(\cdot)$.

$\square$

[72] Also note that at this point $L_i(MRS_{G_i x_i}) = 1$ holds in each of the regions by FOCS (3)–(6), which is what is assumed to have in **m**.

B Appendix 2

Proof of Lemma 2. Let us assume that $\mathbf{m} = (k_1, k_2, P_1, P_2)$ is the social optimum point, where $k_1 > k_2$, $P_1 > P_2$, $MB_{P_i} = 1$,[73] and $P_1, P_2, k_1, k_2 \geq 0$.

The problem to be solved is as follows -

$$\max_{\{k_1,k_2,P_1,P_2\}} Z(\cdot) = L_1 I(P_1) f(k_1) + L_2 I(P_2) f(k_2) - P_1 - P_2$$

subject to:

$$k^* = S_1 k_1 + S_2 k_2$$
$$0 \leq k_1, k_2, P_1, P_2$$

Stage 1 - $Z(\cdot)$ is continuous and defined over a closed and bounded set $S \in R^n$. Therefore, by the *Extreme-Value Theorem*: $\exists$ a maximum $\mathbf{a} = (k_1, k_2, P_1, P_2)$, in S, s.t: $Z(\mathbf{x}) \leq Z(\mathbf{a})\ \forall\ x \in S$. Since $Z(\cdot)$ is continuous and differentiable, $\mathbf{a}$ would be either a stationary or a boundary point, and would thus be located by the Kuhn Tucker method.

Stage 2 - Showing that point $\mathbf{m}$ is a potential maximum.

From the above problem, it is possible to derive the lagrangian expression, as follows:

$\mathcal{L} = L_1 I(P_1) f(k_1) + L_2 I(P_2) f(k_2) - P_1 - P_2 \;-\; \lambda(S_1 k_1 + S_2 k_2 - k^*) \;+\; \alpha(k_1) \;+\; \beta(k_2) \;+\; \gamma(P_1) \;+\; \delta(P_2)$

FOCS:
(1) $\mathcal{L}_{k_1} : \ L_1 I(P_1) f_{k_1} - \lambda S_1 + \alpha = 0$
(2) $\mathcal{L}_{k_2} : \ L_2 I(P_2) f_{k_2} - \lambda S_2 + \beta = 0$
(3) $\mathcal{L}_{P_1} : \ L_1 f(k_1) I_{P_1} - 1 + \gamma = 0$
(4) $\mathcal{L}_{P_2} : \ L_2 f(k_2) I_{P_2} - 1 + \delta = 0$

Kuhn-Tucker Conditions:
(5) $\lambda(S_1 k_1 + S_2 k_2 - k^*) = 0$
(6) $\alpha(-k_1) = 0$
(7) $\beta(-k_2) = 0$
(8) $\gamma(-P_1) = 0$
(9) $\delta(-P_2) = 0$

Given the constraints of the problem (having interior or corner solutions, and capital fully employed), at the optimum we must have:

$$\alpha, \beta, \gamma, \delta \geq 0 \text{ and } \lambda > 0$$

[73] For regions with $(k_i > 0) \in \mathbf{m}$.

Therefore, the FOCS become:
(10) $\mathcal{L}_{k_1}:\ I(P_1)f_{k_1} = (\lambda - \alpha)/L$
(11) $\mathcal{L}_{k_2}:\ I(P_2)f_{k_2} = (\lambda - \beta)/L$
(12) $\mathcal{L}_{P_1}:\ I_{P_1} = (1-\gamma)/F(K_1, L_1)$
(13) $\mathcal{L}_{P_2}:\ I_{P_2} = (1-\delta)/F(K_2, L_2)$

In case we have an interior solution (so that $\alpha, \beta, \gamma, \delta = 0$), by FOCS (12)–(13) we get $P_i(F(K_i, L_i))$ for $i \in (1, 2)$. For convenience, let us denote $P_i(F(K_i, L_i))$ by a_i. In case we have a corner solution in k_2 (so that $k_2 = 0$ and $\beta > 0$), we get the same, only additionally we impose $\lambda = \beta$.

The point - ($k_1 > k_2$ (so that $k_1 > 0, k_2 \geq 0$), $P_1 = a_1 > P_2 = a_2$ (so that $a_1 > 0, a_2 \geq 0$), $\lambda > 0$, $\alpha, \beta, \gamma, \delta \geq 0$) follows conditions (1)–(13),[74] as well as all constraints in S. Therefore, by Stage 1 and the above, point **m** is a *potential* maximum of $Z(\cdot)$.

Stage 3 - Showing that point **m** is the maximum of $Z(\cdot)$.

Proof by contradiction:
Let us assume that $\exists$ a maximum **m'** $\neq$ **m** s.t $Z(\mathbf{x}) \leq Z(\mathbf{m'})\ \forall\ x \in S$.
This would mean that there could be two options:

Option 1:
In **m'** $\exists\ k_1^{'}$ and $k_2^{'}$, s.t: $k_2^{'} = k_1^{'}$ (and $k_1^{'}, k_2^{'} > 0$, since otherwise the value function would be valued at 0, which is certainly not the maximum).
$\therefore$ by FOCS (10)–(11): $I(P_1)/I(P_2) \neq 1$. $\Rightarrow$ FOCS (10)–(11) can not hold together.
$\therefore$ [Option 1 is not a maximum of $Z(\cdot)$] (I)

Option 2:
In **m'** $\exists\ k_1^{'}$ and $k_2^{'}$, s.t: $k_2^{'} > k_1^{'}$.

Case 1: Conditions (1)–(13) do not hold.
$\therefore$ [Option 2 is not a maximum of $Z(\cdot)$] (II)

Case 2: The point - ($k_2^{'} > k_1^{'}$ (so that $k_2^{'} > 0, k_1^{'} \geq 0$), $P_2^{'} = a_2^{'} > P_1^{'} = a_1^{'}$ (so that $a_2^{'} > 0, a_1^{'} \geq 0$), $\lambda > 0$, $\alpha, \beta, \gamma, \delta \geq 0$) follows conditions (1)–(13), as well as all constraints in S. Thus, **m'** is a *potential* maximum of $Z(\cdot)$.

Conjecture 1: In an interior solution $Z(\mathbf{m}) - Z(\mathbf{m'}) > 0$

[74] Also note that at this point $MB_{P_i} = 1$ (for regions with $(k_i > 0) \in \mathbf{m}$) by FOCS (12)–(13), which is what is assumed to have in **m**.

Proof 1: From FOCs (10)–(11), we get:

$$\frac{I(a_1)f_{k_1}}{I(a_2)f_{k_2}} = 1$$

This condition must hold at the optimum. By this condition, and since $f(\cdot)$ and f_k are continuous, differentiable and monotonous, $f_k > 0$, $f_{kk} < 0$, and the Inada conditions hold - then it must mean, by $S_1k_1 + S_2k_2 = k^*$, that:
$(k_1\ ,\ k_2) \in$ **m** and $(k_1^{'}\ ,\ k_2^{'}) \in$ **m'** are each unique.

Also, by definition: $k_1 > k_1^{'}$ and $k_2^{'} > k_2 \Rightarrow K_1 > K_1^{'}$ and $K_2^{'} > K_2$; therefore, by $S_1k_1 + S_2k_2 = k^*$ and the former condition: $(K_1 - K_1^{'}) = (K_2^{'} - K_2)$.

Plugging **m** and **m'** to $Z(\cdot)$, we get:
$Z(\mathbf{m}) = I(a_1)F(K_1, L_1) + I(a_2)F(K_2, L_2) - a_1 - a_2$
$Z(\mathbf{m'}) = I(a_1^{'})F(K_1^{'}, L_1^{'}) + I(a_2^{'})F(K_2^{'}, L_2^{'}) - a_1^{'} - a_2^{'}$

Let:
$A = I(a_1)F(K_1, L_1)$, $B = I(a_2)F(K_2, L_2)$, $C = a_1$, $D = a_2$
$E = I(a_1^{'})F(K_1^{'}, L_1^{'})$, $F = I(a_2^{'})F(K_2^{'}, L_2^{'})$, $G = a_1^{'}$, $H = a_2^{'}$

Therefore:
$Z(\mathbf{m}) - Z(\mathbf{m'}) = (A - E) + (B - F) + (C - G) + (D - H)$

By $L_1 > L_2$, and since we have $(K_1 - K_1^{'}) = (K_2^{'} - K_2)$, we get:
$[(F(K_1, L_1) - F(K_1^{'}, L_1)) > (F(K_2^{'}, L_2) - F(K_2, L_2))]$

Therefore:
$(A - E) + (B - F) > 0$
$(C - G) + (D - H) > 0$

Thus, we have:
$Z(\mathbf{m}) - Z(\mathbf{m'}) > 0$, QED.

$\therefore$ [Option 2 is not a maximum of $Z(\cdot)$] (III)

Conjecture 2: In a corner solution $Z(\mathbf{m}) - Z(\mathbf{m'}) > 0$

Proof 2: In a corner solution we have -
$Z(\mathbf{m}) = I(a_1)F(K^*, L_1) - a_1$
$Z(\mathbf{m'}) = I(a_2^{'})F(K^*, L_2) - a_2^{'}$

Since $L_1 > L_2 \Rightarrow F(K^*, L_1) > F(K^*, L_2)$, and $a_1 > a_2^{'}$.

Thus, we have:
$Z(\mathbf{m}) - Z(\mathbf{m'}) > 0$, QED.

$\therefore$ [Option 2 is not a maximum of $Z(\cdot)$] (IV)

By (I)–(IV), **m'** is not the maximum of $Z(\cdot)$.
$\therefore$ The initial supposition is contradicted.
$\therefore$ Point **m** is the single, and thus global, maximum of $Z(\cdot)$.

$\square$

C Appendix 3

Proof of Lemma 3. Let us assume that $\mathbf{m} = (k_1, k_2^1, k_2^2, P_1, P_2, L_2^1, L_2^2)$ is the social optimum point, where $k_1 > k_2^1$, $P_1 > P_2$, $MB_{P_i} = 1$,[75] and $P_1, P_2, k_1, k_2^1, k_2^2, L_2^1, L_2^2 \geq 0$.

The problem to be solved is as follows -

$$\max_{\{k_1, k_2^1, k_2^2, P_1, P_2, L_2^1, L_2^2\}} Z(\cdot) = L_1 I(P_1) f(k_1) + L_2^1 I(P_2) f(k_2^1) + L_2^2 h(k_2^2, q) - P_1 - P_2$$

subject to:

$$\begin{aligned} k^* &= S_1 k_1 + S_2^1 k_2^1 + S_2^2 k_2^2 \\ L_2 &= L_2^1 + L_2^2 \\ 0 &\leq k_1, k_2^1, k_2^2, P_1, P_2, L_2^1, L_2^2 \end{aligned}$$

Stage 1 - $Z(\cdot)$ is continuous and defined over a closed and bounded set $S \in R^n$. Therefore, by the *Extreme-Value Theorem*: $\exists$ a maximum $\mathbf{a} = (k_1, k_2^1, k_2^2, P_1, P_2, L_2^1, L_2^2)$, in S, s.t: $Z(\mathbf{x}) \leq Z(\mathbf{a})\ \forall\ x \in S$. Since $Z(\cdot)$ is continuous and differentiable, $\mathbf{a}$ would be either a stationary or a boundary point, and would thus be located by the Kuhn Tucker method.

Stage 2 - Showing that point $\mathbf{m}$ is a potential maximum.

From the above problem, it is possible to derive the lagrangian expression, as follows:

$$\mathcal{L} = L_1 I(P_1) f(k_1) + L_2^1 I(P_2) f(k_2^1) + L_2^2 h(k_2^2, q) - P_1 - P_2 - \lambda(S_1 k_1 + S_2^1 k_2^1 + S_2^2 k_2^2 - k^*) - \mu(L_2^1 + L_2^2 - L_2) + \alpha(k_1) + \beta(k_2^1) + \varsigma(k_2^2) + \varphi(L_2^1) + \zeta(L_2^2) + \gamma(P_1) + \delta(P_2)$$

FOCS:

(1) $\mathcal{L}_{k_1}: \ L_1 I(P_1) f_{k_1} - \lambda S_1 + \alpha = 0$
(2) $\mathcal{L}_{k_2^1}: \ L_2^1 I(P_2) f_{k_2^1} - \lambda S_2^1 + \beta = 0$
(3) $\mathcal{L}_{k_2^2}: \ L_2^2 h_{k_2^2} - \lambda S_2^2 + \varsigma = 0$
(4) $\mathcal{L}_{P_1}: \ L_1 f(k_1) I_{P_1} - 1 + \gamma = 0$
(5) $\mathcal{L}_{P_2}: \ L_2^1 f(k_2^1) I_{P_2} - 1 + \delta = 0$
(6) $\mathcal{L}_{L_2^1}: \ f(k_2^1) I_{P_2} - \lambda k_2^1 / L - \mu + \varphi = 0$
(7) $\mathcal{L}_{L_2^2}: \ h(k_2^2, q) - \lambda k_2^2 / L - \mu + \zeta = 0$

Kuhn-Tucker Conditions:

(8) $\lambda(S_1 k_1 + S_2^1 k_2^1 + S_2^2 k_2^2 - k^*) = 0$
(9) $\mu(L_2^1 + L_2^2 - L_2) = 0$
(10) $\alpha(-k_1) = 0$
(11) $\beta(-k_2^1) = 0$

[75] For regions with positive amounts of capital in their manufacturing sector, in $\mathbf{m}$.

(12) $\varsigma(-k_2^2) = 0$
(13) $\gamma(-P_1) = 0$
(14) $\delta(-P_2) = 0$
(15) $\varphi(-L_2^1) = 0$
(16) $\zeta(-L_2^2) = 0$

Given the constraints of the problem (having interior or corner solutions, and capital fully employed), at the optimum we must have:

$$\alpha, \beta, \gamma, \delta, \varsigma, \varphi, \zeta \geq 0 \text{ and } \lambda, \mu > 0$$

Therefore, the FOCS become:
(17) $\mathcal{L}_{k_1}: \ I(P_1)f_{k_1} = (\lambda - \alpha)/L$
(18) $\mathcal{L}_{k_2^1}: \ I(P_2)f_{k_2^1} = (\lambda - \beta)/L$
(19) $\mathcal{L}_{k_2^2}: \ h_{k_2^2} = (\lambda - \varsigma)/L$
(20) $\mathcal{L}_{P_1}: \ I_{P_1} = (1 - \gamma)/F(K_1, L_1)$
(21) $\mathcal{L}_{P_2}: \ I_{P_2} = (1 - \delta)/F(K_2^1, L_2^1)$
(22) $\mathcal{L}_{L_2^1}: \ f(k_2^1)I_{P_2} = \lambda k_2^1/L + \mu - \varphi$
(23) $\mathcal{L}_{L_2^2}: \ h(k_2^2, q) = \lambda k_2^2/L + \mu - \zeta$

In case we have an interior solution (so that $\alpha, \beta, \gamma, \delta, \varsigma, \zeta, \varphi = 0$), by FOCS (20)–(21) we get $P_i(F(K_i^1, L_i^1))$ for $i \in (1, 2)$. For convenience, let us denote $P_i(F(K_i^1, L_i^1))$ by a_i. In case we have a corner solution in k_2^1 (so that $k_2^1 = 0$, $L_2^1 = 0$ and $\beta, \varphi > 0$), we get the same, only additionally we impose $\lambda = \beta$, and $\varphi = \lambda k_2^1/L + \mu$.

The point - ($k_1 > k_2^1$ (so that $k_1 > 0, k_2^1 \geq 0$), $k_2^2 > 0$, $P_1 = a_1 > P_2 = a_2$ (so that $a_1 > 0, a_2 \geq 0$), $\lambda, \mu > 0$, $\alpha, \beta, \gamma, \delta, \varsigma, \zeta, \varphi \geq 0$) follows conditions (1)–(23),[76] as well as all constraints in S. Therefore, by Stage 1 and the above, point $\mathbf{m}$ is a *potential* maximum of $Z(\cdot)$.

Stage 3 - Showing that point $\mathbf{m}$ is the maximum of $Z(\cdot)$.

Proof by contradiction:
Let us assume that $\exists$ a maximum $\mathbf{m'} \neq \mathbf{m}$ s.t $Z(\mathbf{x}) \leq Z(\mathbf{m'})$ $\forall$ $x \in S$.
This would mean that there could be two options:

Option 1:
In $\mathbf{m'}$ $\exists$ $k_1^{'}$ and $k_2^{1'}$, s.t: $k_2^{1'} = k_1^{'}$ (and $k_1^{'}, k_2^{1'} > 0$, since otherwise the value function would certainly not be at the maximum).
$\therefore$ by FOCS (17)–(18): $I(P_1)/I(P_2) \neq 1$. $\Rightarrow$ FOCS (17)–(18) can not hold together.
$\therefore$ [Option 1 is not a maximum of $Z(\cdot)$] (I)

[76] Also note that by FOCS (20)–(21), at this point $MB_{P_i} = 1$ (for regions with positive capital in the manufacturing sector, in $\mathbf{m}$).

Option 2:
In **m'** $\exists\, k_1^{'}$ and $k_2^{1'}$, s.t: $k_2^{1'} > k_1^{'}$.

Case 1: Conditions (1)–(23) do not hold.
$\therefore$ [Option 2 is not a maximum of $Z(\cdot)$] (II)

Case 2: The point - ($k_2^{1'} > k_1^{'}$ (so that $k_2^{1'} > 0, k_1^{'} \geq 0$), $P_2^{'} = a_2^{'} > P_1^{'} = a_1^{'}$ (so that $a_2^{'} > 0, a_1^{'} \geq 0$), $\lambda, \mu > 0$, $\alpha, \beta, \gamma, \delta, \varsigma, \zeta, \varphi \geq 0$) follows conditions (1)–(23), as well as all constraints in S. Thus, **m'** is a *potential* maximum of $Z(\cdot)$.

Conjecture 1: In an interior solution $Z(\mathbf{m}) - Z(\mathbf{m'}) > 0$

Proof 1: From FOCS (17)–(18), we get:

$$\frac{I(a_1)f_{k_1}}{I(a_2)f_{k_2^1}} = 1$$

This condition must hold at the optimum. By this condition, and since $f(\cdot)$ and f_k are continuous, differentiable and monotonous, $f_k > 0$, $f_{kk} < 0$, and the Inada conditions hold - then it must mean, by $S_1k_1 + S_2^1k_2^1 + S_2^2k_2^2 = k^*$, that:
$(k_1\ ,\ k_2^1\ ,\ k_2^2) \in$ **m** and $(k_1^{'}\ ,\ k_2^{'}\ ,\ k_2^2) \in$ **m'** are each unique; also, due to equation (17) k_2^2 would be at the same level in both **m** and **m'**.

Also, by definition: $k_1 > k_1^{'}$ and $k_2^{1'} > k_2^1 \Rightarrow K_1 > K_1^{'}$ and $K_2^{1'} > K_2^1$; therefore, by $S_1k_1 + S_2^1k_2^1 + S_2^2k_2^2 = k^*$ and the former condition: $(K_1 - K_1^{'}) = (K_2^{1'} - K_2^1)$.

Plugging **m** and **m'** to $Z(\cdot)$, we get:
$Z(\mathbf{m}) = I(a_1)F(K_1, L_1) + I(a_2)F(K_2^1, L_2^1) + H(K_2^2, L_2^2, Q) - a_1 - a_2$
$Z(\mathbf{m'}) = I(a_1^{'})F(K_1^{'}, L_1^{'}) + I(a_2^{'})F(K_2^{'}, L_2^{'}) + H(K_2^2, L_2^2, Q) - a_1^{'} - a_2^{'}$

Let:
$A = I(a_1)F(K_1, L_1)$, $B = I(a_2)F(K_2^1, L_2^1)$, $C = a_1$, $D = a_2$
$E = I(a_1^{'})F(K_1^{'}, L_1^{'})$, $F = I(a_2^{'})F(K_2^{1'}, L_2^{1'})$, $G = a_1^{'}$, $H = a_2^{'}$
$J = K = H(K_2^2, L_2^2, Q)$

Therefore:
$Z(\mathbf{m}) - Z(\mathbf{m'}) = (A - E) + (B - F) + (C - G) + (D - H) + (J - K)$

Thus, by $L_1 > L_2^1$, and since we have $(K_1 - K_1^{'}) = (K_2^{1'} - K_2^1)$, we get:
$[(F(K_1, L_1) - F(K_1^{'}, L_1)) > (F(K_2^{1'}, L_2^1) - F(K_2^1, L_2^1))]$

Therefore:
$(A - E) + (B - F) > 0$
$(C - G) + (D - H) > 0$
$(J - K) = 0$

Thus, we have: $Z(\mathbf{m}) - Z(\mathbf{m'}) > 0$, QED.

$\therefore$ [Option 2 is not a maximum of $Z(\cdot)$] (III)

Conjecture 2: In a corner solution $Z(\mathbf{m}) - Z(\mathbf{m'}) > 0$

Proof 2: In a corner solution we have -
$Z(\mathbf{m}) = I(a_1)F(K^* - K_2^2, L_1) + H(K_2^2, L_2^2, Q) - a_1$
$Z(\mathbf{m'}) = I(a_2^{'})F(K^* - K_2^2, L_2^1) + H(K_2^2, L_2^2, Q) - a_2^{'}$

Since $L_1 > L_2^1 \Rightarrow F(K^* - K_2^2, L_1) > F(K^* - K_2^2, L_2^1)$, and $a_1 > a_2^{'}$.

Thus, we have:
$Z(\mathbf{m}) - Z(\mathbf{m'}) > 0$, QED.

$\therefore$ [Option 2 is not a maximum of $Z(\cdot)$] (IV)

By (I)–(IV), **m'** is not the maximum of $Z(\cdot)$.
$\therefore$ The initial supposition is contradicted.
$\therefore$ Point **m** is the single, and thus global, maximum of $Z(\cdot)$.

$\square$

D Appendix 4

Proof of Lemma 4. The two regions operate under identical technology, so that both use the same production function to produce output. Therefore, if $k_1 = k_2$ then by equation (21) we have:

$$\frac{dk_1}{dT_1} = \frac{dk_2}{dT_2}$$

Thus, in that case by equation (26) we have:

$$\left[L_1 m(x_1, G_1) = \frac{1}{1 + \frac{T_1}{k_1}\frac{dk_1}{dT_1}}\right] = \left[L_2 m(x_2, G_2) = \frac{1}{1 + \frac{T_2}{k_2}\frac{dk_2}{dT_2}}\right]$$

As a result, we get an identical reaction function in each of the regions, so that in case $k_1 = k_2$, we have:

$$T_1(k_1) = T_2(k_2)$$

By the same reasoning, if $k_1 \neq k_2$ then all three equations presented above appear with an inequality. Therefore, by the above and because $S_1 + S_2 = 1$ and $f_{k_1} = f_{k_2}$ (if $k_1 = k_2$), then in case $k_1 = k_2$, we have:

$$k^* = S_1 k_1 + S_2 k_2$$
$$f_{k_1} - T_1(k_1) = f_{k_2} - T_2(k_2)$$

Thus, all equilibrium conditions hold, which makes $k_1 = k_2$ a Nash Equilibrium outcome.

Conjecture: $k_1 = k_2$ is the unique Nash Equilibrium outcome in this case.

Proof:

Substituting equation (4) to equation (8), and dividing by L_i, we get:

$$x_i = f(k_i) - T_i k_i$$

Rearranging this, we get:

$$T_i = \frac{f(k_i) - x_i}{k_i}$$

From this, we derive the following:

$$\frac{dT_i}{dk_i} = \frac{T_i k_i - f(k_i) + x_i}{2k_i^2}$$

However, since by the above $x_i > f(k_i)$, then:

$$\frac{dT_i}{dk_i} > 0$$

Given that $dT_i/dk_i > 0$ and $f_{k_ik_i} < 0$, then starting at the Nash Equilibrium outcome where $k_1 = k_2$, if we move capital from region 2 to region 1 then f_{k_1} decreases, T_1 increases, f_{k_2} increases, and T_2 decreases, so that the following holds $\forall\ k_1 > k_2$:

$$f_{k_2} - T_2 > f_{k_1} - T_1$$

Otherwise, if we move capital from region 1 to region 2 then f_{k_2} decreases, T_2 increases, f_{k_1} increases, and T_1 decreases, so that the following holds $\forall\ k_2 > k_1$:

$$f_{k_1} - T_1 > f_{k_2} - T_2$$

By that we conclude that $k_1 = k_2$ represents the unique Nash Equilibrium in this case.

$\square$

E Appendix 5

Proof of Lemma 6. In each region the following must hold in equilibrium:

$$X_i + P_i = L_i I(P_i) f(k_i)$$

Substituting equation (10) to the above, and dividing by L_i, we get:

$$x_i = I(P_i) f(k_i) - T_i k_i$$

Rearranging this, we get:

$$T_i = \frac{I(P_i) f(k_i) - x_i}{k_i}$$

From this, we derive the following:

$$\frac{dT_i}{dk_i} = \frac{T_i k_i - I(P_i) f(k_i) + x_i}{2k_i^2}$$

However, since by the above $x_i > I(P_i) f(k_i)$, then:

$$\frac{dT_i}{dk_i} > 0$$

Then, by equation (10):

$$\frac{dP_i}{dk_i} > 0$$

Given that $dT_i/dk_i > 0$, $dP_i/dk_i > 0$, and $f_{k_i k_i} < 0$, then starting at the point where $k_1 = k_2$ and $P_1 = P_2$, if we move capital from region 2 to region 1 then f_{k_1} decreases, $I(P_1)$ increases, T_1 increases, f_{k_2} increases, $I(P_2)$ decreases, and T_2 decreases, so that at some point we might have:

$$I(P_1) f_{k_1} - T_1 = I(P_2) f_{k_2} - T_2$$

Otherwise, if we move capital from region 1 to region 2 then f_{k_2} decreases, $I(P_2)$ increases, T_2 increases, f_{k_1} increases, $I(P_1)$ decreases, and T_1 decreases, so that at some point we might have:

$$I(P_1) f_{k_1} - T_1 = I(P_2) f_{k_2} - T_2$$

Therefore, since under either of the cases all other equilibrium conditions hold (besides the free capital mobility condition), we conclude that there exist two possible equilibria, one having agglomeration in region 1 (where $k_1 > k_2$, and thus by $dP_i/dk_i > 0$ and equation (10), $P_1 > P_2$), and the other having agglomeration in region 2 (where $k_2 > k_1$, and thus by $dP_i/dk_i > 0$ and the free capital mobility condition, $P_2 > P_1$).

□