A Theoretical Analysis of Passive Investment Strategies on Financial Market Losses during a 'Fire-sale'

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I. Introduction

The true value of an asset can be a difficult concept to pin down. The concept of price efficiency seems to hold during normal market activity; however, during a market crash it can be difficult to reconcile that the true value of a security is drastically less than it was on the previous day. Should an efficient price be defined simply as what someone is willing to pay for it?

"A horse, a horse! My kingdom for a horse!"
Richard The Third Act 5, scene 4, 7–10

In the case of King Richard, can we say that a kingdom is the efficient price for a horse? In this quotation Shakespeare succinctly shows that the value of an asset (a horse) can depend entirely on the need of the purchaser and their current bargaining power. When King Richard is forced into a fire sale situation, he was willing to give up his entire kingdom for the asset he required. This obviously was a massive deviation from the typical market price of horse in medieval England, but this extreme example of price movement is not unlike what results during a credit crunch or a market collapse.

When institutions are forced to raise cash their activity places downward pressure on market prices of the assets they are selling, and if their needs are dire enough they will be willing to accept almost any price. In equity markets this selling activity has historically been conducted by fund managers choosing which assets to liquidate in a way that will minimize their expected losses. Since 2000 we have seen a dramatic rise in the prevalence of passive investing in U.S. equity markets, and it is important for investment practitioners to understand how this change in the macroeconomic structure of the market can affect the behavior of markets during crashes.

The ETF and index mutual fund redemption mechanisms are inherently a non-selective mechanism and during a crash they will create downward price pressure on ETF constituents based upon the relative weightings of that index in excess of what you would observe from a loss minimization liquidation process. The redemption of index linked products has two potential primary problems. One, the co-movement of index constituent returns experienced during normal market activity can break down during a market crash and this breakdown has the potential to exacerbate the magnitude of a crash. Two, the nature of passive investment strategies serves to amplify the pro-cyclical aspects of market cycles; however, this volatility enhancing aspect of passive investments has been mostly ignored when discussing the validity of active management.[1]

This paper sets out to review the evolution of the structure of equity markets and proposes a theoretical framework in order to illustrate new risks that may arise from this financial innovation. This review does not cover the possibilities of a run on an ETF and largely avoids the implications of the breakdown of return co-movement in index constituents during a market panic. The major focus of this paper is how passive investment strategies affect market cycles, how volatility and price formation is impacted, and whether the growth of index linked products poses any threats to financial markets.

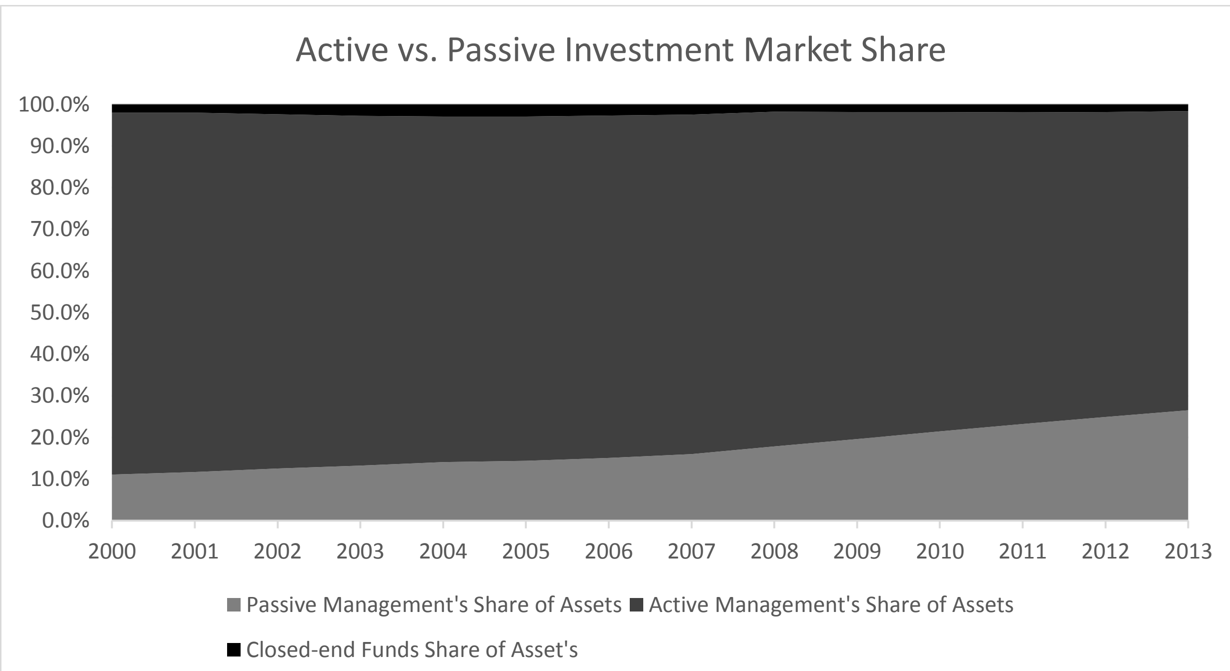
Market Evolution towards Passive Investing

As of February 2015 the Investment Company Institute reported the mutual fund assets under management of their members accounted for 16.24 trillion dollars, while ETFs accounted

[1] The majority of the literature on active investments is entirely focused upon whether managers are capable of outperforming an index. Carhart (1997), Cremers and Petajisto (2009), Daniel and Grinblatt (1997), Fama and French (2009), and many other canonical papers on investment management have not touched upon what effect a recommendation for passive investment strategies will have on the broader market.

for 2.06 trillion dollars. This represents a substantial increase in the market share of passive investing in relation to active management over the past fifteen years. Using TNA data from the Investment Company Institute (ICI) I have computed the implied market share of active and passive management styles in the U.S. equity market from 2000 to 2013 as seen in Figure I.[2] Passive Management's share of assets is calculated using the total net assets of ETFs, UITs and index mutual funds, while Active Management's share of assets is calculated by taking the total net assets of all mutual funds in the dataset and subtracting mutual funds which describe themselves as index funds.

Figure I



[2]*ICI Investment Company Total Net Assets by Type, Billions of dollars; year-end, 1996–2013. http://www.icifactbook.org/fb_ch1.html#investment*
Percentage of equity mutual funds' total net assets, 2000–2013. http://www.icifactbook.org/fb_ch2.html#index

It can be clearly seen that the share of money devoted to a passive investment strategy has doubled since 2000; however, the literature on the implications of this change to market behaviour has been sparse. If active investment managers behave differently than passive investment managers and face different optimization problems how can we determine what effect the shift towards rules based asset allocation will have on the anatomy of a market crash?

According to Cremers and Petajisto (2009) the mutual fund industry has been trending towards lower levels of active share and a growing share of mutual fund dollars are mirroring broad based market indices instead of relying upon active investment decisions and manager discretion. If this is the case, it may be that Figure I under represents the prevalence of passive investment strategies in the market.   Passive investing is a growing trend in the United States and as more assets shift away from valuation based active management, it is necessary to identify where, if any, risks have been shifted or created.

How Active Is Your Fund Manager? A New Measure That Predicts Performance

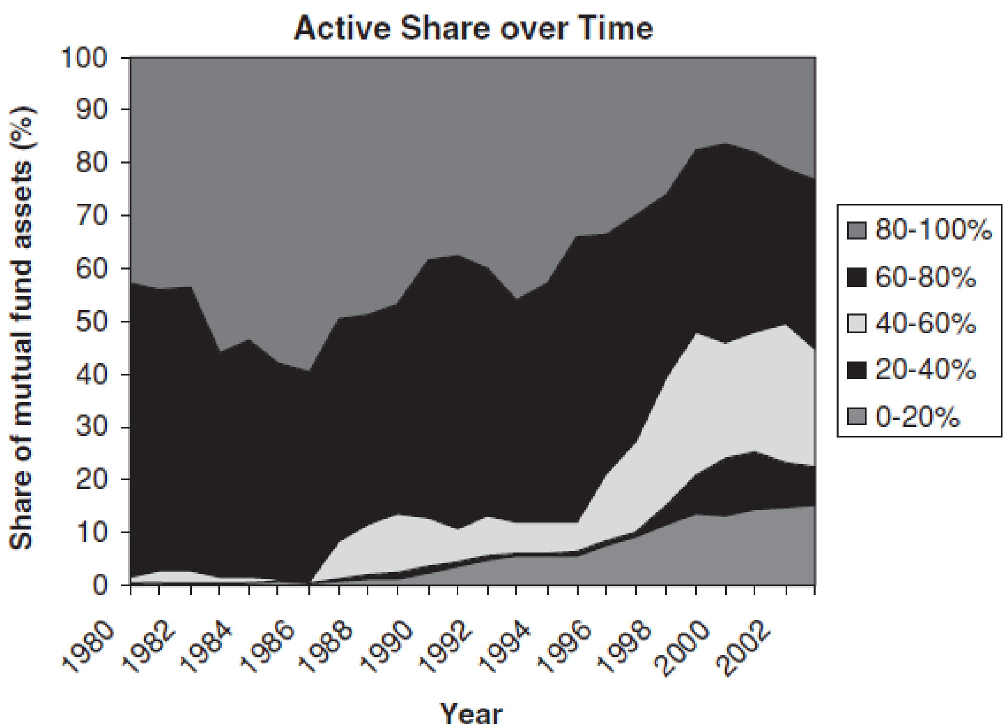


Figure 3
The share of US all-equity mutual fund assets in each Active Share category in 1980–2003

[3]

II. Motivations for Paper

In order for a regulator to minimize the risks within a financial system, it is imperative that risks be identified before the problems manifest themselves. In addition to this, the financial system continues to evolve and change making ‘pinning down’ the risks more difficult. In this light, I hope to propose a model which illustrates what I believe is a potentially growing risk in capital markets. The growth of passive investing has brought many economic benefits to investors such as lower fees and higher liquidity than active investment vehicles; however, the

[3] Chart from Cremers and Petajisto (2009) pg. 3347

increased market share of passive investing carries with it potential downside risks that may not be fully understood. There are two primary factors that lead me to believe that the rise of passive investing may cause negative distortions in how the financial business cycle behaves.

One, exchange traded ETFs currently trade much more frequently than their underlying constituents and are often more liquid as a product than when traded as their individual constituents. Currently ETFs can be used to gain quick exposure to a market, but the very act of investing in a market through ETFs instead of individual companies can have distortionary effects on how assets are priced. Inclusion of a new constituent to an index can fundamentally change the return profile of that security as it begins to exhibit co-movement with the index as a whole.[4] Passive investment vehicles appear to cause constituent security returns to trend towards the average of the index during times of contributions; however, during a market panic it is unlikely that the buyers of securities will have the same solution to an asset allocation optimization problem as a passive investment manager.

During times of market stress passive investment vehicles will undoubtedly be facing redemptions and the buyers of securities will likely be active investors who are holding cash or very defensive securities. These active buyers will likely have differing levels of interest for different securities, making it unlikely that the co-movement of the constituents to be maintained through a market panic.

Two, active managers facing redemptions are able to solve a loss minimization problem by selecting how much they wish to sell of each security, while passive investment managers must meet their redemptions according to the predetermined weights of the index their portfolio

[4] Ben-David(2011)

tracks. This difference between the redemption mechanisms of passive and active managers help determine how pro or counter cyclical the investment market is to a set level of redemptions. This paper hopes to show how the optimization problems of active and passive managers affect the cyclicality of the market using simple demand curves for securities, and solving Lagrange optimization problems for both types of managers. Through this theoretical framework, I hope to show how passive investing strategies are not a 'free lunch' for the broader market, and that the growth of the passive investment industry is having an effect on security prices and trading.

III. Literature Review

Cremers and Petajisto (2009) introduce the metric of active share as a way of differentiating active portfolio management from index tracking mutual funds. Their research primarily focuses on the skill of active managers; however, the measure provides an intuitive way of tracking the amount of passive investing occurring in a market.

Bradley and Litan (2010) voice concerns that ETFs can drain liquidity from stocks, especially if a short squeeze occurs and ETF sponsors rush to create new ETF shares. Their paper presents the idea that trading in ETFs is setting the prices of the underlying basket of securities and not the other way around. They present the case that the structure of the market is radically changing and needs to be addressed by regulators.

Da and Shive (2013) find that ETF ownership has a positive effect on the co-movement of stocks in the same basket and this causes the diversifying effects of holding many different securities to diminish. As the index inclusion effect takes over the return structure of the underlying securities, the diversifying benefits of owning an index decrease.

Madhavan (2011) relates market fragmentation in ETF trading to the Flash Crash of 2010 and examines the distortions in the underlying basket of securities that an ETF tracks. He identifies that a disproportionate amount of the products affected during the flash crash were exchange traded products and identifies that market participants are likely using these products as proxies for exposure to that market.

Wurgler(2011) provides an overview of the economic consequences of index investing and uses his paper to discuss index constituent co-movement, changes in stock returns based on index inclusion, and the formation of index bubbles. The paper discusses how index investing creates both positive and negative feedback loops which can lead to bubble formation as well as market crashes.

IV. Theoretical Pricing Model

This has been built to simulate the effects of passive investment strategies during a market crash. Assuming a fire sale scenario, the model outlines the optimization problems facing active and passive fund managers. The subsequent solutions to the fund manager loss minimization problem are used to illustrate the effect the growth of passive investment strategies can have on liquidation and price activities in the market. Broadly, the model intends to outline how passive investment strategies affect deleveraging cycles and show how the growth of passive strategies can amplify downside losses during a credit crunch.

This paper is not intended to be an argument for or against passive investment vehicles, nor is it aimed at whether active managers are capable of earning their fees. The primary intent of the paper is to demonstrate that while passive investment vehicles have provided investors with low cost access to equity markets, there are risks associated with the markets migration away from active management. Innovation in financial markets often brings new and unforeseen risks and this paper hopes to identify those associated with the rise of passive investment. The risks that stem from the rise of passive investment strategies are not isolated to individual investors and they can have consequences that affect the entire financial system. This paper looks at the primary systemic risks associated with passive investment vehicles and develops a model to identify how changes in the market share of passive investment managers affects market crashes.

This model will be assuming that securities face linear demand curves that vary from security to security and that both active and passive investment managers are facing identical contributions (negative) or redemptions (positive) equal to *R*. To simplify the analysis we will use a representative active and passive investment manager and weight their contributions. We can view these redemptions as the difference between the expected and actual redemptions faced by an investment manager. For most of the analysis of the model, we will assume *R* to be positive, and that managers are facing redemptions. Towards the end of the model we will begin to relax assumptions and see how the results react.

The model assumes a continuum of 1 to N market securities in the market with their respective prices determined by a fire-sale pricing process. It is assumed there is a fire-sale buyer which will purchase the securities sold by asset managers according to the below pricing process.

$$P_i = K_i - A_i S_i \qquad \textbf{(i)}$$

Where K_i is a random positive scalar and denotes the pre-crash market price of security *i* (the price at time *t-1)*, A_i is a random scalar which represents how the securities price is affected by sales pressure, and S_i is the total amount of security *i* sold to the representative fire-sale buyer. S_i is defined as the change in holdings of a security by the investment management community weighted according to the percentage of active and passive managers. P_i is the price at time *t* after asset allocation decisions have been made by all managers.

$$S_i \equiv \theta\left(S_i^A\right) + (1-\theta)(S_i^P) \quad \textbf{(ii)}$$

$$where\ 1 \geq \theta \geq 0$$

In equation **(ii)**, θ represents the percentage of the investment community that is following active management practices, and (1-θ) is the percentage of passive investment managers. The sales activities of the two types of managers are further defined as the difference between the chosen level of holdings at time *t-1* and *t*. For clarity, *t* denotes the time at which the managers are making their asset allocation decisions, and *t-1* is the asset allocations of the previous period and is viewed as a scalar.

$$Sales\ by\ Active\ Managers \quad S_i^A \equiv q_{i,t-1}^A - q_{i,t}^A \quad \textbf{(iii)}$$

$$Sales\ by\ Passive\ Managers \quad S_i^P \equiv (q_{t-1}^{ETF} - q_t^{ETF})w_i \quad \textbf{(iv)}$$

$$\text{Where } \textstyle\sum_{i=1}^{N} w_i = 1$$

Equation **(iv)** differs from **(iii)** because Passive managers are constrained in their sales decisions by the weights of their underlying index w_i. Passive managers choose a total level of asset sales q_t^{ETF} and assets are liquidated according to the exogenous index weights. For simplicity, it is assumed that the index weights do not change from time *t-1* to time *t*. Combining equations (i) through to (iv) provides us with the detailed fire-sale pricing process (v).

$$P_i = K_i - A_i[\theta\left(q_{i,t-1}^A - q_{i,t}^A\right) + (1-\theta)(q_{t-1}^{ETF} - q_t^{ETF})w_i] \quad \textbf{(v)}$$

Before moving on to the optimization problems of the two styles of manager and the analysis of the optimal allocation decisions, it is worthwhile to outline the underlying assumptions and specifications of the model, assess their reasonability, and to discuss the various components of the fire sale pricing process.

V. Assumptions of the Model

1. $K_i > 0$ and represents the pre fire sale price.

Since we are interested in how the market changes from its pre fire sale conditions, it is reasonable for us to begin to measure price changes from the pre panic price. We can safely assume that the pre panic price is greater than zero as a price of zero would imply the security has defaulted already and cannot be positively or negatively impacted by a market panic. The pre fire sale price differs from security to security allowing for a heterogeneous security market.

2. $-A_i < 0$ and represents how the price at time *t* responds to sales of security *i*.

It is not unreasonable for us to assume that the price pressure on market securities during the panic will be wholly negative and that the price movement will be unambiguously down for all securities. Since this paper is focused on the effect the structure of the investment industry has on market crashes, it is reasonable for us to have all securities prices responding in the same direction but with differing price sensitivity to security liquidation.

3. $P_i \geq 0$

The combination of a negative slope $-A_i$ and a positive intercept K_i indicates that there exists a value of S_i where price falls to zero and the security defaults. The model imposes a limit on the potential quantity sold because we would not expect firms to be trading securities once the firm has defaulted. If sales pressure on a security forces the price to zero, the firms write off the value of that position.

4. $S_i^A, S_i^P \geq 0$ preventing short sale strategies as well as positive price pressure.

The restriction of no short sale strategies is there to avoid the unnecessary complication of the model and to focus the results on 'vanilla' investment strategies. This restriction also precludes the price function from negative values of S_i that would imply positive price pressure, and fund inflows instead of redemptions.

5. $\theta, q_{i,t-1}^A, q_{t-1}^{ETF} \geq 0$ and are exogenous.

The values from *t-1* can be viewed as exogenous as the investment managers have no control over their values once time *t* occurs. Similarly, θ, the percentage of active managers in the market, is an exogenous variable between 0 and 1 as neither active nor passive fund managers can choose their respective market share. The quantity held of each security is also restricted to non-negative values to prevent short sale strategies.

6. w_i is exogenous and does not change between periods

Since w_i represents index weights, it is reasonable to assume that the passive investment managers do not have the ability to influence the weights of the benchmark they are pegged to. It is also assumed that the index weights do not change between *t* and *t-1* because we assume the benchmark provider will not be updating their constituent weightings during the middle of a market crash. This assumption both simplifies the model and also reflects the fact that changes to indices constituent weightings do not occur seamlessly and instantaneously.

7. Passive investment vehicles continue to trade at NAV during the fire sale and the selling pressure does not cause q_t^{ETF} to trade at a discount to its individual components. No possibilities of arbitrage.

To avoid complications of the model and to focus entirely on the cyclicality of the results, we assume that the ETF redemption process occurs without price distortions relative to the prices of the ETF's underlying constituents. This simplification allows us to combine ETF managers with mutual fund index trackers as their redemption process only significantly differs when there are arbitrage opportunities available to the authorized participant. By removing the possibility of arbitrage and "runs on an ETF", the model becomes much more intuitive while maintaining the relevance of its interpretation.

8. Passive investment managers minimize losses by choosing q_t^{ETF} and cannot choose interim portfolio weights

It is assumed passive investment managers are bound by their investment mandates to follow the weightings of their stated benchmark and manage their portfolios by minimizing costs and creating and redeeming ETF units based on the volume of contributions/redemptions to their respective fund. The assumption ensures a passive investment manager cannot behave as an active manager during a market crash out of convenience.

9. Homogeneous Investment Managers

For simplicity, we assume a representative active manager and a representative passive manager and weight their impact on prices and sales pressure based on the total market share of their respective management style. This assumption precludes the model from having multiple passive investor types each representing an individual index and implies that the starting holdings of the representative active manager are average holdings across the industry. The model can be further generalized by introducing a continuum of active and passive mangers; however, this step is not necessary to draw conclusions from the solutions to the optimization problems facing the two types of manager.

10. Sales activities of managers affect market prices and the managers are able to predict the impact of their activities.

This assumption ensures that the choices of the managers affect the market and will have an impact on the price function. This implies that there is predictability to the impact of their choices and that the managers do not view prices as a random variable. While an investment manager would not be able to identify exactly how their sales activity will impact prices in an empirical setting, they would have an estimate of how their choices will affect prices and will adjust their sales strategy accordingly.

VI. Optimization Problem Facing Investment Managers

In the model both types of fund managers face a loss minimization optimization problem where they are attempting to satisfy client redemptions while minimizing the short term price impact to their portfolio. To prevent a run on the fund manager, the manger must satisfy client redemptions while minimizing the one period loss to the portfolio. The manager minimizes the current period loss because poor portfolio performance during a fire sale can trigger further redemptions and create further risks to the livelihood of the fund.

VII. Active Fund Manager Optimization

The Active Fund Manager minimizes their loss function **(vi)** subject to raising enough cash to meet their redemptions as shown in **(vii)**. The active manager takes the level of redemptions R as given and chooses how to rebalance their portfolio by choosing $q_{i,t}^{A}$ for each security. The representative active manager's choice of $q_{i,t}^{A}$ affects the market price of security i by determining S_{i}^{A} through equation **(iii).**

$$\min_{q_{i,t}^{A}} \sum_{i=1}^{N} K_i q_{i,t-1}^{A} - \sum_{i=1}^{N} P_i q_{i,t}^{A} \qquad \textbf{(vi)}$$

$$s.t. \sum_{i=1}^{N} P_i \,(q_{i,t-1}^{A} - q_{i,t}^{A}) \geq R \qquad \textbf{(vii)}$$

Using the loss function (**vii)** and the redemption constraint **(vii)** we can set up the Lagrange **(viii).** By substituting the pricing process **(v)** into **(viii)** we can see how the active manager's choice of $q_{i,t}^{A}$ will affect all of the components of the Lagrange in equation **(ix).**

$$L = \sum_{i=1}^{N} K_i q_{i,t-1}^{A} - \sum_{i=1}^{N} P_i q_{i,t}^{A} + \lambda[R - \sum_{i=1}^{N} P_i \left(q_{i,t-1}^{A} - q_{i,t}^{A}\right)] \qquad \textbf{(viii)}$$

$$L = \sum_{i=1}^{N} K_i q_{i,t-1}^{A} - \sum_{i=1}^{N}\{K_i - A_i[\theta\left(q_{i,t-1}^{A} - q_{i,t}^{A}\right) + (1-\theta)(q_{t-1}^{ETF} - q_{t}^{ETF})w_i]\}q_{i,t}^{A} + \lambda[R -$$

$$\sum_{i=1}^{N}\{K_i - A_i[\theta\left(q_{i,t-1}^{A} - q_{i,t}^{A}\right) + (1-\theta)(q_{t-1}^{ETF} - q_{t}^{ETF})w_i]\}\,(q_{i,t-1}^{A} - q_{i,t}^{A})] \qquad \textbf{(ix)}$$

F.O.C.

$$\frac{\partial L}{\partial q_{i,t}^{A}}: -K_i(1-\lambda) + A_i\theta(1-2\lambda)q_{i,t-1}^{A} + A_i w_i(1-\theta)(1-\lambda)(q_{t-1}^{ETF} - q_{t}^{ETF}) -$$

$$2A_i\theta(1-\lambda)q_{i,t}^{A} = 0 \; \textbf{(x)}$$

By taking the first order conditions of equation **(ix)** and simplifying, we get the optimality condition for the active manager represented by equation (**x).** We can further rearrange **(x)** to obtain the optimal choice of $q_{i,t}^{A}$ by the active manager represented by the best response function **(xi)**. We can see from this equation that the active manager's optimal allocation depends on the choices of the passive manager through q_{t}^{ETF}.

$$q_{i,t}^{A} = \frac{-K_i}{2A_i\theta} + \frac{(1-2\lambda)q_{i,t-1}^{A}}{2(1-\lambda)} + \frac{w_i(1-\theta)}{2\theta}(q_{t-1}^{ETF} - q_t^{ETF}) \quad \textbf{(xi)}$$

Before we can draw conclusions about the active manager's rebalancing choice we will have to solve for q_t^{ETF} by working through the passive manager's optimization problem and substitute it into the response function **(xi)**.

VIII. Passive Fund Manager Optimization

The Passive Fund Manager minimizes their loss function **(xii)** subject to raising enough cash to meet their redemptions as shown in **(xiii)**. To ensure that the results of the optimization are not biased for or against the passive manager they face the same redemption constraint *R* as the active manager in equation **(vii)**.

$$\min_{q_t^{ETF}} \sum_{i=1}^{N} K_i q_{t-1}^{ETF} w_i - \sum_{i=1}^{N} P_i q_t^{ETF} w_i \quad \textbf{(xii)}$$

$$s.t. \sum_{i=1}^{N} P_i\,(q_{t-1}^{ETF} - q_t^{ETF})w_i \geq R \quad \textbf{(xiii)}$$

The primary difference between the active and passive investment manager's optimization is that the passive manager is unable to rebalance his portfolio to new holding weights in period *t*. The passive manager's choice is restricted to selecting the level of q_t^{ETF} they wish to hold because their investment mandate restricts them to mirroring the constituent weights of their stated index. We can define this relationship with equation **(xiv)** and **(xv).**

$$q_t^{ETF} = \sum_{i=1}^{N} q_{i,t}^{ETF} \quad \textbf{(xiv)}$$

$$\sum_{i=1}^{N} w_i q_t^{ETF} = q_t^{ETF} \quad \textbf{(xv)}$$

In order for the passive manager to meet redemptions and avoid violating its investment mandate it must choose how many units of the ETF or index portfolio it wishes to sell instead of rebalancing for each security. Using these restrictions we can set up the Lagrange **(xvi)** and model how the passive manager responds to redemptions.

$$L = \sum_{i=1}^{N} K_i q_{t-1}^{ETF} w_i - \sum_{i=1}^{N} P_i q_t^{ETF} w_i + \mu[R - \sum_{i=1}^{N} P_i \, (q_{t-1}^{ETF} - q_t^{ETF}) w_i] \quad \textbf{(xvi)}$$

By inserting equation **(v)** into **(xvi)** we get the detailed Lagrange **(xvii)** which includes the pricing process in the investment manger's decision.

$$L = \sum_{i=1}^{N} K_i q_{t-1}^{ETF} w_i - \sum_{i=1}^{N} \{K_i - A_i[\theta\big(q_{i,t-1}^{A} - q_{i,t}^{A}\big) + (1-\theta)(q_{t-1}^{ETF} - q_t^{ETF}) w_i]\} q_t^{ETF} w_i +$$

$$\mu[R - \sum_{i=1}^{N} \{K_i - A_i[\theta\big(q_{i,t-1}^{A} - q_{i,t}^{A}\big) + (1-\theta)(q_{t-1}^{ETF} - q_t^{ETF}) w_i]\} \, (q_{t-1}^{ETF} - q_t^{ETF}) w_i] \quad \textbf{(xvii)}$$

F.O.C.

$$\frac{\partial L}{\partial q_t^{ETF}}: - K_i w_i (1-\mu) + A_i w_i \theta (1-\mu)\big(q_{i,t-1}^{A} - q_{i,t}^{A}\big) + A_i w_i^2 (1-\theta)(1-2\mu) q_{t-1}^{ETF} =$$

$$2A_i w_i^2 (1-\theta)(1-\mu) q_t^{ETF} \quad \textbf{(xviii)}$$

By differentiating **(xvii)** with respect to q_t^{ETF} and simplifying the equation we obtain the optimality condition **(xviii)** for the passive manger's selection of q_t^{ETF}. We can further rearrange **(xviii)** to obtain the optimal choice of q_t^{ETF} by the passive manager represented by the best response function **(xix)**. We can see from this equation that the passive manager's optimal allocation depends on the choices of the active manager through $q_{i,t}^{A}$.

$$q_t^{ETF} = \frac{-K_i}{2A_i w_i (1-\theta)} + \frac{\theta\big(q_{i,t-1}^{A} - q_{i,t}^{A}\big)}{2w_i (1-\theta)} + \frac{(1-2\mu) q_{t-1}^{ETF}}{2(1-\mu)} \quad \textbf{(xix)}$$

IX. Best Response Functions for Active and Passive Mangers

If we substitute **(xix)** into **(xi)** we obtain the reduced form best response function **(xx)** for the active manager (unconstrained response) and by substituting **(xx)** into **(xix)** we obtain the reduced form best response function **(xxi)** for the passive manager (constrained response).

$$q_{i,t}^{A*} = \frac{-K_i}{3A_i\theta} + \frac{(1-3\lambda)}{3(1-\lambda)} q_{i,t-1}^{A} + \frac{w_i(1-\theta)}{\theta(1-\mu)} q_{t-1}^{ETF} \quad \textbf{(xx)}$$

$$q_t^{ETF*} = \frac{-K_i}{3A_i w_i(1-\theta)} + \frac{\theta}{3w_i(1-\theta)(1-\lambda)} q_{i,t-1}^{A} + \frac{(\mu)}{(\mu-1)} q_{t-1}^{ETF} \quad \textbf{(xxi)}$$

We can see that the quantity both the active and passive manager chooses to hold of security *i* in period *t* depends on the price sensitivity of the security (A_i), the price of the security before the crash (K_i), the index constituent weight of security *i* (w_i), the quantity of the security held in the previous period by the active manager ($q_{i,t-1}^{A}$) and the passive manager (q_{t-1}^{ETF}), the shadow prices (λ and μ), as well as the market share of active investment managers in the industry (θ).

X. Total Sales Pressure during Market Crash

We could analyze how these various factors affect the holdings choices of the managers; however, we are less concerned with the aggregate level of security holdings by the fund managers and we are much more interested in the amount of securities each manager must sell to meet redemptions. By substituting **(xx)** and **(xxi)** into **(ii)** we can establish the total sales pressure for each security in the market during the fire sale which is represented by equation **(xxii)**.

$$S_i^* \equiv \theta\left(S_i^{A*}\right) + (1-\theta)(S_i^{P*}) \ \textbf{(ii)}$$

$$S_i^* \equiv \theta\left(q_{i,t-1}^{A} - q_{i,t}^{A*}\right) + (1-\theta)(q_{t-1}^{ETF} - q_t^{ETF*})w_i$$

$$S_i^* \equiv q_{i,t-1}^{A} \frac{\theta(2w_i-1)}{3w_i(1-\lambda)} + q_{t-1}^{ETF} \frac{(1-\theta)(1-w_i)}{(1-\mu)} + \frac{K_i(1+w_i)}{3A_i w_i} \quad \textbf{(xxii)}$$

By differentiating **(xxii)** with respect to θ we can establish how changes to the market share of the representative active investment manager will affect the total sales of security *i* during the market crash. After differentiating with respect to θ and simplifying the expression we obtain the gradient **(xxiii)**.

$$\frac{\partial S_i^*}{\partial \theta}\colon q_{i,t-1}^{A}\frac{(2w_i-1)}{3w_i(1-\lambda)}+q_{t-1}^{ETF}\frac{(w_i-1)}{(1-\mu)} \qquad \textbf{(xxiii)}$$

The gradient represented by **(xxiii)** describes how a small increase in the market share of active investment managers will affect the total amount of security *i* sold at time *t* to cover redemptions. For us to determine the direction of the gradient we will examine each component individually and determine if this gradient is positively or negatively sloped.

First term in the gradient (xxiii)

Component	Positive/Negative	Reasoning
$q_{i,t-1}^{A}$	Positive	Security positions were restricted to non-negative values in assumption 5.
$(2w_i-1)$	Negative	In cases where index constituent weights (w_i) account for less than 50% of the index, the expression will be negative. Since indices typically do not have constituents that make up over half the index, we can safely assume this is a negative component.
$3w_i(1-\lambda)$	Positive	The value of the Lagrange multiplier at the solution of the problem is equal to the rate of change in the maximal value of the objective function as the constraint is relaxed.[5] As we relax the constraint, the maximal value of the Active Manager's loss function decreases as fewer securities need to be sold to cover redemptions. This indicates that our Lagrange multiplier is a negative term and $3w_i(1-\lambda)$ is a positive term.
$q_{i,t-1}^{A}\frac{(2w_i-1)}{3w_i(1-\lambda)}$	Negative	$\frac{(\boldsymbol{Positive})(\boldsymbol{Negative})}{\boldsymbol{Positive}}=\boldsymbol{Negative}$

[5] For a detailed discussion of Lagrange Multiplier Methods see Bertsekas (1996)

Second term in the gradient (xxiii)

Component	Positive/Negative	Reasoning
q_{t-1}^{ETF}	Positive	Security positions were restricted to non-negative values in assumption 5.
$(w_i - 1)$	Negative	Since security positions were restricted to non-negative values in assumption 5, it implies that $w_i \leq 1$
$(1 - \mu)$	Positive	The value of the Lagrange multiplier at the solution of the problem is equal to the rate of change in the maximal value of the objective function as the constraint is relaxed.[6] As we relax the constraint, the maximal value of the Passive Manager's loss function decreases as fewer securities need to be sold to cover redemptions. This indicates that our Lagrange multiplier is a negative term and $(1 - \mu)$ is a positive term.
$q_{t-1}^{ETF} \dfrac{(w_i - 1)}{(1 - \mu)}$	Negative	$\dfrac{(\boldsymbol{Positive})(\boldsymbol{Negative})}{\boldsymbol{Positive}} = \boldsymbol{Negative}$

$$\frac{\partial S_i^*}{\partial \theta} : q_{i,t-1}^{A} \frac{(2w_i - 1)}{3w_i(1-\lambda)} + q_{t-1}^{ETF} \frac{(w_i - 1)}{(1-\mu)} < 0 \quad \textbf{Result (A)}$$

From the above tables we can conclude that gradient **(xxiii)** is negatively sloped or as θ increases the total sales pressure during the fire sale decreases. We will define this gradient relationship as **Result (A).** What implications does this result have on our view of market panics? In an investment management industry with both active and passive money managers we can see that the total level of sales in a fire sale are increased as the percentage of passive investment strategies in that market increase. The amount that the sales increase in response to a decrease in active management in a system is dependent on the weightings of the underlying

[6] For a detailed discussion of Lagrange Multiplier Methods see Bertsekas (1996)

index which the passive investment managers follow as well as how strongly the redemption constraint binds.

To check the reasonability of the above relationship we must ask ourselves is **Result (A)** consistent with empirical research on the effects of passive investment strategies and ETFs on market volumes? Madhavan (2011) finds that during the flash crash on May 6th 2010 (which can certainly be defined as a market panic or fire sale), the largest proportion of equity transactions that were eventually cancelled were exchange traded products.

> Our analysis provides insight into why ETPs were differentially affected (ETPs accounted for 70% of equity transactions ultimately cancelled on May 6), even though ETP trading is less fragmented than that of other equities. For ETPs whose components are traded contemporaneously, widespread distortion of the prices of underlying basket securities prices can confound the arbitrage pricing mechanism for ETPs, thus delinking price from value.[7]

It appears that our first **Result** (**A**) is supported by the events of May 6th 2010, as a much larger percentage of sales occurred in the index linked section of the market. It is important to note that an event such as the flash crash is much more complicated than this theoretical model, and that there were certainly more factors at play on May 6th than can be modeled in this paper; however, the increased volume of trades on that day were stemming from index linked sources, which matches with what the model would expect. This indicates that the model may be correctly showing the intuition and direction behind passive manager's behavior during market crashes.

XI. Security *i* Price Change during Market Crash

[7] Madhavan 2011, pp. 2-3

The question of how passive investment strategies affect downside volatility during a crash is certainly of interest in our analysis, and by substituting the optimal value of S_i^*, into our pricing process **(i)** we obtain equation **(xxiv)** which depicts equilibrium price level based on the best response functions of the representative active and passive investment managers. By expanding equation **(xxiv)** using equation **(xxii)** we can see the detailed best response price **(xxv).**

$$P_i^* = K_i - A_i S_i^* \qquad \textbf{(xxiv)}$$

$$P_i^* = K_i - A_i[q_{i,t-1}^A \frac{\theta(2w_i-1)}{3w_i(1-\lambda)} + q_{t-1}^{ETF} \frac{(1-\theta)(1-w_i)}{(1-\mu)} + \frac{K_i(1+w_i)}{3A_i w_i}] \ \textbf{(xxv)}$$

We can easily see from **(xxv)** that the best response price depends on the index constituent weights as well as θ, but how do constituent security prices change with θ? By differentiating **(xxv)** with respect to θ we obtain the gradient of the best response price represented by **(xxvi)**.

$$\frac{\boldsymbol{\partial P_i^*}}{\boldsymbol{\partial \theta}}: A_i[q_{i,t-1}^A \frac{(1-2w_i)}{3w_i(1-\lambda)} + q_{t-1}^{ETF} \frac{(1-w_i)}{(1-\mu)}] \qquad \textbf{(xxvi)}$$

The gradient represented by **(xxvi)** describes how a small increase in the market share of active investment managers will affect the price of security *i* sold at time *t* to cover redemptions. For us to determine the direction of the gradient we will examine each component individually and determine if this gradient is positively or negatively sloped.

First term in the gradient (xxvi)

Component	Positive/Negative	Reasoning
$A_i q_{i,t-1}^A$	Positive	Security positions were restricted to non-negative values in assumption 5, and assumption 2 indicates that $A_i > 0$.
$(1 - 2w_i)$	Positive	In cases where index constituent weights (w_i) account for less than 50% of the index, the expression will be positive. Since indices typically do not have constituents that make up

		over half the index, we can safely assume this is a positive component.
$3w_i(1-\lambda)$	Positive	The value of the Lagrange multiplier at the solution of the problem is equal to the rate of change in the maximal value of the objective function as the constraint is relaxed.[8] As we relax the constraint, the maximal value of the Active Manager's loss function decreases as fewer securities need to be sold to cover redemptions. This indicates that our Lagrange multiplier is a negative term and $3w_i(1-\lambda)$ is a positive term.
$A_i q^A_{i,t-1} \frac{(1-2w_i)}{3w_i(1-\lambda)}$	Positive	All components of the first term are positive indicating that the whole term is positive.

Second term in the gradient (xxvi)

Component	Positive/Negative	Reasoning
$A_i q^{ETF}_{t-1}$	Positive	Security positions were restricted to non-negative values in assumption 5, and assumption 2 indicates that $A_i > 0$.
$(1-w_i)$	Positive	Since security positions were restricted to non-negative values in assumption 5, it implies that $w_i \leq 1$ preventing this term from being negative.
$(1-\mu)$	Positive	The value of the Lagrange multiplier at the solution of the problem is equal to the rate of change in the maximal value of the objective function as the constraint is relaxed.[9] As we relax the constraint, the maximal value of the Passive Manager's loss function decreases as fewer securities need to be sold to cover redemptions. This indicates that our Lagrange multiplier is a negative term and $(1-\mu)$ is a positive term.
$A_i q^{ETF}_{t-1} \frac{(1-w_i)}{(1-\mu)}$	Positive	All components of the second term are positive indicating that the whole term is positive.

$$\frac{\partial \boldsymbol{P}_i^*}{\partial \boldsymbol{\theta}}: A_i[q^A_{i,t-1} \frac{(1-2w_i)}{3w_i(1-\lambda)} + q^{ETF}_{t-1} \frac{(1-w_i)}{(1-\mu)}] > 0 \textbf{ Result (B)}$$

[8] For a detailed discussion of Lagrange Multiplier Methods see Bertsekas (1996)

[9] For a detailed discussion of Lagrange Multiplier Methods see Bertsekas (1996)

From the above tables we can conclude that gradient **(xxvi)** is positively sloped or as θ increases the sale price of security *i* during the fire sale increases. We will define this gradient relationship as **Result (B).** We can see from **Result (B)** that the best response price established by the investment managers' redemption activities increases as the share of active management increases and the price faced by the manager decreases as the percentage of passive investment's market share grows.

This result indicates that the total downside loss of security *i* for a given level of redemptions is decreasing with θ, and as the total market share of passive investment strategies increases we can expect the downside loss to grow. The rate at which the price decreases as the share of passive investments increases depends on the weights of the benchmark index, the relative price sensitivity of the security to sales pressure, as well as how strongly the redemption constraints bind for the passive and active managers.

Result (B) appears to indicate that passive investment strategies behave pro-cyclically with asset prices. Wurgler (2010) finds that passive investments have return chasing attributes and the way they allocate assets tends to build bubbles and further crashes through feedback loops.[10] This is the very definition of pro-cyclical activity. **Result** (**B**) supports his evidence on the topic and suggests that the trend towards passive investment strategies does carry some systemic risk. The result also suggests that index constituent weights play a role in price formation, which is not what we would expect if we had assumed securities are priced entirely off their intrinsic value. We can see from **(xxvi)** that a portion of the price formation is a result of money flows through the Lagrange multipliers and how passive instruments allocate assets.

[10] Wurgler (2010) pp. 9-11

In short, as θ increases investment managers need to sell fewer securities to meet their redemption needs and prices are less affected by redemption shocks to the market. Since we have defined *R* as the difference between the expected and actual future redemptions, we can see that passive investment strategies act to amplify the effect a set level of redemptions has on the price level of a market.

XII. Security *i* Total Return

We have established with **Result (B)** that as passive strategies increase in the model, the sales price of security *i*, decreases. We can easily transform this result into percentage gains/losses by dividing **(xxv)** through by K_i, the pre-panic price, subtracting 1 and multiplying by 100. This gives us equation **(xxvii)** which is the total return calculation for security *i*.

$$Total\ Return_i = 100x\left(\frac{P_i^*}{K_i} - 1\right)\% = 100x\left(-A_i\left[q_{i,t-1}^{A}\frac{\theta(2w_i-1)}{3K_iw_i(1-\lambda)} + q_{t-1}^{ETF}\frac{(1-\theta)(1-w_i)}{K_i(1-\mu)} + \frac{(1+w_i)}{3A_iw_i}\right]\right)\% \quad \textbf{(xxvii)}$$

By differentiating the total return with respect to θ we can find how the securities returns are affected by the structure of the investment industry.

$$\frac{\partial \boldsymbol{Total\ Return_i}}{\partial \boldsymbol{\theta}} : \frac{A_i}{K_i}\left[q_{i,t-1}^{A}\frac{(1-2w_i)}{3w_i(1-\lambda)} + q_{t-1}^{ETF}\frac{(1-w_i)}{(1-\mu)}\right] \qquad \textbf{(xxviii)}$$

As in **Result (B),** the Total Return gradient **(xxviii)** is positively sloped if our assumptions hold and it tells us how security *i*'s total return will change as the structure of the industry changes. The change in total return is dependent not only on the price sensitivity A_i, but also the investment manager's holdings in the previous period and the weights of the benchmark index.

XIII. Benchmark Index Total Return

Since all securities returns decrease as θ decreases, it follows that the benchmark index's return will similarly fall. The index total return is defined as the weighted average of the constituent total returns as shown in equation **(xxix).** The Benchmark Index's return is of primary importance to us because it is typically how individuals judge the severity of a market downturn. If the Index's total return is amplified by the growth of passive investing we will be able to draw conclusions about how the structure of the investment industry should affect our expectations about market crashes.

$$Index\ Total\ Return = \sum_{i=1}^{n} w_i Total\ Return_i \qquad \textbf{(xxix)}$$

By substituting in equation **(xxvii)** to **(xxix)** we establish the detailed Index Return equation **(xxx).**

$$Index\ Total\ Return = 100x \sum_{i=1}^{n} w_i \left(-A_i \left[q_{i,t-1}^{A} \frac{\theta(2w_i-1)}{3K_i w_i(1-\lambda)} + q_{t-1}^{ETF} \frac{(1-\theta)(1-w_i)}{K_i(1-\mu)} + \frac{(1+w_i)}{3A_i w_i}\right]\right)\%$$

(xxx)

To determine how the benchmark index responds to changes in the structure of the investment industry we can differentiate **(xxx)** with respect to θ giving us the gradient **(xxxi).**

$$\frac{\boldsymbol{\partial Index\ Total\ Return}}{\boldsymbol{\partial\theta}} : \sum_{\boldsymbol{i}}^{\boldsymbol{N}} \frac{A_i w_i}{K_i} [q_{i,t-1}^{A} \frac{(1-2w_i)}{3w_i(1-\lambda)} + q_{t-1}^{ETF} \frac{(1-w_i)}{(1-\mu)}] \qquad \textbf{(xxxi)}$$

Using the same logic as **Result (B),** we can see that the Index Total Return gradient increases as θ increases and decreases as $(1-\theta)$ increases. This provides us with our final result which is defined as **Result (C).**

$$\frac{\boldsymbol{\partial Index\ Total\ Return}}{\boldsymbol{\partial\theta}} : \sum_{\boldsymbol{i}}^{\boldsymbol{N}} \frac{A_i w_i}{K_i} [q_{i,t-1}^{A} \frac{(1-2w_i)}{3w_i(1-\lambda)} + q_{t-1}^{ETF} \frac{(1-w_i)}{(1-\mu)}] > 0 \qquad \textbf{Result (C)}$$

Result (C) indicates that as long as A_i through to A_N is uniformly positive, as in Assumption 4, the Total Return of the index during a market panic will fall as the percentage of active manager's in the market fall. This result carries with it the alarming implication that as the market share of passive investment managers increases in a financial system the size of the market crash increases. Passive investment vehicles are inherently tied to the total return of their underlying benchmark, and the fact that the more prevalent passive strategies are in the market the greater their downside risk during a market crash may cause us to pause and wonder about the risks associated with this type of investment strategy. While evaluating the expected value of both passive and active investment instruments, we can underestimate the downside risk associated with the down state if we avoid taking the structure of the investment market into account in our estimation.

XIV. Total Return Estimation Bias and Market Structure Assumptions

We can see through equation **(xxv)** and **(xxx)** that both the constituent security prices and the total return of the index are dependent on our measure of active management in the market through the θ term. If we take the expected values of the total return of either the index constituents or the entire index using the values of θ from the previous market crash we find that our expected returns become biased. We further define active managements share into the current market share of active managers as represented by θ^{True}, and the share of active management during the last market crash $\theta^{Previous}$.

When financial analysis is performed relying on historical values of previous market crashes to establish the maximal losses to an investment strategy during a fire sale, estimates of the potential fire-sale can be biased if the model does not account for the prevalence of both investment strategies in the current market. If a firm is attempting to estimate the maximum downside loss during a crash and does not account for the current market structure, they will introduce a bias to the prediction as seen in equation **(xxxii)**.

$$Index\ Total\ Return\ Estimation\ Bias = 100x\{\sum_{i=1}^{n} w_i \left(-A_i \left[q_{i,t-1}^{A} \frac{\theta^{Previous}(2w_i-1)}{3K_i w_i(1-\lambda)} + q_{t-1}^{ETF} \frac{(1-\theta^{Previous})(1-w_i)}{K_i(1-\mu)} + \frac{(1+w_i)}{3A_i w_i}\right]\right) - \sum_{i=1}^{n} w_i \left(-A_i \left[q_{i,t-1}^{A} \frac{\theta^{True}(2w_i-1)}{3K_i w_i(1-\lambda)} + q_{t-1}^{ETF} \frac{(1-\theta^{True})(1-w_i)}{K_i(1-\mu)} + \frac{(1+w_i)}{3A_i w_i}\right]\right)\}\ \% \quad \textbf{(xxxii)}$$

We can further simplify **(xxxiii)** to get **Result (D).**

$$Index\ Total\ Return\ Estimation\ Bias = 100x\left\{\sum_{i=1}^{n} w_i \left(-\frac{A_i}{K_i}\left[q_{i,t-1}^{A} \frac{\left(\theta^{Previous} - \theta^{True}\right)(2w_i - 1)}{3w_i(1-\lambda)} + q_{t-1}^{ETF} \frac{\left(\theta^{True} - \theta^{Previous}\right)(1-w_i)}{(1-\mu)}\right]\right)\right\}\% = 0$$

$$iff\ \ \theta^{True} - \theta^{Previous} = 0 \quad \textbf{Result (D)}$$

Result (D) indicates that if the percentage of passive investment strategies has changed since the previous market crash, using historical returns to estimate future returns will be biased through the $\theta^{True} - \theta^{Previous}$ term. What does this mean in practice? If a firm was to use the worst three month returns during the crash of 2008 to forecast the worst case scenario that their investments are exposed to and did not adjust for changes to the structure of the investment market, according to this simple model their appraisal of the risk of their investments will be overstated or understated depending on how the structure of the market has changed from the previous crash.

At the time of writing this paper passive investment strategies have been growing in market share[11] and based on **Result (D)**, models intended to forecast a worst case scenario are likely underestimating the risks to their investments. The underestimation of risk to a portfolio can make it difficult to correctly prepare for anticipated redemptions and many financial crashes have been a result of such underestimation; AIG clearly underestimated the risks to its CDS obligations and helped to plunge the world into a recession. Leveraged investment strategies which do not correctly evaluate the worst case scenario are prone to failure, and failing to recognize changes to the structure of the investment market can be disastrous to not only an individual firm, but to the Financial System as a whole.

XV. Summary of Results, Relaxing of Assumptions, and Implications

$$\frac{\partial \boldsymbol{S}_i^*}{\partial \boldsymbol{\theta}}: q_{i,t-1}^{A}\frac{(2w_i-1)}{3w_i(1-\lambda)} + q_{t-1}^{ETF}\frac{(w_i-1)}{(1-\mu)} < 0 \quad \textbf{Result (A)}$$

Based on the model and the ten assumptions we can see from **Result (A)** that as the percentage of active management in the market increases, the total quantity of security *i* sold during a market downturn decreases. This indicates that the volume of securities traded in the model increases as passive investment strategies become more popular. This implies that the optimization choices of passive strategies are acting pro-cyclically, and the solution to the active management optimization problem is acting counter-cyclically to the amount of securities traded in the model. Extending **Result (A)** into the real world, we would expect that as passive investment vehicles grow in popularity, market selloffs will become larger and a higher amount of securities will be sold to cover a set amount of redemptions.

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[11] *ICI Investment Company Total Net Assets by Type, Billions of dollars; year-end, 1996–2013. http://www.icifactbook.org/fb_ch1.html#investment*
Percentage of equity mutual funds' total net assets, 2000–2013. http://www.icifactbook.org/fb_ch2.html#index

$$\frac{\boldsymbol{\partial P_i^*}}{\boldsymbol{\partial \theta}}: A_i[q_{i,t-1}^A \frac{(1-2w_i)}{3w_i(1-\lambda)} + q_{t-1}^{ETF} \frac{(1-w_i)}{(1-\mu)}] > 0 \textbf{ Result (B)}$$

Result (B) shows that as the share of passive investment vehicles increases, the sales price of security *i* is expected to decrease given a set level of redemptions. If we relax assumptions two and four, allowing for $-A_i > 0$ and $S_i^A, S_i^P \leq 0$, and assume that instead of redemptions the investment managers face contributions, the model can depict the situation of positive price pressure and depict how passive and active investment strategies affect market prices during an upward trend in the market. It can be seen from **Result (B)** that in either case, active management's share of the market behaves counter-cyclically to the direction of the market, while the growth of passive management amplifies the direction of the price change and causes prices to behave pro-cyclically to redemptions and contributions.

$$\frac{\boldsymbol{\partial Index\ Total\ Return}}{\boldsymbol{\partial \theta}}: \sum_{\boldsymbol{i}}^{\boldsymbol{N}} \frac{A_i w_i}{K_i}[q_{i,t-1}^A \frac{(1-2w_i)}{3w_i(1-\lambda)} + q_{t-1}^{ETF} \frac{(1-w_i)}{(1-\mu)}] > 0 \qquad \textbf{Result (C)}$$

While **Result (B)** provides us with the intuition of how individual security prices respond to changes in the structure of the investment industry, it is also important for us to also look at how these changes affect the market as a whole. Using the total return of the benchmark index as a proxy for the performance of the overall market and differentiating with respect to θ, we arrived at **Result (C)** which indicates that an increase in the share of active management will increase the index total return during a time of redemptions, and conversely that an increase in the share of passive investment strategies will decrease the index total return. When we relax assumptions two, four, and allow for contributions instead of redemptions as in the above paragraph, we can see that during times of positive price pressure, all else equal, an increase in the percentage of passive investment strategies will increase the total return of the index, while an increase in the share of active management will decrease the total return of the index.

This confirms that the benchmark index total return is responding pro-cyclically to increases in passive management strategies and counter-cyclically to active management strategies in both up and down markets. We can see that **Result (B) and (C)** are moving in conjunction with each other, and that changes to the structure of the passive market is affecting the pricing behavior of both the index constituents as well as the index as a whole.

$$Index\ Total\ Return\ Estimation\ Bias$$

$$= 100x\left\{\sum_{i=1}^{n} w_i\left(-\frac{A_i}{K_i}\left[q_{i,t-1}^{A}\frac{\left(\theta^{Previous}-\theta^{True}\right)(2w_i-1)}{3w_i(1-\lambda)}\right.\right.\right.$$

$$\left.\left.\left.+\,q_{t-1}^{ETF}\frac{\left(\theta^{True}-\theta^{Previous}\right)(1-w_i)}{(1-\mu)}\right]\right)\right\}\%=0$$

$$iff\ \ \theta^{True}-\theta^{Previous}=0 \quad \textbf{Result (D)}$$

Result (D) is a simple but important result, since the model indicates that structure of the investment market appears to affect how trading volume, prices, and total return of both the market index and index constituents are established, estimating the expected return of an investment strategy without adjusting for changes to the market structure will produce biased results. If one wishes to accurately estimate the expected return of a security, using historical return values will either underestimate or overestimate the risks depending on how the market shares of the investment managers have changed. It is obvious that it is important to have accurate forecasts, and, if this model is correct, it is important to adjust one's expectations around increases or decreases in the market share of passive investment vehicles.

All of these results combined suggest that passive investments act to increase the cyclical nature of the market. Since redemptions can trigger decreases in prices which trigger further redemptions, the amplification effect of passive managers should be taken into account when establishing our expectations of redemptions as well as the maximum losses or gains associated with the purchase of a security. These results also illustrate that the constrained response of the ETF operators cannot produce greater maxima than that of the unconstrained response. Since the ETF operator is less flexible in its decision making than an active manager, we see that an increase in the percentage of constrained actors in the market produces more pro-cyclical behavior.

XVI. Conclusions

This result seems to confirm the concerns voiced by Madhavan (2011) and Wurgler (2011) that passive investment vehicles may carry systemic risk that is not being accounted for. Passive investments certainly offer advantages to the cost conscious investor; however, the fact ETF returns may be driving the returns of its basket of securities is concerning. The increased co-movement of stocks may be what is driving the decrease in mutual fund active share over the decades and it may be a symptom of securities being valued off of the average of an index instead of their fundamental value.

According to the model's results, we can expect financial markets to become increasingly pro-cyclical as passive investment strategies continue to gain popularity. Regulators and risk managers need to be aware of how the changes to the structure of the investment market will affect trade volume, security prices, and index returns in both bull and bear markets. Coming back to our quote from King Richard, the price of an asset is based on the environment in which it is being sold; for an investor to understand the risks of capital markets, it is imperative that they understand how the investment environment is evolving over time.

XVII. References

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