Cryptocurrency Market and Weak Form Efficiency

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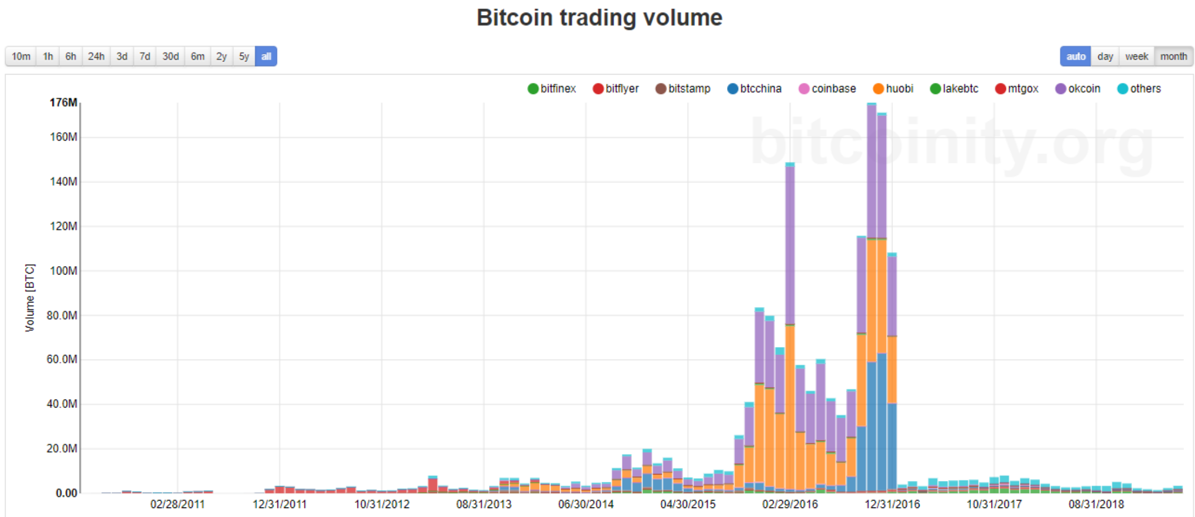
1. Introduction

Cryptocurrency is a digital coin that uses blockchain techniques to generate, transfer, store and verify currency; independent from government and central authorities. Bitcoin and Ethereum are considered a prime example. Identifying weak form efficiency of the cryptocurrency market is important because Cryptocurrency is an unprecedented form of financial asset. It does not have an underlying asset. It does not pay out dividends. Unlike conventional commodities, balance sheets and cash flow do not exist for the cryptocurrency market. Thus, Stock price fair value models such as dividend discount model and capital asset pricing model are inapplicable to estimate the fair value of cryptocurrency. Hence it is natural for investors to rely on fair value estimation of cryptocurrency based on historical return and events such as the listing of Bitcoin futures on Chicago Board Options Exchange (CBOE). It is important for investors to examine and understand whether historical price is useful for their strategy development. In this paper, I attempted to identify whether cryptocurrency price movement has any predictive power, based on Fama's (1970) weak form efficiency. The theory argues asset price movements are random, thus an investors' best prediction of a price of an asset at time t+1 is equal to the price of an asset at time t. The results indicate that cryptocurrency market, which is represented by Bitcoin and Ethereum are weak form efficient.

2. Cryptocurrency History

Following the inception of Bitcoin on January 3rd 2009, cryptocurrency market has evolved rapidly in different ways. The nationality of cryptocurrency holders have dramatically changed. The price of Bitcoin differed among different countries for various reasons at various times. Prior to 2014, cryptocurrency trading was mainly conducted in United States Dollar (USD) in exchange platforms based on the United States and Japan. In 2014, Mt. Gox, a Japanese cryptocurrency platform that was handling over 70% of the world trade volume at its pinnacle, was forced to liquidate firm assets after hackers stole $450 million-dollar worth of Bitcoin from the firm wallet. Not long after the catastrophic bankruptcy of Mt.Gox, haphazardly the Chinese economy started to crumble. China experienced 7.4% growth in 2014, missing its official target 7.5% for the first time in 24 years. Chinese Yuan (CNY) which enjoyed a long run appreciation over two decades, reverted its path to depreciation reflecting the economic trouble China was facing. In June 2015, the Shanghai Stock market crashed alongside with devalued housing prices in many cities of China. China reported 6.9% growth rate in 2015, which was the lowest reported figure in a quarter of a century. Local Chinese wanted to protect their assets and China based cryptocurrency platforms were ready to offer services to their locals in 2015. The Bitcoin trading volume chart displays the dominance of Chinese platforms, OKCoin, BTCChina and Huobi in Bitcoin trading volume from 2014 to 2016. In January 2017, People's Bank of China (PBOC) stepped up to regulate cryptocurrency trading. An article published



on January 18th 2017 by Reuters reports that although the PBOC did not formalize the illegality of cryptocurrency margin trading, after a constructive discussion with PBOC and the three biggest China based cryptocurrency platforms, the platforms decided to stop offering cryptocurrency margin trading. In February of 2017, under heavy pressure from the PBOC to raise commission fees for exchange, cryptocurrency firms increased their fees. Also, few firms started to halt their operation as a trading platform and offered mere wallet service for clients. In September 2017, the PBOC officially ruled cryptocurrency trading illegal. The regulation prohibited cryptocurrency trading platforms from converting fiat currency to cryptocurrency or vice-versa, purchasing or selling cryptocurrency and settling cryptocurrency price for clients. The regulation also banned cryptocurrency mining, which was another domain China showed its dominance in the cryptocurrency market. Simultaneous with Chinese investors ebbing away from the cryptocurrency market, Koreans surged in, consequently creating the term Kimchi Premium in the cryptocurrency market. Nonetheless, the phenomenon also evaporated quickly, due to Korean government regulations regarding foreign currency law. As of 2019, cryptocurrency currency trading is evenly distributed around the globe, which leaves investors with minimal national cryptocurrency market distortion.

Although the cryptocurrency market has attracted investors at an accelerated pace, investment characteristics that could assist investors in making sensible investment decisions were limited until recently. Bouoiyour and Selmi (2015) revealed the general public's interest on Bitcoin proxied by Google Search Index, which was the main driver of Bitcoin price. The Chinese market index, more specifically Shanghai market index, was identified as the second main driver of Bitcoin price. Chinese investors who were looking for an alternative investment other than the traditional Chinese financial market fled to the Bitcoin market, explaining the second market driver. The authors conclude that there is a large portion of unexplained movement of Bitcoin price, which can be attributed to speculation. Dyhrberg (2016) argued that Bitcoin hedges against financial times weighted average stock exchange index and the American dollar exhibiting similar behavior as gold. Bouri et al

(2017) shows the inverse relationship between Bitcoin price and Volatility Index (VIX) and argues Bitcoin had a safe haven property prior to 2013 crash but not after the post-crash. In the same vein as Bouri et al (2017), Demir et al (2018) use the same technique to investigate the relationship between Bitcoin and economic policy uncertainty index and conclude that economic uncertainty has a significant predictability of Bitcoin price and the two variables move inversely.

In a more general approach, this paper aims to distinguish the type of information that can be used to predict cryptocurrency price based on examining the weak form efficiency of Bitcoin and Ethereum. I chose to examine Bitcoin and Ethereum because these two assets are unquestionably leading commodities in the cryptocurrency market based on market cap and rich history compared to other cryptocurrency commodities. The only cryptocurrency assets that have over 10-billion-dollar market cap are Bitcoin and Ethereum.

Some pundits will argue XRP issued from Ripple corporation also exceeds 10-billion-dollar market cap. However, Ripple is a centralized coin that is mainly used for corporate transaction purposes. XRP is not listed in many of the trading platforms. Thus, there is a wide disagreement in the cryptocurrency community about XRP's identity as a cryptocurrency. Despite the fact the term cryptocurrency encompasses Bitcoin, Ethereum and many other coins, it is important to direct attention to its differences of origin, since the types of information that are useful for price formation might be different.

Bitcoin was created in 2009 by pseudonym Satoshi Nakamoto with the purpose of serving as medium of exchange like fiat currency. The currency has a maximum supply cap of 21 million Bitcoin; due to the limited supply, many people compared Bitcoin to gold, conferring the nickname digital gold to Bitcoin. Anyone can participate in Bitcoin mining and miners are rewarded Bitcoin for their contribution to the network. Bitcoin laid the foundation of the novel idea: decentralization of currency.

Ethereum was launched in 2015 by Ethereum Project Consortium led by Vitalik Buterin and it implemented many of the novel ideas which were first introduced by Satoshi Nakamoto in 2009. In parallel to Bitcoin, anyone can mine Ethereum and the miners are awarded with Ethereum for maintaining the network. Ethereum operates a decentralized community. Ethereum's transaction speed is double that of Bitcoin because of Ethereum's advanced technology, allowing the network to execute 15 trades per second. Nonetheless, the maximum supply of Ethereum is unlimited unlike Bitcoin. In addition to functioning as a medium of exchange, Ethereum allows smart contracts. Akin to regular contracts, two parties code an agreement and once the agreed conditions are executed the payment is automatically transferred to the service provider. Ethereum network offers third party surety service at minimal cost.

3. Literature Review

When examining the validity of information on future price and expected return forecasting, the efficient market hypothesis has been the theoretical ground for many scholars, practitioners and investors. Efficient market hypothesis (EMH) asserts financial products' current price reflects all available information if three conditions are satisfyingly met, (i) none or minimal transaction cost, (ii) none or minimal cost for acquiring information, and (iii) all market participants concur on the implication of new market information (Fama, 1970). Under this theory, investors are not able to generate risk-adjusted excess return. Investment professionals such as investment advisors and technical traders cannot yield abnormal excess return for their clients. Efficient market hypothesis is classified into three levels; weak, semi-strong, and strong. Weak form efficiency asserts, historical financial asset information is not helpful in predicting the future asset price. Hence technical analysis has no value in predicting the future asset price. Semi-strong efficiency asserts publicly available information is instantly reflected to the current price, thus it has no value in predicting future price. Strong form efficiency asserts that even private information such as insider information has no value in predicting the future price as market price adjustment is extremely frictionless and agile to any type of information. Efficient market hypothesis which encapsulates all three levels of efficiency, is an amalgamation of martingale hypothesis and random walk theory. (1) $P_t - P_s = \epsilon_t, t > s$, the development of price of an asset at period s to period t is random; independent, and identically distributed from the past information. In other words, the difference of prices of an asset at different time periods is not serially correlated (Malkiel, 1973). If (1) holds, then

$$\mathrm{E}(P_t - P_s | F_s) = 0,\ F_s = \{P_s, P_{s-1}, P_{s-2}..\},$$

future price movement forecast is theoretically not possible. The best prediction one can make about time t future asset price in time s is P_s. This paper focuses on weak form efficiency because cryptocurrency investors' general tendency is to heavily rely on past historical price and return information due to the limited types of information.

The ideal approach to address the efficiency of cryptocurrency market would be using martingale hypothesis testing developed by Philips and Jin (2014). The test is error free from the broad scope of time series data such as non-linear and non-martingale processes. However, as it will be discussed in the data section, the cryptocurrency dataset violates the first assumption of the test which is the convergence of conditional heteroskedasticity of the innovation in the asymptotic theory. Therefore, a time change approach, which would be discussed in the next section, would be needed for accurate statistics. Moreover, gaussian process without a correlated first differences occurs when and only the underlying process is martingale, which implies unit root that contains no correlation in the innovations is generally regarded as a martingale process (Park & Whang, 1999). Based on this definition, one can carefully conduct unit root test to argue for martingale process of the series. In fact, although martingale hypothesis testing/unit root hypothesis testing is different, due to the technical difficulties on examining a continuous time series and ascertaining the type of process of the time series, unit root tests have been the main statistical tool to examine weak form efficiency in the past literature.

Mananyi and Struthers examined Cocoa spot and futures price on London commodity market mainly using Augmented-Dickey-Fuller (ADF) and cointegration. They concluded both Cocoa spot and futures markets are not weak form efficient and suggested that arbitrage opportunity may exist (Mananyi & Struthers, 1997). Poshakwale tested the weak form efficiency of Bombay Stock Exchange using Serial Correlation Coefficients test and concluded the market is not weak form efficient. Investors should not rely on fair return for risk strategy (Poshakwale, 1996). In examining cryptocurrency market's efficiency, Bitcoin and Litecoin have been subject to scrutiny for weak form market efficiency in 2017. The authors used Augmented-Dickey-Fuller (ADF) test, Philips-Perron (PP) test, Dickey-Fuller GLS (DF-GLS) test, Kwiatkowski–Phillips–Schmidt–Shin (KPSS) test and auto-correlation statistics to argue both Bitcoin and Litecoin are not weak form efficient. Past information is a useful indicator to predict future cryptocurrency market prices (Latif et al, 2017).

Nonetheless, similar to martingale hypothesis testing, cryptocurrency market's unconventionally large volatility yields a large problem when calculating accurate t-statistics for unit root testing. If one thinks about how t-statistics is calculated, the problem of non-constant variance becomes obvious. The statistic is equal to $\frac{\beta_1-\beta_0}{\frac{Standard\ Deviation}{\sqrt{n}}}$. Therefore, if the variance is non-constant, the standard deviation will also not be constant, consequentially nullifying the statistics. The effect of time varying variance change and its effect on unit root tests has been investigated for a lengthy time. However, a simple convenient test that could encapsulate all types of variance change has not yet been developed. Himamori and Tokihisa (1997) concluded that standard unit root tests such as Augmented-Dickey-Fuller (ADF) are prone to reject unit roots as ratio of standard deviation enlarges and the change of variance occurs in the earlier period of the series. Kim et al (2002) concluded a large variance increase that leads to a significant innovation variance decrease weakens the precision of Augmented-Dickey-Fuller (ADF) test. Cavaliere (2005) provides a more widely applicable work as he does not limit the abrupt variance changes to a discrete number. He examines the sensitivity of unit root tests on time series including unconditional permanent variance. In summary, econometrics tools based on t-statistics such as Augmented-Dickey-Fuller (ADF) test are in danger of rejecting the null hypothesis; existence of unit root. The findings suggest changes in variance incline the chances of rejection of unit root. The conclusion strongly advocates the necessity of examination of variance changes in time series prior to conducting unit root tests. The authors who acknowledged the potential weakness of unit-root tests were forced to consider a more cautious approach when examining efficiency of cryptocurrencies. For example, Ciaian et al (2016) used Augmented-Dickey-Fuller (ADF) test, the Dickey-Fuller GLS (DF-GLS) test, the Zivot-Andrews (ZA) test and Clemente-Montanes-Reyes (CMR) test. The authors note that they are wary of credibility decrease of Augmented-Dickey-Fuller (ADF) and Dickey-Fuller GLS (DF-GLS) tests when exogenous shocks have a permanent effect on time series.

Even for researchers who are not researching efficiency of cryptocurrency, it has been a common practice to examine the unit root of the cryptocurrency series prior to delving into more complex problems to prevent spurious regressions. Van Wijk (2013) who stressed the significant effect of long term global economic variables such as stock market prices and oil prices on Bitcoin price development used Augmented-Dickey-Fuller (ADF) prior to constructing his Vector-Error-Correction Model (VECM). Baur et al (2018), who strongly criticized Dyhrberg (2016) for potential misspecification of GARCH model because of integrated order of one covariates in the model, also reported Augmented-Dickey-Fuller (ADF) for their covariate's stationarity testing. Nonetheless, the fact that unit-root tests are vulnerable to misreporting that a time series has no unit root under non-constant variance, raises eyebrows for many previously written papers on cryptocurrency. The past cryptocurrency literatures' heavy reliance on conventional t-statistics based unit root testing tools such as Augmented-Dickey-Fuller (ADF) is very alarming, as it could indicate many of the past researches could have been unjustified and inaccurate information could have been disseminated to the public.

4. Time Change Approach

As discussed earlier in the literature section, martingale hypothesis testing and unit root testing malfunctions in the presence of non-constant variance. In order to correct for non-constant second moment of the series, this paper takes the time change approach. In contrary to common time series plots, where observations are recorded at equally spaced time intervals, in this paper time change approach uses volatility as the chronometer. In high volatility periods, observations are more frequent while in low volatility periods, observations are seldom recorded. Intuitively, this will result in similar amount volatility between every observation in the series. The usage of this approach is limited to continuous martingale which has two presuppositions. First, all the past information that could influence the future price is already accounted in the price. Hence the best future prediction that an investor could assign to an asset price is equal to the current price, which in the long term makes abnormal return impossible. Second, the high frequency discrete data which almost replicates the continuous data can be used in deducing quadratic variation.

This is often modeled as such: $\mathrm{E}(Y_{t+h} - Y_t | \emptyset_t, \emptyset_{t-1}, \emptyset_{t-2} \dots) = 0$. Consider a regression model

$$Y_{t+1} - Y_t = (\alpha + \beta X_t)dt + \mu_{t+1} - \mu_t,$$

theoretically if X_t does not contain any useful information for predicting change of Y in the future then $\alpha = 0,\ \beta = 0$. This implies

$$Y_{t+1} - Y_t = \mu_{t+1} - \mu_t,$$

which suggests every information regarding the change of Y can be deduced from μ. A quadratic transformation of

$$Y_{t+1} - Y_t = \mu_{t+1} - \mu_t \text{ yields } \int_{t=0}^{z}(Y_{t+1} - Y_t)^2 = \int_{t=0}^{z}(\mu_{t+1} - \mu_t)^2,$$

the result can be redefined in a simpler form as $A_t = U_t$. If one constraints μ as a continuous martingale then the above equation can be re-written as $dA_t = dU_t = \sigma_t dB_t$, B_t is standard

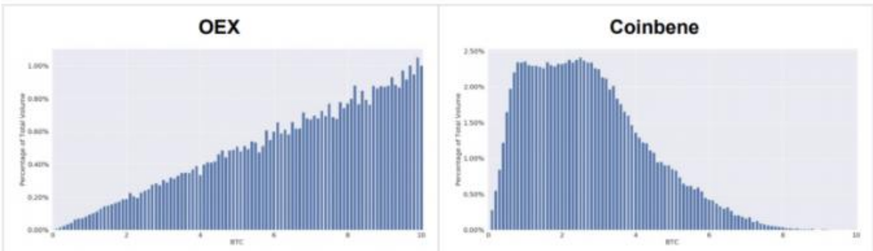
Brownian motion, using Dambis, Dubins and Schwarz Theorem (DDS). The theorem states for any continuous martingale Y, there exists a standard Brownian motion B such that $Y_{T_t} = B_t$, where T is a time change defined by $T_t = \inf\{z \geq 0 | Y_z > t\}$. Y_z represents quadratic transformed Y up to time z like the above example. $dA_t = dU_t = \sigma_t dB_t$, can be naturally re-written as

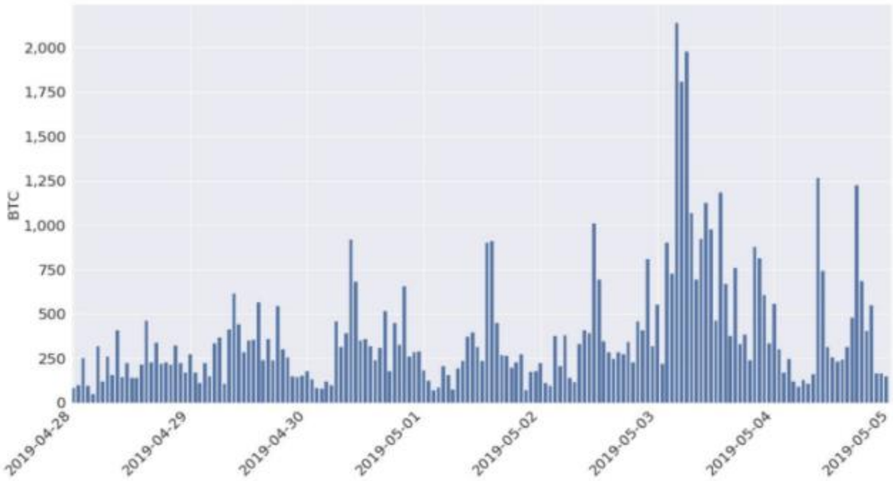
$$dA_t = dU_t = {\sigma_t}^2 d_t,$$

because the randomness of standard Brownian motion is still bounded by the set standard deviation (Choi et al, 2016). One can show through the transformation, second moment becomes constant and by the definition of standard Brownian motion, one can also show the transformed data is gaussian process (Yu & Phillips, 2001).

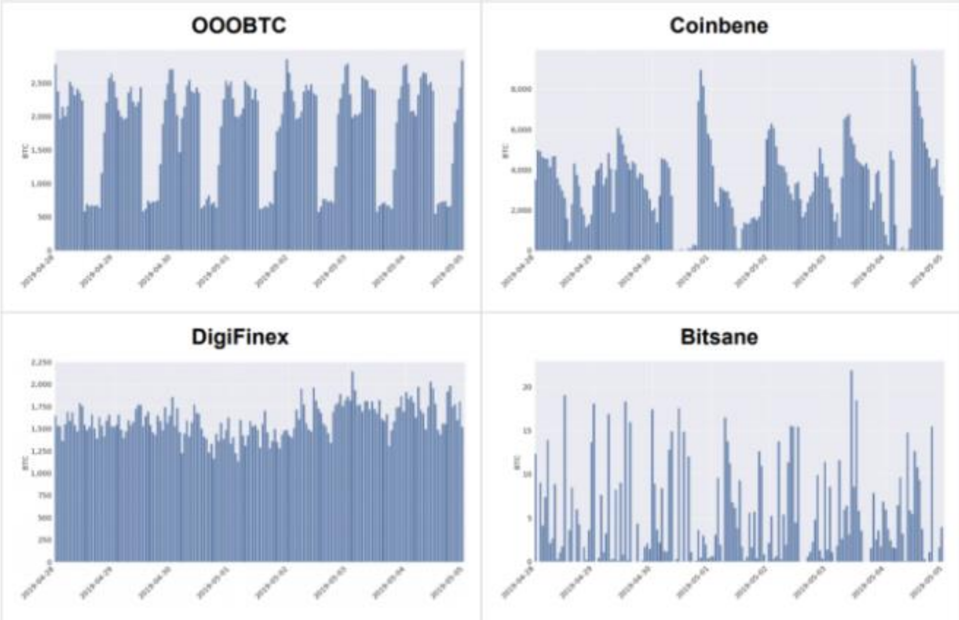
5. Data

Cryptocurrency price's and volume data's reliability has received enormous criticism from many professional investors and Bitwise's report to United States Securities and Exchange Commission (US-SEC) corroborates to the criticism. Bitwise wrote their report after investigating 83 cryptocurrency exchange platforms' trading behavior. It reports regulated exchanges producing true data, has a trading volume pattern that resembles Belford's graph with random spikes. Whereas non-regulated exchanges like OEX and Coinbene shows non-random trading volumes patterns. It is reasonable to believe that the cryptocurrency market, which consist~~s~~ of many small investors, should exhibit larger trade volumes with small amount of coins than large amount of coins. Nonetheless, if one looks at the trading volume chart created based on OEX, it displays rich investors traded large amounts of coin numerous times, which is unnatural.





Bitwise lists trade time graph as one of the tools to distinguish real exchange platforms from fake platforms. Unlike the graph above, where trade volumes are random and spikes occur during the weekends. The fake exchange platforms' trade volumes exhibit a non-random pattern.



Lastly, Bitwise points out the abnormal spread differences of the fake exchanges. The real exchanges had a median spread difference of $1.31 United States Dollar (USD), whereas the fake exchanges at the extreme displayed $700 USD. Bitwise lists two reasons for fake volume:

1. Higher trading volume leads to higher ranking on websites like Bitcoinity or CoinMarketCap, which are websites investors often refer to when they are looking for information about cryptocurrency. Therefore, companies that report fake trading volume will have more exposure to their consumers for free of charge.

2. It is easier to convince coin producers to conduct initial coin offering at their website by emphasizing the liquidity of the exchange platform (Wan, 2019).

Bitwise identified 10 exchange platforms conducting business with integrity which includes; Binance, Bitfinex, bitFlyer, Bitstamp, Bittrex, Coinbase Pro, Gemini, itBit, Kraken and Poloniex (Adeyanju, 2019).

To examine the efficient market hypothesis on Bitcoin and Ethereum, Gemini platform's per minute price data was obtained from Kaiko, a digital asset management firm in Paris, France. The per minute data sometimes isn't reported as no trade occurred. Unfortunately, due to lack of public interest in the first few years of Gemini's establishment, there are many gaps in the data. Therefore, I chose the period between January 1st 2017 and June 30th 2019 to examine the hypothesis.

Table 1. Summary of Variables

Variable	Observation	Min	Max
Bitcoin	992,837	752.01	19998.8
Ethereum	771,630	8.15	1420

Figure 1. Bitcoin and Ethereum Level-Price

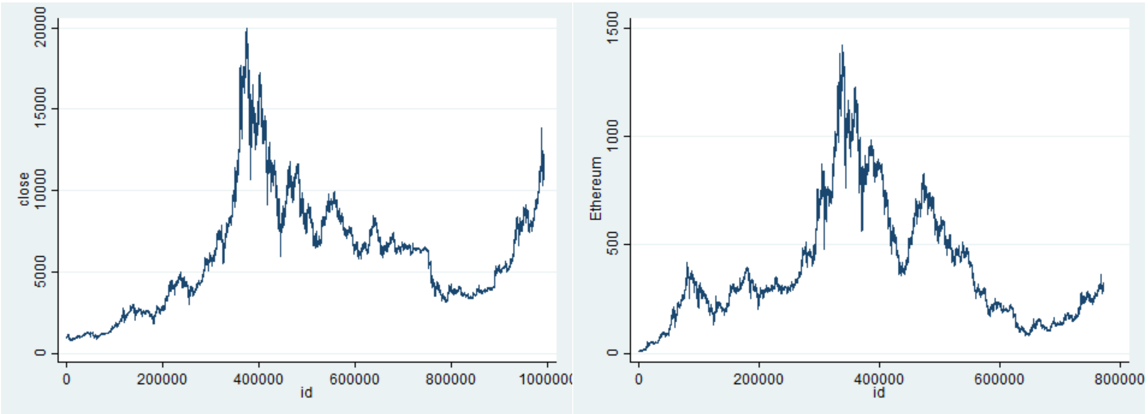
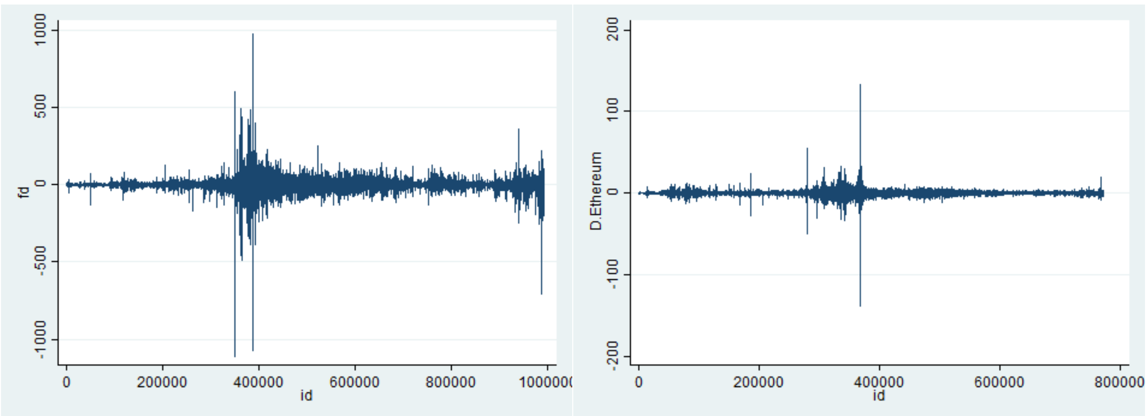


Figure 2. Bitcoin and Ethereum First Differenced Price



The key assumption for martingale hypothesis test is that innovations can be unconditionally/conditionally heteroskedastic nonetheless it requires convergence under the law of large numbers.

$$\frac{1}{n}\sum\nolimits_{t=1}^{n}\mathbb{E}(u_t^2|\mathcal{F}_{t-1}) \rightarrow \sigma^2 > 0$$

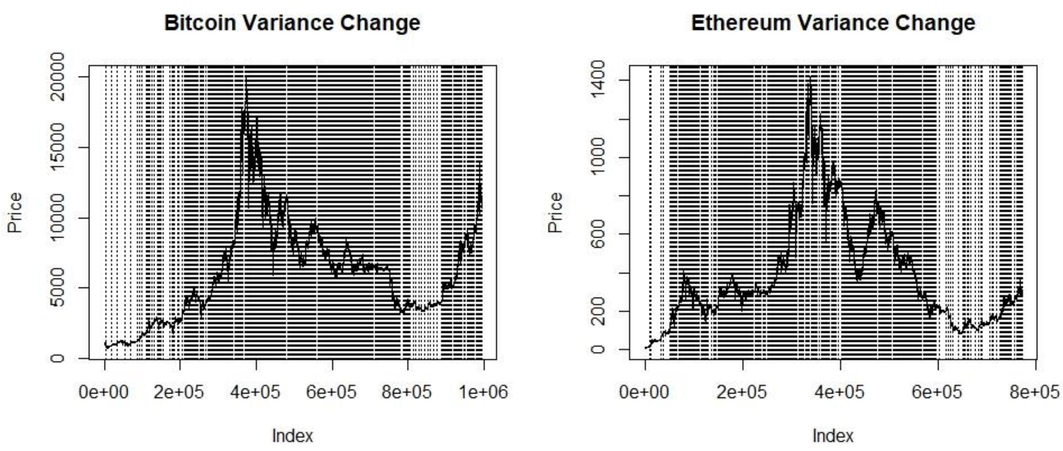
To examine heterogeneous variance, Engel's ARCH test was performed. Both Bitcoin and Ethereum was found to have conditional heteroskedastic variance. Pruned Exact Linear Time (PELT) function was used in software R to calculate the number of change points of variance on Bitcoin and Ethereum per minute price data. The function yielded 1228 change points for Bitcoin and 1003 change points for Ethereum. Vertical lines in below graph represents the variance change points. The graphs show numerous large abrupt changes followed by brief steady price movements.

Figure 3. ARCH Test

Variable	Bitcoin	Ethereum
Lag(1)	84514.368***	190559.440***
Lag(2)	84575.443***	235958.647***

***<1% **<5% *<10%

Figure 4. Bitcoin and Ethereum Variance Change Point



Although martingale hypothesis test developed by Phillips and Jin (2014) and/or Park and Whang (1999) works properly under conditional heteroskedastic variance, which is the case for our dataset, it does not work well with non-converging variance. I believe that the, cryptocurrency dataset clearly violates the convergence of conditional heteroskedastic variance of the innovation [first difference]. First differences of Bitcoin and Ethereum both exhibits explosive characteristics without any significant portents. The price movement and first difference of the price do not exhibit patterns. If the first differences do not exhibit a pattern of convergence, naturally, the variance which is the square of innovation will not converge. Therefore, one cannot use conventional martingale hypothesis testing tools on original cryptocurrency level data. Moreover, the heteroskedastic variance exhibited from cryptocurrency data informs us that the usage of t-statistics should be done with caution as potential distortion of t-statistic is very plausible with our dataset.

6. Methodology

Figure 5. Bitcoin and Ethereum Transformed Price Data

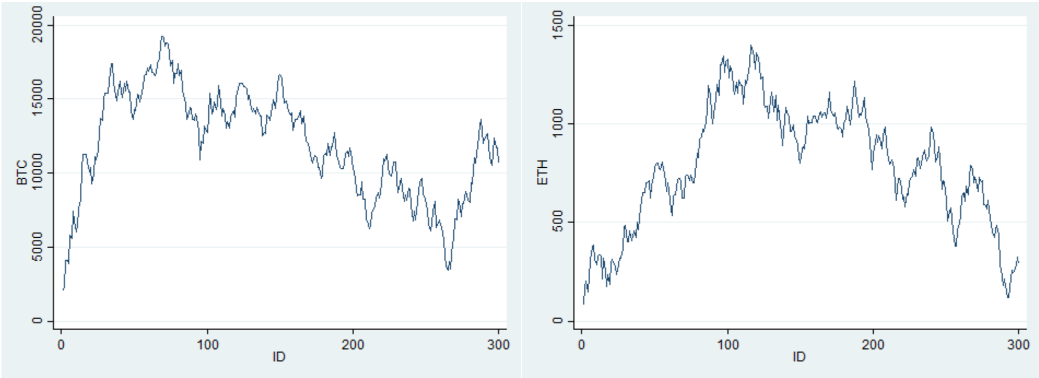
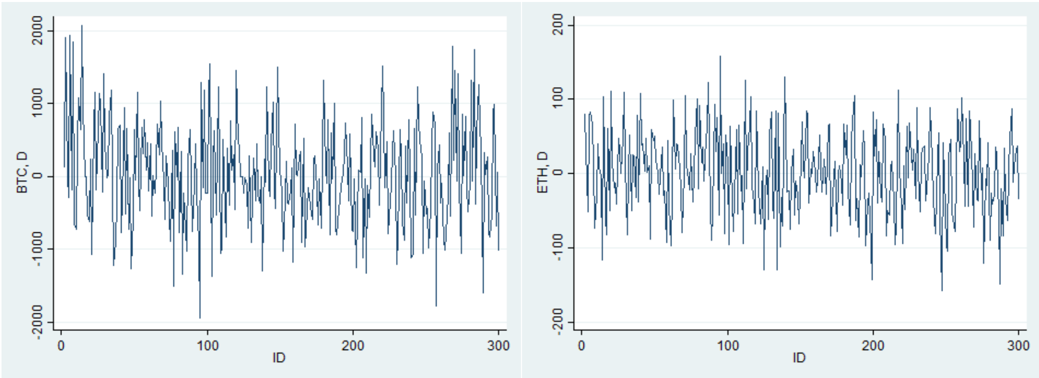


Figure 6. Bitcoin and Ethereum Transformed Price Data First Differenced



As a result of exponential volatility of cryptocurrency, in very volatile intervals the transformed dataset heavily relied on few observations. Therefore, the number of observations this paper could extract from the original level dataset was 300. The transformed data still captures the maximum price of the cryptocurrencies. However, compared to the original data the spikes in first differences are clearly smaller in magnitude and exhibits a pattern of second moment convergence. This conforms with the theory that the transformed data will be independently and identically distributed. Thus, one can rely on t-statistics obtained from the transformed data.

7. Parametric and Non-Parametric Unit Root Tests

Variable	ADF	ADF-Drift	DFGLS-Drift	Breitung's Unit Root Test- No Drift, No Trend
Bitcoin-lag(0)	-2.806*	-2.806***	-0.496	0.008831817
Bitcoin-lag(1)	-2.834*	-2.834***	-0.521	N/A
Bitcoin-lag(2)	-2.526	-2.526***	-0.563	N/A
Ethereum-lag(0)	-2.018	-2.018**	-0.751	0.01971046
Ethereum-lag(1)	-1.830	-1.830**	-0.722	N/A
Ethereum-lag(2)	-1.709	-1.709**	-0.681	N/A

***<1% **<5% *<10%

At first, like many other papers, Augmented-Dickey-Fuller (ADF) test and modified Dickey-Fuller t-test (DFGLS) were conducted. The two results conflicted and yielded results that are contrary to the theory. As written in the paper that introduced modified Dickey-Fuller t-test (DFGLS), 'Efficient Tests for an Autoregressive Unit Root', compared to Augmented-Dickey-Fuller, the modified Dickey-Fuller t-test has higher power and performs better in small sample size. Hence if the Augmented-Dickey-Fuller test was able to reject the null, the modified Dickey-Fuller t-test ought to reject the null as well. But this is not what is presented in the above table. The only plausible way that the system of tests could fail to accurately depict the presence of unit root with a contrary result like the above is when the model specification of the parametric unit root test is incorrectly specified.

Model misspecification is the problem in two ways. The parametric model assumes that the parametric equation can accurately capture the process. Nonetheless, due to unconventional price formation process of cryptocurrency, I believe this is not the case. Moreover, normally, without the time change the simplest unit root test (ADF) with drift can be represented as such:

$$\Delta y_t = \alpha + b y_{t-1} + u_t.$$

Nonetheless in case of time change, if we consider the primary ordinary least square (OLS) model as

$$y_t = \alpha + \rho y_{t-q} + u_t;$$

instead of q equal to 1, under time change q will vary based on the volatility level. Hence, α the drift term needs to be adjusted by the different time intervals such as:

$$\Delta y_t = (t - t_k)\alpha + b y_{t-k} + u_t.$$

This adjustment is currently unavailable through generic software. Therefore, Breitung's non-parametric Variance Ratio Unit Root test had to be employed to examine the presence of unit root in the cryptocurrency series under the assumption that the drift term equals to zero.

The variance unit root test is a ratio between square of cumulative sum of innovations and sum of square of innovations. The test can be denoted such:

$$\frac{T^{-2}\sum_{t=1}^{T} Y_t^2}{T^{-1}\sum_{t=1}^{T} y_t^2}; Y_t = y_1 + y_2 \dots y_t.$$

I would like to make a note that non-parametric unit root tests such as Breitung's variance ratio test is not a panacea. The adjustment on the drift term is still needed. However, if we assume that the drift term $\alpha = 0$, then the Breitung's ratio test statistic becomes reliable.

Breitung's paper shows in small sample properties (t=200), the variance ratio test has smaller type 1 error in all four non-linear models he tested, compared to Augmented-Dickey-Fuller (ADF). His paper also demonstrates variance ratio test has higher power compared to Augmented-Dickey-

Fuller (ADF) in two of the four non-linear models he has tested. As indicated in the above table, at this point the best logical reasoning one can form from the test results is that if one believes the drift term equals to zero both Bitcoin and Ethereum price processes contains unit root and it is weak form efficient (Breitung, 2002).

8. Conclusion

Cryptocurrency commodities' price formation does not exhibit any clear pattern. Although Engel's ARCH test rejects the null that variance is not conditionally heteroskedastic, because the variance does not show a clear path to convergence the usage of conventional martingale tests should be done carefully. Thus, this paper uses high-frequency cryptocurrency data to transform the time intervals to volatility intervals to obtain independently and identically distributed dataset where we could test t-statistics without distortion. Nonetheless, because parametric model is unable to encapsulate the cryptocurrency price formation process correctly and the time change approach distorts the parametric models, non-parametric tests with non-trivial adjustments on drift term and trend term are necessary. Hence to settle for the second best, I attempted to resolve the former issue by using Breitung's Variance Ratio Unit Root test. I find that both Bitcoin and Ethereum price formation process contains unit root under no drift term assumption, which indicates these two commodities are weak form efficient. The corroboration of parametric models such as Augmented-Dickey-Fuller (ADF) would have been more convincing in regard to the presence of unit root in the price formation process. Therefore, development of parametric and non-parametric models that could encapsulate time changed process seems to be an urgent matter. Moreover, it is worthwhile to note, the best approach to obtaining any statistics is to use the original raw data and time change approach was used in necessity to correct for potential distortion in statistics due to non-constant second moments. Due to the rapidly changing financial market which price formation processes are quite different to traditional financial assets, it seems to be an important task to develop statistical tools that can be used on non-constant variance price formations.

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