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Information Transparency of Firm Financing

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Abstract

We propose an information-based theory of capital structure to address the diversity of firm financing behavior and the variety of optimal financial contracts. Our model features nested information problems of adverse selection and agency cost. We prove that there exists a unique perfect Bayesian equilibrium with novel features: First, three types of optimal contracts arise endogenously, i.e., equity, transparent debt, and opaque debt. Equity and transparent debt are both informationally transparent because these contracts require firms to take on a costly technology for verifying types. Opaque debt, however, merely reflects the general information of firms seeking external funds. Any signaling contract that does not involve costly verification does not survive the equilibrium. Second, the unique equilibrium is either pooling on opaque debt, or mixing with transparent and opaque financing. Third, partial capital structure irrelevance exists in a mixing equilibrium. Fourth, debt weakly dominates equity for all firms that seek external financing. Finally, the optimal debt-to-equity ratio is unique for all firms in a pooling equilibrium, but only for a strict subset of firms in a mixing equilibrium.

JEL codes: D82, D86, G32.

Keywords: Capital Structure, Optimal Contracts, Asymmetric Information, Information Transparency

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1 Introduction

We propose a theory of capital structure to address the diversity of firm financing behavior, the variety of financial contracts and information transparency associated with each type of contracts. In reality, businesses have different ways to finance their operations, through internal funds, debt, external equity, or a combination of two or three aforementioned methods. Interestingly, different financing methods tend to offer different degrees of information transparency from an external investor's perspective. We define information transparency as whether investors require business information[1] that can be considered confidential to the firm, to aid their investment decisions. For example, equity investors often seek all kinds of commercial information regarding the firm, *e.g.* core technology, product lines, business outlook, *etc.* Debt can also be informationally transparent. For instance, credit-rating agencies examine firms in depth in order to rank the quality of their corporate bonds. Similarly, banks investigate firms' business backgrounds and financial conditions before approving commercial loans[2]. A salient feature of debt forms such as corporate bonds and commercial loans is that they typically have interest rates that are firm specific. In contrast, there are also forms that seems to be less informationally transparent. Business credit lines[3] and business credit cards[4], for example, have general qualification standards and charge interest rates that are common to users of the same credit service.

To understand these fascinating observations, we construct a theoretical model that combines adverse selection and agency cost in an environment of heterogeneous firms. With this model, we address three fundamental questions on corporate finance. First, what fac-

[1] Business information is all information, including commercial, financial, technical or other types, relating to the firm.

[2] To qualify for a business loan, the firm may be required to submit financial statements for past few years of business, recent business and personal tax returns and a business plan.

[3] Requirements for a business credit line may include business license, tax returns, recent months of bank statements, business bank account, standard financial documents such as cash flow, *etc.*

[4] Requirements for a business credit card may include information on industry type, business entity or legal structure, time in business and number of employees, tax returns, annual business revenue and estimated monthly spend, total annual income, and other information such as personal credit history.

tors drive a firm's optimal financing choice? Second, why do equity, transparent debt and opaque debt coexist as optimal financial contracts? Last but not least, what is the optimal debt-to-equity ratio of a firm?

In our model, there are *ex-ante* heterogeneous firms seeking external funding and a pool of investors providing funds. Firms are heterogeneous in terms of business quality and the amount of internal funds available. The business quality has two dimensions: the firm's productivity (*i.e.*, commercial information) and its survival rate (*i.e.* credit/financial information). Individual firms design optimal contracts to attract funding. All firms face a random output, which can be zero. Conditional on being strictly positive, output strictly increases in productivity given the investment level.

By design, our model has two layers of information problems. The first is an adverse-selection problem, which is associated with *ex-ante* private information of firm quality. There exists a technology that can ascertain the true firm quality (*i.e.*, both productivity and survival rate) at a cost. Firms may choose whether to pay this verification cost to have its quality revealed to investors. We deem this choice to be an important feature of firm financing. One could interpret this cost as the amount of time and effort a firm spends on trying to convince investors of its true value/potential as a business. Alternatively, it could be representative of the cost of hiring an agency for a credit rating. The second information problem is associated with the agency cost. Specifically, there is *ex-post* information asymmetry because a firm can hide from investors a fraction of the actual output. Thus it is also costly to induce truth-telling on this front.

Given the challenges posed by the two information problems, firms have three fundamental types of contracts to choose for external financing: either a separating contract, an *informationally-opaque* contract (opaque contract henceforth), or an *informationally-transparent* contract (transparent contract henceforth). By pursuing a separating contract, the firm does not take on the costly technology to help reveal its quality, but instead designs the contract to successfully signal its actual quality. With an opaque contract, a firm neither signals or costly reveals its type. Instead, the contract terms are based on investors' knowledge of the general characteristics of the set of firms using opaque contracts. This is essentially a pooling contract. Finally, with a transparent contract the firm uses the costly technology to reveal its type. Hence the terms of a transparent contract are based on the true quality of the firm.

We prove the existence of a unique perfect Bayesian equilibrium with several novel features: First, equity, transparent debt and opaque debt contracts arise endogenously as alternative methods of external financing. The optimal transparent contract is either an equity or a transparent-debt contract. The equity contract optimally entitles investors to a part of the output, hence ownership. The transparent-debt contract is one with a firm-specific interest rate that reflects the firm's survival rate. In contrast, the optimal opaque contract is associated with an interest rate common to all opaque-contract users. It is opaque debt because investors do not require information of specific firm quality. Instead, the interest rate of opaque debt depends on the lowest productivity and the expected survival rate of all firms that qualify for external financing.

Second, the separating contract does not survive in equilibrium. No firm, with the only possible exception of the lowest type, can successfully design a separating contract to signal its own type, without being copied by some other firms. Moreover, firms of the lowest type find themselves strictly better off with an opaque-debt contract because they can benefit from masking their inferior quality among all other (higher) types of firms who choose opaque debt.[5]

Third, the unique equilibrium is either pooling on opaque debt or mixing with both transparent and opaque contracts, but never pooling on transparent contracts. There exists a threshold on the cost of verifying quality, above which the equilibrium switches from mixing

[5] It is worth pointing out that we restrict our attention on parameters such that all projects in this environment are worth investing on and are better off with some external financing. Therefore, the term "the lowest quality" here does not literally refer to any conceivably inferior quality of a business.

to pooling as the cost rises.

Fourth, firm characteristics, such as internal funds, productivity and survival rate, drive its optimal financing choice. This optimal choice is diverse across firm-specific characteristics. All firms with sufficiently low expected productivity are strictly better off with opaque debt regardless of their levels of internal funds available at the time of financing. Moreover, conditional on its expected productivity being sufficiently high, a firm will only choose to use a transparent contract if it has an intermediate level of internal funds. Firms with lower-intermediate funds are indifferent between equity and transparent debt, and those with higher-intermediate funds are strictly better off with transparent debt. Firms with very low or very high funds are better off with opaque debt.

Fifth, there is partial capital structure irrelevance in that a strict subset of firms are indifferent between equity and transparent debt contracts in a mixing equilibrium. Moreover, there is no "one-size-fits-all" when it comes to the optimal debt-to-equity ratio for a firm. This ratio varies along the diverse firm characteristics. It is not a smooth function of a firm's internal funds and may not even be unique due to the indeterminacy for some firms.

Finally, debt weakly dominates equity because the former can implement the optimal contract for all firms, while the latter only does so for a strict subset of firms. Intuitively, an equity contract must be informationally transparent and thus, is costly to take on. Therefore, firms who can only afford, or only need, a relatively small amount of external funds are strictly better off with an opaque contract, which can only be implemented by opaque debt. However, firms who find equity optimal also find transparent debt equivalently optimal. Overall, debt is a strategy that weakly dominates equity.

We take the stance that each time a firm decides to adjust its capital structure is a reaction to new business conditions or opportunities that call for new financial arrangements. Our theory can help reconcile a variety of real-life observations on aggregate and individual firm-financing behaviors, for which Section 6 has a detailed discussion. Our paper provides an information-based theory that connects two strands of literature on capital structure:

First, our theory speaks to the seminal work of Modigliani and Miller (1958), which prompted the modern theoretical literature on capital structure. The capital structure irrelevance theorem proposed by Modigliani and Miller (1958) suggests that firms should be indifferent between financing through equity and financing through debt in a context with a perfect capital market, no bankruptcy cost or taxation, and a common interest rate for lenders and borrowers. In other words, there is no optimal debt-to-equity ratio. Hirshleifer (1966) uses the state-preference approach and concludes that when unrealistic conditions do not hold, there exists an optimal debt-to-equity ratio. Stiglitz (1969) extends the results found in Modigliani and Miller (1958) in a general-equilibrium framework, and shows that even when the lending rate is not equal to the borrowing rate and that bankruptcy is a possibility, the value of a levered firm will be the same as the value of an unlevered firm. Our theory proves theoretically that partial capital structure irrelevance exists in a mixing equilibrium. It is partial in the sense that only a strict subset of firms find equity and transparent debt equivalent in helping implement the optimal allocations.

Second, we prove that debt weakly dominates equity, which lends theoretical support to the pecking order hypothesis first suggested by Donaldson (1961) and then modified by Myers and Majluf (1984). In addition, our theory can reconcile the puzzling discrepancy in the two sets of empirical evidence presented respectively by Myers (2001) and Frank and Goyal (2003). In particular, Myers (2001) reports external financing, and especially equity finance, of much smaller magnitudes than the statistics from Frank and Goyal (2003). As far as we can tell, Myers (2001) collects observations from a wider range of U.S. firms while Frank and Goyal (2003) restrict attention to publicly traded ones. Our theory shows that the financing behavior of firms who find external equity optimal can appear very differently from that of firms of other business quality and internal finances, which provides an explanation for the discrepancy in the findings by Myers (2001) and Frank and Goyal (2003). Moreover, the quality-verification cost is likely to vary over time, from industry to industry, and from country to country, *etc.* Therefore, this may also contribute to the fact that firms documented in various datasets display somewhat different behavioral patterns when it comes to external financing.

In addition to capital structure, our theory complements the literature on optimal financial contracts. Models within this theoretical genre are generally characterized by some types of information problems, be it adverse selection, costly state verification or agency theory. For example, Leland and Pyle (1977) develop a model of adverse selection in which entrepreneurs sell shares to the public. Townsend (1979) offers a model of costly state verification that contributes to the understanding of closely held firms. Moreover, Diamond (1984), Gale and Hellwig (1985) and Williamson (1986) provide models of costly monitoring that have debt contracts emerge as the optimal contracts. Stiglitz and Weiss (1981) provide an important paper for the literatures on credit rationing and on adverse selection in capital structure. Myers and Majluf (1984) argue that there exists adverse selection between owners and investors. More papers, namely de Meza and Webb (1987), Noe (1988), Dybvig and Zender (1991), Viswanath (1993), Ravid and Spiegel (1997) and Halov and Heider (2011), advance studies on capital structure under adverse selection. Jensen and Meckling (1976), Morrelec (2004) and Atkeson and Cole (2005) use agency theory to explain the capital structure of firms. DeMarzo and Sannikov (2006) and DeMarzo and Fishman (2007) use dynamic agency models to show that the optimal contract can be implemented by a combination of long-term debt, a line of credit and equity.

In contrast to all of the papers mentioned above, our approach nests two types of information problems, namely adverse selection and agency problem, in a model of heterogeneous firms. Combining two information problems allows us to prove that signaling with a separating contract does not work in this environment and that a firm must rely on a transparent contract to help verify its type, if transparency is desired for overcoming adverse selection. In our model, an equity contract and two types of debt contracts, respectively transparent and opaque, can coexist in equilibrium. An equity contract is a form driven by the binding constraint for a truthful report of output, *i.e.*, the agency problem. A debt contract takes its form from a binding participation constraint of investors. Finally, by considering heterogeneity along both firm quality and internal finances, our theory formalizes the results that debt weakly dominates equity, and that the optimal debt-to-equity ratio is a complex function of firm characteristics and may not be unique for some firms.

Finally, our paper also contributes to the literature regarding the optimal control of information, which was pioneered by Hirshleifer (1971) and Hirshleifer (1972). Some explain why it may be optimal for firms or banks to withhold information, e.g., Hirshleifer (1971), Hirshleifer (1972), Dang, Gorton, Holmström, and Ordoñez (2017) and Monnet and Quintin (2017), etc. Some demonstrate that firms may find it optimal to disclose information, e.g., Diamond (1985), etc. Others show that information can be ranked by favorableness and that the highest types may be willing to pay to separate from the low ones through quality certification, e.g., Viscusi (1978), Grossman (1981) and Milgrom (1981), etc.

Our work incorporates the decision of whether to release information into the optimal design of financial contracts. With this, we show that the optimal contracts vary in terms of information disclosure. The equity contract is optimally associated with making information transparent. The debt contract, however, can be divided into two categories, one with optimal release of information and the other optimally making information opaque. Moreover, we study the contracting decision in an equilibrium context with heterogeneous firms. With this, we identify what characteristics of a firm will lead to the respective contract choices, as well as the conditions given which the equilibrium has all three varieties.

The rest of the paper is organized as follows. Section 2 presents our model. Section 3 analyzes the optimal opaque, separating and transparent contracts. Section 4 details how firms choose the optimal capital structure. Section 5 proves existence and uniqueness of the perfect Bayesian equilibrium, and then characterizes the equilibrium. Section 6 provides an in-depth discussion of firm financing behavior both in cross sections and individual evolutions over time. Finally, Section 7 concludes the paper.

2 Model

The economy has a continuum of firms, each endowed with internal funds n and a technology called a *project*. The project is characterized by its productivity, θ, and survival/success rate, α. All of these firm characteristics, (n, α, θ), are *i.i.d.* with respective supports, $[0, \bar{\imath}]$, $[\underline{\alpha}, \bar{\alpha}]$ and $\left[\underline{\theta}, \bar{\theta}\right]$. With an investment i, the output of a project is given by

$$y(i) = \begin{cases} \theta \min[i, \bar{\imath}], & \text{w.p. } \alpha \\ 0, & \text{w.p. } 1-\alpha, \end{cases} \tag{1}$$

where $\bar{\imath} > 0$ is a parameter common to all projects. The economy is also populated by a continuum of risk neutral investors. All firms and investors have access to a storage technology with an exogenous net rate of return $r > 0$.

A firm's level of internal funds, *i.e.* net worth, is publicly known. However, there are two layers of private information regarding a firm's characteristics. The first one is the realization of (α, θ). These two statistics together represent the quality of the firm's investment opportunity. There exists a verification technology[6] that is available to all to help detect the true type of a firm (α, θ) at a cost of $\gamma > 0$. The second layer of private information is on the realization of the project outcome $y(i)$. Verifying the true output is costless for everyone, although such an effort can only help recover a fraction up to $\sigma \in (0, 1)$ of any hidden output.

Given this environment, obviously all firms with $n < \bar{\imath}$ potentially have the incentive to obtain funding externally. Nevertheless, any contract offered by a firm to investors must address the issue of private information over both the idiosyncratic project quality and the stochastic output. Note that any verification of quality is only meaningful if it takes place *prior to* investors signing the contract. This way the contract terms can be based on the true

[6] One can think of this technology as a third-party, *e.g.*, a rating agency, a mutual fund, or a security analyst, etc., to whom investors can delegate the role of quality verification, to avoid duplication of such efforts. Alternatively, this technology can be interpreted as the fund-seeking firm's own effort in trying to convince potential investors of its true quality. For example, firms can host information sessions for interested investors to explain the business idea, core technology, expertise of its R&D team, *etc.*

project quality. Otherwise, there is no need for investors to know the true quality because they can simply wait and later verify the actual output if necessary. Therefore, a firm seeking external funds must first choose from the following three contract types: an opaque contract, a separating contract or a transparent contract. The former two do not involve information acquisition prior to production while the latter does. The opaque contract is essentially a pooling contract, with which the investors do not know the exact quality of firms using this type of contract. Thus all terms of the contract, such as the size of external funding and the associated returns to funding, etc., are independent of the firm's actual realization of (α, θ). The separating contract is one with which the firm signals its exact quality, (α, θ), without taking on the verification technology. By offering a transparent contract, however, the firm pays the cost γ to have the true quality, (α, θ), revealed to investors. Accordingly, the contract terms are made based on the firm's true level of α and θ. Let $\Omega \subseteq [\underline{\alpha}, \bar{\alpha}] \times \left[\underline{\theta}, \bar{\theta}\right]$ denote the set of quality measures of firms with funds n who choose an opaque contract for external financing. We consider the following signaling game between firms and investors:

Step 1. Given a prior of Ω and its own characteristics of (n, α, θ),[7] each firm chooses its optimal capital structure. If external financing is involved, the firm also decides on which type of contract to offer.

Step 2. After observing all contracts offered by firms, investors form the posterior belief of Ω and choose which contract to accept.

To choose its optimal capital structure, a firm first calculates the respective expected maximized profits of the following four financing options: (i) external funding with an opaque

[7] In our model, each firm has one project to operate. Thus the concept of "firm" and "project" can be used interchangeably. In reality, though, a firm may have multiple business projects going on at the same time, or may need to raise funding once in a while to finance a new project or a new step of an existing project. The financing decision of a firm in our environment can probably be best interpreted as such a problem faced by a firm every time it attempts to finance a new project in reality. The main status of the firm at the time of raising funds can be summarized by the amount of internal funds available, and the quality of the new project as in the expected productivity and the expected success rate. These are the exact key characteristics we had in mind in designing this model.

contract, (ii) external funding with a separating contract, (iii) external funding with a transparent contract, and (iv) self-funding. Finally, the firm selects the financing method with the highest expected profit. For the first three that involve external financing, the firm must respectively design the optimal contract to attract investors while maximizing its own expected profit.

We define the perfect Bayesian equilibrium as one in which the prior and the posterior of Ω coincide for all $n \in [0, \bar{\imath}]$. That is, in equilibrium the optimal choices of firms and investors are consistent with Ω. Let $j = O, S, T, SF$ be the index of a firm's funding method, which respectively represents an opaque contract, a separating contract, a transparent contract and self-funding. We maintain the following assumption throughout the rest of the paper:

Assumption 1. $\underline{\alpha}\ \underline{\theta} > 1 + r > \sigma\bar{\alpha}\bar{\theta}$.

Later it will become clear that the first inequality ensures that all projects are worth investing (as opposed to the storage technology), and the second one guarantees that all firms find it optimal to use (some kind of) external financing. We can interpret this assumption as the *qualification standards* for external financing. Any firm/project that does not meet the assumed conditions is either of inferior quality or is strictly better off with internal funding only.

3 Optimal Financial Contracts

The analysis in this section is conditional on that a firm has decided to raise external financing for its project. First, we introduce the following definition of a contract:

Definition 1. A contract between a firm and external investors[8] is given by (i^j, z^j), where i^j is the total investment in the project with method $j = O, S, T$, and z^j is the payout

[8] Note that here it does not matter whether a firm's contract is offered to one or multiple investors. First, in equilibrium all contracts offer the same expected rate of return. Second, verifying project output entails no cost. For these reasons, there is no complication (as in conflicts of interest, for example) if more than one investor agrees to invest in a particular project.

to investors upon a successful outcome. The firm must have its quality revealed via the verification technology, when offering a contract with $j = T$. If the firm reports zero output, it will be audited by investors. If zero output is confirmed, then the firm is free of any liabilities. Otherwise, the investors will forfeit a fraction $\sigma \in (0, 1)$ of the hidden output, where σ is exogenous.[9] Given the contract, the amount of external funds from investors is $i^j - n$. $\square$

3.1 Transparent Contract

Consider a firm who uses a transparent contract for external financing. In this case, the firm pays the cost γ to have the true quality of its project revealed.[10] Thus any external investor will have perfect information about this firm's realizations of (α, θ). Taking (n, α, θ, r) as given, the firm seeks to maximize its own expected profit from a transparent contract:

$$
\begin{aligned}
\Pi^T(n, \alpha, \theta) \quad &= \quad \max_{(i^T \leq \bar{\imath},\ z^T)} \left[\alpha \left(\theta i^T - z^T \right) - \gamma - (1 + r) n \right] \\
s.t. \qquad & \theta i^T - z^T \geq (1 - \sigma) \theta i^T \qquad (2) \\
& \alpha z^T \geq (1 + r) \left(i^T - n \right). \qquad (3)
\end{aligned}
$$

The objective function is the expected revenue less the expected payout to investors, the cost of revealing true quality (α, θ), and the opportunity cost of investing n in the storage technology. Thus a necessary condition for the firm to use a transparent contract is $\Pi^T(n, \alpha, \theta) \geq 0$ because this implies financing with this contract dominates taking on the outside option, the storage technology. The above problem is subject to the incentive compatibility constraint of the firm (2), and the participation constraint of investors (3). The former ensures that the firm has the incentive to truthfully report a successful project outcome. The left-hand side

[9] Essentially here we impose the most severe penalty available in this environment for misrepresentation, in order to keep incentive compatibility the least costly for any contract.

[10] Alternatively, one can also interpret it as the investors share the cost γ for the third-party to verify the true quality. In an equilibrium where some transparent contracts are accepted by investors, it must be the case that those firms compensate the investors fairly for this cost of information acquisition. In other words, this alternative case would be equivalent to having firms directly take up on the cost γ.

of (2) is the firm's payoff after making the payout to investors according to the contract. The right-hand side is the firm's payoff from falsely claiming zero output and thus triggering verification, in which case the firm will lose a fraction σ of total output. Condition (3) requires that the expected net rate of return to investors must be at least as high as that of the storage technology, r. Solving the above contracting problem leads to the following proposition:

Proposition 1. *Given Assumption 1, the optimal transparent contract* $\left(i^T, z^T\right)$ *is*

$$i^T(n,\alpha,\theta) = \begin{cases} \frac{1+r}{1+r-\sigma\alpha\theta} n, \text{for } n \leq n^T(\alpha\theta) \\ \bar{\imath}, \qquad \text{for } n > n^T(\alpha\theta) \end{cases} \tag{4}$$

$$z^T(n,\alpha,\theta) = \begin{cases} \frac{\sigma\theta(1+r)}{1+r-\sigma\alpha\theta} n, \quad \text{for } n \leq n^T(\alpha\theta) \\ \frac{1+r}{\alpha}(\bar{\imath}-n), \text{for } n > n^T(\alpha\theta) \end{cases} \tag{5}$$

where the threshold of a non-binding incentive compatibility constraint is defined as

$$n^T(\alpha\theta) \equiv \bar{\imath}\left(1 - \frac{\sigma\alpha\theta}{1+r}\right) < \bar{\imath}. \tag{6}$$

A firm's maximized expected profit associated with this contract is given by

$$\Pi^T(n,\alpha,\theta) = \begin{cases} [\alpha\theta - (1+r)] \frac{1+r}{1+r-\sigma\alpha\theta} n - \gamma, \text{for } n \leq n^T(\alpha\theta) \\ [\alpha\theta - (1+r)]\,\bar{\imath} - \gamma, \qquad \text{for } n > n^T(\alpha\theta) \end{cases} \tag{7}$$

which strictly increases in $\alpha\theta$.

All proofs, unless otherwise stated, can be found in the Appendix. An interesting observation from (4) and (5) is that the optimal transparent contract $\left(i^T, z^T\right)$ depends on both α and θ for $n \leq n^T$, yet only on α for $n > n^T$. Based on this observation, we have the next proposition to further characterize the optimal transparent contract. All results of Proposition 2 follow directly from Proposition 1. Given $n \leq n^T(\alpha\theta)$, the first equality in (8) is the binding incentive compatibility constraint and the second the binding participation constraint. The rest of the proposition is straightforward.

Proposition 2. *Provided that* $n \leq n^T(\alpha\theta)$*, the firm's incentive compatibility constraint binds. The payout to external investors can be expressed by*

$$z^T(n,\alpha,\theta) = \sigma \cdot \left[\theta i^T(n,\alpha,\theta)\right] = \frac{1+r}{\alpha} \cdot \left[i^T(n,\alpha,\theta) - n\right]. \tag{8}$$

This means the optimal transparent contract can be implemented by two types of contracts: either an equity contract with the external investors receiving a fraction of the actual output, or a transparent-debt contract with the payout to investors being a firm-specific interest rate, $(1+r)/\alpha$*, times total external investment. Moreover, the amount of external investment is given by*

$$i^T(n,\alpha,\theta) - n = \frac{\sigma\alpha\theta}{1+r-\sigma\alpha\theta} n,$$

which strictly increases in expected productivity $\alpha\theta$*. Provided that* $n > n^T(\alpha\theta)$*, the firm's incentive compatibility constraint does not bind, and the optimal transparent contract can be implemented by a transparent-debt contract with the payout to investors being a firm-specific interest rate,* $(1+r)/\alpha$*, times total external investment, i.e.,*

$$z^T(n,\alpha) = \frac{1+r}{\alpha} \cdot (\bar{\imath} - n).$$

It is important to understand what gives rise to the above proposition. When $n \leq n^T(\alpha\theta)$, the contract $\left(i^T, z^T\right)$ is solved by two binding constraints, (2) and (3). Note that (2) is directly affected by θ and (3) by α. With this, the firm willfully reveals both types of information in order to attract funding. Clearly this is a case where the firm's internal funds are far away from the maximum investment $\bar{\imath}$, and thus the firm seeks external funds to the extent that the incentive compatibility constraint binds. The equity contract (the first equality in [8]) is derived from expressing z^T by the binding (2), and the debt contract (the second equality in [8]) from deriving z^T given the binding (3). The two implementations are equivalent for firms with $n \leq n^T(\alpha\theta)$ and deliver the same outcome, $\left(i^T, z^T\right)$ as specified by (4) and (5).

When $n > n^T(\alpha\theta)$, the contract $\left(i^T, z^T\right)$ is solved by two binding constraints, (3) and

$i^T = \bar{\imath}$. In this case, the firm itself invests a sufficiently high amount of funds into the project (relative to external funds) such that incentive compatibility is not an issue. This is why θ does not matter here. The optimal contract being transparent debt is driven by condition (3). With this, the investors require information of the firm's credit risk α in order to determine the interest rate.

3.1.1 Equity *vs.* Transparent Debt

Proposition 2 sheds light on the nature of financing with a transparent contract: First, both equity and transparent debt are informationally-transparent contracts because they reveal true quality (α, θ) of a firm. Conditional on the internal funds n, better-quality firms are more likely to use transparent financing. The amount of external funds raised by a transparent contract strictly increases in expected productivity $\alpha\theta$, for any given $n \leq n^T(\alpha\theta)$. This suggests that better-quality yet funds-constrained firms are more likely to benefit from using a transparent contract.

Second, investors value different types of information under different types of contracts. Recall (8). For the same given amount of investment $(i^T - n)$, the (successful) payout to investors z^T depends on θ with equity and α with transparent debt. In other words, conditional on firm survival, equity investors care about θ but debt investors focus on α. These results are consistent with real-life observations. To acquire shares, investors are sensitive about all kinds of commercial information regarding the firm, *e.g.* core technology, product lines, business outlook, *etc.*, all of which contribute to θ. In contrast, firms that seek to initiate corporate bonds or obtain a business loan are typically under scrutiny over their financial situations (more so than commercial information) by either credit-rating agencies or banks. Such investigation provides information on the firm's creditworthiness, essentially reflecting α. Moreover, corporate bonds and business loans generally have firm-specific interest rates, which is consistent with our finding that transparent debt has a firm-specific interest rate reflecting α.

Third, debt is more versatile (as in more widely used) than equity. That is, only firms with $n \leq n^T(\alpha\theta)$ may find it optimal to use equity financing, but debt can be optimal for a firm regardless of n. Proposition 9 in Section 5 will formally establish the generality of debt relative to equity.

Next, the result that the optimal transparent contract can be implemented by either equity or transparent debt, is a manifestation of the Modigliani-Miller Theorem. Nevertheless, we obtain this result in an environment with asymmetric information. Moreover, we show that only a strict subset of firm may find themselves indifferent between debt and equity financing. Later Theorem 1 will identify these firms as having quality (α, θ) satisfying condition (27) and funds satisfying $n \in \left[n_1(\alpha, \theta), n^T(\alpha\theta)\right]$.

Finally, it is worth mentioning that the equivalence result can still hold if verifying α and θ involves different costs. The key to equivalence is that incentive compatibility constraint and the participation constraint are both binding. As long as there are firms that find it optimal to reveal both α and θ, they will be indifferent between equity and transparent-debt financing as long as both constraints bind.

3.2 Separating Contract

Consider a firm who uses a contract to signal its true quality (α, θ). That is, the firm does not take on the verification cost, γ, to reveal its quality, but instead designs a contract that no firm of another type $\left(\tilde{\alpha}, \tilde{\theta}\right)$ is willing to mimic. Taking (n, α, θ, r) as given, the firm seeks to maximize its own expected profit from this separating contract:

$$\begin{aligned}
\Pi^S(n, \alpha, \theta) \quad &= \quad \max_{(i^S \leq \bar{\imath},\ z^S)} \left[\alpha\left(\theta i^S - z^S\right) - (1+r)\, n\right] \\
s.t. \quad & \quad \theta i^S - z^S \geq (1-\sigma)\,\theta i^S && (9) \\
& \quad \alpha z^S \geq (1+r)\left(i^S - n\right) && (10) \\
& \quad \tilde{\alpha}\left(\tilde{\theta} i^S\left(\left(\tilde{\alpha}, \tilde{\theta}\right)\right) - z^S\left(\tilde{\alpha}, \tilde{\theta}\right)\right) \geq \tilde{\alpha}\left(\tilde{\theta} i^S\left((\alpha, \theta)\right) - z^S(\alpha, \theta)\right) && (11) \\
& \qquad \forall\ \left(\tilde{\alpha}, \tilde{\theta}\right) \neq (\alpha, \theta).
\end{aligned}$$

The first condition in the above is the incentive compatibility constraint of the firm reporting output honestly. The second condition is the participation constraint of lenders. The third condition is the incentive compatibility constraint of other firms, with $\left(\tilde{\alpha}, \tilde{\theta}\right) \neq (\alpha, \theta)$, not mimicking the firm in question. That is, this condition guarantees that no other firm with $\left(\tilde{\alpha}, \tilde{\theta}\right) \neq (\alpha, \theta)$ will have the incentive to use this particular contract.

Immediately we have two observations from comparing the respective problems of designing transparent and separating contracts. The first one is that the first two constraints for designing the separating contract are exactly the same as those for the transparent contract. The second observation is that the two objective functions are the same except for the constant term $-\gamma$ for the transparent contract. Thus, it follows immediately that any optimal separating contract must also be the optimal solutions to the optimal transparent contract, but not *vice versa*. Therefore, any firm that can successfully design a separating contract will never find it optimal to use a transparent contract because $\gamma > 0$. Thus to characterize the separating contract, it suffices to check if the contract $\left(i^T(n, \alpha, \theta), z^T(n, \alpha, \theta)\right)$ given by (4) and (5) satisfies condition (11). This is detailed in Appendix A.2, which provides a proof of the following proposition:

Proposition 3. *The separating contract does not exist for any firm with $(\alpha, \theta) \neq (\underline{\alpha}, \underline{\theta})$. None of such firms can successfully signal their true quality without being mimicked by some other firms.*

Note that Proposition 3 does not eliminate the possibility of a separating contract for the firms of type $(\underline{\alpha}, \underline{\theta})$. However, even if such a firm can successfully signal its true type, it will find itself strictly better off with an opaque contract in equilibrium, as will be proven by Proposition 7 and Theorem 1. It is interesting to note that the separating contract (the one that does not involve a costly verification of types) collapses for all firms, except for firms of the worst quality (among those qualifying for external financing). The intuition is the following: In this environment with both adverse selection and agency problem, any separating contract must satisfy two sets of incentive compatibility constraints, one ensuring that the financing firm itself truthfully reports output and the other guaranteeing that no other firms can benefit from pretending to be of the same quality as the firm in question. The former condition renders the latter impossible to uphold for all firms except perhaps those of $(\underline{\alpha}, \underline{\theta})$.

3.3 Opaque Contract

Consider a firm who uses an opaque contract for external financing. In this case, investors do not know the true quality (α, θ) because the contract $\left(i^O, z^O\right)$ does not reflect such information. As far as an investor is concerned, the firm offering an opaque contract has some quality $(\tilde{\alpha}, \tilde{\theta}) \in \Omega$. Taking $(n, \alpha, \theta, r, \Omega)$ as given, the firm seeks to maximize its own expected profit from designing an opaque contract:

$$\Pi^O (n, \alpha, \theta) = \max_{(i^O \leq \bar{\imath},\ z^O)} \left[\alpha \left(\theta i^O - z^O\right) - (1 + r)\, n\right] \tag{12}$$

$$s.t. \quad \tilde{\theta} i^O - z^O \geq (1 - \sigma)\, \tilde{\theta} i^O, \quad \forall\ (\tilde{\alpha}, \tilde{\theta}) \in \Omega \tag{13}$$

$$\alpha^\Omega z^O \geq (1 + r) \left(i^O - n\right). \tag{14}$$

Conditions (13) and (14) are respectively the incentive compatibility constraint and the participation constraint. Note that both conditions must be satisfied *given the investors' perspective*. In other words, both conditions must hold given the knowledge that the quality of this firm is some $(\tilde{\alpha}, \tilde{\theta}) \in \Omega$.

Condition (13) ensures that any firm in the set Ω will make a truthful payout to the investors, which is consistent with the information available to investors. The left-hand side of this condition is the firm's payoff from truthfully revealing a successful project outcome. The right-hand side is the firm's payoff from falsely claiming zero output and thus facing audit, which will lead to a loss of a fraction σ of output. Obviously, the incentive to report truthfully requires that the former is at least as high as the latter.

Condition (14) ensures that this contract offers a net rate of return at least as high as r, which is needed to attract external investment. In particular, the left-hand side is the expected payoff to the investors, where α^{Ω} denotes the *expected* survival rate of firms in set Ω. Note that given an incentive compatible contract, there will be no hidden output and thus no recovered output from auditing. Obviously, the right-hand side of this condition gives the required level of returns for an investment of the amount $\left(i^{O}-n\right)$. Moreover, denote θ^{Ω} as the *lowest* level of productivity among firms of set Ω. In this section and the next, α^{Ω} and θ^{Ω} will be taken as given. We will solve for these two equilibrium variables in Section 5. Given the problem presented in (12), we have another proposition:

Proposition 4. *Given Assumption 1, the optimal opaque contract* $\left(i^{O},z^{O}\right)$ *is given by*

$$i^{O}\left(n;\alpha^{\Omega},\theta^{\Omega}\right) = \begin{cases} \frac{1+r}{1+r-\sigma\alpha^{\Omega}\theta^{\Omega}}n, \text{for } n\leq n^{O} \\ \bar{\imath}, \qquad\qquad \text{for } n>n^{O} \end{cases} \tag{15}$$

$$z^{O}\left(n;\alpha^{\Omega},\theta^{\Omega}\right) = \begin{cases} \frac{\sigma\theta^{\Omega}(1+r)}{1+r-\sigma\alpha^{\Omega}\theta^{\Omega}}n, \text{for } n\leq n^{O} \\ \frac{1+r}{\alpha^{\Omega}}\left(\bar{\imath}-n\right), \text{ for } n>n^{O} \end{cases} \tag{16}$$

where the threshold of a non-binding incentive compatibility constraint is

$$n^{O}\left(\alpha^{\Omega}\theta^{\Omega}\right)\equiv\bar{\imath}\left(1-\frac{\sigma\alpha^{\Omega}\theta^{\Omega}}{1+r}\right)<\bar{\imath}. \tag{17}$$

A firm's maximized expected profit of using this contract is given by

$$\Pi^{O}\left(n,\alpha,\theta;\alpha^{\Omega},\theta^{\Omega}\right)=\begin{cases} \frac{(1+r)\left[\alpha\theta-(1+r)-\sigma\theta^{\Omega}\left(\alpha-\alpha^{\Omega}\right)\right]}{1+r-\sigma\alpha^{\Omega}\theta^{\Omega}}n, & \text{for } n\leq n^{O} \\ \left[\alpha\theta-\frac{\alpha}{\alpha^{\Omega}}\left(1+r\right)\right]\bar{\imath}+\left(1+r\right)\left(\frac{\alpha}{\alpha^{\Omega}}-1\right)n, \text{for } n>n^{O}. \end{cases} \tag{18}$$

Results of (15) and (16) imply

$$z^{O}\left(n;\alpha^{\Omega},\theta^{\Omega}\right)=\begin{cases} \frac{(1+r)\sigma\theta^{\Omega}}{1+r-\sigma\alpha^{\Omega}\theta^{\Omega}}n=\frac{1+r}{\alpha^{\Omega}}\left(i^{O}-n\right), \text{for } n\leq n^{O} \\ \frac{1+r}{\alpha^{\Omega}}\left(\bar{\imath}-n\right), \qquad\qquad\qquad \text{for } n>n^{O} \end{cases}$$

which is just (19). Thus it is straightforward to establish the following proposition:

Proposition 5. *The optimal opaque contract can be implemented by an opaque debt contract for all $n < \bar{\imath}$ with the payout to investors being a general interest rate, $(1+r)/\alpha^{\Omega}$, times total external investment. That is,*

$$z^{O}\left(n;\alpha^{\Omega},\theta^{\Omega}\right)=\frac{1+r}{\alpha^{\Omega}}\cdot\left[i^{O}\left(n;\alpha^{\Omega},\theta^{\Omega}\right)-n\right]. \tag{19}$$

This proposition is straightforward and equation (19) is derived from condition (14), which binds for *all* $n < \bar{\imath}$. Here with opaque financing, the optimal contract cannot be implemented by an equity contract because it is impossible to convince an investor to take on shares of ownership without revealing firm quality to them. Technically, this means here a binding incentive compatibility constraint (13) boils down to $z^{O} = \sigma\theta^{\Omega}i^{O}$, which is not an equity contract because it does not entitle the investor to a part of the ownership.

3.3.1 Opaque Debt Financing

It is obvious from Proposition 5 that using the optimal opaque contract is essentially financing with opaque debt because the firm commits to a general interest rate, $(1+r)/\alpha^{\Omega}$, which is independent of any firm-specific information. This type of debt resembles business lines of credit in reality. To qualify for a line of credit, a firm is required to present evidence to the bank that it meets some general criteria. In our model, we can interpret these criteria as those conditions imposed by Assumption 1. Once qualified, the firm faces an interest rate common to all credit-line users, who in our model corresponds to firms within set Ω. Finally, another salient feature of credit lines is that banks impose credit limits on them. On this point, we will show in Section 5 that in equilibrium opaque debt is used by firms seeking smaller amounts of external funds, as opposed to those who choose equity or transparent-debt financing.

It is worth mentioning that the literature has seen models where the optimal financial contract is a debt contract. For example, Diamond (1984), Gale and Hellwig (1985), Williamson (1986), *etc.*, feature costly state verification in an environment with homogeneous entrepreneurs. In contrast, our paper combines adverse selection and agency cost in a model of heterogeneous firms. We will show that not only debt and equity can coexist in equilibrium, but also debt itself has variety by being either transparent or opaque in terms of firm-specific information.

4 Optimal Capital Structure

In the previous section, we have respectively established the optimal transparent and opaque contracts, as well as the result that no firm can successfully use a separating contract except for those of quality $(\underline{\alpha}, \underline{\theta})$. We now examine a firm's overall financing choice at Step 1 of the signaling game. At this stage, firms in general face three investment options: either to use a transparent contract, to use an opaque contract, or to fund the project with its internal funds. Firms with $(\underline{\alpha}, \underline{\theta})$ have one more option, which is to use a separating contract. The firm will calculate the expected profits associated with each of these options and then choose the one that offers the highest profit.

4.1 Self-Funding

Before comparing the three investment options, we need to define the expected profit a firm can get by self-funding its project. A firm's expected profit of self-funding with n is given by

$$\Pi^{SF}(n, \alpha, \theta) = [\alpha\theta - (1+r)]\, n.$$

Proposition 6. *Given Assumption 1, the opaque contract dominates self-funding for all* $(n, \alpha, \theta) \in [0, \bar{\imath}] \times \left[\underline{\theta}, \bar{\theta}\right] \times [\underline{\alpha}, \bar{\alpha}]$.

From Proposition 6, it is clear that all firms will seek external financing since the opaque debt contract is always preferred to the self-funding option.

4.2 Opaque *vs.* Separating Contracts for $(\underline{\alpha}, \underline{\theta})$

Recall from Proposition 3 that only firms of quality $(\underline{\alpha}, \underline{\theta})$ can use a separating contract. Given the fact that these are the firms of the lowest quality, it must be true that $\alpha^{\Omega} \geq \underline{\alpha}$

and $\theta^{\Omega} \geq \underline{\theta}$ regardless of what set Ω is in equilibrium.

Proposition 7. *The opaque contract dominates the separating contract for a firm with $(\underline{\alpha}, \underline{\theta})$, and the dominance is strict if $\alpha^{\Omega} > \underline{\alpha}$.*

The opaque contract is one that firms of various qualities choose to pool on. Intuitively, it is beneficial for the lowest-quality firms to pool with other types of firms on this contract because the resulted pool of firms cannot have an aggregate quality worse than $(\underline{\alpha}, \underline{\theta})$, which means this pool of firms must be evaluated no worse than just $(\underline{\alpha}, \underline{\theta})$ by investors. Later in Section 5, Theorem 1 establishes that $\alpha^{\Omega} = E(\alpha) > \underline{\alpha}$ in the perfect Bayesian equilibrium. Therefore, the separating contract is indeed strictly dominated by the opaque contract for $(\underline{\alpha}, \underline{\theta})$.

4.3 Opaque *vs.* Transparent Contracts

Given Propositions 6 and 7, it is optimal for all firms (with quality satisfying Assumption 1) to seek external funds, either by an opaque contract or a transparent contract. A firm's maximized expected profit is thus given by

$$\Pi(n, \alpha, \theta) = \max\left[\Pi^{O}(n, \alpha, \theta), \Pi^{T}(n, \alpha, \theta)\right]$$

where $\Pi^{O}(n, \alpha, \theta)$ and $\Pi^{T}(n, \alpha, \theta)$ are respectively given by (18) and (7). The following proposition establishes how individual firms make financing choices, taking $\left(\alpha^{\Omega}, \theta^{\Omega}\right)$ as given:

Proposition 8. *Given $\left(\alpha^{\Omega}, \theta^{\Omega}\right)$ and Assumption 1, a firm's optimal financing choice is characterized by the following: (i) If the firm's quality (α, θ) satisfies*

$$\alpha\theta > \alpha^{\Omega}\theta^{\Omega} \tag{20}$$

$$\frac{\gamma}{\sigma\alpha\theta\bar{\imath}} < \frac{\left(\alpha\theta - \alpha^{\Omega}\theta^{\Omega}\right) - (1+r)\left(1 - \theta^{\Omega}/\theta\right) - \sigma\theta^{\Omega}\left(\alpha - \alpha^{\Omega}\right)}{1 + r - \sigma\alpha^{\Omega}\theta^{\Omega}}, \tag{21}$$

there exist unique levels of $n_1(\alpha,\theta)$ *and* $n_2(\alpha,\theta)$ *such that* $0 < n_1(\alpha,\theta) < n_2(\alpha,\theta) < \bar{\imath}$ *and*

$$\Pi(n) = \begin{cases} \Pi^O(n), \textit{ for } n < n_1(\alpha,\theta) \\ \Pi^T(n), \textit{ for } n \in [n_1(\alpha,\theta), n_2(\alpha,\theta)] \\ \Pi^O(n), \textit{ for } n > n_2(\alpha,\theta). \end{cases} \tag{22}$$

That is, such a firm optimally uses the transparent contract for $n \in [n_1(\alpha,\theta), n_2(\alpha,\theta)]$ *and the opaque contract otherwise. (ii) If the firm's quality* (α,θ) *satisfies*

$$\alpha\theta \leq \alpha^\Omega\theta^\Omega \tag{23}$$

$$\frac{\gamma}{\sigma\alpha\theta\bar{\imath}} < \frac{\alpha}{\alpha^\Omega} - 1, \tag{24}$$

there exist unique levels of $n_1'(\alpha,\theta)$ *and* $n_2'(\alpha,\theta)$ *such that* $0 < n_1'(\alpha,\theta) < n_2'(\alpha,\theta) < \bar{\imath}$ *and*

$$\Pi(n) = \begin{cases} \Pi^O(n), \textit{ for } n < n_1'(\alpha,\theta) \\ \Pi^T(n), \textit{ for } n \in [n_1'(\alpha,\theta), n_2'(\alpha,\theta)] \\ \Pi^O(n), \textit{ for } n > n_2'(\alpha,\theta). \end{cases} \tag{25}$$

That is, such a firm optimally uses the transparent contract for $n \in [n_1'(\alpha,\theta), n_2'(\alpha,\theta)]$ *and the opaque contract otherwise. (iii) The rest of the firms optimally uses the opaque contract for all* $n \in [0, \bar{\imath}]$*, and do not use the transparent contract at all.*

Proposition 8 formalizes the conditions under which the transparent contract is preferred over the opaque contract. Note that (20) and (23) are requirements on the firm's quality $\alpha\theta$ relative to the general quality of firms in set Ω, $\alpha^\Omega\theta^\Omega$. Moreover, (21) and (24) are requirements on the verification cost γ relative to the most severe punishment possible for mis-reporting output, $\sigma\alpha\theta\bar{\imath}$. Both conditions stipulate that this ratio cannot be too high, that is, the verification cost cannot be too high relative to the maximum disciplinary measure possible. The conditions (20) to (24) will be used in the next section to characterize the perfect Bayesian equilibrium. We will show that the second case by (23) and (24) does not survive in equilibrium. That is, in equilibrium no firm of inferior quality $\alpha\theta \leq \alpha^\Omega\theta^\Omega$ finds it optimal to finance with a transparent contract.

5 Equilibrium

5.1 Perfect Bayesian Equilibrium

From our analysis thus far, it has become clear that it suffices for investors to know these two characteristics of set Ω, α^{Ω} and θ^{Ω}. With this, we first define the perfect Bayesian equilibrium and then follow up with a theorem that characterizes such an equilibrium.

Definition 2. A perfect Bayesian equilibrium consists of optimal contracts,

$$\{i^T(n,\alpha,\theta), z^T(n,\alpha,\theta); i^O\left(n;\alpha^{\Omega},\theta^{\Omega}\right), z^O\left(n;\alpha^{\Omega},\theta^{\Omega}\right); i^S(n,\underline{\alpha},\underline{\theta}), z^S(n,\underline{\alpha},\underline{\theta})\},$$

values, $\left\{\Pi^T(n,\alpha,\theta), \Pi^O\left(n,\alpha,\theta;\alpha^{\Omega},\theta^{\Omega}\right), \Pi^{SF}(n,\alpha,\theta), \Pi^S(n,\underline{\alpha},\underline{\theta})\right\}$, and set characteristics, $\left(\theta^{\Omega},\alpha^{\Omega}\right)$, such that: (i) All firms and investors optimize given $\left(\theta^{\Omega},\alpha^{\Omega}\right)$; (ii) $\left(\theta^{\Omega},\alpha^{\Omega}\right)$ are consistent with the optimal choices of firms and investors. □

Theorem 1. *There exists a unique perfect Bayesian equilibrium with $\theta^{\Omega}=\underline{\theta}$ and $\alpha^{\Omega}=E[\alpha]$, provided that*

$$\frac{\gamma}{\sigma\bar{\alpha}\underline{\theta}\bar{\imath}} \geq \frac{(1-\sigma)\,\underline{\theta}\,(\bar{\alpha}-E[\alpha])}{1+r-\sigma\underline{\theta}E[\alpha]}. \tag{26}$$

Moreover, the equilibrium is characterized by the following:

(i) No firm uses a separating contract.

(ii) The set of opaque-debt users is non-empty in that all firms with $\alpha\theta \leq \underline{\theta}E[\alpha]$ choose opaque debt regardless of their internal funds. However, the set of transparent-contract users is non-empty iff there exists some quality (α,θ) that satisfies

$$\alpha\theta-\underline{\theta}E[\alpha] > \max\left[0,\ \frac{\gamma\,(1+r-\sigma\underline{\theta}E[\alpha])}{\sigma\alpha\theta\bar{\imath}}+(1+r)\left(1-\frac{\underline{\theta}}{\theta}\right)+\sigma\underline{\theta}\,(\alpha-E[\alpha])\right]. \tag{27}$$

(iii) For a firm with (α,θ) that satisfies the above condition, there exists a unique set $[n_1(\alpha,\theta), n_2(\alpha,\theta)] \subset [0,\bar{\imath}]$ with which the firm chooses transparent financing, and opaque

debt otherwise. Firms with $n \in \left[n_1(\alpha,\theta), n^T(\alpha\theta)\right]$ are indifferent between equity and transparent debt, while those with $\left(n^T(\alpha\theta), n_2(\alpha,\theta)\right]$ are strictly better off with transparent debt. All firms that do not satisfy condition (27) choose to use opaque debt.

(iv) All else equal, the equilibrium set of transparent-contract users shrinks and the set of opaque-debt users expands as γ rises. There exists a threshold $\bar{\gamma} \in (0,\infty)$, above which the equilibrium set of firms with transparent financing becomes empty. Therefore, the equilibrium can only be one of two types: either pooling on opaque debt, or mixing with transparent and opaque contracts.

Theorem 1 states that if exists, the perfect Bayesian equilibrium must have $\theta^\Omega = \underline{\theta}$ and $\alpha^\Omega = E[\alpha]$. This is a key result for solidifying the equilibrium, and it is challenging to establish because the set characteristics, $\left(\theta^\Omega, \alpha^\Omega\right)$, can potentially be any level respectively over $\left[\underline{\theta}, \bar{\theta}\right]$ and $[\underline{\alpha}, \bar{\alpha}]$. Appendix A.7 provides a detailed proof of this result, which is carried out in four steps where we eliminate all other possibilities regarding θ^Ω and α^Ω.

Existence of the unique Bayesian equilibrium requires condition (26) to hold. This condition stipulates that the verification cost γ relative to the tightest discipline on a firm with the maximum investment given the lowest productivity and survival risk, $\sigma\bar{\alpha}\underline{\theta}\bar{\imath}$, must be sufficiently high. Under this condition, all firms with $\underline{\theta}$ and any risk level $\alpha \in [\underline{\alpha}, \bar{\alpha}]$ find it optimal to finance with opaque debt, which ensures $\theta^\Omega = \underline{\theta}$ and $\alpha^\Omega = E[\alpha]$ in equilibrium. Given $\alpha^\Omega = E(\alpha) > \underline{\alpha}$, Proposition 7 implies that the separating contract is strictly dominated by the opaque contract for $(\underline{\alpha}, \underline{\theta})$. This result, together with Proposition 3 show that no firm uses a separating contract in equilibrium. Hence part (i).

Part (ii) of Theorem 1 shows that the equilibrium cannot be pooling on transparent financing. In particular, firms with $\alpha\theta \leq \underline{\theta}E[\alpha]$ will always choose to use opaque debt no matter what level of internal funds they have. Intuitively, using the opaque contract can help these firms mask their inferior quality. Part (iii) establishes that firms who use transparent financing are those with sufficiently high quality, and intermediate levels of internal funds.

Part (iv) of Theorem 1 points out that, all else equal, the equilibrium can be switched from one mixing with transparent and opaque contracts to one pooling on opaque debt, as the verification cost γ rises.

The results in parts (ii) to (iv) of Theorem 1 can be understood with Figure 1, which illustrates the financing choices of an individual firm given $\left(\alpha^{\Omega}, \theta^{\Omega}\right)$. In all panels of this figure, the red curve is the value of opaque financing, $\Pi^{O}(n)$, and the blue curve is the value of transparent financing, $\Pi^{T}(n)$, taking the firm quality (α, θ) as given. Each panel represents a particular case of (α, θ). The two panels in each row are for the same given level of expected productivity, $\alpha\theta$.

All panels have two common features: (a) Each curve has a kink connecting two linear segments. The kink is at the threshold $n = n^{j}$, where $j = O, T$. The incentive compatibility condition binds for the segment to the left of the kink and does not bind for the segment to the right. (b) At the two end points, $n = 0$ and $n = \bar{\imath}$, the red curve is strictly above the blue by a distance of γ. That is, $\Pi^{O}(0) - \Pi^{T}(0) = \Pi^{O}(\bar{\imath}) - \Pi^{T}(\bar{\imath}) = \gamma$. With this, the opaque contract always strictly dominates the transparent contract for a firm with a rather low or rather high level of internal funds. The intuition is that these firms are only seeking a small amount of external funds, and thus it does not justify using the transparent contract because the verification cost γ outweighs the benefit for these firms. Regardless of quality (α, θ), firms with n in the right neighborhood of $n = 0$ are not able to attract a large amount of funds because their internal investment is low. Moreover, firms with n in the left neighborhood of $n = \bar{\imath}$ do not need a large amount of external funds because their internal funds are close to the maximal investment required.

There are also three main differences among the panels in Figure 1: (a) All panels on the left have a positively-sloped right segment of the red curve. This occurs when $\alpha < E[\alpha]$, that is, for riskier firms. All panels on the right have a negatively-sloped right segment of the red, corresponding to $\alpha > E[\alpha]$. Without loss of generality, we ignore the knife-edge case of $\alpha = E[\alpha]$, which obviously would have a flat right segment in red.

(b) The blue curve crosses over the red for some levels of n only for the two top panels. These are the two possible cases in which the transparent contract strictly dominates the opaque contract, when their quality (α, θ) is sufficiently high, satisfying (27). Note that only firms with an intermediate level of internal funds can be better off with the transparent contract. Again this is because of the second common feature discussed above. For the firms characterized by these two top panels, their optimal financing choice, as funds n ranked from low to high, will follow a route of opaque debt, indifference between equity and transparent debt, transparent debt and finally opaque debt.

(c) The four panels in the middle and bottom rows are all cases that the opaque contract strictly dominates the transparent one for all levels of n. The two middle panels are for firms of higher quality ($\alpha\theta > \underline{\theta} E[\alpha]$) that does not satisfy (27). Note that $n^T(\alpha\theta) < n^O$ for these firms. The two bottom panels of Figure 1c are for firms of inferior quality ($\alpha\theta \leq \underline{\theta} E[\alpha]$).

Next we will introduce and discuss two propositions that arise from the results of Theorem 1, one establishing the versatility of debt and the other characterizing the optimal debt-to-equity ratio.

5.2 Observed "Pecking Order" or Not?

Proposition 9. *Debt weakly dominates equity. Both the optimal opaque and the optimal transparent contracts can be implemented by a debt contract. However, an equity contract can only implement the optimal transparent contract for firms with quality* (α, θ) *satisfying (27) and funds* $n \in \left[n_1(\alpha, \theta), n^T(\alpha\theta)\right]$.

Proposition 9 proves itself given Theorem 1. The main message from this proposition is that debt is more versatile than equity in the sense that the former is a weakly dominating choice. The versatility of debt is manifested in four aspects of the equilibrium: (a) Opaque debt is used by firms of all quality measures. (b) Debt can be used to implement the optimal contract for all firms, regardless of their quality and internal funds. Some firms are strictly better off with transparent debt and others with opaque debt. (c) In a mixing equilibrium, the only set of firms that find it optimal to use equity are those with quality (α, θ) satisfying (27) and funds $n \in \left[n_1(\alpha, \theta), n^T(\alpha\theta)\right]$. Even so, these firms are indifferent between equity and transparent debt as both can help implement the same optimal contract. (d) In a pooling equilibrium, no firms find it optimal to use equity.

This result directly speaks to the literature on the pecking-order theory. Myers (2001) reports that for U.S. nonfinancial corporations, external finance covers a small proportion (mostly less than 20%) of capital formation and that equity issues are minor, with the bulk of external finance being debt. This finding can be explained by our theory. First of all, (4) and (15) imply that external financing, be it debt or equity, relies on the existence of internal funds. That is, total investment $i^T = i^O = 0$ for all firms if $n = 0$. Depending on parameter values, it is possible to have that internal equity outweighs external investment. Moreover, keep in mind that our paper focuses on the challenge of external financing and thus has used Assumption 1 to rule out self-funding. In general, the amount of internal investment will certainly be larger if self-funded projects are considered. Secondly, it is also reasonable to have debt financing to be of greater magnitude than equity, considering that debt can be in either opaque or transparent forms.

That being said, Frank and Goyal (2003) find that the aforementioned evidence documented by Myers (2001) does not match the evidence for publicly traded American firms, particularly during the 1980s and 1990s. Their study shows that external finance is much more significant than is usually recognized in that it often exceeds investments. Equity finance is a significant component of external finance. On average, net equity issues commonly exceed net debt issues. This set of evidence presented by Frank and Goyal (2003) can also be reconciled by our theory. Given that the dataset used by Frank and Goyal (2003) covers publicly traded firms, these are firms of higher quality that find the transparent contract optimal. It is not surprising that one would find significant external finance among such firms. Moreover, it is also reasonable to find significant equity finance because our theory shows that the higher the firm quality, the higher the amount of external funds raised by a transparent contract such as an equity contract.

It is important to note that our theory can reconcile the discrepancy in the two sets of empirical evidence respectively by Myers (2001) and Frank and Goyal (2003). As far as we can tell, Myers (2001) reports observations from U.S. nonfarm and nonfinancial corporate businesses, not particularly restricting to publicly traded ones[11] as Frank and Goyal (2003) do. The latter obviously excludes businesses that are either entirely self-funded, with equity but not publicly listed, or only having debt as external financing. According to our theory, the last type of firms can be significant in number because it may contain not only firms that are of relatively lower quality, but also higher-quality firms that find either opaque or transparent debt uniquely optimal given the internal funds available to them. Therefore, it is sensible that Myers (2001) reports external financing, and especially equity finance, of much smaller magnitudes than the statistics from Frank and Goyal (2003). Finally, the quality-verification cost is likely to vary over time, from industry to industry, and from country to country, *etc.* Therefore, this may also contribute to the fact that firms documented in various datasets display somewhat different behavioral patterns when it comes to external financing. Later in Section 6, we will provide an in-depth discussion on how to apply our theory to interpret both aggregate and individual financing behaviors.

5.3 Optimal Debt-to-Equity Ratio

Denote the debt-to-equity ratio of a firm as D/E, where the denominator includes both internal and external equity. Then Theorem 1 leads to another proposition:

Proposition 10. *Given* (n, α, θ) *that satisfies Assumption 1, the optimal debt-to-equity ratio of a firm varies with firm-specific characteristics such as quality and internal funds. (i) In*

[11] Financial firms, regulated utilities, and firms involved in major mergers are excluded, according to page 225 of Frank and Goyal (2003).

a pooling equilibrium with opaque debt,

$$\text{Optimal } \frac{D}{E} = \begin{cases} \frac{\sigma\underline{\theta}E[\alpha]}{1+r-\sigma\underline{\theta}E[\alpha]}, \text{for } n \leq n^O \\ \frac{\bar{\imath}}{n} - 1, \quad\quad \text{for } n > n^O \end{cases} \tag{28}$$

for all firms. That is, regardless of quality, the optimal debt-to-equity ratio is constant for a firm with $n \leq n^O$ *and strictly decreases in* n *for one with* $n > n^O$*; (ii) In a mixing equilibrium, the optimal debt-to-equity ratio of a firm with quality* (α, θ) *that does not satisfy (27) is also given by (28). Nevertheless, the optimal debt-to-equity ratio of a firm with* (α, θ) *satisfying condition (27) is given by*

$$\text{Optimal } \frac{D}{E} = \begin{cases} \frac{\sigma\underline{\theta}E[\alpha]}{1+r-\sigma\underline{\theta}E[\alpha]}, & \text{for } n < n_1(\alpha, \theta) \\ \in \left[0, \frac{\sigma\alpha\theta}{1+r-\sigma\alpha\theta}\right], & \text{for } n \in \left[n_1(\alpha, \theta), n^T(\alpha\theta)\right] \\ \frac{\bar{\imath}}{n} - 1, & \text{for } n \in \left(n^T(\alpha\theta), n_2(\alpha, \theta)\right] \\ \frac{\sigma\underline{\theta}E[\alpha]}{1+r-\sigma\underline{\theta}E[\alpha]}, & \text{for } n \in \left(n_2(\alpha, \theta), n^O\right] \\ \frac{\bar{\imath}}{n} - 1, & \text{for } n > n^O. \end{cases} \tag{29}$$

There exists a unique optimal level of debt-to-equity ratio for most of the firms, with the exception of those who are indifferent between financing with equity or transparent debt. Because of the indifference, there exists a range of levels that are optimal for these firms.

Proposition 10 is remarkable. It clearly shows how complex the notion of the *optimal debt-to-equity ratio* can be. This optimal ratio varies along the diverse firm characteristics of internal funds, productivity and survival rate. It is not a smooth function in terms of these characteristics. Moreover, it may not even be unique for a given combination of characteristics. The optimal ratios specified by (28) and (29) are respectively depicted by the two panels in Figure 2. Figure 2a shows the optimal ratio for a firm that never finds transparent financing optimal regardless of its internal funds. For these firms, the optimal ratio is independent of n when it is sufficiently low such that the incentive compatibility constraint binds. When n is high enough such that the constraint does not bind, the optimal ratio strictly decreases in funds n. Figure 2b corresponds to a firm that sometimes is strictly better off with transparent financing. For such a firm, it is indifferent between equity and transparent debt given funds $n \in \left[n_1(\alpha, \theta), n^T(\alpha\theta)\right]$. Therefore, there are a continuum of ratios that are all optimal for this type of firms. Note that these firms only constitute a strict set of all firms seeking external financing. In other words, not all firms in a mixing equilibrium find equity and transparent debt equivalent, which marks partial capital structure irrelevance. Figure 2b also contains segments where the optimal ratio is either flat or strictly decreasing in n depending on whether the incentive compatibility constraint binds for the optimal opaque/transparent contract. It is obvious that the optimal debt-to-equity ratio is affected by a variety of general parameters. Appendix A.9 provides a detailed discussion of the comparative statics with respect to each of these parameters.

Theorem 1 has provided us a rich set of results regarding the financing behavior of heterogeneous firms.

6 Understanding Firm Financing Behavior

6.1 Cross-Sectional Behavior of Firm Financing

Theorem 1 has provided us a rich set of results regarding the financing behavior of heterogeneous firms. Table 1 summarizes the characteristics associated with each of the three financing methods that arise in equilibrium, namely, equity, transparent debt and opaque debt: First, information transparency varies across the three types of financial contracts. Equity and transparent-debt contracts are transparent in firm-specific information, with firms voluntarily revealing both productivity and survival rate (*i.e.* riskiness). In contrast, the opaque-debt contract is opaque in terms of firm-specific information and only reflects general knowledge of firms that *qualify* for seeking external funds, *i.e.*, firms with quality that satisfies Assumption 1. Accordingly, transparent debt is associated with firm-specific interest rates reflecting α while opaque debt has a common interest rate reflecting $E[\alpha]$.

Second, the financing behavior can signal a firm's quality. In particular, the better-quality firms, those with $\alpha\theta > \underline{\theta}E[\alpha]$ (and satisfying [27]), choose transparent financing either in the form of equity or transparent debt. In contrast, firms of all possible qualities may choose to use opaque debt.

Third, the financing behavior may also be reflective of a firm's internal finances. Transparent financing tends to be used by firms with an intermediate level of funds, with equity users being particularly funds-constrained ($n \in \left[n_1(\alpha, \theta), n^T(\alpha\theta)\right]$). Although firms of any level of internal funds may use opaque debt, those that are either very funds-constrained or very funds-rich are more likely to do so.

It is important to clarify at this point that whether a firm has ample funds or not is a relative term, specifically relative to the investment capacity of a project, that is, $\bar{\imath}$ in our model. It is straightforward to extend the model by making $\bar{\imath}$ vary across projects. All our main results would still carry through. Keep in mind that a low-n firm in our model can be, but does not necessarily correspond to, a small firm or a young firm in reality. Indeed, a firm with low funds in our model simply means the level of internal funds available is low relative to capacity $\bar{\imath}$.

Finally, equity users tend to raise a larger amount of external funds for any given internal equity n, while debt in general is associated with relatively lower external funds (and thus leverage). This can be easily seen given Propositions 1 and 4. From these propositions, we define the ratio of external funds raised relative to internal funds available as the *financial leverage*, which is given by

$$\frac{i^T(n, \alpha, \theta) - n}{n} = \begin{cases} \frac{\sigma\alpha\theta}{1+r-\sigma\alpha\theta}, \text{for } n \leq n^T(\alpha\theta) \\ \frac{\bar{\imath}}{n} - 1, \quad \text{for } n > n^T(\alpha\theta) \end{cases} \tag{30}$$

$$\frac{i^O\left(n; \alpha^\Omega, \theta^\Omega\right) - n}{n} = \begin{cases} \frac{\sigma\underline{\theta}E[\alpha]}{1+r-\sigma\underline{\theta}E[\alpha]}, \text{for } n \leq n^O \\ \frac{\bar{\imath}}{n} - 1, \qquad \text{for } n > n^O. \end{cases}$$

As we know, in equilibrium any firm with equity financing must have $\alpha\theta > \underline{\theta}E[\alpha]$ and

Table 1: **Comparison of Financial Contracts**

		Debt	
Characteristic	**Equity**	**Transparent**	**Opaque**
Information transparency	α, θ	α, θ	$E[\alpha], \underline{\theta}$
Firm quality $(\alpha\theta)$	Higher	Higher	All types
Internal funds	Lower interm.	Interm.	All levels Often, very low/high
Financial leverage	Higher	General	Lower

$n \leq n^T(\alpha\theta)$, which implies a higher amount of external funds raised for a given level of internal funds than opaque debt. Transparent debt, however, may or may not have a higher leverage than opaque debt depending on the firm's internal funds n.

6.2 Firm Financing Behavior Over Time

Our model highlights the fundamental problem faced by a firm who considers external financing at a particular point in time. A firm's financing choice depends on its internal funds available and the specific quality of the investment opportunity *at the time of raising funds*. Firm heterogeneity drives the diversity of firm financing choices. This holds true both for a cross-section of firms (as discussed previously) and for the same firm's evolving behavior over time. Each time that a firm decides to adjust its capital structure is a reaction to new business conditions and/or opportunities that calls for new financing arrangements given the updated levels of (n, α, θ). Therefore, although the model is static in nature, our findings can help reconcile various dynamic patterns of the observed firm financing behavior.

1. *It is not uncommon for startup firms to choose equity financing.* Startups are known as new businesses that intend to grow large. Although startups face high uncertainty and have high rates of failure, a minority of them do go on to be successful and influential. Thus the startups are examples of firms with high $\alpha\theta$ in our model. More-

over, lack of sufficient funds by the founder(s) is common to a new business, meaning low n (again, in a term relative to investment capacity; some startups require an astronomical size of initial investment that may dwarf the amount any single investor contributes). As our theory suggests, firms with higher $\alpha\theta$ and lower n tend to seek equity financing.

2. *At some point a firm may decide to buy back some of its own shares.* Our theory offers an interpretation of such behavior: Conditional on quality staying the same, the firm's profitability may improve over a period of time such that its cash balances have increased. The firm now has ample cash to invest in itself (high n, relative to less n available at the time of the previous financing decision). Our theory suggests that previously the firm might have found it optimal to use equity financing given $n \in \left[n_1(\alpha,\theta), n^T(\alpha\theta)\right]$. But with n rising into $\left(n^T(\alpha\theta), n_2(\alpha,\theta)\right]$ or perhaps even beyond $n_2(\alpha,\theta)$, the firm is now strictly better off with debt financing (see possible cases depicted in Figure 1a). Therefore, the stock buybacks could be an indication of the firm's new decision given the improved cash positions. This argument also applies to a case where quality $\alpha\theta$ has improved along with n.

3. *Small businesses tend to start with a line of credit and then at some point also adopt other types of external funding, such as external equity, business loans or corporate bonds.* According to our theory, this corresponds to the decision of a firm initially with low internal funds, *i.e.*, $n \in (0, n_1(\alpha,\theta))$ and hence financing with opaque debt. As the business grows, any one of the following three scenarios may trigger the firm's decision to switch to some form of transparent financing: (i) internal funds move into $n \in [n_1(\alpha,\theta), n_2(\alpha\theta)]$; (ii) firm quality has improved and thus higher $\alpha\theta$. According to (55), it is possible that $n_1(\alpha,\theta)$ decreases as α or θ rises. Thus given the newly-

improved quality the firm may find itself now better off with transparent financing even though the level of internal funds has not changed; (iii) a combination of both (i) and (ii).

4. *Firms often issue new equity when their equity prices are high.* To understand this behavior, note that higher equity prices can be indicative of improved firm quality. Our theory suggests that firms may take advantage of this better signal of quality to raise more funds, which is consistent with our result that the amount of funds raised through equity strictly increases in firm quality $\alpha\theta$, as is obvious from (30) for $n \leq n^T(\alpha\theta)$.

5. *The same firm may be using two or three types of external financing at the same time.* Our theory suggests that every time a firm adjusts its capital structure reflects its *latest* business status on (n, α, θ). In this sense, it is not surprising that the same firm may be using multiple types of external financing at the same time. One possibility is that these different methods are used for different purposes, for example, on various investment projects. Another possibility is that large amounts of external financing are usually implemented over a prolonged period of time. Therefore, some of the previous undertakings may still be running its due course at the time of the latest adjustment in capital structure, resulting in multiple methods being used simultaneously. Perhaps the most perplexing question is that, is it possible for a firm to use transparent and opaque contracts at the same time? The answer is affirmative. For example, publicly-traded firms or firms with corporate bonds may also have business credit cards or lines of credit. How does our theory explain this? Actually, our theory is about the financing decision for *one* project. In general, the same firm may use different financial contracts for different purposes. A firm may find it optimal to use a transparent contract to finance its main operation, while also finding an opaque

contract optimal for financing items such as routine liquidity needs and employee expenses, *etc*. One might ask how one firm can use both types of contracts when the firm-specific information has already been revealed? In the model context, the information is revealed to all investors through the verification technology. In reality, however, this may only mean that the firm-specific information is revealed to interested parties. For example, this could be a scenario in which an entrepreneur makes effort to convince a group of selected investors. In terms of publicly-available credit ratings of firms with corporate bonds, these ratings do not in any way prevent these firms from accessing business credit cards or credit lines because the latter can be obtained on a qualification basis.

7 Concluding Remarks

We have proposed an information-based theory of capital structure. Our model nests adverse selection and agency cost in an environment with ex-ante heterogeneous firms. With this model, we tackle some fundamental questions of corporate finance through the lens of optimal contracting. We prove that there exists a unique perfect Bayesian equilibrium, in which equity, transparent debt and opaque debt contracts may coexist as optimal contracts. As we have shown, the way these three types of optimal contracts are structured mimics how external equity, corporate bonds/business loans and credit lines respectively work in reality. We also prove that debt weakly dominates equity because the former can help all firms implement the optimal contract while the latter only does so for a strict subset of firms. The intuition is that an equity contract must be informationally transparent and therefore, is costly to take on. Thus firms that can only afford, or only need, a relatively small amount of external funds are better off with an opaque contract (which can only be an opaque-debt contract) instead. Finally, we establish that partial capital structure irrelevance exists in a mixing equilibrium where some firms find equity and transparent debt equivalent. Overall, our theory provides insights on why firm financing behavior is complex, and why a variety of optimal financial contracts are needed to meet the diverse financing demand of distinct types of firms.

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A Appendix

A.1 Proof of Proposition 1

First note that the incentive compatibility constraint (2) can be re-arranged to $\sigma\theta i^T \geq z^T$. Let λ_1, λ_2 and λ_3 respectively denote the Lagrangian multipliers associated with the two conditions in the maximization problem and $i^T \leq \bar{\imath}$. The corresponding Lagrangian function is given by:

$$\mathcal{L} = \alpha\left(\theta i^T - z^T\right) - \gamma - (1+r)\, n + \lambda_1\left(\sigma\theta i^T - z^T\right) + \lambda_2\left[\alpha z^T - (1+r)\left(i^T - n\right)\right] + \lambda_3\left(\bar{\imath} - i^T\right).$$

Assume interior solutions. The first-order conditions are given by:

$$\alpha\theta + \lambda_1\sigma\theta - \lambda_2(1+r) - \lambda_3 = 0, \tag{31}$$

$$-\alpha - \lambda_1 + \alpha\lambda_2 = 0. \tag{32}$$

We consider all possible cases: (i) Suppose $\lambda_2 = 0$. Then (32) cannot hold because $\alpha > 0$ and $\lambda_1 \geq 0$. Thus $\lambda_2 > 0$ must be true and (3) must bind. (ii) Suppose $\lambda_1 = \lambda_3 = 0$. Given $\lambda_2 > 0$, (31) and (32) imply that $1 = \lambda_2 = \alpha\theta/(1+r)$, which requires the firm's quality (α, θ) to satisfy $\alpha\theta = 1+r$. This is a knife-edge case, which we will ignore without loss of generality. (iii) Suppose $\lambda_3 > 0$ and $\lambda_1 = 0$. Then $\lambda_3 > 0$ implies $i^T = \bar{\imath}$. Recall $\lambda_2 > 0$, which implies (3) holds with equality, which solves for $z^T = (1+r)(\bar{\imath} - n)/\alpha$. Finally, $\lambda_1 = 0$ implies (2) does not bind and thus $\sigma\theta i^T > z^T$. This requires $n > \bar{\imath}\left(1 - \frac{\sigma\alpha\theta}{1+r}\right)$. (iv) Suppose $\lambda_1 > 0$ and $\lambda_3 = 0$. Given $\lambda_1, \lambda_2 > 0$, both (2) and (3) bind, and thus we can use them to solve for

$$i^T = \frac{(1+r)}{1+r-\sigma\alpha\theta} n \tag{33}$$

$$z^T = \sigma\theta i^T = \frac{\sigma\theta(1+r)}{1+r-\sigma\alpha\theta} n. \tag{34}$$

Assumption 1 guarantees that $i^T > 0$ for all $(\alpha, \theta) \in \left[\underline{\theta}, \bar{\theta}\right] \times [\underline{\alpha}, \bar{\alpha}]$. Then $\lambda_3 = 0$ implies $i^T < \bar{\imath}$, which requires $n < \bar{\imath}\left(1 - \frac{\sigma\alpha\theta}{1+r}\right)$. (v) Suppose $\lambda_1, \lambda_3 > 0$. Again, $\lambda_1, \lambda_2 > 0$ imply (33) and (34). Moreover, $\lambda_3 > 0$ implies $i^T = \bar{\imath}$ and thus $n = \bar{\imath}\left(1 - \frac{\sigma\alpha\theta}{1+r}\right)$. Combining all of the above cases, (4) and (5) summarize the optimal transparent contract. Finally, since (3) always binds, we use it to eliminate z^T in the objective function and obtain (7).

A.2 Proof of Proposition 3

Recall that the first two constraints for designing the separating contract are exactly the same as those for the transparent contract. Therefore, any optimal separating contract must be an optimal transparent contract, as given by (4) and (5), that also satisfies the incentive-compatibility condition, (11). Given the firm's quality (α, θ), consider another firm of $\left(\tilde{\alpha}, \tilde{\theta}\right)$ with $\tilde{\alpha}\tilde{\theta} \leq \alpha\theta$. Given (4) and (5), which are the contract terms that satisfy conditions (9) and (10), condition (11) becomes

$$\tilde{\alpha}\left(\tilde{\theta}\frac{1+r}{1+r-\sigma\tilde{\alpha}\tilde{\theta}}n - \frac{\sigma\tilde{\theta}\left(1+r\right)}{1+r-\sigma\tilde{\alpha}\tilde{\theta}}n\right) \geq \tilde{\alpha}\left(\tilde{\theta}\frac{1+r}{1+r-\sigma\alpha\theta}n - \frac{\sigma\theta\left(1+r\right)}{1+r-\sigma\alpha\theta}n\right), \tag{35}$$

for $n \leq n^T\left(\alpha\theta\right)$,

$$\tilde{\alpha}\left(\tilde{\theta}\frac{1+r}{1+r-\sigma\tilde{\alpha}\tilde{\theta}}n - \frac{\sigma\tilde{\theta}\left(1+r\right)}{1+r-\sigma\tilde{\alpha}\tilde{\theta}}n\right) \geq \tilde{\alpha}\left(\tilde{\theta}\bar{\iota} - \frac{1+r}{\alpha}\left(\bar{\iota}-n\right)\right), \tag{36}$$

for $n \in \left(n^T\left(\alpha\theta\right), n^T\left(\tilde{\alpha}\tilde{\theta}\right)\right)$, and

$$\tilde{\alpha}\left(\tilde{\theta}\bar{\iota} - \frac{1+r}{\tilde{\alpha}}\left(\bar{\iota}-n\right)\right) \geq \tilde{\alpha}\left(\tilde{\theta}\bar{\iota} - \frac{1+r}{\alpha}\left(\bar{\iota}-n\right)\right), \tag{37}$$

for $n \geq n^T\left(\tilde{\alpha}\tilde{\theta}\right)$. The left-hand sides of all in the above are the expected payoff to the firm if it signals its true quality $\left(\tilde{\alpha}, \tilde{\theta}\right)$. The right-hand sides are the expected payoff if it pretends to be of quality (α, θ). Note that the threshold n^T must be based on the quality of the firm with (α, θ), whom a firm with $\left(\tilde{\alpha}, \tilde{\theta}\right)$ might attempt to mimic. Given $\tilde{\alpha}\tilde{\theta} \leq \alpha\theta$, (6) implies that $n^T\left(\alpha\theta\right) \leq n^T\left(\tilde{\alpha}\tilde{\theta}\right)$. Condition (35) simplifies to

$$\frac{1}{1+r-\sigma\alpha\theta} - \frac{1}{1+r-\sigma\tilde{\alpha}\tilde{\theta}} \leq \sigma\left(\frac{\theta/\tilde{\theta}}{1+r-\sigma\alpha\theta} - \frac{1}{1+r-\sigma\tilde{\alpha}\tilde{\theta}}\right), \quad \text{for } n \leq n^T\left(\alpha\theta\right).$$

Recall that $\sigma < 1$. Moreover, note that the function $1/\left(1+r-\sigma\alpha\theta\right)$ strictly increases in $\alpha\theta$. If $\tilde{\theta} = \theta$, then the above implies $\sigma \geq 1$, which is a contradiction. This also indicates that the above condition never holds for $\tilde{\theta} > \theta$ because $\theta/\tilde{\theta} < 1$, which makes the right-hand side strictly lower than the left-hand side. Together, we have that (35) never holds for firms with $\tilde{\theta} \geq \theta$ and $\tilde{\alpha}\tilde{\theta} \leq \alpha\theta$. That is, given lower internal funds such as $n \leq n^T\left(\alpha\theta\right)$, firms with lower expected quality ($\tilde{\alpha}\tilde{\theta} \leq \alpha\theta$) but higher actual productivity ($\tilde{\theta} \geq \theta$) have the incentive to mimic a firm of (α, θ) by using this contract. Next, condition (36) simplifies to

$$\sigma\tilde{\theta}\left(\tilde{\alpha}-\alpha\right)(1+r)\,n \geq \left[\alpha\tilde{\theta}-(1+r)\right]\left[\left(1+r-\sigma\tilde{\alpha}\tilde{\theta}\right)\bar{\imath}-(1+r)\,n\right], \tag{38}$$

for $n \in \left(n^T(\alpha\theta), n^T\left(\tilde{\alpha}\tilde{\theta}\right)\right)$. By (4),

$$i^T\left(n, \tilde{\alpha}, \tilde{\theta}\right) = \frac{1+r}{1+r-\sigma\tilde{\alpha}\tilde{\theta}}n \leq \bar{\imath}, \quad \text{for } n \leq n^T\left(\tilde{\alpha}\tilde{\theta}\right)$$

where the equality only holds for $n = n^T\left(\tilde{\alpha}\tilde{\theta}\right)$. Thus,

$$\left(1+r-\sigma\tilde{\alpha}\tilde{\theta}\right)\bar{\imath}-(1+r)\,n \geq \left(1+r-\sigma\tilde{\alpha}\tilde{\theta}\right)i^T\left(n, \tilde{\alpha}, \tilde{\theta}\right)-(1+r)\,n = 0.$$

Moreover, $\alpha\tilde{\theta}-(1+r) > 0$ by Assumption 1. Together, we know that the RHS of (38) is non-negative for $n \leq n^T\left(\tilde{\alpha}\tilde{\theta}\right)$. Therefore, the above condition will not hold for any firm with $\tilde{\alpha} < \alpha$. Finally, condition (37) can be simplified to $\alpha \leq \tilde{\alpha}$ for $n \geq n^T\left(\tilde{\alpha}\tilde{\theta}\right)$, which obviously never holds for firms with $\tilde{\alpha} < \alpha$. Combining the results derived from (36) and (37), we can conclude that given higher internal funds such as $n > n^T(\alpha\theta)$, all firms with higher risk ($\tilde{\alpha} < \alpha$) have the incentive to pretend to be of (α, θ). In sum, the incentive-compatibility condition (11) never holds for any firm at any n, except for *maybe* a firm of the worst quality. The latter is because there does not exist any $\left(\tilde{\alpha}, \tilde{\theta}\right)$ such that $\tilde{\alpha}\tilde{\theta} \leq \underline{\alpha\theta}$ and $\tilde{\alpha} < \underline{\alpha}$.

A.3 Proof of Proposition 4

Re-arrange (13) to obtain $\sigma\theta i^O \geq z^O$ for all $(\alpha, \theta) \in \Omega$. Because this must hold for all $(\alpha, \theta) \in \Omega$, the above condition boils down to $\sigma\theta^\Omega i^O \geq z^O$, where θ^Ω is the lowest level of θ among all of firms in set Ω. Note that θ^Ω is taken as given by the firm. Let λ_1, λ_2 and λ_3 respectively denote the Lagrangian multipliers associated with conditions (13) and (14) and $i^O \leq \bar{\imath}$. The corresponding Lagrangian function is thus given by:

$$\mathcal{L} = \alpha\left(\theta i^O - z^O\right) - (1+r)\,n + \lambda_1\left(\sigma\theta^\Omega i^O - z^O\right) + \lambda_2\left[\alpha^\Omega z^O - (1+r)\left(i^O - n\right)\right] + \lambda_3\left(\bar{\imath} - i^O\right).$$

Assume interior solutions. The first-order conditions are given by:

$$\alpha\theta + \lambda_1\sigma\theta^\Omega - \lambda_2(1+r) - \lambda_3 = 0, \tag{39}$$

$$-\alpha - \lambda_1 + \alpha^\Omega\lambda_2 = 0. \tag{40}$$

We analyze all possible cases: (i) Suppose $\lambda_2 = 0$. Then the left-hand side of (40) is strictly negative and thus this condition cannot hold. Thus it must be true that $\lambda_2 > 0$ and that (14) must bind. (ii) Suppose $\lambda_1 = \lambda_3 = 0$. Then (39) and (40) imply that this case requires $\frac{\alpha\theta}{1+r} = \lambda_2 = \frac{\alpha}{\alpha^\Omega}$. That is, $\theta = (1+r)/\alpha^\Omega$. This is clearly a knife-edge case, which we will ignore without loss of generality. (iii) Suppose $\lambda_3 > 0$ and $\lambda_1 = 0$. Then $\lambda_3 > 0$ implies $i^O = \bar{\imath}$. Recall that $\lambda_2 > 0$, which implies (14) holds with equality, which solves for

$$z^O = (1+r)(\bar{\imath} - n)/\alpha^\Omega.$$

Finally, $\lambda_1 = 0$ implies (13) does not bind and thus $\sigma\theta^\Omega i^O > z^O$. The above requires $n > \bar{\imath}\left(1 - \frac{\sigma\alpha^\Omega\theta^\Omega}{1+r}\right)$. (iv) Suppose $\lambda_1 > 0$ and $\lambda_3 = 0$. Given $\lambda_1, \lambda_2 > 0$, both (13) and (14) bind, and thus we can solve for

$$i^O = \frac{(1+r)\,n}{1+r-\sigma\alpha^\Omega\theta^\Omega} \tag{41}$$

$$z^O = \sigma\theta^\Omega i^O = \frac{(1+r)\,\sigma\theta^\Omega n}{1+r-\sigma\alpha^\Omega\theta^\Omega}. \tag{42}$$

Note that $i^O, z^O > 0$ given the definitions of $\left(\alpha^\Omega, \theta^\Omega\right)$ and Assumption 1. Then $\lambda_3 = 0$ implies $i^O < \bar{\imath}$, which requires $i^O = \frac{(1+r)n}{1+r-\sigma\alpha^\Omega\theta^\Omega} < \bar{\imath}$, that is, $n < \bar{\imath}\left(1 - \frac{\sigma\alpha^\Omega\theta^\Omega}{1+r}\right)$. (v) Suppose $\lambda_1, \lambda_3 > 0$. This, together with $\lambda_2 > 0$, implies $i^O = \frac{(1+r)n}{1+r-\sigma\alpha^\Omega\theta^\Omega} = \bar{\imath}$ which solves for $n = \bar{\imath}\left(1 - \frac{\sigma\alpha^\Omega\theta^\Omega}{1+r}\right)$. Combining all of the above cases, (15) and (16) summarize the optimal opaque contract. Finally, given the binding constraint (14), we can substitute (15) and (16) into the objective function and obtain (18).

A.4 Proof of Proposition 6

For a firm of any (n, α, θ) with $n < n^O$, we have

$$\begin{aligned} & \Pi^O(n,\alpha,\theta) - \Pi^{SF}(n,\alpha,\theta) \\ = \; & \left[\alpha\theta - (1+r) - \sigma\theta^\Omega\left(\alpha - \alpha^\Omega\right)\right]\frac{(1+r)\,n}{1+r-\sigma\alpha^\Omega\theta^\Omega} - \left[\alpha\theta - (1+r)\right]n \\ = \; & \frac{\alpha\sigma\theta^\Omega\left[\alpha^\Omega\theta - (1+r)\right]}{1+r-\sigma\alpha^\Omega\theta^\Omega}n \geq 0, \quad \text{“=” iff } n = 0, \end{aligned} \tag{43}$$

where the inequality is derived given Assumption 1. If $n \geq n^O$, then

$$
\begin{aligned}
& \Pi^O(n,\alpha,\theta) - \Pi^{SF}(n,\alpha,\theta) \\
= \; & \frac{\alpha}{\Omega}\left[\alpha^\Omega\theta - (1+r)\right]\bar{\imath} + (1+r)\, n\frac{\alpha-\alpha^\Omega}{\alpha^\Omega} - \left[\alpha\theta - (1+r)\right] n \\
= \; & \frac{\alpha}{\alpha^\Omega}\left[\alpha^\Omega\theta - (1+r)\right](\bar{\imath} - n) \geq 0, \text{ ``='' iff } n = \bar{\imath}
\end{aligned}
$$

where the inequality is derived given Assumption 1. It follows that $\Pi^O(n) \geq \Pi^{SF}(n)$ for all $n \in [0, \bar{\imath}]$.

A.5 Proof of Proposition 7

Recall from Proposition 3 that the optimal separating contract for a firm with $(\underline{\alpha}, \underline{\theta})$, if it exists, is a subset of its optimal transparent contract because the problems of designing respective contracts involve two identical conditions. Given Proposition 1, we have

$$
\Pi^S(n,\underline{\alpha},\underline{\theta}) = \Pi^T(n,\underline{\alpha},\underline{\theta}) + \gamma = \begin{cases} \left[\underline{\alpha\theta} - (1+r)\right]\frac{1+r}{1+r-\sigma\underline{\alpha\theta}}n, & \text{for } n \leq n^T(\underline{\alpha\theta}) \\ \left[\underline{\alpha\theta} - (1+r)\right]\bar{\imath}, & \text{for } n > n^T(\underline{\alpha\theta}) \end{cases}.
$$

By definition, $\alpha^\Omega \geq \underline{\alpha}$ and $\theta^\Omega \geq \underline{\theta}$. Thus comparing the above with (15), (16) and (18), it is straightforward to show that $\Pi^O\left(n,\underline{\alpha},\underline{\theta};\alpha^\Omega,\theta^\Omega\right) \geq \Pi^S(n,\underline{\alpha},\underline{\theta})$ and the inequality is strict, if $\alpha^\Omega > \underline{\alpha}$.

A.6 Proof of Proposition 8

To compare the respective profits from debt and equity contracts, define

$$
\begin{aligned}
O(n) &\equiv \left[\alpha\theta - (1+r) - \sigma\theta^\Omega\left(\alpha - \alpha^\Omega\right)\right]\frac{1+r}{1+r-\sigma\alpha^\Omega\theta^\Omega}n && (44)\\
\bar{\Pi}^O(n) &\equiv \left[\alpha\theta - \frac{\alpha}{\alpha^\Omega}(1+r)\right]\bar{\imath} + (1+r)\left(\frac{\alpha}{\alpha^\Omega} - 1\right)n && (45)\\
T(n) &\equiv \frac{\alpha\theta - (1+r)}{1+r-\sigma\alpha\theta}(1+r)\, n - \gamma && (46)\\
\bar{\Pi}^T &\equiv \left[\alpha\theta - (1+r)\right]\bar{\imath} - \gamma. && (47)
\end{aligned}
$$

Note that $O(n)$, $\bar{\Pi}^O(n)$ and $T(n)$ are all linear functions of n, and $\bar{\Pi}^T$ is constant in n. Thus both profit functions consist of two linear segments. Together they can be expressed as:

$$\Pi^O(n) = \begin{cases} O(n), & \text{if } n \leq n^O \\ \bar{\Pi}^O(n), & \text{if } n > n^O \end{cases} \tag{48}$$

$$\Pi^T(n) = \begin{cases} T(n), & \text{if } n \leq n^T(\alpha\theta) \\ \bar{\Pi}^T, & \text{if } n > n^T(\alpha\theta). \end{cases} \tag{49}$$

It follows that: (a) It is obvious that $O(0) = 0 > -\gamma = T(0)$ and

$$\begin{aligned} \bar{\Pi}^O(\bar{\imath}) &= \left[\alpha\theta - \frac{\alpha}{\alpha^\Omega}(1+r)\right]\bar{\imath} + (1+r)\left(\frac{\alpha}{\alpha^\Omega} - 1\right)\bar{\imath} \\ &= [\alpha\theta - (1+r)]\,\bar{\imath} \\ &> [\alpha\theta - (1+r)]\,\bar{\imath} - \gamma = \bar{\Pi}^T, \text{ for all } (\alpha,\theta). \end{aligned}$$

That is, $O(0) - T(0) = \gamma = \bar{\Pi}^O(\bar{\imath}) - \bar{\Pi}^T$. Therefore, opaque financing strictly dominates transparent financing in the right neighborhood of $n = 0$ and the left neighborhood of $n = \bar{\imath}$. Moreover, the derivation in (44) implies that given Assumption 1,

$$\begin{aligned} O'(n) &= \left[\alpha\theta - (1+r) - \sigma\theta^\Omega\left(\alpha - \alpha^\Omega\right)\right]\frac{1+r}{1+r-\sigma\alpha^\Omega\theta^\Omega} \\ &> \alpha\theta - (1+r) > 0, \text{ for all } (\alpha,\theta) \end{aligned}$$

and

$$T'(n) = \frac{\alpha\theta - (1+r)}{1+r-\sigma\alpha\theta}(1+r) > 0, \text{ for all } (\alpha,\theta).$$

And obviously, $\bar{\Pi}^{O\prime}(n) > 0$ if $\alpha > \alpha^\Omega$, $\bar{\Pi}^{O\prime}(n) = 0$ if $\alpha = \alpha^\Omega$, and $\bar{\Pi}^{O\prime}(n) < 0$ if $\alpha < \alpha^\Omega$. (b) Recall that the respective kinks from equations (6) and (17). It follows that

$$\begin{cases} n^T(\alpha\theta) > n^O, \text{if } \alpha\theta < \alpha^\Omega\theta^\Omega \\ n^T(\alpha\theta) = n^O, \text{if } \alpha\theta = \alpha^\Omega\theta^\Omega \\ n^T(\alpha\theta) < n^O, \text{if } \alpha\theta > \alpha^\Omega\theta^\Omega. \end{cases} \tag{50}$$

To have some firms using equity, it requires that there exists some $\tilde{n} \in (0, \bar{\imath})$ such that $\Pi^O(\tilde{n}) < \Pi^T(\tilde{n})$. Given the preliminary results obtained above, it boils down to considering the following two cases: (i) If $n^T(\alpha\theta) < n^O$, *i.e.*, $\alpha\theta > \alpha^\Omega\theta^\Omega$, then equity use requires

$O\left(n^T(\alpha\theta)\right) < \bar{\Pi}^T$, which is given by

$$\left[\alpha\theta - (1+r) - \sigma\theta^\Omega\left(\alpha - \alpha^\Omega\right)\right]\frac{1+r}{1+r-\sigma\alpha^\Omega\theta^\Omega}\left(1-\frac{\sigma\alpha\theta}{1+r}\right)\bar{\imath} < \left[\alpha\theta - (1+r)\right]\bar{\imath} - \gamma.$$

This boils down to requiring (21). With these conditions being satisfied, firms with $n \in [n_1(\alpha,\theta), n_2(\alpha,\theta)]$ are better off with transparent financing, where $n_1(\alpha,\theta)$ and $n_2(\alpha,\theta)$ are such that $\Pi^O(n) = \Pi^T(n)$. Given that these profit functions respectively have two linear segments, $n_1(\alpha,\theta)$ and $n_2(\alpha,\theta)$ are both unique. (ii) If $n^T(\alpha\theta) \geq n^O$, *i.e.*, $\alpha\theta \leq \alpha^\Omega\theta^\Omega$, then equity use requires $\bar{\Pi}^O\left(n^T(\alpha\theta)\right) < \bar{\Pi}^T$, that is,

$$\left[\alpha\theta - \frac{\alpha}{\alpha^\Omega}(1+r)\right]\bar{\imath} + (1+r)\left(\frac{\alpha}{\alpha^\Omega}-1\right)\left(1-\frac{\sigma\alpha\theta}{1+r}\right)\bar{\imath} < \left[\alpha\theta-(1+r)\right]\bar{\imath}-\gamma.$$

This is equivalent to requiring (24). Similar to the previous case, firms with $n \in [n_1'(\alpha,\theta), n_2'(\alpha,\theta)]$ are better off with transparent financing and the boundary points $n_1'(\alpha,\theta)$ and $n_2'(\alpha,\theta)$ are both unique.

A.7 Proof of Theorem 1

We will proceed with the proof in four stages described as follows.

A.7.1 Stage 1

In this stage, we will prove that any perfect Bayesian equilibrium (PBE henceforth in this proof) must have $\theta^\Omega = \underline{\theta}$ and $\alpha^\Omega = E[\alpha]$. Recall that $\Omega \subseteq \left[\underline{\theta}, \bar{\theta}\right] \times [\underline{\alpha}, \bar{\alpha}]$ represents the set of quality measures of firms who find opaque debt optimal. By definition, θ^Ω is the lowest θ among firms of set Ω and α^Ω is the expected survival rate of these firms. Note that in general these set characteristics can depend on n, which is a firm's characteristic that is publicly known. In other words, the set of firms participating in opaque debt could vary for different levels of n. Thus this first stage is to prove that $\theta^\Omega(n) = \underline{\theta}$ and $\alpha^\Omega(n) = E[\alpha]$ for all n in PBE. We prove this result in the following four steps:

Step 1: $\theta^\Omega(n) = \underline{\theta}$ and $\alpha^\Omega(n) = E[\alpha]$ for all n that is both in the right neighborhood of 0 *and in the left neighborhood of $\bar{\imath}$.* From the proof of Proposition 8, we have $O(0) > T(0)$ and $\bar{\Pi}^O(\bar{\imath}) > \bar{\Pi}^T$ for all (α,θ). Therefore, for *any* given quality $(\alpha,\theta) \in \left[\underline{\theta}, \bar{\theta}\right] \times [\underline{\alpha}, \bar{\alpha}]$, firms with

n that is both in the right neighborhood of 0 and in the left neighborhood of $\bar{\imath}$ are strictly better off with opaque debt, which yields $\theta^{\Omega}(n) = \underline{\theta}$ and $\alpha^{\Omega}(n) = E[\alpha]$ for all such n.

Step 2: $\theta^{\Omega}(n) = \underline{\theta}$ for all n. Suppose $\theta^{\Omega}(\tilde{n}) > \underline{\theta}$ for some $\tilde{n}$, which means that at $\tilde{n}$ all firms that find the opaque contract dominating the transparent one have $\tilde{\theta} \geq \theta^{\Omega}(\tilde{n}) > \underline{\theta}$. Because the slope of $\Pi^{T}(n;\alpha,\theta)$ strictly increases in θ (see [7]), it follows that $\Pi^{T}\left(\tilde{n};\tilde{\alpha},\tilde{\theta}\right) > \Pi^{T}(\tilde{n};\tilde{\alpha},\theta)$ for any $\theta \in \left[\underline{\theta},\theta^{\Omega}(\tilde{n})\right)$ and any given $\tilde{\alpha}$ of a firm with $\tilde{n}$ that uses opaque contract. That is, the opaque contract must also dominate the transparent contract for a firm with exact same levels of $(\tilde{n},\tilde{\alpha})$, but a lower productivity than $\theta < \theta^{\Omega}(\tilde{n})$. This means $\theta^{\Omega}(\tilde{n})$ cannot be the lowest productivity for firms with $\tilde{n}$ that are better off with opaque debt. This is a contradiction. Therefore there cannot exist any n such that $\theta^{\Omega}(n) > \underline{\theta}$. It must be true that $\theta^{\Omega}(n) = \underline{\theta}$ for all n.

Step 3: There cannot be any n such that $\alpha^{\Omega}(n) > E[\alpha]$. Given $\theta^{\Omega} = \underline{\theta}$ for all n, (23) and (24) boil down to

$$\alpha\theta \leq \underline{\theta}\alpha^{\Omega}(n) \tag{51}$$

$$\frac{\gamma}{\sigma\alpha\theta\bar{\imath}} < \frac{\alpha}{\alpha^{\Omega}(n)} - 1. \tag{52}$$

The first inequality in the above implies

$$\alpha \leq \frac{\underline{\theta}}{\theta}\alpha^{\Omega}(n) \leq \alpha^{\Omega}(n), \quad \text{as } \theta \geq \underline{\theta}.$$

This further implies the right-hand side of (52) is strictly negative and thus (52) never holds for any $\gamma \geq 0$. This establishes that all firms with $\alpha\theta \leq \underline{\theta}\alpha^{\Omega}(n)$, that is, $\alpha \leq (\underline{\theta}/\theta)\,\alpha^{\Omega}(n)$, are strictly better off with opaque debt regardless of their levels of n. This is due to adverse selection. That is, firms with lowest qualities are strictly better off with opaque debt because it helps mask the inferiority of the quality. With the inferior firms always being opaque-debt users, it must be true that $\alpha^{\Omega}(n) \leq E[\alpha]$ for all n. That is, there cannot be any n such that $\alpha^{\Omega}(n) > E[\alpha]$.

Step 4: There does not exist any n such that $\alpha^{\Omega}(n) < E[\alpha]$. Therefore, it must be true that $\alpha^{\Omega}(n) = E[\alpha]$ for all n. Suppose there exists some $\tilde{n} \in (0,\bar{\imath})$ such that $\alpha^{\Omega}(\tilde{n}) < E[\alpha]$. Recall (18) and it is straightforward to verify that Π^{O} strictly increases in α^{Ω} for all n.

Moreover, Π^O is linear in n. Given $\theta^\Omega(n) = \underline{\theta}$ for all n, it follows that an opaque-debt-financing firm with $\tilde{n}$ such that $\alpha^\Omega(\tilde{n}) < E[\alpha]$ is strictly better off splitting its funds $\tilde{n}$ into small amounts that qualify for $\alpha^\Omega(n) = E[\alpha]$. We know such small amounts exist because all n in the right neighborhood of zero have $\alpha^\Omega(n) = E[\alpha]$. Because Π^O strictly increases in α^Ω and is linear in n, splitting investment into the smaller pieces will only provide such firms a strictly higher profit because they get to enjoy a strictly higher α^Ω with these smaller investments. This means the set of firms using opaque debt is empty at such $\tilde{n}$, which is a contradiction. Therefore, there cannot be any n such that $\alpha^\Omega(n) < E[\alpha]$ in a PBE. In other words, it must be true that $\alpha^\Omega(n) = E[\alpha]$ for all n.

In sum, the only possible PBE must have $\theta^\Omega(n) = \underline{\theta}$ and $\alpha^\Omega(n) = E[\alpha]$ for all n, if it exists. Given $\alpha^\Omega = E(\alpha) > \underline{\alpha}$, Proposition 7 implies that the separating contract is strictly dominated by the opaque contract for $(\underline{\alpha}, \underline{\theta})$. This result, together with Proposition 3 show that no firm uses a separating contract in equilibrium.

Given $\theta^\Omega = \underline{\theta}$ and $\alpha^\Omega = E[\alpha]$, it is obvious from Propositions 1 and 4 that the optimal transparent contract and the optimal opaque contract are both unique for all qualifying (n, α, θ). We will show in the rest of the proof that all other equilibrium variables are uniquely determined, too. Thus, the PBE must be unique if it exists. We will also derive the condition under which the PBE exists later in this proof.

A.7.2 Stage 2

As the second stage, we will prove parts (ii) - (iii) of the theorem. Recall that given $\theta^\Omega = \underline{\theta}$ and $\alpha^\Omega = E[\alpha]$, all firms with $\alpha\theta \leq \underline{\theta}E[\alpha]$ are strictly better off with opaque debt regardless of their levels of n. Moreover, conditions (20) and (21) become

$$\alpha\theta > \underline{\theta}E[\alpha] \tag{53}$$

$$\frac{\gamma}{\sigma\alpha\theta\bar{\imath}} < \frac{(\alpha\theta - \underline{\theta}E[\alpha]) - (1+r)(1-\underline{\theta}/\theta) - \sigma\underline{\theta}(\alpha - E[\alpha])}{1 + r - \sigma\underline{\theta}E[\alpha]}. \tag{54}$$

These two conditions boil down to (27) by re-arranging (54). Note that (27) implies

$$\bar{\imath}\left(1 - \frac{\sigma\alpha\theta}{1+r}\right) = n^T(\alpha\theta) < n^O = \bar{\imath}\left(1 - \frac{\sigma\underline{\theta}E[\alpha]}{1+r}\right)$$

by (50) because $\alpha\theta > \underline{\theta}E[\alpha]$. Then according to Proposition 8, a firm with (α,θ) that satisfies condition (27), there exists a unique set $[n_1(\alpha,\theta), n_2(\alpha,\theta)] \subset [0,\bar{\imath}]$, such that the firm uses transparent financing if $n \in [n_1(\alpha,\theta), n_2(\alpha,\theta)]$, and opaque debt financing otherwise. Among these firms, those with $n \in \left[n_1(\alpha,\theta), n^T(\alpha\theta)\right]$ choose equity or transparent debt and those with $\left(n^T(\alpha\theta), n_2(\alpha,\theta)\right]$ transparent debt financing. All firms that do not satisfy condition (27) use debt financing in equilibrium. Note that $n_1(\alpha,\theta)$ solves $O(n_1) = T(n_1)$ and $n_2(\alpha,\theta)$ solves $O(n_2) = \bar{\Pi}^T$ where $O(\cdot)$ and $T(\cdot)$ are defined by (44) and (46) and are both linear in n. It is straightforward to derive

$$n_1(\alpha,\theta) = \frac{\gamma}{\frac{\alpha\theta-(1+r)}{1+r-\sigma\alpha\theta}(1+r) - [\alpha\theta-(1+r)-\sigma\underline{\theta}(\alpha-E[\alpha])]\frac{1+r}{1+r-\sigma\underline{\theta}E[\alpha]}} \quad (55)$$

$$n_2(\alpha,\theta) = \frac{[\alpha\theta-(1+r)]\bar{\imath}-\gamma}{[\alpha\theta-(1+r)-\sigma\underline{\theta}(\alpha-E[\alpha])]\frac{1+r}{1+r-\sigma\underline{\theta}E[\alpha]}}, \quad (56)$$

which are obviously unique boundary points.

A.7.3 Stage 3

In the third stage, we derive the existence condition for PBE. The result of $\theta^\Omega = \underline{\theta}$ and $\alpha^\Omega = E[\alpha]$ means that for any given n, firms with $\underline{\theta}$ and any $\alpha \in [\underline{\alpha},\bar{\alpha}]$ must be strictly better off with opaque debt. Recall that all firms with $\alpha\theta \leq \underline{\theta}E[\alpha]$ are strictly better off with opaque debt regardless of their levels of n. This implies that given $\underline{\theta}$, all firms with $\alpha \leq E[\alpha]$ are strictly better off with opaque debt. Thus to ensure that $\alpha^\Omega = E[\alpha]$, it suffices to show that condition (54) does not hold for firms with $\underline{\theta}$ and $\alpha > E[\alpha]$, which boils down to

$$\frac{\gamma}{\sigma\alpha\underline{\theta}\bar{\imath}} \geq \frac{(1-\sigma)\underline{\theta}(\alpha-E[\alpha])}{1+r-\sigma\underline{\theta}E[\alpha]}.$$

Note that the left-hand side of the above strictly decreases in α and the RHS strictly increase. Thus it suffices to require

$$\frac{\gamma}{\sigma\bar{\alpha}\underline{\theta}\bar{\imath}} \geq \frac{(1-\sigma)\underline{\theta}(\bar{\alpha}-E[\alpha])}{1+r-\sigma\underline{\theta}E[\alpha]}.$$

This condition ensures that the transparent-financing condition, (54), does not hold for any $\alpha \in [\underline{\alpha},\bar{\alpha}]$ given $\underline{\theta}$.

A.7.4 Stage 4

Finally, we prove part (iv) of the theorem. Denote the equilibrium set of equity users:

$$\mathcal{E}\left(\gamma\right)=\left\{\left(\alpha,\theta,n\right)\in\left[\underline{\alpha},\bar{\alpha}\right]\times\left[\underline{\theta},\bar{\theta}\right]\times\left[0,\bar{\imath}\right]:\left(\alpha,\theta\right)\text{ satisfies (27); }\Pi^{T}\left(n;\alpha,\theta\right)>\Pi^{O}\left(n;\alpha,\theta\right)\right\}$$

First note that condition (27) is less likely to hold as γ increases. Moreover, it is clear from equations (44) to (47) that the function $\Pi^{O}\left(n\right)$ is independent of γ, while $\Pi^{T}\left(n\right)$ strictly decreases in γ. Thus as γ rises, the set $\left[n_1\left(\alpha,\theta\right),n_2\left(\alpha,\theta\right)\right]$ shrinks for any $\left(\alpha,\theta\right)$ such that condition (27) is satisfied. Altogether, we have that an increase in γ will make fewer quality measures to qualify for equity financing, and at the same time, for those that qualify a smaller range of internal financing such that it is optimal to use equity financing. Finally, it is obvious that there exists some $\bar{\gamma}\in\left(0,\infty\right)$ such that no quality $\left(\alpha,\theta\right)$ satisfies condition (27), and thus no firm uses equity financing.

A.8 Proof of Proposition 10

For all firms that are strictly better off with opaque debt or transparent debt, it is straightforward to calculate $D/E=\left(i^{j}-n\right)/n$, where $j=O,T$. The numerator is the amount of debt and the denominator the amount of internal equity. Given the optimal contracts $\left(i^{T},z^{T}\right)$ and $\left(i^{O},z^{O}\right)$ established by Propositions 1 and 4, the above leads to (28), and (29) for all firms except those with $n\in\left[n_1\left(\alpha,\theta\right),n^{T}\left(\alpha\theta\right)\right]$. For those with $n\in\left[n_1\left(\alpha,\theta\right),n^{T}\left(\alpha\theta\right)\right]$, they are indifferent between an equity contract, a transparent-debt contract or any linear combination of both. The last case is because the optimal contract here, $\left(i^{T},z^{T}\right)$, is linear in n. External financing with pure equity has a debt-to-equity ratio of zero because the capital structure consists of internal and external equity only. Financing with pure transparent debt has the debt-to-equity ratio being $\frac{\sigma\alpha\theta}{1+r-\sigma\alpha\theta}$, which is calculated given $\left(i^{T},z^{T}\right)$. Thus any hybrid of debt and equity financing for this type of firms renders a ratio between 0 and $\frac{\sigma\alpha\theta}{1+r-\sigma\alpha\theta}$. All of these ratios are optimal because of the firm's indifference between equity and transparent debt.

A.9 Effects on the Optimal D/E Ratio

Figure 3 displays the respective effects of σ, $\underline{\theta}$, $E[\alpha]$, $\bar{\imath}$, and r on the optimal debt-to-equity ratio for firms that are strictly better-off with opaque debt. Figure 3a illustrates the effects of a change in σ, $\underline{\theta}$, or $E[\alpha]$. An increase in σ implies that the investor could recover a higher fraction of the total investment times $\underline{\theta}$. When $\underline{\theta}$ increases, the lowest-productivity project becomes more productive. An increase in $E[\alpha]$ causes the projects to be more likely to succeed on average. All three effects means that the size of the loan made by the investor will increase for a given level of internal funds. Therefore, the optimal debt-to-equity ratio rises when the amount of internal funds is less than the threshold n^O. A firm can now reach the maximum amount of investment with a smaller amount of internal funds since the size of the loan made by the investor will increase. Therefore, the threshold n^O will shift to the left. If the internal funds is at least n^O, there is no effect on the optimal debt-to-equity ratio. Figure 3b displays the effects of a change in $\bar{\imath}$. A rise in $\bar{\imath}$ means that a firm will need a larger amount of internal funds to reach the maximum amount of investment. This will shift the threshold n^O to the right. Also, a rise in $\bar{\imath}$ will increase the slope of the curve for the optimal debt-to-equity ratio when the internal funds are above the threshold n^O. Figure 3c shows the effects of a change in r. An increase in r means that the outside option of the investor improves. Therefore, the firm will have to pay more for each unit of funds provided by the investor. Furthermore, an increase in r has the effect to shift the threshold n^O to the right since the size of the loan made by the investor decreases for a given level of internal funds. Reaching the maximum level of investment will therefore require more internal funds. If the amount of internal funds is above the threshold, there is no effect.

Figures 4, 5, 6, and 7 display the effects of the characteristics analyzed in Figure 3, the verification cost (γ), the success rate (α), and the productivity (θ) on the optimal debt-to-equity ratio for a firm that finances with either the transparent contract or the opaque contract, depending on the amount of internal funds. Table 2 summarizes how the thresholds n^O, $n^T(\alpha\theta)$, $n_1(\alpha,\theta)$, and $n_2(\alpha,\theta)$ vary according to the changes in the characteristics mentioned earlier. Figure 4a shows the effects of a change in γ. An increase in γ means that using the transparent contract is more costly. Therefore, the interval, in terms of internal funds, for which the transparent contract gives a higher expected profit than the opaque contract shrinks. Thus, the threshold $n_1(\alpha, \theta)$ increases and the threshold $n_2(\alpha, \theta)$ decreases. Everything else remains the same. Figure 4b illustrates the effects of a change in $\underline{\theta}$ and $E[\alpha]$. An increase in one of the two does not affect the optimal debt-to-equity ratio of a firm that keeps the transparent contract as the optimal financing method. However, the rise means that the size of the loan made by the investor increases for a given level of internal funds. This has the effect of making the transparent contract optimal for a shorter interval which means that the threshold $n_1(\alpha, \theta)$ increases and the threshold $n_2(\alpha, \theta)$ decreases. Furthermore, as discussed earlier the debt-to-equity ratio will increase for those that find the opaque contract optimal and the threshold n^O decreases. Figure 4c shows the effects of a change in r. An increase in r means that the outside option of the investor increases. Therefore, for a given level of internal funds, the amount of investment received through debt, or equity will both decrease. This fall will decrease the optimal debt-to-equity ratio. Furthermore, both thresholds n^O and $n^T(\alpha\theta)$ shift to the right since it will require more internal funds to reach the maximum level of investment. Also, both thresholds $n_1(\alpha, \theta)$ and $n_2(\alpha, \theta)$ increase. Figure 4d displays the effects of a change in $\bar{\imath}$. An increase in $\bar{\imath}$ means that a firm will need more internal funds to reach the maximum level of investments. This shifts both thresholds n^O and $n^T(\alpha\theta)$ to the right. Also, this will make the slope of the curve steeper between $n^T(\alpha\theta)$ and $n_2(\alpha, \theta)$ as well as between n^O and $\bar{\imath}$.

Figure 5 illustrates the effects of a change in σ conditional on its effects on the threshold $n_1(\alpha, \theta)$. An increase in σ can reduce, increase, or not affect $n_1(\alpha, \theta)$. Also, it implies that the investor gets a higher repayment which means that the amount invested in the project rises for a given level of internal funds. Therefore, the optimal debt-to-equity ratio increases if the firm chooses debt. Furthermore, both thresholds n^O and $n^T(\alpha\theta)$ decrease since it takes less internal funds to reach the maximum amount of investment.

Figure 6 displays the effects of a change in α conditional on its effects on the threshold $n_1(\alpha, \theta)$. An increase in α has no clear impact on the change in direction of $n_1(\alpha, \theta)$.

Table 2: **Comparative Statics for a firm that finances with either transparent or opaque contract depending on n**

Boundary	γ	σ	$\bar{\imath}$	$\underline{\theta}, E[\alpha]$	r	α	θ
$n_1(\alpha, \theta)$	+	+/−	No	+	+	+/−	+/−
$n^T(\alpha\theta)$	No	−	+	No	+	−	−
$n_2(\alpha, \theta)$	−	−	+	−	+	+	+/−
n^O	No	−	+	−	+	No	No

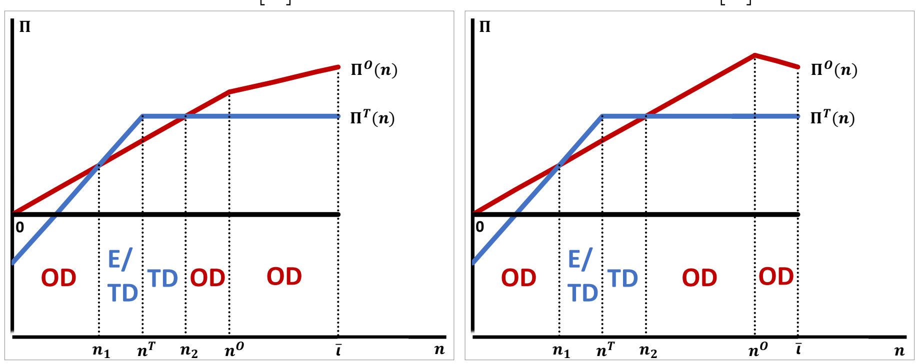
Also, a change in α has no direct effect on the optimal debt-to-equity ratio of the opaque contract. Because a firm has more chance to be successful, the upper bound of the interval $[n_1(\alpha, \theta), n_2(\alpha, \theta)]$ increases. The repayment to the lender if the firm chooses transparent debt increases for a given amount of internal funds. Therefore, the amount invested by the lender increases for a given level of internal funds which raises the optimal debt-to-equity ratio if the firm chooses transparent debt. Lastly, the rise in the investment for a given level of internal funds reduces the amount of internal funds needed to reach the maximum level of investment.

Figure 7 shows the effects of a change in θ conditional on its effects on the thresholds $n_1(\alpha, \theta)$ and $n_2(\alpha, \theta)$. An increase in θ means that the investor gets a higher repayment and so the amount invested in the project increases for a given level of internal funds. Therefore, the optimal debt-to-equity ratio rises and the threshold $n^T(\alpha\theta)$ decreases.

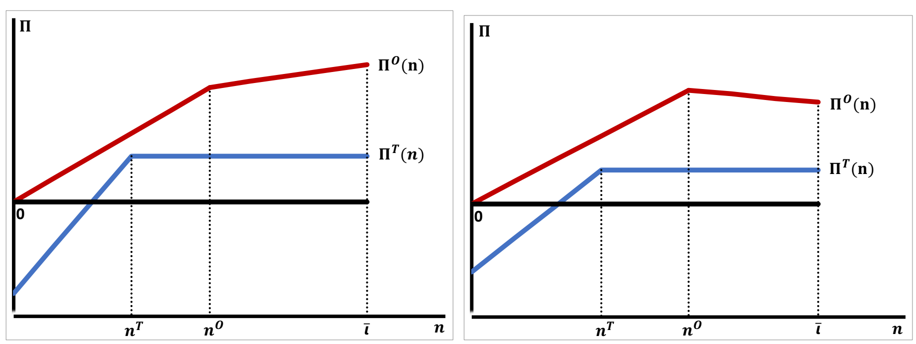
A.10 Figures

Figure 1: **Optimal Financing Choice of Individual Firms**

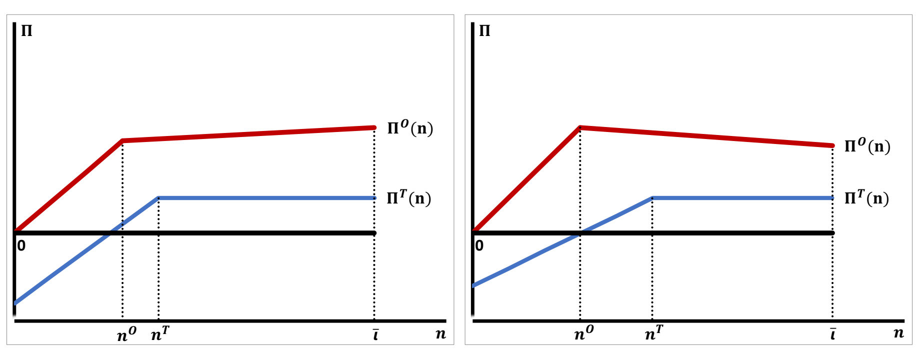
$\alpha < E[\alpha]$ $\alpha > E[\alpha]$



(a) **Case 1**

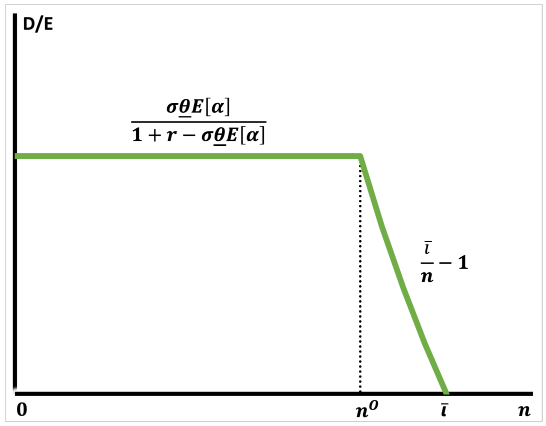


(b) **Case 2**

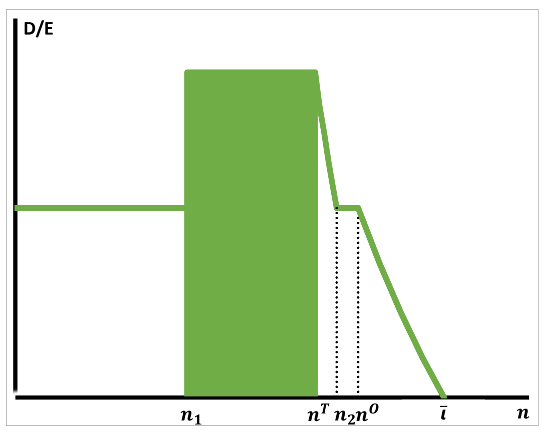


(c) **Case 3**

Figure 2: **Optimal Debt-to-Equity Ratios**

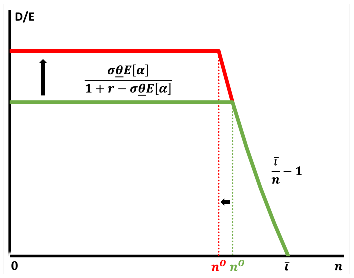


(a) **A firm strictly better off with opaque debt**

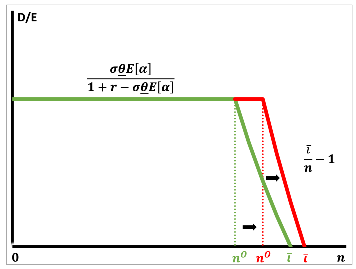


(b) **A firm that finances with either transparent or opaque contract depending on** n

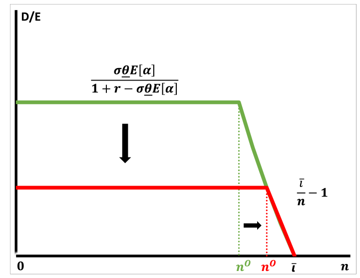
Figure 3: **Comparative Statics for a Firm Strictly Better Off with Opaque Debt**



(a) **Effects from an increase in σ, $\underline{\theta}$, or $E[\alpha]$**

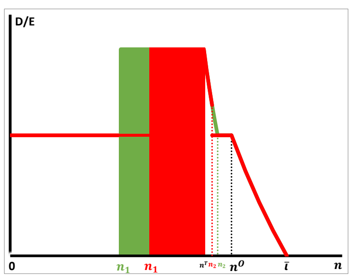


(b) **Effects from an increase in $\bar{\iota}$**

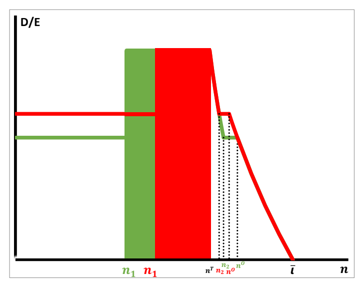


(c) **Effects from an increase in r**

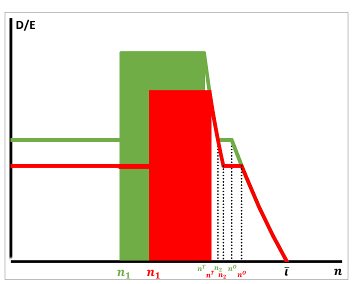
Figure 4: **Comparative Statics for a Firm That Finances with Either Transparent or Opaque Contract Depending on n**



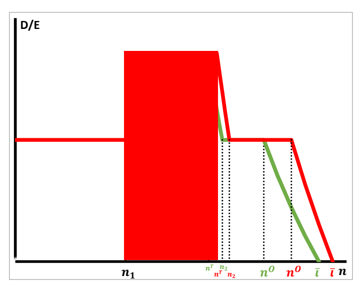
(a) **Effects from an increase in γ**



(b) **Effects from an increase in $\underline{\theta}$, or $E[\alpha]$**

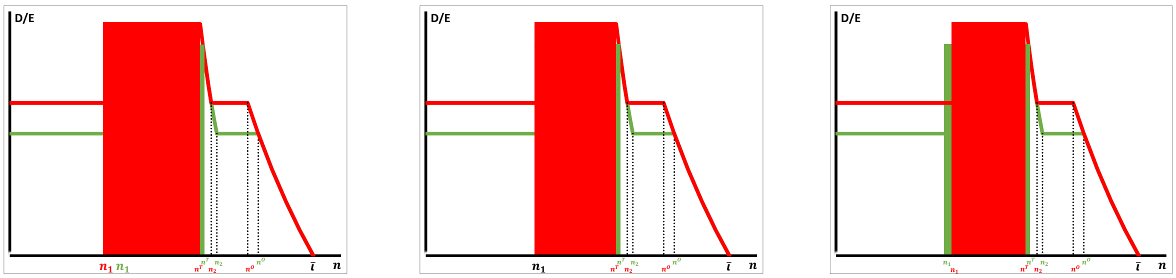


(c) **Effects from an increase in r**



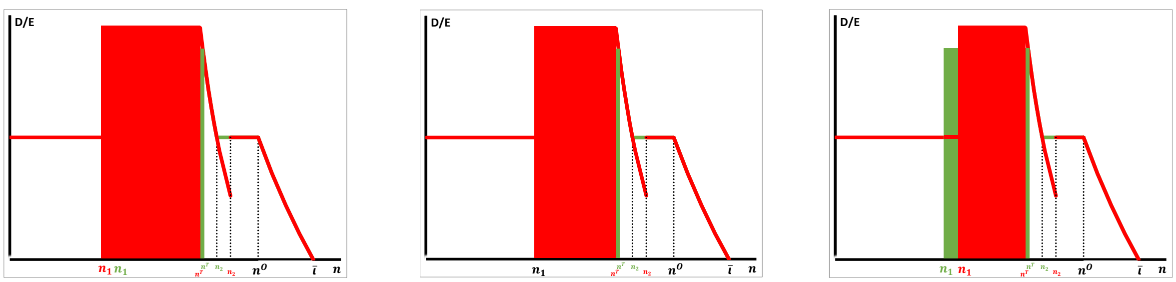
(d) **Effects from an increase in $\bar{\iota}$**

Figure 5: **Effects of an Increase in σ on a Firm That Finances with Either Transparent or Opaque Contract Depending on n**



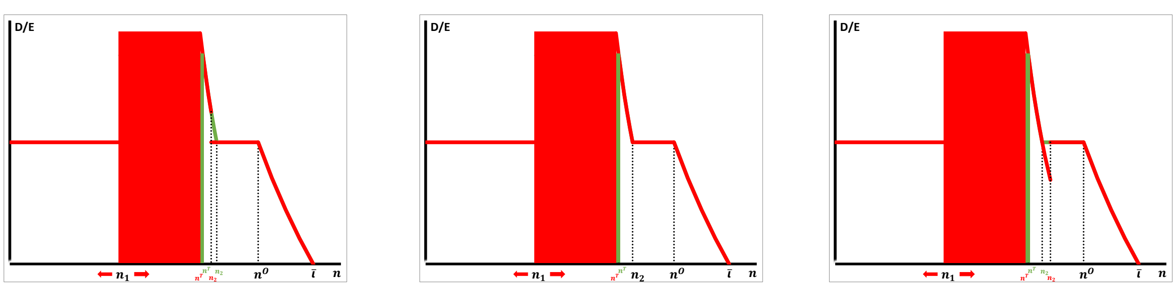
(a) **Negative increase in n_1** (b) **No change in n_1** (c) **Positive increase in n_1**

Figure 6: **Effects of an Increase in α on a Firm That Finances with Either Transparent or Opaque Contract Depending on n**



(a) **Negative increase in n_1** (b) **No change in n_1** (c) **Positive increase in n_1**

Figure 7: **Effects of an Increase in θ on a Firm That Finances with Either Transparent or Opaque Contract Depending on n**



(a) **Negative increase in n_2** (b) **No change in n_2** (c) **Positive increase in n_2**