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Money and Imperfectly Competitive Credit

Allen Head
Queen's University

Timothy Kam
Australian National University

Sam Ng
Australian National University

Isaac Pan
University of Sydney

Department of Economics
Queen's University
94 University Avenue
Kingston, Ontario, Canada
K7L 3N6

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Allen **Head**† Timothy **Kam**‡ Sam **Ng**§ Guangqian **Pan**¶

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Abstract

We develop a monetary economy in which market power in lending is endogenous and responds to policy. The theory can account for both dispersion of loan interest rates and incomplete pass-through of monetary policy to them. The model implies positive and negative relationships, respectively, between the dispersion of loan rate spreads as measured by their standard deviation and coefficient of variation and the average spread. This is a distinguishing feature of our search-based theory of market power and is consistent with new micro-level evidence on U.S. consumer loans. Imperfect competition in lending also creates a novel channel from monetary policy to loan-rate spreads, and thus, to real consumption and welfare. At low inflation, banks tend to demand higher rates from existing loan customers rather than compete for additional loans. As a result, banking activity need not improve welfare if inflation is sufficiently low. Under a given inflation target, welfare gains arise if a central bank uses state-contingent monetary injections (or manages the nominal interest rate) to reduce lenders' market power when aggregate demand is high at the cost of allowing it increase when demand is low.

JEL codes: E41; E44; E51; E63; G21
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†Queen's University. E-mail: heada@econ.queensu.ca
‡Australian National University; Sungkyunkwan University. E-mail: tcy.kam@gmail.com
§University of Melbourne. E-mail: samiengmanng@gmail.com
¶University of Sydney. E-mail: guangqian.pan@sydney.edu.au

1 Introduction

In this paper we construct a theory of dispersion in loan rates associated with market power in lending in a general equilibrium monetary economy. Imperfect information regarding available loan rates generates market power and loan rate dispersion in equilibrium, and this has implications for welfare and policy. There is imperfect pass-through of both shocks and monetary policy to loan rates in equilibrium, and a stabilization policy can improve welfare by offsetting to an extent the resulting fluctuations in loan spreads. Banking plays a potentially welfare-improving role in the economy by reallocating liquidity across agents in response to both idiosyncratic and aggregate shocks. Market power in lending, however, may result in welfare losses in goods markets sufficient to more than offset these gains, particularly in low-inflation environments.

Our work is motivated both by the observation of dispersion in loan rates and evidence of market power in banking and financial services. First, we measure dispersion in *posted* loan rates for identical consumer loan products, controlling for geography and other confounding factors.[1] We refer to the remainder or unexplained dispersion as *residual* or *orthogonalized dispersion* in loan rates. We view the dispersion of loan rates, how it responds to changes in demand and to monetary policy as an interesting phenomenon in its own right: By generating differences in borrowing costs across households, the equilibrium dispersion affects welfare directly. We also see dispersion in loan rates and spreads as indications of the existence of market power.

In that vein, the literature has identified sizable profit margins in the banking sector (with mark-ups approaching 90%) and imperfect pass-through (with a Rosse-Panzar H-statistic of 50%).[2] It is also well documented that there is substantial concentration in the banking industry in all developed countries. For example, post-2007, the market share of the top-three banks is about 78% in Germany, roughly 58% in the U.K., 44% in Japan, and 35% in the U.S.. These statistics are averages across annual 2007-2019 time-series data from Bankscope (2020).[3]

A large literature has studied aspects of the nature and implications of market power in banking. Several authors have identified substantial spreads in both lending (in the form of loan-rate spreads over bank funding costs) and in bank funding (in the form of deposit "markdowns"). See, for example, Wang, Whited, Wu and Xiao (2022). Others have studied links between market power in the financial sector and macroeconomic stability (*e.g.*, Brunnermeier and Sannikov (2014), Coimbra, Kim and Rey (2022), and Corbae and Levine (2022)). In this paper we focus on the *dispersion* of loan rates specifically and its links to market power. This distinguishes our work from most of the theoretical literature, which generally focuses on cases with uniform lending and deposit rates.

There has also been increased policy interest in the link between market power in the financial sector and monetary policy in both the academic literature (*e.g.*, Duval, Furceri, Lee and Tavares (2021), Godl-

[1] Martín-Oliver, Vicente and Saurina (2007) and Martín-Oliver, Salas-Fumás and Saurina (2009) also find price dispersion in loan rates for identical loan products in the case of Spanish banks.

[2] See Corbae and D'Erasmo (2015, 2018). The Rosse-Panzar H-statistic measures the degree of competition in the banking market. It measures the elasticity of banks' revenues relative to input prices. Under perfect competition, the H-statistic equals one since an increase in input prices raise marginal cost and total revenue by the same amount. Under a monopoly, an increase in input prices raises marginal costs, lowers output and revenues, so that the H-statistic is less than or equal to zero. When the H-statistic is between zero and unity, one usually presumes a monopolistically competitive industry.

[3] In this paper, however, we do not focus on concentration as notions of market share and oligopolistic market structure are beyond the scope of our theory.

Hanisch (2022), Bellifeime, Jamilov and Monacelli (2022) and Wang (2022)) and policy circles (*e.g.*, Sims (2016), Productivity Commission (2018), Wilkins (2019), and Executive Order 14036 (2021) for Australia, Canada and the U.S.). Specifically, U.S. President Biden Executive Order 14036 (2021) has recently called for, *inter alia*, a review of banking-sector market power with the following motivation:

> [O]ver the last several decades, as industries have consolidated, competition has weakened in too many markets .... [F]ederal Government inaction has contributed to these problems, with workers, farmers, small businesses, and consumers paying the price. ...
>
> [I]n the financial-services sector, consumers pay steep and often hidden fees ... .

Likewise, in other countries such as Australia. Sims (2016), Chairperson of the *Australian Competition and Consumer Commission*, commented that:

> [I]t is the way market participants gain, maintain and use their market power that may lead to poor consumer outcomes. . . . Reforms that alter *incentives of banks* [*sic*] ... [a]imed directly at bolstering consumer power in markets, and reforms to the governance of the financial system, should be the prime focus of policy action.

A prevailing view, evidently, is that market power matters for monetary policy and welfare.

In this paper, we focus on the liquidity transformation role of banks (*i.e.*, of insuring households against liquidity risk) and how it is affected by market power in *lending*. We characterize the effects of inflation on the dispersion of loan rates and spreads and ask if and how monetary policy should respond. Specifically, in a setting where bank market power is itself an *equilibrium phenomenon* we ask: How does imperfect competition among lenders affect the pass-through of monetary policy to the distribution of loan rates? And, in the presence of market power, is it always the case that banks—as insurers of liquidity risk—improve economic welfare? Finally, if market power in banking responds endogenously to policy, should monetary policy attempt to temper the response of banks' loan rate spreads to aggregate demand shocks?

Our baseline model inherits the following features from that of Berentsen, Camera and Waller (2007) (BCW): In each period, there are two sequential markets. We will follow the New Monetarist tradition and label the first as the *decentralized market* (DM). In the DM, agents are anonymous and lack the ability to commit to honoring private contracts. This contractual friction is the source of market incompleteness and renders fiat money valuable for sustaining private exchange. Also, in the DM, households that are sellers transform their own labor services into goods on the spot, taking fiat money as payment from buyers of their goods.

The second market is a *centralized market* (CM) that has Walrasian features. The CM enables *ex-post* heterogeneous agents to re-balance their asset portfolios through labor supply and consumption demand choices. Households produce a homogeneous good using their own labor and sell to other households in perfectly competitive market. The government implements monetary and tax policies in the DM and CM. Absent banking institutions, the model is isomorphic to a DM price-taking variant in Rocheteau and Wright (2005).

Our focus on banks as intermediaries between agents with different liquidity needs is similar to BCW. Heterogeneity among depositors and borrowers arises because *ex-ante* identical households experience idiosyncratic shocks that determine whether an agent is an *ex-post* "active" or an "inactive" buyer. "Inactive" buyers end up holding idle money which they can deposit in the banking system to avoid the inflation tax. "Active" buyers consume using previously acquired money and may also borrow from banks. Repayments of loans and returns on deposits occur in the CM. Banks can perfectly monitor and enforce these contracts thus help to insure individual liquidity risks by accepting nominal deposits and extending loans in the DM.

We depart from the model of BCW in one respect: Whereas BCW study a perfectly competitive credit market, in our environment active buyers must search among banks for potential lines of credit. We adapt the noisy consumer search model of Burdett and Judd (1983) to rationalize policy-dependent market power, which in turn, induces loan-rate dispersion that have welfare consequences. Banks post loan rates anticipating that their potential customers will observe only a random sample of posted rates. In equilibrium, active buyers who have made contact with at least one lender choose optimally whether and how much to borrow. This equilibrium mechanism implies *ex-post* heterogeneity among active buyers depending on whether they have the opportunity to borrow and the rate at which they borrow.

For simplicity, our model features loan-rate heterogeneity driven solely from the optimal loan-pricing strategy of banks, all of which have the same cost of funds. First, banks engage in perfect competition for deposits. Second, we assume that to the extent that a given bank's loan demand exceeds its deposits, it can raise funds in an external interbank market at a constant rate. Thus we abstract from both imperfect competition for deposits, which are studied by Andolfatto (2021) and others, and from frictions in the interbank market, which are studied in detail by Bianchi and Bigio (2022), and others.[4] Our model thus nests BCW's equilibrium with perfect competition in banking as the limit when all active buyers have at least two borrowing opportunities and monopoly banking as the other extreme when all active buyers have at most one such opportunity.

Calibrated to match aggregate money demand and the average loan rate spread, the model is consistent with observed relationships between the dispersion and average level of the spread. Using publicly-available U.S. data, including loan-rates at the bank branch level, we find new empirical evidence at both the national and state levels, of positive (negative) relationships between the standard deviation (coefficient of variation) of bank loan rate spreads and their averages.[5]

In the theory, inflation affects market power in lending through a novel channel and we identify the possibility of a negative welfare effect of banking at low inflation rates stemming directly from imperfect competition. Effectively, imperfect competition among lenders distorts and potentially overcomes the traditional welfare-improving role of banks in transforming liquidity. At low inflation rates, an increase in inflation causes the dispersion of loan rates to diminish as banks exploit their ability to raise rates (the *intensive margin*) at the expense of attracting borrowers (the *extensive margin*). In this way banks effectively extract surplus from consumers in goods trades. As agents' need to insure liquidity risk is low at low inflation, welfare in an economy with active banks can thus be lower than if there were no banking at all. As inflation rises, insurance of liquidity risk becomes more important and banks respond to increased loan demand by reducing their lending spreads in order to attract more loan customers. In such cases, banks improve welfare just as they do in BCW's environment with perfect competition.

Chiu, Dong and Shao (2018) also identify a potentially negative welfare effect of banking, but for a different reason. In their setting banks are competitive, and a pecuniary externality may arise if "too

[4]We also study imperfect competition for deposits in Head, Kam, Ng and Pan (2022).
[5]Although not the focus of this paper, we have also documented similar evidence in terms of mortgage loan rates.

many" agents have access to credit, raising marginal cost and thus prices. In contrast, our result is driven by the market power of banks in equilibrium. Policy can diminish or even eliminate the effect, and under perfect competition banking activity can never be welfare-reducing.[6]

Within the New Monetarist framework our model effectively endogenizes the notion of costly credit in Wang, Wright and Liu (2020) along the lines of BCW. As we nest Bertrand pricing as a parametric limit, we can replicate the competitive banking setting of BCW as a special case. We thus decompose money demand into several components, one capturing the liquidity insurance role of banks that is identical to that arising in BCW. We also, however, isolate new marginal benefit and cost terms that capture the effects of equilibrium market power and its attendant loan rate risk on agents' decisions to accumulate money.

We then use this decomposition of money demand to show the effects of monetary policies that redistribute liquidity. We study this in a version of the model with shocks to aggregate demand, an exercise in the spirit of Berentsen and Waller (2011) and Boel and Waller (2019). We shut down the fluctuations in the deposit rate that are the focus of Berentsen and Waller (2011) and isolate welfare improvements arising from the effects of policy on market power in bank lending. With perfectly competitive lending, while redistributive tax instruments do not directly affect individual agents' money demand in equilibrium, they are useful for counteracting sub-optimal interest rate movements by raising the deposit rate when aggregate demand is low (Berentsen and Waller, 2011). Here, in contrast, the optimal stabilization policy exploits the endogeneity of market power in banking.

While both in Berentsen and Waller (2011) and here optimal policy redistributes liquidity among *ex-post* heterogeneous agents and is akin to the maintenance of an "elastic currency" it does so through different channels in the two settings.[7] Whereas in Berentsen and Waller (2011) the optimal policy improves welfare by counteracting sub-optimal interest rate movements, here it counteracts movements in interest rate spreads. Specifically, it reduces lenders' market power (lowering the average spread) in periods of high aggregate demand and allows it to increase when demand is low.

Several other recent papers also consider imperfect competition in banking. Drechsler, Savov and Schnabl (2017) show that banks' ability to mark down on deposits is empirically important. Choi and Rocheteau (2021) assume depositors have private information in deposit-contracts bargaining. This allows banks to second-degree price discriminate. The authors use this mechanism to rationalize the bank-deposit channel of monetary policy documented in Drechsler et al. (2017).

Alternatively, others have studied oligopoly in the banking industry. Corbae and D'Erasmo (2021) model a market structure where big banks interact with small fringe banks and other non-bank lenders. The theory can generate an empirically relevant bank-size distribution and shows how regulatory policies

[6]Specifically, in Chiu et al. (2018), having more agents with access to credit results in more DM consumption demand which raises marginal cost and thus the goods price (under competitive pricing or under a more general mechanism design approach). A necessary condition for their result is that the marginal cost of producing the DM good is strictly increasing. The resulting higher goods price under perfect competition effectively tightens the liquidity constraint of agents who have to pay for goods using money. While in principle a similar result could arise in our environment, it is precluded by our assumption of constant marginal production cost in the DM. As such, the potentially negative welfare effects of lending in our model arises only from the loan spreads that arise in equilibrium, and depend on inflation and active monetary policy.

[7]This harks back to the *Aldrich-Vreeland Act* of 1908. The Act was enacted to implement elastic or emergency currency in response to the Bankers' Panic or Knickerbocker Crisis of 1907. The Act also led to the creation of a decentralized Federal Reserve model under the *Federal Reserve Act* of 1913. In the official title of the *Federal Reserve Act*, one finds the phrase: "[A]n Act to provide for the establishment of Federal reserve banks, to furnish an elastic currency ... [*etc*] (*sic*)." (We thank Randy Wright for suggesting this interpretation.)

affect banking stability through the market structure. Altermatt and Wang (2021) show how oligopoly among banks affects both the monetary policy transmission mechanism and bank defaults. Dong, Huangfu, Sun and Zhou (2021), endogenize the number of banks. Chiu, Davoodalhosseini, Jiang and Zhu (2019) focus on oligopolistic competition on the deposit side to study the effects of central bank digital currency. Our approach complements these papers by accounting for imperfect competition on the lending side (see also, Allen, Clark and Houde, 2019; Clark, Houde and Kastl, 2021, for more evidence using mortgage data).

Our theory is distinguished by its focus on the equilibrium extent of market power embodied in the distribution of loan rate spreads, whereas most others feature homogeneous bank lending rates. An exception is Corbae and D'Erasmo (2021) who study a non-monetary model in which banks face exogenous idiosyncratic and aggregate shocks. There, banks economize on monitoring costs and diversify borrowers' idiosyncratic project risk. Also, their focus is on the effects of market power in banking on financial stability. In contrast, we study a monetary model in order to connect inflation and monetary policy directly to market power and loan-rate dispersion. The role of banks on which we focus, insuring against the costs of having idle liquidity, arises only in monetary equilibria.[8]

The remainder of the paper is organized as follows. In Section 2, we present the details of the environment and the decision problems of households, firms, banks and the government. In Section 3, we describe a stationary monetary equilibrium and discuss its novel features. In Section 4, we calibrate the model to U.S. data and illustrate numerically the quantitative effects of equilibrium market power in the banking sector. Here we identify a relationship between the dispersion and level of loan rate spreads implied by the theory. In Section 5, we provide micro evidence on this relationship that lends support to our theoretical mechanism. In Section 6, we study an optimal monetary stabilization policy in response to aggregate demand shocks and their attendant fluctuations in money and loan demand. We conclude in Section 7.

2 A Monetary Model of Imperfectly Competitive Lending

Our model builds on the perfectly competitive banking model of BCW and nests it as a special case. As do they, we focus solely on the role banks play in insuring individuals against liquidity risk: Banks take deposits from *ex-post* holders of idle money and make loans to those who require additional liquidity.[9] We generalize the loan market to one where there is noisy consumer search for loans along the lines of Burdett and Judd (1983). This new feature generates dispersion in loan rates in equilibrium and also results in an

[8]Of course, there are other aspects of banking that may improve welfare. Chang and Li (2018) consider the same mechanism for banks but extend it to incorporate fractional reserves and liquidity buffers (see also, Kashyap, Rajan and Stein, 2002). This gives rise to a non-neutral liquidity channel of monetary policy in their model. Gu, Mattesini, Monnet and Wright (2013) consider a setting with limited commitment in exchange. In their model, banks improve welfare since limited commitment in private contractual obligations prevents more efficient allocative outcomes in the absence of banks. Also, bank liabilities can serve as payment instruments. He, Huang and Wright (2008) consider the safe-keeping role of banks when there is a risk of asset theft. These various reasons imply that banks can support a more efficient allocation in equilibrium. We eschew these factors in our model and focus solely on the role of banks as potential institutions for insuring private liquidity risk.

[9]Following BCW, we abstract from means of consumption smoothing other than banks and individually held money. In general, we could allow agents to own other assets (*e.g.*, claims to private equity or bonds). In order to rationalize equilibrium coexistence of fiat money alongside other asset claims, we could introduce costly asset liquidation in the frictional secondary asset market. This could be modelled, for example, as frictional over-the-counter trades as in Rocheteau and Rodriguez-Lopez (2014) and Duffie, Gârleanu and Pedersen (2005). This would render demand for multiple assets that have different liquidity premia in equilibrium. For the purposes of this paper, these are unnecessary features that would not alter our main insights.

endogenous degree of market power in lending that responds to both shocks and policy changes.

2.1 Overview

In the model, time is discrete and infinite. We use the following notational convention for date-dependent variables: $X \equiv X_t$ and $X_{+1} \equiv X_{t+1}$. As in Lagos and Wright (2005) or BCW, each date is divided into two sequential markets— respectively, a *Decentralized Market* (DM) and a *Centralized Market* (CM). At the beginning of each date $t = 0, 1, ...$, the aggregate stock of money in circulation is M. Let $\boldsymbol{\tau} = (\tau_b, \tau_s, \tau_2)$ denote a list of constant policy (tax or transfer) outcomes, where τ_b and τ_s are implemented in the DM and τ_2 is applied in the CM. We will denote the list of initial aggregate stock of money and policy outcomes by the vector notation $\mathbf{s} := (M, \boldsymbol{\tau})$. An individual with money balance m will have value and decision functions that depend on the vector $(m, \mathbf{s})$. The sequencing of events and actions are as follows:

1. A unit measure of households enter the period each carrying individual money balance, m.

2. Each household observes the outcome of an individual shock:

   (a) With probability n, the household becomes an *active buyer*. That is, the household wishes to consume goods q in the DM of the current period. As agents are anonymous in the DM, they cannot trade with sellers using promises of future repayment. Exchange will thus be supported by fiat money. Agents can borrow additional money from a *lending agent*, if they succeed in matching with one.

   (b) With probability $1-n$, the household becomes an *inactive buyer* and does not want to consume in the current DM. This buyer may be holding idle money, and if there is a banking sector, they can deposit it possibly in return for interest.[10]

3. There is a continuum of institutions we refer to as *banks*. Each bank has a *depository agent* that competes with those of other banks for deposits in a perfectly competitive market.[11] These agents take deposits in the DM and commit to repaying with nominal interest i_d in the upcoming CM. They supply funds to lending agents within the bank and can also invest any extra deposits in an external market in return for nominal interest r at the beginning of the period. Previewing equilibrium, $i_d = r$.

   Each bank also has a *lending agent* that makes loans in an imperfectly competitive market characterized by noisy search. Lending agents post loan rates i, given their anticipation of being matched with active buyers with loan demand, $l^{\star}(i, m, \mathbf{s})$. Active buyers search among the lending agents for loans, resembling lines of credit, and may match with one, two or no lenders. Funds borrowed in the DM are repayed (with interest i) in the CM of the same period.

4. Sellers in the DM produce non-storable goods on the spot and trade with active buyers who are heterogeneous *ex-post* regard to their access to loans at potentially different rates. These buyers' demands vary with their borrowing costs.

[10]This is a slight variation of the original setup of BCW. In BCW, there is an equivalent measure n of agents who become buyers in the DM. The remaining $1-n$ become sellers in the DM. Here, we fix a unit measure of agents as always being sellers and re-label the $1-n$ measure of agents as "inactive buyers". Substantively, this is still of the same form as BCW.

[11] We shall see later that this assumption allows us to compare with Berentsen et al. (2007) as a special case.

5. In the DM, the government can transfer nominal amounts to households and sellers in the DM. These, respectively, are denoted by $\tau_b M$ and $\tau_s M$.[12]

6. In the CM, markets are perfectly competitive. All households both consume and can produce a homogeneous good using labor. Heterogeneity at this point is due to different DM experiences only. Depending on their individual state, households may collect interest on deposits, pay interest on loans, work and/or consume. Finally, households accumulate a money balance, m_{+1}, to carry into the following period.

7. In the CM, the government can make transfers and/or collect taxes, lump-sum, uniformly on all households ($\tau_2 M$).

8. Also in the CM, lending agents enforce loan repayments return funds to depository agents. These in turn repay depositors and external lenders with interest at rate i_d.[13]

This sequence of events repeats with households carrying m_{+1} at the start of date $t+1$. Following Lagos and Wright (2005) and BCW we assume households' utilities are quasi-linear so that all carry the same money balance, m_{+1} into period $t+1$. We now turn to detailed descriptions of the decision problems of each type of agent. In each case, we work backwards from the CM to the DM.

2.2 Households

Households' period utility is given by

$$\mathcal{U}(q,x,h) = u(q) + U(x) - h, \tag{2.1}$$

where $u(q)$ denotes the utility flow from consumption of the DM good q, $U(x)$ is the utility of consumption good x in the CM, and $-h$ is the disutility of labor. We assume that $u' > 0$, $u'' < 0$ and that u satisfies the usual Inada conditions. Likewise for U. For concreteness now, and anticipating the quantitative analyses later, we restrict our attention to the constant-relative-risk-aversion (CRRA) family of functions:

$$u(q) = \lim_{\hat{\sigma}\to\sigma} \frac{q^{1-\hat{\sigma}} - 1}{1-\hat{\sigma}}, \tag{2.2}$$

and, we will assume that $\sigma < 1$.[14]

2.2.1 Households in the Centralized Market

Consider a household at the beginning of the CM, with money, loan and deposit balances (m, l, d). Households discount payoffs between two time periods using subjective discount factor $\beta \in (0,1)$. In the

[12] For the most part we rule out taxes in the DM. Policy is described in detail below.

[13] The assumption that lending agents all face the same cost of funds, i_d, is for simplification only. An extension to the case where lenders have heterogeneous costs is straightforward. For examples of models of this type (in contexts other than lending) see Herrenbrueck (2017) and Baggs, Fung and Lapham (2018).

[14] This restriction is empirically consistent with our calibration later, as comes out of our fitting of long-run money-demand data. It is not required theoretically. We also consider the case of $\sigma > 1$ but we do not discuss it here for brevity. The knife-edge case of $\sigma = 1$ is not well-defined in terms of equilibrium characterization. The restriction with $\sigma < 1$ is also the case studied by Head, Liu, Menzio and Wright (2012).

preceding DM, the agent may have been an active buyer (with $m \geq 0$, $l \geq 0$ and $d = 0$) or inactive buyer (with $l = 0$ and $m \geq d \geq 0$). Let $V(\cdot)$ denote the value function of the household at the beginning of the next period. The household's value at the beginning of the CM is

$$W(m, l, d, \mathbf{s}) = \max_{x,h,m_{+1}} \left[U(x) - h + \beta V(m_{+1}, \mathbf{s}_{+1}) \right], \tag{2.3}$$

subject to

$$x + \phi m_{+1} = h + \phi m + \phi(1 + i_d) d - \phi(1 + i) l + \pi + T, \tag{2.4}$$

where ϕ is the date-t value of a unit of money in units of CM good x, i_d is the market interest rate on deposits d, i is the interest rate on the buyer's outstanding loan l, π is aggregate profit from bank ownership, and $T = \tau_2 M$ is any lump-sum tax or transfer from the government in the CM.

Using Equations (2.4) and (2.3), the problem may be rewritten as

$$W(m, l, d, \mathbf{s}) = \phi\left[m - (1 + i) l + (1 + i_d) d\right] + \pi + T + \max_{x,m_{+1}} \left\{U(x) - x - \phi m_{+1} + \beta V(m_{+1}, \mathbf{s}_{+1})\right\}. \tag{2.5}$$

The first-order conditions with respect to the choices of x and m_{+1}, respectively, are

$$U_x(x) = 1, \tag{2.6}$$

and,

$$\beta V_m(m_{+1}, \mathbf{s}_{+1}) = \phi, \tag{2.7}$$

where $V_m(m_{+1}, \mathbf{s}_{+1})$ is the marginal value of an additional unit of money taken into period $t+1$. The envelope conditions are

$$W_m(m, l, d, \mathbf{s}) = \phi, \quad W_l(m, l, d, \mathbf{s}) = -\phi(1 + i), \quad \text{and,} \quad W_d(m, l, d, \mathbf{s}) = \phi(1 + i_d). \tag{2.8}$$

Note that $W(\cdot, \mathbf{s}_{+1})$ is linear in (m, l, d) and optimal decisions characterized by Equations (2.6) and (2.7) are independent of the agent's wealth. Moreover, each household supplies labor in the CM exactly sufficient to produce enough of the CM good to repay their loan (if necessary) and acquire the optimal money balance m_+ and consumption of x from (2.7) and (2.6), respectively.

2.2.2 Households in the Decentralized Market

We begin with households' problems in the DM after having realized their status as either inactive or active and after having potentially matched with one or more lenders.

Inactive buyers. Conditional on being inactive in the DM (with probability $1 - n$), a household with money holdings, m, can deposit d of this money with the depository agents. She then has the *ex-post* value of continuing to the CM having also received transfer τ_b: $W(m + \tau_b M - d, 0, d, \mathbf{s})$.

Active buyers with no line of credit. Consider an active household that has not succeeded in meeting a lending agent and thus cannot borrow money in addition to her holdings, m, plus their transfer, $\tau_b M$. Let p denote the money price of a DM good. Such a buyer has the following value:

$$B^0(m, \mathbf{s}) = \max_{0 \leq q_b \leq \frac{m+\tau_b M}{p}} \{u(q_b) + W(m + \tau_b M - pq_b, 0, 0, \mathbf{s})\}. \tag{2.9}$$

Using (2.2), the agent's optimal demand for goods can be derived as

$$q_b^{0,\star}(m, \mathbf{s}) = \begin{cases} \frac{m+\tau_b M}{p} & \text{if } p < \hat{p} \\ (p\phi)^{-1/\sigma} & \text{if } p \geq \hat{p} \end{cases}, \tag{2.10}$$

where $\hat{p}$ is the price below which the agent spends all of their money and in equilibrium is

$$\hat{p} \equiv \hat{p}(m, \mathbf{s}) = \phi^{\frac{1}{\sigma-1}} (m + \tau_b M)^{\frac{\sigma}{\sigma-1}}. \tag{2.11}$$

Active buyers with one or more line of credit. Next, consider the post-match value of a buyer who has contacted at least one lending agent:

$$B(m, \mathbf{s}) = \max_{q_b \leq \frac{m+l+\tau_b M}{p}, l \in [0, \bar{l}]} \{u(q_b) + W(m + \tau_b M + l - pq_b, l, 0, \mathbf{s})\}. \tag{2.12}$$

Under the assumption that loan contracts are perfectly enforceable as in the baseline case of BCW, the borrowing limit, $\bar{l}$, can be set sufficiently high so as never to bind in equilibrium. In the calibrated economy of Section 4 we allow for borrowers to default on loans at an exogenous probability. This does not change the equilibria we study qualitatively; it is useful only in matching certain calibration targets.[15]

Using the Karush-Kuhn-Tucker conditions derived from (2.12), we obtain the demands for the DM good and loans. The demand for the DM good is given by:

$$q_b^\star(i, m, \mathbf{s}) = \begin{cases} [p\phi(1+i)]^{-1/\sigma} & \text{if } 0 < p \leq \tilde{p}_i \text{ and } 0 \leq i \leq \hat{i} \\ \frac{m+\tau_b M}{p} & \text{if } \tilde{p}_i < p < \hat{p} \text{ and } i > \hat{i} \\ (p\phi)^{-1/\sigma} & \text{if } p \geq \hat{p} \text{ and } i > \hat{i} \end{cases}, \tag{2.13}$$

where

$$\hat{p} \equiv \hat{p}(m, \mathbf{s}) = \phi^{\frac{1}{\sigma-1}} (m + \tau_b M)^{\frac{\sigma}{\sigma-1}} \qquad \text{and} \qquad \tilde{p}_i = \hat{p}(1+i)^{\frac{1}{\sigma-1}}, \tag{2.14}$$

respectively, correspond to a maximal DM price at which the household will use both her own liquidity and credit line from the bank, and, a maximal price at which her purchase results in her being liquidity constrained. Since $\sigma < 1$, we have: $0 < \tilde{p}_i < \hat{p} < +\infty$.

[15] The analysis can also be extended to the case of an endogenous borrowing limit determined by limited commitment to repay loans, as in a version of the model studied by BCW. Unlike there, however, in our setting bank-specific lending limits would have to be determined simultaneously with the equilibrium distribution of loan rates. While this is complicated, it is not intractable. Just as in BCW, however, default will never occur in equilibrium and thus the implications both qualitative and quantitative are minimal. Thus, here we opt for the more parsimonious setup with exogenous random default.

The maximal interest rate at which a buyer is willing to borrow is given by

$$\hat{i} \equiv \hat{i}(m, \mathbf{s}) = (p\phi)^{\sigma-1} \left[\phi(m + \tau_b M)\right]^{-\sigma} - 1. \tag{2.15}$$

For any interest rate $i \in [0, \hat{i}]$, the buyer's loan demand is:

$$l^{\star}(i, m, \mathbf{s}) = \begin{cases} p^{\frac{\sigma-1}{\sigma}} \left[\phi(1+i)\right]^{-\frac{1}{\sigma}} - (m + \tau_b M) & p \in (0, \tilde{p}_i];\ i \in [0, \hat{i}] \\ 0 & p \in (\tilde{p}_i, \hat{p});\ i > \hat{i} \\ 0 & p \geq \hat{p};\ i > \hat{i}. \end{cases} \tag{2.16}$$

From the respective first cases of Equations (2.13) and (2.16), we can see that if the DM good's relative price ($p\phi$) and interest on bank loans (i) are sufficiently low, the agent borrows to augment her money balance and her goods and loan demands are decreasing in both i and $p\phi$. If, however, the DM good's relative price and interest on borrowing are higher (*i.e.*, the intermediate case), the agent prefers not to borrow, but rather to spend all her money on the DM good and be liquidity constrained. In this case the loan rate does not matter for demand. Finally, if $p\phi$ and i are sufficiently high, the buyer not only doesn't borrow but does not spend all her money balance on the DM good. The cutoff prices $(\hat{p}, \tilde{p}_i, \hat{i})$ are functions of the state in equilibrium.

Households in the DM, *ex-ante*. Now consider the beginning of period t prior to the both search and the determination of which households are active and inactive. Given the money balance, m, *all* households have *ex ante* value:

$$V(m, \mathbf{s}) = n \left\{ \alpha_0 B^0(m, \mathbf{s}) + \alpha_1 \int_{[\underline{i}, \bar{i}]} B(i, m, \mathbf{s}) \, \mathrm{d}F(i, m, \mathbf{s}) \right.$$
$$\left. + \alpha_2 \int_{\underline{i}(m,\mathbf{s})}^{\bar{i}(m,\mathbf{s})} B(i, m, \mathbf{s}) \, \mathrm{d}\left[1 - (1 - F(i, m, \mathbf{s}))^2\right] \right\} + (1-n) W(m + \tau_b M - d, 0, d, \mathbf{s}). \tag{2.17}$$

Conditional on being an active DM buyer with probability n, a household searches for a lender from which to obtain a line of credit, taking the distribution $F(\cdot, m, \mathbf{s})$ of posted loan rates, i, as given. The CDF $F(\cdot)$ is an equilibrium object.[16] With probability $\alpha_0 \in (0, 1)$, the buyer fails to find a lender. Her value is then $B^0(m, \mathbf{s})$, which is given in (2.9). With probability $\alpha_1 \in (0, 1 - \alpha_0)$, the buyer makes contact with one lender and has *ex post* value then $B(i, m, \mathbf{s})$, from (2.12), where i is a single draw from the distribution F. With probability $\alpha_2 = 1 - \alpha_0 - \alpha_1$, the buyer randomly and independently meets two lenders and has *ex-post* value $B(i, m, \mathbf{s})$, again from (2.12), but with i is the lower of two rates drawn from $F(\cdot, m, \mathbf{s})$, distributed as $1 - (1 - F(\cdot, m, \mathbf{s}))^2$.[17]

[16] We assume for now a compact support for $F(\cdot, m, \mathbf{s})$ as $\left[\underline{i}(m, \mathbf{s}), \bar{i}(m, \mathbf{s})\right]$. This will be a result proved in Lemma A.7 in Online Appendix A.6.

[17] The maximal number of lenders that can be contacted by introducing a cost of search in various ways (see, *e.g.* Burdett and Judd (1983), Head and Kumar (2005), and Wang (2016)). We abstract from endogenous search intensity as it has little effect on the issues on which we focus here.

2.3 Sellers in the Decentralized Market

In the DM, a unit measure of sellers of goods behave much like households, except that they can produce the DM good on demand and do not value consuming it. DM sellers are analogous to Walrasian price-taking producers in Rocheteau and Wright (2005). Each DM seller has value:

$$S(m, \mathbf{s}) = \max_{q_s} \{-c(q_s) + W(m + \tau_s M + pq_s, 0, 0, \mathbf{s})\}. \tag{2.18}$$

Here, $c(q)$ represents the cost of producing quantity q of goods, where $c(0) = 0$, $c_q(q) > 0$ and $c_{qq}(q) \geq 0$. The sellers' optimal production plan satisfies

$$c_q(q_s) = p\phi. \tag{2.19}$$

That is, the DM sellers produce to the point where the marginal cost of producing good q_s equals its relative price. It is straightforward to show that in equilibrium their valuation will be $S(0, \mathbf{s})$ at the start of each DM—*i.e.*, sellers optimally carry no money into the DM.

2.4 Banking

The banking system consists of a continuum of institutions, each comprised of a depository agent and a lending agent as described above. Depository agents are price-takers in a perfectly competitive market for deposits. Lending agents, in contrast, compete in an imperfectly competitive loan market. These agents post loan rates subject to ex-post heterogeneous borrower demand for loans, anticipating the noisy nature of borrower search under imperfect information. We will use the term *banks* to discuss both depository and lending agents when the context makes the difference clear.

It would be straightforward to introduce bargaining between inactive households and depository agents in order to introduce a wedge between the external cost of funds and the deposit rate, i_d. Qualitatively, this would have little effect on behavior of the loan rate spreads on which we focus and so we abstract from it here. Moreover, competition for deposits aids in making comparisons to the previous results of BCW and Berentsen and Waller (2011). In Head et al. (2022), we take up the issue of imperfect competition for deposits in order to study the deposits channel of monetary policy and its interaction with capital accumulation and long-run growth.

2.4.1 Depository agents

First, consider the interaction between households, sellers and depository agents in the DM, where the latter take deposits and supply them to lending agents within the bank and/or the interbank market. In the CM, each bank can enforce repayment of loans and thus depository agents can commit to return these deposits with interest to individual depositors. In the event that the bank's lending agent faces loan demand exceeding their deposits, the depository agent can borrow from in the interbank market. As lending is imperfectly competitive with some lending agents meeting more, just enough, or too few borrowers, it may be the case in equilibrium total deposits exceed the loans extended by lending agents. The reverse, however, will not occur in equilibrium.

Note that DM sellers will not hold money in equilibrium as it is costly to do so. Thus, all deposits will come from inactive DM households carrying idle money. Each bank whose lending agent has posted loan rate i, has a balance-sheet constraint:

$$(1-n)d + e_{i,f} = \left[\alpha_1 + 2\alpha_2(1 - F(i,m,\mathbf{s}))\right] l^\star(i,m,\mathbf{s}). \tag{2.20}$$

That is, the bank's expected deposits, $(1-n)d$, plus borrowed funds $e_{i,f} > 0$ (or loaned-out surplus deposits $e_{i,f} \leq 0$) will equal its expected loans.

We assume that the interbank market for funds e_f is perfectly competitive and open to the rest of the world. (This implies that $\int e_{i,f} dF(i,m,\mathbf{s}) \leq 0$.) Let i_f be the interbank rate, which is thus exogenous. As depository agents are perfectly competitive, any equilibrium will have them offering a deposit rate of $i_d = i_f$ and so banks make zero profits from deposits.

We assume also that monetary policy is pegged to i_f. Thus, $i_f = i_d = (\phi/\phi_{+1})/\beta - 1$, *i.e.*, the inflation-adjusted risk-free rate. Under this assumption, i_d equals perfectly-competitive lending and borrowing rate that arises in the baseline version of BCW. As such, households will be insured against holding idle balances here to the same extent that they are in BCW's environment.

2.4.2 Lending agents

Lending agents contract with prospective borrowers before the latter trade with sellers in the DM and can enforce loan contracts in the CM. These agents behave similarly to the sellers in the basic model of Burdett and Judd (1983). That is, they post a lending rate i, and commit to satisfying the demand for loans at that rate. This commitment is credible as each bank's lending agent has access not only to the bank's own deposits but also to the interbank market, if necessary. Thus, the lending agent obtains funds from either source at constant gross marginal cost $1 + i_d$.

Active buyers observe randomly zero, one or two loan rates and are able to borrow at the lowest rate they observe.[18] Let α_k for $k \in \{0,1,2\}$ denote the probability with which an individual buyers has k borrowing opportunities. The details of the interest rate (price)-posting problem are described below. Again previewing equilibrium, lending agents will, on average, earn positive profits as all posted lending rates in equilibrium will exceed i_d. Lending agents can either retain these profits or return them lump-sum to households in the CM.

Consider a lending agent that takes the distribution of posted rates, F, as given and has net marginal cost of funds i_d. The lender's expected profit from posting net loan rate i is

$$\Pi(i,m,\mathbf{s}) = n\left[\alpha_1 + 2\alpha_2\left(1 - F(i,m,\mathbf{s})\right) + \alpha_2 \zeta(i,m,\mathbf{s})\right] R(i,m,\mathbf{s}), \tag{2.21}$$

where

$$\zeta(i,m,\mathbf{s}) = \lim_{\varepsilon \searrow 0}\left\{F(i,m,\mathbf{s}) - F(i-\varepsilon,m,\mathbf{s})\right\}, \tag{2.22}$$

$$R(i,m,\mathbf{s}) = l^\star(i,m,\mathbf{s})\left[(1+i) - (1+i_d)\right], \tag{2.23}$$

$R(i,m,\mathbf{s})$ is the profit per customer served, $l^\star(i,m,\mathbf{s})$ is the demand for loans, and $n\alpha_2\zeta(i,m,\mathbf{s})$ is the

[18] Sellers have no need for loans in equilibrium, so we ignore the possibility of them borrowing.

measure of consumers that contact this bank and another that has posted the same rate, i.[19]

Consider first a hypothetical lender posting rate i and serving borrowers who have contacted *only* that lender. The lender's *realized* profit is

$$\Pi^m(i, m, \mathbf{s}) = n\alpha_1 R(i, m, \mathbf{s}), \tag{2.24}$$

where the superscript, m, indicates that the lender is a monopolist, as its customers have only this one opportunity to borrow.

Second, consider a lender facing customers who potentially observe more than one rate due to noisy search. The lender's realized profit is given by

$$\Pi^\star(m, \mathbf{s}) = \max_{i \in \text{supp}(F(\cdot, m, \mathbf{s}))} \Pi(i, m, \mathbf{s}) \tag{2.25}$$

subject to Equations (2.21), (2.22), (2.23) and (2.16).

Restricting attention to linear pricing rules, we can prove that $\Pi^m(\cdot, m, \mathbf{s})$ is twice continuously differentiable, strictly concave and positive-valued. Moreover, any lending agent facing more than one customer will also earn strictly positive profit. There is a maximal loan interest rate that is the smaller of either the monopolist's optimal loan rate $i^m(m, \mathbf{s})$ or the consumer's maximum willingness to pay $\hat{i}(m, \mathbf{s})$, where the latter is endogenous. The natural lower bound on loan rates is $i_d = (1 + \tau)/\beta - 1$, and so the support of the distribution of posted loan rates is bounded. The support is also connected, as in Burdett and Judd (1983).[20]

From the definition of a lending agent's expected profit in (2.21), it can be seen that the lender faces the following trade-off: It can raise its profit *per loan* by raising its loan rate relative to its cost of funds, i_d (*i.e.* its *loan spread*). On the other hand it can increase the *measure* of borrowers it serves by *lowering* its posted rate. Since lenders are *ex-ante* identical, we may think of the distribution $F(\cdot, m, \mathbf{s})$ as representing different pure-strategy choices or we may think of lending agents as mixing symmetrically over a range of interest rates that yield the same expected profit. In either interpretation, each borrower faces distribution $F(\cdot, m, \mathbf{s})$ of random loan rates.[21]

The distribution, F, may be characterized analytically:

Lemma 2.1. *Suppose that the aggregate money stock grows by the factor $\gamma > \beta$.*

1. *If $\alpha_1 \in (0, 1)$, each borrower $(m, \mathbf{s})$ faces a unique non-degenerate, posted-loan-rate distribution $F(\cdot, m, \mathbf{s})$. This distribution is continuous with connected support:*

$$F(i, m, \mathbf{s}) = 1 - \frac{\alpha_1}{2\alpha_2}\left[\frac{R(\bar{i}, m, \mathbf{s})}{R(i, m, \mathbf{s})} - 1\right], \tag{2.26}$$

where $\text{supp}(F(\cdot, m, \mathbf{s})) = [\underline{i}(m, \mathbf{s}), \bar{i}(m, \mathbf{s})]$, $R(\underline{i}, m, \mathbf{s}) = \frac{\alpha_1}{\alpha_1 + 2\alpha_2} R(\bar{i}, m, \mathbf{s})$ *and* $\bar{i}(m, \mathbf{s}) = \min\{\hat{i}(m, \mathbf{s}), i^m(m, \mathbf{s})\}$.

[19] We assume that in such cases prospective borrowers randomize between the two lenders. In equilibrium, the probability of a borrower observing two identical lending rates goes to zero.

[20] These results are derived formally in the Online Appendix in Sections A.1 to A.6.

[21] Note that the existence of a distribution of posted loan rates, $F(\cdot, m, \mathbf{s})$ does not depend on banks being *ex ante* identical. See, for examples, Herrenbrueck (2017) and Baggs et al. (2018).

2. *If* $\alpha_2 = 1$*, then* $F(\cdot, m, \mathbf{s})$ *is degenerate at* i_d*:*

$$F(i, m, \mathbf{s}) = \begin{cases} 0 & \text{if } i < i_d \\ 1 & \text{if } i \geq i_d \end{cases}. \tag{2.27}$$

3. *If* $\alpha_1 = 1$*,* $F(\cdot, m, \mathbf{s})$ *is degenerate at the largest possible loan rate* $\bar{i}$ *such that*

$$F(i, m, \mathbf{s}) = \begin{cases} 0 & \text{if } i < \bar{i}(m, \mathbf{s}) \\ 1 & \text{if } i \geq \bar{i}(m, \mathbf{s}) \end{cases}. \tag{2.28}$$

We relegate the proof to Online Appendix A.7. This result is akin to the original notion of "firm equilibrium" in Burdett and Judd (1983, Lemma 2) and in the monetary version of Head and Kumar (2005, Proposition 3). For empirical relevance, we restrict attention to the first part of Lemma 2.1. That is, we focus on an equilibrium in which the distribution of rates is non-degenerate. With regard to the extent of market power, this case is sandwiched between the two familiar extremes: A Bertrand equilibrium and a monopoly-price equilibrium. These are described, respectively, in the second and third parts of Lemma 2.1.

2.5 Government

We maintain the notation for government policies from BCW. The government and monetary authority can make a lump-sum monetary injection or extraction in the CM, and can make targeted positive transfers, to active and inactive households in the DM. These policy instruments are denoted as τ_1, and τ_2, in the DM and CM, respectively.

The total change to the money supply, $(\gamma - 1)M \equiv \tau M$, is split between DM and CM:

$$M_{+1} - M = (\gamma - 1)M = \tau_1 M + \tau_2 M, \tag{2.29}$$

where

$$\tau_1 = n\tau_b + (1 - n)\tau_b + \tau_s \geq 0 \tag{2.30}$$

Transfers $\tau_b M$ and $\tau_s M$ are to buyers and sellers in the DM, respectively, and (2.30) rules out lump-sum *taxes* in the DM.[22]

3 A Stationary Monetary Equilibrium

We focus on a *stationary monetary equilibrium* (SME), in which the price level and money supply grow at the same constant rate: $\phi/\phi_{+1} = M_{+1}/M = \gamma \equiv 1 + \tau$. In this section, we characterize the components of a SME, with a particular focus on an equilibrium with money and credit. This is the equilibrium configuration that emerges in the calibrated model later.

[22] We restrict attention to identical transfers to all "active" and "inactive" buyers. This can be relaxed.

As the price level $(1/\phi)$ is non-stationary, to obtain a well-defined stationary equilibrium we multiply nominal variables by ϕ. Let $z = \phi m$ and $Z = \phi M$ denote individual and aggregate real balances, respectively. Also, let $\rho = \phi p$ denote the relative price of DM to CM goods, and, $\xi = \phi l$ the real value of a loan. In a SME, DM sellers neither accumulate money in the CM nor borrow. Inactive DM households deposit all their money with depository agents. Thus, we need need consider the loan demand of active buyers. The stationary counterpart to the state-policy vector $(m, \mathbf{s})$ will now be $(z, \mathbf{z})$, where $\mathbf{z} = (Z, \boldsymbol{\tau})$.

3.1 The distribution of posted lending rates

Consider the case of $\alpha_1 \in (0,1)$ as stated in Lemma 2.1. Rewriting the distribution of loan rates in (2.26) in terms of stationary variables, we have:

$$F(i, z, \mathbf{z}) = 1 - \frac{\alpha_1}{2\alpha_2}\left[\frac{R(\bar{i}, z, \mathbf{z})}{R(i, z, \mathbf{z})} - 1\right], \tag{3.1}$$

where $\text{supp}(F) = \left[\underline{i}(z, \mathbf{z}), \bar{i}(z, \mathbf{z})\right]$, $\underline{i}(z, \mathbf{z})$ solves

$$R(\underline{i}, z, \mathbf{z}) = \frac{\alpha_1}{\alpha_1 + 2\alpha_2} R(\bar{i}, z, \mathbf{z}), \qquad \bar{i}(z, \mathbf{z}) = \min\{i^m(z, \mathbf{z}), \hat{i}(z, \mathbf{z})\}, \tag{3.2}$$

and,

$$R(i, z, \mathbf{z}) = \left[\rho^{\frac{\sigma-1}{\sigma}}(1+i)^{-\frac{1}{\sigma}} - (z + \tau_b Z)\right](i - i_d) \tag{3.3}$$

is real bank profit per customer served.

We now have the following useful comparative static result regarding the relationship between household-level real balances and the distribution of posted lending rates:

Lemma 3.1. *Fix a long-run inflation rate $\gamma > \beta$, and let $\alpha_0, \alpha_1 \in (0,1)$. Consider any two real money balances z and z' such that $z < z'$. The induced loan-price distribution $F(\cdot, z, \mathbf{z})$ first-order stochastically dominates $F(\cdot, z', \mathbf{z})$.*

The proof can be found in Online Appendix B.1. In short, in a SME where households carry higher (lower) real balances into the DM, they are more (less) likely to draw lower loan-rate quotes, *ceteris paribus*. This reflects the fact that when potential borrowers carry low real balances into the period, demand for loans will be relatively high. All else equal, given strong loan demand, lending agents' optimal loan rates and spreads rise. Hence, the conclusion of Lemma 3.1.

3.2 The demand for money and bank credit

We now derive an equation describing CM agents' optimal money demand.[23] For clarity, we restrict attention to a stationary monetary equilibrium (SME) in which both *ex-ante* demand for money balances and *ex-post* demand for loans in the DM are positive. This will turn out to be the equilibrium configuration

[23] This is done by taking the partial derivative of (2.17) (i.e., marginal valuation of money) one period ahead, combining this with the first-order condition with respect to next-period money balance in (2.7) and the optimal DM-good and loan demand functions in Equations (2.10) and (2.13).

that emerges under our calibration. This is also the case when we consider a range of computational experiments later.[24]

Lemma 3.2. *Fix long-run inflation $\gamma \equiv 1+\tau > \beta$ and let $\alpha_0, \alpha_1 \in (0,1)$. Assume that there is an SME in which real balances, $z^\star \in \left(0, \left(\frac{1}{1+\bar{i}(z^\star,\mathbf{z})}\right)^{\frac{1}{\sigma}}\right)$. Then,*

1. *the relative price of DM goods satisfies*

$$\rho = 1 < \tilde{\rho}_i(z^\star, \mathbf{z}) \equiv (z^\star)^{\frac{\sigma}{\sigma-1}}(1+i)^{\frac{1}{\sigma-1}}, \tag{3.4}$$

for any $i \in \text{supp}(F(\cdot; z^\star, \mathbf{z}))$; $\tilde{\rho}_i = \phi\tilde{p}_i$ is the stationary transform of cut-off pricing function $\tilde{p}_i$, defined in (2.14);

2. *loan demand is always positive; and,*

3. *money demand is given by the Euler equation*

$$\begin{aligned} \frac{\gamma-\beta}{\beta} \quad = \quad & \underbrace{(1-n)i_d}_{[A]} + \underbrace{n\alpha_0\left(u'[q_b^0(z^\star,\mathbf{z})]-1\right)}_{[B]} \\ & \underbrace{+\,n\int_{\underline{i}(z^\star,\mathbf{z})}^{\bar{i}(z^\star,\mathbf{z})} i\left[\alpha_1 + 2\alpha_2(1-F(i,z^\star,\mathbf{z}))\right] dF\left(i,z^\star,\mathbf{z}\right).}_{[C]} \end{aligned} \tag{3.5}$$

At the end of each period (CM), each agent anticipates that in the following period (DM) he may be an active DM buyer with probability $1-n$. In this case, regardless of the agent's access to loans *ex post*, he has incentive to carry money given the potential cost of borrowing.[25] The left-hand side of (3.5) is the forgone nominal risk-free interest rate due to demanding money–*i.e.* the marginal cost of real balances.

The terms on the right-hand side of (3.5) constitute the expected marginal benefit of carrying money into the next DM. The first term (A) captures the benefit of banking in reducing the cost of holding money balances. Agents can deposit their idle money balances in the bank to earn interest if they do not value DM consumption, *i.e.* are inactive in the upcoming DM. Here, banks help households insure against their liquidity/consumption risks (via increasing the marginal value of money) to the same extent as in Berentsen et al. (2007). The second term (B) is the liquidity premium on own money holding in delivering consumption value (in the case of no contact with a lender). The third term (C) comes about because an extra unit of money carried into the next period saves the agent an expected interest per unit of money not borrowed.

Because it is costly to carry money into the next period ($\gamma > \beta$), agents may find it welfare improving *ex ante* to count on using bank credit to "top up" liquidity contingent on being active. This is partially

[24] These equilibrium properties rely on sufficient conditions that are *per se* not purely characterized by model primitives. (See Proposition 3.1 further below for the details.) However, the sufficient conditions are satisfied automatically in the computational experiments.

[25] Here as in BCW, households are insured against the cost of carrying idle balances by the availability of bank deposits. Their incentive to carry money is increased here relative to BCW by 1) the existence of positive loan spreads (loans are expensive) and 2) the possibility of having no access to loans (if $\alpha_0 > 0$).

reflected in the last term (C) in (3.5). Implicit in this same term (C), however, market power among banks raises the cost of these funds, essentially making DM consumption more costly and thus reducing the *ex-ante* value of real balances. As such, whether or not the presence of banking improves household welfare in equilibrium is ambiguous.[26]

To illustrate this further (and we return to the issue in Section 4.3, we rewrite (3.5) as an asset pricing relation:

$$1 = \underbrace{\frac{\alpha_0 \left(u'[q_b^0(z^\star, \mathbf{z})] - 1 \right)}{i_d}}_{\text{Self-insurance: benefit-cost ratio}} + \underbrace{\int_{\underline{i}(z^\star,\mathbf{z})}^{\bar{i}(z^\star,\mathbf{z})} \underbrace{[\alpha_1 + 2\alpha_2 (1 - F(i, z^\star, \mathbf{z}))]}_{\text{Extensive margin}} \underbrace{\left(\frac{i}{i_d} \right)}_{\text{Intensive margin}} \mathrm{d}F(i, z^\star, \mathbf{z})}_{\text{Benefit-to-cost ratio from reduced borrowing} \equiv \text{Expected transactions spread, } \hat{\mu}}. \tag{3.6}$$

The left-hand side of the no-arbitrage condition in (3.6) is the normalized, relative price of giving up CM consumption for real money balance today. On the right, we have the discounted expected real return from carrying money into the next-period DM. This return has two components inherited from the equivalent expression in (3.5). These components—now measured relative to the opportunity cost of holding money (i_d)—are associated with the ability to consume without credit and with a reduced loan-interest burden, respectively. The latter term reflects the fact that households must consider both the cost of a particular loan (the *intensive* margin) and the likelihood of being able to choose among multiple lenders (the *extensive* margin).

3.3 Two special cases

It is useful at this point to consider two special cases. First, let $\alpha_2 = 1$ and $\alpha_0 = 0$. In this case $F(\cdot, z, \mathbf{z})$ is degenerate on the singleton set $\{i = i_d\}$ (by Proposition 2.1) and (3.5) becomes:

$$1 + \frac{\gamma - \beta}{\beta} = u'(q^{BCW}), \tag{3.7}$$

where q^{BCW} will be a non-decreasing function of money balance in equilibrium. This case corresponds to that of Berentsen et al. (2007) with perfectly competitive banks.[27]

Second, consider a pure-currency economy without banks ($\alpha_0 = 1$). In this case, agents must self-insure by carrying money and money demand is implicitly described by

$$1 + \frac{1}{n} \left(\frac{\gamma - \beta}{\beta} \right) = u'(q^{\text{no-bank}}). \tag{3.8}$$

On the left-hand sides of (3.7) and (3.8) are buyers' gross costs of carrying additional money into

[26] This is in contrast to BCW where the only potential gains from banking arise from the payment of interest to *inactive* buyers. Their result that the welfare gains from banking are always non-negative depends on perfect competition for both loans and deposits. With an endogenous degree of imperfect competition associated with dispersion of loan spreads, the welfare-gain result will be a conditional one. We return to this issue in Section 4.3.

[27] (3.7) yields the same expression as Equation (22) in BCW when DM production is linear, *i.e.*, $c'(q_s) = 1$.

the DM, in the BCW and no-bank (or self-insurance) economies, respectively. On the right are their respective gross (utility) returns from consuming in the DM using their real money balance. Since perfectly competitive banks serve to insure agents who end up inactive and not requiring money, the liquidity premium on money $u'(q^{BCW}) - 1$ is exactly the competitive interest rate $i = i_d = (\gamma - \beta)/\beta$. In the case of a no-bank (or self-insurance) environment, however, the cost of carrying money is higher, as can be seen directly from the additional term, $1/n > 1$, on the left-hand side of (3.8). This raises the liquidity premium on money when agents cannot access banks to insure the risk of having idle liquidity. Thus, perfectly competitive banks enable higher consumption than under self insurance if money has a return that is inferior to that on a risk-free asset. Thus, in BCW banking always weakly raises welfare.[28]

From (3.6) and (3.7), it is immediate that with regard to consumption, the welfare will be lower in the search economy than in one with perfectly competitive lending. The comparison is less clear, however, when comparing the search economy to one with no banks at all. In the former, carrying a lower real money balance into the DM does not necessarily result in lower welfare since buyers can increase their consumption by borrowing. More borrowing, however, entails more surplus being extracted by lenders with market power. To see these additional, opposing effects arising in an agent's *ex-ante* money balance decision, we can rewrite (3.6) more compactly as:

$$1 + \frac{1}{\alpha_0}\left[1 - \hat{\mu}(\gamma)\right]\left(\frac{\gamma - \beta}{\beta}\right) = u'(q^{0,HKNP}), \tag{3.9}$$

where $i_d = (\gamma - \beta)/\beta$,

$$\hat{\mu}(\gamma) = \int_{\underline{i}(z^\star,\mathbf{z})}^{\bar{i}(z^\star,\mathbf{z})} \left[\alpha_1 + 2\alpha_2\left(1 - F\left(i, z^\star, \mathbf{z}\right)\right)\right]\left(\frac{i}{i_d}\right) \mathrm{d}F(i, z^\star, \mathbf{z}),$$

and $q^{0,HKNP}$ is DM consumption of an *ex-post* active buyer who has no line of credit in our search (HKNP) environment (see (3.12) below). Notice that on the right-hand side of (3.9), this is now *only* the gross return to consuming with real balances in the event that the agent fails to make contact with any bank. The additional return from carrying money now depends on the expected loan-rate *spread*, represented by the term $\hat{\mu}$ on the left-hand side. As agents do not know *ex ante* the interest rate at which they will be able to borrow, their money balance decision must take into account the distribution of such rates. The result is the distortion term $[1 - \hat{\mu}(\gamma)]/\alpha_0$ to the basic gross cost of holding money, $(\gamma - \beta)/\beta$.

On the one hand, inflation (γ) increases the gross cost of holding money—*i.e.*, the term $(\gamma-\beta)/\beta$ on the left-hand side of (3.9). We can visualize this effect of inflation as shifting up the cost of holding money—*i.e.*, the left-hand side of (3.9). On the other hand, since $\hat{\mu}$ is strictly greater than one, $1 - \hat{\mu}(\gamma) < 0$ is always negative. This works in the opposite direction, shifting down the left-hand side of (3.9). Moreover, we show that $\hat{\mu}(\gamma)$ is larger (smaller) for lower (higher) inflation (see Proposition D.1 in Online Appendix D.) Hence, the net effects of long-run inflation are ambiguous. That is, the effects of changes in the long-run inflation rate on both relative money holdings and welfare in the search economy may be non-monotone relative to their counterparts in either an economy with perfectly-competitive banking or one with no banks.[29] We discuss this new trade-off and evaluate it quantitatively in Section 4 below.

[28] That is, comparing (3.7) and (3.8), it is clear that $q^{BCW} > q^{\text{no-bank}}$ if $\gamma > \beta$. Since $n < 1$, the left-hand side of (3.8) is a shifted relative to that of (3.7).

[29] Chiu et al. (2018) also study a model in which access to banks may result in aggregate welfare losses. Their result hinges

3.4 Goods market equilibrium in the DM and CM

Sellers in the DM are Walrasian price takers, and so inn equilibrium the real price of the DM good equals its marginal cost: $\rho = c'(q_s)$. Supply, q_s, equals demand for the DM good:

$$q_s(z, \mathbf{z}) \equiv c'^{-1}(\rho) = n\alpha_0 q_b^{0,\star}(z, \mathbf{z}) + n\left[\int_{\underline{i}(z,\mathbf{z})}^{\bar{i}(z,\mathbf{z})} [\alpha_1 + 2\alpha_2 - 2\alpha_2 F(i, z, \mathbf{z})]\, q_b^\star(i, z, \mathbf{z})\, \mathrm{d}F(i, z, \mathbf{z})\right]. \tag{3.10}$$

Given $x^\star = 1$, we can also verify that aggregate CM labor equals $x^\star$ due to the assumption that all households have access to a linear production technology in the CM.

3.5 Deposits and interest in equilibrium

In equilibrium lenders must earn non-negative profits. In aggregate, this requires that total interest collected on real loans ($\xi^\star(i, z, \mathbf{z})$) weakly exceeds that paid on total real deposits:

$$(1-n)i_d\delta^\star(z, \mathbf{z}) \equiv (1-n)i_d(z + \tau_b Z) \leq n\left\{\int_{\underline{i}(z,\mathbf{z})}^{\bar{i}(z,\mathbf{z})} [\alpha_1 + 2\alpha_2 - 2\alpha_2 F(i, z, \mathbf{z})]\, i\, \xi^\star(i, z, \mathbf{z})\, \mathrm{d}F(i, z, \mathbf{z})\right\}. \tag{3.11}$$

3.6 Summary: SME with money and credit

Definition 3.1. *A* stationary monetary equilibrium with money and credit *is a steady-state allocation $(x^\star, z^\star, Z)$, allocation functions $\{q_b^{0,\star}(z^\star, \mathbf{z}), q_b^\star(\cdot, z^\star, \mathbf{z}), \xi^\star(\cdot, z^\star, \mathbf{z}\}$, and (relative) pricing functions $(\rho, F(\cdot; z^\star, \mathbf{z}))$ such that given government policy $\boldsymbol{\tau}$ satisfying (2.30),*

1. $x^\star = 1$;
2. $z^\star \equiv z^\star(\boldsymbol{\tau}) = Z$ *solves* (3.5);
3. *given $z^\star$, $q_b^{0,\star}(z^\star, \mathbf{z})$ and $q_b^\star(\cdot, z^\star, \mathbf{z})$, respectively, satisfy*

$$q_b^{0,\star}(z^\star, \mathbf{z}) = \frac{z^\star + \tau_b Z}{\rho}, \qquad \text{for } \rho < \hat{\rho}(z^\star, \mathbf{z}), \tag{3.12}$$

and,

$$q_b^\star(i, z^\star, \mathbf{z}) = [\rho(1+i)]^{-\frac{1}{\sigma}}, \qquad \text{for } 0 < \rho \leq \tilde{\rho}_i(z^\star, \mathbf{z}) \text{ and } 0 \leq i < \hat{i}(z^\star, \mathbf{z}); \tag{3.13}$$

4. $\xi^\star(\cdot, z^\star, \mathbf{z})$ *satisfies:*

$$\xi^\star(i, z^\star, \mathbf{z}) = \rho^{\frac{\sigma-1}{\sigma}}(1+i)^{-\frac{1}{\sigma}} - (z^\star + \tau_b Z), \quad \text{for } \rho \in (0, \tilde{\rho}_i(z^\star, \mathbf{z})], \quad i \in [0, \hat{i}(z^\star, \mathbf{z})); \tag{3.14}$$

on increasing marginal cost of production in the DM. Here we generalize the result to a case where goods are produced at constant marginal cost. Our result can obtain if all buyers have access to credit, as long as lenders have sufficient market power and inflation is sufficiently low.

5. *ρ solves (3.10);*

6. *$F(\cdot; z^{\star}, \mathbf{z})$ is determined by (3.1); and,*

7. *aggregate loans supplied is feasible according to (3.11).*

Note that τ_s does not materially affect equilibrium determination, and so we can set $\tau_s = 0$ without changing our basic results. For the baseline calibration of the model, we also set $\tau_b = 0$, so that there is no redistributive tax or transfer policy in place. Later we will consider counterfactual analyses involving differential tax policies.

Under sufficient conditions, there exists a unique SME with money and credit:

Proposition 3.1. *Assume loan contracts are perfectly enforceable. If $1 + \tau \equiv \gamma > \beta$, $z^{\star} \in (0, \overline{z})$, where $\overline{z} = \left[1 + \overline{i}(z^{\star}, \mathbf{z})\right]^{-\frac{1}{\sigma}}$, and there is a $N(z^{\star}, \mathbf{z}) \in [0, 1]$ such that $n \geq N(z^{\star}, \mathbf{z})$, then there exists a unique SME with both money and credit.*

For a proof, see Appendix B.4. Formal proofs of intermediate results can be found in Appendices B.1, B.2, and B.3. Here we sketch the basic idea. Fix $\gamma > \beta$. First we show that lending banks' posted loan-price distribution $F(\cdot, z, \mathbf{z})$ is decreasing (in the sense of first-order stochastic dominance) in households' real balance, z. The intuition is that as households carry more money into the DM, the marginal benefit of bank credit falls. See Lemma 3.1 for details. As such, when households have higher real balances, they are more likely to observe (and be able to borrow at) a lower interest rate.

Second, with probability α_0, a household contacts no lending agent and so its marginal benefit from holding an extra dollar falls as real balances rise. Together, these factors establish that the right-hand side of (3.5) is a continuous and monotone decreasing function of z. Since the left-hand side of (3.5) is constant in z, there exists a unique real money balance $z^{\star}$ for a given $\gamma > \beta$.

The second condition ensures that $z^{\star}$ is bounded and that the maximal loan interest is not too high, guaranteeing positive loan demand. The third condition requires the measure of active DM buyers not be too small. Although these conditions are not determined solely by model primitives, but rather depend on equilibrium objects, we can easily verify them in our numerical calculations.

Proposition 3.1 requires $\gamma > \beta$. Now consider the case of $\gamma = \beta$ (*i.e.* the Friedman Rule):

Proposition 3.2. *If $1 + \tau \equiv \gamma = \beta$, then there is no SME with loan interest rate dispersion. Moreover, if $\alpha_0 > 0$, the Friedman rule attains the first-best allocation $q^{\star,FB}$.*

As under the Friedman rule it is costless to carry money across periods, banking of this type is redundant. Households can insure themselves perfectly against the risk of being inactive and there is no gain to redistributing liquidity in an SME. From this point on, we restrict attention to $\gamma > \beta$.

Remark. In a SME, we have that $z = z^{\star}(\boldsymbol{\tau}) = Z$, so we can collapse the state-policy vector $(z, \mathbf{z})$ into just $\mathbf{z}$ when we discuss results pertaining to an SME below.

4 Quantitative Analysis

We now further analyze the model numerically, disciplining the analysis by calibrating the model to macro-level data. We then use the model to investigate the effects of various parameters and alternative policies.

We will also relate the model's predictions to micro-level empirical observations on the dispersion and levels of loan rate spreads.

To account for observations of default in our loans data, we augment the basic model to allow for exogenous random default. In the extended model, banks lose the whole amount of the loan if a borrower defaults on repayment in the CM, and this happens with probability $0 < \hat{\delta} < 1$. This feature generates a default risk premium on loans in equilibrium that can be calibrated to the data. We assume that banks can punish defaulters by excluding them from the financial system (with probability $\psi = 1$) in all future periods.[30]

4.1 Baseline calibration

Our approach is to match the empirical money demand and average loan spread in the macro data, where we measure the latter in a SME by

$$\mu(\gamma) = (1 - \hat{\delta}) \int_{\underline{i}(\mathbf{z})}^{\bar{i}(\mathbf{z})} \left[\frac{i}{i_d(\tau)} - 1 \right] \mathrm{d}F(i, \mathbf{z}), \tag{4.1}$$

where $\gamma = 1 + \tau$.[31]

For identification, the bank contact probabilities (α_0, α_1) affect directly the loan rate distribution, $F(\cdot, \mathbf{z})$, and thus banks' average loan-rate spread over their cost of funds $i_d = (\gamma - \beta)/\beta$. The CM utility function, U, is assumed to be logarithmic. With quasi-linear preferences, real CM consumption is then given by $x^\star = (U^{'})^{-1}(A)$, where the scaling parameter, A, determines the relative importance of CM and DM consumption. The DM utility function is given by (2.2). The DM production function is linear and has no parameter to be estimated. The parameters (A, σ) are identified through the model-implied aggregate real money demand relationship with i_d.

We set the model time period to a year and calibrate to annual data. There are eight parameters: $(\tau, \beta, \hat{\delta}, \sigma, A, n, \alpha_0, \alpha_1)$, as we assume in the baseline that there are no redistributive policies ($\tau_b = \tau_s = 0$), just the long-run inflation target, γ, with $\tau = (\gamma - 1) = \tau_2$.

External calibration. Some parameters can be determined directly by observable statistics. We use the Fisher relation to determine the money growth rate, τ, and discount factor, β. The share of inactive buyers (depositors) $\tilde{n} \equiv 1 - n$ is set to match the average share of household depositors with commercial banks per thousand adults in the United States.[32] We set the default rate, $\hat{\delta}$, to match the national

[30] We relegate the details of the extended model to Online Appendix E. The restriction on deterministic punishment does not alter the main insight on default risk premium. Since we consider a single asset economy, we argue that excluding defaulters from the banking system is the hardest punishment the bank can implement in such an economy. By varying the parameter ψ we can examine three special cases: (1) the model presented in the Section 3; (2) a model with deterministic punishment, and (3) a model without punishment. We have also checked that the following numerical results are essentially the same qualitatively in all these cases.

[31] We use $(\frac{\text{bank prime loan rate}}{\text{federal funds rate}} - 1)$ as a proxy for the average loan spread. As a robustness check, we also consider $(\frac{\text{finance rate on personal loans}}{\text{federal funds rate}} - 1)$. The two measures are qualitatively similar. The data for the finance rate on personal loans at commercial banks can be found in the FRED Series (TERMCBPER24NS). We use the data on the bank prime loan rate as it is a longer time series. Alternatively, we could use the three-month T-bill rate to be consistent with the empirical money demand in Lucas and Nicolini (2015). Since the time series for the three-month T-bill rate and fed funds rates behave similarly, this would not alter the general shape of our average loan spread.

[32] Source: FRED Series USAFCDODCHANUM, "Use of Financial Services—key indicators".

average percentage of consumers with new bankruptcies in the United States.[33]

Internal calibration. We jointly choose the pairs (σ, A) and (α_0, α_1) to match, respectively, the aggregate relationships between nominal interest and money demand, and between nominal interest and average gross loan spread. These empirical relations are estimated by auxiliary fitted-spline functions. Intuitively, each pair of these parameters are identified by the shift (or position) and the overall shape of the respective spline approximations of the empirical relations.

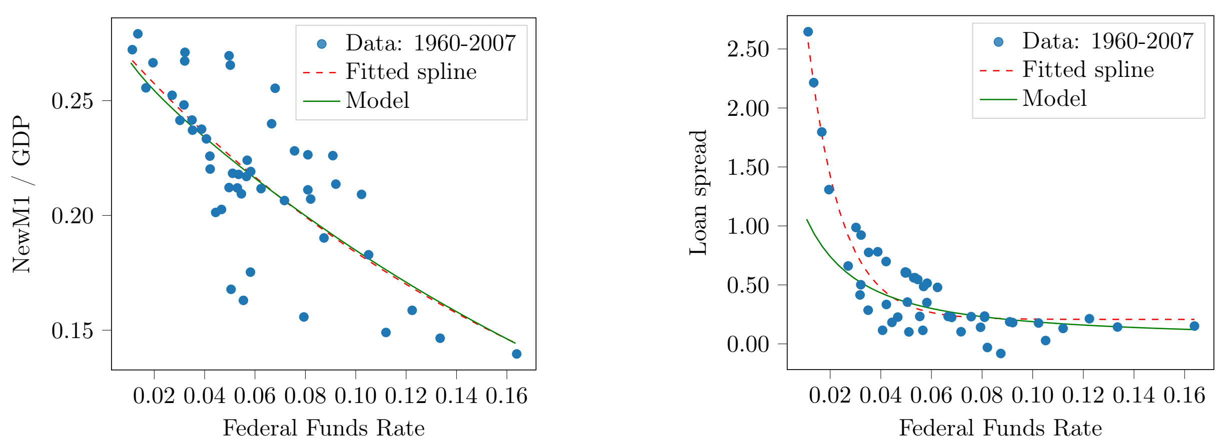
Our parameter values and targets are summarized in Table 1. Figure 1 provides the respective scatterplots of the two empirical relationships (*blue circles*) just mentioned, the empirical spline models (*dashed-red lines*, "Fitted Model"), and our calibrated model's predictions (*solid-green lines*, "Model") for these relations.

Table 1: Calibration and targets.

Parameter	**Value**	**Empirical Targets**	**Description**
$1+\tau$	$(1+0.042)$	Inflation rate^a	Inflation rate
$1+i^{\text{f}}$	$(1+0.061)$	Effective federal funds rate^a	Nominal interest rate
β	0.982	-	Discount factor, $(1+\tau)/(1+i^{\text{f}})$
$\hat{\delta}$	0.15%	National consumers new bankruptcies rate^b	Default rate
σ	0.1895	Aux reg. $(i^{\text{f}}, M/PY)^c$	CRRA (DM q)
A	0.6285	Aux reg. $(i^{\text{f}}, M/PY)^c$	CM preference scale
$\tilde{n}$	0.35	household depositors^d	Proportion of inactive DM buyers
α_0, α_1	0.0324, 0.0995	Aux reg. $(i^{\text{f}}$,spread$)^e$	Prob. $k = 0, 1$ bank contacts

[a] Annual nominal interest and inflation rates.
[b] National average percent of consumers with new brankruptcies.
[c] Auxiliary statistics (data) via spline function fitted to the annual-data relation between the federal funds rate (i^{f}) and Lucas and Nicolini (2015) New-M1-to-GDP ratio (M/PY).
[d] Household depositors with commercial banks per 1000 adults for the United States.
[e] Auxiliary statistics (data) via spline function fitted to the annual-data relation between the federal funds rate (i^{f}) and average loan spread, (bank loan prime rate - i^{f})/i^{f}.

Figure 1: Aggregate money demand and average loan spread—model and data.



(a) Calibration and money demand data

(b) Calibration and average loan spread data

In Figure 1, the model's fit to the average loan spread (the solid green line in Panel b) is not perfect,

[33] Source: Quarterly report on household debt and credit, May 2022, Federal Reserve Bank of New York.

especially at low nominal interest rates. This is due to a tension between matching both real money demand and the average loan-spread. In the model, a lower nominal policy interest leads to a lower cost of holding money, and thus, higher real money demand, reducing the average loan rate by Lemma 3.1. Since the cost of funds $i_d(\tau)$ is fixed by the inflation rate, τ, the average loan spread has to be lower. Nonetheless, we view the fit under the benchmark calibration to be reasonable.

4.2 Comparative steady states

In an SME setting the inflation rate at $\gamma = 1+\tau$ and fixing the nominal interest rate at $i^{\mathrm{f}} = (1+\tau-\beta)/\beta$ are equivalent. From here on, we use the inflation rate as the monetary policy instrument and consider SMEs indexed by different net inflation rates, τ.

We now ask: First, what mechanisms are at work and how do they affect loan rate spreads and the pass-through of monetary policy? And second, what are the testable empirical predictions of these mechanisms? And finally, under what circumstances are agents *ex-ante* better off in an economy with banks than in one without them?

As noted above, in contrast to BCW, financial intermediation of the type studied here need not always be welfare improving. Moreover, in our theory the design of optimal cyclical policy depends on the long-run inflation target and its effects on the state-dependent loan rate distribution, $F(\cdot, \mathbf{z})$. We take up the optimal policy problem in Section 6.

Banks' intensive-extensive profit margin trade-off. Figure 2 depicts realized profit per customer and posted loan rate densities for steady-state inflation rates at zero (*solid-red*), five (*dashed-green*) and ten percent (*dotted-blue*). Changes in the width in the domains of these graphs indicate the shifts in the bounds of the distributions' supports as inflation changes.

In the figure, we see lenders' trade-off between profit per customer (the intensive margin) which is increasing in the posted loan rate, and the number of customers that it successfully serves (the extensive margin) which is *decreasing* in the posted rate. As inflation, τ, rises, not only does the equilibrium support of F shift to the right, but the mass of the density also shifts rightward relative to the lower bound. We identify this latter effect as the extensive margin: As inflation rises lenders raise their loan rates relative to the lower bound, increasing profit per loan, but losing loan customers.

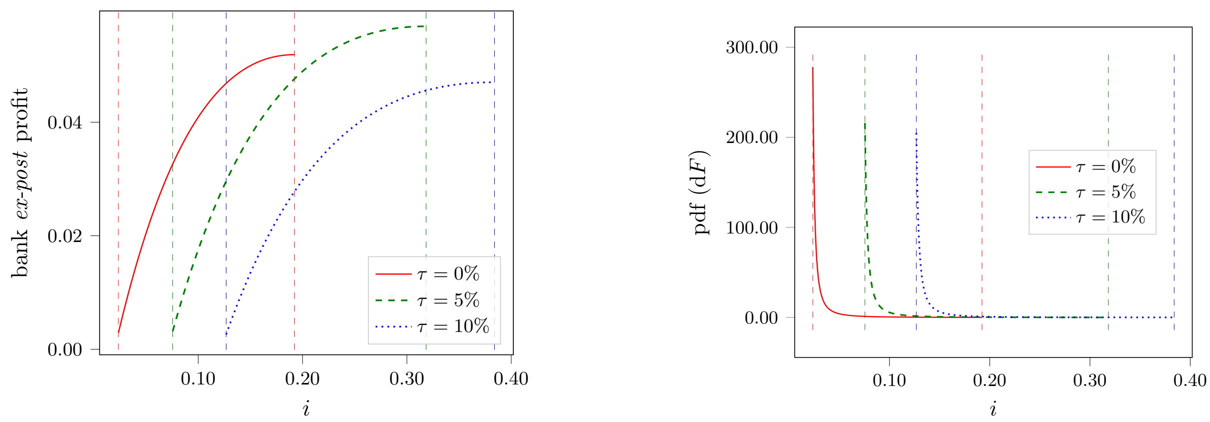
Inflation, dispersion and loan-rate spreads. As noted above, the distribution of posted loan rates, $F(i, \mathbf{z})$, gives rise to an associated distribution of loan rate spreads. We measure the dispersion of these spreads by their standard deviation and coefficient of variation. Let

$$\breve{\mu}(i, \mathbf{z}) := \frac{(1-\hat{\delta})i}{i_d(\tau)} - 1, \tag{4.2}$$

and then the standard deviation of the loan rate spread is:

$$\sigma_\mu = \left[\int_{\underline{i}(\mathbf{z})}^{\bar{i}(\mathbf{z})} [\breve{\mu}(i, \mathbf{z}) - \overline{\mu}]^2 \mathrm{d}F(i, \mathbf{z}) \right]^{\frac{1}{2}}, \tag{4.3}$$

Figure 2: Lenders' extensive-versus-intensive margin trade-off for various levels of long-run inflation, $\gamma = 1 + \tau$.



(a) intensive margin: profit per customer

(b) extensive margin: posted loan-rate density

where the average spread, $\overline{\mu} := \mu(\gamma)$, was defined in (4.1). Alternatively, the coefficient of variation of the spread is:

$$CV_\mu = \frac{\sigma_\mu}{\overline{\mu}}. \tag{4.4}$$

The mean and coefficient of variation of the loan spread as functions of the trend inflation rate are depicted in Panels (a) and (b), respectively, of Figure 3. The standard deviation of the spread is in Panel (c). The relationship between average loan spread and both of these measures of dispersion agree with the empirical evidence in Section 5.[34]

As trend inflation rises, the average loan spread declines, and it declines especially sharply at low inflation. The average spread in (4.1) is the ratio of two parts that are both increasing in the rate of inflation. First, consider the denominator of the term in brackets, which reflects the cost of loans. Higher inflation translates into a higher nominal interest rate and hence a higher deposit rate. This increases the cost of funds to banks and puts upward pressure on loan rates.

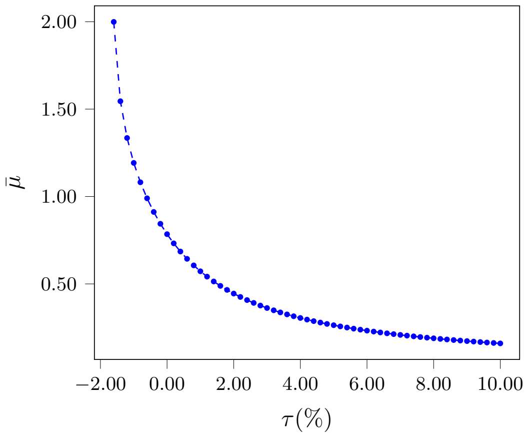
Second, the entire distribution of loan rates shifts upward. Higher inflation reduces real money balances (see Proposition D.1 in Online Appendix D) and lowers consumption, raising marginal utility for active buyers. Their low demand thus increases—*i.e.* they are willing to pay more for loans, and so lenders raise their posted rates. This reasoning also underlies the conclusion of Lemma 3.1.

For the average loan rate *spread* to fall with inflation, the average loan rate itself must rise by less than the deposit rate. In Online Appendix D, we identify sufficient conditions under which this can be proven. Intuitively, borrowers demand smaller loans when the rates they face are higher. Lenders must then "compete harder" for both borrowers and loan size, as inflation rises mitigating the *pass-through* of the increase in the cost of funds represented by the deposit rate to loan rates.[35]

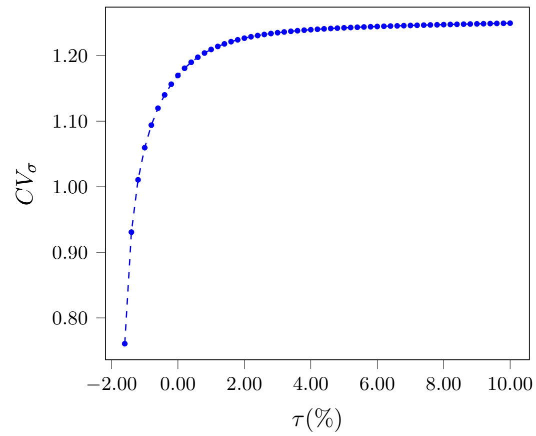
[34] Similar results to those in Figure 3 arise in the U.S. data (for the sample period consistent with our use of *RateWatch* data on bank-level loan rates). See Online Appendix F.

[35] The support of $F(\cdot, z, \mathbf{z})$ shifts right and becomes wider, reflecting an increase in dispersion. The scale-free, coefficient-of-variation measure in Figure 3 depicts exactly this point. Banks posting relatively low rates, post closer to their marginal cost in an attempt to serve a large number of borrowers (many of whom have made contact with an alternative lender). Those posting high rates (which mostly serve only customers with no alternative) raise their rates by a large amount to take

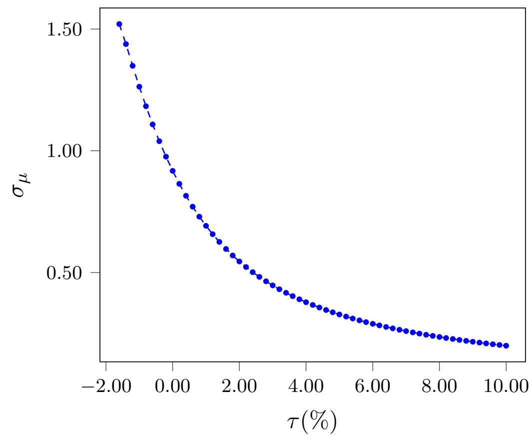
Figure 3: The effects of inflation on lenders' market power for $\tau \in (\beta - 1, \bar{\tau}]$.



(a) Loan spread average



(b) Loan spread dispersion (CV)



(c) Loan spread dispersion (SD)

4.3 The welfare consequences of inflation

The tension discussed in Section 3.2 leads to the welfare effects of banking varying non-monotonically with the rate of trend inflation. On the one hand, access to the banking system provides insurance against the cost of inflation in the event of inactivity. This is the channel through which the competitive banks in BCW raise welfare. On the other hand, significant loan rate spreads effectively lower buyers' surplus in goods trades in spite of the fact that the DM goods market is competitive. In equilibrium, this distortion offsets to some extent the insurance benefits of banks, and in some cases may overcome them. As such, here the presence of a banking system of this type may actually lower welfare relative to a non-bank economy.

When inflation is away from the Friedman rule (*i.e.*, $\tau > \beta - 1$), the presence of an imperfectly competitive banking system as modeled here affects money demand in a number of ways. First, the possibility of obtaining bank credit enables agents to accumulate less money in expectation of taking

advantage of their borrowers' higher marginal utility of consumption. At higher inflation, the mass of $F(\cdot, z, \mathbf{z})$ spreads out from its lower bound so that posted interest rates become more dispersed. At the same time, that lower bound falls toward banks' marginal cost of funds. The average spread is highest at low inflation (although it collapses to zero at the Friedman Rule) and tends to zero as inflation rises.

out a loan in the upcoming DM to minimize the inflation tax. Second, in the event of inactivity, bank deposits provide insurance against carrying *ex post* unwanted balances. In BCW, only the second channel is operative, as the cost of borrowing is exactly equal to the cost of carrying cash into the next period and spending it. Here, however, there are two additional forces at work.

First, positive loan-spreads effectively tighten the liquidity constraint of *ex post* active buyers with positive demand for credit.[36] No longer, as in BCW is the *realized* cost of borrowed funds equal to the cost implied by inflation. Second, as the trend rate of inflation *changes*, changes in the dispersion of loan rates influence the change in the average loan rate spread. This effect depends on both the elasticity of loan demand and the nature of the search process and determines the overall pass-through of inflation to the cost of borrowing relative to the insurance provided by deposits.

Overall, whether the presence of the banking system here improves welfare depends on parameters and, importantly, monetary policy. Lower trend inflation (see Proposition D.1 in Online Appendix D) both exacerbates the negative effects of banks' market power in lending and reduces the insurance benefits of liquidity reallocation. We consider the quantitative welfare effects of trend inflation here and take up stabilization policy in Section 6.

We calculate welfare in terms of the consumption equivalent variation (CEV) measure. A negative (or positive) CEV outcome tells us how much consumption the household needs to be compensated (or willing to forgo) in a pure monetary economy (without banks) to move to an economy with banks (perfectly or imperfectly competitive credit).

The welfare function in a pure monetary economy (equivalent to our baseline search economy with $\alpha_0 = 1$) at a given net inflation rate τ is:

$$W^{no-banks}(\tau) = \frac{1}{1-\beta}\left[nu^{\star}[q(\hat{z})] - c[q^{\star}(\hat{z})] + U(x^{\star}) - x^{\star}\right], \tag{4.5}$$

where $\hat{z}$ is an optimal money balance determined by Equation (3.8).

Without an exogenous default, the welfare function in our baseline economy is:[37]

$$\begin{aligned} W^{HKNP}(\tau) = & \frac{1}{1-\beta}\left[U(x^{\star}) - x^{\star} - c[q_s^{\star}(\mathbf{z})]\right] \\ & + \frac{n}{1-\beta}\left[\alpha_0 u[q_b^{0,\star}(\mathbf{z})] + \int_{\underline{i}(\mathbf{z})}^{\bar{i}(\mathbf{z})} [\alpha_1 + 2\alpha_2 - 2\alpha_2 F(i,\mathbf{z})] u[q_b^{\star}(i,\mathbf{z})] \mathrm{d}F(i,\mathbf{z})\right], \end{aligned} \tag{4.6}$$

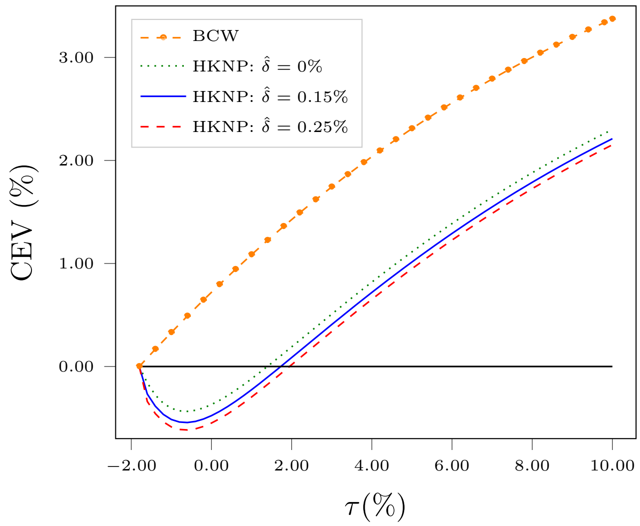
where the functions $q_b^{0,\star}$, $q_b^{\star}$ and $q_s^{\star}$ are characterized by Equations (3.12), (3.13) and (3.10), respectively. The perfectly competitive lending case is equivalent to setting $\alpha_2 = 1$ in Equation (4.6).

We then calculate the *ex-ante* lifetime utility in our baseline economy with consumption reduced by a

[36] This effect is particularly strong for households that have no access to credit, but exists even if $\alpha_0 = 0$.

[37] See Appendix E for the welfare criterion in an economy with random default on loans and punishment on defaulters.

Figure 4: Welfare Comparison: Competitive and non-competitive banks versus no banks.



factor of Δ as:

$$W_{\Delta}^{HKNP}(\tau) = \frac{1}{1-\beta}\Bigg[U(\Delta x^{\star}) - x^{\star} - c[q_s^{\star}(\mathbf{z})]\Bigg] + \frac{n}{1-\beta}\Bigg[\alpha_0 u[\Delta q_b^{0,\star}(\mathbf{z})] + \int_{\underline{i}(\mathbf{z})}^{\bar{i}(\mathbf{z})} [\alpha_1 + 2\alpha_2 - 2\alpha_2 F(i,\mathbf{z})] u[\Delta q_b^{\star}(i,\mathbf{z})] \mathrm{d}F(i,\mathbf{z})\Bigg], \tag{4.7}$$

and similarly, setting $\alpha_2 = 1$ in Equation (4.7), we measure the utility in a perfectly competitive lending economy with consumption reduced by a factor of Δ as:

$$W_{\Delta}^{BCW}(\tau) = \frac{1}{1-\beta}\Bigg[nu^{\star}[\Delta q(i_d, z^{BCW})] - c[q^{\star}(i_d, z^{BCW})] + U(\Delta x^{\star}) - x^{\star}\Bigg], \tag{4.8}$$

where z^{BCW} is the equilibrium real money balance determined by Equation (3.7) and in the BCW equilibrium, the competitive loan rate equals the deposit rate i_d.

To assess the welfare contributions of financial intermediation *per se*, we measure the welfare gains in CEV as the value $1-\Delta$ that yields the same utility under the economy with banking as the pure monetary economy, $W_{\Delta}^{e}(\tau) = W^{no-banks}(\tau)$, for each economy $e \in \{HKNP, BCW\}$ under consideration.

Figure 4 depicts the welfare contribution of banking as inflation changes in four different economies. The search economy with random default under the benchmark calibration is depicted by the *solid-blue* line (HKNP). The BCW economy (*dash-dotted-orange*, BCW) is nested in our model by setting $\alpha_2 = 1$ and $\hat{\delta} = 0$. The other two cases are that of no random default (*dotted-green*) and with a higher default probability (*dashed-red*).

Gains from banking are always positive and increasing in trend inflation in the BCW economy. Inflation reduces the return on money balances and is a cost for agents carrying money into the DM for consumption purchases. This cost is exacerbated by the possibility of being inactive in the DM. As noted above, bank deposits provide insurance against this additional cost. The higher the rate of trend inflation, the more valuable this insurance is. As such, banks not only improve welfare, but have a larger effect the higher is the inflation rate.

As noted above, imperfect competition in lending introduces a wedge between the deposit and average loan rates, eroding the welfare gains from insurance described above. Moreover, imperfectly competitive banking can actually lower welfare relative to an economy with no banking if it generates costs which outweigh the insurance benefits of liquidity reallocation.

At a given inflation rate, banking raises the nominal price level even as it increases real balances by providing insurance through the deposit rate. With competitive banking, this effect is compensated for by loans which are no more costly than carrying money into the DM in the event that it is spent. With imperfectly competitive banking, however, these additional *nominal* balances are more costly than the cash carried into the DM by the amount of the loan spread. At low inflation, the insurance value of deposits is low and, as seen above in Figure 3, loan spreads are high. The cost of nominal balances effectively extracts surplus from active DM buyers in goods markets, lowering welfare. Effectively, households must either carry excessive nominal balances in to the CM or pay lenders a high rate on loans in the event that they are liquidity constrained. From households' point of view, this is akin to facing DM sellers who exercise market power. As the DM goods market is competitive, however, this surplus goes to the banks, rather than to the sellers.

While this effect always reduces welfare relative to the competitive lending case, it can only dominate if banks have sufficient market power (*i.e.* if loan spreads are high enough) and inflation is sufficiently low (the insurance value of deposits is low). As inflation rises, loan spreads fall and deposit interest rises. For any configuration of parameters consistent with an SME, at some inflation rate the presence of banking of this type raises welfare. These welfare gains, however, are always lower than they would be if lending were competitive.

The possibility of default lowers welfare for standard reasons. Default risk in lending implies higher lending costs and thus *larger* loan spreads over the deposit rate. This increases the extent to which buyers' surplus in goods trades is reduced, although this component is a pure loss rather than a transfer to lenders. In Figure 4 accounts for both the *solid-blue* and the *dashed-red* lines lying below the case of no default (*dotted-green*).[38]

Overall, trend inflation has two opposing effects on welfare. First, as in BCW, banks improve welfare by providing insurance against holding idle money in the DM as an inactive buyer. Second, banks' market power in the loan market reduces households surplus from goods trades in the DM, lowering the value of real balances. This happens as banks both increase the nominal price level and raise the cost of additional funds. As such, with constant marginal cost of production this can only happen in the presence of a loan spread, and occurs even if all active buyers have access to banks. The overall welfare effect of banking depends on the relative sizes of these effects. In our baseline calibrated economy when inflation is sufficiently low, loan spreads are big enough and the gains to insurance are low enough that banking of this type can reduce aggregate household welfare.

[38]We have considered a *hyperinflationary regime*, with $\tau = 100\%$ as a robustness check. In this case, both the BCW economy and our baseline, the relationship between trend inflation and welfare gain from banking is non-monotonic. As $\tau \to \infty$, the welfare gains from banking in both economies approaches zero. The reason for this is that at sufficiently high inflation, values of real balances in both economies tend to zero. In our working paper, we also consider an alternative welfare-comparison in wihch we replace the no-bank economy here with one in which banks exist, but agents are restricted not to being able to borrow or deposit excess liquidity. We label this as a *financially-autarkic* equilibrium. We can prove that welfare under any HKNP equilibrium always dominates its corresponding financially-autarkic equilibrium if the long-run inflation target is away from the Friedman rule. In the interest of brevity, we do not present these alternative results here.

5 Inflation, pass-through and dispersion: Empirics

There are at least two distinguishing features of our theory that bear empirical consideration. First, there is imperfect pass-through of inflation to lending rates at a rate that increases with inflation. Second, there is a positive (negative) correlation between equilibrium average loan spread and the standard deviation (coefficient of variation) of spreads in the loan market. That is, as inflation rises, banks pass through the increase in costs *differentially* to their lending rates in a manner analogous to that described by Head, Kumar and Lapham (2010).

Since the first result is a well-known fact, we focus here on the second. The model's equilibrium dispersion of loan rates for an identical loan product suggests the need to document new empirical relationships between measures of dispersion and the average level of the loan spread. In an attempt to maintain a match between model and measurement, we will focus on consumer loan rates in the U.S. data (obtained from *RateWatch*).[39] In this data, we have information starting from a granular level of specific bank branches. We aggregate this to the national level in our main regression results, but find similar results at the state level.[40]

In this data, we measure dispersion using both the standard deviation and the coefficient of variation. The main empirical insights are as follows: First, there is a positive relationship between the standard deviation and the average level of the loan spread at monthly frequency. Second, we find a *negative* relationship between the coefficient of variation of the loan spread and its average at the national level. For both measures of dispersion, these empirical results are consistent with the theoretical predictions of the model.

Below, we detail how these empirical insights are obtained. In our Online Appendix G.2, we also consider alternative loan product classes, including mortgages. We find that the main results here are robust to these alternative measurements.

5.1 Data

Branch-level interest rate data. *RateWatch* provides monthly interest rate data at the branch level for several types of consumer lending products. Our baseline analysis focuses on unsecured consumer loans within a particular class. By focusing on posted loan rates (rather than the rates on specific loans) we minimize the effects of both observed and unobserved heterogeneity across borrowers and loans. Also, this measure is the most consistent with our theoretical model's setting where there is equilibrium rate dispersion for a single type of consumer loan product. Specifically, we choose the most commonly used product for personal loans: Personal Unsecured Loan for Tier 1 borrowers.[41] Our primary sample includes 496,942 branch-month observations from January 2003 to December 2017, involving 11,855 branches. To

[39] See `https://www.rate-watch.com/`. We have also checked that similar results obtain when we consider alternative classes of loan products (*e.g.*, mortgages) and different borrower risk groups. These extended results are available from the authors upon request. Since the simple model is about liquidity risk at the consumer level, it is appropriate here to just present results for the consumer-loan case.

[40] In theory, one could perform the empirical analysis at the bank-branch or county level. However, in practice, the information is too sparse at many branches or counties to be informative at such levels.

[41] As a robustness check, we also use mortgage rates as the alternative variable to calculate loan spreads. Specifically, we choose 30-Year Fixed Mortgage rate with an origination size of $175,000. Our key results still hold when we use mortgage rates. Our results continue to hold if we use rates on personal loans with different borrower qualities (*i.e.*, different borrower "tier" definition).

calculate each branch's loan spread over the federal funds rate, we collect daily effective federal funds data from Federal Reserve H15 report.

Bank and county controls. We obtain commercial banks' information from their call reports. Specifically, we collect information on each commercial bank's reliance on deposit financing, leverage ratio, credit risk and bank size.

The Federal Deposit Insurance Corporation (FDIC) provides branch-level deposit holdings information, for all FDIC-insured institutions. This can be found in the Summary of Deposits (SOD) dataset. We use this data set to approximate each branch's local market competition and the impact of its commercial-bank-branch network. To control for potential local-market competition effects, we calculate each branch's deposit share in its county, the Herfindahl-Hirschman Index (HHI) in each county's deposit holdings and the number of branches in the county. To measure one branch's parent commercial bank's branch network, we calculate one branch's deposit share in its parent bank, the Herfindahl-Hirschman Index (HHI) of the commercial bank's deposit holdings and the number of branch counts in the commercial bank.

We also control for county-level socioeconomic information. This includes median income, the poverty rate, population and the average house price, all obtained from census data. We also have county-level unemployment and number of business establishments from the Bureau of Labor Statistics, county-level real GDP and GDP growth from the Bureau of Economic Analysis to control for local economic activity.

5.2 Loan rate spreads

We use two measures of loan rate spreads: (1) the raw spread of lending rates over the federal funds rate; and (2) an orthogonalized spread using a set of control variables.

The raw spread. We calculate each branch's loan spread relative to the federal fund rate (FF). Specifically, the branch-level raw spread is calculated as

$$Spread_{b,i,c,s,t} = \frac{(1 + Rate_{b,i,c,s,t}) - (1 + FF_t)}{1 + FF_t}. \tag{5.1}$$

In this definition, b stands for a bank branch, i for the parent bank to which the branch belongs, c for the county in which branch is located, s for the state and t for the date that *RateWatch* reports the branch rate information.

Residual or orthogonalized spreads. Differences in branch-level loan pricing could simultaneously be explained by local socioeconomic factors, deposit market competition, bank-branch networks, characteristics of banks and other fixed effects. These factors could determine locally different demands for loans and costs of bank funds. These confounding features, however, will not be captured in our simpler model structure. In our model, the distribution of loan rate spreads will result from the single feature of noisy consumer search in equilibrium. To maintain consistency with our model, it is useful to focus on an empirical measure of the residual spread accounting for as many of these factors as possible.

We thus orthogonalize the branch-level spread with respect to these potential factors to obtain a measure of a residual loan spread. We use this OLS regression to obtain the residual $\epsilon_{b,i,c,s,t}$:

$$Spread_{b,i,c,s,t} = a_0 + a_1 X_{b,i,c,s,t} + a_2 X_{i,t} + a_3 X_{c,s,t} + \epsilon_{b,i,c,s,t}. \tag{5.2}$$

Here, $X_{b,i,c,s,t}$ represents branch-specific control variables including local deposit market competition and bank branch networks, $X_{i,t}$ represents commercial bank control variables and $X_{c,s,t}$ represents county-level socio-economic control variables. We then re-scale $\epsilon_{b,i,c,s,t}$ to match the mean and standard deviation of raw spreads in our full sample and use it as our alternative specification for the loan rate spread. A detailed summary can be found in our Online Appendix G.1 (Table 5).

5.3 The dispersion and mean of the loan spread

We estimate OLS regressions of the dispersion of spreads ($Dispersion_t$) on their monthly average, ($\overline{Spread}_t$):

$$Dispersion_t = b_0 + b_1 \overline{Spread}_t + \epsilon_t. \tag{5.3}$$

In the (5.3), b_1 is the coefficient of interest and standard errors are clustered by month. The average spread, $\overline{Spread}_t$, refers to either the raw or orthogonalized spread. We consider two measures of dispersion: the monthly standard deviation (SD_t) coefficient of variation (CV_t) of the spreads.

5.4 Results

We illustrate first our main empirical findings graphically using simple scatter plots. In Figure 5, we have the correlations between our two measures of loan spread dispersion and the average spread.

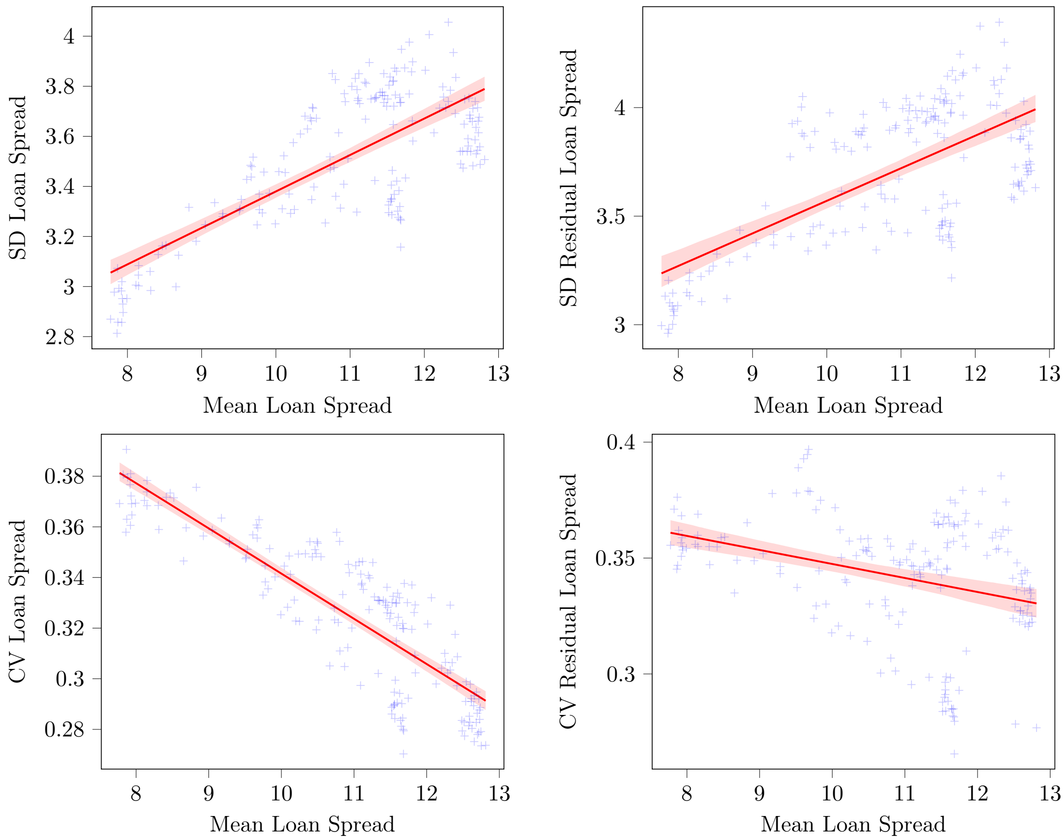
Consider now the relationship between the standard deviation and mean of spreads in the top two panels of Figure 5. The left panel depicts the relationship using the raw spreads, while the right uses the orthogonalized spread, *i.e.*, our residual measure after controlling for various local, market, and social confounding factors. The standard deviations of both measures of loan spreads are positively correlated with their averages. In particular, the correlation is 0.752.

The bottom two panels of Figure 5, show that the coefficients of variation of spreads are *negatively* correlated with their averages. This holds for both raw (*left panel*) and orthogonalized (*right panel*) measures. The correlation for the case of the raw loan spread is -0.857.

Next, we report regression results for our two measures of loan spreads from estimating (5.3). The results are summarized in Table 2. From Columns (1) and (3) of the table we can see positive and statistically significant relationships between standard deviations and averages for the respective raw and orthogonalized loan spread measures. For the raw spread, the coefficient in Column (1) indicates that a one-percentage-point increase in the average spread is associated with a 0.146-percentage-point increase in its standard deviation. Alternatively, for the orthogonalized spread, Column (3) indicates that a one-percentage-point increase in the average is associated with a 0.192-percentage-point increase in the standard deviation. Similarly, from Columns (2) and (4), we can see *negative* and statistically significant relationships between the coefficients of variation and averages, again for both measures of the spread.

We report state-level results in Online Appendix G. We consider the dispersion of spreads at the state level using the standard deviation of branch spreads from state s in month t. As at the national level,

Figure 5: Spread dispersion and averages at the national level (January 2003 to December 2017). Dispersion measures: SD (*standard deviation*) and CV (*coefficient of variation*). Data source: *RateWatch*, "Personal Unsecured Loans (Tier 1)." Least squares regression lines with 95% (bootstrapped) confidence bands (shaded patches) superimposed.



the standard deviations of spreads are positively related to their average levels in the state-month panel data after controlling for state and time fixed effects. There is a corresponding, but noisier result for the coefficient of variation at the national level.

Table 2: Regression of spread dispersion on averages (national data).

	Spread dispersion: $Dispersion_t$			
	Raw loan spread		Orthogonalized spread	
	(1)	(2)	(3)	(4)
	SD_t	CV_t	SD_t	CV_t
$\overline{Spread}_t$	0.146***	−0.018***	0.192***	−0.014***
	(0.004)	(0.000)	(0.010)	(0.001)
Constant	1.924***	0.520***	1.621***	0.492***
	(0.039)	(0.004)	(0.111)	(0.009)
N	180	180	180	180
adj. R^2	0.554	0.733	0.333	0.259

Note: Standard errors in parentheses. $^{*}\ p < 0.1$; $^{**}\ p < 0.05$; $^{***}\ p < 0.01$

6 Optimal stabilization policy

We now study an optimal stabilization policy in response to aggregate demand shocks by solving a version of the Ramsey problem considered by Berentsen and Waller (2011).[42] Specifically, we consider the problem of a central bank that varies the timing of its monetary injections (or alternatively, manages the nominal interest rate) in response to fluctuations in the share of active buyers in the DM.

Our model, however, has a new perspective on why stabilization policy matters and how it functions. While the optimal policy here in some ways resembles that of Berentsen and Waller (2011), it does so for different reasons owing to differences in the nature of competition among banks. The stabilization policy in Berentsen and Waller (2011) counteracts sub-optimal interest rate movements arising from the effect of demand fluctuations on competitively determined deposit and loan rates. Specifically, it raises (lowers) the deposit rate when aggregate demand is low (high). Here the optimal policy reduces banks' loan spreads when aggregate demand is high. We isolate this channel here by holding the deposit rate fixed as aggregate demand varies.

6.1 Aggregate demand shocks

The model admits two simple types of aggregate demand shocks. Let n, the fraction of active DM buyers now fluctuate randomly. An increase of this fraction raises the demand for DM goods and increases the number of potential borrowers. Let ϵ be a multiplicative shock to the utility of DM consumption for active buyers. An increase in ϵ raises demand for both goods and loans by each active DM buyer. Let $n < 1$ lie in $[\underline{n}, \overline{n}]$, and, $\epsilon > 0$ in $[\underline{\epsilon}, \overline{\epsilon}]$. Define $\omega = (n, \epsilon) \in \Omega$ denote the aggregate state vector, and $\psi(\Omega)$ be its probability density.

6.2 Monetary policy

The central bank commits to an overall long-run inflation target τ (or equivalently, a price path) and engages in state-contingent liquidity management which varies prices and loan interest rates in the DM.

[42]Details of the problem setup can be found in Appendix H.

The sequence of central bank actions are as follows. First, a uniform monetary injection, τM, is made to all buyers at the beginning of the period (before ω is realized). Second, contingent on ω, the central bank makes a non-negative lump-sum transfer to buyers in the DM, $\tau_1(\omega)$.[43] We assume the central bank can tax only in the CM, hence the restriction that $\tau_1(\omega) \geq 0$. Next, DM interactions among buyers, banks and sellers take place as described above followed similarly by CM interactions. Lastly, we assume that any state-contingent injection of liquidity made in the DM is undone in the CM: *i.e.*, $\tau_2(\omega) = -\tau_1(\omega)$.[44]

Given the assumption that the DM state-contingent policy will be undone in the subsequent CM, the total change to the aggregate money stock is deterministic and given by

$$M_{+1} - M = (\gamma - 1)M = \tau M, \tag{6.1}$$

where $\gamma = 1+\tau$ is the growth in money supply. As such, we consider only a stationary monetary equilibrium where end-of-period real money balances are both time and state invariant, *i.e.* $\phi M = \phi_{+1} M_{+1} = z$, for all $\omega \in \Omega$. In a stationary monetary equilibrium, money supply growth is

$$\frac{\phi}{\phi_{+1}} = \frac{M_{+1}}{M} = \frac{p_{+1}}{p} = \gamma = 1 + \tau. \tag{6.2}$$

Thus, the central bank follows a price-level targeting policy via a given trajectory for the money stock, as in Berentsen and Waller (2011).

6.3 The central bank

We compare an *active* central bank conducting an optimal policy of the type described above to two alternatives. First, to a *passive* central bank that undertakes no policy actions in response to shocks (*i.e.* $\tau_1(\omega) = \tau_2(\omega) = 0$ for all $\omega \in \Omega$). And second, to an active central bank under the assumption that $\alpha_2 = 1$ and with the restriction that the deposit rate remain constant removed. This later case replicates the policy experiment conducted in Berentsen and Waller (2011).

An active central bank commits to an *ex-ante* optimal policy that maximizes social welfare in a

[43] The central bank could treat the active and inactive buyers differently and could make transfers to sellers. We ignore this channel because such policies are redundant. Moreover, it is without loss of generality that we let $\tau_s(\omega) = 0$.

[44] This policy can be thought of as a repo agreement where the central bank sells money in the DM and promises to buy it back in the CM. We also consider a redistributive transfer-and-tax scheme as a robustness check. The result is qualitatively similar and hence we do not present it here. The key difference between our main exercise and the alternative is the relaxation of the constraint on taxing in the DM. Under the alternative policy, the government commits to an overall inflation target τ and splits the overall transfer, τM between the DM and the CM depending on the realization of the state. Injections of liquidity to agents in the DM are followed by transfers/taxes in the CM to maintain the price-level target, i.e., $\tau_2(\omega)M = [\tau - \tau_1(\omega)]M$ for all ω.

stationary (Markov) monetary equilibrium:[45]

$$\begin{aligned}
\max_{\{q_b^0(\cdot,\omega),\ q_b(\cdot,\omega),\tau_1(\omega)\}_{\omega\in\Omega}} \quad & U(x) - x - c(q_s(\mathbf{z},\omega)) \\
& + \int_{\omega\in\Omega} n\alpha_0 \epsilon u \left[q_b^0(\mathbf{z},\omega)\right] \psi(\omega)\, \mathrm{d}\omega \\
& + \int_{\omega\in\Omega} n \int_{\underline{i}(\mathbf{z},\omega)}^{\bar{i}(\mathbf{z},\omega)} \left[\alpha_1 + 2\alpha_2 (1 - F(i,\mathbf{z},\omega))\right] \\
& \times \epsilon u \left[q_b(i,\mathbf{z},\omega)\right] \mathrm{d}F(i,\mathbf{z},\omega)\, \psi(\omega)\, \mathrm{d}\omega
\end{aligned} \tag{6.3}$$

subject to the constraint on policy: $\frac{\gamma-\beta}{\beta} = \tau + \tau_1(\omega) + \tau_2(\omega)$, and, $\tau_2(\omega) = -\tau_1(\omega)$. The optimal policy prescribes a set of state-contingent liquidity injections, $\tau_1(\omega)$.[46]

6.4 The optimal policy: An example

For illustration, we consider a policy exercise using only shocks to the number of active DM buyers, n, and holding ϵ fixed at one. We do this for simplicity as optimal policy in response to both types of shock is qualitatively similar. The key difference between shocks to marginal utility (ϵ) and to the measure of active buyers (n) is that in the absence of a policy response the former shifts the distribution of loan rates whereas the latter leaves it unchanged. As such the effect of the optimal policy on the average loan-rate spread is simpler in the case of shocks to n. We assume that n is distributed uniformly on $\{n_1, \ldots n_4\}$ where $n_i < n_{i+1}, i = 1, ..., 3$.

Table 3 depicts the result of our optimal policy exercise. As noted above we compare three economies, the first in the table being our Benchmark calibrated economy with imperfectly competitive lending and four aggregate demand states as described above. We compare this economy under the optimal policy with an active central bank to two alternatives. The first being the same economy with the central bank remaining passive in response to shocks and the second being the case considered by Berentsen and Waller (2011). In this case lending is perfectly competitive, the lending rate equals the deposit rate and varies in response to both shocks and the policy response to them. In all cases we fix the long-run inflation target at $\tau > \beta - 1$, and set it to 0.042, reflecting the average of 4.2% annual inflation over the period of our calibration. Thus, banks' marginal cost of funds is also fixed at $i_d = \gamma/\beta - 1$.[47]

The first line for each case in the table reports the optimal DM transfer of additional liquidity for each state. When the central bank is passive, this is of course zero in all states. Before describing the optimal active policy in the benchmark economy, it is useful to review briefly the optimal policy in the economy of Berentsen and Waller (2011). In the absence of active policy in that case, as the measure of active buyers increases deposits decline and loan demand increases, putting upward pressure on the loan rate. This, however, is sub-optimal, as the return on deposits rises precisely when relatively few inactive buyers hold them, limiting their insurance function. As such, the optimal policy counteracts this—increasing liquidity

[45] The equilbrium definition in Section 3.6 is expanded straightforwardly to account for variation in the state, ω, and policy, $\tau_1(\omega)$.

[46] We write these as functions of an SME state-policy vector augmented by ω—*i.e.*, $(\mathbf{z}, \omega)$. Details of the solution can be found in Online Appendix H.

[47] If $\tau = \beta - 1$, *i.e.* the *Friedman Rule*, then holding money is costless and there is no need for either banking or stabilization policy of any kind.

when the supply of deposits would otherwise be low and lowering (raising) the loan (and deposit) rate when aggregate demand is high (low).

Table 3: **Optimal policy in response to aggregate demand shocks**

	States			
Measure of active buyers n:	$n_1 = 0.7$	$n_2 = 0.75$	$n_3 = 0.8$	$n_4 = 0.85$
I. Benchmark Economy with Active Central Bank:				
Amount of transfer $z\tau_1$	0.0002	0.0040	0.0384	0.0344
DM consumption q_b	0.5276	0.5657	0.6067	0.6442
Average loan interest rate i	0.0874	0.0871	0.0846	0.0849
Average loan interest spread μ	0.4300	0.4249	0.3846	0.3889
II. Benchmark Economy with Passive Central Bank:				
Amount of transfer $z\tau_1$	0	0	0	0
DM consumption q_b	0.5285	0.5663	0.6040	0.6418
Average loan interest rate i	0.0866	0.0866	0.0866	0.0866
Average loan interest spread μ	0.4168	0.4168	0.4168	0.4168
III. Perfect Competition with Active Central Bank:				
Amount of transfer $z\tau_1$	0.0211	0.0423	0.0581	0.0740
DM consumption q_b	0.5512	0.6065	0.6596	0.7143
Loan interest rate i	0.0470	0.0389	0.0320	0.0256
Loan interest spread μ	0	0	0	0
Welfare gains: Consumption Units				
Active vs. Passive Policy in the Benchmark (I vs. II):	0.0249%			
Perfect vs. Imperfect Competition with Active Policy (III vs. I):	0.6106%			

Note: $\tau = 0.042$. Each row depicts the state-contingent variable in level. The *ex-ante* welfare gain is the percentage deviation from the benchmark with a passive central bank, measured as compensating variation in consumption units.

In our benchmark economy, in the absence of active policy fluctuations in aggregate demand have no effect on either the return on deposits (because it is determined in the external interbank market) or on the distribution of loan rates (because fluctuations in n alone have no effect on the upper bound of the loan rate distribution). Optimal policy in this case hinges on the effect of banks' market power on consumption per active buyer in the DM.

As aggregate demand increases, the aggregate welfare cost of a given loan-rate spread increases as lenders extract surplus from a larger share of the population. The central bank can counteract this to an extent by injecting liquidity in the DM, inducing banks to reduce their loan market spreads in hopes of making more and larger loans. A higher DM liquidity injection ($\tau_b(n)$) thus lowers directly both the average loan spread and its dispersion by reducing the maximum (*i.e.* monopoly) loan rate. There is, however, a counteracting force which can dominate when the fraction of active buyers becomes sufficiently large.[48] This can be seen in the non-monotonicity of the optimal policy. Liquidity injections *lower* all buyers' real money balance, inducing increased dispersion of the loan spread. The net welfare consequence of a

[48]These can be deduced from Equations (H.5) and (H.6) in Appendix H.

given liquidity injection, and thus the optimal state-contingent policy depends on the relative magnitude of these two opposing forces, the first of which raises welfare when aggregate demand increases and the second of which mitigates these gains.

In our example exercise here, as long as the demand shock is not too big the injection is increasing in n. The central bank thus increases the transfer as aggregate demand increases, lowering the average loan spread in the higher demand states at the expense of allowing it to rise when demand is lower. Only in the highest demand state (with $n = .85$) is the latter effect sufficient to blunt this trend. The result is a non-monotonicity in the DM liquidity injection. It increases with n to a point and then declines. The decline is, however, rather small. It remains the case that in the highest demand state the average loan-spread is lower (and DM consumption per active buyer higher) than it would be under the passive policy.

Overall, the optimal policy raises DM consumption and lowers the average loan spread when aggregate demand is high and does the opposite when it is low. The policy thus raises welfare, although the gains are small relative to the overall losses from imperfect competition in lending. The latter can be seen by comparing the imperfectly competitive benchmark with the economy of Berentsen and Waller (2011) under their respective optimal policies.

7 Conclusion

We construct and study a monetary economy in which market power in lending is endogenous and responds to policy. The theory can account for both dispersion of loan interest rates and incomplete pass-through of monetary policy to them. The model generates positive and negative relationships, respectively, between the dispersion of loan rate spreads as measured by their standard deviation and coefficient of variation and the average spread. This is a distinguishing feature of our theory of market power based on search frictions and is consistent with new evidence from micro-level data on U.S. consumer loans.

We also show that imperfect competition in lending may render an otherwise useful banking system detrimental to welfare when inflation is sufficiently low. That is, an economy with no banking whatsoever may achieve higher welfare than one in which banks provide insurance against random liquidity needs. In our model this effect arises solely from the market power of lenders and suggests that this power may be of particular concern in periods of low inflation.

We also study an optimal monetary policy in which the central bank reallocates liquidity differentially in response to aggregate demand shocks under the constraint of a long-run inflation target. For a given inflation target, the optimal stabilization policy reduces loan spreads in periods of high demand and allows them to increase when aggregate demand is low. Policy makers' ability to erode market power, both under stabilization policy and in the long run is limited by the need to maintain the inflation target or incur excessive costs associated with violating it.

References

Allen, Jason, Robert Clark, and Jean-François Houde, "Search Frictions and Market Power in Negotiated-Price Markets," *Journal of Political Economy*, 2019, *127* (4), 1550–1598. Cited on page(s): [5]

Altermatt, Lukas and Zijian Wang, "Oligopoly Banking, Risky Investment, and Monetary Policy," Technical Report July 2021. Cited on page(s): [5]

Andolfatto, David, "Assessing the Impact of Central Bank Digital Currency on Private Banks," *Economic Journal*, February 2021, *131* (634), 535–40. Cited on page(s): [3]

Baggs, Jenn, Loretta Fung, and Beverly Lapham, "Exchange rates, cross-border travel, and retailers: Theory and empirics," *Journal of International Economics*, November 2018, *115*, 59–79. Cited on page(s): [7], [13]

Bankscope, "Bank Concentration (%)," 2020. World Bank Database, Annual Time Series GFDD.OI.01. Cited on page(s): [1]

Bellifeime, Marco, Rustam Jamilov, and Tommaso Monacelli, "HBANK: Monetary Policy with Heterogeneous Banks," Technical Report, LSE, Oxford University and Bocconi University July 2022. Cited on page(s): [2]

Berentsen, Aleksander and Christopher Waller, "Price-Level Targeting and Stabilization Policy," *Journal of Money, Credit and Banking*, 2011, *43* (s2), 559–580. Cited on page(s): [4], [11], [33], [34], [35], [37], [OA.22], [OA.27]

_ , Gabriele Camera, and Christopher Waller, "Money, Credit and Banking," *Journal of Economic Theory*, 2007, *135* (1), 171–195. Cited on page(s): [2], [6], [16], [17], [OA.22]

Bianchi, Javier and Saki Bigio, "Banks, Liquidity Management and Monetary Policy," *Econometrica*, January 2022, *90* (1), 391–454. Cited on page(s): [3]

Boel, Paola and Christopher J. Waller, "Stabilization Policy at the Zero Lower Bound," *International Economic Review*, 2019, *60* (4), 1539–1563. Cited on page(s): [4]

Brunnermeier, Markus K. and Yuliy Sannikov, "A Macroeconomic Model with a Financial Sector," *American Economic Review*, February 2014, *104* (2), 379–421. Cited on page(s): [1]

Burdett, Kenneth and Kenneth L. Judd, "Equilibrium Price Dispersion," *Econometrica*, 1983, *51* (4), 955–969. Cited on page(s): [3], [5], [10], [12], [13], [14], [OA.1], [OA.5]

Chang, Te-Tsun and Yiting Li, "Banking, Liquidity Effects and Monetary Policy," *Macroeconomic Dynamics*, 2018, *22* (5), 1267–1297. Cited on page(s): [5]

Chiu, Jonathan, Mei Dong, and Enchuan Shao, "On the Welfare Effects of Credit Arrangements," *International Economic Review*, 2018, *59* (3), 1621–1651. Cited on page(s): [3], [4], [18]

_ , Mohammad Davoodalhosseini, Janet Hua Jiang, and Yu Zhu, "Bank Market Power and Central Bank Digital Currency: Theory and Quantitative Assessment," Staff Working Papers 19-20, Bank of Canada May 2019. Cited on page(s): [5]

Choi, Michael and Guillaume Rocheteau, "A Model of Retail Banking and the Deposits Channel of Monetary Policy," Technical Report November 2021. Cited on page(s): [4]

Clark, Robert, Jean-François Houde, and Jakub Kastl, "Chapter 15 - The industrial organization of financial markets," in Kate Ho, Ali Hortaçsu, and Alessandro Lizzeri, eds., *Handbook of Industrial Organization, Volume 5*, Vol. 5 of *Handbook of Industrial Organization*, Elsevier, 2021, pp. 427–520. Cited on page(s): [5]

Coimbra, Nuno, Daisoon Kim, and Héléne Rey, "Central Bank Policy and the Concentration of Risk: Empirical Estimates," *Journal of Monetary Economics*, 2022, *125*, 192–98. Cited on page(s): [1]

Corbae, Dean and Pablo D'Erasmo, "Discussion of "Large Banks, Loan Rate Markup, and Monetary Policy"," *International Journal of Central Banking*, 2015, *June*, 1–14. Cited on page(s): [1]

_ and _ , "Capital Requirements in a Quantitative Model of Banking Industry Dynamics," *Unpublished*, 2018. SED2018. Cited on page(s): [1]

_ and _ , "Capital Buffers in a Quantitative Model of Banking Industry Dynamics ," *Econometrica*, 2021. Forthcoming. Cited on page(s): [4], [5]

_ and Ross Levine, "Competition, Stability, and Effciency in the Banking Industry," manuscript, University of Wisconsin and UC Berkeley-Hass August 2022. Cited on page(s): [1]

Dong, Mei, Stella Huangfu, Hongfei Sun, and Chenggang Zhou, "A Macroeconomic Theory of Banking Oligopoly," *European Economic Review*, 2021, *138*, 103864. Cited on page(s): [5]

Drechsler, Itamar, Alexi Savov, and Philipp Schnabl, "The Deposits Channel of Monetary Policy," *The Quarterly Journal of Economics*, 05 2017, *132* (4), 1819–1876. Cited on page(s): [4]

Duffie, Darrell, Nicolae Gârleanu, and Lasse Heje Pedersen, "Over-the-Counter Markets," *Econometrica*, 2005, *73* (6), 1815–1847. Cited on page(s): [5]

Duval, Romain A, Davide Furceri, Raphael Lee, and Marina M. Tavares, "Market Power and Monetary Policy Transmission," Technical Report WP/21/184 July 2021. Cited on page(s): [1]

Executive Order 14036, "Executive Order 14036: Promoting Competition in the American Economy," 2021. Joseph R. Biden, July 9. *Federal Register*, 86 (36987). Cited on page(s): [2]

Godl-Hanisch, Isabel, "Bank Concentration and Monetary Policy Pass-Through," manuscript, LMU Munich and CESifo September 2022. Cited on page(s): [1]

Gu, Chao, Fabrizio Mattesini, Cyril Monnet, and Randall Wright, "Banking: A New Monetarist Approach," *The Review of Economic Studies*, 01 2013, *80* (2), 636–662. Cited on page(s): [5]

He, Ping, Lixin Huang, and Randall Wright, "Money, Banking, and Monetary Policy," *Journal of Monetary Economics*, 2008, *55* (6), 1013–1024. Cited on page(s): [5]

Head, Allen, Alok Kumar, and Beverly Lapham, "Market Power, Price Adjustment, And Inflation," *International Economic Review*, February 2010, *51* (1), 73–98. Cited on page(s): [29]

_ and _ , "Price Dispersion, Inflation, and Welfare," *International Economic Review*, 2005, *46* (2), 533–572. Cited on page(s): [10], [14]

_ , Lucy Qian Liu, Guido Menzio, and Randall Wright, "Sticky prices: A New Monetarist Approach," *Journal of the European Economic Association*, 2012, *10* (5), 939–973. Cited on page(s): [7]

_ , Timothy Kam, Sam Ng, and Guangqian Pan, "Banking Market Power, the Deposits Channel of Monetary Policy and Capital," *Manuscript*, August 2022. Cited on page(s): [3], [11]

Herrenbrueck, Lucas, "An Open-economy Model with Money, Endogenous Search, and Heterogeneous Firms," *Economic Inquiry*, October 2017, *55* (4), 1648–70. Cited on page(s): [7], [13]

Kashyap, Anil K., Raghuram Rajan, and Jeremy C. Stein, "Banks as Liquidity Providers: An Explanation for the Coexistence of Lending and Deposit-taking," *The Journal of Finance*, 2002, *57* (1), 33–73. Cited on page(s): [5]

Lagos, Ricardo and Randall Wright, "A Unified Framework for Monetary Theory and Policy Analysis," *Journal of Political Economy*, 2005, *113* (3), 463–484. Cited on page(s): [6], [7]

Lucas, Robert E. and Juan Pablo Nicolini, "On the Stability of Money Demand," *Journal of Monetary Economics*, 2015, *73* (C), 48–65. Cited on page(s): [21], [22]

Martín-Oliver, Alfredo, Salas-Fumás Vicente, and Jesús Saurina, "A Test of the Law of One Price in Retail Banking," *Journal of Money, Credit and Banking*, 2007, *39* (8), 2021–2040. Cited on page(s): [1]

_ , Vicente Salas-Fumás, and Jesús Saurina, "Informational Differentiation, Interest Rate Dispersion and Market Power," *Applied Economics Letters*, 2009, *16* (16), 1645–1649. Cited on page(s): [1]

Productivity Commission, "Competition in the Australian Financial System," Technical Report, Canberra 2018. Report No. 89. Cited on page(s): [2]

Rocheteau, Guillaume and Antonio Rodriguez-Lopez, "Liquidity Provision, Interest Rates, and Unemployment," *Journal of Monetary Economics*, 2014, *65* (C), 80–101. Cited on page(s): [5]

_ and Randall Wright, "Money in Search Equilibrium, in Competitive Equilibrium, and in Competitive Search Equilibrium," *Econometrica*, 2005, *73* (1), 175–202. Cited on page(s): [2], [11]

Sims, Rod, "Committee Hansard," 2016. Chairman of the Australian Competition and Consumer Commission, 14 October. Cited on page(s): [2]

Wang, Liang, "Endogenous search, price dispersion, and welfare," *Journal of Economic Dynamics and Control*, 2016, *73* (C), 94–117. Cited on page(s): [10]

_ , Randall Wright, and Lucy Qian Liu, "Sticky Prices and Costly Credit," *International Economic Review*, 2020, *61* (1), 37–70. Cited on page(s): [4]

Wang, Olivier, "Banks, Low Interest Rates, and Monetary Policy Transmission," Technical Report, NYU-Stern February 2022. Cited on page(s): [2]

Wang, Yifei, Toni M. Whited, Yufeng Wu, and Kairong Xiao, "Bank Market Power and Monetary Policy Transmission: Evidence from a Structural Estimation," *Journal of Finance*, August 2022, *LXXVII* (4), 2093–2141. Cited on page(s): [1]

Wilkins, Carolyn, "Why Do Central Banks Care About Market Power?," April 2019. G7 Conference at Banque du France. Cited on page(s): [2]

Online Appendix

Appendices

A Omitted proofs: Lending with noisy search for loans

In this section, we collect the intermediate results and proofs that lead to the characterization of an equilibrium distribution of loan rates in the noisy-search model for loans. Most of the proofs in this section are standard in the Burdett and Judd (1983) model. We revisit them here for completeness.

Remark on notation. In the paper, functions such as Π, Π^m, R, $l^\star$—respectively, *ex-ante* profit, monopoly profit, per-customer profit and optimal loan demand functions—all depend on a vector of individual state m, aggregate state M and policies $\boldsymbol{\tau}$, which we summarize as $(m, \mathbf{s}) = (m, (M, \boldsymbol{\tau}))$. Since the noisy-search banking equilibrium is an intratemporal or static one, in the proofs below, we dispense with explicit dependencies on $(m, \mathbf{s})$ to keep proofs more readable. For example, we will write $l^\star(i)$ in place of the explicit notation $l^\star(i, m, \mathbf{s})$.

Summary of results. We begin by proving for Case 1 in Lemma 2.1 (since the case where $\alpha_1 \in (0, 1)$ is our main focus of the model). Then we lay out the proof for the remaining cases of a pure monopoly bank in one limit and competitive banks in the other.

The characterization is arrived at in a few intermediate steps. First, in Section A.1, we show any bank faced with just one customer ex post will earn strictly positive profit. Second, in Section A.2 we show that banks that ex post face more than one customer will also earn strictly positive profit. Third, in Section A.3 we show that there is a unique upper bound on loan prices. Fourth, if the upper bound loan rate is the monopoly rate, we show (in Section A.4) that this rate is uniquely determined as a function of the state of the economy. There is a natural lower bound on loan rates, which is i_d. These results help establish that the equilibrium support on the distribution of loan rate F is bounded.

In a noisy search equilibrium, the banks will be indifferent between a continuum of pure-strategy price posting outcomes. For example, a bank can choose some lower rate in return for attracting a larger measure of borrowers. Or it can post some higher rate to increase its profits per loan but attract a smaller measure of borrowers. Or it can charge a monopolist price. The intermediate results in Lemmata A.5 to A.7 (in Section A.5 to A.6) show that there is a continuum of pure-strategy price posting outcomes that deliver the same maximal monopoly profit. Thus, banks can ex-ante mix over these pure strategies, and in equilibrium, borrowers face a lottery over loan rates, given by a distribution function F. Finally, we can summarize F as an analytical expression in Lemma 2.1. The proof of this is in Section A.7.

A.1 Positive monopoly bank profit

Lemma A.1. $\Pi^m(i) > 0$ *for* $i > i_d$.

Proof. For any positive loan spread $i - i_d$, $\Pi^m(i) = n\alpha_1 R(i) = \Pi^m(i) = n\alpha_1 l^\star(i)[(1+i) - (1+i_d)]$. Since $l^\star(i) > 0$ and $i - i_d > 0$, then $\Pi^m(i) > 0$. □

A.2 All banks earn positive expected profit

Now, we prove that banks will earn strictly positive expected profits:

Lemma A.2. $\Pi^\star > 0$.

Proof. Since pricing rules are linear then for any loan rate exceed the marginal cost of funds, $\mu > 1$, the profit from positing $i = \mu i_d$ is $\Pi(\mu i_d) = n[\alpha_1 + 2\alpha_2(1 - F(\mu i_d)) + \alpha_2 \xi(\mu i_d)] R(\mu i_d) > n\alpha_1 R(\mu i_d) = \Pi^m(\mu i_d) > 0$, where $R(i) = l^\star(m; i, p, \phi, M, \tau_b)[(1+i) - (1+i_d)]$. The last inequality is from Lemma A.1. From the definition of the max operator in (2.25), $\Pi^\star = \max_{i \in \text{supp}(F)} \Pi(i) \geq \Pi(\mu i_d) > \Pi^m(\mu i_d) > 0$. □

A.3 Maximal loan pricing

Third, we can also show that:

Lemma A.3. *The largest possible price in the support of F is the smaller of the monopoly price and ex-post borrower's maximum willingness to pay:* $\bar{i} := \min\{i^m, \hat{i}\}$.

Although the monopoly rate i^m is the maximal possible price in defining an arbitrary support of F, it may be possible in some equilibrium that this exceeds the maximum willingness to pay by households, $\hat{i}$. We condition on this possibility when characterizing an *equilibrium* support of F later.

Proof. First assume the case that $\hat{i} \geq i^m$. Suppose there is a $\bar{i} \neq i^m$ which is the largest element in supp (F). Then $\Pi^m(\bar{i}) = n\alpha_1 R(\bar{i})$. Since $F(i^m) \geq 0$ amd $\zeta(i^m) \geq 0$, then $\Pi(i^m) = n[\alpha_1 + 2\alpha_2(1 - F(i^m)) + \alpha_2 \zeta(i^m)] R(i^m) \geq n\alpha_1 R(i^m) = \Pi^m(i^m) > \Pi^m(\bar{i})$. The last inequality is true by the definition of a monopoly price i^m. Therefore $\Pi(i^m) > \Pi^m(\bar{i})$. The equal profit condition would require that, $\Pi^m(\bar{i}) = \Pi^\star \geq \Pi^m(i^m)$. Therefore $\bar{i} = i^m$ if $\hat{i} \geq i^m$.

Now assume $\hat{i} < i^m$. In this case, the most that a bank can charge for loans is $\hat{i}$, since at any higher rate, no ex-post buyer will execute his line of credit (i.e., he will not borrow). Thus trivially, $\bar{i} = \hat{i}$ if $\hat{i} < i^m$. □

A.4 Unique monopoly loan rate

Fourth, under a mild parametric regularity condition on preferences, we show that there is a unique monopoly loan rate.

Lemma A.4. *Assume $\sigma < 1$. For an arbitrarily small constant bounded below by zero, i.e., $\epsilon > 0$, if $\sigma \geq \epsilon/(2+\epsilon)$, then there is a unique monopoly-profit-maximizing price i^m that satisfies the first-order condition* $\frac{\partial \Pi^m(i)}{\partial i} = n\alpha_1 \left[\frac{\partial l^\star(i)}{\partial i}(1+i) + l^\star(i) - \frac{\partial l^\star(i)}{\partial i}(1+i_d)\right] = 0$.

Proof. Assume $\hat{i} > i^m$. Using the demand for loans from (2.16) the first-order condition at $i = i^m$ is explicitly

$$-\underbrace{\frac{m + \tau_b M}{p^{\frac{\sigma-1}{\sigma}} \phi^{-\frac{1}{\sigma}}}}_{f(i)} + \underbrace{\frac{1}{\sigma}(1+i)^{-\frac{1}{\sigma}} \left[(\sigma - 1) + \frac{1+i_d}{1+i}\right]}_{g(i)} = 0. \tag{A.1}$$

Note that given individual state m, aggregate state M, and policy/prices (τ_b, p, ϕ), the term $f(i)$ is a constant with respect to i. Given i_d, the term $g(i)$ has these properties: (1) $g(i)$ is continuous in i; (2) $\lim_{i \searrow 0} g(i) = +\infty$; (3) $\lim_{i \nearrow +\infty} g(i) = 0$, and, (4) the RHS is monotone decreasing, $g'(i) < 0$.

The first three properties are immediate from (A.1). Since $\Pi^m(i)$ is twice-continuously differentiable, the last property can be shown by checking for a second-order condition: For a maximum profit at $i = i^m$, we must have $\left.\frac{\partial^2 \Pi^m(i)}{\partial i^2}\right|_{i=i^m} \leq 0$. Observe that the second-derivative function is

$$\frac{\partial^2 \Pi^m(i)}{\partial i^2} = g'(i) = -\underbrace{\frac{1}{\sigma^2}(1+i)^{-\frac{1}{\sigma}-1}}_{>0}\left[(\sigma-1) + \frac{(1+\sigma)(1+i_d)}{(1+i)}\right]. \tag{A.2}$$

For (A.2) to hold with ≤ 0, Case A ($\sigma < 1$) would require $\frac{(1+\sigma)(1+i_d)}{(1+i)} \geq 1 - \sigma$ for all $i \geq i_d$. Let $1 + i \equiv (1+\epsilon)(1+i_d)$ since $i^m \geq i > i_d$. The above inequality can be re-written as $\frac{1}{1+\epsilon} \geq \frac{1-\sigma}{1+\sigma}$, which implies $1 > \sigma \geq \frac{\epsilon}{2+\epsilon}$. This is a sufficient condition on parameter σ to ensure that a well-defined and unique monopoly profit point exists with monopoly price $i^m \geq \underline{i} > i_d$.

□

A.5 Distribution is continuous

In the next two results, we show that the loan pricing distribution is continuous with connected support.

Lemma A.5. *F is a continuous distribution function.*

We will prove Lemma A.5 in two parts. First, we document a technical observation that the per-customer profit difference is always bounded above:

Lemma A.6. *Assume there is an $i' < i$ and an $i'' < i'$, with $\zeta(i) = \lim_{i' \nearrow i}\{F(i) - F(i')\} > 0$, and $\zeta(i') = \lim_{i'' \nearrow i'}\{F(i') - F(i'')\} > 0$, and that $R(i') > 0$. The per-customer profit difference is always bounded above: $\Delta := R(i) - R(i') < \frac{\alpha_2 \zeta(i) R(i)}{\alpha_1 + 2\alpha_2}$.*

Proof. The expected profit from posting i is

$$\Pi(i) = n[\alpha_1 + 2\alpha_2(1 - F(i)) + \alpha_2\zeta(i)]R(i).$$

The expected profit from posting i' is

$$\Pi(i') = n\left[\alpha_1 + 2\alpha_2\left(1 - F(i')\right) + \alpha_2\zeta(i')\right]R(i').$$

A firm would be indifferent to posting either prices if $\Pi(i) - \Pi(i') = 0$. This implies that

$$(\alpha_1 + 2\alpha_2)\left[R(i) - R(i')\right] + \alpha_2\zeta(i)R(i) - \alpha_2\zeta(i')R(i') \\ - 2\alpha_2\left[F(i)R(i) - F(i')R(i')\right] = 0.$$

Rearranging and using the definition of $\zeta(i) = \lim_{i' \nearrow i}\{F(i) - F(i')\} > 0$:

$$\begin{aligned}(\alpha_1 + 2\alpha_2)\left[R(i) - R(i')\right] &= \alpha_2\left[F(i)R(i) - F(i')R(i')\right] - \alpha_2\zeta(i')R(i') \\ &< \alpha_2\left[F(i)R(i) - F(i')R(i')\right] \\ &\leq \alpha_2 \lim_{i' \nearrow i}\left\{F(i) - F(i')\right\}R(i).\end{aligned}$$

The strict inequality is because $R(i') > 0$ and $\zeta(i') > 0$. The subsequent weak inequality comes from the fact that $R(i)$ is continuous, so that we can write

$$\lim_{i' \nearrow i} \left\{ F(i) R(i) - F(i') R(i') \right\} = \lim_{i' \nearrow i} \left\{ F(i) - F(i') \right\} R(i).$$

Since $\zeta(i) = \lim_{i' \nearrow i} \{F(i) - F(i')\}$, the last inequality implies that $R(i) - R(i') < \frac{\alpha_2 \zeta(i) R(i)}{\alpha_1 + 2\alpha_2}$. ☐

The following is the proof of Lemma A.5.

Proof. Assume $i \in \text{supp}(F)$ such that $\zeta(i) > 0$ and $\Pi(i) = n[\alpha_1 + 2\alpha_2(1 - F(i)) + \alpha_2 \zeta(i)] R(i)$. R is clearly continuous in i. Hence there is a $i' < i$ such that $R(i') > 0$ and from Lemma A.6, $\Delta := R(i) - R(i') < \frac{\alpha_2 \zeta(i) R(i)}{\alpha_1 + 2\alpha_2}$. Then

$$\begin{aligned}
\Pi(i') &= n\left[\alpha_1 + 2\alpha_2\left(1 - F(i')\right) + \alpha_2 \zeta(i')\right] R(i') \\
&\geq n[\alpha_1 + 2\alpha_2(1 - F(i)) + \alpha_2 \zeta(i)][R(i) - \Delta] \\
&\geq \Pi(i) + n\{\alpha_2 \zeta(i)[R(i) - \Delta] - (\alpha_1 + 2\alpha_2)\Delta\}.
\end{aligned}$$

The first weak inequality is a consequence of $F(i) - F(i') \geq \zeta(i)$. Since $R(i) > \Delta$ and $\Delta < \frac{\alpha_2 \zeta(i) R(i)}{\alpha_1 + 2\alpha_2}$, then the last line implies $\Pi(i') > \Pi(i)$. This contradicts $i \in \text{supp}(F)$. ☐

A.6 Support of distribution is connected

Lemma A.7. *The support of F, $\text{supp}(F)$, is a connected set.*

Proof. Pick two prices i and i' belonging to the set $\text{supp}(F)$, and suppose that $i < i'$ and $F(i) = F(i')$. The expected profits are, respectively, $\Pi(i) = n[\alpha_1 + 2\alpha_2(1 - F(i))] R(i)$ and $\Pi(i') = n[\alpha_1 + 2\alpha_2(1 - F(i'))] R(i')$. Since $F(i) = F(i')$, then the first terms in the profit evaluations above are identical: $n[\alpha_1 + 2\alpha_2(1 - F(i))] = n[\alpha_1 + 2\alpha_2(1 - F(i'))]$. However, since i and i' belonging to the set $\text{supp}(F)$, then clearly, $i_d < i < i' \leq i^m$. From Lemma A.4, we know that $R(i)$ is strictly increasing for all $i \in [i_d, i^m]$, so then, $R(i) < R(i')$. From these two observations, we have $\Pi(i) < \Pi(i')$. This contradicts the condition that if firms are choosing i and i' from $\text{supp}(F)$ then F must be consistent with maximal profit $\Pi(i) = \Pi(i') = \Pi^\star$ (viz. the equal profit condition must hold). ☐

A.7 Proof of Lemma 2.1

Proof. Consider the case where $\alpha_1 \in (0, 1)$. Since F has no mass points by Lemma A.7, and is continuous by Lemma A.5, then expected profit from any $i \in \text{supp}(F)$ is a continuous function over $\text{supp}(F)$,

$$\Pi(i) = n[\alpha_1 + 2\alpha_2(1 - F(i))] R(i),$$

where the image $\Pi[\text{supp}(F)]$ is also a connected set. From Lemma A.3, profit is maximized at $\Pi^m(i^m) = n\alpha_1 R(i^m)$. For any $i \in \text{supp}(F)$, the induced expected profit must also be maximal, i.e.,

$$\Pi(i) = n[\alpha_1 + 2\alpha_2(1 - F(i))] R(i) = n\alpha_1 R(i^m).$$

Solving for F yields the analytical expression in (2.26).

Proofs for the remaining Case 2 and Case 3 in Lemma 2.1 follow directly from Lemma 1 and Lemma 2 in Burdett and Judd (1983). The pricing outcomes, $\bar{i}$ and i_d are, respectively, the upper bound (the monopoly price) and the lower bound (Bertrand price) on the support of F. □

B Omitted proofs: SME

We provide the intermediate results and proofs for establishing existence and uniqueness of a stationary monetary equilibrium with co-existing money and credit.

The conclusion is arrived at in a few intermediate steps. First, in Section B.1 we show that a posted loan-price distribution with lower real money balance first-order stochastic dominance a distribution with higher real money balance, given a monetary policy rule $\gamma > \beta$. Second, in Section B.2 we show that the money demand Euler Equation simplifies to Condition (3.5), and the candidate real money balance solution to the money demand Euler equation is bounded. Third, we use results from Section B.1 and Section B.2 together in section B.3 to show there exists a unique real money balance that solves the money demand Euler (3.5). This establishes existence. Finally, we prove for the uniqueness of a SME with co-existing money and credit in Section B.4.

B.1 First-order stochastic dominance: Proof of Lemma 3.1.

Proof. The analytical formula for the loan-price distribution $F(i, z, \mathbf{z})$ is characterized in (3.1). Suppose we fix $\bar{i}(z, \mathbf{z}) = \bar{i}(z', \mathbf{z})$, and denote it as $\bar{i}$. In general, the lower and upper support of the distribution F is changing with respect to z and policy γ. By fixing the upper support at both z and z' here, we are checking whether the curve of the cumulative distribution function, $F(\cdot, z, \mathbf{z})$, is lying on top or below for z relative to z'. We have $\frac{\partial F(i,z,\mathbf{z})}{\partial z} = \underbrace{\frac{\alpha_1}{2\alpha_2}}_{>0} \left[\frac{(\bar{i}-i_d)R(i,z,\mathbf{z})-(i-i_d)R(\bar{i},z,\mathbf{z})}{\underbrace{(R(i,z,\mathbf{z}))^2}_{>0}} \right]$. For $\partial F(i, z, \mathbf{z})/\partial z > 0$ to hold, one needs to show the numerator is positive. Suppose this were not the case. Then we have

$$
\begin{aligned}
&(\bar{i} - i_d)R(i, z, \mathbf{z}) - (i - i_d)R(\bar{i}, z, \mathbf{z}) \leq 0 \\
\implies &(\bar{i} - i_d) \underbrace{\left[(1 + i)^{\frac{-1}{\sigma}} - z\right](i - i_d)}_{=R(i,z,\mathbf{z})} \leq (i - i_d) \underbrace{\left[(1 + \bar{i})^{\frac{-1}{\sigma}} - z\right](\bar{i} - i_d)}_{=R(\bar{i},z,\mathbf{z})} \\
&\qquad \implies \left[(1 + i)^{\frac{-1}{\sigma}} - z\right] \leq \left[(1 + \bar{i})^{\frac{-1}{\sigma}} - z\right]
\end{aligned}
$$

The last inequality contradicts the fact that the loan demand curve is downward sloping in i, and $\bar{i}$ is the highest possible loan-price posted by banks (lending agents). Thus, the numerator must be positive and $\partial F(i, z, \mathbf{z})/\partial z > 0$. This shows that a loan-price distribution $F(\cdot, z, \mathbf{z})$ first-order stochastically dominates $F(\cdot, z', \mathbf{z})$, for $z < z'$. □

B.2 Money and credit: Proof of Lemma 3.2

Proof. We want to show equivalence in the three claims in Lemma 3.2. The proof relies on a CRRA(σ) preference representation and linear cost of producing the DM good $c(q) = q$.

1. We say that the DM relative price ρ is sufficiently low if real money balance z is such that

$$\rho = 1 < \tilde{\rho}_i(z, \mathbf{z}) \equiv (z)^{\frac{\sigma}{\sigma-1}}(1+i)^{\frac{1}{\sigma-1}}, \qquad 0 < \sigma < 1. \tag{B.1}$$

   The following is a sufficient requirement: If $z < \left(\frac{1}{1+i}\right)^{\frac{1}{\sigma}}$, then inequality (B.1) holds. From Lemma 2.1, if $\alpha_1 \in (0,1)$, the distribution $F(\cdot, z, \mathbf{z})$ is non-degenerate and $\text{supp}(F(\cdot, z, \mathbf{z})) = [\underline{i}(z, \mathbf{z}), \bar{i}(z, \mathbf{z})]$ exists. This implies that for all $i \in \text{supp}(F(\cdot, z, \mathbf{z}))$, the inequality $z < \left(\frac{1}{1+\bar{i}(z,\mathbf{z})}\right)^{\frac{1}{\sigma}}$ is also true. Since SME $z = z^\star$ exists and $z^\star < \left(\frac{1}{1+\bar{i}(z^\star,\mathbf{z})}\right)^{\frac{1}{\sigma}}$, then ρ is sufficiently low and satisfies inequality (B.1).

2. From Claim 1 above, the DM relative price ρ satisfies inequality (B.1). From (3.14), there is ex-post positive loan demand by the active DM buyers who meet at least one bank. In the opposite direction: If there is ex-post positive loan demand, then condition (B.1) must hold, thus implying Claim 1.

3. Combining Claim 2 with agents' first-order condition for optimal money demand, their money-demand Euler Equation reduces to (3.5). In reverse, (3.5) implies that there is positive demand for loans and money (Claim 2).

□

B.3 Unique real money balance

Lemma B.1. *Fix long-run inflation as $\gamma = 1 + \tau > \beta$. Assume $\alpha_0, \alpha_1 \in (0,1)$. In any SME, there is a unique real money demand, $z^\star \equiv z^\star(\boldsymbol{\tau})$.*

Proof. Consider the case where the long-run inflation target is set away from the Friedman rule, i.e., $\gamma > \beta$. From Lemma 3.2, the money demand Euler equation is characterized by

$$i_d = \frac{\gamma - \beta}{\beta} = \underbrace{\alpha_0 \left(u'[q_b^0(z^\star, \mathbf{z})] - 1\right)}_{=:A} + \underbrace{\int_{\underline{i}(z^\star,\mathbf{z})}^{\bar{i}(z^\star,\mathbf{z})} i \mathrm{d}J(i, z^\star, \mathbf{z})}_{=:B}, \tag{B.2}$$

where

$$\begin{aligned} \mathrm{d}J(i, z^\star, \mathbf{z}) &= \underbrace{\{\alpha_1 + 2\alpha_2(1 - F(i; z^\star))\} f(i, z^\star, \mathbf{z})}_{=:j(i,z^\star,\mathbf{z})} \mathrm{d}i \\ &\equiv \alpha_1 + 2\alpha_2(1 - F(i, z^\star, \mathbf{z}))\mathrm{d}F(i, z^\star, \mathbf{z}). \end{aligned}$$

Recall that $1 \equiv \rho < \tilde{\rho}_i(z^\star, \mathbf{z})$ from Lemma 3.2, the ex-post DM goods demand function for the event where the active DM buyer failed to meet with a lending bank is given by $q_b^0 = \frac{z}{\rho}$, i.e., she is liquidity constrained with own money balance. Thus, $\partial q_b^0 / \partial z > 0$. Since $u'' < 0$, then $u' \circ q_b^0(z, \mathbf{z})$ is continuous and decreasing in z. Thus, term A is continuous and decreasing in z.

Next, let $H(z,\mathbf{z}) := \int_{\underline{i}(z,\mathbf{z})}^{\bar{i}(z,\mathbf{z})} i\mathrm{d}J(i,z,\mathbf{z})$. Applying integration by parts, we obtain $H(z,\mathbf{z}) = \bar{i}(z,\mathbf{z}) - \tilde{H}(z)$, where $\int_{\underline{i}(z,\mathbf{z})}^{\bar{i}(z,\mathbf{z})} J(i,z,\mathbf{z})\mathrm{d}i$. Applying Leibniz' Rule to $\tilde{H}(z)$, we have $\tilde{H}^{'}(z,\mathbf{z}) = \bar{i}^{'}(z,\mathbf{z}) + \int_{\underline{i}(z,\mathbf{z})}^{\bar{i}(z,\mathbf{z})} \frac{\partial J(i,z,\mathbf{z})}{\partial z}\mathrm{d}i$. Overall, we have $H^{'}(z,\mathbf{z}) = \bar{i}^{'}(z,\mathbf{z}) - \tilde{H}^{'}(z,\mathbf{z}) = -\int_{\underline{i}(z,\mathbf{z})}^{\bar{i}(z,\mathbf{z})} \frac{\partial J(i,z,\mathbf{z})}{\partial z}\mathrm{d}i$. From Lemma 3.1, we know that $J(\cdot,z,\mathbf{z})$ first-order stochastically dominates $J(\cdot,z^{'},\mathbf{z})$ for all $z < z^{'}$. Thus, $\partial J(i,z,\mathbf{z})/\partial z > 0$, which implies $H^{'}(z,\mathbf{z}) < 0$. Thus, both terms A and B on the RHS of (B.2) are continuous and monotone decreasing in z. Moreover, the LHS of (B.2) is constant with respect to z. Therefore, there exists a unique real money demand $z^{\star}(\boldsymbol{\tau})$ that solves the money-demand Euler (B.2). Moreover, $z^{\star}(\boldsymbol{\tau})$ is bounded, by Lemma 3.2. □

B.4 SME with money and credit: Proof of Proposition 3.1

Proof. From Lemmata 3.1, 3.2, and B.1, we have established existence of solution to both money and credit. In particular, we have shown that there exists a unique money demand $z^{\star} \equiv z^{\star}(\boldsymbol{\tau})$ such that $z^{\star} \in \left(0,\ [1+\bar{i}(z^{\star})]^{\frac{-1}{\sigma}}\right)$, for a given $\gamma > \beta$. This condition ensures that the optimal real money balance $z^{\star}$ is bounded and that the maximal loan interest of the posted loan-price distribution is not too high. Moreover, this guarantees positive loan demand.

To establish a unique SME with both money and credit, what remains is to show that the following equilibrium requirements also hold, when evaluated at $z = z^{\star}$. That is,

1. Deposit interest is feasible (i.e., interest on loans weakly exceeds that on deposits):

$$\underbrace{(1-n)\delta^{\star}(z,\mathbf{z})\, i_d}_{:=A} \equiv (1-n)\,(z+\tau_b Z)\, i_d$$
$$\leq \underbrace{n\int_{\underline{i}(z,\mathbf{z})}^{\bar{i}(z,\mathbf{z})} \left[\alpha_1 + 2\alpha_2 - 2\alpha_2 F(i,z,\mathbf{z})\right] i\xi^{\star}(z,\mathbf{z})\,\mathrm{d}F(i,z,\mathbf{z})}_{:=B}, \tag{B.3}$$

   where $z = Z$ at equilibrium, and term A and term B are respectively the interest on deposits and loans.

2. DM (competitive price-taking) goods market clears:

$$q_s(z,\mathbf{z}) = n\alpha_0 q_b^{0,\star}(z,\mathbf{z})$$
$$+\, n\left[\int_{\underline{i}(z,\mathbf{z})}^{\bar{i}(z,\mathbf{z})} \left[\alpha_1 + 2\alpha_2 - 2\alpha_2 F(i,z,\mathbf{z})\right] q_b^{\star}(z;\rho,Z,\gamma)\,\mathrm{d}F(i,z,\mathbf{z})\right]. \tag{B.4}$$

3. Both CM goods and labor market clear.

Next, we focus on Condition 1. From the households' problem, we can show that depositors (i.e., *inactive DM buyers*) deposit all of their idle money balance at the perfectly competitive depository institution as long as deposit interest is positive. That is, $\delta(z,\mathbf{z}) > 0$ if and only if $i_d > 0$. The deposit interest rate is pinned down by $i_d = r = \frac{\gamma-\beta}{\beta}$ (where r is given by the Fisher equation under our underlying "small open economy" assumption on depository agents). Since we focus on monetary policy $\gamma > \beta$, and it follows that the deposit interest is positive. Moreover, n is the probability of agent wanting to consume in the DM (i.e., the measure of *active DM buyers*). Thus, for $n \in [0,1]$, we have $0 \leq (1-n)\delta^{\star}(z,\mathbf{z})\, i_d$.

Now, we have to consider the remaining term on the RHS of the Loans-feasibility Constraint (B.3). We have shown there is a positive loan demand evaluated at z, i.e., $\xi^{\star}(z,\mathbf{z}) > 0$, and the loan-price distribution $F(\cdot, z, \mathbf{z})$ is continuous on a connected support, with the lower support being $\underline{i}(z,\mathbf{z}) > i_d$. Thus, for $n \in [0,1]$, we have

$$0 \leq n \int_{\underline{i}(z,\mathbf{z})}^{\bar{i}(z,\mathbf{z})} \left[\alpha_1 + 2\alpha_2 - 2\alpha_2 F\left(i, z, \mathbf{z}\right)\right] i \xi^{\star}\left(z, \mathbf{z}\right) \mathrm{d}F\left(i, z, \mathbf{z}\right).$$

Since both sides of the loans feasibility constraint are non-negative, then rearranging, we have

$$N(z,\mathbf{z}) := \frac{\delta^{\star}\left(z,\mathbf{z}\right) i_d}{\delta^{\star}\left(z,\mathbf{z}\right) i_d + \int_{\underline{i}(z,\mathbf{z})}^{\bar{i}(z,\mathbf{z})} \left[\alpha_1 + 2\alpha_2 - 2\alpha_2 F\left(i, z, \mathbf{z}\right)\right] i \xi^{\star}\left(z, \mathbf{z}\right) \mathrm{d}F\left(i, z, \mathbf{z}\right)} \leq n.$$

From this, we can deduce the bound on n as follows.

$$0 \leq \lim_{[\underline{i}(z,\mathbf{z}), \bar{i}(z,\mathbf{z})] \nearrow \infty} N(z,\mathbf{z}), \tag{B.5}$$

and,

$$\lim_{[\underline{i}(z,\mathbf{z}), \bar{i}(z,\mathbf{z})] \searrow i_d} N(z,\mathbf{z}) \leq 1. \tag{B.6}$$

Condition (B.5) says if the support of the distribution of posted loan interest rates is sufficiently wide in the sense that $\frac{\bar{i}(z,\mathbf{z})}{\underline{i}(z,\mathbf{z})} \to \infty$, then the equilibrium restriction on n is *relaxed* as long as it exceeds a small number (closed to zero). Intuitively, if the average (total) revenue from loans is very high, banks do not need to rely on a large share of active DM buyers (i.e., high n) to be able to (weakly) cover the interest paid on deposits.

Likewise, Condition (B.6) says if the support of the posted loan interest rate distribution is sufficiently narrow in the sense that $\frac{\bar{i}(z,\mathbf{z})}{\underline{i}(z,\mathbf{z})} \to i_d$, then the restriction on n is *tight*. That is, the interest from loans is low and n needs to be at least as great as a large number (closed to one) for banks to be able to cover the interest paid on deposits.

In summary, we have derived a sufficient condition that there exists an endogenous lower bound $N(z,\mathbf{z}) \in [0,1]$ such that $n \geq N(z,\mathbf{z})$. This condition requires that the measure of *active DM buyers* not be too small for feasible deposit interest at equilibrium. While these sufficient conditions to ensure positive loan demand and feasible deposit interest above depend on equilibrium objects, they both can be easily verified by numerical calculations.

Now, we turn to the DM goods market clearing requirement in Condition 2. Since the DM firms' optimal production rule is pinned down by a constant marginal cost (due to linear production technology), then the aggregate supply equals to the aggregate demand in the DM goods market.

Finally, we consider Condition 3. In any equilibrium we have constant optimal CM consumption $x^{\star}$ (due to quasi-linear preference). Given real money balance $z^{\star} \equiv z^{\star}(\boldsymbol{\tau})$ and DM allocations $(q_b^{0,\star}(z^{\star},\mathbf{z}), q_b^{\star}(\cdot, z^{\star}, \mathbf{z}))$, we can verify that the CM goods and labor market also clear. Hence, the details are omitted here. In equilibrium $z = z^{\star}(\boldsymbol{\tau}) = Z$, so we could further reduce the characterizations above by rewriting $(z, \mathbf{z})$ as just $\mathbf{z}$ in a SME. $\square$

C Friedman Rule and the first-best: Proof of Proposition 3.2

Proof. Suppose that $\gamma = \beta$ but that there is an SME with a non-degenerate distribution of loan interest rates, $F(\cdot, z, \mathbf{z})$.

Since we focus on $\alpha_1 \in (0, 1)$, from Lemma 2.1 (part 1), we know that if there is an SME, then the posted loan-rate distribution $F(\cdot, z, \mathbf{z})$ is non-degenerate and continuous with connected support, $\text{supp}(F(\cdot, z, \mathbf{z})) = [\underline{i}(\cdot, z, \mathbf{z}), \bar{i}(\cdot, z, \mathbf{z})]$.

If there is an SME, then the Euler condition for money demand holds. However the marginal cost of holding money—i.e., LHS of the Euler condition—is zero at the Friedman rule ($\gamma = \beta$). Also, the liquidity premium of carrying more real money balance at the margin into next period is always non-negative—*i.e.*, for any $q > 0$, $u'(q)/c'(q) - 1 \geq 0$. What remains on the RHS of the Euler condition are all the (net) marginal benefit of borrowing less at the margin when one has additional real balance, *i.e.*, the integral terms. These terms are also non-negative measures. Thus, for an SME to hold, it must be that $F(\cdot, z, \mathbf{z})$ is degenerate on a singleton set.

Since the Euler condition must hold in a SME, then our previous reasoning must further imply that the integral terms reduce to the condition $u'(q^f) = c'(q^f)$. We can compare this with the first best allocation. Given our CRRA preference representation assumption, the first-best allocation solving $u'(q^*) = c'(q^*)$ will yield $q^* = 1$.

Thus if there is an SME at the Friedman rule, then $F(\cdot, z, \mathbf{z})$ must be degenerate. Moreover, at the Friedman rule, the allocation is Pareto efficient: $q^f = q^* = 1$.

$\square$

D Omitted proofs - Loan spreads and inflation

Recall that gross inflation is $\gamma = 1 + \tau$. How does the average, posted loan spread ($\mu(\gamma)$) change with respect to inflation γ? Also, from a household's perspective, how does the *ex-ante* loan spread ($\hat{\mu}(\gamma)$) change with respect to inflation γ? We will show below that successively higher-inflation SME economies having higher average loan rates and higher deposit rates (*i.e.*, banks' common marginal cost of funds). However, in our comparative stationary monetary equilibrium (SME) experiments, higher inflation is associated with successively lower average spreads in the banking (loans) sector.

For the result that average loan-rate spread falls with inflation, it must be that average loan rate itself is rising slower than the deposit rate. In this part, we provide a theoretical proof of this result under quite mild regularity conditions. The following proposition says that if the support of an SME loan-rate distribution is not too wide, and, the gap between the lowest posted loan rate and banks' (common) marginal cost of fund is not too large, then one can show that the average loan spread measure is a decreasing function of long-run inflation.

We will use the notation $f_x(x; y) := \frac{\partial f(x,y)}{\partial x}$ to denote the partial derivative of function $f(x, y)$ with respect to argument x. The results below are with regard to an equilibrium, so we have $z = Z = z^{\star}(\boldsymbol{\tau})$ and we can also write $\mathbf{z} = (z^{\star}, \mathbf{z})$.

Proposition D.1. *Assume $\gamma = 1 + \tau > \beta$, and $\alpha_1 \in (0, 1)$. Let the average loan-rate spread be*

$$\mu(\gamma) := \frac{\int_{\underline{i}(\mathbf{z})}^{\bar{i}(\mathbf{z})} i dF(i, \mathbf{z})}{i_d(\gamma)} =: \frac{g(\gamma)}{h(\gamma)},$$

and let the ex-ante *loan rate spread be*

$$\hat{\mu}(\gamma) := \frac{\int_{\underline{i}(\mathbf{z})}^{\bar{i}(\mathbf{z})} i \cdot [\alpha_1 + 2\alpha_2(1 - F(i, \mathbf{z}))] \, dF(i, \mathbf{z})}{i_d(\gamma)} =: \frac{g(\gamma)}{h(\gamma)},$$

where $i_d(\gamma) = \frac{\gamma - \beta}{\beta}$. If (1): $\bar{i}(\mathbf{z}) - \underline{i}(\mathbf{z}) < \frac{1}{\beta}$, and, (2): $\underline{i}(\mathbf{z}) - i_d(\gamma) < \epsilon(\gamma)$, where $\epsilon(\gamma) := \sqrt{\frac{1}{\beta} \frac{\alpha_1}{2\alpha_2} \frac{\xi(\bar{i}(\mathbf{z}), \mathbf{z})}{\xi(\underline{i}(\mathbf{z}), \mathbf{z})} \frac{1}{\mu(\gamma)}} > 0$, then both the average loan rate spread and the ex-ante *loan rate spread are monotone decreasing in inflation γ. Respectively, $\mu_\gamma(\gamma) < 0$ and $\hat{\mu}_\gamma(\gamma) < 0$.*

We should point out that the sufficient conditions behind Proposition D.1 are perhaps not the most general ones, but they suffice practically: For plausible experiments around the empirically calibrated model, the sufficient conditions always hold. For extremely high, hyperinflationary scenarios, these specific sufficient conditions may not hold. Nevertheless, we will see that average loan spread is still decreasing with inflation in our numerical experiments.

Proof. Fix $\gamma > \beta$ (i.e., inflation target away from the Friedman rule) and $\alpha_1 \in (0, 1)$ (i.e., agents can meet more than one lending agent). Consider an SME with co-existence of money and bank loans at the given γ. In such an equilibrium, the distribution of loan rates is non-degenerate.

The average posted loan rate spread. First, we prove this for $\mu(\gamma)$. At each γ, $g(\gamma) > h(\gamma)$, since average spread is strictly greater than unity $\mu(\gamma) > 1$.

Since the average loan spread function μ is differentiable with respect to γ, then we have

$$\mu_\gamma(\gamma) = \frac{g_\gamma(\gamma) h(\gamma) - g(\gamma) h_\gamma(\gamma)}{[h(\gamma)]^2}. \tag{D.1}$$

To show that average loan spread is decreasing in inflation, $S_\gamma(\gamma) < 0$, it suffices to verify that $\frac{g_\gamma(\gamma)}{g(\gamma)} < \frac{h_\gamma(\gamma)}{h(\gamma)}$. This requires that the percentage change in average loan rate with respect to inflation is strictly smaller than that of banks' marginal cost of funds.

Using the definition of g and h, we can also rewrite the last inequality as $g_\gamma(\gamma) < \frac{1}{\beta}\mu(\gamma)$. Applying integration by parts, we can rewrite average loan-rate $g(\gamma)$ as

$$g(\gamma) = [iF(i, \mathbf{z})]_{\underline{i}(\mathbf{z})}^{\bar{i}(\mathbf{z})} - \int_{\underline{i}(\mathbf{z})}^{\bar{i}(\mathbf{z})} \frac{\partial i}{\partial i} F(i, \mathbf{z}) \, \mathrm{d}i = \bar{i}(\mathbf{z}) - \underbrace{\int_{\underline{i}(\mathbf{z})}^{\bar{i}(\mathbf{z})} F(i, \mathbf{z}) \, \mathrm{d}i}_{=:\tilde{g}(\gamma)}. \tag{D.2}$$

Differentiating Expression (D.2) with respect to γ yields

$$g_\gamma(\gamma) = \bar{i}_\gamma(\gamma) - \tilde{g}_\gamma(\gamma) = \bar{i}_\gamma(\gamma) - \left[\bar{i}_\gamma(\gamma) + \int_{\underline{i}(\mathbf{z})}^{\bar{i}(\mathbf{z})} F_\gamma(i, \mathbf{z}) \, \mathrm{d}i\right] = -\int_{\underline{i}(\mathbf{z})}^{\bar{i}(\mathbf{z})} F_\gamma(i, \mathbf{z}) \, \mathrm{d}i, \tag{D.3}$$

where

$$F_\gamma(i,\mathbf{z}) = \frac{\alpha_1}{2\alpha_2}\frac{1}{\beta}\left\{\frac{\xi(\bar{i},\mathbf{z})R(i,\mathbf{z}) - \xi(i;\gamma)R(\bar{i},\mathbf{z})}{[R(i,\mathbf{z})]^2}\right\} = \frac{1}{\beta}\frac{\alpha_1}{2\alpha_2}\frac{\xi(\bar{i},\mathbf{z})}{\xi(i,\mathbf{z})}\frac{i-\bar{i}(\mathbf{z})}{[i-i_d(\gamma)]^2} < 0. \tag{D.4}$$

The last term $\tilde{g}_\gamma(\gamma)$ in (D.3) is obtained by Leibniz' rule: $\tilde{g}_\gamma(\gamma) = \bar{i}_\gamma(\gamma) + \int_{\underline{i}(\mathbf{z})}^{\bar{i}(\mathbf{z})} F_\gamma(i,\mathbf{z})\,\mathrm{d}i$.

Observe that $F_\gamma(\cdot,\mathbf{z})$ is negative valued for all i in the equilibrium support of $F(\cdot,\mathbf{z})$, since $i < \bar{i}$ and since the event that two banks post the same interest rate, $\{i\}$, has zero probability measure in any SME. Thus, from Equations (D.3) and (D.4), we have that average loan rate is increasing with inflation, or, $g_\gamma(\gamma) > 0$.

Consider Expression (D.4). Since loan demand ξ is decreasing in i, $\bar{i}(\mathbf{z}) > \underline{i}(\mathbf{z})$, and, $\underline{i}(z,\mathbf{z}) - i_d(\gamma) < i - i_d(\gamma)$, then the relative demand terms are always bounded in $(0,1)$:

$$0 < \frac{\xi(\bar{i}(\mathbf{z}),\mathbf{z})}{\xi(\underline{i}(\mathbf{z}),\mathbf{z})} < \frac{\xi(\bar{i}(\mathbf{z}),\mathbf{z})}{\xi(i,\mathbf{z})} < 1, \tag{D.5}$$

and,

$$0 < \frac{1}{[i-i_d(\gamma)]^2} < \frac{1}{[\underline{i}(z,\mathbf{z}) - i_d(\gamma)]^2} < 1, \tag{D.6}$$

for all $i \in \left(\underline{i}(\mathbf{z}), \bar{i}(\mathbf{z})\right)$.

The bounds in Inequalities (D.5) and (D.6) allow us to look at the extreme case by setting $i = \bar{i}(z,\mathbf{z})$ so that the sufficient bound is independent of the endogenous i. From sufficient condition (1), we can deduce

$$0 < \frac{\bar{i}(z,\mathbf{z}) - i}{\beta} < \frac{\bar{i}(z,\mathbf{z}) - \underline{i}(z,\mathbf{z})}{\beta} < 1. \tag{D.7}$$

Using Inequalities (D.5), (D.6) and (D.7), $0 < \alpha_1/2\alpha_2 < 1$, Sufficient Conditions (1) and (2) and (D.4), we have an upper bound on how fast the average loan rate varies with inflation:

$$0 < g_\gamma(\gamma) := -\int_{\underline{i}(\mathbf{z})}^{\bar{i}(\mathbf{z})} F_\gamma(i,\mathbf{z})\,\mathrm{d}i < \left[\bar{i}(z,\mathbf{z}) - \underline{i}(z,\mathbf{z})\right]\mu(\gamma) < \frac{1}{\beta}\mu(\gamma). \tag{D.8}$$

The result above says that the upper bound on $g_\gamma(\gamma)$ is given by the rate of change in the deposit rate with respect to inflation, $1/\beta$, times the average loan spread, $\mu(\gamma)$. Therefore, we have that average loan spread decreases with inflation, $\mu_\gamma(\gamma) < 0$.

Note that at any $\gamma > \beta$, the second last term in Condition (D.8) gives the area of a rectangle whose height is $\mu(\gamma)$, and width is $\left[\bar{i}(\mathbf{z}) - \underline{i}(\mathbf{z})\right]$. Under sufficient condition (1) and (2), and the fact that $F_\gamma(i,\mathbf{z})$ is monotone decreasing in i, we have that the maximal value of $F_\gamma(i,\mathbf{z})$ is bounded above by $\mu(\gamma)$. Sufficient condition (1) bounds the limits of the integral above by $1/\beta$. Hence the definite integral $g_\gamma(\gamma)$ is bounded: $0 < g_\gamma(\gamma) < \frac{1}{\beta}\mu(\gamma)$. This suffices for the conclusion that average spread is decreasing with inflation, i.e., $\mu_\gamma(\gamma) < 0$ as desired.

The *Ex-ante* loan spread. We now prove the second part. Observe that the only difference between $\mu(\gamma)$ and $\hat{\mu}(\gamma)$ is that in the latter, an additional probability weighting function, $\alpha_1 + 2\alpha_2(1 - F(i, \mathbf{z}))$ appears in the definition of the *ex-ante* or mean transaction rate buyers face. Let this be $\hat{g}(\gamma) := \int_{\underline{i}(\mathbf{z})}^{\bar{i}(\mathbf{z})} i \cdot [\alpha_1 + 2\alpha_2(1 - F(i, \mathbf{z}))] \, \mathrm{d}F(i, \mathbf{z})$. It is immediate that $0 < \hat{g} \leq g$. Under the same sufficient conditions above, we also have $\frac{\hat{g}_\gamma(\gamma)}{\hat{g}(\gamma)} \leq \frac{g_\gamma(\gamma)}{g(\gamma)} < \frac{h_\gamma(\gamma)}{h(\gamma)}$. That is, since the integrand in the integral function $\hat{g}$ is dominated by the integrand in g, then $\hat{g}(\gamma)$ can grow no faster than $g(\gamma)$ with respect to inflation γ. Finally, since we concluded that $g(\gamma)$ grows slower than the deposit rate $h(\gamma)$ as γ increases, then so must $\hat{g}(\gamma)$. Thus, $\hat{g}(\gamma)$ is also decreasing with γ under the same sufficient condition. □

E Exogenous default on loans

In this section, we consider an economy having exogenous random default on repaying consumer loans. Moreover, we allow the punishment of defaulters by excluding them from the banking system. Since we consider a single asset economy, we argue that financial exclusion is the hardest punishment that the bank can implement.

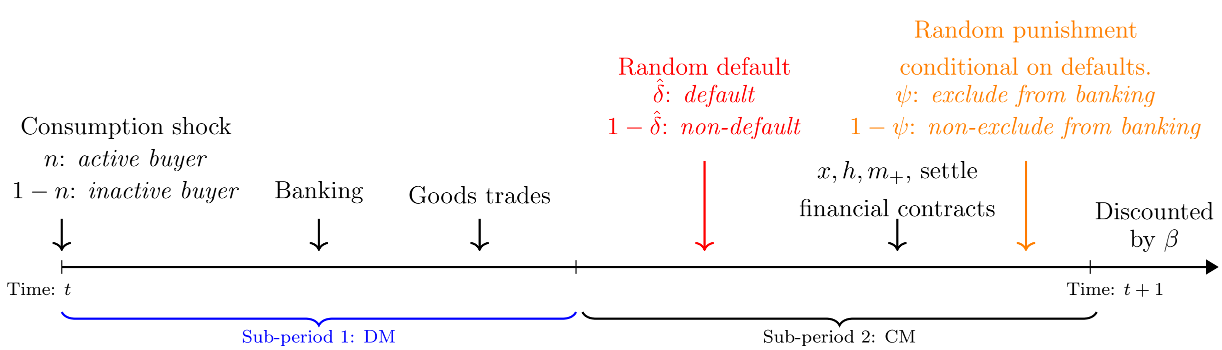
The basic model structure is similar to that presented in Section 3. The difference is that banks lose the whole amount of loans if borrowers default on repayment in the centralized market with a probability of $0 < \hat{\delta} < 1$. This feature generates a default risk premium on loans in equilibrium. Moreover, banks can punish defaulters by excluding them from the banking system with a probability of $0 < \psi < 1$. As such, agents may or may not have access to banking in the next period, depending on their default status in the current period. Hence, agents exiting the centralized market could have different continuation valuations entering the next decentralized market. The parametric limits of this extended setup have three special cases: (1) the model presented in the Section 3; (2) a model with deterministic punishment ($\psi = 1$), and (3) a model without punishment ($\psi = 0$).

We first outline the model timeline in Section E.1. We then present agents' problem and characterize an equilibrium (co-existing with money and credit) respectively in Section E.2 and Section E.3.

E.1 Overview

Figure 6 displays the model timeline. Text highlighted in red color and orange color are both the new features relative to the baseline model presented in the main text.

Figure 6: Timing



Preview of new features. From the banks' perspective, loans are risky assets: The gross return on loans is given by $R^L = 0$ with probability $\hat{\delta}$ and $R^L = 1 + i$ with probability $1 - \hat{\delta}$. We use $\mathbb{I}_{\{1-\hat{\delta}\}} = 1$ to denote non-default status of agents in the centralized market. The default status is given by $\mathbb{I}_{\{\hat{\delta}\}} := 1 - \mathbb{I}_{\{1-\hat{\delta}\}} = 0$. Given non-default status $\mathbb{I}_{\{1-\hat{\delta}\}} = 1$, non-defaulters have continuation valuation $V(m_+)$ entering the next decentralized market with banking opportunity, where m_+ is the money balance chosen by non-default agents.

Given default status $\mathbb{I}_{\{\hat{\delta}\}} = 0$, banks could punish defaulters (with probability ψ) by excluding them from the banking system in the next decentralized market. As such, defaulters exiting the centralized market have continuation valuation, $\psi \hat{V}(\hat{m}_+) + (1-\psi)V(\hat{m}_+)$, where $\hat{V}(\hat{m}_+)$ is the continuation valuation entering the next DM without banking opportunity, and $\hat{m}_+$ is the money balance chosen by default agents. We will be more explicit about the continuation value functions, $\hat{V}$ and V, when we present the model setup.

In what follows, we will present the setup with both random defaults and punishment, i.e., $0 < \delta < 1$ and $0 < \psi < 1$. We can always set $\psi = 1$ to allow for a deterministic punishment on defaulters by excluding them from the banking system. This restriction on $\psi = 1$ would not alter the insights on default risk premium. The basic model setup is similar to that discussed in the main text. In what follows, we only highlight the new features to avoid repetition.

E.2 Model setup

Let $\tau = (\tau_b, \tau_s, \tau_2)$ denote a list of constant policy (tax or transfer) outcomes, where τ_b and τ_s are respectively levied on active buyers and inactive buyers in the DM, and τ_2 is applied in the CM. We will denote the list of initial aggregate stock of money and policy outcomes by the vector notation $\mathbf{s} := (M, \tau)$. We use m to denote the individual money balance for non-defaulters and $\hat{m}$ to denote the individual money balance for defaulters. Next, we describe the DM problem and then the CM problem.

E.2.1 First sub-period: DM

Defaulters DM valuation.

$$
\begin{aligned}
\hat{V}(\hat{m}) = & \underbrace{\psi}_{\text{punish by banking excluded}} \Bigg[\underbrace{nB^0(\hat{m}, \mathbf{s})}_{\text{without bank credit}} + \underbrace{(1-n)W(\hat{m} + \tau_s M, 0, 0, 0)}_{\text{can't deposit idle money}} \Bigg] \\
& + \underbrace{(1-\psi)}_{\text{not punish}} \Bigg\{ \underbrace{(1-n)W(\hat{m} + \tau_s M - \hat{d}, 0, \hat{d})}_{\text{deposit idle money}} \\
& + n \Bigg[\underbrace{\alpha_0 B^0(\hat{m}, \mathbf{s})}_{\text{without bank credit}} + \underbrace{\int_{\underline{i}(m;\tau)}^{\bar{i}(\hat{m};\mathbf{s})} [\alpha_1 + 2\alpha_2(1 - F(i;\hat{m},\mathbf{s}))] B^1(i,\hat{m},\mathbf{s}) \mathrm{d}F(i;\hat{m},\mathbf{s}) \Bigg]}_{\text{with bank credit}} \Bigg\}.
\end{aligned}
\tag{E.1}
$$

Non-defaulters DM valuation.

$$V(m) = n\Bigg[\underbrace{\alpha_0 B^0(m,\mathbf{s})}_{\text{without bank credit}} + \underbrace{\int_{\underline{i}(m;\mathbf{s})}^{\bar{i}(m;\mathbf{s})} [\alpha_1 + 2\alpha_2(1 - F(i;m,\mathbf{s}))] B^1(i,m,\mathbf{s}) \mathrm{d}F(i;m,\mathbf{s})}_{\text{with bank credit}} \Bigg] + \underbrace{(1-n)W(m + \tau_s M - d, 0, d)}_{\text{deposit idle money}}. \tag{E.2}$$

In general, money balances ($\hat{m}$) held by defaulters could be different to the amount held by non-defaulters (m). We also maintain the assumption of quasi-linear preference in the CM. As such, all defaulters are identical at the beginning of the DM. Likewise for the non-defaulters. The post-match problem for each type of agent, $B^0(\cdot,\mathbf{s})$, and $B^1(\cdot,\mathbf{s})$ is similar to that described in the main text. Hence, we omit the details here.

E.2.2 Second sub-period: CM

Agents entering the CM are indexed by $(m, l, d, R^L \cdot \mathbb{I}_{\{1-\hat{\delta}\}}; \mathbf{s})$. As described in the main text, we can verify that inactive buyers have no incentive to borrow additional funds from banks, and active buyers have no incentive to deposit funds in the banking system. As such, the measure of $1-n$ agents are the natural depositors, and the measure of n agents are the potential borrowers in the economy. We now break down the CM problem by depositors, non-defaulted borrowers and defaulted borrowers as follows.

Depositors CM valuation.

$$W(m, 0, d, 0) = \max_{x, m_+} \Big\{ U(x) - h + \beta V(m_+) \Big\}, \tag{E.3}$$

subject to

$$\phi m_+ + x = h + \phi m + \Pi + T + \phi(1 + i_d),$$

where $V(m_+)$ is the continuation value entering the next DM with banking opportunity.

Non-defaulters CM valuation.

$$W(m, l, 0, R^L \cdot \mathbb{I}_{\{1-\hat{\delta}\}}) = \max_{x, m_+} \Big\{ U(x) - h + \beta V(m_+) \Big\}, \tag{E.4}$$

subject to

$$\phi m_+ + x = h + \phi m + \Pi + T - \phi \mathbb{I}_{\{1-\hat{\delta}\}}(1 - \hat{\delta})(1 + i)l.$$

Defaulter's CM valuation.

$$W(\hat{m}, l, 0, R^L \cdot \mathbb{I}_{\{\hat{\delta}\}}) = \max_{x, \hat{m}_+} \Big\{ U(x) - h + \beta \Big[\psi \hat{V}(\hat{m}_+) + (1 - \psi) V(\hat{m}_+) \Big] \Big\}, \tag{E.5}$$

subject to

$$\phi \hat{m}_+ + x = h + \phi \hat{m} + \Pi - \underbrace{\phi \mathbb{I}_{\{\hat{\delta}\}}(1-\hat{\delta})(1+i)l}_{=0} + T,$$

where $\hat{V}(\hat{m}_+)$ is the continuation value entering the next DM without banking opportunity, and $V(\hat{m}_+)$ is the continuation value entering the next DM with banking opportunity.

Note: If $\psi = 1$, banks could punish defaulters by excluding them from the financial system in all future periods. The implication of punishment is as follows. Defaulters (with probability n wanting to consume) can only purchase the DM goods with their own money balances $\hat{m}_+$ in the next DM. If they end up having idle money balances (i.e., with probability $1-n$ that they are inactive for DM consumption), they can't deposit money balances in the bank.

The first order condition with respect to x is identical to depositors, defaulters and non-defaulters as

$$U_x(x) = 1. \tag{E.6}$$

For depositors and non-defaulters, their first order conditions with respect to m_+ are the same:

$$\phi = \beta V_m(m_+). \tag{E.7}$$

The first order conditions (for defaulters) with respect to $\hat{m}_+$ is given by:

$$\phi = \beta \left[\psi \hat{V}_{\hat{m}}(\hat{m}_+) + (1-\psi) V_{\hat{m}}(\hat{m}_+) \right]. \tag{E.8}$$

The envelop conditions are:

$$\begin{aligned}
&W_m(m, l, d, \mathbb{I}_{\{1-\hat{\delta}\}}, \mathbf{s}) = W_{\hat{m}}(\hat{m}, l, d, \mathbb{I}_{\{\hat{\delta}\}}, \mathbf{s}) = \phi, \\
&W_l(m, l, d, \mathbb{I}_{\{1-\hat{\delta}\}}, \mathbf{s}) = -\phi(1-\hat{\delta})(1+i) \\
&W_l(\hat{m}, l, d, \mathbb{I}_{\{\hat{\delta}\}}, \mathbf{s}) = \psi \cdot 0 - (1-\psi)\phi(1-\hat{\delta})(1+i), \\
&W_d(m, l, d, \mathbb{I}_{\{1-\hat{\delta}\}}, \mathbf{s}) = \phi(1+i_d), \\
&W_d(\hat{m}, l, d, \mathbb{I}_{\{\hat{\delta}\}}, \mathbf{s}) = \psi \cdot 0 + (1-\psi)\phi(1+i_d).
\end{aligned} \tag{E.9}$$

For ease of illustrating additional default risks in lending, we consider the economy with random default $0 < \delta < 1$ and deterministic punishment on defaulters $\psi = 1$ for the remaining of this section.

E.2.3 Lending banks

Extended feature. Banks now need to take into account the risky nature of loans when they choose the loan rate to post to compete with other banks. Also, banks commit to supply loans to customers at their posted rate.

Consider the problem of a lending bank that takes the distribution of posted interest rates F, as given, and has common marginal cost of funds i_d and default risk $\hat{\delta}$. If the bank posts a loan rate i, its' expected profit is

$$\Pi(i,m,\mathbf{s}) = \max_i n\left[\alpha_1 + 2\alpha_2(1-F(i,m,\mathbf{s})) + \alpha_2\eta(i,m,\mathbf{s})\right]R(i,m,\mathbf{s}), \tag{E.10}$$

where

$$\eta(i,m,\mathbf{s}) = \lim_{\epsilon\searrow 0} F(i,m,\mathbf{s}) - F(i-\epsilon,m,\mathbf{s}), \tag{E.11}$$

$$R(i,m,\mathbf{s}) = l^\star(i,m,\mathbf{s})\left[\underbrace{\hat{\delta}\cdot 0 + (1-\hat{\delta})i}_{\text{new feature}} - i_d\right], \tag{E.12}$$

where $R(i,m,\mathbf{s})$ is the profit per customer served, $l^\star(i,m,\mathbf{s})$ is the optimal demand for loans by buyers, and $n\alpha_2\eta(i,m,\mathbf{s})$ is the measure of customers contacting both banks that are posting the same interest rate i.

E.3 Stationary monetary equilibrium

We restrict attention to stationary monetary equilibrium (SME) co-existing with money and credit. The stationary counterpart to the state-policy vector $(m,\mathbf{s})$ now becomes $(z,\mathbf{z})$, where $\mathbf{z}=(Z,\tau)$. Given policy $\gamma = 1+\tau > \beta$, an SME consists of a system of equations:

1. Non-defaulters money demand decision:

$$\begin{aligned}\frac{1+\tau-\beta}{\beta} &= (1-n)i_d + n\alpha_0\left(u'[q_b^0(z^\star,\mathbf{z})] - 1\right) \\ &\quad + n\int_{\underline{i}(z^\star,\mathbf{z})}^{\bar{i}(z^\star,\mathbf{z})}\left[\alpha_1 + 2\alpha_2(1-F(i,z^\star,\mathbf{z})\right]\left([1-\hat{\delta}]i - \hat{\delta}\right)\mathrm{d}F(i,z^\star,\mathbf{z})\end{aligned} \tag{E.13}$$

2. Defaulters money demand decision:

$$\frac{1+\tau-\beta}{\beta} = n\left[u'[q_b^0(\hat{z}^\star,\mathbf{z})] - 1\right] \tag{E.14}$$

   Given policy $\gamma = 1+\tau$, aggregate money is given by

$$Z = (1-\hat{\delta})z^\star + \hat{\delta}\hat{z}^\star, \tag{E.15}$$

   where z and $\hat{z}$ are respectively determined by (E.13) and Equation(E.14).

3. DM goods market clearing:

$$\begin{aligned} q_s &= \hat{\delta}nq_{b,\star}^0(\hat{z},\mathbf{z}) \\ &\quad + (1-\hat{\delta})n\left[\alpha_0 q_{b,\star}^0(z,\mathbf{z}) + \int_{\underline{i}(z,\mathbf{z})}^{\bar{i}(z,\mathbf{z})}[\alpha_1 + 2\alpha_2(1-F(i,z,\mathbf{z})]q_{b,\star}^1(i,z,\mathbf{z})\mathrm{d}F(i;z,\tau)\right].\end{aligned} \tag{E.16}$$

Given $\psi = 1$, there are measure of $\hat{\delta}$ defaulters who are punished by exclusion from the financial system. In this case, the total demand for DM goods is a weighted average of demand by defaulters (with own money balance $\hat{z}$) and demand by non-defaulters (with own money balance and loans).

4. Loans feasibility:

$$(1-n)i_d d^{\star}(z,\mathbf{z}) \leq n \int_{\underline{i}(z,\mathbf{z})}^{\bar{i}(z,\mathbf{z})} [\alpha_1 + 2\alpha_2(1-F(i,z,\mathbf{z})]i\xi(i,z,\mathbf{z})\mathrm{d}F(i,z,\mathbf{z}). \tag{E.17}$$

5. CM goods market clearing:

$$\begin{aligned} x = (1-\hat{\delta})\Bigg\{ n\Bigg[\alpha_0 h^0(z,\mathbf{z}) + \int_{\underline{i}(z,\mathbf{z})}^{\bar{i}(z,\mathbf{z})} [\alpha_1 + 2\alpha_2(1-F(i,z,\mathbf{z})]h^1(i,z,\mathbf{z})\mathrm{d}F(i;z,\tau)\Bigg] \\ + (1-n)h^{\text{inactive}}(z,\mathbf{z})\Bigg\} + \hat{\delta}\Big[nh^0(\hat{z},\mathbf{z}) + (1-n)h^{\text{inactive}}(\hat{z},\mathbf{z})\Big]. \end{aligned} \tag{E.18}$$

E.4 Welfare

Our welfare criterion is measured in terms of the ex-ante lifetime utility of homogeneous households. Given policy τ, exogenous random default on loans, $0 < \hat{\delta} < 1$ and a deterministic punishment on defaulters, $\psi = 1$, the welfare function in the baseline SME with noisy search frictions for loans, is

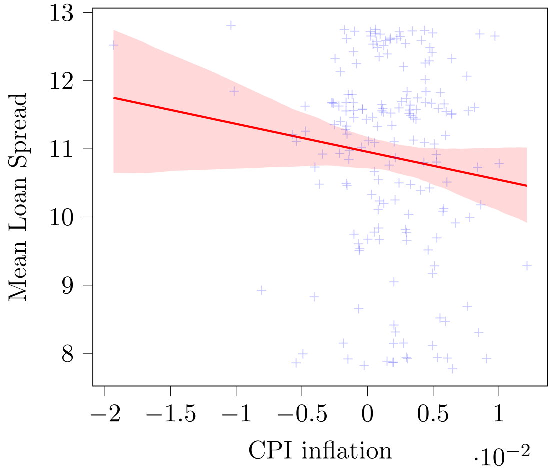
$$\begin{aligned} W^{\text{HKNP}}(\tau) = & \frac{1}{1-\beta}\Bigg[U(x) - x - c[q_s(z,\hat{z},\mathbf{z})]\Bigg] \\ & + \underbrace{\frac{(1-\hat{\delta})}{1-\beta}\Bigg[n\alpha_0 u[q_b^{0,\star}(z,\mathbf{z})] + n\int_{\underline{i}(z,\mathbf{z})}^{\bar{i}(z,\mathbf{z})} [\alpha_1 + 2\alpha_2(1-F(i,z,\mathbf{z}))]u[q_b^{1,\star}(i,z,\mathbf{z})]\mathrm{d}F(i,z,\mathbf{z})\Bigg]}_{\text{non-defaulters}} \\ & + \underbrace{\frac{\hat{\delta}}{1-\beta}\Bigg[nu[q_b^{0,\star}(\hat{z},\mathbf{z})]\Bigg]}_{\text{defaulters: exclude from banking}} \end{aligned} \tag{E.19}$$

If $\hat{\delta} = 0$, then the welfare function is identical to the one presented in the main text.

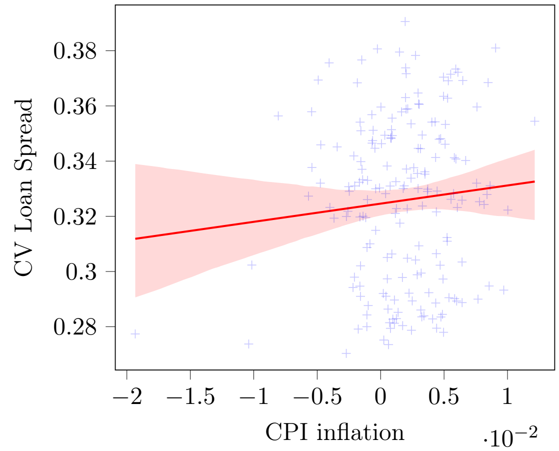
F Inflation, the average and dispersion of the loan spread: Data

Figure 7 depicts the correlations between monthly CPI inflation and average loan-rate spreads, and two dispersion measures in *RateWatch* data—standard deviation (SD) and coefficient of variation (CV)—for January 2003 to December 2017. These three panels provide U.S. data counterparts to those for the model in Figure 3.

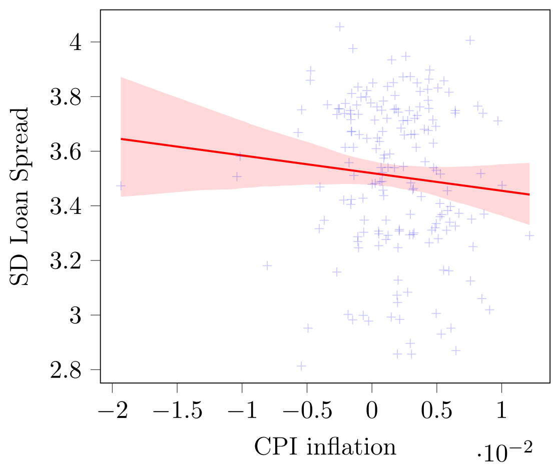
Figure 7: Correlations between inflation, the average loan spread and the dispersion of spreads.



(a) The average loan spread (%)



(b) The dispersion of loan spreads (CV)



(c) The dispersion of loan spreads (SD)

G Empirical analysis of loan spreads at the state level

In this section, we calculate the standard deviations and means of loan spreads. There are 8,464 usable observations of the variables at the state and month level. This allows us to construct a panel dataset. In Figure 8, we can see that the spreads' standard deviation and average are positively correlated at the state-month level.

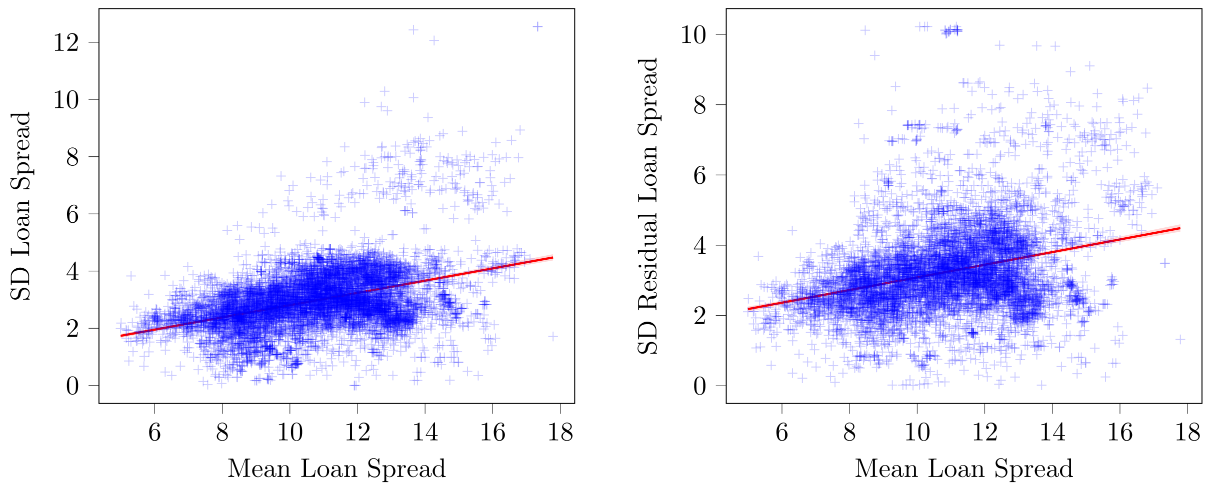
We estimate by OLS the relationship between the standard deviation and average of the loan-spread:

$$Dispersion_{s,t} = b_0 + b_1 \overline{Spread}_{s,t} + b_2 Z_s + b_3 Z_t + \epsilon_{s,t} \tag{G.1}$$

The index s stands for a particular state and t stands for the month of observation. We cluster standard errors by state and month.

Table 4 reports the regression results. Columns (1) to (3) use the raw spreads and Columns (4) to (6) the orthogonalized ones. (Table 5 in Section G.1 for the controls used to define the orthogonalized spreads.)

Figure 8: State-month-level relationship between the dispersion and average of the loan spread. Dispersion measures: SD: *standard deviation*. Data source: *RateWatch*, "Personal Unsecured Loans (Tier 1)."



All columns show positive and statistically significant relationships between the standard deviations and averages. The magnitude of the coefficient is also economically significant. From column (6), the coefficient indicates that a one-percentage-point increase in orthogonalized spread average is associated with a 0.286-percentage-point increase in the standard deviation after controlling for state and time fixed effects.

Table 4: OLS regressions: Averages and Std. Deviations of State Level Loan Spreads, Jan. 2003 to Dec. 2017.

	Spread dispersion: $Dispersion_{s,t}$					
	Raw spread			Orthogonalized spread		
	(1)	(2)	(3)	(4)	(5)	(6)
	State FE	Time FE	Both FE	State FE	Time FE	Both FE
$\overline{Spread}_{s,t}$	0.179***	0.290***	0.353***	0.220***	0.304***	0.286***
	(0.030)	(0.094)	(0.077)	(0.055)	(0.079)	(0.084)
State fixed effects	X		X	X		X
Time fixed effects		X	X		X	X
N	8237	8237	8237	7463	7463	7463
adj. R^2	0.618	0.178	0.646	0.538	0.203	0.577

Note: Standard errors in parentheses. $^{*}p < 0.1$; $^{**}p < 0.05$; $^{***}p < 0.01$

G.1 Control variables list

In Table 5, we describe the controls used in constructing our orthogonalized loan spreads.

Table 5: Control variables to obtain the orthogonalized spread.

(a) Panel A: County variables

Variable	Data source	Frequency	Details
Real GDP	BEA	Annual	Annual county real GDP
GDP growth	BEA	Annual	Real GDP growth
Establishments	BLS	Annual	Number of establishments within county
Unemployment	BLS	Annual	County unemployment rate
House price	U.S. Census	Annual	Average housing pricing in the county
Median income	U.S. Census	Annual	Median Household Income
Population	U.S. Census	Annual	ln(Total population)
Poverty	U.S. Census	Annual	Proportion of county population under poverty line

(b) Panel B: Local competition

Variable	Data source	Frequency	Details
Within county share	SOD	Annual	Total branch deposits / Total county deposits
County deposit HHI	SOD	Annual	HHI of county's deposit holdings
County branch count	SOD	Annual	Number of branch counts in the county

(c) Panel C: Bank branch network

Variable	Data source	Frequency	Details
Within bank share	SOD	Annual	Total branch deposits / Total bank deposits
Bank deposit HHI	SOD	Annual	HHI of bank's deposit holdings across its branches
Bank branch count	SOD	Annual	Number of branch counts in the commercial bank

(d) Panel D: Commercial bank controls

Variable	Data source	Frequency	Details
Deposit reliance	Call reports	Quarter	Total deposits / Total liabilities
Leverage	Call reports	Quarter	Total equity / Total assets
Credit risk	Call reports	Quarter	Allowance for Loan and Lease Losses/Total Loans
Bank size	Call reports	Quarter	ln(Total assets)

G.2 Different household loan products

In this section, we show that the main evidence, based on a particular high-quality-consumer loan product in Section 5, is robust to alternative loan-product definitions. Here, we redo the analysis using other household loan products, namely personal unsecured loan, credit card, fixed rate mortage, variable rate mortgage, new vehicle auto loan and used vehicle auto loan.

Table 6 provides details of each loan product. Figure 9 and 10 corroborate the raw spread results in Figure 5, respectively, for standard deviation and coefficient of variation measures of dispersion.

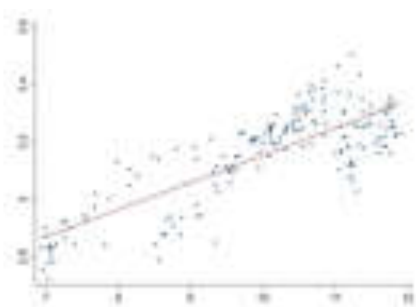
Consistent with our main finding, there is a positive (negative) relationship between the standard deviation (coefficient of variation) and the average in loan spreads for all six different household loan products. While these figures are graphical summaries, more formal regression results confirming the same relationships are also available upon request.

Table 6: Different household loan products information.

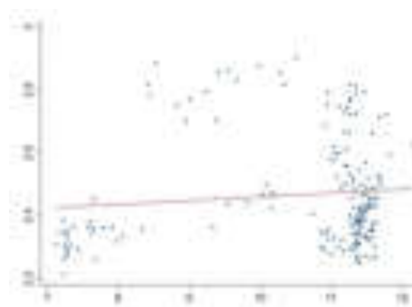
Product type	Observations	Descriptions
Personal Unsecured Loan	718,748	Personal unsecured loan with tier 1 borrowers
Credit Card	182,118	Credit card with Visa
Mortgage (fixed rates)	331,558	30-Year fixed rate mortgage ($175k loan amount)
Mortgage (variable rates)	194,740	5-Year adjustable-rate mortgage ($175k loan amount)
Auto Loan (New)	878,797	Auto loan for new vehicles (60 mths term)
Auto Loan (Used)	804,871	Auto loan for (<24 mths) used vehicles (36 mths term)

Figure 9: Loan spread dispersion measures (Y-axis) and average (X-axis) at the national level (January 2003 to December 2017). Dispersion measures: SD (*standard deviation*). Both variables are shown in percentage points. Data source: *RateWatch*.

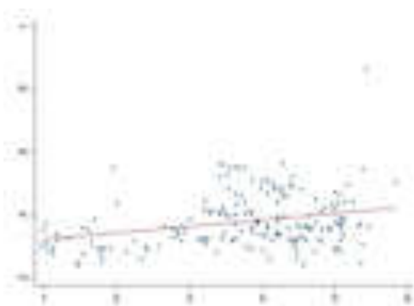
(a) Personal Unsecured Loan



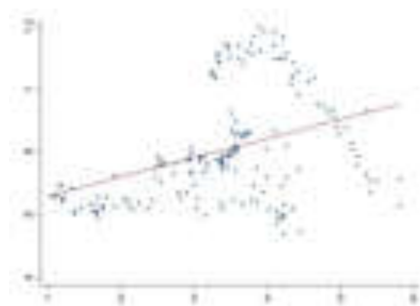
(b) Credit Card



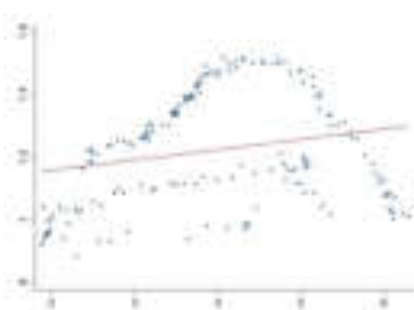
(c) Mortgage (fixed rates)



(d) Mortgage (variable rates)



(e) Auto Loan (New)



(f) Auto Loan (Used)

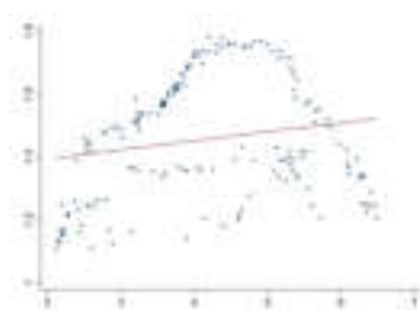
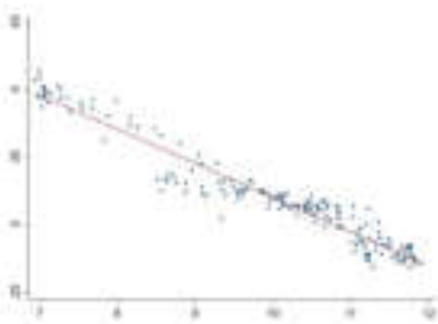
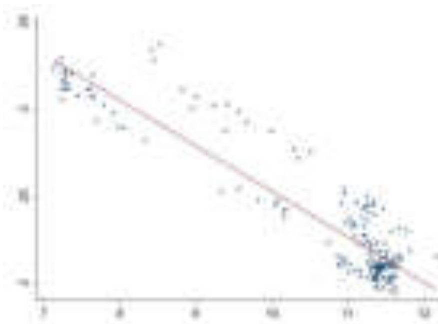


Figure 10: Loan spread dispersion measures (Y-axis) and average (X-axis) at the national level (January 2003 to December 2017). Dispersion measures: CV (*coefficient of variation*). Spread means are shown in percentage points. Data source: *RateWatch*.

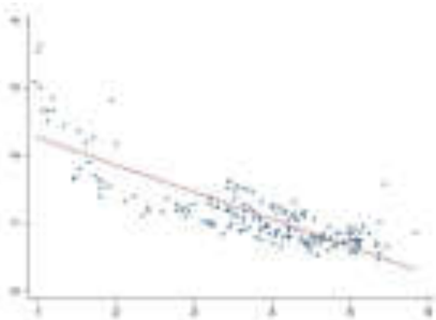
(a) Personal Unsecured Loan



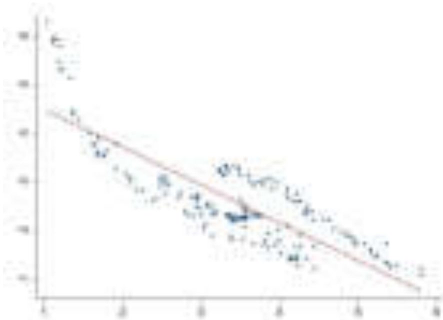
(b) Credit Card



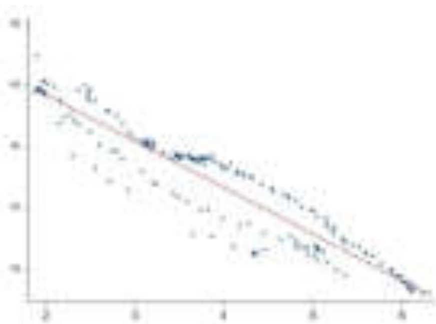
(c) Mortgage (fixed rates)



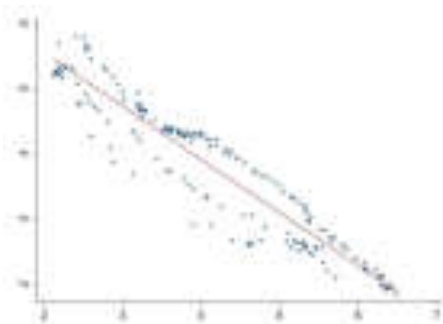
(d) Mortgage (variable rates)



(e) Auto Loan (New)



(f) Auto Loan (Used)



H Aggregate demand shocks in the baseline model

We provide the details of the stochastic version of the baseline model used in Section 6 of the paper here. In particular, we characterize the SME with aggregate demand shocks and we set up the Ramsey optimal policy problem for aggregate demand stabilization. The optimal policy exercise here is in the same spirit as Berentsen and Waller (2011). In contrast to the perfectly-competitive banking environment of Berentsen et al. (2007) and Berentsen and Waller (2011), our model now has non-trivial consequences for the design of optimal monetary policy in response to aggregate demand shocks.

H.1 Shocks

We can consider n and/or ϵ as random variables to capture aggregate demand fluctuation in the DM. The random variable ϵ is interpreted as marginal utility shock of the DM DM goods, and the random variable n is interpreted as shock that affects the number of active DM buyers (or we can interpert that as shocks to the probability of *early consumption*). In particular, $n \in [\underline{n}, \overline{n}]$, where $0 < \underline{n} < \overline{n} < 1$, and $\epsilon \in [\underline{\epsilon}, \overline{\epsilon}]$, where $0 < \underline{\epsilon} < \overline{\epsilon} < \infty$. Let $\omega = (n, \epsilon) \in \Omega$ denote the aggregate state (vector).

H.2 Monetary policy

The central bank commits to an overall long-run inflation target τ (or equivalently, a price path) and conducts state-contingent liquidity policy. The timing and sequence of actions is as follows. First, monetary injection, τM occurs at the beginning of the period (before shocks are realized). Second, the central bank injects a lump-sum amount of money to the DM agents up to $\tau_1(\omega)M = n\tau_b(\omega) + (1-n)\tau_b(\omega) + \tau_s(\omega)$ after shocks are realized. We assume the central bank can only tax in the CM but not the DM, so only transfers are premitted in the DM, i.e., $\tau_1(\omega) \geq 0$. Third, agents conduct banking arrangement, exchange and production of DM goods take place. Lastly, agents trade goods, work and settle the financial claims (loans and deposits) in the subsequent CM. Here, we have also assumed any state-contingent injection of liquidity received by the DM agents will be undone in CM, i.e., $\tau_2(\omega) = -\tau_1(\omega)$. The state-contingent policy plan here can be thought as a repo agreement made by the central bank: The central bank sells money in DM and promised to buy that back in the CM.

Given the assumption of DM state-contingent policy will be undone in the subsequent CM, the total change to the aggregate money stock is deterministic, which is given by $M_{+1} - M = (\gamma - 1)M = \tau M$, where $\gamma = 1 + \tau$ is the gross growth rate of money supply.

H.3 Characterization of SME with shocks

The markets structure of the model is the same as in baseline except that ϵ and n are random variables now.[49] We also work with stationary variables and restrict attention to stationary monetary equilibrium (SME).

Ex-post households with at least one lending bank contact. In events with probability measure α_1 and α_2, and for all $\epsilon \in \omega \in \Omega$, the buyer's optimal demand for DM consumption and loan is respectively characterized by

$$q_b^{1,\star}(z, \mathbf{z}, \omega) = \begin{cases} \epsilon^{\frac{1}{\sigma}} \left[\rho(1+i)\right]^{-\frac{1}{\sigma}} & \text{if } 0 < \rho \leq \tilde{\rho}_i \text{ and } 0 \leq i \leq \hat{i} \\ \frac{z + \tau_b Z}{\rho} & \text{if } \tilde{\rho}_i < \rho < \hat{\rho} \text{ and } i > \hat{i} \\ \epsilon^{\frac{1}{\sigma}} \rho^{-\frac{1}{\sigma}} & \text{if } \rho \geq \hat{\rho} \text{ and } i > \hat{i} \end{cases}, \tag{H.1}$$

[49] If we treat ϵ and n as parameter, and set $\epsilon = 1$, then we are back to the deterministic baseline case.

and,

$$\xi^{\star}\left(z,\mathbf{z},\omega\right)=\begin{cases}\epsilon^{\frac{1}{\sigma}}\rho^{\frac{\sigma-1}{\sigma}}\left(1+i\right)^{-\frac{1}{\sigma}}-\left(z+\tau_b Z\right) & \text{if } 0<\rho\leq\tilde{\rho}_i \text{ and } 0\leq i\leq\hat{i}\\ 0 & \text{if } \tilde{\rho}_i<\rho<\hat{\rho} \text{ and } i>\hat{i}\\ 0 & \text{if } \rho\geq\hat{\rho} \text{ and } i>\hat{i}\end{cases}, \tag{H.2}$$

where $\hat{\rho}:=\hat{\rho}(z,\mathbf{z},\omega)=\epsilon^{-\left(\frac{1}{\sigma-1}\right)}\left(z+\tau_b Z\right)^{\frac{\sigma}{\sigma-1}}$, $\tilde{\rho}_i:=\hat{\rho}\left(1+i\right)^{\frac{1}{\sigma-1}}$, and, $\hat{i}=\epsilon\left(z+\tau_b Z\right)^{-\sigma}\rho^{\sigma-1}-1>0$.

Ex-post households with zero lending bank contact. The buyer's optimal demand for DM consumption (for events with probability measure α_0) is

$$q_b^{0,\star}\left(z,\mathbf{z},\omega\right)=\begin{cases}\frac{z+\tau_b Z}{\rho} & \text{if } \rho\leq\hat{\rho}\\ \epsilon^{\frac{1}{\sigma}}\rho^{-\frac{1}{\sigma}} & \text{if } \rho\geq\hat{\rho}\end{cases}, \tag{H.3}$$

where $\hat{\rho}:=\hat{\rho}(z,\mathbf{z},\omega)=\epsilon^{-\left(\frac{1}{\sigma-1}\right)}\left(z+\tau_b Z\right)^{\frac{\sigma}{\sigma-1}}$.

Firms. The firm's optimal production plan satisfies $c_q\left(q_s\right)=p\phi$.

Hypothetical monopolist lending bank. We can derive the closed-form loan-price posting distribution similar to the baseline, except that the distribution is both state and policy dependent now. Given a realization of shock ω, this bank's "monopoly" profit function is $\Pi^m\left(i\right)=n\alpha_1 R\left(i\right)$. To pin down a monopoly loan price, differentiate the bank's "monopoly" profit function wrt. i, the (stationary variable version) FOC is

$$-\underbrace{z+\tau_b Z}_{f(i)}+\underbrace{\frac{1}{\sigma}\epsilon^{\frac{1}{\sigma}}\left(1+i\right)^{-\frac{1}{\sigma}}\left[\left(\sigma-1\right)+\frac{1+i_d}{1+i}\right]}_{g(i)}=0, \tag{H.4}$$

which needs to hold for each realization of state $\omega\in\Omega$.

Observe that in Condition (H.4), for a given individual state z, aggregate state Z, trend inflation rate τ, state ω, and $\omega\mapsto\tau_b(\omega)$, $f\left(i\right)$ is a constant w.r.t. i, and $g\left(i\right)$ is decreasing in i. Thus, as in the earlier, baseline model, there exists a unique monopoly-profit-maximizing price i^m that satisfies the above FOC for each realization of state $\omega\in\Omega$.

Once we pin down this $i^m(\mathbf{z},\omega)$ in a SME, then we use the equal profit condition combining with the upper support of the distribution $\bar{i}(\omega):=\min\{i^m(\mathbf{z},\omega),\hat{i}(\mathbf{z},\omega)\}$ to derive the lower support of the distribution $\underline{i}(\mathbf{z},\omega)$, which together pin down the closed-form loan-price posting distribution for each realization of state $\omega\in\Omega$.

Real money demand. Similar to the baseline case, we differentiate the DM value function with respect to m, update one period and substitute that into the CM first-order condition. Convert the result using stationary variables and combining that with the *ex-post* optimal goods demand functions in Equations

(H.1) and (H.3) in DM, and we get the Euler equation for real money demand as

$$
\begin{aligned}
\frac{\gamma-\beta}{\beta} &= \theta\left(z,\mathbf{z},\omega\right)-1 \\
&+\int_{\omega\in\Omega} n\mathbb{I}_{\{\rho<\hat{\rho}\}}\alpha_0\left[\frac{1}{\rho}\epsilon\left(\frac{z+\tau_b(\omega)z}{\rho}\right)^{-\sigma}-1\right]\psi\left(\omega\right)\mathrm{d}\omega \\
&+\int_{\omega\in\Omega} n\int_{\underline{i}}^{\bar{i}}\mathbb{I}_{\{\rho<\tilde{\rho}_i\}}\left[\alpha_1+2\alpha_2\left(1-F\left(i,z,\mathbf{z},\omega\right)\right)\right]i\mathrm{d}F\left(i,z,\mathbf{z},\omega\right)\psi\left(\omega\right)\mathrm{d}\omega \\
&+\int_{\omega\in\Omega} n\int_{\underline{i}}^{\bar{i}}\mathbb{I}_{\{\tilde{\rho}_i\leq\rho<\hat{\rho}\}}\left[\alpha_1+2\alpha_2\left(1-F\left(i,z,\mathbf{z},\omega\right)\right)\right] \\
&\times\left[\frac{1}{\rho}\epsilon\left(\frac{z+\tau_b(\omega)z}{\rho}\right)^{-\sigma}-1\right]\mathrm{d}F\left(i,z,\mathbf{z},\omega\right)\psi\left(\omega\right)\mathrm{d}\omega,
\end{aligned}
\tag{H.5}
$$

and,

$$
\begin{aligned}
\theta\left(z,\mathbf{z},\omega\right)-1 :=& \int_{\omega\in\Omega}\left(1-n\right)\left(1+i_d\right)\psi\left(\omega\right)\mathrm{d}\omega \\
&+\int_{\omega\in\Omega} n\alpha_0\psi\left(\omega\right)\mathrm{d}\omega \\
&+\int_{\omega\in\Omega} n\int_{\underline{i}}^{\bar{i}}\mathbb{I}_{\{\rho<\tilde{\rho}_i\}}\left[\alpha_1+2\alpha_2\left(1-F\left(i,z,\mathbf{z},\omega\right)\right)\right]\mathrm{d}F\left(i,z,\mathbf{z},\omega\right)\psi\left(\omega\right)\mathrm{d}\omega \\
&+\int_{\omega\in\Omega} n\int_{\bar{i}}^{i^{\mathrm{m}}}\mathbb{I}_{\{\tilde{\rho}_i\leq\rho<\hat{\rho}\}}\left[\alpha_1+2\alpha_2\left(1-F\left(i,z,\mathbf{z},\omega\right)\right)\right]\mathrm{d}F\left(i,z,\mathbf{z},\omega\right)\psi\left(\omega\right)\mathrm{d}\omega \\
&+\int_{\omega\in\Omega} n\int_{\bar{i}}^{i^{\mathrm{m}}}\mathbb{I}_{\{\hat{\rho}\leq\rho\}}\left[\alpha_1+2\alpha_2\left(1-F\left(i,z,\mathbf{z},\omega\right)\right)\right]\mathrm{d}F\left(i,z,\mathbf{z},\omega\right)\psi\left(\omega\right)\mathrm{d}\omega \\
&-1.
\end{aligned}
$$

Note that the integral limits $(\bar{i}, i^{\mathrm{m}}, \underline{i})$ and cut-off prices $(\tilde{\rho}_i, \hat{\rho})$ are also functions of $(z, \mathbf{z}, \omega)$. The LHS of Condition (H.5) captures the marginal cost of accumulating an extra unit of real money balance at the end of each CM, and the RHS captures the expected marginal utility value of that extra unit of money balance (evaluated at the beginning of next DM before shock is realized and before buyer types, matching and trading occurs).

Loan price-posting distribution. We restrict to the case $\alpha_1 \in (0,1)$ for the stochastic version here. The distribution of loan (interest-rate) price posts is given by:

$$
F\left(i,z,\mathbf{z},\omega\right)=1-\frac{\alpha_1}{2\alpha_2}\left[\frac{R\left(\bar{i}(z,\mathbf{z},\omega)\right)}{R\left(i(z,\mathbf{z},\omega)\right)}-1\right],
\tag{H.6}
$$

and, $\mathrm{supp}\left(F(\cdot,z,\mathbf{z},\omega)\right)=\left[\underline{i}(z,\mathbf{z},\omega),\bar{i}(z,\mathbf{z},\omega)\right]$, and, given $\bar{i}(z,\mathbf{z},\omega)=\min\{i^m(z,\mathbf{z},\omega),\hat{i}(z,\mathbf{z},\omega)\}$, $\underline{i}(z,\mathbf{z},\omega)$ solves: $R\left(\underline{i}(z,\mathbf{z},\omega)\right)=\frac{\alpha_1}{\alpha_1+2\alpha_2}R\left(\bar{i}(z,\mathbf{z},\omega)\right)$, where the (real) bank profit per customer served is $R\left(i,z,\mathbf{z},\omega\right)=\left[\epsilon^{\frac{1}{\sigma}}\rho^{\frac{\sigma-1}{\sigma}}\left(1+i\right)^{-\frac{1}{\sigma}}-\left(z+\tau_b Z\right)\right]\left(i-i^d\right)$.

Observe that in Equations (H.1) and (H.2), all the cutoff functions (in terms of relative price of DM goods or lending interest rate) are all now depend on the optimal policy function, $\omega \mapsto \tau_b(\omega)$ function, and also on the $\omega := (\epsilon, n)$ states of the economy.

Similarly, the support of the posted loan interest rate distribution in Equation (H.6) now also depends on a given $\omega \mapsto \tau_b(\omega)$ function, and also on $\omega := (\epsilon, n)$. This can be seen from the optimal monopoly rate that solves the Condition (H.4), from households' reservation interest rate $\hat{i}(z, \mathbf{z}, \omega)$, and from the associated lowest possible loan rate of the distribution $\underline{i}(z, \mathbf{z}, \omega)$.

The key difference between ϵ shocks and n shocks is that the former induces one extra moving part—a direct effect of policy outcomes $\tau_b(\omega)$ on the support of $F(\cdot, z, \mathbf{z}, \omega)$. The latter shock implies one less moving part.

Competitive price taking and goods market clearing. DM goods market clears for all ω:

$$\begin{aligned} q_s(z, \mathbf{z}, \omega) &\equiv c'^{-1}(\rho) \\ &= \int_\Omega n\alpha_0 q_b^{0,\star}(z, \mathbf{z}, \omega)\, \psi(\omega) \mathrm{d}\omega \\ &+ \int_\Omega n \int_{\underline{i}(z,\mathbf{z},\omega)}^{\bar{i}(z,\mathbf{z},\omega)} \left[\alpha_1 + 2\alpha_2 \left(1 - F(i, z, \mathbf{z}, \omega)\right)\right] q_b^\star(i, z, \mathbf{z}, \omega)\, \mathrm{d}F(i, z, \mathbf{z}, \omega)\, \psi(\omega) \mathrm{d}\omega. \end{aligned} \tag{H.7}$$

We can also verify that the CM labor and goods market clear given the SME solutions $\{z^\star, q_b^{0,\star}, q_b^\star\}$.

Aggregate feasibility of loanable funds in banking market. Interests on total loans weakly exceed that on total deposits

$$\begin{aligned} &\int_\Omega n \int_{\underline{i}(z,\mathbf{z},\omega)}^{\bar{i}(z,\mathbf{z},\omega)} \left[\alpha_1 + 2\alpha_2 \left(1 - F(i, z, \mathbf{z}, \omega)\right)\right] \xi^\star(i, z, \mathbf{z}, \omega)\, i \mathrm{d}F(i, z, \mathbf{z}, \omega)\, \psi(\omega) \mathrm{d}\omega \\ &\quad \geq \int_\Omega (1-n)\, i_d \delta^\star(z, \mathbf{z}, \omega) \psi(\omega) \mathrm{d}\omega \equiv \int_\Omega (1-n)\, i_d \left(\frac{z + \hat{\tau}_b(\omega) Z}{\rho}\right) \psi(\omega) \mathrm{d}\omega, \end{aligned} \tag{H.8}$$

for each realization of state $\omega \in \Omega$.

We now summarize the description of a SME with aggregate demand shocks below.

Definition H.1. *Assume $\sigma < 1$. Given money supply growth $\gamma = 1 + \tau$, and redistributive policy plan $\{\boldsymbol{\tau}(\omega)\}_{\omega\in\Omega} := \{\tau_1(\omega), \tau_2(\omega)\}_{\omega\in\Omega}$ a* Stationary Monetary Equilibrium (SME) *is a list of time- and state-invariant CM consumption allocation and residual real money balance outcomes $\{x^\star \equiv 1, z^\star\}$, $z^\star = Z$ so that $\mathbf{z} = (z^\star, \boldsymbol{\tau}(\omega))$ and time-independent allocation functions for DM goods and loans, $\{q^\star(\mathbf{z}^\star, \omega), \xi^\star(\mathbf{z}^\star, \omega)\}$, and distribution, $F(\cdot; \mathbf{z}^\star, \omega)$ such that: (1) household optimization satisfies the money-demand Euler Equation (H.5); (2) the distribution of posted loan (interest-rate) price satisfies Equation (H.6); (3) DM goods market clearing satisfies Condition (H.7); (4) loans feasibility satisfies Requirement (H.8); and (5) the government budget constraint holds for each ω, i.e.,*

$$\frac{\gamma - \beta}{\beta} = \tau + \tau_1(\omega) + \tau_2(\omega), \quad \tau_1(\omega) = -\tau_2(\omega). \tag{H.9}$$

H.4 Optimal stabilization policy over SME with shocks

To understand how the stabilization policy in response to demand fluctuation may work, we compare two types of government policy:

1. **Active central bank**. The policymaker commits to an ex-ante, optimal policy plan that maximizes social welfare over a steady-state equilibrium (i.e., a SME). In particular, the active central bank solves

$$\begin{aligned} \max_{\{q_b^0(\cdot,\omega),\ q_b(\cdot,\omega),\tau_b(\omega)\}_{\omega\in\Omega}} & U(x) - x - c(q_s(\mathbf{z},\omega)) \\ & + \int_{\omega\in\Omega} n\alpha_0 \epsilon u\left[q_b^0(\mathbf{z},\omega)\right]\psi(\omega)\,\mathrm{d}\omega \\ & + \int_{\omega\in\Omega} n \int_{\underline{i}(\mathbf{z},\omega)}^{\bar{i}(\mathbf{z},\omega)} \left[\alpha_1 + 2\alpha_2\left(1 - F(i,\mathbf{z},\omega)\right)\right] \\ & \times \epsilon u\left[q_b(i,\mathbf{z},\omega)\right]\mathrm{d}F(i,\mathbf{z},\omega)\,\psi(\omega)\,\mathrm{d}\omega \end{aligned} \tag{H.10}$$

   subject to optimal money demand in (H.5) the distribution of loan interest rates in (H.6), the DM goods market clearing in Condition (H.7), the loans feasibility Condition (H.8) and government budget feasibility in Condition (H.9), where q_s is given by (H.7).

   Note: The policy plan prescribes ω-contingent liquidity injections, i.e., $\tau_1(\omega) = \tau_b(\omega) \geq 0$, and assume $\tau_2(\omega) = -\tau_1(\omega)$ for all $\omega \in \Omega$.

2. **Passive central bank**. In this regime, the policymaker is constrained by $\tau_1(\omega) = \tau_2(\omega) = 0$ for all $\omega \in \Omega$. The outcomes will be very similar to our deterministic, baseline SME.

The objective of the active central bank is similar to Berentsen and Waller (2011). New insights arises from the equilibrium varying dispersion of loan spreads since $F(i;\omega)$ is now both state and policy dependent. We explain what the new insights are in Section 6 of the paper.