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James G. MacKinnon
Queen's University

Morten Ørregaard Nielsen
Aarhus University

Matthew D. Webb
Carleton University

Department of Economics
Queen's University
94 University Avenue
Kingston, Ontario, Canada
K7L 3N6

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Leverage, Influence, and the Jackknife in Clustered Regression Models: Reliable Inference Using summclust[*]

James G. MacKinnon[†]
Queen's University
`mackinno@queensu.ca`

Morten Ørregaard Nielsen
Aarhus University
`mon@econ.au.dk`

Matthew D. Webb
Carleton University
`matt.webb@carleton.ca`

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Abstract

We introduce a new `Stata` package called `summclust` that summarizes the cluster structure of the dataset for linear regression models with clustered disturbances. The key unit of observation for such a model is the cluster. We therefore propose cluster-level measures of leverage, partial leverage, and influence and show how to compute them quickly in most cases. The measures of leverage and partial leverage can be used as diagnostic tools to identify datasets and regression designs in which cluster-robust inference is likely to be challenging. The measures of influence can provide valuable information about how the results depend on the data in the various clusters. We also show how to calculate two jackknife variance matrix estimators efficiently as a byproduct of our other computations. These estimators, which are already available in `Stata`, are generally more conservative than conventional variance matrix estimators. The `summclust` package computes all the quantities that we discuss.

Keywords: summclust, clustered data, cluster-robust variance estimator, CRVE, grouped data, high-leverage clusters, influential clusters, jackknife, partial leverage, robust inference.

JEL Codes: C10, C12, C21, C23, C87.

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[†] Corresponding author. Address: Department of Economics, 94 University Avenue, Queen's University, Kingston, Ontario K7L 3N6, Canada. Email: `mackinno@queensu.ca`. Tel. 613-533-2293. Fax 613-533-6668.

1 Introduction

It is now standard in many fields of economics and other disciplines to employ cluster-robust inference for the parameters of linear regression models. In the most common case, each of the N observations is assigned to one of G disjoint clusters, which might correspond to, for example, families, schools, villages, hospitals, firms, industries, years, cities, counties, or states. The assignment of observations to clusters is assumed to be known, and observations in different clusters are assumed to be independent, but any pattern of heteroskedasticity and/or dependence is allowed within each cluster. Under these assumptions, a cluster-robust variance matrix, or CRVE, yields asymptotically valid t-tests, Wald tests, and confidence intervals. However, even when N is very large, the resulting inferences may be unreliable when G is not large or the clusters are not sufficiently homogeneous.

The literature on cluster-robust inference has grown rapidly in recent years. Cameron and Miller (2015) is a classic survey article. Conley, Gonçalves, and Hansen (2018) surveys a broader class of methods for dependent data. MacKinnon, Nielsen, and Webb (2023a) is a comprehensive guide to empirical practice. As it discusses, there are two situations in which cluster-robust t-tests and Wald tests are at risk of over-rejecting to an extreme extent, even when G is not small. The first is when one or a few clusters are much larger than the rest, and the second is when the only "treated" observations belong to just a few clusters; Djogbenou, MacKinnon, and Nielsen (2019) discusses the first case, and MacKinnon and Webb (2017a, b, 2018) discuss the second. In both of these cases, one cluster (or a few of them) has high leverage, in the sense that omitting this cluster has the potential to change the OLS estimates substantially. When that actually happens, a cluster is said to be influential.

The concepts of leverage and influence are normally applied at the observation level (Belsley, Kuh, and Welsch 1980), but they are equally applicable at the cluster level. Just as high-leverage observations can make heteroskedasticity-robust inference unreliable (Chesher 1989), so too can high-leverage clusters make cluster-robust inference unreliable. Just as highly influential observations may lead us to suspect that there is something wrong with the model or the data, so too may highly influential clusters. Any situation in which a few clusters have high leverage or high influence should be worrying.

There are at least two different concepts of leverage. The usual one focuses on fitted values or, equivalently, residuals. A cluster is said to have high leverage if removing it has the potential to change the fitted values for that cluster by a lot. The second concept is partial leverage (Cook and Weisberg 1980). A cluster is said to have high partial leverage for the j^{th} coefficient if removing that cluster has the potential to change the estimate of the j^{th} coefficient by a lot. We discuss both concepts in Section 2.1.

Whether a cluster has high leverage, high partial leverage, or is influential can depend on the sample in rather complicated ways. We provide a new Stata package called summclust that implements computationally-efficient ways to identify high-leverage and influential clusters and provides a number of statistics that collectively summarize the cluster structure of the dataset. These can be useful for detecting cases in which cluster-robust inference may not be reliable. Our leverage and influence calculations also allow us to compute two cluster jackknife variance matrix estimators, which we refer to as CV_3 and $\mathrm{CV}_{3\mathrm{J}}$, at little additional cost. These estimators are already available in Stata by using either the vce(jackknife) option or the jackknife prefix. Recent work (Hansen 2022; MacKinnon, Nielsen, and Webb 2023b) suggests that CV_3 and $\mathrm{CV}_{3\mathrm{J}}$ generally perform better in finite samples than more widely-used CRVEs; see also Section 7.

The remainder of the paper is organized as follows. The next section begins with a brief review of cluster-robust inference for linear regression models. Then Section 2.1 introduces our new measures of leverage, partial leverage, and influence at the cluster level. Section 2.2 shows how our results can be used to compute the CV_3 and $\mathrm{CV}_{3\mathrm{J}}$ jackknife variance matrix estimators. Section 2.3 discusses what quantities are reported by summclust and should, at least in some cases, be reported by the investigator.

Section 3 provides a detailed description of the summclust package which computes these variance estimators and diagnostic measures. The command uses the syntax:

```
summclust varlist, cluster(varname) [options]
```

The package has quite a few options and can even be used by itself to estimate a linear regression model with clustered disturbances. The last few sections of the paper illustrate the use of summclust and provide evidence on the value of the measures that it calculates. Section 4 presents an empirical illustration in which measures of leverage, partial leverage, and influence are highly informative. Section 5 discusses several special cases in which some or all of these measures can be determined analytically. Section 6 briefly discusses two-way clustering, where summclust can be valuable even though it is not explicitly designed to handle this case. Section 7 describes some simulation experiments which suggest that it may be desirable to report many of the quantities calculated by summclust, and Section 8 concludes.

2 Clustering, Leverage, Influence, and the Jackknife

We focus on the linear regression model

$$\boldsymbol{y}_g = \boldsymbol{X}_g\boldsymbol{\beta} + \boldsymbol{u}_g, \quad g = 1, \ldots, G, \tag{1}$$

where the data have been divided into G disjoint clusters. The g^{th} cluster has N_g observations, so that the sample size is $N = \sum_{g=1}^{G} N_g$. In (1), $\boldsymbol{X}_g$ is an $N_g \times k$ matrix of regressors, $\boldsymbol{\beta}$ is a k-vector of coefficients, $\boldsymbol{y}_g$ is an N_g-vector of observations on the regressand, and $\boldsymbol{u}_g$ is an N_g-vector of disturbances (or error terms). The $\boldsymbol{X}_g$ may of course be stacked into an $N \times k$ matrix $\boldsymbol{X}$, and likewise the $\boldsymbol{y}_g$ and $\boldsymbol{u}_g$ may be stacked into N-vectors $\boldsymbol{y}$ and $\boldsymbol{u}$, so that (1) can be rewritten as $\boldsymbol{y} = \boldsymbol{X}\boldsymbol{\beta} + \boldsymbol{u}$.

Dividing the sample into clusters only becomes meaningful if we make assumptions about the disturbance vectors $\boldsymbol{u}_g$ and, consequently, the score vectors $\boldsymbol{s}_g = \boldsymbol{X}_g^\top \boldsymbol{u}_g$. For a correctly specified model, $\mathrm{E}(\boldsymbol{s}_g) = \boldsymbol{0}$ for all g. We further assume that

$$\mathrm{E}(\boldsymbol{s}_g \boldsymbol{s}_g^\top) = \boldsymbol{\Sigma}_g \quad \text{and} \quad \mathrm{E}(\boldsymbol{s}_g \boldsymbol{s}_{g'}^\top) = \boldsymbol{0}, \quad g, g' = 1, \ldots, G, \quad g' \neq g, \tag{2}$$

where $\boldsymbol{\Sigma}_g$ is the symmetric, positive semidefinite variance matrix of the scores for the g^{th} cluster. The second assumption in (2) is crucial. It says that the scores for every cluster are uncorrelated with the scores for every other cluster. We take the number of clusters G and the allocation of observations to clusters as given. The important issue of how to choose the clustering structure, perhaps by testing for the correct level of clustering, is discussed in detail in MacKinnon, Nielsen, and Webb (2023c).

The OLS estimator of $\boldsymbol{\beta}$ is

$$\hat{\boldsymbol{\beta}} = (\boldsymbol{X}^\top \boldsymbol{X})^{-1} \boldsymbol{X}^\top \boldsymbol{y} = \boldsymbol{\beta}_0 + (\boldsymbol{X}^\top \boldsymbol{X})^{-1} \boldsymbol{X}^\top \boldsymbol{u},$$

where the second equality depends on the assumption that the data are actually generated by (1) with true value $\boldsymbol{\beta}_0$. It follows that

$$\hat{\boldsymbol{\beta}} - \boldsymbol{\beta}_0 = (\boldsymbol{X}^\top \boldsymbol{X})^{-1} \sum_{g=1}^{G} \boldsymbol{X}_g^\top \boldsymbol{u}_g = \left(\sum_{g=1}^{G} \boldsymbol{X}_g^\top \boldsymbol{X}_g \right)^{-1} \sum_{g=1}^{G} \boldsymbol{s}_g. \tag{3}$$

From the rightmost expression in (3), we see that the distribution of $\hat{\boldsymbol{\beta}}$ depends on the disturbance subvectors $\boldsymbol{u}_g$ only through the distribution of the score vectors $\boldsymbol{s}_g$. Asymptotic inference commonly uses the empirical score vectors $\hat{\boldsymbol{s}}_g = \boldsymbol{X}_g^\top \hat{\boldsymbol{u}}_g$, in which the $\boldsymbol{u}_g$ are replaced by the residual subvectors $\hat{\boldsymbol{u}}_g$, to estimate the variance matrix of the $\boldsymbol{s}_g$. This should work well if the sum of the $\boldsymbol{s}_g$, suitably normalized, is well approximated by a multivariate normal distribution with mean zero, and if the $\boldsymbol{s}_g$ are well approximated by the $\hat{\boldsymbol{s}}_g$. However, asymptotic inference can be misleading when either of these approximations is poor.

It follows immediately from (3) that an estimator of the variance of $\hat{\boldsymbol{\beta}}$ may be based on the usual sandwich formula,

$$(\boldsymbol{X}^\top\boldsymbol{X})^{-1}\Big(\sum_{g=1}^{G}\boldsymbol{\Sigma}_g\Big)(\boldsymbol{X}^\top\boldsymbol{X})^{-1}. \tag{4}$$

The natural way to estimate (4) is to replace the $\boldsymbol{\Sigma}_g$ matrices by their empirical counterparts, that is, the $\hat{\boldsymbol{s}}_g\hat{\boldsymbol{s}}_g^\top$. If, in addition, we multiply by a correction for degrees of freedom, we obtain the cluster-robust variance estimator, or CRVE,

$$\text{CV}_1\text{:}\qquad \frac{G(N-1)}{(G-1)(N-k)}(\boldsymbol{X}^\top\boldsymbol{X})^{-1}\Big(\sum_{g=1}^{G}\hat{\boldsymbol{s}}_g\hat{\boldsymbol{s}}_g^\top\Big)(\boldsymbol{X}^\top\boldsymbol{X})^{-1}. \tag{5}$$

This is by far the most widely used CRVE in practice, and it is the default one implemented in `Stata`; alternatives to this estimator will be discussed in Section 2.2. When $G = N$, the CV_1 estimator reduces to the familiar HC_1 estimator (MacKinnon and White 1985) that is robust only to heteroskedasticity of unknown form.

The fundamental unit of inference for clustered observations is not the observation but the cluster; this is evident from (3), (4), and (5). The asymptotic theory for cluster-robust inference has been analyzed by Djogbenou, MacKinnon, and Nielsen (2019) and Hansen and Lee (2019) under the assumption that $G \to \infty$. The quality of the asymptotic approximation depends on the number of clusters G and the heterogeneity of the score vectors (MacKinnon, Nielsen, and Webb 2023a). The more the distributions of the scores vary across clusters, the worse the asymptotic approximation will likely be. Heterogeneity can arise from variation in cluster sizes and/or from variation in the distributions of the disturbances, the regressors, or both. As we discuss in Sections 2.1, 2.3 and 7, leverage, partial leverage, and summary statistics based on them provide useful measures of heterogeneity across clusters.

Inference about $\boldsymbol{\beta}$ is typically based on cluster-robust t-statistics and Wald statistics. If β_j denotes the j^{th} element of $\boldsymbol{\beta}$ and β_{0j} is its value under the null hypothesis, then the appropriate t-statistic is

$$t_j = \frac{\hat{\beta}_j - \beta_{0j}}{\text{s.e.}(\hat{\beta}_j)},$$

where $\hat{\beta}_j$ is the OLS estimate, and $\text{s.e.}(\hat{\beta}_j)$ is the square root of the j^{th} diagonal element of (5). Under extremely strong assumptions (Bester, Conley, and Hansen 2011), it can be shown that t_j asymptotically follows the $t(G-1)$ distribution. Conventional inference in `Stata` and other programs is based on this distribution.

As the articles cited in the second paragraph of Section 1 discuss, inference based on t_j and the $t(G-1)$ distribution can be unreliable when G is small and/or the clusters are severely heterogeneous. This is true to an even greater extent for Wald tests of two or more restrictions (Pustejovsky and Tipton 2018). The measures of leverage and partial leverage at the cluster level that we introduce in the next section may help to identify the sort of heterogeneity that is likely to make inference unreliable.

Instead of using the $t(G-1)$ distribution, we can obtain both P values for t_j and confidence intervals for β_j by employing the wild cluster restricted (or WCR) bootstrap (Cameron, Gelbach, and Miller 2008). It can sometimes provide much more reliable inferences than the conventional approach; see Section 7. Roodman, MacKinnon, Nielsen, and Webb (2019) describes a computationally efficient implementation of this method in the `Stata` package `boottest`. MacKinnon, Nielsen, and Webb (2023b) proposes new versions of the wild cluster bootstrap that involve transforming the empirical scores. When G is reasonably large and the clusters are not very heterogeneous, inferences based on the WCR bootstrap and inferences based on CV_1 t-statistics combined with the $t(G-1)$ distribution will often be very similar. When they differ noticeably, neither should be relied upon without further investigation.

Section 2.2 discusses two CRVEs, which we refer to as CV_3 and $\mathrm{CV}_{3\mathrm{J}}$, that are both based on the cluster jackknife. In practice, these estimators are often extremely similar. CV_3 and $\mathrm{CV}_{3\mathrm{J}}$ tend to yield more reliable inferences in finite samples than does CV_1, especially when the clusters are quite heterogeneous; see Section 7 and MacKinnon, Nielsen, and Webb (2023b). Based on this simulation evidence, we recommend computing either CV_3 or $\mathrm{CV}_{3\mathrm{J}}$ essentially all the time. This is easy to do using `summclust`.

2.1 Identifying High-Leverage and Influential Clusters

At the observation level, there are three classic measures of heterogeneity, namely, leverage, partial leverage, and influence (Belsley, Kuh, and Welsch 1980; Chatterjee and Hadi 1986). In this section, we propose analogous measures at the cluster level.

Measures of leverage at the observation level are based on how much the residual for observation i changes when that observation is omitted from the regression. If h_i denotes the i^{th} diagonal element of the "hat matrix" $\boldsymbol{H} = \boldsymbol{P}_{\boldsymbol{X}} = \boldsymbol{X}(\boldsymbol{X}^\top\boldsymbol{X})^{-1}\boldsymbol{X}^\top$, then omitting the i^{th} observation changes the i^{th} residual from $\hat{u}_i$ to $\hat{u}_i/(1-h_i)$. Because $0 < h_i < 1$, this delete-one residual is always larger in absolute value than $\hat{u}_i$. The factor by which the delete-one residual exceeds $\hat{u}_i$ increases with h_i. Since the average of the h_i is k/N, observations with values of h_i substantially larger than k/N may reasonably be said to have high leverage.

Dropping the g^{th} cluster when we estimate $\boldsymbol{\beta}$ yields the delete-one-cluster estimate $\hat{\boldsymbol{\beta}}^{(g)}$. Using $\hat{\boldsymbol{\beta}}^{(g)}$ in place of $\hat{\boldsymbol{\beta}}$ changes the residual vector for the g^{th} cluster from $\hat{\boldsymbol{u}}_g$ to $\hat{\boldsymbol{u}}_g^{(g)}$. These delete-one-cluster residual vectors can be written in two ways:

$$\hat{\boldsymbol{u}}_g^{(g)} = \boldsymbol{y}_g - \boldsymbol{X}_g\hat{\boldsymbol{\beta}}^{(g)} = (\mathbf{I} - \boldsymbol{H}_g)^{-1}\hat{\boldsymbol{u}}_g.$$

In the rightmost expression above,

$$\boldsymbol{H}_g = \boldsymbol{X}_g(\boldsymbol{X}^\top\boldsymbol{X})^{-1}\boldsymbol{X}_g^\top$$

is the $N_g \times N_g$ diagonal block of $\boldsymbol{H}$ that corresponds to cluster g. The matrix $\boldsymbol{H}_g$ is the cluster analog of the scalar h_i. Of course, it is not feasible to report the $\boldsymbol{H}_g$. In fact, when any of the clusters is sufficiently large, even computing and storing these matrices may be challenging. As a measure of leverage, we therefore suggest using a matrix norm of the $\boldsymbol{H}_g$. Specifically, we suggest the scalar

$$L_g = \mathrm{Tr}(\boldsymbol{H}_g) = \mathrm{Tr}\Big(\boldsymbol{X}_g^\top\boldsymbol{X}_g(\boldsymbol{X}^\top\boldsymbol{X})^{-1}\Big). \tag{6}$$

When the g^{th} cluster contains just one observation, say the i^{th}, then $L_g = h_i$. Thus, in this special case, the leverage measure that we are proposing reduces to the usual measure of leverage at the observation level.

The trace in (6) is the nuclear norm of the matrix $\boldsymbol{H}_g$. In general, the nuclear norm of a matrix $\boldsymbol{A}$ is the sum of the singular values of $\boldsymbol{A}$. When $\boldsymbol{A}$ is symmetric and positive semidefinite, the singular values are equal to the eigenvalues, which are non-negative. Since the trace of any square matrix is equal to the sum of the eigenvalues, the trace of a symmetric and positive semidefinite matrix is also its nuclear norm. In principle, we could report any norm of the $\boldsymbol{H}_g$ matrices, but the nuclear norm is particularly easy to compute. Also, because it is linear, we can sum over g and take the sum inside the norm just as if the $\boldsymbol{H}_g$ were scalars. Since $\sum_{g=1}^G \boldsymbol{X}_g^\top\boldsymbol{X}_g = \boldsymbol{X}^\top\boldsymbol{X}$, this result means that $G^{-1}\sum_{g=1}^G \mathrm{Tr}(\boldsymbol{H}_g) = k/G$, which is analogous to the result that the average of the h_i over all observations is k/N.

High-leverage clusters can be identified by comparing the L_g to k/G, their average. If, for some cluster h, L_h is substantially larger than k/G, then cluster h may be said to have high leverage. Just how much larger L_h has to be is a matter of judgement. A cluster with $L_h = 2k/G$ probably does not qualify, but a cluster with $L_h = 5k/G$ probably does. Cluster h can have high leverage either because N_h is considerably larger than G/N or because the matrix $\boldsymbol{X}_h$ is somehow extreme relative to the other $\boldsymbol{X}_g$ matrices, or both. We can compare the leverage of any two clusters by forming ratios. For example, if $L_1 = 3$ and $L_2 = 1$, then we can say that the first cluster has three times the leverage of the second cluster.

The leverage measure we suggest in (6) shows the potential impact of a specified cluster on residuals and fitted values, but not on any particular regression coefficient. When interest focuses on just one such coefficient, say the j^{th}, it may be more interesting to calculate the partial leverage of each cluster. The concept of partial leverage was introduced, for individual observations, in Cook and Weisberg (1980). Let

$$\acute{\boldsymbol{x}}_j = \Big(\mathbf{I} - \boldsymbol{X}_{[j]}\Big(\boldsymbol{X}_{[j]}^\top \boldsymbol{X}_{[j]}\Big)^{-1}\boldsymbol{X}_{[j]}^\top\Big)\boldsymbol{x}_j,$$

where $\boldsymbol{x}_j$ is the vector of observations on the j^{th} regressor, and $\boldsymbol{X}_{[j]}$ is the matrix of observations on all the other regressors. Thus $\acute{\boldsymbol{x}}_j$ denotes $\boldsymbol{x}_j$ after all the other regressors have been partialed out. The partial leverage of observation i is simply the i^{th} diagonal element of the matrix $\acute{\boldsymbol{x}}_j(\acute{\boldsymbol{x}}_j^\top \acute{\boldsymbol{x}}_j)^{-1}\acute{\boldsymbol{x}}_j^\top$, which is just $\acute{x}_{ji}^2/(\acute{\boldsymbol{x}}_j^\top \acute{\boldsymbol{x}}_j)$, where $\acute{x}_{ji}^2$ is the i^{th} element of $\acute{\boldsymbol{x}}_j$.

The analogous measure of partial leverage for cluster g is

$$L_{gj} = \frac{\acute{\boldsymbol{x}}_{gj}^\top \acute{\boldsymbol{x}}_{gj}}{\acute{\boldsymbol{x}}_j^\top \acute{\boldsymbol{x}}_j}, \tag{7}$$

where $\acute{\boldsymbol{x}}_{gj}$ is the subvector of $\acute{\boldsymbol{x}}_j$ corresponding to the g^{th} cluster. This is what (6) reduces to if we replace $\boldsymbol{X}$ and $\boldsymbol{X}_g$ by $\acute{\boldsymbol{x}}_j$ and $\acute{\boldsymbol{x}}_{gj}$, respectively. It is easy to calculate the partial leverage for every cluster for any coefficient of interest. The average of the L_{gj} is evidently $1/G$, so that if cluster h has $L_{hj} >> 1/G$, it has high partial leverage for the j^{th} coefficient. Moreover, as we will see in Section 7, the empirical distribution of the L_{gj} across clusters seems to provide useful diagnostic information.

Young (2022) derives a measure of cluster-level leverage for the first-stage regression used to obtain a linear instrumental variables estimator. The paper calls L_{gj} the group g "share of coefficient leverage" for instrument j and then uses the maximum of the L_{gj} over all the instruments excluded from the structural equation as a measure of the leverage of cluster g. Using simulations based on 1309 IV regressions from 30 published papers, the paper finds that inference is much less reliable for models where one or two clusters have high leverage in the first-stage regression than for models where no clusters do so.

One possible consequence of heterogeneity is that the estimates may change a lot when certain clusters are deleted. It can therefore be illuminating to delete one cluster at a time, so as to see how influential each cluster is. To do this in a computationally efficient manner, `summclust` first computes the cluster-level matrices and vectors

$$\boldsymbol{X}_g^\top \boldsymbol{X}_g \quad \text{and} \quad \boldsymbol{X}_g^\top \boldsymbol{y}_g, \quad g = 1, \ldots, G. \tag{8}$$

These are then used to construct $\boldsymbol{X}^\top \boldsymbol{X}$ and $\boldsymbol{X}^\top \boldsymbol{y}$, and the vector of least squares estimates when cluster g is deleted is computed as

$$\hat{\boldsymbol{\beta}}^{(g)} = (\boldsymbol{X}^\top \boldsymbol{X} - \boldsymbol{X}_g^\top \boldsymbol{X}_g)^{-1}(\boldsymbol{X}^\top \boldsymbol{y} - \boldsymbol{X}_g^\top \boldsymbol{y}_g). \tag{9}$$

Unless k is extremely large, it should generally not be expensive to compute $\hat{\boldsymbol{\beta}}^{(g)}$ for every cluster using (9). summclust simply has to invert G matrices, each of them $k \times k$, and then multiply each of those matrices by a k-vector.

Especially when they vary a lot, the $\hat{\boldsymbol{\beta}}^{(g)}$ can reveal a great deal about the sample. In addition, as we shall see in Section 2.2, they may be used to calculate jackknife variance matrices. When there is a parameter of particular interest, say β_j, it may be a good idea to report the $\hat{\beta}_j^{(g)}$ for $g = 1, \ldots, G$ in either a histogram or a table. By default, summclust creates several figures with these and other cluster-level statistics. If $\hat{\beta}_j^{(h)}$ differs greatly from $\hat{\beta}_j$ for some cluster h, then cluster h is evidently influential.

In a few extreme cases, there may be a cluster h for which it is impossible to compute $\hat{\beta}_j^{(h)}$. This will happen, for example, when the regressor corresponding to β_j is a treatment dummy and cluster h is the only treated one. This is an extreme example of the problem of few treated clusters, and inferences based on either the $t(G-1)$ distribution or the WCR bootstrap are likely to be completely unreliable in this case (MacKinnon and Webb 2017b, 2018, 2020).

Identifying influential clusters by comparing the $\hat{\boldsymbol{\beta}}^{(g)}$ with $\hat{\boldsymbol{\beta}}$ is very similar to identifying influential observations using the classic methods discussed in Belsley, Kuh, and Welsch (1980) and Chatterjee and Hadi (1986); for an interesting recent extension, see Broderick, Giordano, and Meager (2021). Unlike the leverage measures, the $\hat{\beta}_j^{(g)}$ may be either positive or negative, must depend on the $\boldsymbol{y}_g$, and necessarily vary across clusters. They may sometimes reveal features of the model or dataset that require further investigation. Perhaps the model does not seem to apply to some clusters, or perhaps there are measurement errors or observations that have been miscoded.

Regression models often include cluster fixed effects. When one of the regressors is a fixed-effect dummy for cluster g, the matrices $\boldsymbol{X}_g^\top \boldsymbol{X}_g$ and $\boldsymbol{X}^\top \boldsymbol{X} - \boldsymbol{X}_g^\top \boldsymbol{X}_g$ are singular. This will cause the calculation in (9) to fail unless a generalized inverse routine, such as the invsym routine in Mata, is used. Although summclust uses this routine, it also provides options to avoid the problem, and save some computer time, by partialing out the fixed-effect dummies prior to computing the cluster-level matrices and vectors in (8); see Section 3.

Partialing out cluster fixed effects means replacing $\boldsymbol{X}$ and $\boldsymbol{y}$ by $\tilde{\boldsymbol{X}}$ and $\tilde{\boldsymbol{y}}$, the deviations from their cluster means. For example, the element of $\tilde{\boldsymbol{y}}$ corresponding to the j^{th} observation in the g^{th} cluster is $y_{g,j} - N_g^{-1} \sum_{i=1}^{N_g} y_{g,i}$. The g^{th} subvector of $\tilde{\boldsymbol{y}}$ is $\tilde{\boldsymbol{y}}_g$, and the g^{th} submatrix of $\tilde{\boldsymbol{X}}$ is $\tilde{\boldsymbol{X}}_g$. Since there is just one fixed effect per cluster, $\tilde{\boldsymbol{y}}_g$ depends solely on $\boldsymbol{y}_g$, and $\tilde{\boldsymbol{X}}_g$ depends solely on $\boldsymbol{X}_g$. The calculations in (6) and (9) are now based on $\tilde{\boldsymbol{X}}^\top \tilde{\boldsymbol{X}}$, $\tilde{\boldsymbol{X}}^\top \tilde{\boldsymbol{y}}$, the $\tilde{\boldsymbol{X}}_g^\top \tilde{\boldsymbol{X}}_g$, and the $\tilde{\boldsymbol{X}}_g^\top \tilde{\boldsymbol{y}}_g$. Importantly, the sum of the L_g is now equal to the number of columns in $\tilde{\boldsymbol{X}}$ instead of the number of columns in $\boldsymbol{X}$.

2.2 Two Jackknife Variance Matrix Estimators

Although the CV_1 variance estimator defined in (5) is very widely used, it often does not have good finite-sample properties. Two alternative CRVEs, which are usually known as CV_2 and CV_3, were proposed in Bell and McCaffrey (2002). They are the cluster analogs of the heteroskedasticity-consistent estimators HC_2 and HC_3, which are appropriate when the u_i are independent. These names were coined in MacKinnon and White (1985), which proposed HC_3 as a jackknife variance estimator. In the remainder of this section, we focus on CV_3, because CV_2 is not a jackknife estimator and is not amenable to the computational methods that we propose; for more on it, see Imbens and Kolesár (2016), Pustejovsky and Tipton (2018), and Niccodemi, Alessie, Angelini, Mierau, and Wansbeek (2020). `Stata 18` added the ability to rapidly calculate CV_2 standard errors, using the option `vce(hc2 clustvar)`. Simulations in MacKinnon, Nielsen, and Webb (2023b) suggest that CV_2 is preferred to CV_1, but that CV_3 is almost always preferred to CV_2.

CV_3 can be written in several ways. One of them is

$$\mathrm{CV}_3\text{:} \qquad \frac{G-1}{G}(\boldsymbol{X}^\top\boldsymbol{X})^{-1}\Bigl(\sum_{g=1}^{G}\ddot{\boldsymbol{s}}_g\ddot{\boldsymbol{s}}_g^\top\Bigr)(\boldsymbol{X}^\top\boldsymbol{X})^{-1}, \tag{10}$$

where the modified score vectors $\ddot{\boldsymbol{s}}_g$ are defined as

$$\ddot{\boldsymbol{s}}_g = \boldsymbol{X}_g^\top\boldsymbol{M}_{gg}^{-1}\hat{\boldsymbol{u}}_g.$$

Here $\boldsymbol{M}_{gg} = \mathbf{I}_{N_g} - \boldsymbol{X}_g(\boldsymbol{X}^\top\boldsymbol{X})^{-1}\boldsymbol{X}_g^\top$ is the diagonal block corresponding to the g^{th} cluster of the projection matrix $\boldsymbol{M}_{\boldsymbol{X}}$, which satisfies $\hat{\boldsymbol{u}} = \boldsymbol{M}_{\boldsymbol{X}}\boldsymbol{u}$. Although computing CV_3 using (10) works well when all the N_g are very small, it becomes expensive, or perhaps computationally infeasible, when one or more of the N_g is large. The problem is that an $N_g \times N_g$ matrix needs to be stored and inverted for every cluster. Niccodemi, Alessie, Angelini, Mierau, and Wansbeek (2020) proposes a method that is much faster for large clusters, versions of which apply to both CV_2 and CV_3. However, recognizing that CV_3 is a jackknife estimator makes a method that is even simpler and usually faster available.

There are actually two cluster jackknife estimators of $\mathrm{Var}(\hat{\boldsymbol{\beta}})$. The simplest is probably

$$\mathrm{CV}_{3\mathrm{J}}\text{:} \qquad \frac{G-1}{G}\sum_{g=1}^{G}(\hat{\boldsymbol{\beta}}^{(g)} - \bar{\boldsymbol{\beta}})(\hat{\boldsymbol{\beta}}^{(g)} - \bar{\boldsymbol{\beta}})^\top, \tag{11}$$

where $\bar{\boldsymbol{\beta}}$ is the sample mean of the $\hat{\boldsymbol{\beta}}^{(g)}$, which were defined in (9). The expression in (11) is the cluster analog of the usual jackknife variance matrix estimator given in MacKinnon and White (1985, eqn. (11)). Each of the $\hat{\boldsymbol{\beta}}^{(g)}$ is obtained by deleting a cluster instead of an observation, and the summation is over clusters instead of observations. If $\bar{\boldsymbol{\beta}}$ in (11) is replaced by $\hat{\boldsymbol{\beta}}$, we obtain instead

$$\mathrm{CV}_3\text{:}\qquad \frac{G-1}{G}\sum_{g=1}^{G}(\hat{\boldsymbol{\beta}}^{(g)}-\hat{\boldsymbol{\beta}})(\hat{\boldsymbol{\beta}}^{(g)}-\hat{\boldsymbol{\beta}})^\top. \tag{12}$$

This version of CV_3 is numerically identical to the one in (10) (MacKinnon, Nielsen, and Webb 2023b, Section 3). Unless all the clusters are very small, computing CV_3 using (12) is much faster than using (10); timings are reported in MacKinnon, Nielsen, and Webb (2023b).

Many discussions of jackknife variance estimation follow Efron (1979) and use $\bar{\boldsymbol{\beta}}$ as in (11), but others, including Bell and McCaffrey (2002), use $\hat{\boldsymbol{\beta}}$ as in (12). Although these two jackknife variance estimators are asymptotically the same, they are rarely equal, since CV_3 minus $\mathrm{CV}_{3\mathrm{J}}$ is a positive semi-definite matrix. In practice, however, they tend to be very similar (MacKinnon, Nielsen, and Webb 2023b), and there seems to be no good reason to expect either CV_3 or $\mathrm{CV}_{3\mathrm{J}}$ to perform better in general. Interestingly, the original HC_3 estimator proposed in MacKinnon and White (1985) is actually the analog of $\mathrm{CV}_{3\mathrm{J}}$. The modern version of HC_3, which is the analog of CV_3, seems to be due to Davidson and MacKinnon (1993, Chapter 16). This version of HC_3 is normally computed by dividing each residual by the corresponding diagonal element of $\boldsymbol{M_X}$, and the factor of $(N-1)/N$ is usually (but incorrectly) omitted.

The factor of $(G-1)/G$ in both (11) and (12) is designed to compensate for the tendency of the $\hat{\boldsymbol{\beta}}^{(g)}$ to be too spread out. This factor is the analog of the usual factor of $(N-1)/N$ for a jackknife variance matrix at the individual level. It implicitly assumes that all clusters are the same size and perfectly balanced, with disturbances that are independent and homoskedastic. In this special case, the estimators CV_3 and $\mathrm{CV}_{3\mathrm{J}}$ would be identical and unbiased (Bell and McCaffrey 2002). These estimators are already available in `Stata`. When used with the `cluster` option, the `vce(jackknife)` option computes $\mathrm{CV}_{3\mathrm{J}}$ standard errors, and the `vce(jackknife,mse)` option computes CV_3 standard errors. Because it is specialized for linear regression models, the implementation in `summclust` is quite a bit faster.

Both jackknife estimators may readily be used to compute cluster-robust t-statistics. Because there are G terms in the summation, it seems natural to compare these with the $t(G-1)$ distribution, as usual. These procedures should almost always be more conservative than t-tests based on the widely-used CV_1 estimator. In an important recent paper, Hansen (2022) shows that CV_3 has much better worst-case theoretical properties than CV_1. This strongly suggests that t-statistics based on CV_3 are likely to yield lower rejection frequencies than ones based on CV_1. The simulation results in Section 7 and in MacKinnon, Nielsen, and Webb (2023b) are consistent with this conjecture.

When a model includes fixed effects, some care needs to be taken when computing CV_3 and $\mathrm{CV}_{3\mathrm{J}}$. As noted in Section 2.1, it is computationally attractive to partial out fixed effects prior to calculating $\hat{\boldsymbol{\beta}}$. However, if we were to partial out any arbitrary regressors prior to computing the delete-one-cluster estimates in (9), then the computed $\hat{\boldsymbol{\beta}}^{(g)}$ would depend on the values of the partialed-out regressors for the full sample, including those in the g^{th} cluster. Consequently, the values of CV_3 and $\mathrm{CV}_{3\mathrm{J}}$ will be incorrect if we partial out any regressor that affects more than one cluster (such as industry-level fixed effects with firm-level clustering). The regressors that are partialed out must be cluster fixed effects or fixed effects at a finer level (such as firm-level fixed effects with industry-level clustering), because each of them affects only one cluster. See the discussion of the `absorb` and `fevar` options in Section 3.

It is possible that the vector $\boldsymbol{\beta}$ is identified for the full sample but not when one cluster is deleted. For example, consider the coefficient on a dummy variable that takes on non-zero values only for observations in the g^{th} cluster. This coefficient cannot be identified when cluster g is omitted. In such a case, the matrix $\boldsymbol{X}^\top\boldsymbol{X} - \boldsymbol{X}_g^\top\boldsymbol{X}_g$ in (9) is singular, and CV_3 and $\mathrm{CV}_{3\mathrm{J}}$ cannot be computed using an ordinary matrix inverse. However, because `summclust` uses the `invsym` function in `Stata`, which implements a generalized inverse, the offending element of $\hat{\boldsymbol{\beta}}^{(g)}$ is simply replaced by 0. The package therefore checks whether any of the $\hat{\beta}^{(g)}$ coefficients of interest are equal to 0 and issues a warning when they are; see Section 3.

There may be more than one set of fixed effect that are invariant at the cluster level. For example, imagine an analysis of students' test scores where the researcher wants to control for both school and neighborhood fixed effects and cluster the standard errors at the state level. In this case, neither of `Stata's` built-in `regress` and `areg` commands can produce an estimate of CV_3, because the fixed effects for schools and neighborhoods in state g cannot be identified when state g is omitted. However, `summclust` can produce such an estimate.

2.3 What Should Be Reported

We believe that investigators should routinely compute the L_g. They should also compute the L_{gj} for any coefficient(s) of particular interest. In some cases, the L_g and the L_{gj} will be roughly proportional to the N_g (the cluster sizes). That in itself would be informative. It may be even more interesting, however, to find that the relative size of L_g and/or L_{gj} for some cluster(s) g is much larger, or much smaller, than the relative size of N_g.

When the number of clusters is small, it is easy enough to look at all the N_g, $\hat{\beta}_j^{(g)}$, L_g, and L_{gj} to see whether any clusters are unusually large, unusually influential, or have unusually high leverage or partial leverage. Once G exceeds 10 or 12, however, it may be more informative to report summary statistics or to plot these quantities. The `summclust` package always reports the minimum, first quartile, median, mean, third quartile, and maximum of the N_g and the L_g. It also reports these quantities for the L_{gj} and the $\hat{\beta}_j^{(g)}$ for the specified regressor j, and by default it provides a figure containing four scatterplots of the L_g and the L_{gj} against the N_g and the $\hat{\beta}^{(j)}$; see Sections 3 and 4.

Another possibility is to report a few summary statistics, as `summclust` also does. Consider a generic (positive) quantity a_g, which might denote any of N_g, L_g, or L_{gj} for $g = 1, \ldots, G$. It seems plausible that inference may be unreliable when any of the a_g vary substantially across clusters, and we provide some evidence to support this conjecture in Section 7.

There are many measures of how much the distribution of the a_g differs from what it would be in the perfectly balanced case. One of these is the scaled variance

$$V_s(a_\bullet) = \frac{1}{(G-1)\bar{a}^2} \sum_{g=1}^{G} (a_g - \bar{a})^2, \tag{13}$$

where the argument $a_\bullet$ is to be interpreted as the entire set of a_g for $g = 1, \ldots, G$, and $\bar{a}$ denotes the arithmetic mean, which is N/G for the N_g, k/G for the L_g, and $1/G$ for the L_{gj}. These are all positive numbers, so it is reasonable to scale by their squares. Larger values of V_s imply that the a_g are more variable across clusters, relative to their mean. We could report either V_s or its square root, which is often called the coefficient of variation. In the perfectly balanced case, $V_s = 0$. By default, `summclust` reports the coefficient of variation for the cluster sizes, the leverages, the partial leverages, and the $\hat{\beta}_j^{(g)}$.

Another possibility, which is only valid for positive quantities, is to report one or more alternative sample means. The more these differ from the arithmetic mean, the more heterogeneous must be the clusters. Three common alternatives to the arithmetic mean are the harmonic, geometric, and quadratic means:

$$\bar{a}_{\text{harm}} = \left(\frac{1}{G}\sum_{g=1}^{G} 1/a_g\right)^{-1}, \quad \bar{a}_{\text{geo}} = \left(\prod_{g=1}^{G} a_g\right)^{1/G}, \quad \text{and} \quad \bar{a}_{\text{quad}} = \left(\frac{1}{G}\sum_{g=1}^{G} a_g^2\right)^{1/2}.$$

Unless all the a_g are the same, the harmonic and geometric means will be less than the arithmetic mean $\bar{a}$, and the quadratic mean (which has the same form as the root mean squared error of an estimator) will be greater than $\bar{a}$. `summclust` optionally reports all three of these alternative means, along with the ratio of each of them to $\bar{a}$. The three ratios provide scale-free measures of cluster heterogeneity; the closer they are to one, the more homogeneous are the clusters.

Another way to quantify the heterogeneity of the cluster sizes and the regressors is to calculate G^*, the "effective number of clusters," as proposed in Carter, Schnepel, and Steigerwald (2017). The value of G^* depends on the coefficient j for which it is being computed and on a parameter ρ to be discussed below, so we denote it $G_j^*(\rho)$. It is defined as

$$G_j^*(\rho) = \frac{G}{1+\Gamma_j(\rho)}, \quad \Gamma_j(\rho) = \frac{1}{G}\sum_{g=1}^{G}\left(\frac{\gamma_{gj}(\rho)-\bar{\gamma}_j(\rho)}{\bar{\gamma}_j(\rho)}\right)^2, \quad \bar{\gamma}_j(\rho) = \frac{1}{G}\sum_{g=1}^{G}\gamma_{gj}(\rho), \tag{14}$$

where $0 \leq \rho \leq 1$, and the $\gamma_{gj}(\rho)$ are given by

$$\gamma_{gj}(\rho) = \boldsymbol{e}_j^\top(\boldsymbol{X}^\top\boldsymbol{X})^{-1}\boldsymbol{X}_g^\top\boldsymbol{\Omega}_g(\rho)\boldsymbol{X}_g(\boldsymbol{X}^\top\boldsymbol{X})^{-1}\boldsymbol{e}_j, \quad g = 1,\ldots,G. \tag{15}$$

Here $\boldsymbol{e}_j$ is a k-vector with 1 in the j^{th} position and 0 everywhere else, so that $\boldsymbol{e}_j^\top(\boldsymbol{X}^\top\boldsymbol{X})^{-1}$ is the j^{th} row of $(\boldsymbol{X}^\top\boldsymbol{X})^{-1}$, and $\boldsymbol{\Omega}_g(\rho)$ is an $N_g \times N_g$ matrix with 1 on the principal diagonal and ρ everywhere else. It is easy to see that

$$\boldsymbol{\Omega}_g(\rho) = \rho\boldsymbol{\iota}\boldsymbol{\iota}^\top + (1-\rho)\mathbf{I}, \tag{16}$$

where $\boldsymbol{\iota}$ is an N_g-vector of 1s, and $\mathbf{I}$ is an $N_g \times N_g$ identity matrix. Notice that $\Gamma_j(\rho)$ is just the scaled variance of the $\gamma_{gj}(\rho)$; compare (13).

The parameter ρ may be interpreted as the intra-cluster correlation coefficient for a model with cluster-level random effects. Since ρ is unknown, Carter, Schnepel, and Steigerwald (2017) suggests calculating $G_j^*(1)$ as a sort of worst case. However, when there are cluster-level fixed effects, or fixed effects at a finer level nested within clusters, they will absorb all of the intra-cluster correlation. Thus it does not make sense to specify $\rho > 0$ in either of these cases. It does seem natural to use $G_j^*(0)$, however, because the amount of intra-cluster correlation that remains in models with cluster fixed effects is often quite small.

From (15) and (16), we see that

$$\boldsymbol{X}_g^\top\boldsymbol{\Omega}_g(\rho)\boldsymbol{X}_g = \rho(\boldsymbol{\iota}^\top\boldsymbol{X}_g)^\top(\boldsymbol{\iota}^\top\boldsymbol{X}_g) + (1-\rho)\boldsymbol{X}_g^\top\boldsymbol{X}_g. \tag{17}$$

This result makes it inexpensive to compute the $\gamma_{gj}(\rho)$ for any value of ρ by first computing them for $\rho = 0$ and $\rho = 1$. The needed equations are

$$\begin{aligned}\gamma_{gj}(0) &= \boldsymbol{w}_j^\top\boldsymbol{X}_g^\top\boldsymbol{X}_g\boldsymbol{w}_j,\\ \gamma_{gj}(1) &= (\boldsymbol{\iota}^\top\boldsymbol{X}_g\boldsymbol{w}_j)^\top(\boldsymbol{\iota}^\top\boldsymbol{X}_g\boldsymbol{w}_j), \text{ and}\\ \gamma_{gj}(\rho) &= \rho\gamma_{gj}(1) + (1-\rho)\gamma_{gj}(0),\end{aligned} \tag{18}$$

where $\boldsymbol{w}_j$ is the j^{th} column of $(\boldsymbol{X}^\top\boldsymbol{X})^{-1}$. After obtaining the $\gamma_{gj}(\rho)$ from (18), it is trivial to compute $G_j^*(\rho)$ using (14). Evidently, $G_j^*(\rho)$ is always less than G. When it is much smaller than G, it can provide a useful warning.

Suppose that we have partialed out cluster fixed effects prior to computing $G_j^*(\rho)$. Then the first term on the right-hand side of (17) should in theory be a zero matrix, because every column of $\boldsymbol{X}_g$ should add to zero. In practice, however, the limitations of floating-point arithmetic mean that this matrix will actually contain extremely small positive numbers. This will cause the computation of $G_j^*(\rho)$ to be numerically unstable. When the fixed effects are not partialed out, similar but more complicated numerical issues arise.

The `Stata` package `clusteff` discussed in Lee and Steigerwald (2018) is designed to calculate $G_j^*(\rho)$, with $\rho = 0.9999$ rather than $\rho = 1$ by default to avoid numerical instabilities. However, the only version of this package that we have used does so in a computationally inefficient way that does not use (18). When any of the N_g is large, it can take a very long time, or even fail because `Stata` runs out of memory. For example, it failed with some of the samples in MacKinnon, Nielsen, and Webb (2023a).

Like $V_s(a_\bullet)$ and the alternative sample means for measures of leverage and partial leverage discussed above, $G_j^*(\rho)$ is sensitive not only to variation in cluster sizes but also to other features of the $\boldsymbol{X}_g$ matrices. But it is not sensitive to heteroskedasticity or to any other features of the disturbances. `summclust` computes $G_j^*(0)$, $G_j^*(1)$, and (optionally) $G_j^*(\rho)$ for a specified covariate. However, when there are cluster fixed effects, or fixed effects nested within clusters, it only computes $G_j^*(0)$. For example, it will not compute $G_j^*(\rho)$ for $\rho \neq 0$ whenever there are state-level fixed effects and clustering at the region level.

The quantity $G_j^*(0)$ is very closely related to $V_s(L_{\bullet j})$, where $L_{\bullet j}$ denotes the entire set of L_{gj}, for $g = 1, \ldots, G$. It is not hard to see that the $\gamma_g(0)$ defined in (15) and (18) are equal to the L_{gj} defined in (7) divided by $\acute{\boldsymbol{x}}_j^\top \acute{\boldsymbol{x}}_j$. Since this makes the $\gamma_g(0)$ proportional to the L_{gj}, $V_s(L_{\bullet j})$ is numerically identical to $\Gamma(0)$; compare (13) and the middle equation in (14). Thus we see from the first equation in (14) that $G_j^*(0)$ is simply a monotonically decreasing function of the scaled variance of our measures of partial leverage at the cluster level. When $V_s(L_{\bullet j})$ is large, $G_j^*(0)$ is necessarily much smaller than G.

3 The `summclust` Package

The `summclust` package may be obtained from `SSC` or https://github.com/mattdwebb/summclust. It implements the `summclust` command, which calculates a large number of statistics to help assess cluster heterogeneity and also provides CV_3 and $\mathrm{CV}_{3\mathrm{J}}$ standard errors. The package does not rely on any other `Stata` packages, but it does require a version of `Stata` that provides `Mata`'s `panelsum()` function (Version 13 or later).

We first present an overview of the `summclust` command, followed by a simple illustration using the `webuse` dataset `nlswork`.

3.1 Syntax and options

`Syntax`

```
summclust varlist, cluster(varname) [options]
```

varlist: the dependent variable, the independent variable of interest, and other (binary or continuous) independent variables. At least one additional regressor must be specified. Time-series operators and factor variables are not permitted.
cluster: the clustering variable, for which the number of unique values equals G.

options	**Description**
`fevar(varlist)`	creates fixed effects for each of the specified variables, using `i.varname`.
`absorb(varname)`	partials out the variable `varname` before computing other statistics. This option should only be used for variables that are nested within the specified clusters. It can often be computationally faster than using `fevar` and should be used when there are cluster-level fixed effects in order to avoid singular omit-one-cluster samples caused by those fixed effects. In cases with an extremely large number of fixed effects, `summclust` may run into memory issues. If so, one can use the `Stata` prefix `jackknife` with the user-contributed command `reghdfe`.
`jackknife`	calculates the jackknife variance estimator $\text{CV}_{3\text{J}}$ in addition to CV_3.
`addmeans`	displays the alternative sample means of the N_g, L_g, L_{gj}, and $\hat{\beta}_j^{(g)}$, as described in Section 2.3. For the N_g, L_g, and L_{gj}, it reports the harmonic, geometric, and quadratic means, as well as the ratio of each of them to the arithmetic mean. For the $\hat{\beta}_j^{(g)}$, which can be negative, only the quadratic mean and its ratio are reported, because the harmonic and geometric means are not defined for negative numbers.
`gstar`	calculates the effective number of clusters G*(0) and, when there are no cluster (or subcluster) fixed effects, G*(1) as well.
`rho(scalar)`	calculates the effective number of clusters, G*(rho), in addition to G*(0) and G*(1). This option can be used with or without the `gstar` option. The value of rho must be between 0 and 1; the program ends with an error message when an invalid value for rho is entered. If it is not valid to display G*(rho), due to variables that are invariant within clusters, it reports that G*(rho) cannot be computed and displays only G*(0). There is no reason to use the `gstar` option when this option is used.

options	**Description**
`table`	displays the cluster-by-cluster values of cluster size, leverage, partial leverage, and the delete-one-cluster coefficient estimate. If $G > 52$, then the unformatted matrix is displayed instead of a table.
`sample`	allows for sample restrictions. The argument(s) for this option are whatever would follow the "if" in a standard `regress` command. For instance, in order to restrict the analysis to individuals 25 years of age or older based on a variable "age", `sample(age>=25)` should be added to the list of options.
`nograph`	suppresses creation of the figure, which is otherwise shown by default.
`regtable`	displays a full table of regression output, similar to Stata's regress table, but with jackknife standard errors. It reports CV_3 standard errors by default, but it instead reports $\mathrm{CV}_{3\mathrm{J}}$ standard errors when the `jackknife` option is also specified. If $k > 52$, then the unformatted matrix is displayed instead of a table.

Description

`summclust` is a stand-alone command for summarizing cluster variability in several ways. It always calculates measures of cluster-level influence and leverage, and it optionally calculates the effective number of clusters. It also always reports CV_1 and CV_3 standard errors for a single coefficient, and it optionally reports a $\mathrm{CV}_{3\mathrm{J}}$ standard error as well. If requested, it can calculate additional measures of cluster-level heterogeneity. Unless it is told not to, it produces a figure which can help identify the source of cluster level heterogeneity. Finally, it can optionally produce a full table of regression results with CV_3 standard errors.

By default, **`summclust`** calculates the CV_3 standard error based on (10). With well-behaved samples, this should match the standard error calculated using either `Stata`'s native **`jackknife: reg y x, cluster(group)`** or **`reg y x, cluster(group) vce(jackknife)`** commands. However, many samples are not well-behaved, in the sense that the regressor matrices for some of the omit-one-cluster subsamples may not have full rank. We will refer to such subsamples, rather informally, as "singular subsamples."

Whenever there are singular subsamples, **`summclust`** calculates two standard errors. One of these drops the singular subsamples, as the native Stata commands do. The other uses a generalized inverse. **`summclust`** provides guidance as to which standard error is likely to be more reliable. When **`regtable`** is specified, and singular subsamples are present, two versions of the regression table are displayed. Similarly, if **`jackknife`** is specified and there are singular subsamples, four different standard errors are shown, either CV_3 or $\mathrm{CV}_{3\mathrm{J}}$, combined with either the generalized inverse or one that drops the singular subsamples.

nograph suppresses creation of the figure, which is otherwise shown by default. The figure shows four scatter plots: leverage against observations per cluster, partial leverage against observations per cluster, leverage against omit-one-cluster coefficients, and partial leverage against omit-one-cluster coefficients. This figure can be quite informative, but it is computationally costly to produce. We recommend invoking this option after the figure has been inspected.

When **jackknife** is specified, **regtable** uses the $\text{CV}_{3\text{J}}$ estimates to produce the regression table. Otherwise, CV_3 estimates are used.

3.2 Illustration with nlswork

To illustrate `summclust`'s functionality and syntax, we consider a simple example using the online dataset `nlswork`, which contains a sample of women who were 14–26 years of age in 1968 from the National Longitudinal Survey of Young Working Women. For the purposes of this exercise, we restrict the sample to individuals who are 20 to 40 years old.

We estimate a simple Mincer regression using the `nlswork` dataset to see whether there is a marriage premium for wages. The variable `msp` is equal to 1 if the person is married and cohabits with their spouse, and equal to 0 otherwise. For the purposes of this example, we cluster by industry. The following code opens the dataset and estimates the regression using `Stata`'s `regress` command:

```
webuse nlswork, clear
keep if inrange(age,20,40)
reg ln_wage i.grade i.age i.birth_yr union race msp, cluster(ind)
```

The `Stata` output from the command above provides CV_1 standard errors. Alternatively, we can estimate CV_3 and $\text{CV}_{3\text{J}}$ standard errors using this code:

```
reg ln_wage i.grade i.age i.birth_yr union race msp, cluster(ind) vce(jackknife, mse)
reg ln_wage i.grade i.age i.birth_yr union race msp, cluster(ind) vce(jackknife)
```

When either of these commands is run, `Stata` displays the warning "Note: One or more parameters could not be estimated in 2 jackknife replicates; standard-error estimates include only complete replications."

The coefficient on `msp` and two or three standard errors can also be obtained using `summclust`. The basic command is:

```
summclust ln_wage msp union race, fevar(grade age birth_yr) cluster(ind)
```

This code results in the default output from summclust, which is mostly contained in two tables. The first one includes the coefficient on the second variable in the varlist (in this case msp), the CV_1 and CV_3 standard errors for this coefficient, and the associated t-statistics, P values, and confidence intervals. In this case, summclust also displays a warning about singular subsamples and thus produces two “Regression Output” tables. The standard errors in the table which drops singular subsamples match those produced natively in Stata.

```
Cluster summary statistics for msp when clustered by ind_code.
There are 17395 observations within 12 ind_code clusters.

Regression Output
s.e. |      Coeff   Sd. Err.   t-stat  P value    CI-lower    CI-upper
------+----------------------------------------------------------------
 CV1 |  -0.026940   0.008248  -3.2663   0.0075   -0.045093   -0.008787
 CV3 |  -0.026940   0.011150  -2.4161   0.0342   -0.051481   -0.002399
-----------------------------------------------------------------------

Regression Output -- Dropping Singular Omit-One-Cluster Subsamples
s.e. |      Coeff   Sd. Err.   t-stat  P value    CI-lower    CI-upper
------+----------------------------------------------------------------
CV3  |  -0.026940   0.006701  -4.0200   0.0030   -0.042099   -0.011780
-----------------------------------------------------------------------
```

In the first table for this example, the CV_1 and CV_3 standard errors are noticeably different, with the latter being considerably larger. However, in the second table, where the two singular subsamples are dropped, the CV_3 standard error becomes much smaller.

The “Cluster Variability” table from summclust (below) provides insight into what is happening. It reports summary statistics for N_g, L_g, L_{gj}, and $\hat{\beta}_j^{(g)}$. Whenever singular subsamples are dropped, two sets of statistics are shown for $\hat{\beta}_j^{(g)}$. The first (second-last column) uses all the jackknife subsamples with a generalized inverse standard error. The second (final column) uses only the non-singular subsamples. We can see that the largest value of $\hat{\beta}_j^{(g)}$ is considerably smaller (and therefore more different from the other values) when none of the subsamples is dropped. This explains why the CV_3 standard error is larger in the first table above than in the second one.

```
Cluster Variability
Statistic |       Ng    Leverage   Partial L.  all bet~g   kept be~g
----------+---------------------------------------------------------
      min |    35.00    0.085945     0.000700  -0.032772   -0.032772
       q1 |   144.50    0.633594     0.004399  -0.027655   -0.027917
   median |   905.00    2.794231     0.038554  -0.026891   -0.027082
     mean |  1449.58    4.583333     0.083333  -0.026398   -0.027571
       q3 |  2112.50    6.190322     0.105043  -0.025268   -0.026587
      max |  5736.00   17.008305     0.353148  -0.019198   -0.024202
   -----------+-----------------------------------------------------
  coefvar |     1.19    1.166238     1.320154   0.131277    0.074100
```

It is evident from this table that the clusters are extremely heterogeneous. The largest cluster contains almost one-third of the sample and is 167 times the size of the smallest. There are also extreme differences in both leverage and partial leverage across clusters. The ratio of the largest to the smallest value is 198 for leverage and 504.5 for partial leverage. The sum of the leverages is $12 \times 4.583333 = 55$, which is the number of estimated coefficients. Although both sets of $\hat{\beta}_j^{(g)}$ vary quite a bit, dropping one cluster never changes the sign of the coefficient.

The option `fevar` is used when there are factor variables, which would be specified as `i.varname` in conventional `Stata` syntax. In the above example, the arguments to `fevar` are `grade`, `age`, and `birth_yr`. For each argument, a set of temporary dummy variables is created. These dummy variables are included in the regression, and there is no constant term if they are present.

The sample code above does not illustrate several additional options. The most important of these is the `absorb` option, which operates like `fevar`. It treats its argument, a single variable, as an additional factor variable to include in the set of regressors. `absorb(varname)` can be used when including `i.varname` in a regression would result in many fixed effects. Speed can often be increased, perhaps substantially, by partialing out the absorbed fixed effects from the dependent and all the independent variables. It is advisable to use `absorb` rather than `fevar` whenever their argument corresponds to a set of cluster fixed effects, since the elements of $\hat{\boldsymbol{\beta}}^{(g)}$ that correspond to the fixed effects cannot be identified in that case; see Section 2.1.

The `absorb` option should be used with care. Partialing out fixed effects is valid for the measures of leverage and influence and for the jackknife variance matrices only when the absorbed variable yields fixed effects that can be partialed out on a cluster-by-cluster basis. That is, `absorb` should only be used for straight cluster fixed effects or for fixed effects at a finer level, such as state × year fixed effects for a panel with clustering at the state level. It is not valid to partial out fixed effects that are not limited to a single cluster. In that case, the $\hat{\boldsymbol{\beta}}^{(g)}$ and quantities based on them would be different for the original data and the data after partialing out, because the partialed-out observations for the g^{th} cluster would depend on other clusters as well. Accordingly, `summclust` checks to ensure that the clustering variable is invariant within each value of the absorbed variable. When it is not invariant, a warning is displayed, and the values of L_g, L_{gj}, $\hat{\beta}_j^{(g)}$, CV$_3$, and CV$_{3\text{J}}$ are not available.

To see the difference between `fevar` and `absorb`, we can estimate an expanded regression that includes industry fixed effects. Consider the following two commands:

```
summclust ln_wage msp union race, fevar(grade age birth_yr ind) cluster(ind)
summclust ln_wage msp union race, fevar(grade age birth_yr) absorb(ind) cluster(ind)
```

For the command which uses `fevar` for all the categorical variables, some of the output is

```
Regression Output
  s.e. |      Coeff   Sd. Err.   t-stat  P value    CI-lower    CI-upper
-------+--------------------------------------------------------------------
   CV1 |  -0.018955   0.007014  -2.7025   0.0206   -0.034392   -0.003517
   CV3 |  -0.018955   0.007586  -2.4987   0.0296   -0.035651   -0.002258
----------------------------------------------------------------------------
```

Because every one of the jackknife subsamples is singular, only the results based on the generalized inverse are reported. In contrast, when `absorb` is used for the industry fixed effects, the corresponding output is instead

```
Regression Output
  s.e. |      Coeff   Sd. Err.   t-stat  P value    CI-lower    CI-upper
-------+--------------------------------------------------------------------
   CV1 |  -0.018955   0.007014  -2.7025   0.0206   -0.034392   -0.003517
   CV3 |  -0.018955   0.007586  -2.4987   0.0296   -0.035651   -0.002258
----------------------------------------------------------------------------


Regression Output -- Dropping Singular Omit-One-Cluster Subsamples
----------------------------------------------------------------------------
  s.e. |      Coeff   Sd. Err.   t-stat  P value    CI-lower    CI-upper
   CV3 |  -0.018955   0.004173  -4.5418   0.0014   -0.028396   -0.009514
----------------------------------------------------------------------------
```

These two tables highlight a key reason for using `absorb`. Because only two of the jackknife subsamples are singular, `summclust` is able to report both standard errors. Observe that, when all 12 jackknife samples are used, the standard errors are the same regardless of whether industry fixed effects are specified using `fever` or `absorb`.

Whether we use `fevar` or `absorb` leads to somewhat different output for the measures of cluster variability.

```
Cluster Variability [using fevar]
 Statistic |       Ng      Leverage     Partial L.  beta no g
-----------+-------------------------------------------------
       min |    35.00      1.079703       0.000276  -0.021394
        q1 |   144.50      1.617131       0.003970  -0.020316
    median |   905.00      3.752372       0.033630  -0.019050
      mean |  1449.58      5.500000       0.083333  -0.018880
        q3 |  2112.50      7.066207       0.092329  -0.018852
       max |  5736.00     17.728424       0.382133  -0.012367
-----------+-------------------------------------------------
   coefvar |     1.19      0.957329       1.422090   0.126464

Cluster Variability [using absorb]
 Statistic |       Ng    Leverage   Partial L.  all bet~g   kept be~g
-----------+---------------------------------------------------------
       min |    35.00    0.079703     0.000700  -0.021394   -0.021394
        q1 |   144.50    0.617131     0.004399  -0.020316   -0.020601
    median |   905.00    2.752372     0.038554  -0.019050   -0.019281
      mean |  1449.58    4.500000     0.083333  -0.018880   -0.019538
        q3 |  2112.50    6.066207     0.105044  -0.018852   -0.019028
       max |  5736.00   16.728424     0.353143  -0.012367   -0.016767
-----------+---------------------------------------------------------
   coefvar |     1.19    1.170068     1.320148   0.126464    0.061639
```

The $\hat{\beta}_j^{(g)}$ when all clusters are retained are identical for both options. But since there are two singular subclusters, there are two versions of the $\hat{\beta}_j^{(g)}$ for the `fevar` results.

The leverage estimates are also smaller when we use the `absorb` option. Recall that, for the original model with no industry fixed effects, the leverages summed to 55. In the first case just above, where the industry fixed effects are included as regressors in `fevar`, the regression has 66 coefficients, and the leverages therefore sum to $12 \times 5.5 = 66$. In the second case, where the industry fixed effects are partialed out using `absorb`, the regression has 54 coefficients, and the leverages therefore sum to $12 \times 4.5 = 54$. Thus for the first case, each of the leverages is larger than the corresponding one for the second case by precisely 1.

Examples

In the examples that follow, we include the `nograph` option to reduce computational time. This example illustrates the `jackknife` and `table` options:

```
summclust ln_wage msp union race, fevar(grade age birth_yr) cluster(ind) nog///
   jack table
```

```
Regression Output
  s.e. |      Coeff   Sd. Err.   t-stat  P value    CI-lower    CI-upper
-------+--------------------------------------------------------------------
   CV1 |  -0.026940   0.008248  -3.2663   0.0075   -0.045093   -0.008787
   CV3 |  -0.026940   0.011150  -2.4161   0.0342   -0.051481   -0.002399
  CV3J |  -0.026940   0.011004  -2.4482   0.0324   -0.051160   -0.002720
----------------------------------------------------------------------------
```

In addition to the two standard tables, it displays the following table:

```
Cluster by Cluster Statistics
  ind_code |     Ng      Leverage     Partial L.  beta no g
-----------+---------------------------------------------------
         1 |      119      0.581881       0.002825  -0.026959
         2 |       35      0.085945       0.000700  -0.027206
         3 |      170      0.685307       0.005341  -0.026823
         4 |     3451     12.753229       0.241651  -0.021861
         5 |      974      2.448713       0.114532  -0.024202
         6 |     2626      7.815303       0.095555  -0.027393
         7 |     1599      4.565341       0.048163  -0.026587
         8 |      513      2.494440       0.018808  -0.029519
         9 |      836      3.131195       0.028945  -0.032772
        10 |      114      0.336320       0.003457  -0.027917
        11 |     5736     17.008305       0.353148  -0.019198
        12 |     1222      3.094021       0.086874  -0.026333
---------------------------------------------------------------
```

This table makes it easy to see whether the high leverage clusters are also the largest clusters. That is clearly the case here. After running the program, this table is stored as the `Mata` matrix `scall`.

To obtain summary statistics on the four (or five) measures of cluster variability, we can use the `addmeans` option:

```
summclust ln_wage msp union race, fevar(grade age birth_yr) cluster(ind) nog add
```

This command produces the following table:

```
Alternative Sample Means and Ratios to Arithmetic Mean
                |          Ng     Leverage  Partial L.  all bet~g   kept be~g
----------------+----------------------------------------------------------
  Harmonic Mean |     206.576     0.608440    0.004988          .           .
 Harmonic Ratio |       0.143     0.132751    0.059853          .           .
 Geometric Mean |     623.091     2.042731    0.025557          .           .
Geometric Ratio |       0.430     0.445687    0.306684          .           .
 Quadratic Mean |    2193.268     6.870062    0.134308   0.026605    0.027654
Quadratic Ratio |       1.513     1.498923    1.611699  -1.007868   -1.003015
---------------------------------------------------------------------------
```

Once again, we see that there is extreme variability across the clusters. This is particularly noticeable for the ratio of the harmonic mean to the arithmetic mean, which is between 0.125 and 0.143 for the cluster size, leverage, and partial leverage measures. Recall that these ratios would be close to one if the clusters were relatively homogeneous. This table is stored in `Mata`'s memory as `bonus`.

To obtain estimates of the effective number of clusters, we can use either the `gstar` option or the `rho()` option. The former displays $G^*_j(0)$ and $G^*_j(1)$. The latter requires a specified value of ρ and displays $G^*_j(0)$ and $G^*_j(1)$ along with $G^*_j(\rho)$. When there are fixed effects at the cluster or subcluster level, only $G^*_j(0)$ is reported.

For the `nlswork` example, the first option may be called as:

```
summclust ln_wage msp union race, fevar(grade age birth_yr) cluster(ind) nog gstar
```

This yields:

```
Effective Number of Clusters
----------------------------
G*(0)  =  5.495
G*(1)  =  1.376
----------------------------
```

The second option, using $\rho = 0.5$ as an illustration, may be called as:

```
summclust ln_wage msp union race, fevar(grade age birth_yr) cluster(ind) nog rho(0.5)
```

This yields:

```
Effective Number of Clusters
----------------------------
G*(0)  =  5.495
G*(.5) =  1.433
G*(1)  =  1.376
----------------------------
```

In this example, it is clear that the effective number of clusters is substantially less than the actual number of clusters. This provides more evidence that inference using the CV_1 standard error together with the $t(G-1)$ distribution is likely to be unreliable. These three quantities can be accessed in `Mata`'s memory as `gstarzero`, `gstarrho`, and `gstarone`, respectively.

By using the `regtable` option, one can display a modified version of the regression table, which is similar to the default output from Stata's `regress` command. The command is:

```
summclust ln_wage msp union race, fevar(grade age birth_yr) cluster(ind) nog regtable
```

When there are singular subsamples, two versions of this table will be displayed. In this example, the table is quite long, so we do not reproduce it here.

3.3 List of Stored Results

All the results that are displayed as output can also be found in `Mata`'s memory. To access one of these after running `summclust`, simply add the following line:

```
mata: object_name
```

The `object_name` can take one of the following values:

`cvstuff`: This matrix stores the table with the title “Regression Output”. It is 2 × 6 when the `jackknife` option is not used (the default), and 3 × 6 when `jackknife` is used.

`scall`: This matrix stores the $G\times 4$ table created by the `table` option with the title “Cluster by Cluster Statistics”.

`bonus`: This 6 × 4 matrix contains the alternative sample means and their ratios to the arithmetic mean created by the `addmeans` option.

`gstarzero`: This scalar contains $G^*(0)$ created by the `gstar` or `rho` options.

`gstarone`: This scalar contains $G^*(1)$ created by the `gstar` or `rho` options.

`gstarrho`: This scalar contains $G^*(\rho)$ created by the `rho` option.

`regresstab`: This matrix contains the table shown when the regtable option is specified.

Scalars within matrices can be referenced on a cell-by-cell basis. For example, the CV_3 standard error is stored in the second row and second column of `cvstuff`, and to display it one can enter the following command:

```
mata: cvstuff[2,2]
```

Additionally, several results are available as scalars or matrices in return memory using `r()`. The available scalars are:

`beta`: The estimate $\hat{\beta}$ for the coefficient of interest.
`cv1se`: The CV_1 standard error for the coefficient of interest.
`cv1t`: The CV_1 t-statistic for the coefficient of interest.
`cv1p`: The P value for the null hypothesis that $\beta = 0$ for the coefficient of interest using the CV_1 standard error.
`cv1lci`: The lower bound of the 95% confidence interval for β using the CV_1 standard error.
`cv1uci`: The upper bound of the 95% confidence interval for β using the CV_1 standard error.
`gstarzero`: The effective number of clusters for the coefficient of interest using $\rho = 0$.
`gstarone`: The effective number of clusters for the coefficient of interest using $\rho = 1$.
`gstarrho`: The effective number of clusters for the coefficient of interest using the value of ρ specified in `rho(`ρ`)`.

The standard error, t-statistic, P value, and confidence interval bounds are also available for the CV_3 and $\mathrm{CV}_{3\mathrm{J}}$ standard errors. To access these, replace "1" in the above with either "3" or "3J"; for example, the P value using $\mathrm{CV}_{3\mathrm{J}}$ is available in `cv3Jp`. In the event of singular subsamples, there are two versions of the CV_3 or $\mathrm{CV}_{3\mathrm{J}}$ results. The ones where singular subsamples have been dropped have a suffix of 'drop'. For instance, `cv3sedrop` is used instead of `cv3se`.

The available matrices are:

`ng`: This $G \times 1$ matrix contains the number of observations, N_g, for each cluster.
`leverage`: This $G \times 1$ matrix contains the leverage, L_g, for each cluster.
`partlev`: This $G \times 1$ matrix contains the partial leverage, L_{gj}, for each cluster.
`betanog`: This $G \times 1$ matrix contains the $\hat{\beta}_j^{(g)}$ for each cluster.

4 Empirical Example

We consider an empirical example from Busso and Galiani (2019), which studies an experiment where retail firms were randomly assigned to enter one of 72 different geographic markets (in Spanish, mercados), within the Dominican Republic. After randomization, 21 markets had no entrants and so were in the control group, 18 had one entrant, another 18 had two, and the remaining 15 had three. The primary analysis only distinguishes between the 51 treated markets and the 21 control markets. The number of observations (stores) per market varies from 20 to 55.

This example is interesting because conventional wisdom (e.g., MacKinnon, Nielsen, and Webb 2023a) suggests that, with 72 clusters that do not vary much in size, and with neither few treated nor few control clusters, inference based on CV_1 standard errors and the $t(71)$ distribution should work well. However, our leverage measures suggest otherwise, and alternative inference methods yield noticeably different results.

The model we estimate is

$$Y_{sd} = \alpha + \gamma Z_d + \boldsymbol{X}_{sd}\boldsymbol{\beta} + \epsilon_{sd}. \tag{19}$$

Here s indexes stores, and d indexes markets. The treatment variable Z_d equals 1 if market d is treated (there was entry) and 0 if it was a control (there was no entry). The coefficient of interest is γ, which measures the causal effect of increased competition on an outcome Y. We focus on just one of several outcomes, namely, the log of demeaned prices after treatment. The results from this regression are found in Table 5, Panel B, column 4, row 1 of Busso and Galiani (2019). The table states that there are 72 clusters and 2,025 observations; however, the replication dataset that we use contains just 1,926 observations.

Regression (19) includes 17 control variables in the row vector $\boldsymbol{X}_{sd}$. These are the first lag of the outcome variable, the number of retailers in each district pre-treatment, a lagged quality index, eight province fixed effects, total district beneficiaries of a conditional cash transfer program, percent beneficiaries of that program, average income in the market, two market education measures, and a binary indicator for the urban status of the market. Thus the total number of regressors is 19.

The OLS estimate of γ, its CV_1 standard error, the P value for a test that $\gamma = 0$, and a .95 confidence interval are shown in the first row of Table 1. Allowing for different numbers of reported digits, these estimates accord with the ones in Busso and Galiani (2019). The estimate of -0.01469 has the expected sign (average prices declined). However, the P value is just slightly less than 0.05, and the confidence interval barely excludes 0.

We next use the `summclust` package to calculate the cluster-level characteristics of the model and dataset. Some key ones are reported in Table 2. It is evident that cluster sizes are well balanced, varying from 20 to 55, with the first and third quartiles equal to 24 and 27. However, both the leverages L_g and the partial leverages L_{g1} vary considerably. The former range from 0.1308 to 0.7378, and the latter from 0.0001 to 0.0642. The coefficients of

Table 1: Estimates of the Treatment Effect

Method	$\hat{\gamma}$	Standard Error	P value	Confidence Interval
CV_1	-0.01469	0.007243	0.0461	$[-0.02913, \ -0.00025]$
CV_2	-0.01469	0.008078	0.0730	$[-0.03080, \ 0.00142]$
CV_3	-0.01469	0.009090	0.1105	$[-0.03281, \ 0.00343]$
$\mathrm{CV}_{3\mathrm{J}}$	-0.01469	0.009087	0.1104	$[-0.03281, \ 0.00343]$
WCR-C bootstrap	-0.01469		0.0891	$[-0.03121, \ 0.00243]$
WCR-S bootstrap	-0.01469		0.0913	$[-0.03121, \ 0.00254]$

Notes: There are $N = 1{,}926$ observations and $G = 72$ clusters. The two WCR bootstraps use $B = 999{,}999$ and a seed of $56{,}829{,}046$. WCR-C is the classic WCR bootstrap of Cameron, Gelbach, and Miller (2008), and WCR-S is the "score" variant proposed in MacKinnon, Nielsen, and Webb (2023b). It involves transforming the restricted empirical scores in a way based on the jackknife, but it still uses CV_1. The bootstrap results were obtained using Version 4.2.0 of `boottest`.

Table 2: Leverage and Partial Leverage for $\hat{\gamma}$

Statistic	N_g	Leverage	Partial Leverage	$\hat{\gamma}^{(g)}$
Minimum	20	0.130842	0.000099	-0.017550
First quartile	24	0.204104	0.003166	-0.015089
Median	26	0.235813	0.009001	-0.014791
Mean	26.75	0.263889	0.013889	-0.014663
Third quartile	27	0.292042	0.020926	-0.014070
Maximum	55	0.737797	0.064242	-0.010723
Coef. of variation	0.21	0.388686	1.059813	0.074061

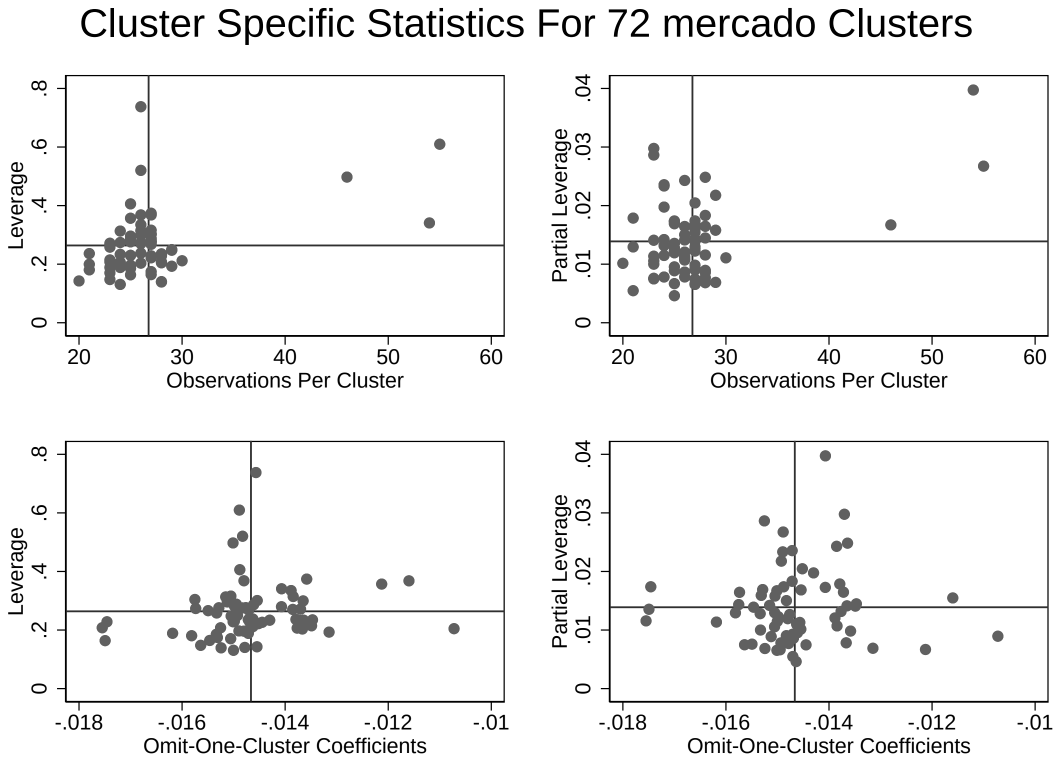
Notes: There are $N = 1{,}926$ observations and $G = 72$ clusters. The effective numbers of clusters are $G^*_\gamma(0) = 34.16$ and $G^*_\gamma(1) = 33.33$.

variation are 0.3887 and 1.0598, respectively. The latter is moderately large, although not enormous. The two values of G^* are slightly smaller than $G/2$, which also suggests that the sample is not well balanced.

Most of the $\hat{\gamma}^{(g)}$ do not vary much, and thus their coefficient of variation is small. However, the most extreme values are notable. The estimate of γ, which is -0.01469, could be as small as -0.01755 or as large as -0.01072 if just one out of 72 clusters were dropped.

These results suggest that CV_1, the default CRVE, may not be particularly reliable in this case. We therefore consider five alternative procedures. The second, third, and fourth rows of Table 1 report the CV_2, CV_3, and $\mathrm{CV}_{3\mathrm{J}}$ standard errors, along with the P values and confidence intervals associated with them. The CV_2 P value is noticeable larger than the CV_1 one and suggests that the estimate is not significant at the .05 level. The CV_3 and $\mathrm{CV}_{3\mathrm{J}}$ rows are almost identical. At 0.1105, the CV_3 P value does not even allow us to reject the null at the .10 level. The fifth and six rows of Table 1 report two WCR bootstrap P values and the

Figure 1: Example summclust Figure



Notes: A figure like this is always produced unless the `nograph` option is specified. It plots both leverage and partial leverage against cluster size and against the omit-one-cluster coefficients for, in this case, 72 clusters specified by a variable called “mercado.”

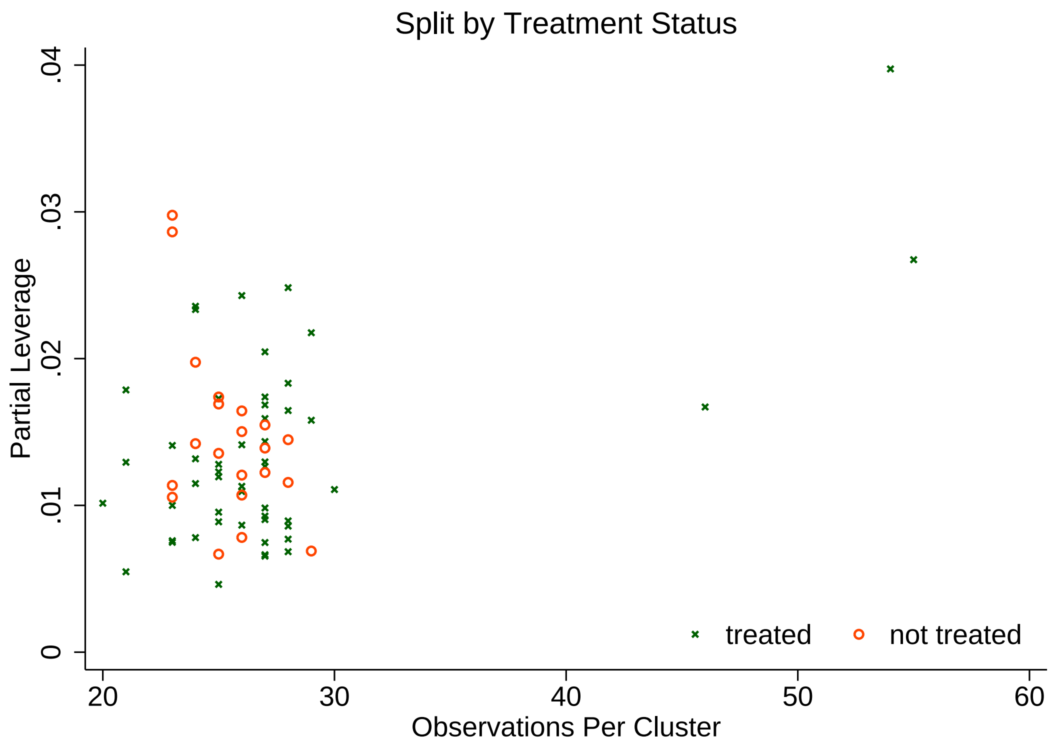
associated .95 confidence intervals. At 0.0891 and 0.0913, these are a bit smaller than the jackknife ones, but they clearly do not allow us to reject the null hypothesis at the .05 level.

In view of the reasonably large number of clusters and the fact that cluster sizes do not vary much, the large discrepancy between the results for CV_1 and the other procedures may seem surprising. However, it is not all that surprising when we note how much the leverages and, especially, the partial leverages vary.

By default, `summclust` produces a figure like Figure 1, with its title created by the program using the name of the clustering variable, in this case “mercado”. This figure plots both leverage and partial leverage against the number of observations per cluster and also against the omit-one-cluster coefficients. These four subfigures may help to reveal the source of cluster-level heterogeneity. For this example, neither the large leverages nor the large partial leverages come exclusively from clusters with large numbers of observations or extreme omit-one-cluster coefficients.

To explore what is driving the differences in partial leverage, we create an additional scatter plot. Figure 2 plots partial leverage against the number of observations per cluster,

Figure 2: Partial Leverage vs Cluster Size



Notes: The figure plots partial leverage against cluster size for 72 clusters. A green X marks a treated cluster, and an orange circle marks a control cluster.

with different colors and symbols depending on whether or not a given market (cluster) was treated. The figure has two interesting features. The first is that the three rather large clusters have fairly small partial leverage. The second is that the 12 clusters with the highest partial leverage are all control markets. The first result is quite surprising, since large clusters often tend to have high leverage. But Figure 2 makes it clear that there is, in general, no simple relationship between cluster sizes and partial leverage. The second result is not so surprising, because only 21 out of the 72 clusters are controls. Many of the control clusters presumably have high partial leverage because control clusters are relatively rare. See (32) in Example 4 in the next section for an explanation.

5 Simple Analytical Examples

In this section, we discuss a number of simple examples in which it is possible to calculate our measures of leverage and influence analytically. These examples are quite revealing.

Example 1 (Estimation of the mean)**.** Finding the sample mean is equivalent to performing a least-squares regression in which the only regressor is $x_i = 1$ for all $i = 1, \ldots, N$. In this case, it is easy to see that $\boldsymbol{X}_g^\top \boldsymbol{X}_g = N_g$ and $\boldsymbol{X}^\top \boldsymbol{X} = N$. Therefore,

$$L_g = \mathrm{Tr}(\boldsymbol{H}_g) = \frac{N_g}{N} = \frac{N_g}{\sum_{h=1}^G N_h}. \tag{20}$$

In this simple case, cluster leverage is exactly proportional to cluster size. In other cases, we can interpret leverage as a generalization of cluster size that takes into account other types of heterogeneity as well.

Evidently, $\hat\beta = \bar y = N^{-1}\sum_{g=1}^G N_g \bar y_g$, where $\bar y$ and $\bar y_g$ denote the sample average for the full sample and for cluster g, respectively. This expression can be rewritten as

$$\hat\beta = \sum_{g=1}^G \frac{N_g}{N}\bar y_g = \sum_{g=1}^G L_g \hat\beta_g, \tag{21}$$

so that $\hat\beta$ is seen to be a weighted average of the G estimates $\hat\beta_g = \bar y_g$, with the weight for each cluster equal to its leverage. Similarly, we find that

$$\hat\beta^{(g)} = \frac{N}{N - N_g}\sum_{h\neq g} L_h \hat\beta_h, \tag{22}$$

where the first factor simply makes up for the fact that we are summing over $G-1$ clusters instead of G as in (21). Subtracting (21) from (22), we conclude that

$$\hat\beta^{(g)} - \hat\beta = \frac{N_g}{N}\Big(\hat\beta^{(g)} - \hat\beta_g\Big) = L_g\Big(\hat\beta^{(g)} - \hat\beta_g\Big). \tag{23}$$

Therefore, cluster g will be influential whenever omitting it yields an estimate $\hat\beta^{(g)}$ that differs substantially from the estimate $\hat\beta_g$ for cluster g itself, especially when cluster g also has high leverage. □

Example 2 (Single regressor plus constant)**.** Consider a regression design with a single regressor, x_i, and a constant term. Then

$$\boldsymbol{X}_g^\top \boldsymbol{X}_g = \begin{bmatrix} N_g & \sum_{i=1}^{N_g} x_{g,i} \\ \sum_{i=1}^{N_g} x_{g,i} & \sum_{i=1}^{N_g} x_{g,i}^2 \end{bmatrix} \quad\text{and}\quad (\boldsymbol{X}^\top \boldsymbol{X})^{-1} = \frac{1}{N^2\hat\sigma_x^2}\begin{bmatrix} \sum_{i=1}^N x_i^2 & -\sum_{i=1}^N x_i \\ -\sum_{i=1}^N x_i & N \end{bmatrix},$$

where $\hat\sigma_x^2$ denotes the sample variance of the x_i. After some algebra, we find that

$$L_g = \frac{N_g}{N\hat\sigma_x^2}\Big(\hat\sigma_x^2 + \hat\sigma_{x,g}^2 + (\bar x_g - \bar x)^2\Big), \tag{24}$$

where $\bar x_g$ and $\hat\sigma_{x,g}^2$ denote the sample mean and sample variance of the x_i within cluster g. Expression (24) is a straightforward generalization of (20). The last two terms within the large parentheses are the sample variance of the $x_{g,i}$ within cluster g and the square of the difference between $\bar{x}_g$ and $\bar{x}$. The sum of these terms is the sample variance of the $x_{g,i}$ around $\bar{x}$ within cluster g. Thus cluster g will have high leverage when the variance of the $x_{g,i}$ around $\bar{x}$ within that cluster is large relative to the variance $\hat{\sigma}_x^2$ for the full sample. If everything except cluster sizes were perfectly balanced, L_g would evidently reduce to $2N_g/N$.

The partial leverage for x is just

$$L_{g2} = \frac{N_g\bigl(\hat{\sigma}_{x,g}^2 + (\bar{x}_g - \bar{x})^2\bigr)}{N\hat{\sigma}_x^2}, \tag{25}$$

the total variation around $\bar{x}$ within cluster g divided by the total variation within the sample. If everything except cluster sizes were perfectly balanced, it would reduce to N_g/N. □

Example 3 (Single regressor plus fixed effects)**.** Suppose there is a single regressor, x_i, and there are cluster-level fixed effects, which have been partialed out. In this case, we can write all quantities as deviations from their cluster averages, and there is no distinction between leverage and partial leverage. Then $\tilde{\boldsymbol{X}}_g^\top\tilde{\boldsymbol{X}}_g = \sum_{i=1}^{N_g}(x_{g,i} - \bar{x}_g)^2 = N_g\hat{\sigma}_{x,g}^2$. Similarly, $\tilde{\boldsymbol{X}}^\top\tilde{\boldsymbol{X}} = \sum_{g=1}^{G} N_g\hat{\sigma}_{x,g}^2$ is the average variance of the x_i across all clusters. We find that

$$L_g = \frac{N_g\hat{\sigma}_{x,g}^2}{\sum_{h=1}^{G} N_h\hat{\sigma}_{x,h}^2}, \tag{26}$$

which is again a straightforward generalization of (20). The leverage of cluster g is proportional to N_g times the variance of the $x_{g,i}$ around $\bar{x}_g$. Thus, for example, doubling the variance of the $x_{g,i}$ has the same effect on leverage as doubling N_g.

In this case, using (26), it is easy to see that

$$\hat{\beta} = \frac{\sum_{g=1}^{G} N_g\hat{\sigma}_{xy,g}}{\sum_{g=1}^{G} N_g\hat{\sigma}_{x,g}^2} = \sum_{g=1}^{G} L_g\frac{\hat{\sigma}_{xy,g}}{\hat{\sigma}_{x,g}^2} = \sum_{g=1}^{G} L_g\hat{\beta}_g, \tag{27}$$

where $\hat{\sigma}_{xy,g} = (1/N_g)\sum_{i=1}^{N_g}(x_{g,i} - \bar{x}_g)(y_{g,i} - \bar{y}_g)$ is the sample covariance of x_i and y_i within cluster g. The rightmost expressions in (21) and (27) are identical. In both cases, $\hat{\beta}$ is seen to be a weighted average of the G cluster estimates, with the weight for each cluster equal to its leverage.

When cluster g is omitted, we obtain

$$\hat{\beta}^{(g)} = \frac{\sum_{h\neq g} N_h\hat{\sigma}_{xy,h}}{\sum_{h\neq g} N_h\hat{\sigma}_{x,h}^2} = \frac{\sum_{h\neq g} L_h\hat{\beta}_h}{\sum_{h\neq g} L_h}, \tag{28}$$

which would specialize to (22) if (20) were true. As before, $\hat{\beta}^{(g)}$ is a weighted average of the $\hat{\beta}_h$, with weights proportional to the L_g, which in this case are also the partial leverages.

Subtracting (27) from (28), we find that

$$\hat\beta^{(g)} - \hat\beta = L_g\big(\hat\beta^{(g)} - \hat\beta_g\big), \tag{29}$$

which is formally identical to the rightmost expression in (23), although of course L_g is defined in (26) not (20). Cluster g will be influential whenever $\hat\beta^{(g)}$ differs substantially from the estimate $\hat\beta_g$ for cluster g itself, especially when cluster g also has high leverage. □

Example 4 (Treatment model with a constant term)**.** Now we specialize Example 2 to the case in which the single regressor is a treatment dummy denoted by d_i. Let $\bar d_g$ and $\bar d$ denote the proportion of treated observations in cluster g and in the sample, respectively. Then (24) becomes

$$L_g = \frac{N_g}{N}\left(\frac{\bar d_g}{\bar d} + \frac{1-\bar d_g}{1-\bar d}\right). \tag{30}$$

The first factor here is the relative size of the g^{th} cluster. The second factor depends on how much $\bar d_g$ differs from $\bar d$. When $\bar d_g = \bar d$, we see that $L_g = 2N_g/N$. Otherwise, the first term inside the parentheses causes leverage to be high whenever $\bar d_g$ is large relative to $\bar d$, and the second term causes leverage to be high whenever $\bar d_g$ is small relative to $\bar d$. As $\bar d$ increases for given $\bar d_g$, the first term becomes smaller relative to the second term. Thus the g^{th} cluster will tend to be influential either when it has a large proportion of treated observations and the overall proportion is small, or when it has a small proportion of treated observations and the overall proportion is large.

We can also obtain the partial leverage of the treatment dummy for this case. Expression (25) simply becomes

$$L_{g2} = \frac{N_g}{N}\left(\frac{\bar d_g}{\bar d} + \frac{\bar d-\bar d_g}{1-\bar d}\right). \tag{31}$$

Once again, the first factor is the relative size of the g^{th} cluster. The second factor reduces to 1 when $\bar d_g = \bar d$, so that $L_{g2} = N_g/N$ in that special case.

We can further specialize (30) and (31) to models in which the treatment is applied at the cluster level. Suppose that all observations in clusters $g = 1, \ldots, G_1$ are treated and no observations in the $G_0 = G - G_1$ control clusters from $G_1 + 1$ to G are treated. Then we find that $\bar d_g = 1$ for $g = 1, \ldots, G_1$, and $\bar d_g = 0$ for $g = G_1 + 1, \ldots, G$. Inserting these into (30) shows that

$$L_g = \begin{cases} \frac{N_g}{N}\frac{1}{\bar d} & \text{for } g = 1, \ldots, G_1, \\ \frac{N_g}{N}\frac{1}{1-\bar d} & \text{for } g = G_1+1, \ldots, G. \end{cases} \tag{32}$$

Inserting them into (31) shows that

$$L_{g2} = \begin{cases} \frac{N_g}{N}\frac{\bar{d}+1}{\bar{d}} & \text{for } g = 1, \ldots, G_1, \\ \frac{N_g}{N}\frac{\bar{d}}{1-\bar{d}} & \text{for } g = G_1+1, \ldots, G. \end{cases}$$

Thus any cluster tends to have high leverage if N_g/N is large. A treated cluster has high leverage and partial leverage if $\bar{d}$ is small. Conversely, a control cluster has high leverage and partial leverage if $\bar{d}$ is large. □

Example 5 (Treatment with fixed effects)**.** Finally, we consider the case of cluster-level fixed effects, where treatment is randomly applied at the individual level. This is a special case of Example 3. We cannot consider cluster fixed effects with cluster-level treatment, because the treatment dummy would be invariant within clusters. We specialize (26) and find that

$$L_g = \frac{N_g \bar{d}_g(1-\bar{d}_g)}{\sum_{h=1}^{G} N_h \bar{d}_h(1-\bar{d}_h)}. \tag{33}$$

Thus, as before, the leverage of cluster g, relative to the average for the other clusters, is proportional to its size, N_g. It also depends on the proportion of treated observations in the cluster. The maximum (relative) leverage for cluster g occurs at $\bar{d}_g = 1/2$ and is symmetric around $1/2$. The result (29) continues to hold. It tells us that cluster g will be influential when its leverage (33) is large and $\hat{\beta}^{(g)}$ differs greatly from $\hat{\beta}_g$. □

6 Two-Way Clustering

Up to this point, we have focused on one-way clustering. However, it is also important to compute measures of leverage, partial leverage, and influence when there is clustering in two or more dimensions (Cameron, Gelbach, and Miller 2011). In the simplest and most commonly-encountered case, where there is two-way clustering, we recommend computing the usual one-way measures of leverage, partial leverage, and influence for each of the two clustering dimensions. This requires calling `summclust` twice.

When the number of clusters in either dimension is small, or when the data are seriously unbalanced in either dimension, conventional inference based on a two-way version of CV_1, together with the $t(\min(G-1, H-1))$ distribution, can be seriously unreliable. MacKinnon, Nielsen, and Webb (2021) therefore suggests using the usual two-way CV_1 estimator and applying the original WCR bootstrap to the dimension with the fewest clusters or the most unbalanced clusters. Simulation evidence suggests that this often provides more reliable inferences than the t distribution, but these inferences may still be problematic.

It may also be interesting to calculate measures of leverage, partial leverage, and influence for the intersection of the two clustering dimensions, especially when the number of non-empty intersections is not large. This means calling `summclust` a third time. Suppose there are two clustering dimensions, with G clusters in the first dimension and H clusters in the second. Then the number of intersection clusters is at most GH, but it can be smaller if some of the intersection clusters are empty. In order to use `summclust` for the intersections, it is necessary to create a new variable that uniquely identifies each of the non-empty intersection clusters. Running `summclust` for this case may be expensive when the number of non-empty intersections is large, especially if k is also large.

It is important to remember that, when `summclust` is invoked three times for each of two clustering dimensions and their intersection, the CV_3 standard error that it reports for each of the three cases is based on a different pattern of one-way clustering. When two-way clustering is appropriate, none of these standard errors is valid. However, what `summclust` reports can be used to compute an asymptotically valid variance as

$$\widehat{\text{Var}}_{2\text{W}}(\hat{\beta}_j) = \widehat{\text{Var}}_G(\hat{\beta}_j) + \widehat{\text{Var}}_H(\hat{\beta}_j) - \widehat{\text{Var}}_{GH}(\hat{\beta}_j). \tag{34}$$

Here $\hat{\beta}_j$ is the OLS estimate of a coefficient of interest, and the three estimated variances on the right-hand side of (34) are the squares of the CV_3 or $\text{CV}_{3\text{J}}$ standard errors reported by `summclust` for clustering dimension G, clustering dimension H, and the intersection of the two clustering dimensions, respectively.

Asymptotically, the two-way variance $\text{Var}_{2\text{W}}(\hat{\beta}_j)$ should not be less than either of the one-way variances. Therefore, if $\widehat{\text{Var}}_{2\text{W}}(\hat{\beta}_j)$ is less than either $\widehat{\text{Var}}_G(\hat{\beta}_j)$ or $\widehat{\text{Var}}_H(\hat{\beta}_j)$, it makes sense to replace it by the larger of those two variance estimates. Doing this also eliminates the risk of having to take the square root of a negative number. The appropriate t distribution has $\min(G-1, H-1)$ degrees of freedom if $\widehat{\text{Var}}_{2\text{W}}(\hat{\beta}_j)$ is used and $G-1$ or $H-1$ degrees of freedom if it is replaced by either $\widehat{\text{Var}}_G(\hat{\beta}_j)$ or $\widehat{\text{Var}}_H(\hat{\beta}_j)$, respectively. We conjecture that, especially when this is done, the two-way standard error based on either jackknife estimator will yield more conservative, and generally more reliable, inferences than the usual two-way standard error based on CV_1.

As we discuss in Section 3, it is often invalid to partial out fixed effects when computing a jackknife CRVE. This can be particularly tricky in the case of two-way clustering. For example, suppose there are G states and H years. Then it may be desirable to partial out the state fixed effects when computing $\widehat{\text{Var}}_G(\hat{\beta}_j)$ but invalid to partial out the year fixed effects. Similarly, it may be desirable to partial out the year fixed effects when computing $\widehat{\text{Var}}_H(\hat{\beta}_j)$ but invalid to partial out the state fixed effects. Finally, it is invalid to partial out either set of fixed effects when computing $\widehat{\text{Var}}_{GH}(\hat{\beta}_j)$. The `absorb` option of `summclust` normally detects cases where partialing out is invalid and refuses to display jackknife standard errors and several other quantities.

7 Simulation Experiments

One of the reasons for calculating leverages and partial leverages is to identify cases in which inference may be problematical. The objective of the simulation experiments in this section is to see whether the rejection frequencies for cluster-robust t-tests can be predicted from the features of the $\boldsymbol{X}$ matrix reported by `summclust`. There are 3000 cases, each corresponding to a particular $\boldsymbol{X}$ matrix. For each case, we generate 10,000 values of $\boldsymbol{y}$ and use them to estimate rejection frequencies for t-tests or bootstrap tests at the .05 level.

In the experiments, there are either 20 clusters and 2000 observations or 30 clusters and 3000 observations. The cluster sizes N_g are determined by a parameter $\gamma \geq 0$, as follows:

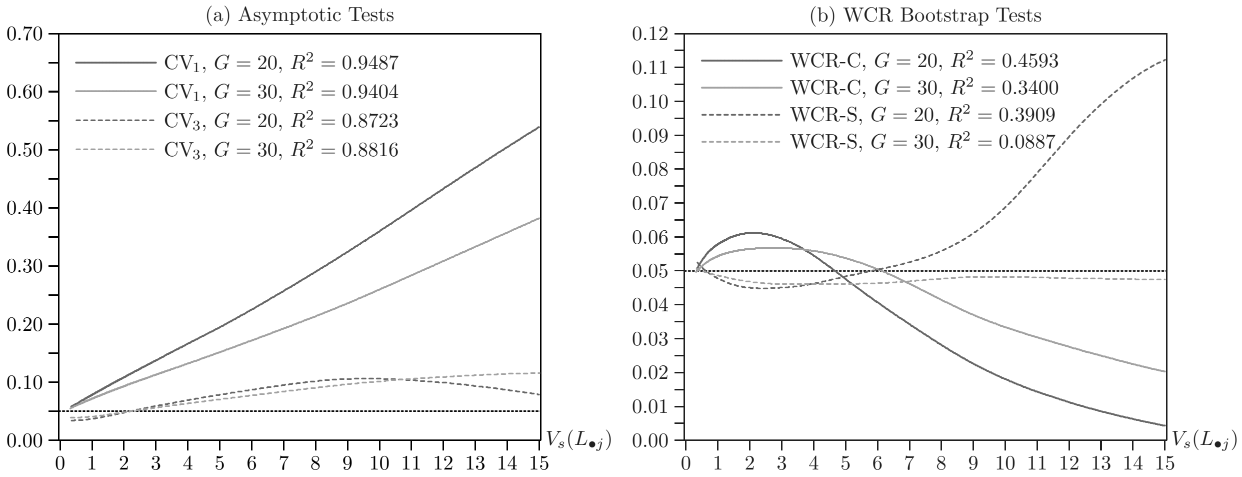
$$N_g = \left[N \frac{\exp(\gamma g/G)}{\sum_{j=1}^{G} \exp(\gamma j/G)} \right], \quad g = 1, \ldots, G-1,$$

where $[\cdot]$ denotes the integer part of its argument, and $N_G = N - \sum_{j=1}^{G-1} N_g$. As γ increases, the cluster sizes become increasingly unbalanced. The value of γ is chosen randomly from the U$[2,4]$ distribution, so that the cluster sizes tend to vary quite a lot. When $G = 20$, the smallest cluster has between 8 and 32 observations, and the largest has between 229 and 378. When $G = 30$, the smallest cluster has between 7 and 32 observations, and the largest has between 237 and 396.

There are five regressors, one of which is the test regressor, plus a constant term. The regressors equal either 0 or 1. With probability $1 - p_c$, all the observations in a cluster are 0. With probability p_c, they randomly equal either 0 or 1, both with probability 0.5. Thus, when $p_c = 1$, all variation is at the individual level, and leverage tends to be proportional to cluster sizes. As p_c declines, the samples become more unbalanced. In the experiments, the values of p_c are chosen to be 0.25, 0.30, 0.35, 0.40, 0.50, and 0.60, each for one-sixth of the cases. Smaller values of p_c tend to be associated with larger discrepancies between actual rejection frequencies and .05, the nominal level of the tests.

For each experiment, we obtain 12,000 estimated rejection frequencies. One-quarter of these are based on CV$_1$ and the $t(G-1)$ distribution, one-quarter on CV$_3$ and the $t(G-1)$ distribution, and one-quarter on each of the WCR-C and WCR-S bootstraps. To predict these rejection frequencies, we use a generalized additive model based on smoothing splines; see James, Witten, Hastie, and Tibshirani (2021, Section 7.7). The base model can be

Figure 3: Predicted rejection frequencies for asymptotic and bootstrap tests at .05 level



Notes: Each of the curves shows fitted values from the generalized additive model (35) that predicts observed rejection frequencies, based on 10,000 replications, using nonlinear functions of the $V_s(L_{\bullet j})$; see the text for details. Bootstrap rejection frequencies are based on $B = 399$. WCR-C is the classic restricted wild cluster bootstrap, and WCR-S is the score variant proposed in MacKinnon, Nielsen, and Webb (2023b).

written as

$$r_i = \beta_0 + f_1(V_{si}) + f_2(V_{si}^{1/2}) + \beta_1 G_{i0}^* + u_i, \tag{35}$$

where r_i is the rejection frequency for case i. Here V_{si} denotes $V_s(L_{\bullet j})$, the scaled variance of the partial leverages L_{gj} for the test regressor for case i, G_{i0}^* denotes $G_j^*(0)$ for the test regressor for case i (recall from Section 2.3 that it is a monotonically decreasing function of the L_{gj}), and $f_1(\cdot)$ and $f_2(\cdot)$ are smoothing splines with five degrees of freedom. Since everything on the right-hand side of (35) is a function of V_{si}, this model is simply using the V_{si} to predict the r_i in a potentially nonlinear way.

Figure 3 shows the fitted values from (35), which are predicted rejection frequencies, plotted against the scaled variance of the partial leverages L_{gj} for four methods of inference and two sample sizes. Panel (a) shows them for t-tests based on both CV_1 (solid lines) and CV_3 (dashed lines) for $G = 20$ and $G = 30$, and Panel (b) shows them for WCR-C and WCR-S bootstrap tests for the same two cases. The model seems to fit quite well, at least for the asymptotic tests, as can be seen from the values of R^2 reported for each of the curves. It also fits well for the bootstrap tests, and in fact it has smaller residuals for them than for the asymptotic tests. The lower R^2 values for the bootstrap tests simply reflect the fact that there is much less variation to explain.

We can see from Figure 3 that t-tests based on CV_1 often over-reject to an extreme degree. For the very smallest values of $V_s(L_{\bullet j})$, the tests tend to over-reject modestly, with predicted rejection frequencies of 0.058 for $G = 20$ and 0.055 for $G = 30$. However, these then rise quite rapidly and almost linearly. For $G = 30$, there are four cases (out of 3000) for which $V_s(L_{\bullet j}) > 15$. These are not shown in the figure, but the approximately linear relationship continues to hold, and the fit for these extreme cases is reasonably good.

In contrast, the t-tests based on CV_3 tend to under-reject for small values of $V_s(L_{\bullet j})$. For the very smallest values, the predicted rejection frequencies are 0.033 for $G = 20$ and 0.039 for $G = 30$. Although it is not obvious from the figure, the CV_3 tests are predicted to under-reject somewhat more than half the time, because, in our experiments, most values of $V_s(L_{\bullet j})$ are quite small. As $V_s(L_{\bullet j})$ increases, rejection frequencies increase, although for $G = 20$ they start to decline again once $V_s(L_{\bullet j})$ exceeds about 9.6. The predicted rejection frequencies never exceed 0.105 for $G = 20$ and 0.118 for $G = 30$. In a few cases (74 for $G = 20$ and 5 for $G = 30$), the matrix that is inverted in (9) was singular for at least one omit-one-cluster subsample. This happened whenever one of the regressors took the same value for all observations in $G - 1$ of the clusters. These cases were dropped.

Panel (b) of Figure 3 shows the fitted values from (35) for WCR-C and WCR-S bootstrap t-tests plotted against the scaled variance of the L_{gj}. Notice that the scale of the vertical axis differs greatly from the one in Panel (a). All tests, especially the WCR-S ones, perform quite well for smaller values of $V_s(L_{\bullet j})$. Except for WCR-S with $G = 30$, however, the rejection-frequency curves are not even close to being linear. This is also the only case for which the fitted values do not deviate greatly from 0.05 for large values of $V_s(L_{\bullet j})$. In every other case, a large value of $V_s(L_{\bullet j})$ tends to be associated with substantial levels of over-rejection or under-rejection.

It is natural to ask whether we can improve the fit of (35) by adding additional explanatory variables that are not simply functions of the $V_s(L_{\bullet j})$. The answer is that we can. In particular, the variables $\bar{a}_{\text{geo}}(L_{\bullet j})$ and $G^*_j(1)$ are often significant when they are added. However, the spline $f_1(V_{si})$ always remains highly significant, even when many other regressors are included. Thus, at least in these experiments, the scaled variance of the partial leverages, which is the square of their coefficient of variation, seems to be particularly revealing.

Based on these results, which are of course extremely dependent on the way in which the regressors are generated, it seems sensible for investigators to look at a number of different summary measures for both leverage and partial leverage. That is why `summclust` reports several of them. In this case, the most informative summary measure appears to be the scaled variance, defined in (13), of the partial leverage measures L_{gj}, defined in (7), for the regressor of interest. `summclust` reports the square root of this in the “Coefvar” line of the “Cluster Variability” table. In general, cluster-robust inference seems to be most reliable when the partial leverages do not vary greatly across clusters.

8 Conclusions

We have discussed a new `Stata` package called `summclust` that is designed to summarize the cluster structure of the dataset for a linear regression model with clustered disturbances. Since the key unit of observation is the cluster, it makes sense to examine measures of influence, leverage, and partial leverage at the cluster level. These are easy to compute and are conceptually very similar to the corresponding classic measures at the observation level (Belsley, Kuh, and Welsch 1980; Chatterjee and Hadi 1986). The `summclust` package calculates all of them and also reports a number of summary statistics.

Our measure of influence at the cluster level can provide valuable information about how empirical results depend on the data in the various clusters. Investigators should be wary if dropping one or two clusters changes the results dramatically. However, apart from such cases, the most interesting quantities that `summclust` calculates generally seem to be the partial leverages and measures that summarize their distribution.

It has long been known that cluster-robust inference can be unreliable when the number of clusters is small. More recent work, including MacKinnon and Webb (2017a, 2018) and Djogbenou, MacKinnon, and Nielsen (2019), has shown that it can also be severely unreliable when cluster sizes vary a lot or when few clusters are treated in the context of difference-in-differences and other treatment models. In both of these cases, leverage and partial leverage tend to vary greatly across clusters. It therefore seems natural to use our measures of leverage and partial leverage as diagnostic tools to identify datasets and regression designs in which cluster-robust inference is likely to be challenging. Simulation results in Section 7 suggest that the extent to which partial leverage varies across clusters can be particularly informative. We believe that investigators should always look at the summary statistics reported by `summclust` and exercise caution whenever they indicate substantial variation across clusters.

As we discuss in Section 2.2, the computations needed for leverage and influence are very similar to the ones needed to compute cluster jackknife variance matrix estimators. The `summclust` package therefore computes two very similar jackknife estimators, which we refer to as CV_3 and $\mathrm{CV}_{3\mathrm{J}}$, almost as a byproduct of other computations. These are the same estimators that `Stata` can produce using the `vce(jackknife,mse)` and `vce(jackknife)` options. However, because `summclust` is designed explicitly for linear regression models estimated by OLS, it is faster than using these `vce` options. Moreover, when `summclust` is already being used to obtain cluster-level measures of influence and leverage for diagnostic purposes, the additional cost of computing the jackknife variance estimators is minimal.

When the number of clusters is reasonably large and the variation of leverage and partial leverage across clusters is small, we would expect conventional inference based on CV_1 standard errors to perform well. If so, the CV_3 standard errors reported by summclust should be very similar to the CV_1 standard errors reported by one of Stata's regression commands. When this is the case, there is probably no need for investigators to worry further about the reliability of their inferences. In many cases, however, the CV_3 and CV_1 standard errors will differ noticeably. This happens for the empirical example in Section 4, where there are 72 clusters but partial leverage varies a lot. In such cases, the CV_3 standard errors are almost certain to be more conservative, and very likely to be more reliable, than the CV_1 ones.

P values and confidence intervals that are even more reliable can often be obtained by using the restricted wild cluster bootstrap, which is implemented natively with wildbootstrap in Stata 18 and in the package boottest (Roodman, MacKinnon, Nielsen, and Webb 2019). Recent versions of that package implement the WCR-S bootstrap (MacKinnon, Nielsen, and Webb 2023b) in addition to the classic WCR-C bootstrap. We strongly recommend that both variants be calculated whenever the CV_3 and CV_1 standard errors disagree. When the two bootstrap P values agree, as they do for the empirical example in Section 4, then it is probably safe to rely on either of them. When they disagree, then neither of them may be entirely reliable, but we would be inclined to use the one given by the WCR-S bootstrap.

Up to this point, everything in this section has been based on the assumption that there is one-way clustering with a known clustering structure. When more than one level of clustering is plausible, investigators need to choose among them, and this can be challenging; see the discussions in MacKinnon, Nielsen, and Webb (2023a, c). The measures of leverage and influence produced by summclust may be helpful in deciding at what level to cluster.

The current version of summclust is not explicitly designed to handle two-way clustering. However, as we discuss in Section 6, it can be called for each clustering dimension so as to produce two sets of diagnostic statistics. If it is called three times, once for each dimension and once for their intersection, then it can also be used to compute two-way cluster jackknife variance matrix estimators. At present, however, little is known about the properties of these estimators.

Software Installation

To install the software files as they exist at the time of publication of this article, type

```
. net sj 23-4
. net install st00!!  (to install program files, if available)
. net get st00!!      (to install ancillary files, if available)
```

The command `summclust` can be installed from the Statistical Software Components archive by typing

```
. ssc install summclust
```

or from GitHub by typing

```
net install summclust, ///
from("https://raw.githubusercontent.com/mattdwebb/summclust/main/")
```

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