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Markups, Pass-Through, and Firm Heterogeneity with Sequentially Mixed Search

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Abstract

We study the determination of market power at the firm and industry levels when heterogeneous firms compete for sales to *ex ante* homogeneous buyers in a market with both directed and random search and free entry of firms that differ in productivity. Search and the distribution of productivity across active firms generate distributions of equilibrium prices and markups that we relate to variation in the elasticity of demand at the firm level. With directed search at the outset, a shock that raises the matching rate for buyers improves conditions for them and tends to lower markups. Random matching follows sequentially, and the same shock can lower the productivity threshold for operation, pushing up prices and markups for all firms. The net effect on market power can be ambiguous depending on the forces driving matching rates. The distributions of prices and markups respond in equilibrium to changes in common and firm-specific costs, consumption utility, and fixed costs of both entry and operation. We characterize the differential pass-through of these changes to prices and markups at both the firm and market levels.

JEL Codes: D21, D43, E31, L11

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1 Introduction

In this paper, we study the determination of firm- and industry-level market power in a market with search frictions driven by matching and incomplete information and in which heterogeneous firms compete for sales to *ex ante* homogeneous buyers. Matching rates and the distribution of productivity across active firms, both of which are determined in equilibrium, generate distributions of prices and markups that we relate to variation in the elasticity of demand at the firm level. These distributions respond endogenously to changes in variable costs, utility from consumption, and fixed costs of both entry and operation. We refer to these responses as *pass-through* and characterize them for both individual firms and the industry average.

Our theory employs the concept of sequentially mixed search, extending that of Shi (2023) to allow for both firm heterogeneity and a more general matching process. This enriched framework enables us to investigate critical issues related to variations in market power among firms and across economic conditions. For example, we characterize how search frictions allow less-productive firms to persist in the market and analyze how their presence influences the pricing and markups of more-productive firms. Additionally, we examine how firms with differing levels of productivity respond distinctly to changes in cost and demand conditions and explore the implications of those responses for average prices and markups.

In our environment with sequentially mixed search and heterogeneous firms, measures of identical households and *ex ante* identical firms first choose to enter one of many potentially available submarkets where each is characterized by matching rates for both buyers and sellers and a maximum allowable price. The latter, here, is effectively a commitment on the part of firms who enter the submarket. This process takes place under conditions of directed search, with agents understanding the relationship between the upper limit on the posted price and the matching probabilities.

After choosing a submarket to enter and paying an entry cost, firms realize their productivity and then decide whether to pay a fixed production cost and to operate in that submarket. Operating firms then post a price contingent on their realized productivity and sell to those households that 1) have an opportunity to trade with them and 2) for which this is the lowest-price opportunity that they have. This process involves random matching and noisy search along the lines of Burdett and Judd (1983) and is extended to incorporate firm heterogeneity as in Herrenbrueck (2017), Baggs et al. (2018), and Chernoff et al. (2024).

In this environment, we specify conditions under which there exists a unique sequential mixed-search equilibrium. We characterize this equilibrium for classes of productivity distributions and matching processes and illustrate the relationships among prices, markups, firm size, and productivity under various conditions. We also characterize the changes in the distributions of both prices and markups that occur in response to changes in costs, both variable and fixed, and demand. In some cases, the effects of these changes on average prices and overall market power are unambiguous; in others, they are not. To shed light on the quantitative forces that can, in principle, resolve any ambiguity, we also consider computational examples computed for parameterizations based on empirical work in Chernoff et al. (2024).

In our theory, directed and random search play distinct roles in the determination of market power. The process of directed search establishes a link between the relative measures of buyers and sellers in the market, that is, market tightness. In general, lower tightness (more sellers relative to buyers) is associated with lower markups. Effectively, buyers receive a higher share of the surplus from trade as sellers have to "compete harder" for these buyers. This is a common finding in directed search models in many applications and is related to the efficiency of matching in these environments, as many authors have noted (see, for example, the survey by Guerrieri et al. (2021)).

Random search in the presence of firms with differing productivities within a submarket, however, introduces a countervailing force. Lower market tightness driven by an increase in the number of firms, due to a reduction in costs or an increase in consumer demand, lowers the productivity threshold for operation. That is, it allows less-productive firms to operate in the market, driving up prices and, therefore, markups overall, thereby moving them in the *opposite* direction from market tightness. This effect, that search frictions may increase market power by allowing less-productive firms to successfully operate, thereby allowing more-productive firms to raise their markups, has been recognized, for example, by Chernoff et al. (2024) and Menzio (2024).

Here directed search, while contributing to a positive relationship between tightness and market power, may also contribute to this countervailing force in the opposite direction. In the random search environments described above, the upper bound of the price distribution is determined solely by demand, being equal either to an exogenously specified buyers' reservation price or to the unrestricted monopoly price in the case of downward-sloping demand (see Burdett and Judd

(1983)). With sequentially mixed search, competition among firms determines the maximum price. Here, that competition is among heterogeneous firms. The presence of less-productive firms in the market, in response to a decline in market tightness, as described above, not only raises markups for their more-productive competitors but also contributes to the increase in average prices and markups by raising the maximum price, that is, the price posted by the least-productive firms in the market.

Overall, average market power may be either increasing or decreasing in market tightness; that is, it is possible for markups to rise (fall) when conditions become, in a sense, more (less) favorable to buyers. As noted above, the overall effect of a given change can depend on quantitative factors. Some general findings, however, do apply. Specifically, changes in marginal costs that are common across firms, and changes in the fixed cost of entry and in buyers' common utility from consumption all cause market tightness and the average markup to move in opposite directions, while changes in the fixed operating costs induce market tightness and the average markup to move in the same direction.

We also focus on differences in market power at the firm level and how it responds to various shocks. Thus, we examine differences in pass-through in different regions of the equilibrium price and the markup distribution. We find that while there are strong forces inducing markups to be increasing in firm size, in many cases markups exhibit a u-shape. That is, markups are falling in size for the smallest, least-productive firms. These firms may also exhibit very large degrees of pass-through relative to their more-productive peers. As those consumers purchasing from these firms tend to be the least informed about prices, heterogeneity in markups and pass-through across firms has implications for welfare and inequality.

Our analysis and results contribute to the literature on search, price dispersion, pass-through, adapting the framework of Burdett and Judd (1983), as do Head et al. (2010), Head et al. (2012) and Wang et al. (2020), and combining it with the directed search following Shi (2023). Our work, here, is distinguished by the inclusion of firm heterogeneity in a manner similar to Herrenbrueck (2017), Baggs et al. (2018), and Chernoff et al. (2024). Many others have extended the framework of Burdett and Judd (1983) in related but different directions. Examples include Kaplan et al. (2019), Menzio and Trachter (2018), and Menzio (2023).

Our work is also related to that of Nord (2023) and Menzio (2024). The former focuses on heterogeneity in consumers' search efforts and characterizes the responses of prices and markups in a model employing the structure of Burdett and Judd (1983), when individual shoppers choose whether to observe one or two prices. The latter studies the distribution of markups in an extension of Burdett and Judd (1983) with a Poisson search process with and without firm heterogeneity. Our work differs from Nord (2023) in that we consider firm heterogeneity and a more general but fixed search process. It differs from that of Menzio (2024) in that we generalize the search process to reflect empirical evidence from Chernoff et al. (2024) and focus on pass-through at both the firm and market/industry level.

Most importantly, however, our work differs from both Nord (2023) and Menzio (2024) in its use of the framework of sequentially mixed search. It is this aspect of the theory that allows an analysis of the potentially conflicting forces affecting market power and tightness described above. Methodologically our work is thus similar to that of Shi (2023), although we diverge from the model in that paper both with regard to the search process and in the inclusion of firm heterogeneity.

We also view our work as complementary to that found in the large literature on industry dynamics following Hopenhayn (1992), Melitz (2003) and others, although it differs substantially in several ways. First, imperfect competition here stems from search frictions, specifically consumers' incomplete information regarding trading opportunities, rather than from product differentiation. Second, markups are endogenous, heterogeneous across firms, and respond differentially to a wide range of parameter changes and shocks. Moreover, the distribution of markups is driven by both the nature of demand, which depends on the search process, and by the entire distribution of firm productivity. Finally, the overall degree of market power here is determined by competition among firms and depends on both demand and the nature and extent of heterogeneity in firms' productivity.

The remainder of the paper is organized as follows. Section 2 describes the basic environment, preferences, and technologies, including the extent and nature of firm heterogeneity and the matching and meeting processes. Section 3 describes the nature of search and interactions among firms and households and proves the existence and uniqueness of a sequential mixed search equilibrium. In Section 4, the equilibrium distributions of prices and markups and the pass-through of various shocks are characterized. We address the potential ambiguity of the qualitative results, discussed above, in a series of computational examples in Section 5, and we conclude in Section 6.

2 The Environment

There is a fixed measure, λ, of *ex-ante* identical households that consume either one or zero units of a homogeneous good and receive utility $\bar{u}$ if they consume. There is also a large measure of prospective firms that can potentially produce the homogeneous good. These firms are *ex ante* homogeneous but, prior to production, each may pay a fixed *entry* cost, f_e, to draw a firm-specific productivity parameter, z, from a common distribution, $J(z)$, which is continuous on a connected support, $[z_L, z_H)$. We further restrict $J(\cdot)$ to have the property that there exists a monotonic function $G(z_L/z)$ such that $J(z) = G(z_L/z)$. We refer to this as the *monotone z-ratio* property and describe its usefulness in our discussion of firm optimization and equilibrium, below. The Pareto, uniform, and triangular distributions are examples of distributions with this property.

Having paid the entry cost and drawn z, in order to be an *active* producer, the firm must pay a common fixed production cost, δ. An active firm's marginal production cost is constant and given by $\frac{\phi}{z}$, where z is the firm's individual productivity parameter and $\phi > 0$ is common across firms. Previewing the equilibria we consider below, there will be a cut-off productivity level, $\tilde{z}$, such that only firms that draw $z \geq \tilde{z}$ will choose to be active. In this case, the distribution of productivity among active firms is

$$F(z) = \frac{J(z) - J(\tilde{z})}{1 - J(\tilde{z})} \tag{1}$$

on support $[\tilde{z}, z_H)$. It is straightforward to show that if $J(z)$ has the monotone z-ratio property on $[z_L, z_H)$, then $F(z)$ also does on $[\tilde{z}, z_H)$. This will be useful as, in this case, a firm's behavior is dependent on its productivity relative to the least-productive operating firm (*i.e.*, those with $z = \tilde{z}$).

The economy is characterized by a search friction whereby households may contact firms only randomly. We think of this as representing limitations on households' information. First, while households know the underlying distribution of productivity, $J(z)$, they never directly observe any firm's costs. Second, they match randomly with at least one firm in a way that depends on the relative measures of firms and households trading. Because there is a positive probability they do not match, some firms and households will *not* trade in equilibrium. Finally, conditional on matching, households receive opportunities to trade (*meetings*) with a random number of firms and may choose the best trading opportunity from this set.

Given the measures of households, λ, and firms active in the market, N, let the probabilities with which households and firms, respectively, match with at least one trading partner be given by

$$b(\theta) \qquad \text{and} \qquad s(\theta). \tag{2}$$

In (2), $\theta = \frac{\lambda}{N}$ represents market tightness, and the matching rates are derived from a constant returns to scale (CRS) matching function, $M(\lambda, N)$, where

$$b(\theta) = \frac{M(\lambda, N)}{\lambda} = M(1, 1/\theta) \qquad \text{and} \qquad s(\theta) = \frac{M(\lambda, N)}{N} = M(\theta, 1). \tag{3}$$

As the matching function is CRS, the elasticity of the firm matching rate, $\epsilon_s^\theta(\theta)$, is less than one. Further, we assume that the matching function is log-supermodular. This implies that $\epsilon_s^\theta(\theta)$ is non-increasing in θ.

Conditional on matching with probability $b(\theta)$, a buyer has k total meetings with firms, for $k = 1, \ldots, K$, with probability q_k, where $\sum_{k=1}^{\infty} q_k = 1$ and K equals an integer greater than one or infinity. It thus follows that conditional on matching with probability $s(\theta)$, a fraction q_k of a firm's meetings are with households with $k-1$ *other* opportunities to trade. Since a household consumes, at most, one unit of the good, it will choose to trade in, at most, one meeting. Firms, however, may produce for and trade with multiple households.

3 Equilibrium

We consider a market organized similarly to that studied by Shi (2023) and referred to as *sequentially mixed search*. Prior to observing their draw of z, a firm may pay the fixed cost to enter one of potentially many submarkets that are characterized by a maximum price, $\tilde{p} \leq \bar{u}$, and matching rates $b(\theta)$ and $s(\theta)$ for households and firms. Given a matching function (3), we may thus identify a submarket by a maximum price, $\tilde{p}$, and market tightness, θ. A firm entering *submarket* $(\tilde{p}, \theta)$ commits, in the event that the firm is active, to post a price, $p \leq \tilde{p}$, and to produce to meet the demand associated with the matching probabilities and meeting rates q_k for all k associated with market tightness θ. Similarly, each household also chooses to enter one submarket characterized by a pair $(\tilde{p}, \theta)$.

Firms take into account that, having entered submarket $(\tilde{p}, \theta)$, they will participate in a price-posting game along the lines of the noisy search environment of Burdett and Judd (1983) as extended to incorporate heterogeneous firm productivity and free entry by Chernoff et al. (2024). Thus, we think of competition as taking place in two stages, with the second stage being the price-posting game characterized by random matching between active firms and households and the first stage being the choice of submarket $(\tilde{p}, \theta)$ by all entering firms.

3.1 The Second Stage: Price-posting with Random Matching

Prospective firms' decisions to become active and choose a posted price, $p \leq \tilde{p}$, are similar to those described in Chernoff et al. (2024). For this reason, we relegate some descriptions of these interactions to Appendix A and describe, here, only the main components of those interactions and the ways in which we deviate from the environment in that paper.

At this stage, having entered submarket $(\tilde{p}, \theta)$, matching and the number of meetings are entirely random for both firms and households. A firm receives its draw of z from distribution $J(z)$ and then, conditional on this draw, decides whether to pay the fixed cost δ and become active. Given $\tilde{p}$ and θ and its beliefs about its competitors' prices within that submarket, each active firm posts a price, $p \leq \tilde{p}$, to maximize its expected profit. Let the endogenous distribution of prices posted by firms in the submarket be denoted $L(p; \tilde{p}, \theta)$. Then, given the matching probability for a firm, $s(\theta)$, and the meeting rates, q_k, for all k, the expected sales revenue for a firm that posts price $p \leq \tilde{p}$ is

$$R(p) = ps(\theta)D(1 - L(p; \tilde{p}, \theta)), \tag{4}$$

where

$$D(1 - L(p; \tilde{p}, \theta)) = \sum_{k=1}^{\infty} q_k k(1 - L(p; \tilde{p}, \theta)) \tag{5}$$

represents the expected demand, conditional on matching, for a firm that posts price p.

A firm with productivity parameter z then chooses its posted price, p, to solve

$$\max_{p \leq \tilde{p}} \left[p - \frac{\phi}{z}\right] s(\theta)D(1 - L(p; \tilde{p}, \theta)) - \delta. \tag{6}$$

Note that a firm that chooses to post the maximum allowable price, $\tilde{p}$, expects to sell $s(\theta)D(0) =$

$s(\theta)q_1$ units for a profit of

$$\Pi(\tilde{p}) = \left[\tilde{p} - \frac{\phi}{z}\right] s(\theta)q_1 - \delta. \tag{7}$$

The first-order condition for an interior solution to the profit maximization problem is given by [1]

$$D\left(1 - L(p(z))\right) - \left[p(z) - \left(\frac{\phi}{z}\right)\right] D'\big(1 - L(p(z))\big)L'(p(z)) = 0. \tag{8}$$

The optimal pricing function, $p(z)$, solves (8). Regarding the dependence of price on firm productivity, z, we have the following proposition:

Proposition 1. *The optimal pricing function, $p(z)$, is monotonically decreasing in z.*

Proof: See Appendix A.

Note that given Proposition 1 and the assumption that $J(z)$ and therefore $F(z)$ are continuous, it follows directly that the endogenous distribution of prices $L(p; \tilde{p}, \theta)$ will also be continuous.

As noted above, having entered and drawn productivity parameter z, a firm pays fixed cost δ to operate in submarket $(\tilde{p}, \theta)$. Thus, all firms that expect to earn positive profits, net of the fixed cost, will operate. Proposition 1 shows that posted prices are strictly decreasing in firm productivity. It is then a straightforward application of the envelope theorem that firm profits are strictly increasing in z. Thus, there will be a single productivity level, $\tilde{z}$, below which expected profits are negative. Given that $\tilde{p} \leq \bar{u}$, a firm in this submarket with productivity parameter $\tilde{z}$ will post price $\tilde{p}$ and earn zero profits, in expectation. To guarantee that such a firm exists and that $\tilde{z} > z_L$, we assume

$$\left[\tilde{p} - \frac{\phi}{z_L}\right] s(\theta)q_1 < \delta < \tilde{p}s(\theta)q_1. \tag{9}$$

Given (9) and Proposition 1, $1 - L(p(z)) = F(z)$ and $-L'(p(z))p'(z) = F'(z)$, where $F(z)$ is the distribution of productivity across active firms on support $[\tilde{z}, z_H)$, as described above. In this case, we may re-write (8):

$$D\left(F(z)\right) + \left[p(z) - \left(\frac{\phi}{z}\right)\right] \left[\frac{D'\big(F(z)\big)F'(z)}{p'(z)}\right] = 0. \tag{10}$$

[1] In (8) we have suppressed the notation for the price distribution's dependence on $L(\cdot)$ on $\tilde{p}$ and θ as prior choice of the submarket is assumed throughout this sub-section.

As $\tilde{p}$ is the maximum price and $\tilde{z}$ is the minimum productivity parameter among firms in submarket $(\tilde{p}, \theta)$, the solution to this first-order differential equation is

$$p(z) = \left(\frac{1}{D\big(F(z)\big)}\right)\left(\tilde{p}q_1 + \phi \int_{\tilde{z}}^{z} D'\big(F(x)\big)\left(\frac{F'(x)}{x}\right) dx\right). \tag{11}$$

For details of this derivation, see Appendix A.

At this point, we use the monotone z-ratio property introduced above to write the pricing function (11) in terms of firms' relative productivities. Recall that for distributions in this class we may write $F(z;\tilde{z}) = G(\tilde{z}/z)$, where $G(\tilde{z}/z)$ is a monotonic function, and we also have $F'(z;\tilde{z}) = -G'(\tilde{z}/z)\left(\tilde{z}/z^2\right)$. For this class of distributions, we may rewrite the pricing function (11) as

$$p(z;\tilde{z}) = \left(\frac{1}{D\big(G(\tilde{z}/z)\big)}\right)\left(\tilde{p}q_1 - \phi \int_{\tilde{z}}^{z} D'\big(G(\tilde{z}/x)\big)G'(\tilde{z}/x)\left(\frac{\tilde{z}}{x^3}\right) dx\right). \tag{12}$$

Letting $w \equiv \tilde{z}/x$, we may then express (12) as

$$p\left(z;\tilde{z}\right) = \left(\frac{1}{D\big(G(\tilde{z}/z)\big)}\right)\left(\tilde{p}q_1 - \left(\frac{\phi}{\tilde{z}}\right)\int_{\tilde{z}/z}^{1} wD'\big(G(w)\big)G'(w)dw\right). \tag{13}$$

From now on we index firms by their *relative marginal cost*, $(\phi/z)/(\phi/\tilde{z}) = (\tilde{z}/z) = v$, allowing us to rewrite the pricing function (13) as

$$p(v) = \left(\frac{1}{D\big(G(v)\big)}\right)\left(\tilde{p}q_1 - \left(\frac{\phi}{\tilde{z}}\right)\int_{v}^{1} h(w)dw\right), \tag{14}$$

where $h(v) = vD'(G(v))G'(v) < 0$, $D(G(1)) = q_1$ and, thus, $p(1) = \tilde{p}$. The firm markup, defined as a firm's price divided by its marginal cost, can then also be written as a function of v,

$$mkup(v) = \frac{p(v)\tilde{z}}{\phi v}, \tag{15}$$

and expected profit:

$$\begin{aligned} \Pi(v;\tilde{p},\theta) &= s(\theta)\left[\mathrm{E}\left[D\left(F(z)\right)\left(p(z) - \frac{\phi}{z}\right);\tilde{p},\theta\right]\right] - \delta & (16)\\ &= s(\theta)\int_0^1 \left[\tilde{p}q_1 - \frac{\phi}{\tilde{z}}\left(\int_v^1 h(w)dw + vD\left(G(v)\right)\right)\right](-G'(v))dv - \delta. & (17) \end{aligned}$$

Note that the zero-profit condition that determines $\tilde{z}$ is

$$s(\theta)\left[\tilde{p}-\frac{\phi}{\tilde{z}}\right]q_1=\delta \qquad \longrightarrow \qquad \tilde{z}(\tilde{p},\theta)=\frac{s(\theta)\phi q_1}{s(\theta)\tilde{p}q_1-\delta}. \tag{18}$$

We can then write a firm's variable expected profits, conditional on matching, as a function of its price, p:

$$\mathrm{E}\left[D(1-L(p))\left(p-\frac{\phi}{z}\right)\right]=\tilde{p}c_1+\left(\frac{\delta}{s(\theta)}\right)d_1. \tag{19}$$

where

$$c_1 = \int_0^1\left(vD(G(v))+\int_v^1 h(w)dw\right)G'(v)dv+q_1 \tag{20}$$

$$d_1 = -\left[\frac{1}{q_1}\right]\int_0^1\left(vD(G(v))+\int_v^1 h(w)dw\right)G'(v)dv \tag{21}$$

depend only on exogenous parameters. Here, $c_1>0$ and $c_1=q_1(1-d_1)$.

Given that only firms with $v\le 1$ ($z\ge\tilde{z}$, as determined by the zero-profit condition (18)), will operate and post price $p(v)$ optimally, expected profits in submarket $(\tilde{p},\theta)$ are

$$\Pi(\tilde{p},\theta)=s(\theta)\left(c_1\tilde{p}+\left(\frac{\delta}{s(\theta)}\right)d_1\right)-\delta. \tag{22}$$

Similarly, a household entering submarket $(\tilde{p},\theta)$ and facing the meeting process given by q_k for all k, conditional on matching, expects to pay price

$$\mathbf{E}\left[p;\tilde{p},\theta\right]\equiv\bar{p}=c_2\tilde{p}+\left(\frac{\delta}{s(\theta)}\right)d_2. \tag{23}$$

Again the following depend only on exogenous parameters:

$$c_2 = \int_0^1\left(\int_v^1 h(w)dw-q_1\right)G'(v)dv \tag{24}$$

$$d_2 = -\left[\frac{1}{q_1}\right]\int_0^1\left(\int_v^1 h(w)dw\right)G'(v)dv, \tag{25}$$

where $c_2>0$, $d_2<0$, and $c_2=q_1(1-d_2)$.

3.2 The First Stage: Free Entry to a Submarket

Taking as given the expected outcomes in the second stage described above, a potential firm chooses a submarket, $(\tilde{p}, \theta)$ to enter at cost f_e. Similarly, each consumer chooses a single submarket to enter in order to maximize their expected utility:

$$V_b = b(\theta)\left[\bar{u} - \mathrm{E}[p; \tilde{p}, \theta]\right], \tag{26}$$

where the price they expect to pay is given by (23) and depends on their matching rate, $b(\theta)$, the maximum price $\tilde{p}$, and the meeting process determined by q_k, for all k.

In this stage, search is directed in the sense that firms, in choosing the submarket to enter, take into account the relationship between the maximum price, $\tilde{p}$, and tightness in the submarket, θ. Following the literature (see, *e.g.*, Shi (2023), Menzio and Shi (2010), and Guerrieri et al. (2010)), we restrict firms' beliefs to require that buyers receive V_b in all potential submarkets. Then, using (19)-(26), an entering firm pays f_e and chooses $(\tilde{p}, \theta)$ to solve

$$V_s = \max_{\tilde{p},\theta} s(\theta)\left[c_1\tilde{p} + \frac{\delta}{s(\theta)}d_1\right] - \delta \tag{27}$$

subject to

$$\frac{\tilde{p}}{\delta} = \frac{1}{c_2}\left[\frac{\bar{u}}{\delta} - \frac{d_2}{s(\theta)} - \frac{\theta}{s(\theta)}\frac{V_b}{\delta}\right]. \tag{28}$$

Using (28), we may define $\tilde{p}(\theta)$ and identify submarkets with θ only. This allows us to represent the firm's optimal submarket choice with θ^*, as follows:

$$\theta^* = \arg\max_{\theta} s(\theta)\left[c_1\tilde{p}(\theta) + \frac{\delta}{s(\theta)}d_1\right] - \delta. \tag{29}$$

Now consider a prospective firm's entry decision. Firms enter until market tightness in the optimally chosen submarket, θ^*, satisfies

$$(1 - J(\tilde{z}(\theta^*)))V_s = f_e. \tag{30}$$

3.3 Equilibrium Definition, Existence, and Uniqueness

Following Shi (2023), a *sequentially mixed search (SMS) equilibrium* is a buyer's value, V_b^*; a collection of active submarkets, I, with associated $\tilde{p}(\theta^*)$ and $\tilde{z}(\theta^*)$ for all $\theta^* \in I$; and firms' beliefs across all submarkets, active or not, such that

1. In all submarkets indexed by θ, firms believe that V_b^* satisfies (26), with $\tilde{p}(\theta)$ and $\tilde{z}(\theta)$ determined by (28) and (18), respectively.

2. In all active submarkets, θ^* satisfies (29).

3. Given that the total measure of buyers entering some active submarket is λ, firms enter all active submarkets so that θ^* satisfies (30).

Demonstrating that the firm's objective in (29) is strictly concave is straightforward, given the assumed properties of $s(\theta)$, the first-order condition is

$$s'(\theta^*)\bar{u} = V_b. \tag{31}$$

As such, given the assumed properties of $s(\theta)$, in any SMS equilibrium there must be a unique active submarket as only a single θ^* can satisfy condition 2. Also, condition 3 determines the equilibrium measure of entering firms.

If an SMS equilibrium exists, $\tilde{z}^*$, p^*, V_b^*, and θ^* are characterized by (18), (28), (31), and (30). Using the first-order condition, (31), to eliminate $\frac{V_b}{\delta}$ from the buyers' utility constraint (28) and the free entry condition (30) gives

$$\frac{p^*}{\delta} = \frac{1}{c_2}\left[\frac{\bar{u}}{\delta}\left(1 - \epsilon_s^{\theta}(\theta^*)\right) - \frac{d_2}{s(\theta^*)}\right] \tag{32}$$

$$\frac{f_e}{\delta} = (1 - J(\tilde{z}(\theta^*))\frac{c_1}{c_2}\left[\frac{\bar{u}}{\delta}s(\theta^*)(1 - \epsilon_s^{\theta}(\theta^*)) - 1\right], \tag{33}$$

where, as noted above, $\epsilon_s^{\theta}(\theta)$ is non-increasing in θ.

Using (32) to eliminate $\tilde{p}/\delta$ from the zero-profit condition, (18), we have

$$s(\theta^*)\left(\frac{q_1}{c_2}\left[\frac{\bar{u}}{\delta}(1 - \epsilon_s^{\theta}(\theta^*)) - \frac{d_2}{s(\theta^*)}\right] - \frac{\phi}{\tilde{z}\delta}q_1\right) = 1. \tag{34}$$

Now, (33) and (34) are a system in $\tilde{z}^*$ and θ^*:

$$H_1(\tilde{z}^*, \theta^*; \phi, \delta, f_e, \bar{u}) = 0 \tag{35}$$

$$H_2(\tilde{z}^*, \theta^*; \phi, \delta, f_e, \bar{u}) = 0 \tag{36}$$

the solution to which is an SMS equilibrium. We then have the following.

Proposition 2. *Under appropriate restrictions on the parameters $\phi, \bar{u}, f_e$ relative to the fixed cost δ, there exists a unique SMS equilibrium. That is, there exists a unique pair, $(\tilde{z}^*, \theta^*)$, that solves the system (35) and (36), with V_b^* and $p^* = \tilde{p}(\theta^*)$ satisfying (26) and (28), supported by firms' beliefs regarding submarkets out of equilibrium, as described in part 1 of the above definition of equilibrium.*

Proof: See Appendix A.

4 Markups, Prices and Pass-Through

4.1 Markups

In this section, we examine the role of productivity heterogeneity in determining firm-level markups in the model. In the absence of productivity heterogeneity, firm-level markups are inversely related to firm size, as they are in the environment of Burdett and Judd (1983). This is because firms that post the lowest prices and, therefore, have the lowest markups make more trades than firms posting higher prices. Since prices and markups are always positively correlated, firm size and the markup are negatively correlated. This reasoning also implies that the elasticity of demand facing a firm is falling in its price and its markup and rising in its size.

With productivity heterogeneity at the firm level, however, an additional factor comes into play. The distribution of productivity determines, at each productivity level, the measure of firms with similar productivities that are therefore choosing a relatively similar price. As productivity increases, not only does a firm's fraction of sales fall to buyers with no alternative but the measure of firms with comparable productivity also changes. Whether this measure rises or falls—and by how much—depends on the shape of the productivity distribution. In particular, if the measure of firms falls as productivity rises, as it does with a Pareto productivity distribution, this decline puts downward pressure on the elasticity of demand.

Thus, while consumer search generally creates a negative correlation between firm size and the elasticity of demand, productivity heterogeneity introduces a potential countervailing positive correlation. Consequently, in our search model, firm-level markups may vary non-monotonically in the presence of firm-level productivity differences.

Henceforth, we find it useful to reference firms using a measure of their relative productivity, rather than their relative cost, v. Specifically, let $1-v = \frac{z-\tilde{z}}{z}$ be our measure of relative productivity. Multiplied by 100, this may be interpreted as the percentage deviation of a firm's productivity from that of the least-productive firm. The following proposition characterizes conditions under which firm-level markups do and do not vary monotonically with relative productivity.

Proposition 3. *The markup function is decreasing in relative productivity at* $v = 1$ *if and only if*

$$\theta^* < \tilde{\theta},$$

where $\tilde{\theta}$ *satisfies*

$$s(\tilde{\theta})\left(1 - \epsilon_s^{\theta}(\tilde{\theta})\right) = \frac{d_2 q_1^2 - 2c_2 q_2 G'(1)}{q_1^2(\bar{u}/\delta)}. \tag{37}$$

Proof: See Appendix A.

As an example, consider the telephone line matching function with $s(\theta) = \frac{\theta}{1+\theta}$. Then the upper bound on θ in Proposition 3 is

$$\tilde{\theta} = \frac{\left[\frac{d_2 q_1^2 - 2c_2 q_2 G'(1)}{q_1^2(\bar{u}/\delta)}\right]^{.5}}{1 - \left[\frac{d_2 q_1^2 - 2c_2 q_2 G'(1)}{q_1^2(\bar{u}/\delta)}\right]^{.5}} \tag{38}$$

In Appendix A we show that this condition (37) will hold provided the elasticity of demand facing the least-productive firm remains below a certain threshold determined by a subset of the economy's parameters. In particular, the markup will be decreasing in relative productivity at $v = 1$, provided the elasticity of demand of the firm with $v = 1$ is less than $\frac{-2q_2 G'(1)}{q_1}$. In this case, the markup of the least-productive firm is above its nearest, slightly more-productive competitors.

In our environment with heterogeneous productivities for larger, more-productive firms, the markup eventually becomes increasing in productivity as the downward effect of rising productivity on the elasticity of demand becomes dominant. Moreover, since the limit of the posted price as z approaches infinity (v approaches zero) is positive, the markup approaches infinity as v approaches zero. Thus in the model, here, with an unbounded productivity distribution, the markup will either have a u-shaped relationship with firm size or it will be monotonically increasing in firm size.

As the elasticity of $s(\theta)$ is non-increasing in θ, the intuition for why the markup function will be decreasing at $v = 1$, if equilibrium tightness is sufficiently low, is the following. With θ relatively low, the matching rate for buyers, $b(\theta)$, is high, while that for sellers, $s(\theta)$, is low. Firms in this case will tend to face a relatively high elasticity of demand. With fixed meeting rates, a low $s(\theta)$ means relatively few sales overall and, thus, a small change in price leads to a relatively large proportionate change in sales. At the same time, with $s(\theta)$ low, the least-productive firm sets a relatively high p^* to cover its fixed cost δ with only a relatively small number of sales. This implies a *low* elasticity of demand for this firm relative to those of its nearest more-productive competitors. Thus, its markup will be higher than those of these slightly more-productive firms.

Note that productivity heterogeneity also alters the relationship between market tightness and the average markup. If the market conditions tend to be favorable to sellers (a high $\bar{u}$ for example), then N^* will be relatively high *but* $\tilde{z}^*$ will be relatively low. A higher N^* is associated with a lower θ^*, a higher probability of matching for a buyer and, thus, as described above, a higher elasticity of demand facing firms. This will tend to lead to a relatively low average markup. At the same time, a low $\tilde{z}^*$ is associated with a "less-competitive" market because of the increased presence of lower-productivity, higher-pricing firms and, therefore, a relatively *high* average markup. Thus, a relatively low θ^* in equilibrium is not necessarily associated with a low average markup. In fact, in our computed examples below, we find that in response to the changes in most of the economy parameters we consider, θ^* and the average markup move in *opposite* directions.

4.2 Pass-Through of Cost and Demand Changes to Prices and Markups

In this section, we characterize the equilibrium variables' responses to changes in the exogenous parameters $\bar{u}$, ϕ, δ, and f_e. To do so, we derive the elasticities of equilibrium variables with respect to each of these exogenous parameters. We use those elasticities to measure the degree of pass-through of parameter $x \in \{\bar{u}, \phi, \delta, f_e\}$ to the equilibrium variables of interest: prices and markups. We denote the elasticity of an equilibrium variable, $y(x)$, with respect to parameter x as ϵ_y^x.

First, we document the responses of firm- and industry-level prices and markups to a change in x through both direct and indirect effects that operate through a change in θ^* and $\tilde{z}^*$. We then examine the effects on market tightness, θ^*, and the cutoff productivity parameter for operation, $\tilde{z}^*$.

Before proceeding, we note that in our presentation of the computational examples in Section 5 we choose values for $\bar{u}$, ϕ, and f_e *relative* to fixed costs, δ. Hence, there we are only able to calculate prices that are scaled by δ, $p(v)/\delta$, for example. In anticipation of those experiments, here we derive pass-through expressions as functions of parameters and of prices scaled by fixed costs.

4.2.1 Firm-Level Responses

We begin with firm-level prices and markups. From (14) and (15), we may derive the following firm-level pass-through equations:

$$\epsilon^x_{p(v)} = \left(\frac{1}{D(G(v))(p(v)/\delta)}\right) \left[q_1 \frac{\bar{p^*}}{\delta} \epsilon^x_{p^*} + \left(\frac{\phi/\delta}{\tilde{z}^*} \int_v^1 h(w)dw - h(v)\left(\frac{p(v)}{\delta} - \frac{(\phi/\delta)v}{\tilde{z}^*}\right)\right)\epsilon^x_{\tilde{z}^*} - \begin{cases} \frac{\phi/\delta}{\tilde{z}^*} \int_v^1 h(w)dw & \text{if } x = \phi, \\ 0 & \text{if } x = \bar{u}, \delta, f_e \end{cases} \right]. \tag{39}$$

$$\epsilon^x_{mkup(v)} = \epsilon^x_{p(v)} - \begin{cases} 1 & \text{if } x = \phi \\ 0 & \text{if } x = \bar{u}, \delta, f_e. \end{cases} \tag{40}$$

Equation (39) decomposes the impact of a change in a parameter on a firm's price into three components. First, a change in a parameter will change the maximum posted price in the submarket, p^*. As expected, this will tend to cause firm-level prices and markups to move in the same direction as p^*, holding other effects constant.

Next, there is a response in the composition of firms as $\tilde{z}^*$ responds to a change in a parameter. Recalling that $h(\cdot) < 0$, we see that the first term in the coefficient on $\epsilon^x_{\tilde{z}^*}$ is negative, resulting in firm-level prices tending to move in the opposite direction as $\tilde{z}^*$ in response to a parameter shift. The intuition for this is that when, say, $\tilde{z}^*$ increases and less-productive firms exit the submarket, this puts downward pressure on the average price, making the conditions in the submarket more "competitive" and inducing firms to lower their prices. The second term in the coefficient on $\epsilon^x_{\tilde{z}^*}$, however, is positive, resulting in firm-level prices that tend to move in the same direction as $\tilde{z}^*$. The intuition here is that when $\tilde{z}$ increases, a firm's relative cost, v, also increases, putting upward pressure on that firm's price. Overall, then, when a parameter changes, the qualitative relationship between movements in $\tilde{z}^*$ and firm-level prices and markups, holding other effects constant, is ambiguous.

The final component is a direct effect that is present only when considering a change in ϕ. As expected, an increase in a firm's marginal cost will tend to increase its price. The effect on a firm's markup, however, is ambiguous. As expected, if pass-through of a change in ϕ to the firm's price is greater than one (more than complete), then the firm's markup will move in the same direction as its price in response to a change in the common marginal cost parameter.

In Section 5, below, we demonstrate in computational experiments that the relationship between firm productivity and firm-level price pass-through varies with the source of the parameter change. In particular, we find generally negative relationships between firm productivity and the degree of pass-through for movements in ϕ and δ and generally positive relationships between productivity and pass-through for movements in f_e and $\bar{u}$.

4.2.2 Industry-Level Responses

We now examine how industry-level variables respond to changes in economy parameters, beginning with the maximum equilibrium price p^*, while in the previous section, we analyzed the responses of firm-level variables. From equation (32), we may derive the following expression for pass-through of a change in parameter x to p^*:

$$\epsilon^x_{p^*} = \left(\frac{1}{c_2(p^*/\delta)}\right)\left[\left(\frac{\bar{u}}{\delta}\left(\epsilon^\theta_s - 1 - \epsilon^\theta_{s'}\right) + \frac{d_2}{s(\theta^*)}\right)\epsilon^\theta_s \epsilon^x_{\theta^*} + \begin{cases} -\frac{d_2}{s(\theta^*)} & \text{if } x = \delta \\ \frac{\bar{u}}{\delta}\left(1 - \epsilon^\theta_s\right) & \text{if } x = \bar{u}, \\ 0 & \text{if } x \in \{\phi, f_e\} \end{cases}\right]. \tag{41}$$

In this expression, the term on $\bar{u}/\delta$ is non-negative because ϵ^θ_s is non-increasing in θ. The composite parameter d_2, however, is negative. Hence, it is unclear if the coefficient on ϵ^*_θ in the above expression is positive or negative. Furthermore, the direct effects of a change in δ or $\bar{u}$ may reinforce or counter the effects of a change in θ^* on p^* in response to a change in a parameter. In our computational examples in Section 5, we find that p^* and θ^* respond in the same direction to most changes in economy parameters.

Next, we consider the effects of parameter changes on two measures of the average price: the average transaction price, $\bar{p}$, and the average posted price, $\bar{p}^P$. The average transaction price is given by (23) and the average posted price, $\bar{p}^P = \int_0^1 p(v)G'(v)dv$, can be written as

$$\bar{p}^P = c_3 p^* + d_3 \frac{\delta}{s(\theta)}, \tag{42}$$

where

$$c_3 = \int_0^1 \left(\int_v^1 h(w)dw - q_1 \right) \frac{G'(v)}{D(G(v))} dv \tag{43}$$

$$d_3 = -\left[\frac{1}{q_1}\right] \int_0^1 \left(\int_v^1 h(w)dw \right) \frac{G'(v)}{D(G(v))} dv, \tag{44}$$

with $c_3 > 0$ and $d_3 < 0$. Using these expressions, we may derive pass-through rates for each of these average price measures:

$$\epsilon_{\bar{p}}^x = \left(\frac{1}{\bar{p}/\delta}\right) \left[c_2 \left(\frac{p^*}{\delta}\right) \epsilon_{p^*}^x - \frac{d_2}{s(\theta^*)} \epsilon_s^\theta \epsilon_{\theta^*}^x + \begin{cases} \frac{d_2}{s(\theta^*)} & \text{if } x = \delta, \\ 0 & \text{if } x\{\bar{u}, \phi, f_e\} \end{cases} \right]. \tag{45}$$

$$\epsilon_{\bar{p}^P}^x = \left(\frac{1}{\bar{p}^P/\delta}\right) \left[c_3 \left(\frac{p^*}{\delta}\right) \epsilon_{p^*}^x - \frac{d_3}{s(\theta^*)} \epsilon_s^\theta \epsilon_{\theta^*}^x + \begin{cases} \frac{d_3}{s(\theta^*)} & \text{if } x = \delta, \\ 0 & \text{if } x \in \{\bar{u}, \phi, f_e\} \end{cases} \right]. \tag{46}$$

These expressions indicate that in response to a change in ϕ, $\bar{u}$, or f_e, if p^* and θ^* change in the same direction, then average prices will also change in that direction ($c_2 > 0$, $c_3 > 0$, $d_2 < 0$, $d_3 < 0$). This is intuitive: an increase in the maximum price will put upward pressure on average prices and an increase in market tightness affords more market power to firms, also putting upward pressure on average prices. The impact of a change in the fixed cost of production is more complicated. We anticipate that an increase in δ will increase p^* and because fewer firms will enter, this will also increase θ^*, both of which will tend to increase average prices. The rise in the fixed costs of production will, however, increase $\tilde{z}$, which tends to put downward pressure on average prices.

Turning to the impact on the firms' average markup, using equation (15), we may write this as

$$\overline{mkup} = c_4 \left(\frac{p^* \tilde{z}^*}{\phi} \right) + d_4, \tag{47}$$

where

$$c_4 = -q_1 \int_0^1 \left(\frac{G'(v)}{vD(G(v))} \right) dv \tag{48}$$

and

$$d_4 = \int_0^1 \left(\int_v^1 h(w)dw \right) \left(\frac{G'(v)}{vD(G(v))} \right) dv, \tag{49}$$

where $c_4 > 0$ and $d_4 > 0$. From this, we can derive pass-through for the average markup:

$$\epsilon^x_{\overline{mkup}} = \left(\frac{c_4}{\overline{\overline{mkup}}} \right) \left(\frac{p^* \tilde{z}^*}{\phi} \right) \left[\epsilon^x_{p^*} + \epsilon^x_{\tilde{z}^*} - \begin{cases} 1 & \text{if } x = \phi, \\ 0 & \text{if } x \in \{\bar{u}, \delta, f_e\} \end{cases} \right]. \tag{50}$$

This expression indicates that if p^* and $\tilde{z}^*$ respond in the same direction to a change in $\bar{u}$, δ, or f_e, then the average markup will also respond in that direction. Furthermore, if p^* and $\tilde{z}^*$ respond in the same direction and if either exhibit more than complete pass-through, then the average markup will move in the same direction as these variables.

We may also characterize the effect of a change in economy parameters on buyers' valuations, using equation (31):

$$\epsilon^x_{V_b} = \epsilon^x_{s'} \epsilon^x_{\theta^*} + \begin{cases} 1 & \text{if } x = \bar{u}, \\ 0 & \text{if } x \in \{\delta, \phi, f_e\}. \end{cases} \tag{51}$$

Now, since $\epsilon^x_{s'} < 0$, we see that tightness and buyers' utility will move in opposite directions in response to a change in ϕ, δ, or f_e, consistent with intuition. Furthermore, we demonstrate, below, that tightness falls with $\bar{u}$, so the above equation implies that an increase in the utility of direct consumption, $\bar{u}$, will raise the buyers' valuations, as expected.

Finally, to characterize the responses of θ^* and $\tilde{z}^*$ to changes in economy parameters, we may differentiate equations (35) and (36) with respect to $x \in \{\phi, \delta, f_e, \bar{u}\}$. Doing so and rearranging gives the following system of linear equations in $\epsilon^x_{\theta^*}$ and $\epsilon^x_{\tilde{z}^*}$:

$$\begin{bmatrix} \epsilon^x_{\theta^*} \\ \epsilon^x_{\tilde{z}^*} \end{bmatrix} = \begin{bmatrix} \theta^* \frac{\partial H_1(\cdot)}{\partial \theta^*} & \tilde{z}^* \frac{\partial H_1(\cdot)}{\partial \tilde{z}^*} \\ \theta^* \frac{\partial H_2(\cdot)}{\partial \theta^*} & \tilde{z}^* \frac{\partial H_2(\cdot)}{\partial \tilde{z}^*} \end{bmatrix}^{-1} \begin{bmatrix} -x \frac{\partial H_1(\cdot)}{\partial x} \\ -x \frac{\partial H_2(\cdot)}{\partial x} \end{bmatrix}. \tag{52}$$

We then have the following proposition, which characterizes the qualitative responses of θ^* and $\tilde{z}^*$ to changes in economy parameters.

Proposition 4. *The qualitative responses of market tightness and the cutoff productivity for operation to a change in economy parameters that can be definitively determined are as follows:*

(i.) $\epsilon^\phi_{\theta^*} > 0$, $\epsilon^\phi_{\tilde{z}^*} > 0$ *(ii.)* $\epsilon^\delta_{\theta^*} > 0$ *(iii.)* $\epsilon^{f_e}_{\theta^*} > 0$, $\epsilon^{f_e}_{\tilde{z}^*} < 0$ *(iv.)* $\epsilon^{\bar{u}}_{\theta^*} < 0$.

Proof: See Appendix A.

Positive pass-through to θ^* and $\tilde{z}^*$ in response to a change in ϕ is consistent with intuition. An increase in the variable cost of production should be associated with less entry; that is, with a fall in N^* and, so, a rise in θ^*. Similarly, it should be associated with a higher level of minimum productivity required for profitable production; that is, with a rise in $\tilde{z}$. A higher entry cost discourages entry, lowering N^* and raising θ^*. The resulting increase in market tightness, however, increases the probability that a firm will match with a buyer, which raises expected profits, allowing less-productive firms to produce ($\tilde{z}$ falls). Finally, an increase in δ or a decrease in a household's utility parameter, $\bar{u}$, should lead to lower profits, discouraging entry and, thereby, raising θ^*.

We end this section by briefly considering pass-through in the context of a specific matching function. Consider, again, the telephone line form (see Appendix C). Now the pass-through equations for p^* and $\bar{p}$ can be written as follows:

$$\epsilon^x_{p^*} = \left(\frac{1}{c_2(p^*/\delta)}\right) \left[\left(\frac{\bar{u}}{\delta} \frac{\theta^*}{(1+\theta^*)^2} + \frac{d_2}{\theta^*}\right) \epsilon^x_{\theta^*} + \begin{cases} -d_2 \frac{(1+\theta^*)}{\theta^*} & \text{if } x = \delta \\ \left(\frac{\bar{u}}{\delta}\right) \frac{\theta^*}{1+\theta^*} & \text{if } x = \bar{u}, \\ 0 & \text{if } x \in \{\phi, f_e\} \end{cases} \right]. \tag{53}$$

$$\epsilon^x_{\bar{p}} = \left(\frac{1}{\bar{p}/\delta}\right) \frac{\bar{u}}{\delta} \frac{\theta^*}{1+\theta^*} \left[\frac{\epsilon^x_{\theta^*}}{1+\theta^*} + \begin{cases} 1 & \text{if } x = \bar{u}, \\ 0 & \text{if } x \in \{\phi, \delta, f_e\} \end{cases} \right]. \tag{54}$$

The second expression makes it clear that with this matching function, the average transaction price (and the average posted price) and tightness will move in the same direction in response to a change in ϕ, δ, or f_e. Average prices may, however, move in opposite directions when $\bar{u}$ changes, since θ^* falls with $\bar{u}$.

The intuition for why market tightness and average prices will move in the same direction with the telephone line matching function is as follows. An increase in θ^* is associated with a rise in the sellers' probability of matching and, therefore, with a fall in firms' elasticity of demand, as discussed above. These forces tend to increase the average price. The increase in $\tilde{z}^*$, however, raises average productivity and this puts downward pressure on the average price. With the telephone line matching function, the first effect dominates and the average price rises in response to a rise in ϕ or δ.

5 Computational Examples

In this section, we delve deeper into the model's properties through a series of computational examples. These exercises enable us to investigate the qualitative behavior of the equilibrium variables in response to changes in the economy parameters for those cases where analytical results could not be derived in the previous section. Additionally, by selecting parameter values that are informed by existing empirical studies, we are able to examine key qualitative features of the model, such as whether the firm-level and average-price pass-through are incomplete. Furthermore, we explore how firms' pricing responses vary according to their productivity level.

5.1 Productivity, Meeting Probabilities, and Other Model Parameters

For our computational exercises, we require a specification of the *ex ante* productivity distribution, $J(z)$. In our computational experiments, we follow a large number of papers in the firm-level heterogeneity literature in assuming that z follows a Pareto distribution with shape parameter γ and use the estimate $\gamma = 3.4764$ from Chernoff et al. (2024) for the Canadian retail clothing subsector.

For the meeting probabilities conditional on matching q_k for all k, we use a distribution that accommodates a fully non-parametric specification of q_1 and q_2, letting these parameters vary freely between 0 and 1, subject to the constraint that $q_1 + q_2 < 1$. We let the maximum number of meetings be infinity and we specify that the meeting probabilities in the tail for $k \geq 3$ decline according to a power function governed by the parameter ν. That is,

$$q_k = q_3 \nu^{k-3} \qquad \text{for } k \geq 4,\ \nu \in (0,1) \text{ and } q_3 \equiv (1 - q_1 - q_2)(1 - \nu). \tag{55}$$

Chernoff et al. (2024) argue that this type of probability distribution over the number of meetings provides a better fit to several salient features of the data when compared to other specifications that are commonly used in the search literature, notably including the Bernoulli and Poisson distributions. In Appendix E, we demonstrate that this is also the case in our setting by comparing our results to those generated from these alternative distributions. For our computational experiments we, again, use the estimates from Chernoff et al. (2024) for the Canadian retail clothing subsector: $q_1 = 0.1842$, $q_2 = 0.5829$, and $\nu = 0.9651$. With this process, the median number of meetings for a matched household is two and the percentage of households that receive three or fewer such trading opportunities is approximately 78%.

For our computational exercises, we also require values for three additional parameters relative to the fixed costs: the household utility parameter, $\bar{u}/\delta$, the common cost parameter, ϕ/δ, and the cost of entry, f_e/δ.[2] Specifying these parameters relative to δ reduces by one the number of parameters we need to specify while still allowing us to calculate the elasticities with respect to each of these three parameters and also δ. We calibrate these three parameters to target the mean markup of 1.4096 for the clothing subsector, as reported in Chernoff et al. (2024). We are able to exactly match this target using $u/\delta = 100$, $\phi/\delta = 34.43$, and $f_e/\delta = 1.475$.

We note that this strategy can only determine one of these three parameters. However, in Appendix D, we demonstrate that our results are qualitatively robust to other configurations of these parameters. There, we show that our main results continue to hold for other combinations of the parameters that also generate an average markup matching our target of 1.4096.

[2]The lower support of the ex-ante firm productivity distribution, z_L, appears only in our free-entry condition. We divide both the free-entry and the zero-profit conditions by z_L, such that the parameter ϕ in our computational examples should be implicitly interpreted as normalized relative to z_L.

5.2 Prices, Markups and Revenues

In this section, we examine the equilibrium distributions of prices, markups, and revenues, given the model parameters chosen above. We also present plots of the markups as functions of the measure of relative productivity described in the previous section, $1 - v = (z - \tilde{z})/z$.

Figure 1 presents the prices, markups, and revenue distributions and plots markups as a function of relative productivity.[3] These figures are qualitatively consistent with several well-documented findings in the literature. For example, in Panel D, the largely positive relationship between markups and firm size is consistent with both empirical and theoretical research on firm heterogeneity and varying endogenous markups. For example, Autor et al. (2020), De Loecker et al. (2020), and De Loecker and Warzynski (2012) use U.S. firm-level data to provide evidence that larger firms tend to charge higher markups than smaller ones do.

In Panel B, we plot the probability density function for the log of scaled revenue. We see that our calibrated model generates a two-tailed distribution with a broadly single-peaked density.[4] While a two-tailed log revenue distribution is a well-documented stylized fact, it is not an inherent feature of our model. In Appendix E, we show that when we change our search process to the commonly used Bernoulli or Poisson distributions, the log revenue distribution becomes single tailed and skewed to the left.

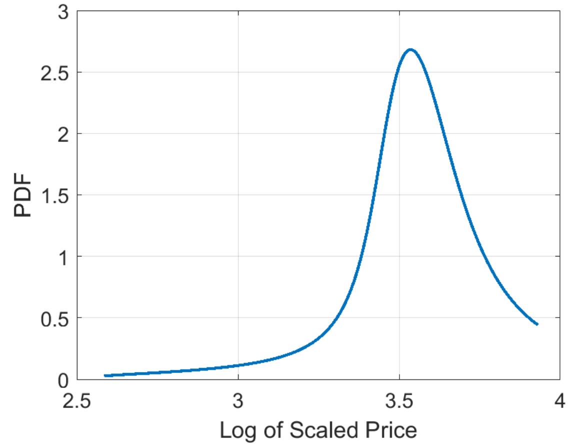
With regard to the PDF of prices in Panel A, note also that when compared to these alternative specifications of the search process, our preferred specification tends to generate a greater degree of price dispersion. As we discuss below, the higher degree of price dispersion in our model arises in part because some consumers are very well-informed (that is, they meet with a larger number of firms and, thus, have many opportunities to trade). This higher degree of price dispersion arises not as an inherent feature of our model but because our baseline parameter configuration implies that there are some highly informed consumers and a large degree of heterogeneity in firm productivity.

In Appendix E, we present figures for various other configurations of the parameters that govern the meeting probabilities and the shape of the productivity distribution. Our objective with those

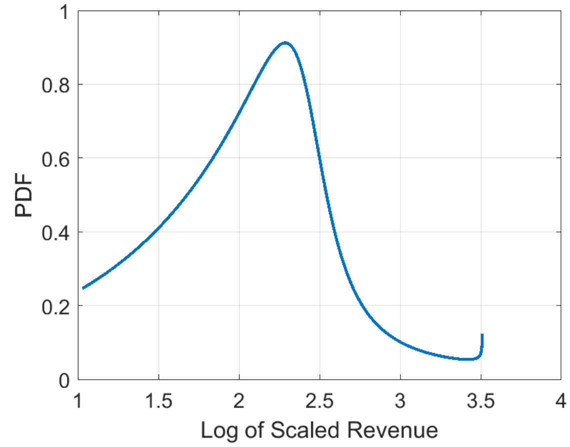
[3] While we are able to derive analytical expressions for revenues and prices, the non-monotonicity of markups in productivity make it difficult to derive the markup PDF. We therefore represent the markup distribution, using a histogram of simulated markups.

[4] The upturn in the density for the very highly productive firms occurs as these firms face very similar costs and demand conditions and so have very similar revenues.

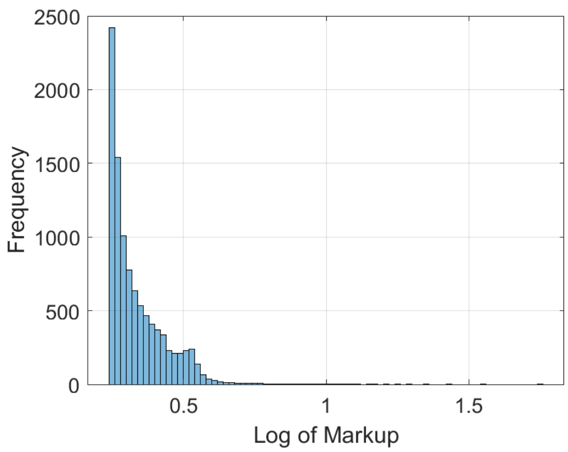
Figure 1: Markups, Prices and Revenues with Baseline Parameters



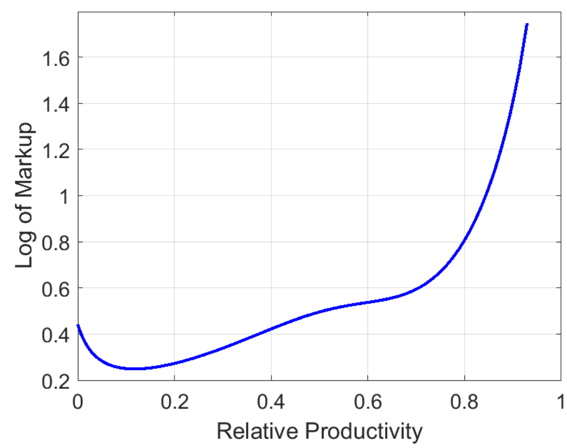
A. PDF of the Log of Scaled Price



B. PDF of Log of Scaled Revenue



C. Histogram of the Log of Markups



D. Log of Markups

Notes: Panels A and B display the probability density functions (PDF) of the log of the scaled prices (Panel A) and the log of the scaled revenue (Panel B). Panel C displays a histogram of the log of markups, and panel D plots the log of markups as a function of relative productivity. All plots use the parameter estimates from Chernoff, Head, and Lapham (2024) for the search process and the Pareto shape parameter. These plots also use the following values: $\phi/\delta = 34.43$, $f_e/\delta = 1.475$ and $u/\delta = 100$.

figures is to demonstrate that when we use some other parameter configurations that are commonly used in the literature, the model exhibits counterfactual results regarding the revenue distributions and/or markups. Thus, we believe that our baseline parameter configuration specified above is well-suited for examining the model's qualitative and quantitative results regarding pass-through in the next section.

Here, we briefly outline our alternative specifications. In Appendix E, we provide more details and graphical results. We first consider two parameter settings in which the maximum number of prices households can observe equals two and then we examine two scenarios in which the meeting probabilities follow a Poisson distribution. In all of these cases, the log-revenue function exhibits highly counterfactual properties in that it is highly left-skewed. Lastly, we dramatically increase the shape parameter in the Pareto productivity distribution (thus greatly reducing the degree of heterogeneity in firm productivity) and demonstrate that this leads to two counterfactual results: low levels of price dispersion and a largely negative relationship between a firm's markup and its size.

5.3 Pass-Through of Changes in Economy Parameters

We now turn to an examination of the firm- and industry-level variables' quantitative responses to changes in the four economy variables we examined analytically in Section 4. We begin with industry-level variables as movements in those variables help us better understand the responses of firm-level variables.

5.3.1 Pass-Through at the Industry Level

Table 1 presents the elasticities for market tightness, the cutoff productivity for operation, the buyers' valuation, the maximum price, our two measures of average prices, and the average markup. For ease of exposition, we frame the discussion around movements that lead to the entry of less-productive firms; that is, to changes in the economy parameters that lead to a fall in $\tilde{z}$. Thus, we focus on adjustments in the parameters that are associated with reductions in fixed and variable production costs, ϕ and δ or, with more-favorable demand conditions, a rise in $\bar{u}$. We also expect that a rise in the entry costs, f_e, will be associated with a fall in $\tilde{z}$. This is the case because higher costs of entry raise θ as fewer firms enter, thereby increasing the probability that a firm will match with a buyer, allowing less-productive firms to produce.

Focusing on the first two rows of Table 1, note that market tightness and the cutoff productivity for operation move in a manner consistent with both intuition and Proposition 4. A fall in the production costs or a rise in the household utility parameter induce entry, causing N^* to rise and θ^* to fall. Any of these changes makes the environment more favorable for sellers and, thus, allows less-productive firms to operate, resulting in a reduction of $\tilde{z}$. A rise in the entry costs reduces entry, raising θ^* and again lowering $\tilde{z}$. Not surprisingly, the buyers' valuation, V_b, moves in the opposite direction as θ^* as a buyer's probability of matching is negatively associated with θ^*.

Turning to industry-level price measures, note that a fall in the production costs lowers both the maximum price and average prices. This is intuitive and as we would expect from our analytical results in the previous section (see (45) and (46)). We note that pass-through of a change in the variable cost of production, ϕ, to average prices is incomplete, with approximately 50% of a variable cost change being passed through. Also, as expected, a rise in the entry costs or a rise in the household utility parameter are reflected in a higher maximum price and higher average prices.

Lastly, we consider the response of average markups. Because pass-through to average prices is incomplete in response to a change in ϕ, the average markup rises when variable production costs fall. This is to be expected in our environment with market power arising from households having imperfect information regarding prices, but it is also affected by the presence of firm heterogeneity. In contrast, average markups fall when fixed production costs fall, a result primarily driven by the rise in the elasticity of demand associated with the fall in market tightness. A small increase in the entry costs slightly lowers the average markup as less-productive firms enter the market. Finally, a rise in $\bar{u}$ is associated a rise in the average markup, which is intuitive: more-favorable demand conditions allow for higher average markups.

5.3.2 Pass-Through at the Firm Level

We now turn our attention to our primary issue of interest, the response of firm-level prices to changes in economy parameters. We seek to quantify how those responses vary across firms according to their size and productivity. Figure 2 depicts firm-level price pass-through for incumbent firms in response to changes in the four economy parameters as functions of firms' relative productivities. The figure decomposes those responses into the three channels described in Section 4; that

Table 1: Pass-Through of Cost and Demand Shocks to Equilibrium Outcomes

	ϕ	δ	f_e	$\bar{u}$
θ^*	0.7492	0.1223	0.2155	-1.0870
$\tilde{z}^*$	0.3409	0.0192	-0.1896	-0.1704
V_b^*	-0.4468	-0.0729	-0.1285	1.6482
p^*	0.2354	0.3145	0.0677	0.3823
Mean transaction price	0.5258	0.0858	0.1513	0.2371
Mean posted price	0.4741	0.1266	0.1364	0.2629
Mean markup	-0.1665	0.1311	-0.0479	0.0832

Notes: The estimates show the elasticity of the variables in the rows with respect to the parameters in the columns. The elasticity estimates use the parameter estimates from Chernoff, Head, and Lapham (2024) for the search process and the Pareto shape parameter. These estimates also use the following values: $\phi/\delta = 34.43$, $f_e/\delta = 1.475$ and $u/\delta = 100$.

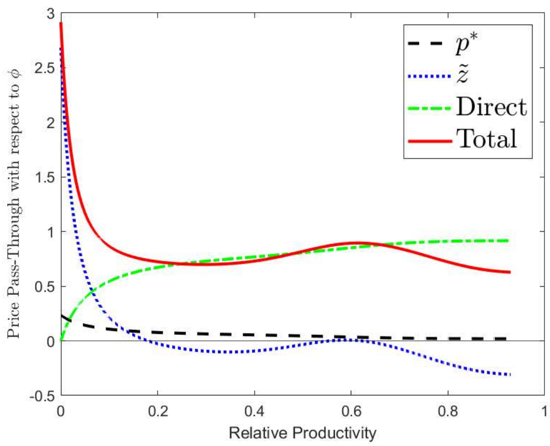
is, through changes in the maximum price, p^*, changes in the composition of firms, $\tilde{z}$, and, in the case of a change in ϕ, through a direct change in the variable costs of production. In each figure, the solid line depicts total pass-through.

Beginning with a change in ϕ in Panel A, we see that there is a non-monotonic relationship between a firm's productivity (and, therefore, its size) and pass-through of a common cost change. Generally, however, the figure shows that small firms change their prices more in response to a common change in the variable production costs than large firms do. These properties are similar to the results presented in Chernoff et al. (2024) for the long-run pass-through of a common cost change.

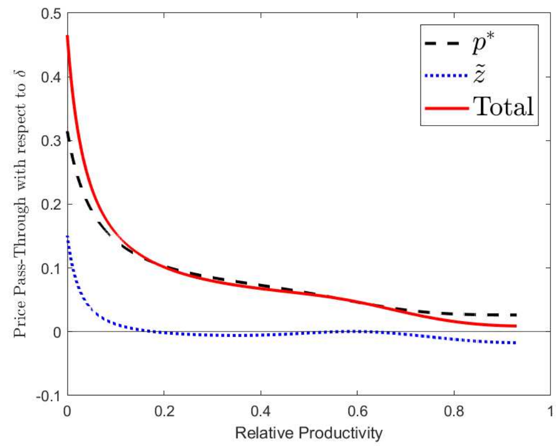
Looking more carefully into the various forces that affect firm-level pass-through, we note that the effects of a change in the maximum price, p^*, are straightforward: a fall in the maximum price due to a fall in ϕ negatively affects all firms' prices but the effects are strongest for the lower-productivity firms that are pricing near that maximum price. Similarly, the direct effect of a change in ϕ causes all firms to lower their prices and the strength of this force rises with a firm's productivity. Hence, the non-monotonic shape of the total response is largely determined by the effects stemming from a change in the composition of the firms operating in the industry.

The generally negative relationship between firm-level pass-through and firm size can be understood as follows: smaller firms tend to be less productive, resulting in higher prices and a customer base consisting primarily of buyers with limited alternatives. Thus, these firms face higher demand elasticity and are more responsive to changes in their costs. For these smaller firms, a reduction in

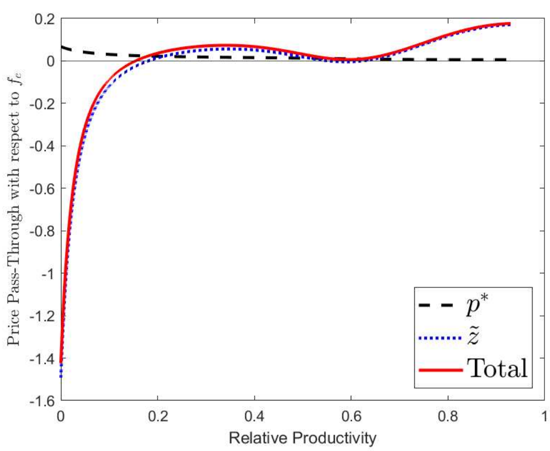
Figure 2: Pass-through to Firm-Level Prices



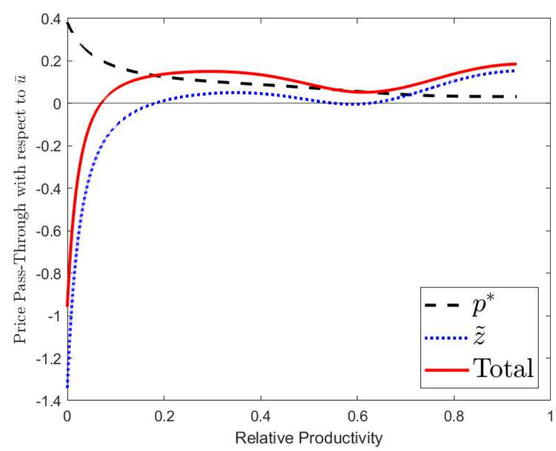
A. Pass-through with respect to ϕ



B. Pass-through with respect to δ



C. Pass-through with respect to f_e



D. Pass-through with respect to $\bar{u}$

Notes: All plots use the parameter estimates from Chernoff, Head, and Lapham (2024) for the search process and the Pareto shape parameter. These plots also use the following values: $\phi/\delta = 34.43$, $f_e/\delta = 1.475$ and $u/\delta = 100$.

price—enabled by a cost reduction—allows them to capture significant sales from competitors.

In contrast, larger, more-productive firms are able to charge lower prices and cater to a broader customer base that includes many households with alternative options. These firms face lower demand elasticities and, thus, respond less aggressively to cost reductions. Their muted price adjustments reflect their ability to retain a substantial portion of their sales, even as less-productive firms make aggressive price reductions.

Next, consider a fall in δ, the fixed cost of operating. In Panel B, we observe a negative relationship between firm size and its pass-through rate in response to a change in fixed costs. While changes in δ have no direct effect on pricing, a reduction in the fixed costs increases profits conditional on z, leading to firm entry, as reflected in a lower θ^* and lower $\tilde{z}$. The figure shows that the responses of both p^* and $\tilde{z}$ play a significant role in determining the overall shape of the pass-through function. Not surprisingly, these effects are strongest for low-productivity firms that are pricing near p^*.

The figure shows that the responses of both p^* and $\tilde{z}$ play a significant role in determining the overall shape of the pass-through function. Not surprisingly, these effects are strongest for low-productivity firms that are pricing near p^*.

In Panels C and D, we present firm-level pass-through in response to a change in the entry cost and a change in the household's utility parameter, $\bar{u}$, respectively. These pass-through functions have similar shapes and share the feature that their shapes are primarily determined by the effects emanating from a change in the composition of the firms in the industry. For those firms above a certain level of productivity, the pass-through rates for both types of parameter changes are positive but relatively modest and those rates do not exhibit a monotonic relationship with firm size. Smaller, less-productive firms exhibit negative pass-through, suggesting that they lower their prices when entry costs increase and when $\bar{u}$ increases. In each case, those firms are strongly responding to the entry of relatively lower-productivity firms. As noted in the discussion following the analytical expression for firm-level pass-through, given by equation (39), the relationship between the movements in $\tilde{z}$ and a firm's price is ambiguous due to opposing forces. In this computational example, the positive effects of a change in $\tilde{z}$ appear to dominate for the least-productive firms.

Firm heterogeneity plays a crucial role in affecting the relationships between firm size and the magnitude and direction of a firm's response to changes in economy parameters. In Appendix E, we present Figure 12, which depicts pass-through rates as a function of relative productivity when we consider a near-degenerate productivity process ($\gamma = 100$). Although the general qualitative relationships are similar between the two economies (pass-through decreasing in firm size for a change in ϕ or a change in δ and increasing in firm size for a change in f_e or a change in $\bar{u}$), the shape of the functions are quite different. Furthermore, in the near-degenerate economy, most firms exhibit either nearly complete or nearly incomplete pass-through in response to changes in economy parameters, which contrasts with the pass-through rates in our model with heterogeneous firms. In the computational exercises presented in this section, we employed a specific matching function, the telephone line matching function. In Appendix F, however, we consider a similar economy but with an alternative matching function, the urn-ball matching function. There, we show that the qualitative and quantitative results we documented in this section continue to hold under that alternate matching technology.

Overall, the results of these computational experiments demonstrate that firms with varying productivity levels–and, consequently, different sizes–respond quite differently to changes in the economic environment. We offer additional insights into these specific responses and their impact on average prices. Hence, our analysis underscores the importance of accounting for the individual firm price and markup adjustments to fully understand pass-through to average prices.

Furthermore, the substantial heterogeneity in pass-through across firms we document may be particularly significant when addressing other issues that are beyond our current theoretical framework. In particular, focusing on heterogeneous firm-level responses highlights the potential distributional impact of shocks across households purchasing in different regions of the price distribution. For example, given that the pass-through function for a change in the variable production costs is increasing in firms' prices (decreasing in relative productivities), households that receive more price quotes will benefit less from the cost reduction than will less-informed households.

6 Conclusion

We construct a model of search frictions driven by consumers' incomplete information regarding trading opportunities and we use it to study the determination of market power at both the firm and industry level in a setting with free entry of firms that are heterogeneous with regard to productivity. Using the framework of sequential mixed search, we show how countervailing forces may make the relationship between market tightness and market power ambiguous. It may be natural to think of the conditions we expect to be favorable to buyers, such as high matching relative to that for firms, to be associated with relatively low markups. Here, we show that given the interaction between directed search and random matching, this may or may not be the case.

In equilibrium, prices fall monotonically with productivity, while markups generally increase with productivity and firm size. Depending on the conditions, however, specifically when market tightness is low and the matching rate for buyers is high, markups may be falling in productivity and firm size for the least-productive (highest-pricing) firms. In contrast to settings with homogeneous firms, in our environment, firms' elasticity of demand tends to fall with productivity and size, due to the interaction between pricing and the distribution of productivity. This results in increasing markups. But, with low market tightness overall, the least-productive firm must set relatively high markups to cover its fixed operating costs, given that it has a relatively small number of sales. This dynamic creates a non-monotonic relationship between firm size and markups, with markups declining at low productivity levels but rising as productivity increases.

Search and productivity heterogeneity combine to determine how both firm- and industry-level average prices respond to a variety of exogenous shocks or changes in parameters. At the outset, directed search determines the maximum price charged in equilibrium, the overall market tightness, the consumer and firm matching rates, and the distribution of markups across firms by productivity. At this stage, the factors we think of as improving the position of buyers tend to raise the matching rate for consumers (lower market tightness) and to lower markups. The random matching that follows, however, may diminish this effect or even overcome it. Lower market tightness can enable less-productive firms to survive in the market, thus, tending to raise markups overall. The net effect depends on the forces driving market tightness. In particular, the fixed costs of entry determine market tightness but not the probabilities with which consumers meet specific firms and, thus, drive tightness and average prices in opposite directions.

Pass-through at the firm level varies dramatically across firms of different productivities and the overall relationship differs substantially by the type of shock. Shocks to fixed operating costs and common marginal costs are passed through positively at rates that tend to decline with productivity. Low-productivity firms have to significantly raise their prices to cover their fixed operating costs in response to these changes. In contrast, shocks to the *entry* costs and consumers' utility from consumption are passed through at rates that *increase* with productivity. Moreover, for relatively unproductive firms, pass-through of these shocks can be negative. That is, those firms at the lower end of the productivity distribution may *lower* their prices, even if average prices rise.

Overall, our findings demonstrate that in the presence of search frictions and firm heterogeneity, the determinants of market power at both the firm and industry level are complicated. Consequently, assuming a stable relationship between market tightness and market power can be misleading. Put differently, market conditions may appear to become more or less favorable to consumers and yet result in either a reduction or an increase in market power. Moreover, pass-through varies dramatically across firms at different points in the productivity and size distributions. To the extent that these firms serve consumers with varying levels of information about trading opportunities, this differential pass-through has welfare implications and may contribute to inequality.

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Appendix A: Proofs and Derivations

Proof of Proposition 1 (Monotonicity of the Pricing Function, $p(v)$)

The approach in this proof follows the approach of the proof of Lemma 1 in Herrenbrueck (2017), which follows the proof in Burdett and Mortensen (1998). To economize on the notation, here, we suppress the dependence of the distribution of the posted prices, $L(p)$, on the submarket $(\tilde{p}, \theta$. Let $z_1 > z_2$ and let $p_j \equiv p(z_j)$ for $j \in \{1, 2\}$. To establish the proposition, it is sufficient to show that $p_1 < p_2$. Let $\tilde{\pi}(p_j, z_i) \equiv \left(p_j - \frac{\phi}{z_i}\right) \mu A \left(1 - L(p_j)\right) - \Psi$. We have the following inequalities:

$$\tilde{\pi}(p_1, z_1) \geq \tilde{\pi}(p_2, z_1) > \tilde{\pi}(p_2, z_2) \geq \tilde{\pi}(p_1, z_2). \tag{56}$$

The first and third inequalities follow from profit maximization and the second inequality follows from $z_1 > z_2$. Subtracting the fourth term from the first term and the third term from the second term, we have

$$\tilde{\pi}(p_1, z_1) - \tilde{\pi}(p_1, z_2) \geq \tilde{\pi}(p_2, z_1) - \tilde{\pi}(p_2, z_2), \tag{57}$$

or

$$\left(p_1 - \frac{\phi}{z_1}\right) s(\theta) D \left(1 - L(p_1)\right) - \left(p_1 - \frac{\phi}{z_2}\right) s(\theta) D \left(1 - L(p_1)\right) \geq \left(p_2 - \frac{\phi}{z_1}\right) s(\theta) D \left(1 - L(p_2)\right) - \left(p_2 - \frac{\phi}{z_2}\right) s(\theta) D \left(1 - L(p_2)\right). \tag{58}$$

We can rearrange this to derive

$$\left(\frac{\phi}{z_2} - \frac{\phi}{z_1}\right) D \left(1 - L(p_1)\right) \geq \left(\frac{\phi}{z_2} - \frac{\phi}{z_1}\right) D \left(1 - L(p_2)\right), \tag{59}$$

or, since $z_1 > z_2$, we have

$$D \left(1 - L(p_1)\right) \geq D \left(1 - L(p_2)\right). \tag{60}$$

Now, because $L(p)$ is a CDF, $L(p)$ is strictly increasing in p. From the definition of $A((1 - L(p))$, we see that this function is strictly decreasing in $L(p)$. It follows, then, that $A((1 - L(p))$ is strictly decreasing in p. Hence, from equation (60), we can conclude that $p_1 \leq p_2$.

Suppose, now, that $p_1 = p_2$. Given that the distribution of productivity, $F(z)$, is continuous with connected support, $z_1 > z_2$ implies that there exists a positive measure of firms maximizing profits by posting the same price. This contradicts the result that $L(p)$ is a continuous distribution with connected support, established by Burdett and Judd (1983), Lemma 1, p.959. ■

Derivation of the Pricing Function, (11)

The first-order condition for profit maximization given by equation (10) can be rewritten as

$$p'(z) + V(z)p(z) = \left(\frac{\phi}{z}\right) V(z), \tag{61}$$

where $V(z) \equiv \frac{D'(F(z))F'(z)}{D(F(z))}$. This is in a standard form of a linear first-order differential equation whose solution is given by

$$p(z) = \left(\frac{1}{\chi(z)}\right) \left(\int \chi(z) \left(\frac{\phi}{z}\right) V(z)dz + C\right), \tag{62}$$

where $\chi(z) = exp\left(\int V(z)dz\right)$. Given the definition of $V(z)$, we have $\chi(z) = exp\left(\ln(A(F(z)))\right) = D(F(z))$. Substituting this and the expression for $V(z)$ into equation (62) gives

$$p(z) = \left(\frac{1}{D(F(z))}\right) \left(\int \left(\frac{\phi}{z}\right) D'(F(z))F'(z)dz\right). \tag{63}$$

Recalling that an indefinite integral can be written as a definite integral plus an arbitrary constant, we can write the above as follows:

$$p(z) = \left(\frac{1}{D(F(z))}\right) \left(C + \int_{\tilde{z}}^{z} \left(\frac{\phi}{x}\right) D'(F(x))F'(x)dx\right), \tag{64}$$

where $C = \int_a^{\tilde{z}} \left(\frac{\phi}{x}\right) D'(F(x))F'(x)dx$ with a as an arbitrary constant. We impose the condition that the maximum price, $\tilde{p}$ is associated with $\tilde{z}$ to solve for C:

$$p(z) = \frac{C}{D(J(\tilde{z}))} = \frac{C}{q_1} \longrightarrow C = \tilde{p}q_1. \tag{65}$$

Substituting this into equation (64) gives the pricing function, (11):

$$p(z) = \left(\frac{1}{D(F(z))}\right) \left(\tilde{p}q_1 + \phi \int_{\tilde{z}}^{z} \frac{D'(F(x))F'(x)}{x} dx\right). \qquad ■$$

Proof of Proposition 2 (Existence and Uniqueness of the SMS Equilibrium)

Let (33) implicitly define a relationship between $\tilde{z}$ and θ, which we refer to as FE as it arises from the following free-entry condition:

$$J(\tilde{z}_{FE}(\theta)) = 1 - \frac{f_e/\delta}{\frac{c_1}{c_2}\left[\frac{\bar{u}}{\delta}s(\theta)(1-\epsilon_s^\theta) - 1\right]} \tag{66}$$

$$= 1 - \frac{a_1}{g(\theta)}. \tag{67}$$

Given that $c_1 > 0$, $c_2 > 0$, $s'(\theta) > 0$, and ϵ_s^θ is non-increasing in θ, we have $g'(\theta) > 0$. Thus,

$$\left[J'(\tilde{z})\right]\left(\tilde{z}'_{FE}(\theta)\right) = \frac{a_1 g'(\theta)}{g(\theta)^2} > 0. \tag{68}$$

As $J'(\tilde{z}) > 0$, we then have $\tilde{z}'_{FE}(\theta) > 0$. That is, FE is increasing in $(\tilde{z}, \theta)$ space.

Similarly, let (34) implicitly define a relationship between $\tilde{z}$ and θ, which we refer to as ZP as it arises from the zero-profit condition in the second stage:

$$\tilde{z}_{ZP}(\theta) = \frac{(\phi/\delta)q_1}{\frac{q_1}{c_2}\left(\frac{\bar{u}}{\delta}(1-\epsilon_s^\theta) - \frac{1}{s(\theta)}\right)} \tag{69}$$

$$= \frac{a_2}{k(\theta)}. \tag{70}$$

Then, as $s'(\theta) > 0$ and ϵ_s^θ is non-increasing in θ, we have $k'(\theta) > 0$. Thus,

$$\tilde{z}'_{ZP}(\theta) = -\frac{a_2 k'(\theta)}{k(\theta)^2} < 0 \tag{71}$$

and ZP is downward sloping in $(\tilde{z}, \theta)$ space.

Thus, if an SMS equilibrium, $(\tilde{z}^*, \theta^*)$, exists, then it must be unique. All that remains to be shown is that FE and ZP cross in an acceptable region of $(\tilde{z}, \theta)$ space. The free-entry condition (30) bounds N and thus θ^* and is required for the derivation of (33). Given a form of $s(\theta)$, the choices of $\bar{u}$, ϕ, q_1 and δ that satisfy (9) can be made to guarantee that $\tilde{z}^* > z_L$. This is required for the derivation of (34). Thus, under these conditions, a sequential mixed-search equilibrium exists and is unique. ■.

Proof of Proposition 3 (Slope of the Markup Function at $v = 1$)

The elasticity of the markup function given by (15) with respect to v is given by

$$\epsilon^v_{mkup(v)} = \epsilon^v_{p(v)} - 1, \tag{72}$$

where

$$\epsilon^v_{p(v)} = \left(\frac{h(v)}{D(G(v))}\right)\left(\frac{\phi/z - p(v)}{p(v)}\right) > 0. \tag{73}$$

Evaluating these expressions for the least-productive firm, $v = 1$, we have

$$\epsilon^v_{mkup(1)} = \frac{-2q_2G'(1)}{q1 \times elas(1)} - 1. \tag{74}$$

Hence, the markup function will be decreasing at $v = 1$ if and only if

$$elas(1) < \frac{-2q_2G'(1)}{q1}. \tag{75}$$

Using the zero-profit condition, we have

$$elas(1) = \frac{q_1p^*s(\theta)}{\delta}. \tag{76}$$

Hence, the above condition can be written as

$$\left(\frac{p^*}{\delta}\right)s(\theta) < \frac{-2q_2G'(1)}{q_1^2}. \tag{77}$$

Recall, from above, that the equilibrium maximum scaled price is given by

$$\frac{p^*}{\delta} = \left(\frac{1}{c_2}\right)\left[\frac{u}{\delta}\left(1 - \epsilon^\theta_s(\theta)\right) - \frac{d_2}{s(\theta)}\right], \tag{78}$$

where $\epsilon^\theta_s(\theta)$ is the elasticity of $s(\theta)$. Hence, the condition above, which guarantees that the markup will be decreasing at $v = 1$, can be written as

$$s(\theta)\left(1 - \epsilon^\theta_s(\theta)\right) < \frac{d_2q_1^2 - 2c_2q_2G'(1)}{q_1^2(\bar{u}/\delta)}. \tag{79}$$

Now, since $\epsilon_s^\theta(\theta)$ is non-increasing in θ, the left-hand side of equation (79) is increasing in θ and the result follows. ■

Proof of Proposition 4 (Qualitative Properties of Responses of θ^* and $\tilde{z}^*$ to Parameter Changes) From (82) - (85), we know that the determinant of the matrix on the right-hand side of equation (52) is positive. Hence, using Cramer's Rule, the sign of $\epsilon_{\theta^*}^x$ is determined by the sign of the determinant of

$$\begin{bmatrix} -x\frac{\partial H_1(\cdot)}{\partial x} & \tilde{z}^*\frac{\partial H_1(\cdot)}{\partial \tilde{z}^*} \\ -x\frac{\partial H_2(\cdot)}{\partial x} & \tilde{z}^*\frac{\partial H_2(\cdot)}{\partial \tilde{z}^*} \end{bmatrix}, \tag{80}$$

and the sign of $\epsilon_{\tilde{z}^*}$ is determined by the sign of the determinant of

$$\begin{bmatrix} \theta^*\frac{\partial H_1(\cdot)}{\partial \theta^*} & -x\frac{\partial H_1(\cdot)}{\partial x} \\ \theta^*\frac{\partial H_2(\cdot)}{\partial \theta^*} & -x\frac{\partial H_2(\cdot)}{\partial x} \end{bmatrix}. \tag{81}$$

Using the signs of these derivatives given in equations (82)-(89), we can definitively determine the signs of the determinants in equation (80) for $x \in \{\phi, f_e, \bar{u}\}$ and the signs of the determinants in equation (81) for $x \in \{\phi, f_e\}$. The results then follow. ■.

Appendix B: Linear System of Elasticities

The elements of the matrix on the right-hand side of equation (52) are given by

$$\theta^*\frac{\partial H_1(\cdot)}{\partial \theta^*} = \frac{c_1}{c_2}\frac{\bar{u}}{\delta}(J(\tilde{z}^*)-1)s(\theta^*)\epsilon_s^\theta\epsilon_{s'}^\theta > 0, \tag{82}$$

$$\tilde{z}^*\frac{\partial H_1(\cdot)}{\partial \tilde{z}^*} = \frac{f_e}{\delta}\left(\frac{\tilde{z}^*J'(\tilde{z}^*)}{J(\tilde{z}^*)-1}\right) < 0, \tag{83}$$

$$\theta^*\frac{\partial H_2(\cdot)}{\partial \theta^*} = \tilde{z}^*\epsilon_s^\theta\frac{q_1}{c_2}\left[\frac{\bar{u}}{\delta}\left(\epsilon_s^\theta - 1 - \epsilon_{s'}^\theta\right) + \frac{1}{s(\theta^*)}\right] > 0, \tag{84}$$

$$\tilde{z}^*\frac{\partial H_2(\cdot)}{\partial \tilde{z}^*} = q_1\frac{\phi}{\delta} > 0. \tag{85}$$

The inequality in (82) holds, given that $s(\theta)$ is concave, and the inequality in equation (84) holds, given that ϵ_s^θ is non-increasing in θ.

The semi-elasticities of these functions with respect to the various parameters in the vector on the right-hand side of (52) are as follows:

$$-\phi\frac{\partial H_1(\cdot)}{\partial \phi} = 0 \qquad -\phi\frac{\partial H_2(\cdot)}{\partial \phi} = q_1\frac{\phi}{\delta} > 0 \tag{86}$$

$$-\delta\frac{\partial H_1(\cdot)}{\partial \delta} = (1 - J(\tilde{z}))\frac{c_1}{c_2} > 0 \qquad -\delta\frac{\partial H_2(\cdot)}{\partial \delta} = \tilde{z}\frac{q_1}{c_2}\frac{1}{s(\theta)} > 0 \tag{87}$$

$$-f_e\frac{\partial H_1(\cdot)}{\partial f_e} = \frac{f_e}{\delta} > 0 \qquad -f_e\frac{\partial H_2(\cdot)}{\partial f_e} = 0 \tag{88}$$

$$-\bar{u}\frac{\partial H_1(\cdot)}{\partial \bar{u}} = \frac{c_1}{c_2}\frac{\bar{u}}{\delta}(1 - J(\tilde{z}^*))s(\theta^*)(\epsilon_s^\theta - 1) < 0 \qquad -\bar{u}\frac{\partial H_2(\cdot)}{\partial \bar{u}} = \tilde{z}^*\frac{q_1}{c_2}\frac{\bar{u}}{\delta}(\epsilon_s^\theta - 1) < 0. \tag{89}$$

Appendix C: Examples of Matching Functions

C.1: The Telephone Line Matching Function

The telephone line matching function is given by

$$M(\lambda, N) = \frac{\lambda N}{\lambda + N}. \tag{90}$$

In this case, the probability that a firm has a match is given by

$$s(\theta) = \frac{\theta}{1 + \theta}. \tag{91}$$

The elasticity of $s(\theta)$ equals

$$\epsilon_s^\theta = \frac{1}{1 + \theta} = \frac{s(\theta)}{\theta}, \tag{92}$$

and

$$1 - \epsilon_s^\theta = s(\theta). \tag{93}$$

The elasticity of $s'(\theta)$ equals

$$\epsilon_{s'}^\theta = \frac{-\theta}{1 + \theta} = -2s(\theta). \tag{94}$$

C.2: The Urn-Ball Matching Function

The urn-ball matching function is given by

$$M(\lambda, N) = N\left(1 - e^{-\alpha(\lambda/N)}\right), \tag{95}$$

where $\alpha > 0$ is a constant. In this case, the probability that a firm has a match is given by

$$s(\theta) = \left(1 - e^{-\alpha\theta}\right). \tag{96}$$

The elasticity of $s(\theta)$ equals

$$\epsilon_s^\theta = \frac{\alpha\theta e^{-\alpha\theta}}{1 - e^{-\alpha\theta}} = \frac{\alpha\theta(1 - s(\theta))}{s(\theta)}. \tag{97}$$

The elasticity of $s'(\theta)$ equals

$$\epsilon_{s'}^\theta = -\alpha\theta < 0. \tag{98}$$

Appendix D: Robustness of the Computational Example to Alternative Values of $\bar{u}/\delta$, ϕ/δ, and f_e/δ

To choose the parameters $\bar{u}/\delta$, ϕ/δ, and f_e/δ for our computational example, we targeted the mean markup for the Canadian retail clothing subsector reported in Chernoff et al. (2024). This calibration strategy is only capable of pinning down one of these parameter values. In this section, we show that our main results are qualitatively robust to two alternative choices of $\bar{u}/\delta$, ϕ/δ, and f_e/δ that also generate a mean markup that matches this target.

In our first parameter robustness specification, we set $\phi/\delta = 45.84$, $f_e/\delta = 0.5455$ and $u/\delta = 100$. Comparing the elasticities calculated using the parameters in Table 2 to those in our main specification in Table 1, we see that there are no sign changes in the elasticities and, in fact, that the quantitative differences are very small in any of the values in the table. The price, markup, and revenue distributions for this alternative specification are shown in Figure 3 and show no discernible differences from the corresponding figures for our main specification in Figure 1. There are also no qualitative differences in the shape of firm-level pass-through rates when comparing the results for this specification in Figure 4 to those for our main specification in Figure 2.

Table 2: Pass-Through to Equilibrium Outcomes, Parameter Robustness Specification I: $\phi/\delta = 45.84$, $f_e/\delta = 0.5455$ and $u/\delta = 100$

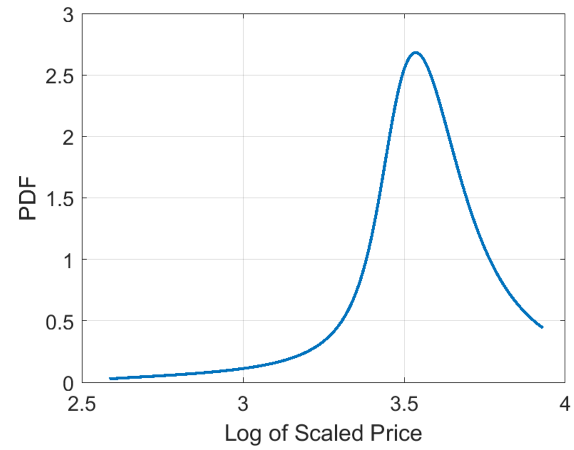
	ϕ	δ	f_e	$\bar{u}$
θ^*	0.7493	0.1223	0.2155	-1.0870
$\tilde{z}^*$	0.3409	0.0192	-0.1896	-0.1704
V_b^*	-0.4468	-0.0729	-0.1285	1.6482
p^*	0.2355	0.3145	0.0677	0.3823
Mean transaction price	0.5258	0.0858	0.1513	0.2371
Mean posted price	0.4741	0.1266	0.1364	0.2629
Mean markup	-0.1665	0.1311	-0.0479	0.0832

Notes: The estimates show the elasticity of the row variable with respect to the parameters in the columns. The elasticity estimates use the parameter estimates from Chernoff, Head, and Lapham (2024) for the search process and the Pareto shape parameter. These estimates also use the following values: $\phi/\delta = 45.84$, $f_e/\delta = 0.5455$ and $u/\delta = 100$.

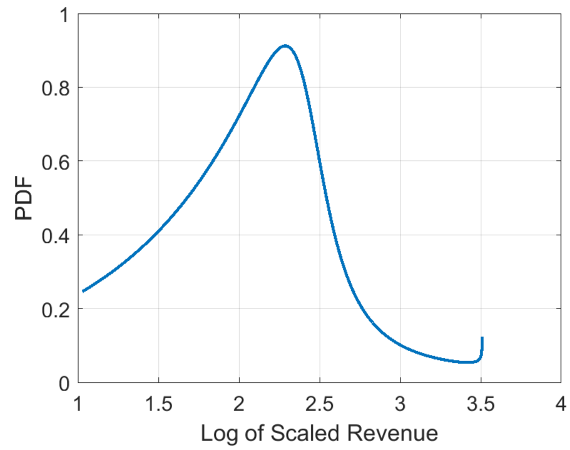
In our second parameter robustness specification, we set $\bar{u}/\delta = 125$, $\phi/\delta = 43.05$, and $f_e/\delta = 1$. Unlike our first alternative specification, where we changed only ϕ/δ and f_e/δ, in this case, we also change $\bar{u}/\delta$. Comparing the elasticities calculated using these parameters in Table 3 to those in our main specification in Table 1, we see some small quantitative differences but, again, there are no sign changes in the elasticities and the magnitudes are largely unchanged. The price, markup, and revenue distributions in Figure 5 are very similar in shape to those of our main specification in Figure 1. The same is true when comparing firm-level pass-through in Figure 6 to our main specification in Figure 2.

For the sake of brevity, we limited ourselves to presenting our main results for only two alternative sets of parameters. Our results in this section and the additional experiments we undertook suggest, however, that our main results are qualitatively robust in this dimension.

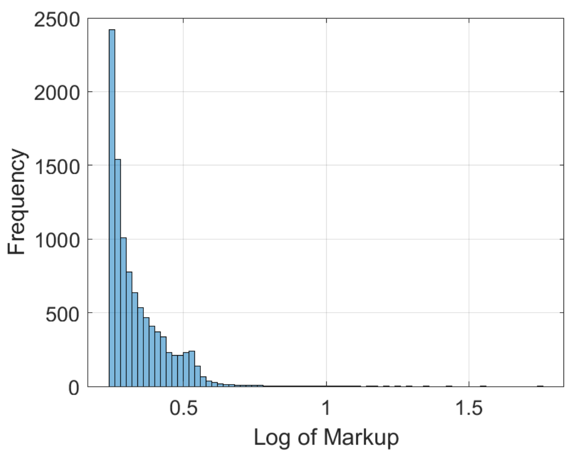
Figure 3: Markups, Prices and Revenues, Parameter Robustness Specification I: $\phi/\delta = 45.84$, $f_e/\delta = 0.5455$ and $u/\delta = 100$



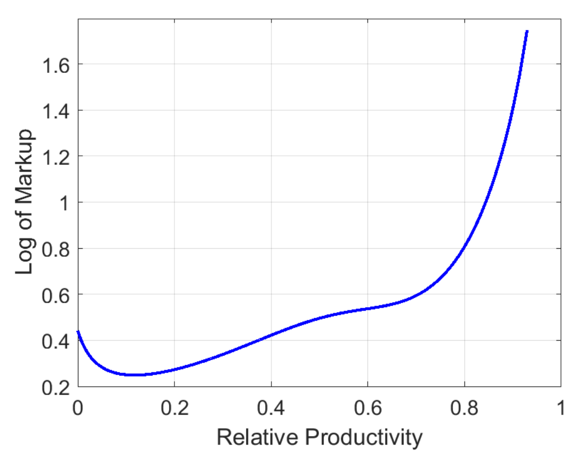
A. PDF of the Log of Scaled Price



B. PDF of the Log of Scaled Revenue



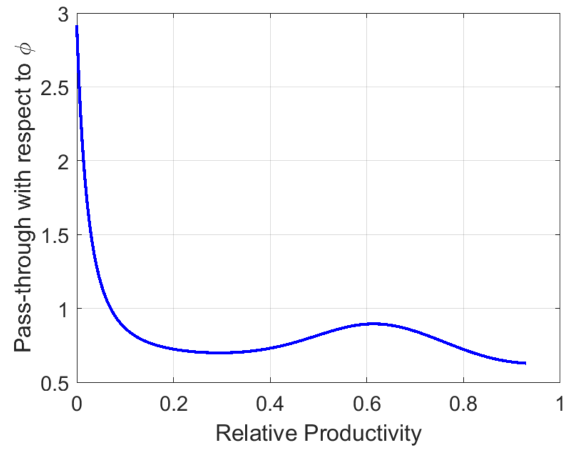
C. Histogram of the Log of Markups



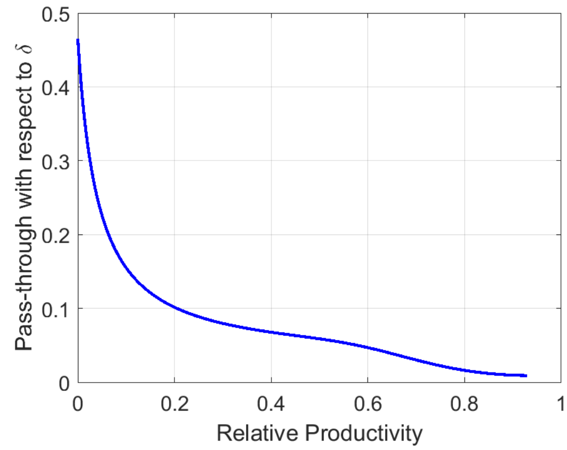
D. Log of Markups

Notes: Panels A and B display the probability density functions (PDF) of the log of the scaled prices (Panel A) and the log of the scaled revenue (Panel B). Panel C displays a histogram of the log of markups, and panel D plots the log of markups as a function of relative productivity. All plots use the parameter estimates from Chernoff, Head, and Lapham (2024) for the search process and the Pareto shape parameter. These plots also use the following values: $\phi/\delta = 45.84$, $f_e/\delta = 0.5455$ and $u/\delta = 100$.

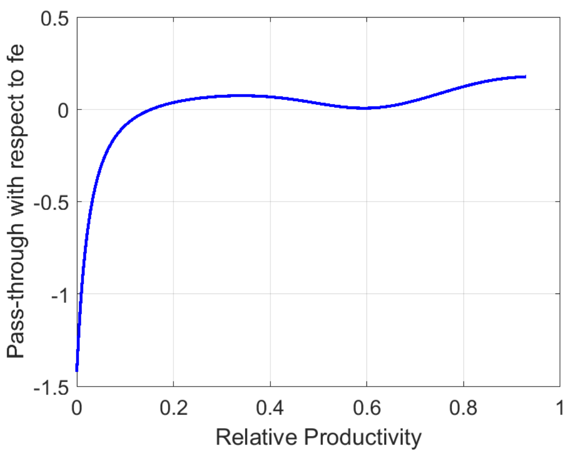
Figure 4: Firm-level Pass-through, Robustness Specification I: $\phi/\delta = 45.84$, $f_e/\delta = 0.5455$ and $u/\delta = 100$



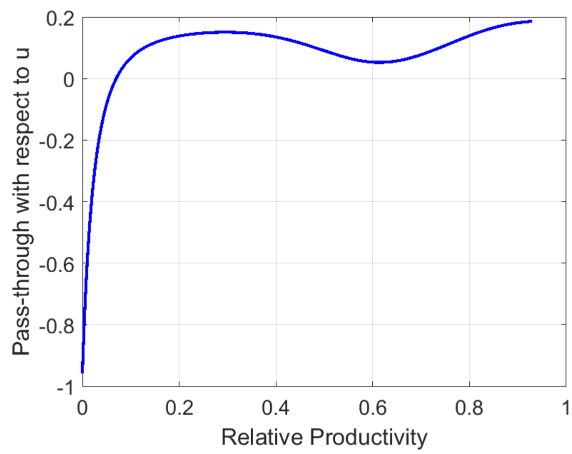
A. Pass-through with respect to ϕ



B. Pass-through with respect to δ



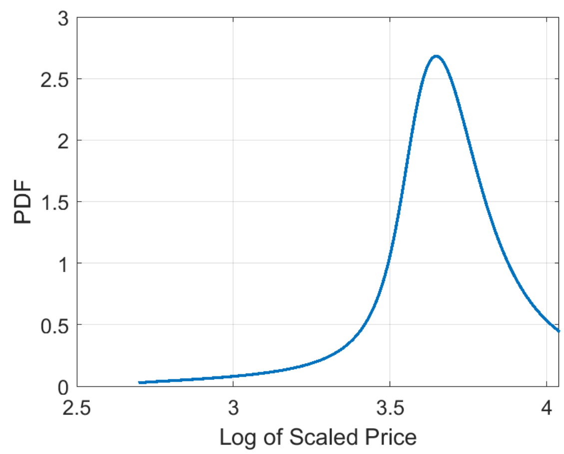
C. Pass-through with respect to f_e



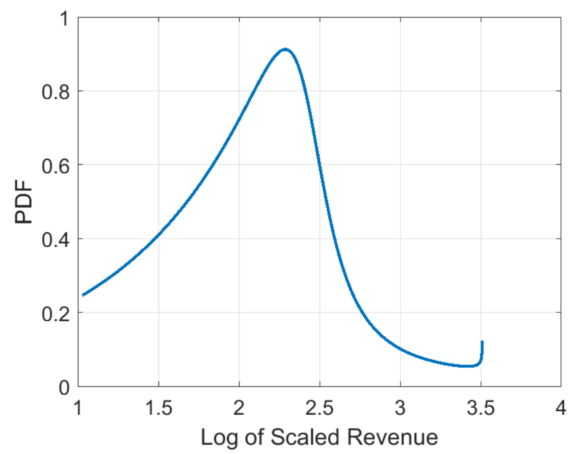
D. Pass-through with respect to $\bar{u}$

Notes: All plots use the parameter estimates from Chernoff, Head, and Lapham (2024) for the search process and the Pareto shape parameter. These plots also use the following values: $\phi/\delta = 45.84$, $f_e/\delta = 0.5455$ and $u/\delta = 100$.

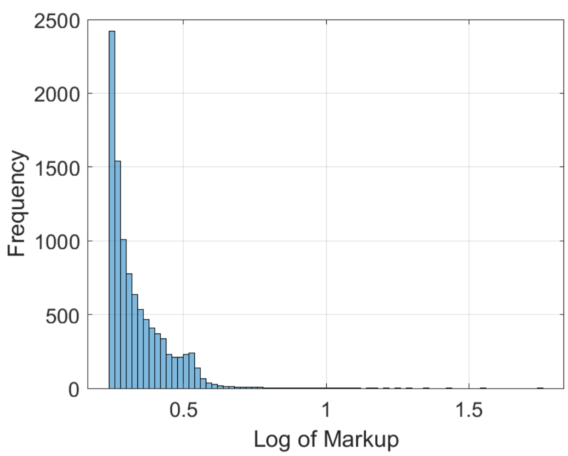
Figure 5: Markups, Prices and Revenues, Robustness Specification II: $\phi/\delta = 43.05$, $f_e/\delta = 1$ and $u/\delta = 125$



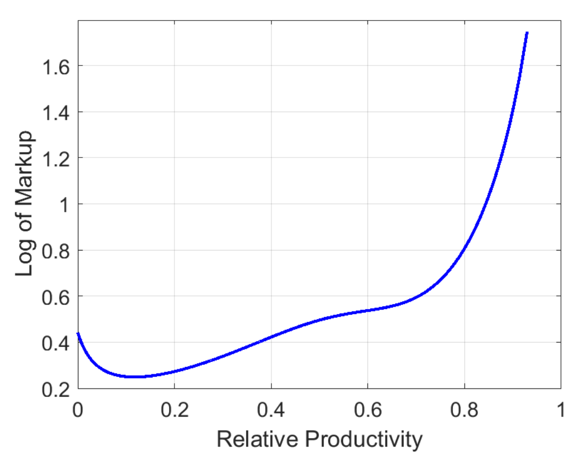
A. PDF of the Log of Scaled Price



B. PDF of the Log of Scaled Revenue



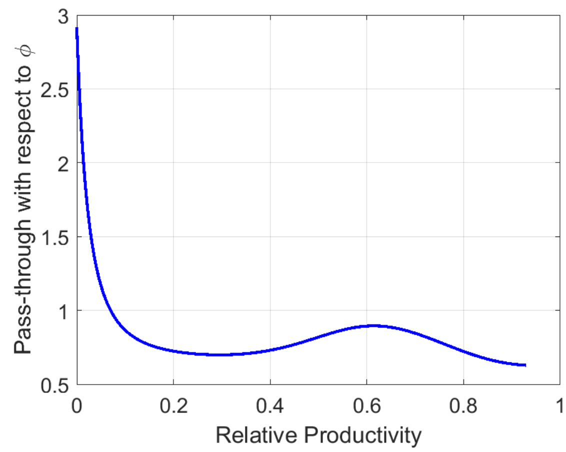
C. Histogram of the Log of Markups



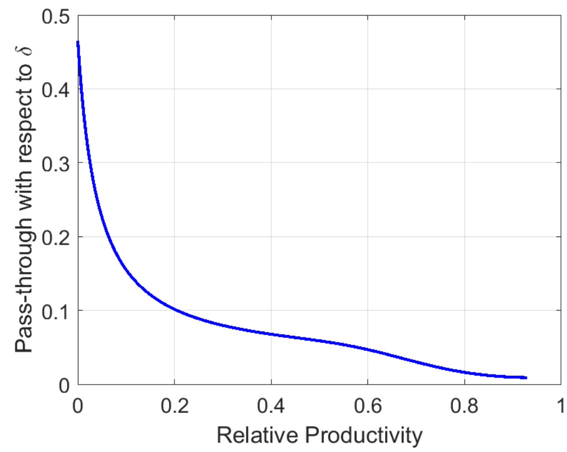
D. The log of Markups

Notes: Panels A and B display the probability density functions (PDF) of the log of the scaled prices (Panel A) and the log of the scaled revenue (Panel B). Panel C displays a histogram of the log of markups, and panel D plots the log of markups as a function of relative productivity. All plots use the parameter estimates from Chernoff, Head, and Lapham (2024) for the search process and the Pareto shape parameter. These plots also use the following values: $\phi/\delta = 43.05$, $f_e/\delta = 1$ and $u/\delta = 125$.

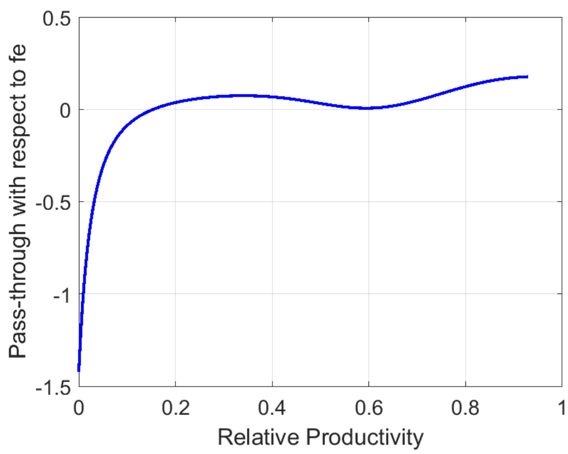
Figure 6: Firm-level Pass-through, Robustness Specification II: $\phi/\delta = 43.05$, $f_e/\delta = 1$ and $u/\delta = 125$



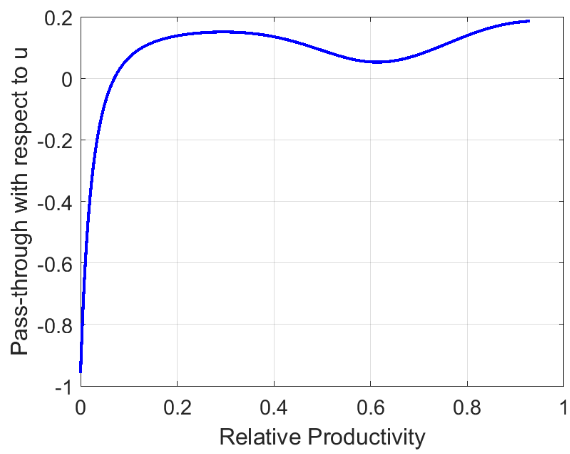
A. Pass-through with respect to ϕ



B. Pass-through with respect to δ



C. Pass-through with respect to f_e



D. Pass-through with respect to $\bar{u}$

Notes: All plots use the parameter estimates from Chernoff, Head, and Lapham (2024) for the search process and the Pareto shape parameter. These plots also use the following values: $\phi/\delta = 43.05$, $f_e/\delta = 1$ and $u/\delta = 125$.

Table 3: Pass-Through to Equilibrium Outcomes, Robustness Specification II: $\phi/\delta = 43.05$, $f_e/\delta = 1$ and $u/\delta = 125$

	ϕ	δ	f_e	$\bar{u}$
θ^*	0.7171	0.1170	0.2063	-1.0404
$\tilde{z}^*$	0.3409	0.0192	-0.1896	-0.1704
V_b^*	-0.3825	-0.0624	-0.1100	1.5549
p^*	0.2355	0.3145	0.0677	0.3823
Mean transaction price	0.5258	0.0858	0.1513	0.2371
Mean posted price	0.4741	0.1266	0.1364	0.2629
Mean markup	-0.1665	0.1311	-0.0479	0.0832

Notes: The estimates show the elasticity of the row variable with respect to the parameters in the columns. The elasticity estimates use the parameter estimates from Chernoff, Head, and Lapham (2024) for the search process and the Pareto shape parameter. These estimates also use the following values: $\phi/\delta = 43.05$, $f_e/\delta = 1$ and $u/\delta = 125$.

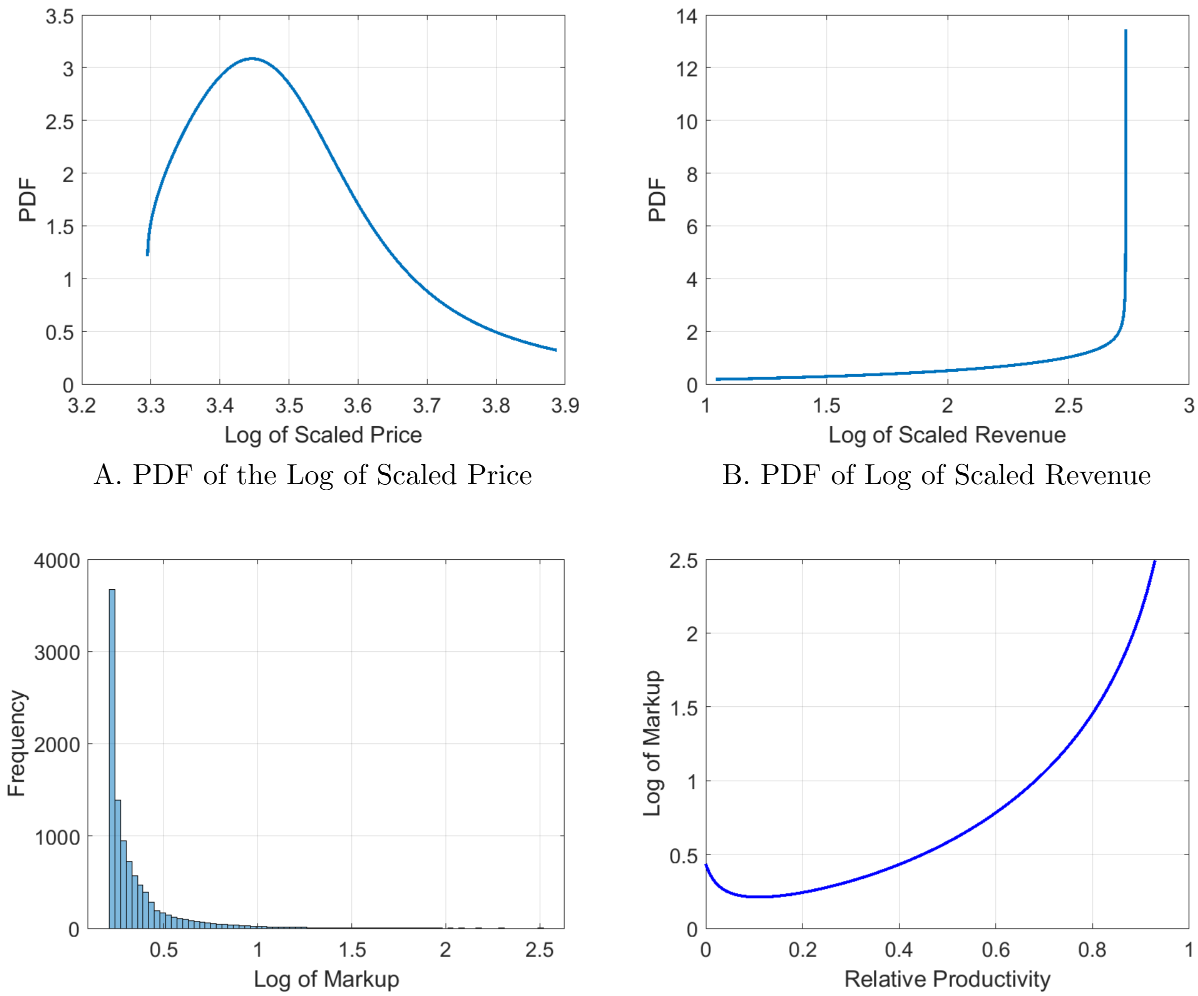
Appendix E: Alternate Meeting Probability Functions and Productivity Distributions

E.1: One or Two Meetings Only

Figures 7 and 8 show the same plots as Figure 1 but for a Bernoulli distribution where, conditional on matching, households have either one or two meetings with firms. Figure 7 is for a model where consumers have a low probability of meeting just one firm, $q_1 \approx 0.18$, while in Figure 8, this probability is much higher, $q_1 = 0.75$.

These parameterizations of the search process are inferior to our baseline specification because they generate log revenue distributions that are highly left skewed with a large mass of firms at the high end of the distribution. This is highly counterfactual to the common observation that, in most industries, there are a relatively small number of dominant firms and a large mass of smaller firms.

Figure 7: Markups, Prices and Revenues with $q_1 = 0.1842$ and $q_2 = 1 - q_1$



A. PDF of the Log of Scaled Price

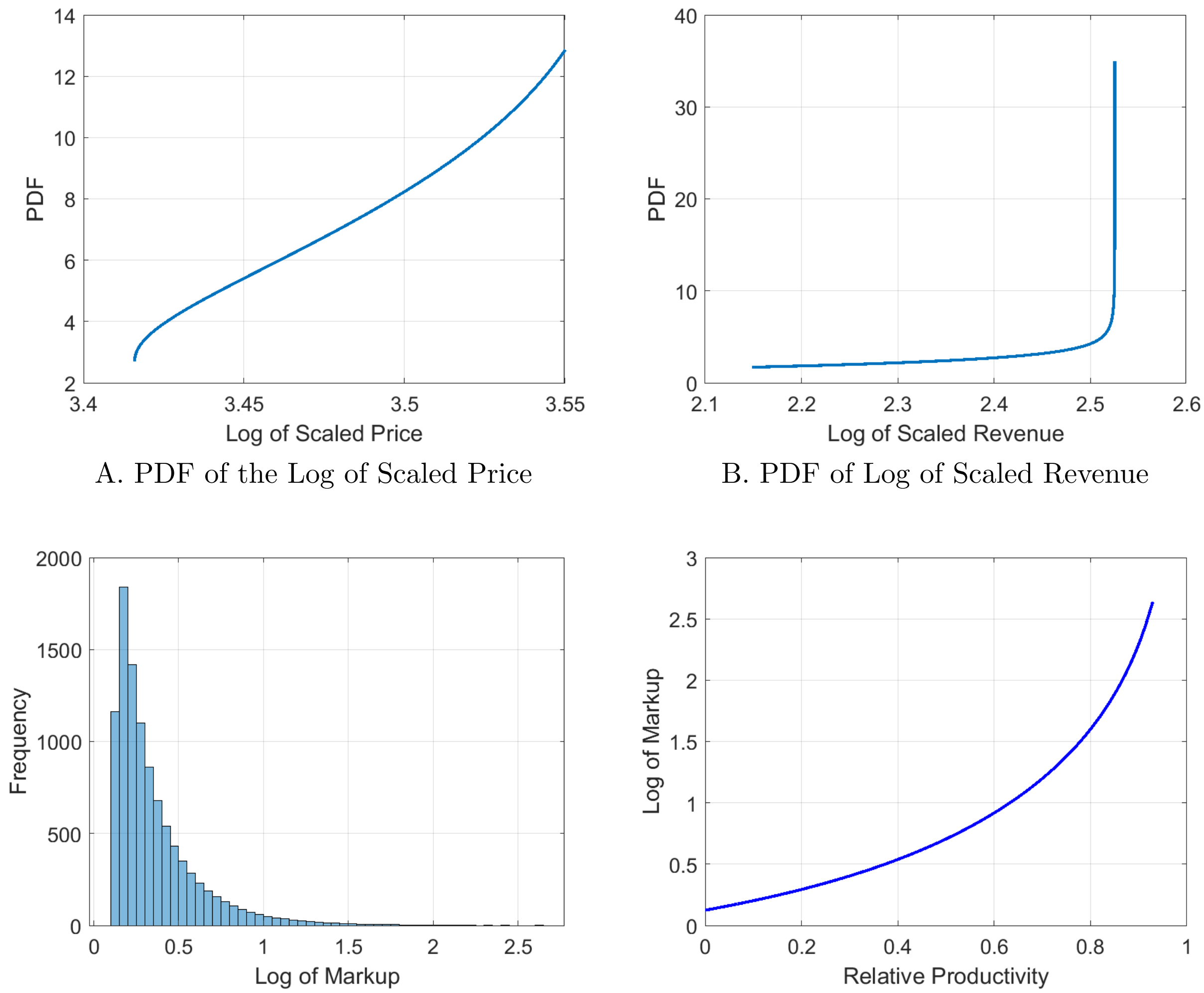
B. PDF of Log of Scaled Revenue

C. Histogram of the Log of Markups

D. Log of Markups

Notes: Panels A and B display the probability density functions (PDF) of the log of the scaled prices (Panel A) and the log of the scaled revenue (Panel B). Panel C displays a histogram of the log of markups, and panel D plots the log of markups as a function of relative productivity. All plots use a search process with $q_1 = 0.1842$ and $q_2 = 1 - q_1$ and the Pareto shape parameter estimate from Chernoff, Head, and Lapham (2024). These plots also use the following values: $\phi/\delta = 34.43$, $f_e/\delta = 1.475$ and $u/\delta = 100$.

Figure 8: Markups, Prices and Revenues with $q_1 = .75$ and $q_2 = 1 - q_1$



A. PDF of the Log of Scaled Price

B. PDF of Log of Scaled Revenue

C. Histogram of the Log of Markups

D. Log of Markups

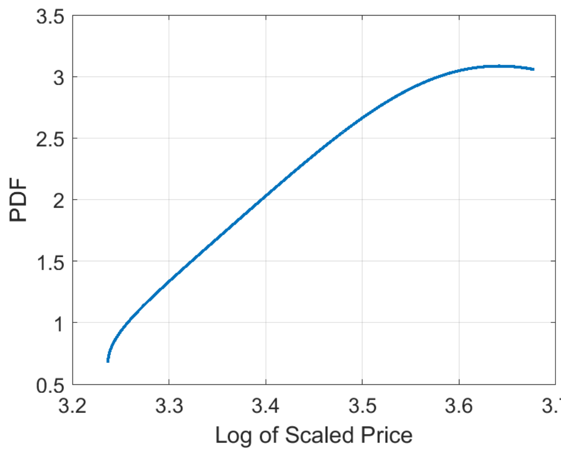
Notes: Panels A and B display the probability density functions (PDF) of the log of the scaled prices (Panel A) and the log of the scaled revenue (Panel B). Panel C displays a histogram of the log of markups, and panel D plots the log of markups as a function of relative productivity. All plots use a search process with $q_1 = 0.75$ and $q_2 = 1 - q_1$ and the Pareto shape parameter estimate from Chernoff, Head, and Lapham (2024). These plots also use the following values: $\phi/\delta = 34.43$, $f_e/\delta = 1.475$ and $u/\delta = 100$.

E.2: Poisson Distribution for Meetings

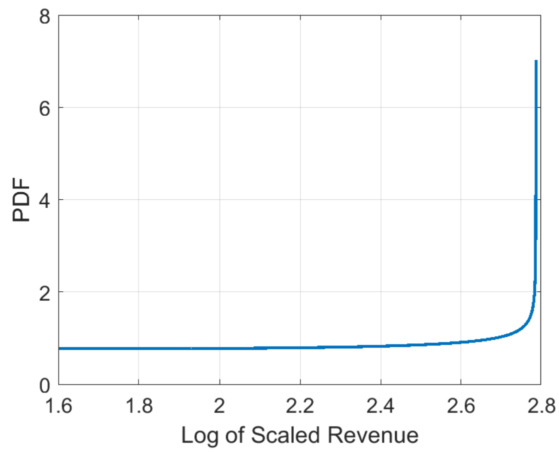
In Figures 9 and 10, we consider Poisson distributions for the meeting rate probabilities. Figure 9 has a lower Poisson arrival rate, using the estimate of 1.624 from Baggs et al. (2018), and Figure 10 uses a higher arrival rate parameter of 5. Under either specification, the log revenue distribution looks similar to the figures that use Bernoulli distributions, they are left-skewed and with an uncharacteristically large mass in the right tail relative to what is commonly observed in the firm-level revenue data.

It is interesting to note that the level of the price dispersion in the Poisson specification with an arrival rate equal to 5 is similar to our baseline specification. The Poisson distribution and our preferred specification share the feature that the maximum number of meetings is infinite. When the arrival rate in the Poisson distribution is high, there is a large mass of highly informed consumers who have many meetings. This is also the case in our baseline and this feature partially explains the relatively higher level of price dispersion that is apparent in both specifications.

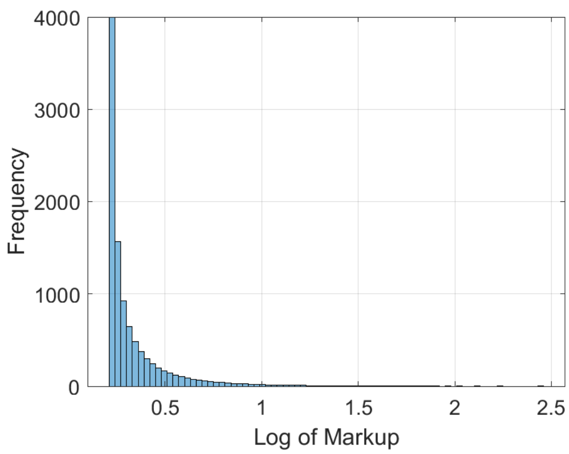
Figure 9: Markups, Prices and Revenues with Poisson Search and Arrival Rate of 1.624



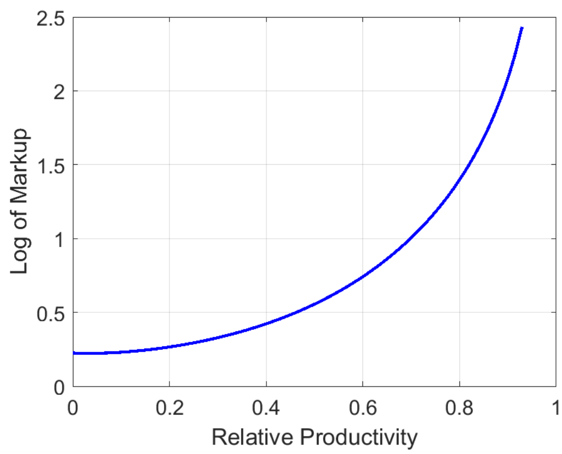
A. PDF of the Log of Scaled Price



B. PDF of Log of Scaled Revenue



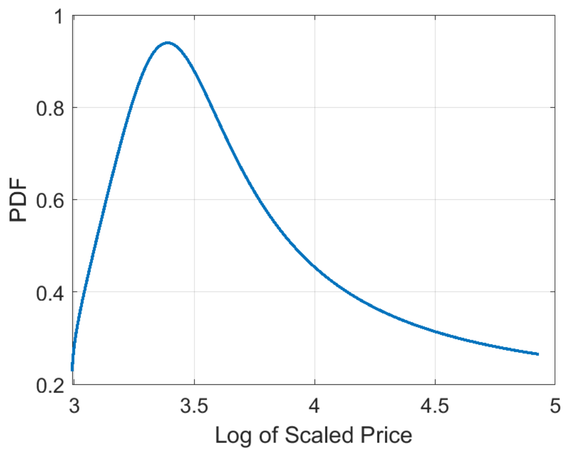
C. Histogram of the Log of Markups



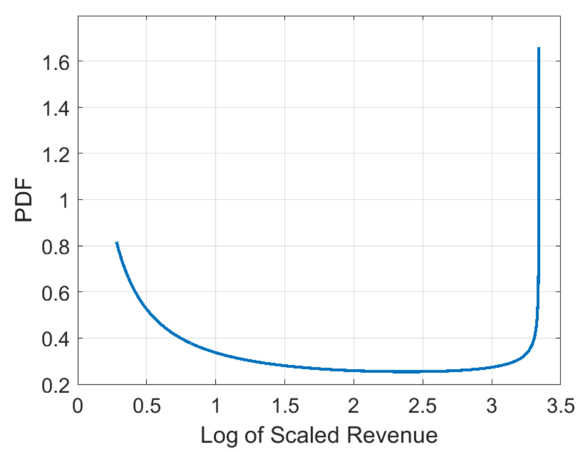
D. Log of Markups

Notes: Panels A and B display the probability density functions (PDF) of the log of the scaled prices (Panel A) and the log of the scaled revenue (Panel B). Panel C displays a histogram of the log of markups, and panel D plots the log of markups as a function of relative productivity. All plots use a Poisson search process with an arrival rate of 1.624 and the Pareto shape parameter estimate from Chernoff, Head, and Lapham (2024). These plots also use the following values: $\phi/\delta = 34.43$, $f_e/\delta = 1.475$ and $u/\delta = 100$.

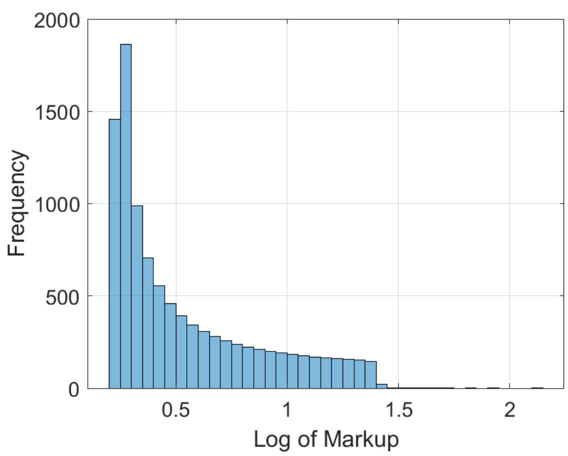
Figure 10: Markups, Prices and Revenues with Poisson Search and an Arrival Rate of 5



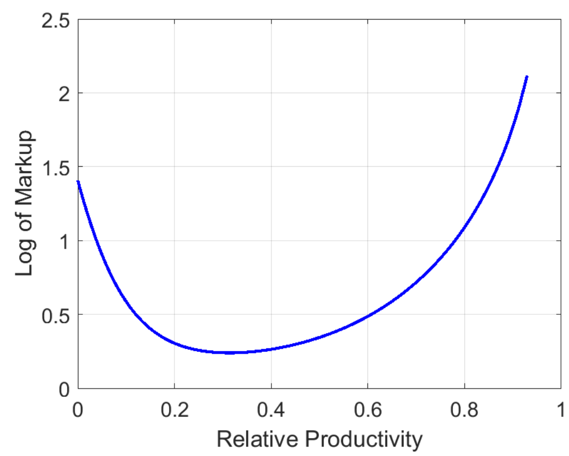
A. PDF of the Log of Scaled Price



B. PDF of Log of Scaled Revenue



C. Histogram of the Log of Markups



D. Log of Markups

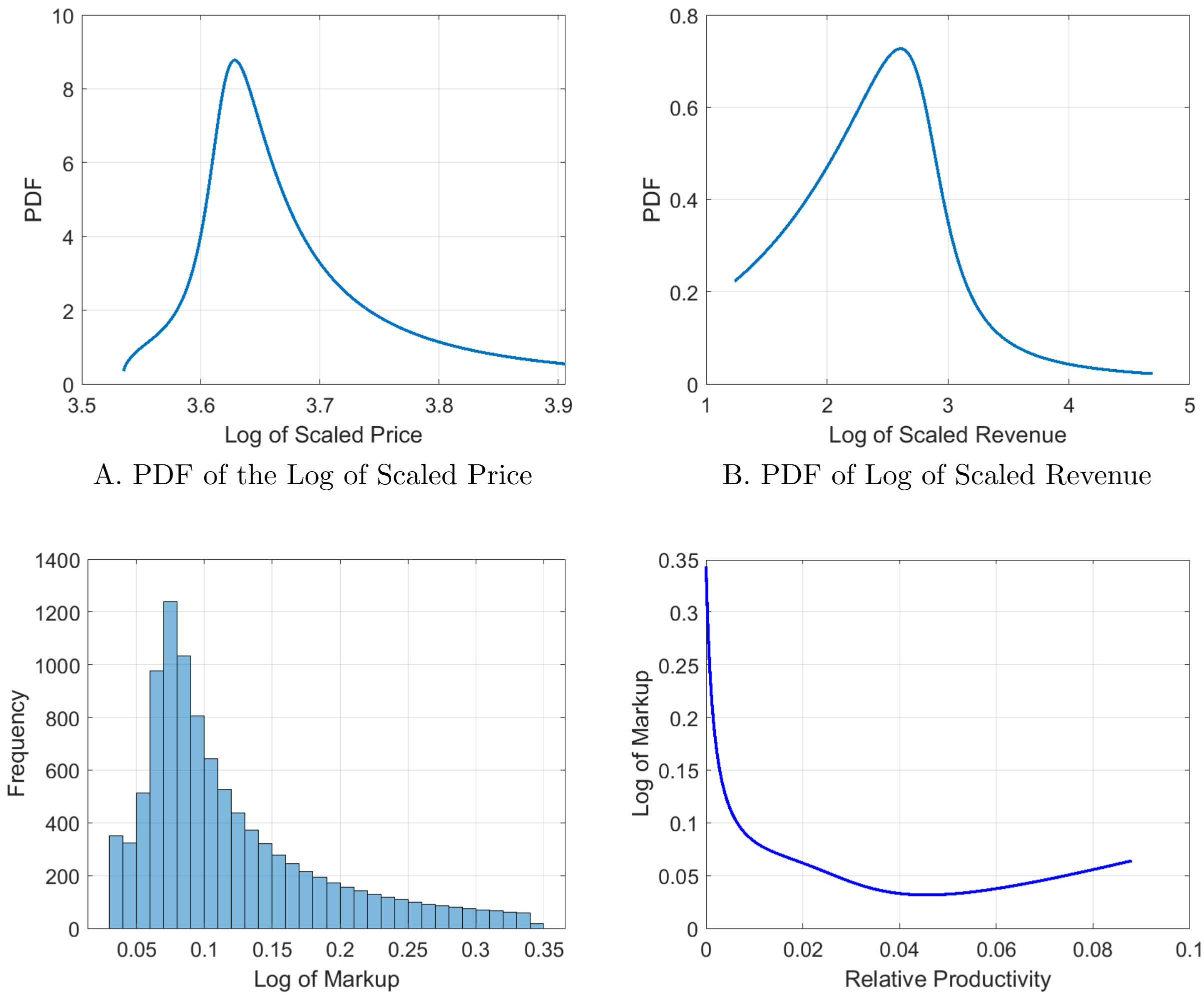
Notes: Panels A and B display the probability density functions (PDF) of the log of the scaled prices (Panel A) and the log of the scaled revenue (Panel B). Panel C displays a histogram of the log of markups, and panel D plots the log of markups as a function of relative productivity. All plots use a Poisson search process with an arrival rate of 5 and the Pareto shape parameter estimate from Chernoff, Head, and Lapham (2024). These plots also use the following values: $\phi/\delta = 34.43$, $f_e/\delta = 1.475$ and $u/\delta = 100$.

E.3: Shape of the Productivity Distribution

In Figure 11 we show the results when we drastically increase the Pareto shape parameter to 100, which drives the productivity distribution toward being nearly degenerate. As can be seen from Panel A, this results in a significant reduction in the degree of price dispersion relative to our baseline. This exercise highlights that firm heterogeneity in productivity is an important feature of our model that generates price dispersion.

In addition to generating lower price dispersion, the model with a near degenerate productivity distribution is inferior to our baseline specification because of the qualitative aspects of markups. In Panel D, we see that markups are decreasing in productivity for much of the productivity distribution. As firm revenues are monotonic in productivity, markups are also decreasing in firm revenues for much of the revenue distribution. This is counterfactual to the previously referenced literature, which consistently finds that markups are increasing in firm size and productivity.

Figure 11: Markups, Prices and Revenues with a Pareto Shape Parameter, $\gamma = 100$



A. PDF of the Log of Scaled Price

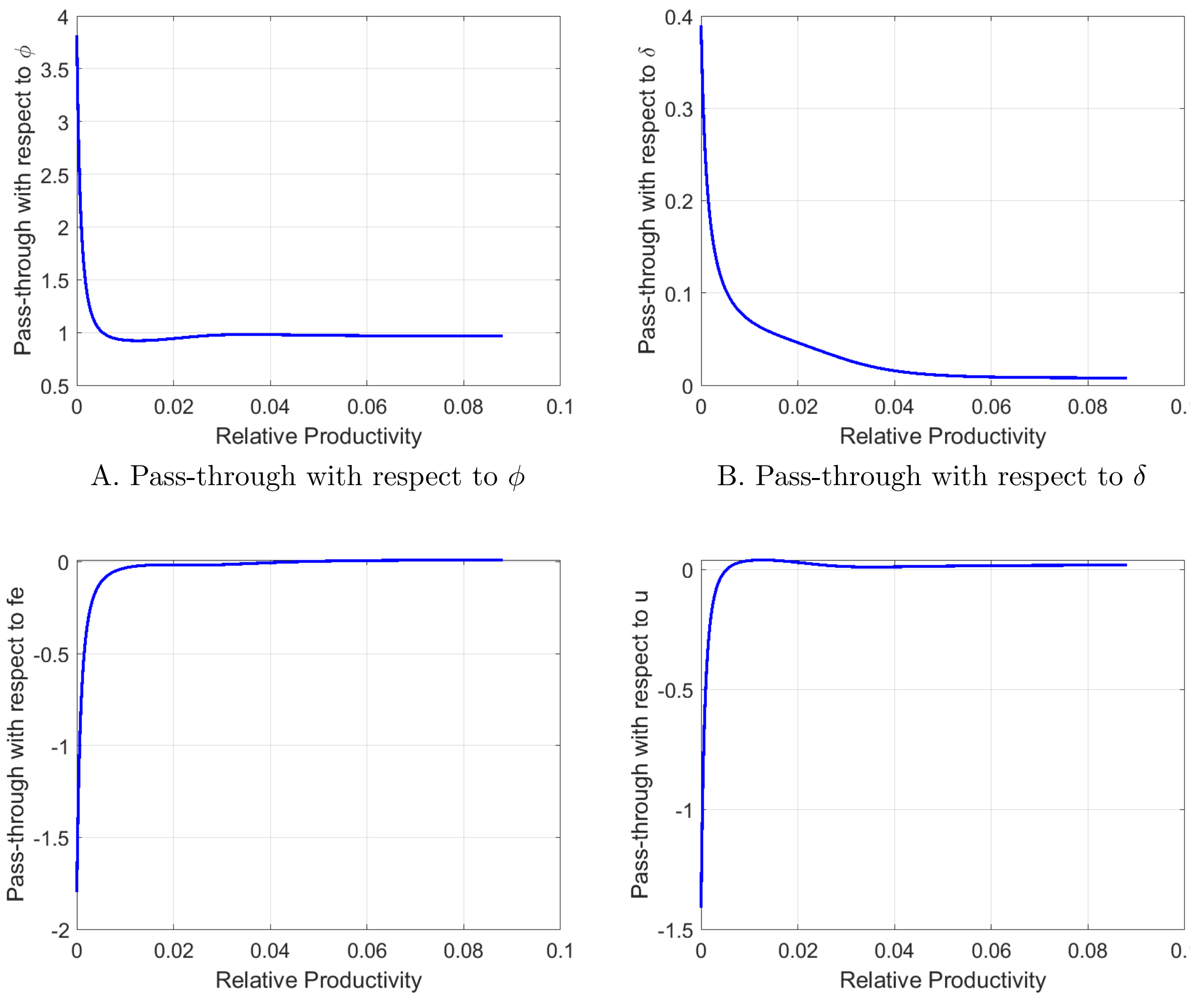
B. PDF of Log of Scaled Revenue

C. Histogram of the Log of Markups

D. Log of Markups

Notes: Panels A and B display the probability density functions (PDF) of the log of the scaled prices (Panel A) and the log of the scaled revenue (Panel B). Panel C displays a histogram of the log of markups, and panel D plots the log of markups as a function of relative productivity. All plots use the Pareto shape parameter $\gamma = 100$, and the parameter estimates from Chernoff, Head, and Lapham (2024) for the search process. These plots also use the following values: $\phi/\delta = 34.43$, $f_e/\delta = 1.475$ and $u/\delta = 100$.

Figure 12: Pass-through to Firm-Level Prices - Near Degenerate Productivity Distribution



A. Pass-through with respect to ϕ

B. Pass-through with respect to δ

C. Pass-through with respect to f_e

D. Pass-through with respect to $\bar{u}$

Notes: All plots use Pareto shape parameter $\gamma = 100$ and the parameter estimates from Chernoff, Head, and Lapham (2024) for the search process. These plots also use the following values: $\phi/\delta = 34.43$, $f_e/\delta = 1.475$ and $u/\delta = 100$.

Appendix F: Robustness of the Computational Examples to Using an Urn-Ball Matching Function

In this section, we show the robustness of our main results to using a different matching function. We use the urn-ball matching function, introduced in Appendix B.2, with the parameter $\alpha = 1$. We use the same parameter values as our main specification except we lower the parameter ϕ/δ to 26.27. This adjustment is made so that the mean markup continues to match the average markup for the Canadian retail clothing subsector that is reported in Chernoff et al. (2024).

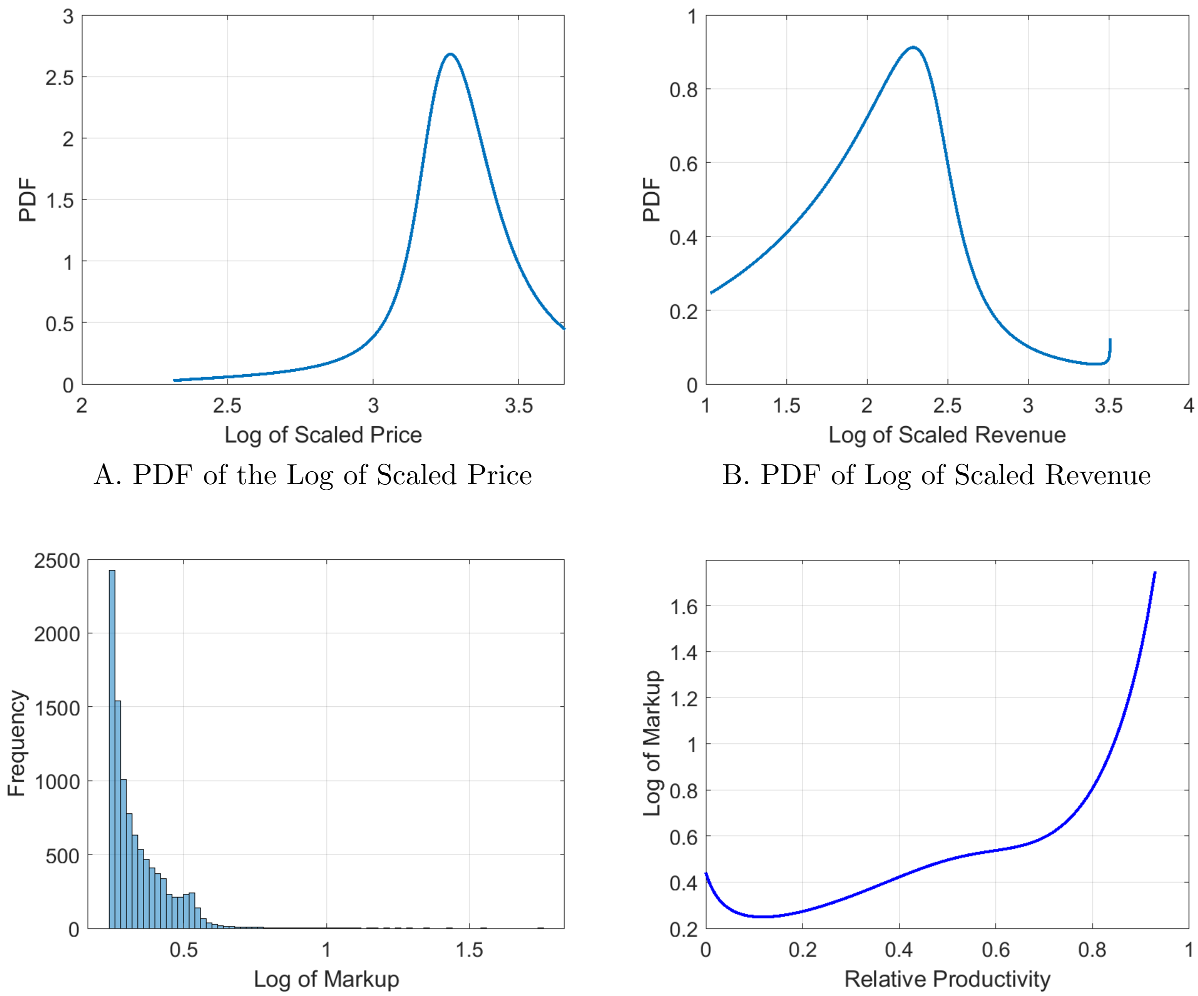
With the urn-ball matching function, we get qualitatively similar results to our main specification. Comparing the elasticities in Table 4 to those for our main specification, we observe that all reported elasticities exhibit similar magnitudes, with no changes in signs. Next, comparing the densities in Figure 13 to those in Figure 1, we see that the shape of the revenue, price, and markup distributions are similar. As with our baseline specification, the markup is increasing with productivity for much of the productivity distribution under the urn-ball matching-function specification. Finally, comparing firm-level pass-through in Figures 2 and 14, we see that firms across the productivity distribution adjust their prices in a qualitatively similar fashion for each of the shocks we consider.

Table 4: Pass-Through to Equilibrium Outcomes, Robustness Specification: Urn-Ball Matching Function

	ϕ	δ	f_e	$\bar{u}$
θ^*	0.5988	0.0977	0.1722	-0.8687
$\tilde{z}^*$	0.3267	0.0169	-0.1937	-0.1499
V_b^*	-0.2968	-0.0484	-0.0854	1.4307
p^*	0.2672	0.3197	0.0769	0.3362
Mean transaction price	0.5455	0.0890	0.1569	0.2085
Mean posted price	0.4959	0.1301	0.1427	0.2313
Mean markup	-0.1595	0.1322	-0.0459	0.0732

Notes: The estimates show the elasticity of the row variable with respect to the parameter in the columns. The elasticity estimates use the urn-ball matching function with parameter $\alpha = 1$. The estimates use the parameter estimates from Chernoff, Head, and Lapham (2024) for the search process and the Pareto shape parameter. These estimates also use the following values: $\phi/\delta = 26.27$, $f_e/\delta = 1.475$ and $u/\delta = 100$.

Figure 13: Markups, Prices and Revenues, Robustness Specification: Urn-Ball Matching Function



A. PDF of the Log of Scaled Price

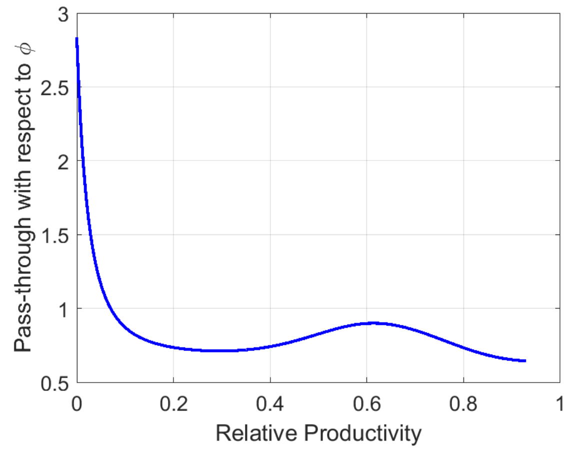
B. PDF of Log of Scaled Revenue

C. Histogram of the Log of Markups

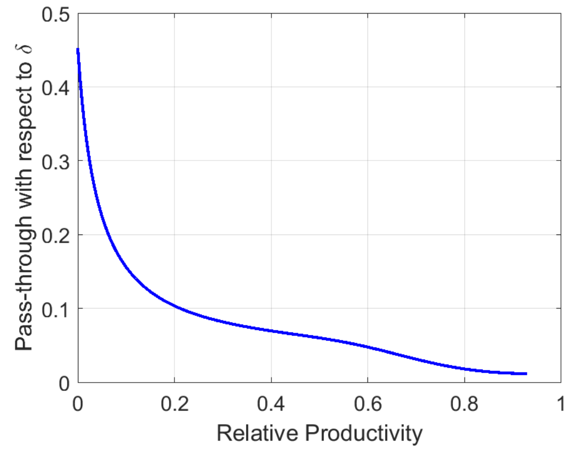
D. Log of Markups

Notes: Panels A and B display the probability density functions (PDF) of the log of the scaled price (Panel A) and the log of the scaled revenue (Panel B). Panel C displays a histogram of the log of markups, and panel D plots the log of markups as a function of the relative productivity. All plots use the urn-ball matching function with parameter $\alpha = 1$. These plots use the parameter estimates from Chernoff, Head, and Lapham (2024) for the search process and Pareto shape parameter. These plots also use the following values: $\phi/\delta = 26.27$, $f_e/\delta = 1.475$ and $u/\delta = 100$.

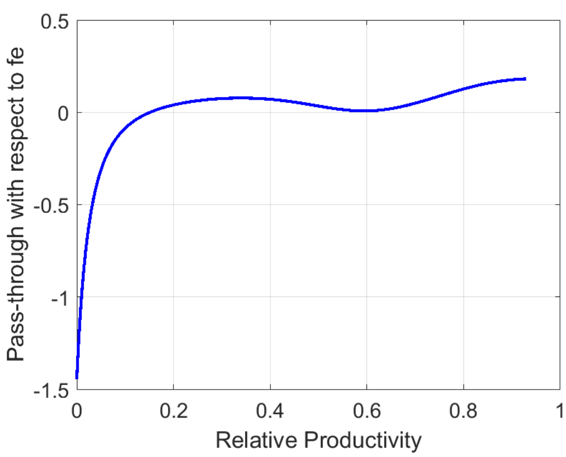
Figure 14: Firm-level Pass-through, Robustness Specification: Urn-Ball Matching Function



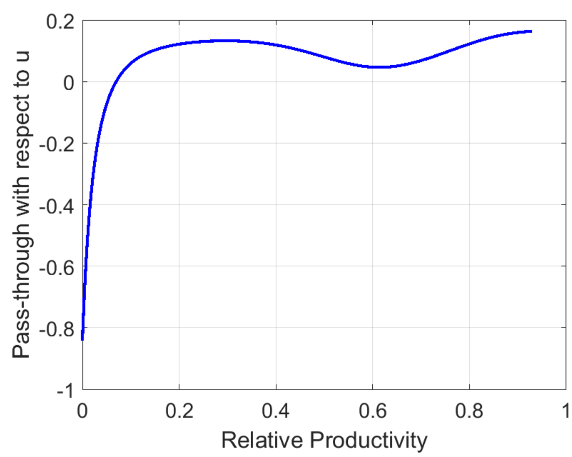
A. Pass-through with respect to ϕ



B. Pass-through with respect to δ



C. Pass-through with respect to f_e



D. Pass-through with respect to $\bar{u}$

Notes: All plots use the urn-ball matching function with parameter $\alpha = 1$. These plots use the parameter estimates from Chernoff, Head, and Lapham (2024) for the search process and the Pareto shape parameter. These plots also use the following values: $\phi/\delta = 26.27$, $f_e/\delta = 1.475$ and $u/\delta = 100$.